First-Order uncertainty Analysis using AD
2D Application

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Abstract—**We present here an efficient method to quantify uncertainty in morphodynamic models. The FOSM/AD method is applied to a complex 2D test case: the long term morphodynamic evolution of a tidal inlet. The sensitivity to grain size and bed roughness has been quantified as well as various model parameters including slope effect and secondary currents using the tangent linear model (TLM) of the Sisyphe/Telemac-2d model for the 7.0 release. The TLM was developed using the AD-enabled Nag Fortran compiler. However the duration of simulation is still limited due to the use of a scalar version. The method needs to be extended to study the effect of mesh size.**

I. INTRODUCTION

The uncertainty associated with morphodynamic simulations is difficult to quantify given the number of variable input parameters and CPU time associated to each simulation. This is particularly true in complex process-based models like the Telemac-2d/Sisyphe morphodynamic model.

An efficient first-order second moment method using Algorithmic Differentiation (FOSM/AD) developed by Villaret et al. (2015) can be applied to quantify uncertainty/sensitivities in morphodynamic models. Changes in the calculated bed evolution with respect to variable flow and sediment input parameters are estimated with machine accuracy using the technique of Algorithmic Differentiation (AD).

The FOSM/AD method has been previously applied and validated in a simple 1D application. Results were found to be consistent with Monte Carlo (MC) simulations for a significant gain in CPU time. Only one run of the Tangent Linear Model (TLM) is required per variable input parameter against hundreds for the MC method even with the use of stratified sampling techniques.

In this paper, the FOSM/AD method is applied to a complex 2D simulation using a recently developed TLM model of the Telemac-2D/Sisyphe model for the 7.0 release (Goeury, 2015). TLM and Adjoints codes were developed using the NAG-enabled Fortran compiler, following a procedure developed for the 6.2 release (Riehme et al., 2010).

The test case selected here is a schematic representation of a tidal inlet which was developed initially by Marciano et al. (2005) in order to represent the typical conditions of the Dutch Warden Sea, and later reproduced by Baaren (2011).

The objectives of the present study are:

1. To provide a new validation test case for the Telemac-2d/Sisyphe morphodynamic model.

2. To apply the FOSM/AD method to identify the key processes and most sensitive input parameters

3. To quantify the total uncertainty

4. To discuss the present limitations and provide guidance for future work

The outline of this paper is as follows: Part 2 gives a brief literature review on the tidal inlet processes and existing models. In Part 3, we describe the test case and Telemac-2D/Sisyphe model. In Part 4, the model is applied to medium term simulations (up to 100 years). This part includes a brief discussion of the model set-up and CPU time. In Part 5, we present a sensitivity and uncertainty analysis using the FOSM/AD method. We finally draw some conclusions on the feasibility of the method for in-situ applications and discuss the present limitations.

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II. LITERATURE REVIEW

Tidal inlets are morphodynamic features commonly observed around the world. Despite numerous observations, theoretical and numerical studies, the key processes governing the ebb delta formation offshore and the development of a complex multi-channel pattern inside the inlet are still unknown.

There are many examples of well developed branching channels in natural inlets where the main characteristic of the channel inlets depend on the geometrical dimensions of the barrier and tidal forcing. The Arcachon Basin in France, the tidal inlets in the Dutch Warden Sea and the Humber estuary in the UK are some examples of the diversity of the features which can be encountered in nature (cf. Stefanon et al., 2010).

In addition to theoretical and experimental studies, there have been a few attempts to model the medium to long term evolution of such complex systems using morphodyamic process-based models. Cayocca (2001) developed a 2D morphodynamic model of the Arcachon Bay, whereas Marciano et al. (20005) developed a 2D model of a schematic inlet using Delft3D. The same test case was later reproduced by Baaren (2011) using the ELCOM 3D model. A more realistic 2D model using Delft 3D is presented in Dessanayake et al. (2009) to represent the tidal network formation in the Ameland inlet (Dutch Warden Sea) including tidal asymmetry and higher harmonics and long shore current. Both 2D and 3D models were able to reproduce the ebb delta formation offshore and the branching system with features typically observed in short basins.

