

A discrete numerical description of the mechanical response of soils subjected to degradation by suffusion

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ABSTRACT: Internal erosion is a major cause of the failure of hydraulic earthen structures. A particular case of such an erosion process is suffusion which constitutes a strongly coupled fluid-solid interaction problem. It is a selective erosion of fine particles from an unstable soil structure leaving behind the granular skeleton which possibly leads to deformations. Such a process may cause modification in the mechanical behavior of the soil. To study this problem numerically, a model is established based on the discrete element method implemented in Yade software (Smilauer et al. 2015). Periodic boundary conditions are adopted and the soil is represented by a 3D assembly of spherical discrete elements. Such an oversimplified particle's shape leads to excessive rolling. To overcome this obstacle, rolling resistance was taken into consideration in the inter-particle contact law. Bearing in mind that numerical modeling of suffusion can constitute a difficult task requiring important computational resources due to the direct description of interactions between solid and liquid phases, a one-way coupling with a fluid phase is considered here. However, effects on the soil due to the loss of a fraction of fine particles is investigated either by modeling soils with different grains size distribution and different initial fines content to characterize its influence on the soil microstructure, or by mimicking the suffusion process by defining an extraction criterion of potentially erodible particles. This extraction criterion is based on the size of the particles, constriction sizes, and the velocity of particles under the effect of fluid forces. From these two approaches, we were able to specify the fines content from which their erosion may have a significant influence on the microstructure. Moreover, the defined extraction criterion was able to describe the effect of erosion on the stability of the soil structure.

1 INTRODUCTION

Suffusion is a particle-scale mechanism characterized by the detachment and migration of fine soil particles by interstitial flow leaving behind the granular skeleton. This can influence the mechanical behavior of the considered soil and may result in catastrophic consequences on civil engineering structures such as embankment dams, dykes and levees.

Many studies have focused on the role of fine particles on the soil behavior. A significant influence on the nature of the microstructure can be noticed.

The participation of particles of different sizes in the interparticle contact force changes with fines content (Thevanayagam 1998, Thevanayagam 2000, Shire et al. 2015, Shire et al. 2014). Moreover, fines content has a significant influence on the shear strength and the onset of mechanical instabilities in soils as well as on the position of the critical state line of silty sands (Benahmed et al. 2015, Yin et al. 2014). Therefore the soil behavior seems to be dependent on the range of fines content, which may explain the

changes in the soil strength after internal erosion.

Besides, several laboratory investigations have been carried out to study the development of suffusion and its induced effects on soil properties. Seepage tests, or real erosion tests, by developing suitable apparatus for that purpose, showed the influence of soil gradation and hydraulic gradient on erosion rates. Changes in the void ratio and hydraulic conductivity were reported. Concerning the mechanical behavior, different conclusions were found. An increase or a decrease in soil strength may occur (Ke and Takahashi 2012, Xiao and Shwiyhat 2012, Ke and Takahashi 2014a, Ke and Takahashi 2014b, Chang and Zhang 2011, Chang et al. 2012). Therefore, further investigations of the microstructure is necessary to explain the mechanical behavior of eroded soils.

In addition to experimental studies, few numerical studies are available to describe the relation between the removal of soil particles and its consequences at the macroscopic scale. Some include direct coupling between solid and liquid phases (Sari et al. 2011) while others immitate erosion by defining a procedure allowing the removal of particles without a liquid phase (Scholtès et al. 2010, Wood and Maeda 2008b, Wood and Maeda 2008a). This latter is an efficient method from a numerical point of view being simple and requiring less computational resources. However, the removal of particles is usually based on the particle's size and the stress that a particle holds. So this method lacks other criteria that play a major role in erosion of fine particles, such as, the constriction sizes to allow particles to pass through or the driving fluid forces to cause particles to move.

Motivated by finding a clear relation between erosion of fine particles and the mechanical behavior of the soil structure, we present in this paper a numerical approach based on the discrete element method (DEM) implemented in Yade software (Smilauer et al. 2015). From one hand, the role of fine particles on the micro-structure of soils is analyzed by modeling soils with different grains size distribution but constant inter-granular void ratio. On the other hand, an extraction criterion of potentially erodible particles is defined to mimic the suffusion process. This extraction criterion is based on the ability of small particles to move under the effect of fluid forces as well as the constriction sizes of the solid granular matrix allowing fine particles to migrate. A one way fluid-solid coupling approach is introduced for this purpose.

2 THE DISCRETE NUMERICAL MODEL

Numerical tests were performed using YADE software. The 3D granular assembly is composed

of spherical discrete elements. Periodic boundary conditions are adopted and gravity is neglected in all simulations which may result in having some floating particles. Such particles are rattlers and can be easily detected. This helps in identifying particles that participate in the force transfer.

The periodic cell was formed as a parallelepipedic block filled with a cloud of spheres. Then, isotropic compaction was applied until reaching the required confining pressure and initial density. Finally, the triaxial compression was performed.

The interparticle interaction is modelled by a linear elastic relationship between forces and displacements with a slip Coulomb model. Since spherical particles cause excessive rolling, rolling resistance was taken into account in the contact law. For the purpose of modeling soils with different initial densities, it was suggested that during the compaction phase only, either to decrease the contact friction angle to reach high initial densities, or to add adhesive normal and tangential contact forces to reach low initial densities. All details concerning our numerical model and the preparation method can be found in (Aboul Hosn et al. 2016). For the one way coupling, DEM-Pore scale finite volume coupling is followed (Chareyre et al. 2012) which allows us to solve the interstitial fluid flow and to compute the fluid forces applied on each soil particle.

