Hydraulic failure by heave and piping

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ABSTRACT: It is generally agreed the same criterion for hydraulic heave and for piping can be used. In vertical upward flow the theory is well established. The weight of the soil must be greater than the seepage force of the water. Despite this well established theory piping and sand boiling occurs at much lower hydraulic gradient than the theoretical value. It is concluded that this discrepancy resulting from a conceptual error by not making distinction between the local and the global equilibrium. In the case of hydraulic heave the equilibrium of the entire soil prism (global equilibrium), while for the initiation of piping on the surface of the soil the equilibrium of one grain (local equilibrium) should be investigated. Investigating the local equilibrium of a single grain physical model for the initiation of piping is proposed. Criterion for piping on the top of a horizontal surface in granular soil subject to an upward flow deduced from first principles. It is concluded that hydraulic gradient alone does not sufficiently describe the hydraulic stability of the soil. The particle size and its distribution also have effect on the hydraulic stability of the soil. Critical grain diameter corresponding to the critical hydraulic gradient is introduced. If the diameters of the grains in a soil matrix are bigger than the critical diameter then the Terzaghi criterion is valid. Grains with smaller diameters lose their stability at lower hydraulic gradient than the critical one. The removals of grains from the soil matrix increase both the hydraulic gradient and the permeability of the soil. Thus the initial grain removal from the surface of the soil can progressively lead to sand boiling; therefore, the stronger criterion of piping should also be required for hydraulic heave.

1 INTRODUCTION

The stability of soil in an upward flow is well defined theoretically. In an upward flow, neglecting the side friction, a soil is fluidized or boils if the uplift force due to seepage exceeds the weight of the soil. If this condition occurs over a large area it is known as quicksand or sand boiling while if the condition occurs in a localized channel/s usually called piping (Powrie, 2014). Terzaghi conducted several model tests (Terzaghi, 1922). He was investigating a single row sheet of pile and found that if the penetration of the sheet pile is D then the failure due to piping occur within a distance of D/2 from the sheet pile (Fig. 1). He suggested calculating the factor of safety (F_s) against heave as:

$$F_s = \frac{W'}{U} \tag{1}$$

where W' is the submerged weight of the unit length soil prism and U is the hydraulic uplifting pressure. The submerged weight of the soil prism is calculated as:

$$W = \frac{1}{2}\gamma' D^2, \tag{2}$$

where γ' is the buoyant unit weight of the soil. The hydraulic uplifting pressure can be calculated as:

$$U = \frac{1}{2}\gamma_w D h_a \tag{3}$$

where h_a is the average hydraulic head at the base of the soil prism and γ_w is the unit weight of water. The results of the model test were in agreement with theory. However, it should be noted that Terzaghi clearly expressed that his derivation intended to model the heave mechanism specifically (Terzaghi & Pack, 1948). Despite his statement the theory, as a rule of thumb, is commonly used to predict factor of safety against piping in engineering practice.

The stability criterion used by contemporary engineering practice does not restrict the stability investigation to a certain pre-defined volume as it was suggested by Terzaghi but rather use the criterion in a broad term and requires satisfying the criterion in any volume or on any surface.

The stability criterion of soil in upward flow is usually given in one of the following ways:

#1./ $S < G,$ (4)

where S is the seepage force on the soil column, and G is the submerged weight of the same column,

#2./ $u < \sigma_z$ or $0 < \sigma'$, since $\sigma' = \sigma_z - u$, (5)

where u is the destabilizing total pore water pressure, σ_z is the stabilizing total vertical stress at the same place, and σ' is the effective stress at the same place, and

#3./ $i_z < i_c$ where $i_c = \frac{\gamma'}{\gamma_w} = (1-n)\frac{\gamma_s - \gamma_w}{\gamma_w}$ (6)

where i_z is the hydraulic gradient in the vertical direction, i_c is the critical hydraulic gradient, and γ_s is the unit weight of the soil particles. Eurocode 7 (2013) requires satisfying criteria #1 and #2 (Eqs 4, 5) for both hydraulic heave and piping.