According to previous limited sensitivity analysis (Marciano et al., 2005 and van Baaren, 2011), the effect of the initial bathymetry is essential and tidal forcing and geometrical constraints govern the channel final equilibrium pattern. The effects of secondary currents and bottom slope are expected to play a minor role, whereas the sediment mobility and initial bed perturbation have a major influence on the development and equilibrium pattern. Model results are expected to be also sensitive to the limiting erosion depth. The effect of the mesh size has not yet been examined and is expected to play a major role.

III.   MORPHODYNAMIC MODEL

A. Schematic Test Case

The test case proposed by Marciano et al. (2005) and Baaren (2011) represents a short tidal embayment, with conditions typically encountered on the Dutch Warden Sea. The model geometry, shown in Figure 1, covers an area of 12x16 km^2 with an offshore area extending 4 km seaward and an inner basin of 8x16 km^2. The two areas are separated by a barrier island of 2.5 km width. In the offshore area the bed slope increases linearly from -8 m below MSL to -6 m at the inlet. Inside the inlet, the bed profile increases from -6 m at the inlet to +1 m at the landward boundary. The initial bathymetry does not have any shoal pattern but small perturbations of +/-0.15 m height are randomly distributed at each node. As in the original simulation (Baaren, 2011), a Chezy friction coefficient is imposed for the long term simulations using a value of 65 m$^{0.5}$/s.

Different grids have been tested. In Figure 1 we show the initial coarse mesh with 32 500 elements and a triangular mesh size of 100 m. Another refined mesh around the inlet (with mesh size down to 30 m and 52 000 elements) was also included to test the sensitivity of the model to the mesh size.
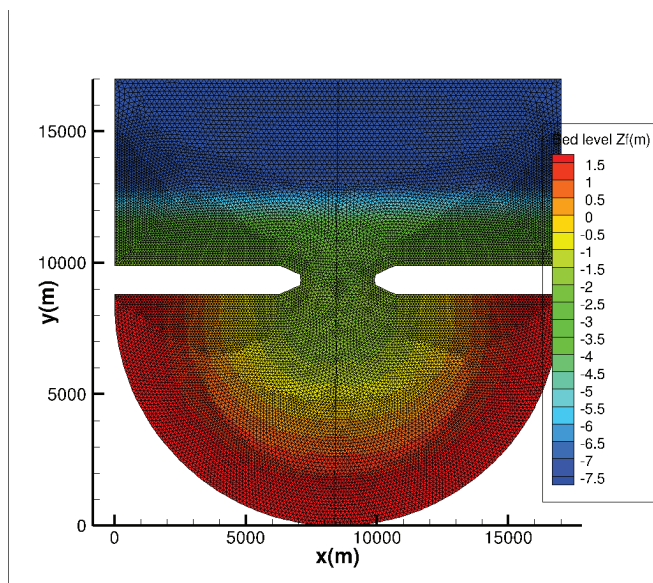

Figure 1.   Geometry and bottom variation. The coarse grid (mesh size of 100 m) is also represented.

B. Hydrodynamic Model

On the seaward boundary, a sinusoidal variation of the water level is imposed with an amplitude of 1 m. At the inlet, the flow velocity reaches its maximum of 0.8 m/s approximately 3 hours before Low Water (LW). As shown in Figure 2, the tidal currents at the tidal inlet are ebb dominant leading to a global loss of sediment from the inlet offshore. Hydrodynamic results obtained with different meshes and using different numerical schemes are found to give the same flow results. This test case shows very little sensitivity to numerical parameters. Diffusion coefficients from 10^{-6} to 10^{-2}, slip or no slip conditions have no effects on the hydrodynamic model results and velocity pattern. Two different advection schemes have been tested: the N-edge based Residual Distributive (NERD) scheme (13) and the recently developed Locally Implicit Psi Scheme (LIPS) second-order predictor corrector scheme which is 10 times less diffusive than the NERD scheme (Hervouet et al., 2015). Both schemes are found to give the same results for a significant increase in the CPU time (35%).