3 DEPENDENCE OF SOIL MICROSTRUCTURE ON FINES CONTENT

As mentioned previously, the first approach followed to account for the effect of the loss of a fraction of fines is done by studying soils with different fines content. We focus in this section on the role of fine particles on the microscopic and macroscopic behavior of the soil. The results of the numerical model built for this purpose are compared with those obtained from an experimental study whose aim was to characterize the mechanical behavior of natural silty sand. Such sand was collected in the adjacent zone of a breach of the Rhone embankment dike during the 2003 flood (Benahmed et al. 2015). The problem that had occurred involved soil in a relatively loose initial state that we search to characterize here.

The particle size distributions, resulting from a mixture of sand and silt, characterized by a fines content FC (i.e. silt content) ranging from 0 to 15%, are displayed in Figure 1. They were simplified in our model to reduce the difference in diameter between large and small particles, else it will require a high number of particles and a high computational time.

The stress-strain curves obtained from triaxial compression tests on initially loose soil samples are

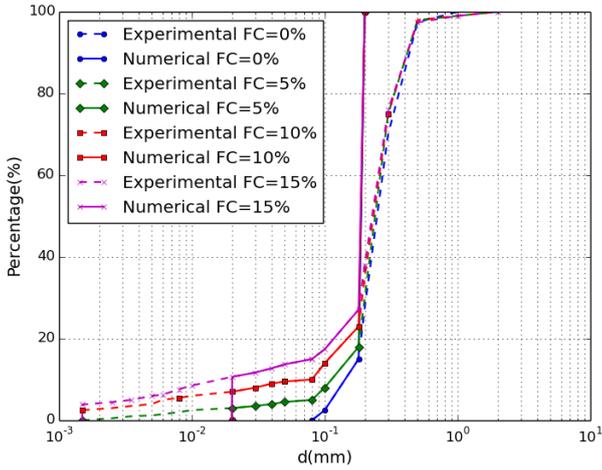


Figure 1: Particle size distributions for fines content, FC, ranging from 0 to 15%.

illustrated in Figure 2. Notice, the effect of fines on the soil strength is reduced in the numerical model with respect to experimental data for $FC \leq 15\%$. This can be related to the modification in the numerical gradations with respect to the experimental ones, but also to the difference between the angular shape of real particles, and the spherical one of numerical particles. However, in all cases, the simulated responses to triaxial compressions show that the numerical model presents a realistic mechanical behavior close to the experimental data.

Therefore, analysis of the microstructure for samples with a wider range of FC (0-100%) is done after reaching an isotropic state in order to understand at which FC fines start to play a significant effect on the microstructure. Different measures of density were studied for this purpose: the global void ratio (e), the intergranular void ratio (e_g , considering all fine particles as voids) and the equivalent intergranular void ratio (e_{eq} , considering only inactive fine particles as voids). The distinction between active and inactive particles was done by checking the number of contacts of each particle. If the number of contacts is less than 2, then the particles are considered inactive. The variation of these parameters with respect to FC is shown in Figure 3b for the granular assembly in a loose state. In Figure 3a, the global void ratio, e , in the current loose state is compared with e_{min} , the void ratio reached for the densest state of the granular assembly.

It is noticed that the global void ratio decreases with the increase of fines content until reaching a threshold value at $FC_{threshold}=40\%$ beyond which it increases again as more small particles fill the voids made by large particles (Thevanayagam 2000). At a certain FC, small particles start to separate large particles; hence increasing the global void ratio again. Similar behavior is noticed for the minimum void ratio (e_{min}), but with $FC_{threshold}=25\%$.

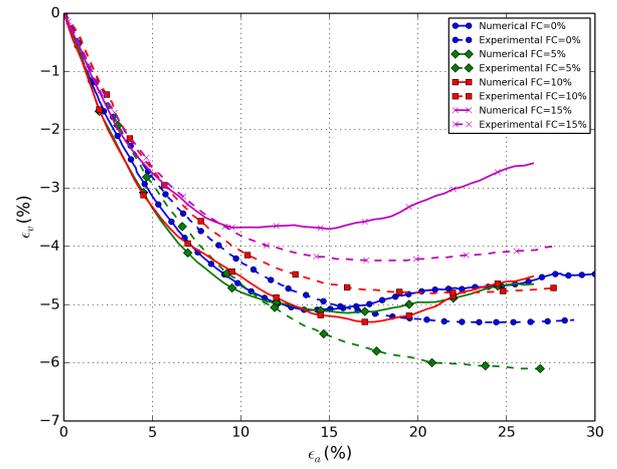
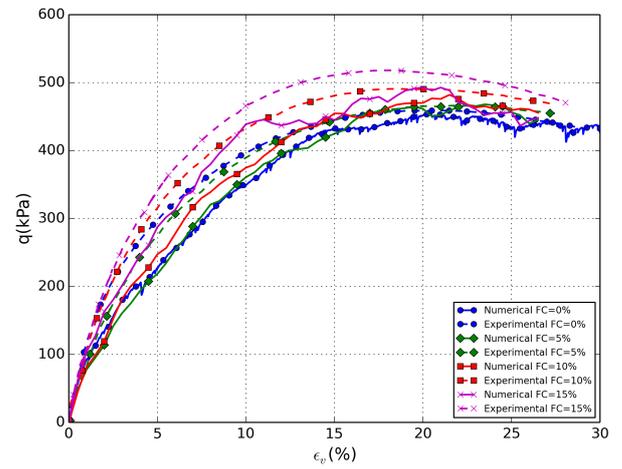


Figure 2: Stress-strain response and volumetric deformation of a loose silty sand subjected to a drained triaxial compression.

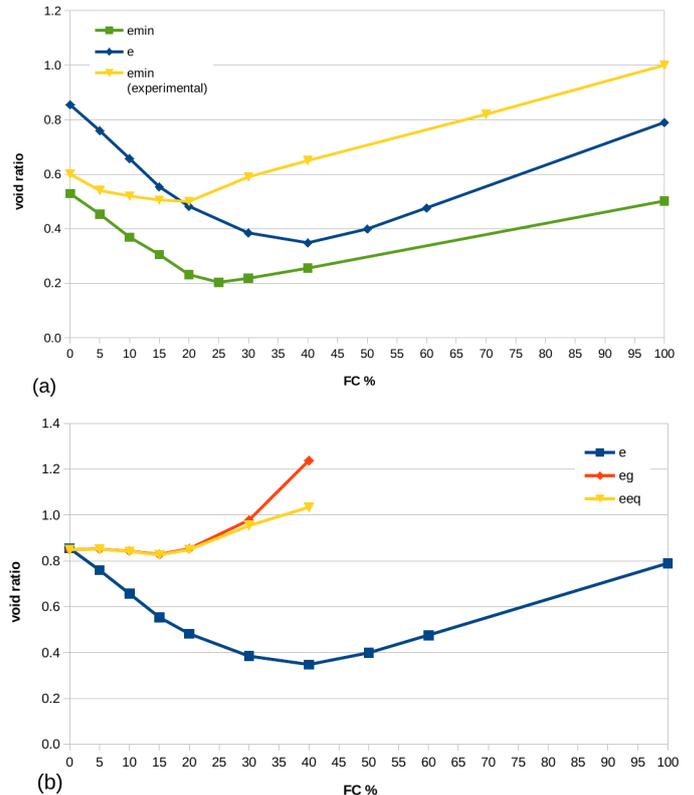


Figure 3: (a) Comparison of the global void ratio (e) reached with e_{min} , the void ratio at the densest state. (b) Comparison of e , e_g and e_{eq} in the current loose sample state.

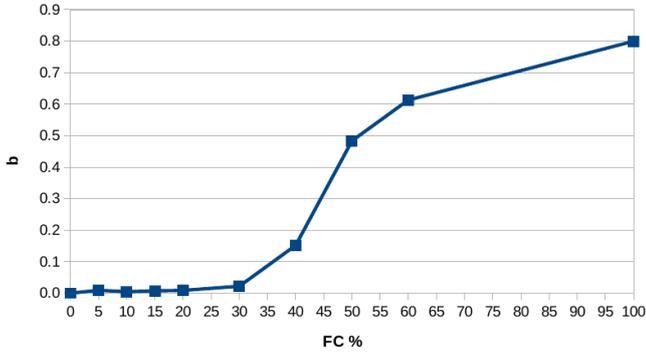


Figure 4: The variation of the parameter b with fines content.

Therefore, we conclude that the threshold fines content beyond which the granular assembly becomes fines-dominated ranges between 25% and 40%. Note that (Benahmed et al. 2015) found $FC_{threshold}=20\%$ based on the evolution of e_{min} with FC. This value is smaller than the one we obtained when studying the variation of e_{min} . This can be a result of the simplified gradation and the spherical grains' shape used in our model. However, assessing $FC_{threshold}$ by considering only e_{min} is only one of the methods that can be found in the literature (Zuo and Baudet 2015). It is possible to determine $FC_{threshold}$ from e_{max} or e_{min} curves, however, it is not necessarily true that both curves give the same $FC_{threshold}$. Therefore, such a method is efficient to define a range in which the value of $FC_{threshold}$ may exist depending on the degree of compaction of the granular assembly. In our numerical model this range is 25-40%. This range could be refined by investigating other FC values in the vicinity of $FC_{threshold}$ identified here.

Figure 3 also shows the variation of the intergranular and equivalent intergranular void ratios. For $FC \leq 20\%$, both parameters are almost constant. At $FC > 20\%$, these void ratios start to increase although the global void ratio is still decreasing. This means that at $FC > 20\%$, a fraction of fine particles starts to separate large particles. In other words, a portion of small particles becomes active with an important effect on the soil structure.

Moreover, the fraction of active particles can be represented through the parameter b which is the fraction of the volume of active fine particles to the total volume of fine particles. Figure 4 shows the variation of b with FC. It is clear that most particles are inactive until $FC=20\%$ where it can be noticed that some small particles start to be involved in the contact force network by forming two or more contacts with their neighbors.