The N-edge scheme No13 which is well adapted to represent tidal inlet has been used in the morphodynamic simulations with a time step Dt=10 s.
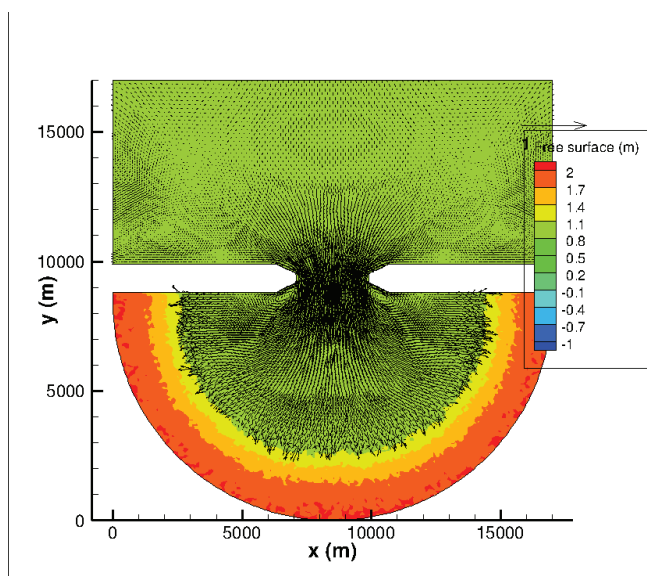
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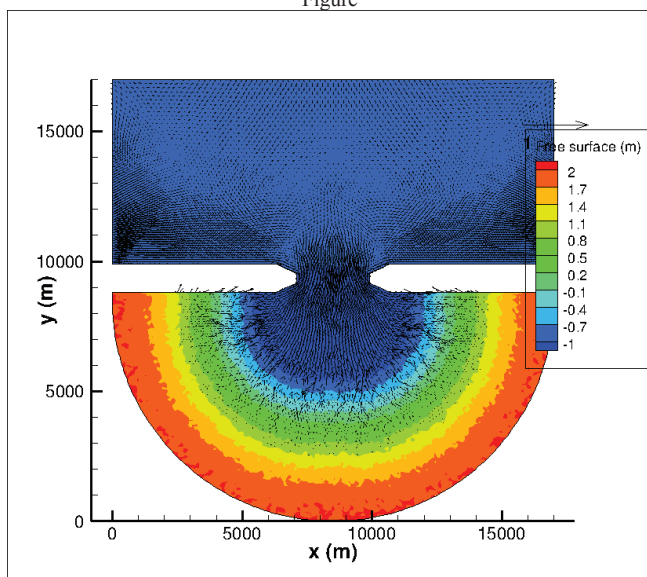
2: Free surface and velocity field at high tide (top) and at low tide (bottom)

C. Sediment Transport Processes

The sediment is made of uniform sand with mean diameter D_{50}=150 µm. The Coriolis force is neglected while the effect of secondary currents and sloping bed effects are parameterized.

The model has been run using the Engelund and Hansen (1967) total load formula (N° 30):

$$Q_s = 0.01 \sqrt{g(s-1)d_{50}^3} \; \theta^{5/2} \qquad (1)$$

Where θ is the adimensional bed shear stress, s the relative sand density, g gravity and d_{50} the sediment grain size. Following Baaren (2011), we added the Kirwan and Murray (2007) sloping bed effects; the current induced sediment transport is modified by adding a slope driven component:

$$Qs = \alpha \nabla Z_f \qquad (2)$$

Where α is a dimensional empirical coefficient (α=1.15 10^{-5}m²/s). The slope driven transport only occurs if the slope is greater than 0.01.

We also included the secondary current parameterization in Sisyphe. Following the empirical method of Engelund (1974) the near bed angular deviation is proportional to h/R, where h is the water depth and R the radius bend:

$$\tan \delta = \beta \frac{h}{R} \qquad (3)$$

with the adimensional coefficient β =7.

IV.   LONG TERM EVOLUTION

A. Morphodynamic Factor

For long term simulation (up to 100 years), we used a morphodynamic factor (MF) to reduce the CPU time. This classical method is equivalent to multiplying the hydrodynamic time step (Dt=10s) by the MF factor in the morphodynamic model. In all simulations, the coupling period is set to 1.

Results obtained for different values of the MF factor from 10 up to 50 are compared in Figure 3. The bed evolution pattern obtained after 30 years are very similar and therefore we used in the simulations MF=25.

B. Long term evolution – Coarse mesh

The bed obtained after 100 years are shown in Figure 4. The ebb delta forms during the first 10 years and then the channel pattern develop rapidly in the next 30 years and continue to extend and deepen more slowly after 100 years.