Further investigations were done at the particles' scale to investigate whether $FC_{threshold}$, as defined above, effectively constitutes a threshold with respect to the constitution of the microstructure

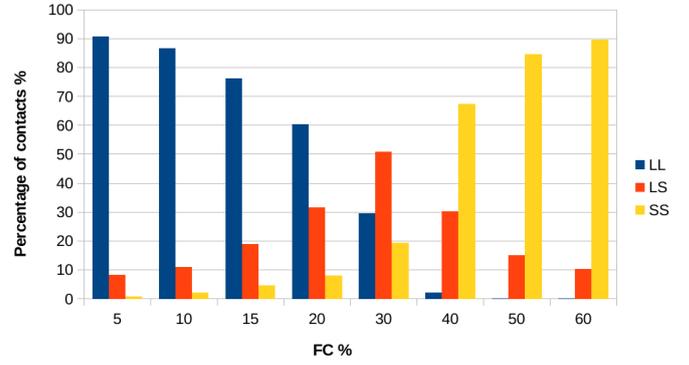


Figure 5: Percentage of contacts: LL(Large-Large particles), LS (Large-Small particles), and SS (Small-Small particles).

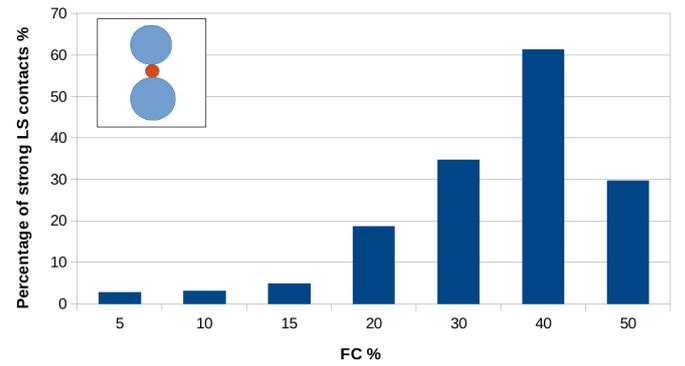


Figure 6: Percentage of the strong LS contacts.

of the granular assembly or if other limits can be identified with respect to the role that the fine fraction plays in the contact force network. Percentages of contacts established between small particles (SS), large particles (LL) and between large and small particles (LS) are illustrated in Figure 5. Notice that, LL contacts decrease with FC and they almost vanish for $FC > 40\%$ meaning that all large particles have contacts with small particles only. In other words, large particles who had contacts with large particles establish new contacts with small particles as FC increases and are separated as well. In addition to LL contacts, Figure 5 shows that as FC increases, large particles establish more contacts with small ones until a maximum percentage of LS contacts is reached at $FC=30\%$. Beyond 30%, large particles become dispersed in the medium where fine particles start to form the major structure. This is proved by SS contacts which increase significantly for $FC > 30\%$.

Thus an evident conclusion is that for FC greater than 20% the microstructure is changing significantly. A portion of small particles starts to play an important role. For further interpretation, it was checked among strong contacts which ones correspond to LS contacts. What is meant here by strong contact is the one with a force higher than the mean contact force in the system. Figure 6 summarizes the results.

It is noticed that for $FC \geq 20\%$, the percentage

of strong LS contacts increases significantly. Thus more small particles separate large particles and contribute to the transfer of forces in the system. Therefore, the configuration that is expected in such a case can be as the one inserted in Figure 6.

As a conclusion, the microscopic analysis showed that fine particles start to be involved in the contact force network, and thus in the macroscopic shear strength of the granular assembly for a fines content lower than $FC_{threshold}=40\%$ and around $FC = 20\%$. We suggest to define this limit as the active fine limit FC_{active} . Note that FC_{active} is here different from $FC_{threshold}$ for the granular assemblies we studied. However, FC_{active} may be equal to $FC_{threshold}$ in other conditions (for other gradings, particle shape, density, ...). Thus if erosion of all fine particles occurs in our studied soil with initial $FC=20\%$, one can expect a more important impact on the macroscopic mechanical properties of the soil than for lower initial fines content.

4 EROSION PROCEDURE

In the previous sections the effect of different percentages of fine particles on the microstructure was presented. It was found that for the studied soil, $FC=30\%$ corresponds to an active fine limit beyond which the soil becomes fines-dominated. Thus one can imagine that if suffusion occurs in such a soil with $FC=20\%$, the soil strength may be influenced seriously. However, several criteria should be satisfied for internal erosion to occur (Kenney and Lau 1985, Wan and Fell 2008):

1. Geometrical criteria: grain sizes and constriction sizes. They affect the potential of internal instability.
2. Hydraulic criteria: hydraulic gradient, flow velocity and flow direction. They govern the onset of erosion.

Such criteria can be described numerically if a coupling is performed between solid and fluid phases. However from a computational point of view this can be an expensive task. Therefore, some simplified numerical extraction procedures were suggested by other authors, allowing them to describe the effect of particles removal on the soil behavior (Scholtès et al. 2010, Wood and Maeda 2008b, Wood and Maeda 2008a). Such procedures present partial description of the problem neglecting other essential criteria of suffusion. Usually in such methods, removing a particle from a soil matrix may be done by assessing its stress state, coordination number and size. Other aspects such as the constriction size and the hydraulic gradient were ignored.

Our purpose here is to improve such extraction

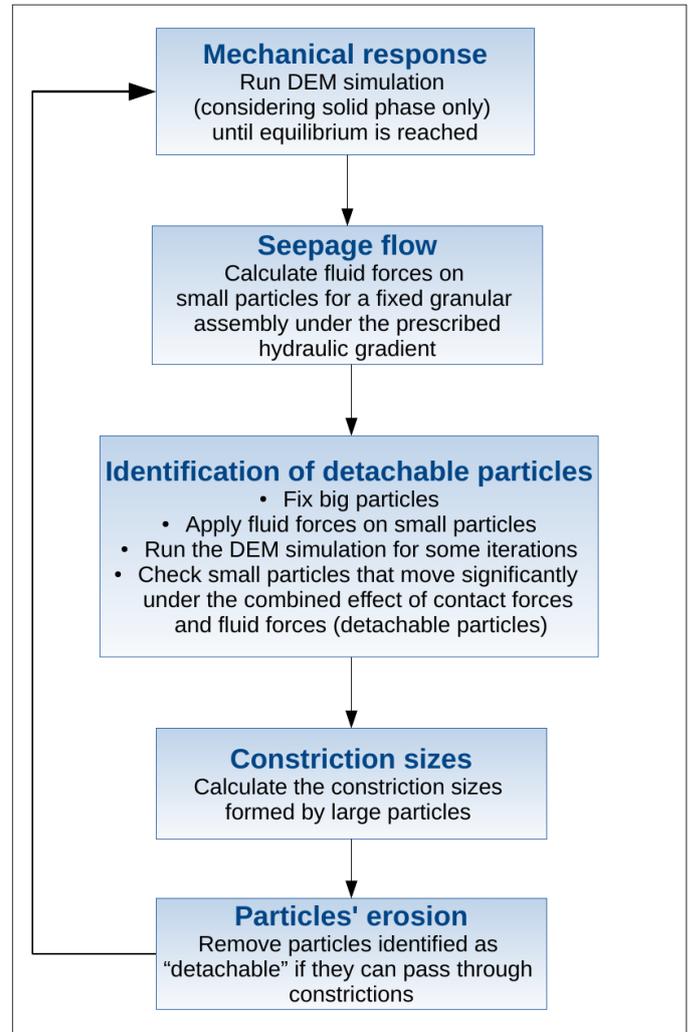


Figure 7: Erosion algorithm.

procedures by taking into account other criteria in our numerical model with least computational cost. Then to apply such a method on the studied soil in the previous section. The suggested algorithm to erode particles is illustrated in Figure 7. Notice that it is a preliminary method which can be modified and developed in future work.

The procedure includes the following steps:

1. Start from an equilibrium state reached for a given stress state (after an isotropic compaction or triaxial compression up to a given stress deviator).
2. Perform one way coupling where seepage flow is considered only in this step to calculate the fluid force on small particles.
3. Fix big particles and apply the fluid forces gradually on small particles. Under such forces which are kept constant, run the calculation (without coupling with the fluid phase) for some iterations. Then check small particles which moved significantly under the combined action of fluid forces and contact forces. These particles are identified as “detachable” particles.

4. Calculate the constriction sizes formed by big particles only.
5. Starting from the equilibrium state remove the small “detachable” particles that can pass through constrictions.
6. Perform DEM simulation (representing the solid phase only) until a new equilibrium is achieved. Then go back to step 1 again.

Following such an algorithm, we were able to take into account more complex erosion criteria keeping the computational time reasonable. However, questions arise:

1. How we define small particles that “move significantly”? It is suggested as a first approach to assess the increase in the magnitude of particles velocity. But then what is the threshold velocity used for comparison?
2. It will be hard to compare the particle’s size with each nearby constriction, so what constriction size to choose for comparison?
3. What is the definition of big particles which will be fixed during the application of fluid forces and which will be only considered to calculate the constriction sizes?
4. How many iterations should we wait to judge if a particle is detached or not?

To answer these questions, Fontainebleau sand (Figure 8) was used to test the proposed method, as it doesn’t require a high number of particles to represent correctly the whole particle size distribution. 10,000 spheres were used. To choose the fine fraction, small particles may be considered based on the method of Kenney and Lau (Kenney and Lau 1985). So small grains correspond to the fraction of the soil gradation that crosses the line $\frac{H}{F}=1$; where H is the increment of % passing that occurs over a designated grain size interval of d to 4d (d being the diameter of the soil grain) and F is the % passing at grain size d. However, with Fontainebleau sand used for illustration only, this criterion can’t be applied on such soil gradation. Therefore, small particles were chosen arbitrary to be those whose diameter is less than 0.26mm.

Several simulations were analyzed. Only 1000 iterations were done after the complete application of the fluid forces. Figure 9 shows the velocity of small particles under different hydraulic gradients ($i=0; 0.01; 0.05; 0.1$), one can consider that the particle’s detachment initiates when the hydraulic gradient reaches 0.01 for which some particles are uniformly accelerated under the effect of the fluid force (whereas the velocity of other particles is

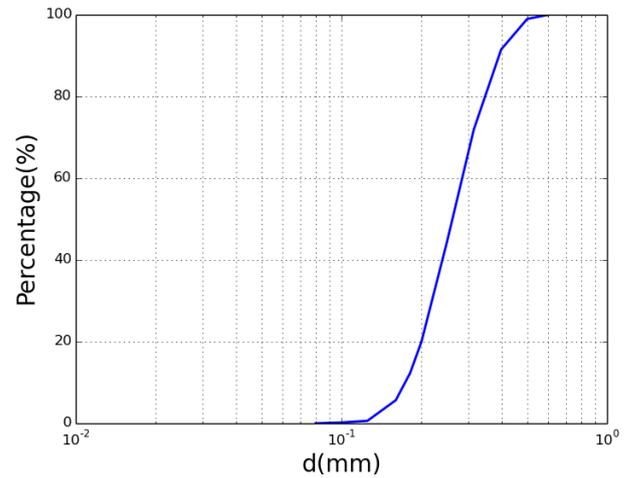


Figure 8: Particle size distribution of Fontainebleau sand.

not affected). This is more explained in Figure 9 for $i=0.05$ where two classes of particles can be distinguished:

1. A group of particles showing a continuous increase in velocity.
2. Another group of particles showing initial increase of velocity under the effect of seepage flow, however, the variation of velocity kept fluctuating.