In the morphodynamic model results shown below, there is no limiting depth to erode. After 100 years, the bed is eroded locally down to 50 m which is rather unrealistic, since in nature the presence of a rigid bottom would limit the depth of erosion.

C. Influence of the Mesh size

The effect of the mesh size after 10 years of bed evolution – including sloping bed effects – is shown on Figure 5. The refined mesh on the right provides a more detailed channel pattern. The mesh size has an important effect on the results whereas the slope effect does not make any difference for the coarser grid and modifies slightly the results for the refined grid.
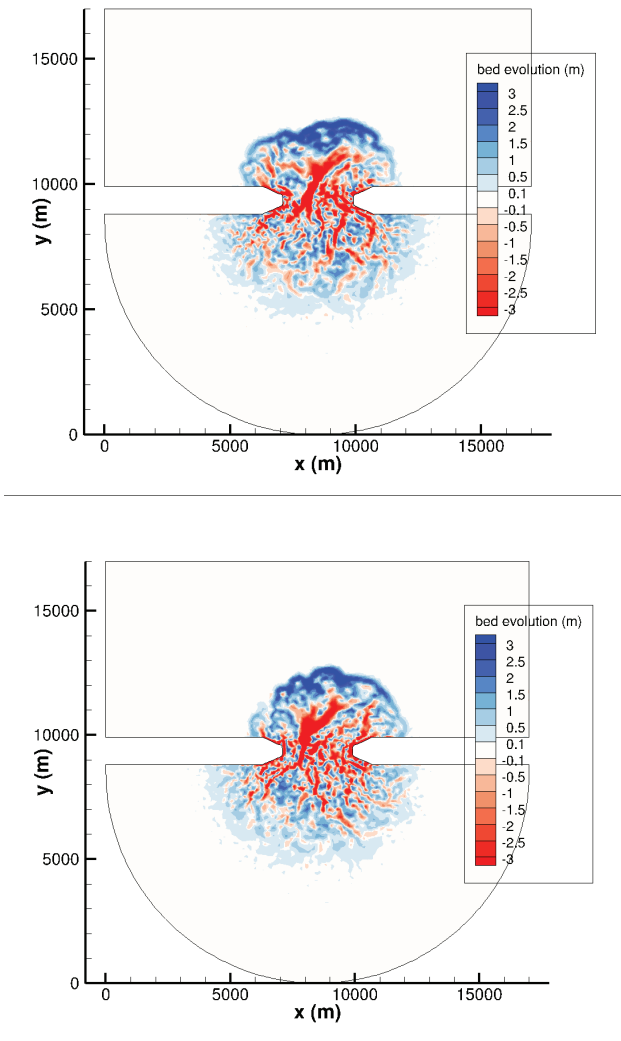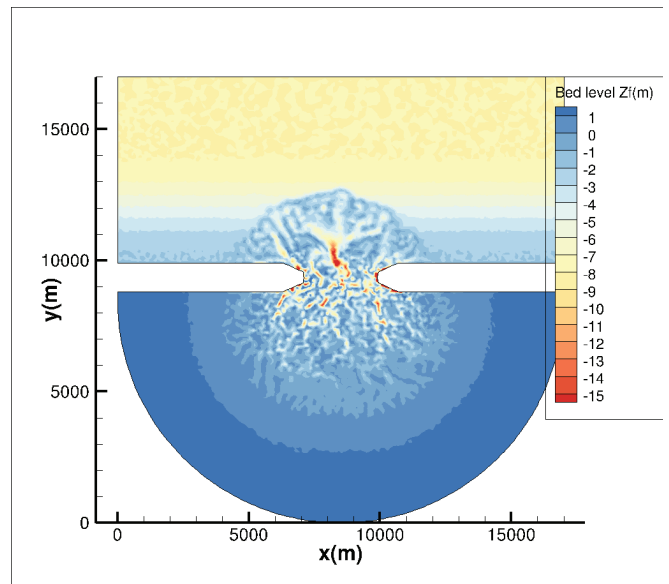
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Figure 4: Bottom after 100 years for the coarse mesh – For 8 processors on a linux Z 600 station, the CPU time is about 1 day 13 hrs.

Variable input parameters	Mean Value	Standard deviation
k_s (m)	0.05	0.005
d_{50}(m)	$1.5\ 10^{-4}$	$1.5\ 10^{-5}$

Table 1: Variable input parameters – mean values and standard deviations

Variable Input parameters	Mean Value	Standard deviation
Secondary current parameter β	7	0.7
Sloping bed parameter α (m²/s)	$1.15\ 10^{-5}$	10^{-6}

Table 2: Variable input model parameters- mean values and standard deviations

Figure 3: Bed evolution obtained after 30 years for a morphodynamic factor MF =10 on top and MF = 50 on the bottom
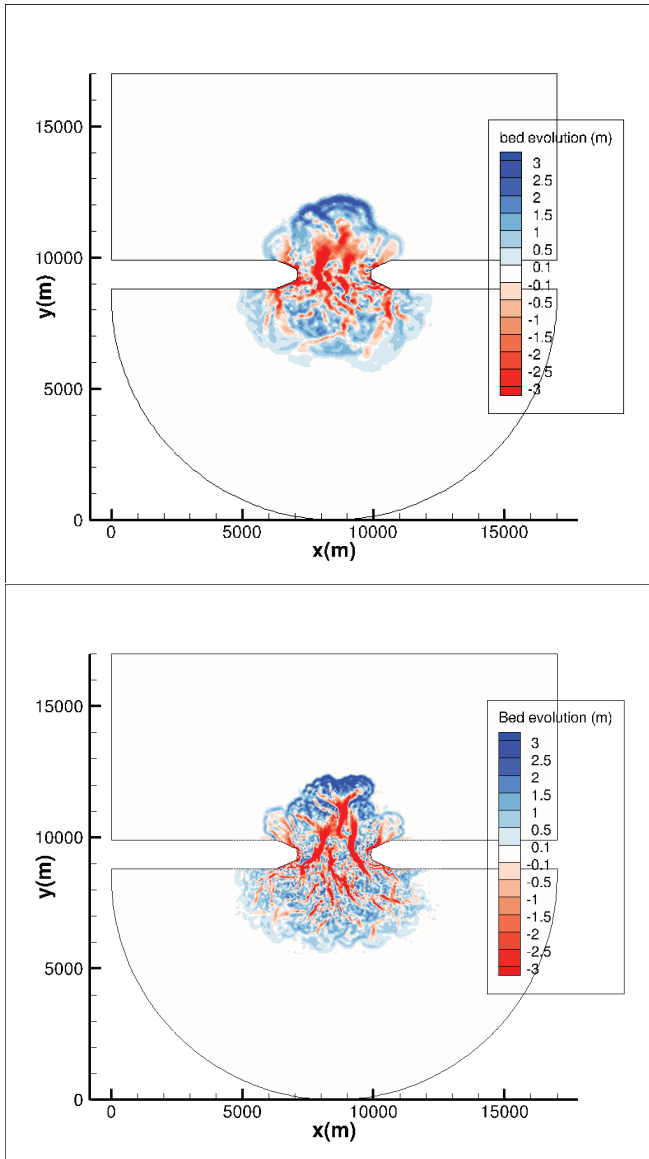
Figure 5: Bottom evolution after 10 years for the 2 meshes with sloping beds included. The top figure is obtained for the coarse grid and the bottom one, with the refined grid.

V.   UNCERTAINTY AND SENSITIVITY ANALYSIS USING AD

A. Presentation of the FOSM/AD Method

The First-Order Second Moment Method using Algorithmic Differentiation (FOSM/AD) presented in Villaret et al. (2015) is applied here to quantify sensitivities and uncertainty in the Telemac-2D/Sisyphe morphodynamic model.

Assuming first-order Taylor expansion and independent input variables X_i, the variance of the model output variable can be expressed as a function of partial derivatives of the morphodynamic model function F according to:

$$Var(Zf) \simeq \sum_{i=1}^{n} \left[\frac{\partial F}{\partial X_i} \big(E(X) \big) \right]^2 \cdot Var(X_i) \qquad (4)$$

Where $Zf = F(X_i)$ is the calculated bed evolution, and n is the number of variable inputs, $E(X)$ is the best estimates or mean values of X. Partial derivatives in Eq. 4 can be calculated exactly up to machine accuracy using Algorithmic Differentiation (AD).