Assuming that particles belonging to the first group are detachable, an erosion process based on the previously described algorithm was performed with a hydraulic gradient of 0.05. This latter was used as it was easier after 1000 iterations to separate detachable particles from non-detachable ones. Therefore, detachable particles were removed if they can pass through constrictions. The average value of the constriction sizes formed by big particles was chosen for comparison with the detachable particles’ size and to judge if such particles are erodible or not. Then after the first erosion process, another erosion process was done by checking the velocity of fine particles (that are still present) under the effect of seepage flow, with $i=0.05$ (Figure 10). The percentage of eroded mass in each case is shown in Figure 11.

Figure 12 demonstrates the coordination number of eroded particles. In the first erosion process, very few particles have a coordination number greater than 2. So most particles are rattlers. Whereas in the second erosion process, most particles are active ones. Thus one can expect a major effect of the second erosion process on the stability of the soil matrix. Such an influence is illustrated in Figures 13 and 14 through the variation of the stress and the volumetric deformation during the erosion processes as well as the change in porosity. It is shown that the first erosion caused a slight change in stress with almost negligible effect on the volumetric deformation. This means no soil grains rearrangement has taken place

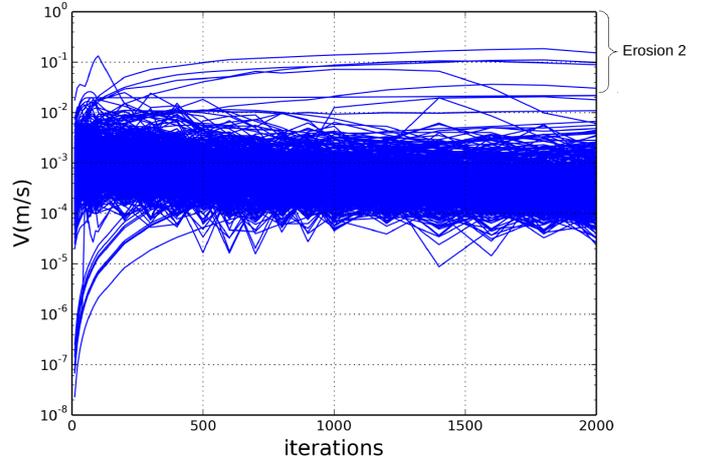
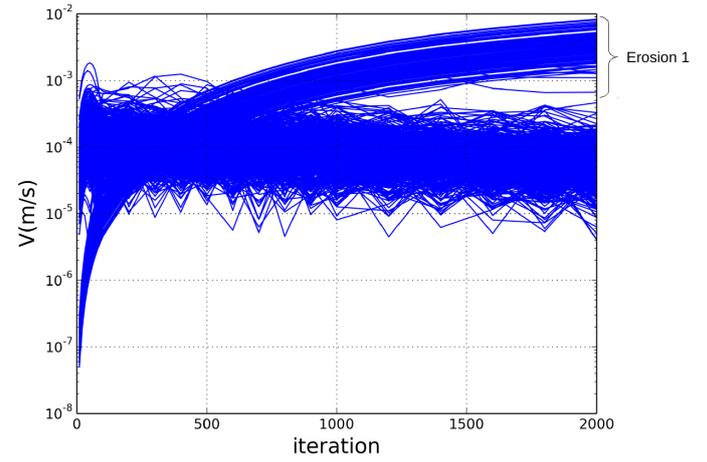
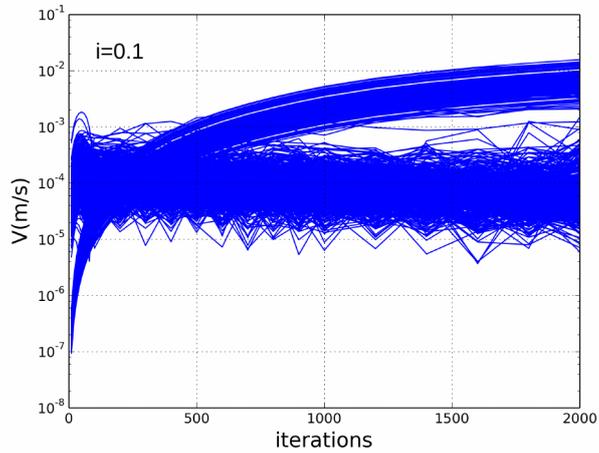
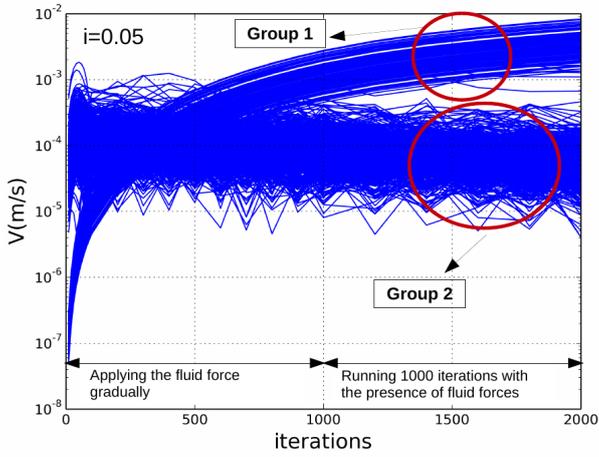
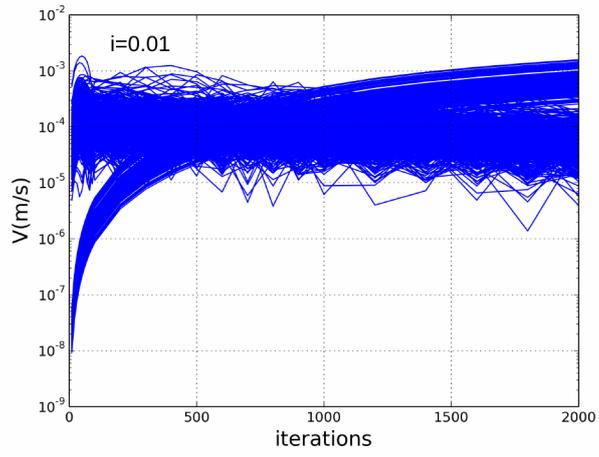
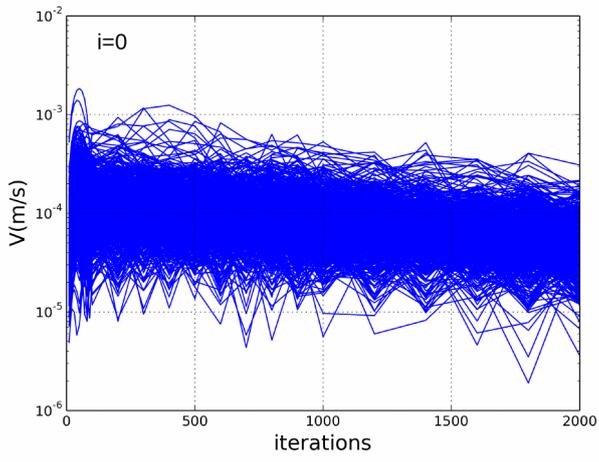