The AD-generated Tangent Linear Model of the Telemac-2D/Sisyphe model (TLM) computes in addition to the bed evolution, a projection of the Jacobian (matrix of partial derivatives). The partial derivatives $\frac{\partial Zf}{\partial X_i}$ of the calculated bed evolution $Zf = F(X_i)$ with respect to each individual uncertain variables X_i for $1 \leq i \leq n$ are obtained by evaluating the TLM $\dot{F}$ repeatedly. Eq. (4) can then be evaluated easily from the stored partial derivatives $\frac{\partial Zf}{\partial X_i}$ to obtain the total variance $Var(Zf)$.

B. Sensitivity Analysis to variable grain size and bed roughness

For the FOSM uncertainty, we use a Nikuradse friction law and variable bed roughness with mean value k_s=0.05 m.

Sediment transport formulas embedded in morphodynamic models are highly sensitive to both grain size d_{50} and bed roughness k_s input parameters. Here we assume 10% for the standard deviation of each input parameter.

The TLM model has been applied twice to calculate the partial derivatives with respect to both grain size and bed roughness. Variations of the bed evolution are then obtained by multiplying the calculated partial derivatives by the standard deviation of each input parameter.

Variations of the bed evolution with respect to both grain size and bed roughness after one year of bed evolution are shown in Figure 6. The pattern obtained for both parameters is overall similar but with opposite signs. An increase in grain size is expected to reduce the transport rates and resulting bed evolutions, which is qualitatively similar to a decrease in bed roughness.

Quantitatively, the effects of both grain size and bed roughness are maximum where the bed evolutions are higher, e.g. at the mouth of the inlet and in the delta where it is about +/-20 cm, which represents overall less than 10 % of the bed evolution.
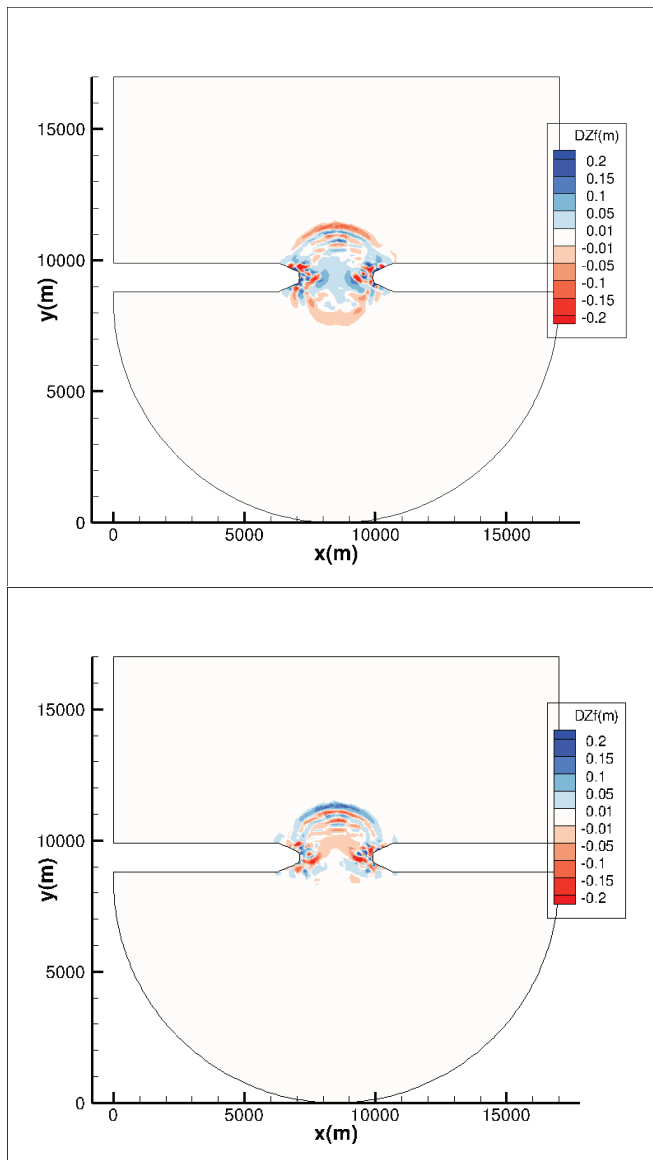
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Figure 6: Sensitivity to grain size (top) and bed roughness (bottom) after 1 year of simulation (130 000 NIT with Dt = 10s and MF=25). In scalar version each run of the TLM model takes approximately 10 to 15 hours on a Linux station

C. Sensitivity Analysis to Model parameterization

Sediment transport models rely on empirical formulations for complex sediment transport processes (bed load formulae, sloping bed effects, secondary currents…). In addition to the set of physical input parameters, a set of semi-empirical coefficients can be also considered as variable input model parameters.