Figure 10: Velocity of small particles for two successive erosion processes under a hydraulic gradient of 0.05.

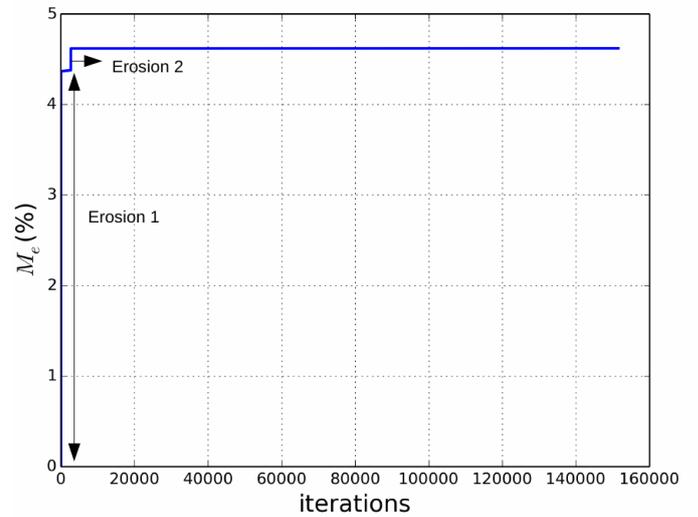


Figure 11: Mass of eroded particles.

Figure 9: Velocity of small particles after the application of fluid forces (step 3 of the erosion process described previously).

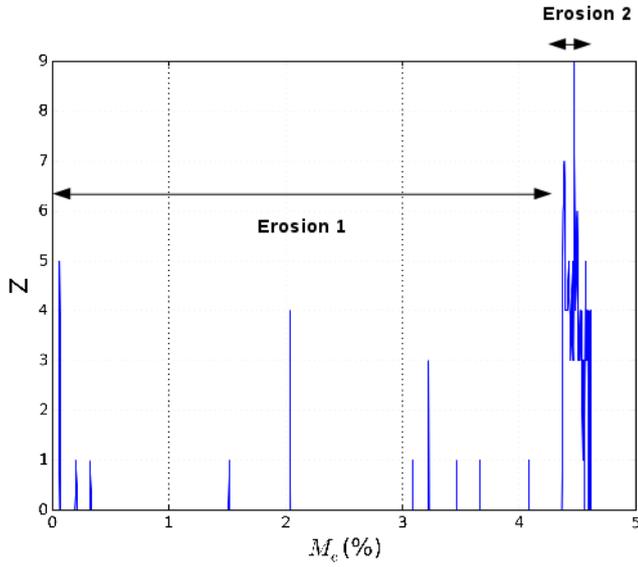


Figure 12: Coordination number of eroded particles.

and the stability of the microstructure wasn't influenced significantly after the first erosion. The porosity increases with the eroded mass with a negligible decrease at the end of erosion. However, the second erosion shows an important influence on the stress and the volumetric deformation (Figure 13). Since active particles were removed, the grains rearrange themselves to reach a new equilibrium state. This is also translated by the decrease in porosity (Figure 14).

Performing such an erosion procedure, one can describe the effect of the removal of fine particles on the soil structure and its behavior. However, further analysis is needed when checking the particles' velocities. (Figure 9).

Particles of the second group may belong to two different cases:

1. Non-detachable particles where they remain blocked without breaking their contacts and so this reflects just the deformation of contacts without being destroyed.
2. Detachable particles where they are transported by seepage flow, therefore the velocity fluctuations may be due to successive bouncing of one grain on another one encountered during its transportation through constrictions.

Therefore to justify our choice of detachable and non-detachable particles on clear basis, the displacement of grains may be a better parameter to work on. This is to be discussed in future work.

5 CONCLUSION

This paper presents microscale analysis of soil samples with different fines content as well as a definition of an extraction criterion. Idealized samples with isotropic fabric and spherical particles are considered.

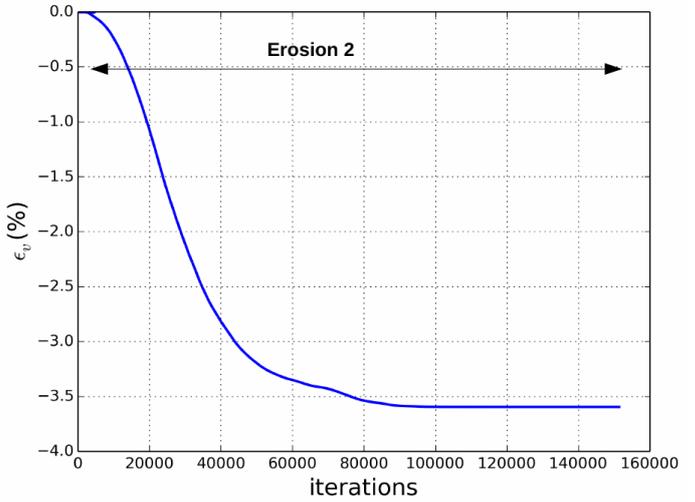
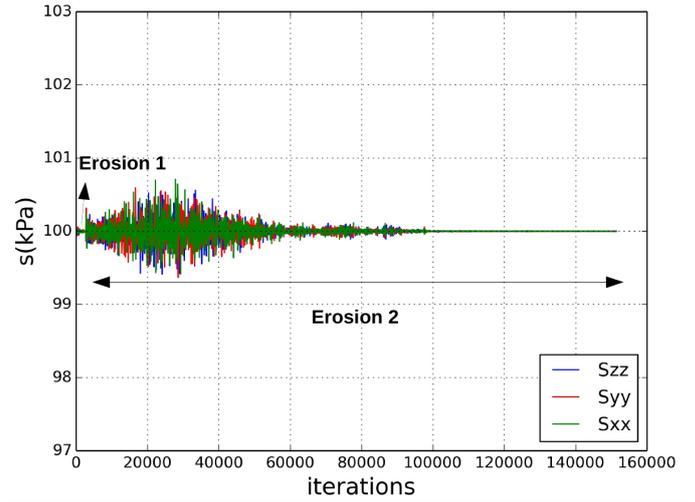


Figure 13: The variation of stress and volumetric deformation after erosion.

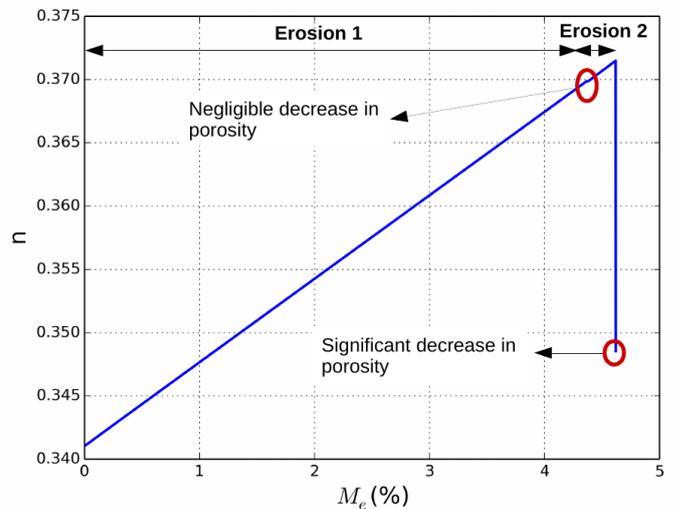


Figure 14: Change of porosity with erosion.

Analysis of the microstructure of soils with different fines content is done by studying the variation of void ratios (e , e_{min} , e_g and e_{eq}) as well as the contacts formed between particles. From such an analysis, an "active" fines content FC_{active} can be distinguished from $FC_{threshold}$. Therefore, if erosion takes place for FC around FC_{active} and if all the fine fraction is eroded, the consequences on the macroscopic behavior can be more important than for soil gradations with lower fines content.

To perform an erosion process, a numerical extraction procedure is defined. It is based on a one-way coupling method so that to keep the computational cost reasonable. Such a procedure is based on: the particles size, the constriction size, the magnitude of the particles velocity and the hydraulic gradient. Adopting this method, it is possible to define two groups of particles behavior based on their velocity. However, checking only the velocity is not sufficient, as some information may not be clear or may be hidden. Therefore, a modified version of this method will be presented in future work where the displacement of particles in the flow direction is considered to differentiate detachable from non-detachable particles.

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