Here we are interested in the sensitivity of model results to sloping bed and secondary currents. Again we assume 10% of variation for the standard deviation of both coefficients α in the sloping bed formula (eq. 2) and β in (eq. 3).

According to the TLM model results shown in Figure 7, both secondary currents and sloping bed effects are found to have only a minor influence on the results, in comparison to the grain size and bed roughness parameters. Sloping bed effects are more important at the mouth of the inlet where gradients in the bottom slope are higher. The effect of secondary currents is found to have a very local influence inside the inlet. Quantitatively their effect is less than 2% of the bed evolution.
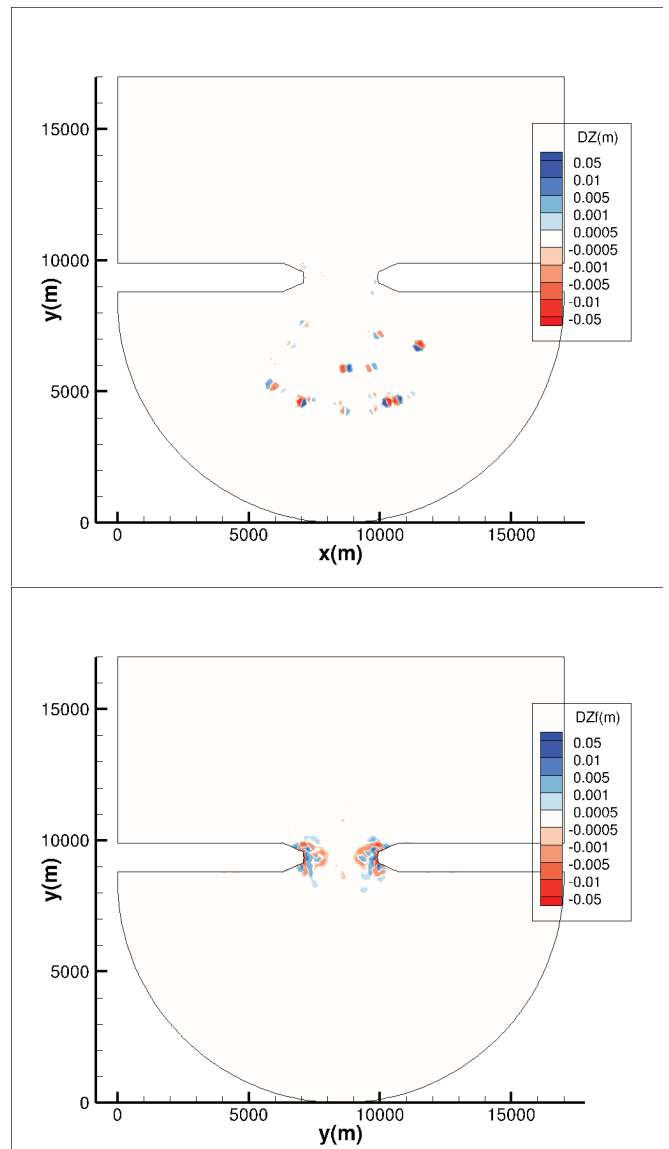

Figure 7: Sensitivity to secondary current (top) and sloping bed (bottom) after one year of simulation

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D. Uncertainty Analysis

The effects of both bed roughness and grain size are dominant and only those two terms have been retained to calculate the total variance. Equation (4) reduces to:

$$Var(Z_f) \simeq \left[\frac{\partial z_f}{\partial D_{50}}\right]^2 \cdot Var(D_{50}) + \left[\frac{\partial z_f}{\partial k_s}\right]^2 \cdot Var(k_s) \quad (5)$$

The total variance has been calculated by post processing results of the TLM results presented in Figure 6. The final results obtained after 1 year of bed evolution are shown in Figure 8 (top figure). Again the uncertainty is larger where the bed evolution (bottom figure) is larger, i.e. at the inlet side boundaries and at the position of the ebb delta offshore. The global uncertainty represents approximately 10 % of the bed evolution.
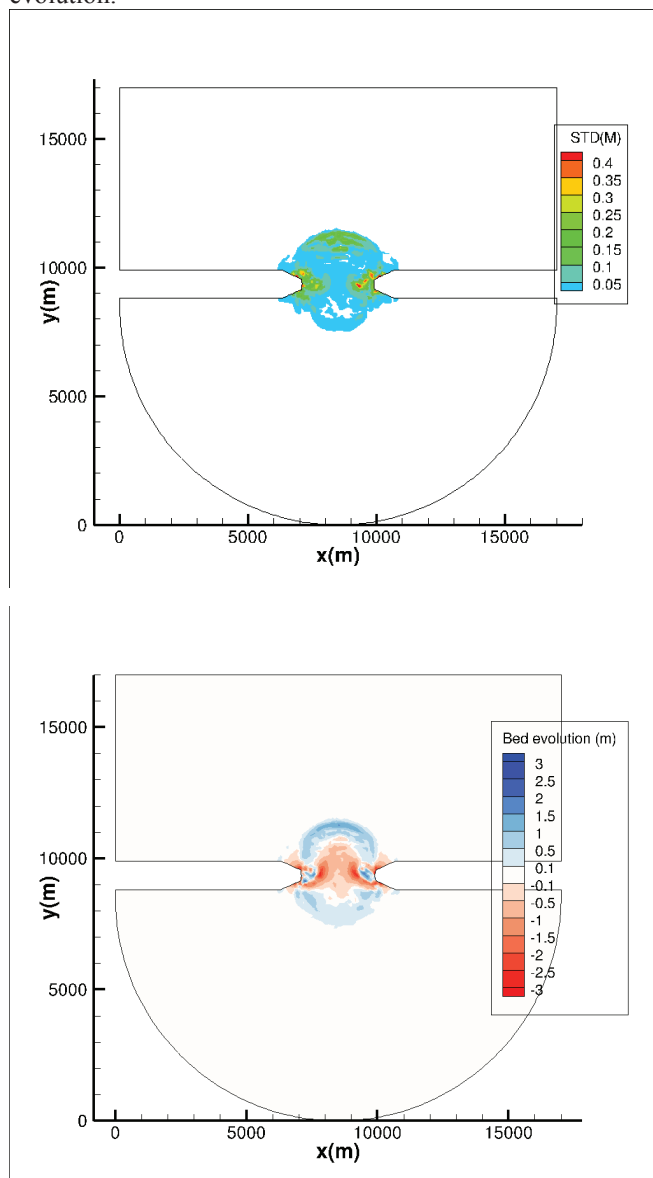

Figure 8: Uncertainty analysis. The standard deviation of the bed evolution (m) as a result of variability in the grain size and bed roughness is shown on the top. The corresponding bed evolution after one year is shown on the bottom.

CONCLUSIONS

The FOSM/AD method has been applied to quantify uncertainty and sensitivities in a complex 2D application: the formation of an ebb delta and channel pattern in a schematic tidal inlet. Here we used a recently developed TLM model of the Telemac-2d/Sisyphe model using release 7.0 and the AD-enabled NAG Fortran compiler.

The effects of secondary currents and sloping beds are found to make only a small local contribution to the total uncertainty whereas the sensitivities to both bed roughness and grain size are dominant. The partial derivatives obtained with respect to grain size and bed roughness are similar but with opposite sign.

Assuming 10 % for the standard deviation of each input parameters, the uncertainty due to both bed roughness and grain size represents approximately 10% of the bed evolution after one year. Uncertainty is larger where the bed evolution is larger, i.e. at the mouth of the inlet along the side boundaries as well as offshore where the ebb delta starts to form.

Despite its efficiency – only one TLM run per variable input parameter – the FOSM/AD method is here limited by the use of a scalar version. A parallel version is under development and needs to be applied to study uncertainties at morphodynamic time scales (10 to 100 years). The effect of other parameters – like the rigid bed level - is expected to be important for the channel pattern formation observed after 30 years.

The mesh dimension is shown to have a major influence on the model results and determine the channel pattern characteristic scales (extension and width). The FOSM/AD method needs to be extended to study the sensitivity to mesh size.

The sensitivity of the bed evolution to the initial bathymetry needs also to be further investigated.

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