All three criteria are expressing the same fundamental fact that is in equilibrium condition the weight of the soil must exceed the uplift force due to seepage. However, there are subtle differences among these three equilibrium criteria. The criterion #1 investigates the equilibrium conditions in a given volume, #2 on a horizontal surface, and #3 in a unit volume. The outcomes of these different criteria are identical if the boundary conditions are the same. Like criterion #2 is applied at the surface of the bottom of the soil column used in #1 etc.

The ratio of the buoyant unit weight of the soil and water is usually around one. Thus soil generally should lose its stability when the hydraulic gradient is higher than one (Eq. 6). The observed sand boils gradients are usually lower than one (Daniel, 1985; Turnbull & Mansur, 1961; U.S. Army Waterways Experimentation Station, 1956; USACE, 2005). Based on field experiences Schmertmann (2000) suggested using 0.5 maximum upward gradients for design to prevent vertical piping. Investigating the initial movement of the grains in test apparatus the critical hydraulic gradient required to initiate piping for vertical flow was 0.2 - 1.0 (Skempton & Brogan, 1994). It can be concluded that based on field studies and experiments piping and sand boiling in many cases occur at much lower hydraulic gradient than predicted by theory. The theoretical value of the critical gradient is derived from first principles. Why field and laboratory experiments disagree with the well established theory? This discrepancy is investigated in this study.

2 GLOBAL AND LOCAL EQUILIBRIUM

It has been shown that the stability criterion in an upward flow deduced from equilibrium investigations of a given volume (#1), a horizontal surface (#2), and a unit volume (#3). All the three equilibrium conditions investigate the stability of a restricted part of space or plane, containing many-many soil particles. Thus these equilibrium conditions used for hydraulic heave and piping can be consider as "global". However, piping or sand boiling starts on the surface of the soil with the removal of an individual grain (Fig. 1).

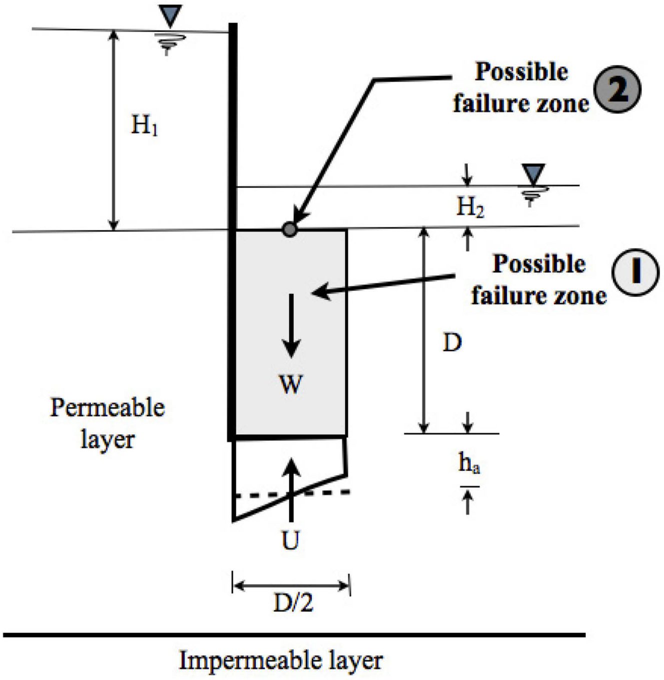

Figure 1 Terzaghi's hydraulic heave model for a single row sheet pile structure. The possible failure zones for heave (1) and for the initiation of piping (2) are shown.

Therefore, the "global" equilibrium conditions investigating the stability of the entire soil prism might not be relevant. This possibility is investigated.

The equilibrium of a soil grain in an upward flow can be defined by the velocity difference between the terminal velocity of the grain and the velocity of the upward flow (Fig. 1a). In a granular soil, a grain situating on the top of the soil matrix loses its position stability, when the velocity of the upward flow exceeds the terminal velocity of the grain (Fig. 2/b). The criterion for the stability of a grain then can be defined as:

$$v_t > v_w \tag{7}$$

where v_t is the terminal velocity of the flow and v_w is the upward flow velocity of the water. It is suggested that this "local" equilibrium criterion should be applied when the stability of an individual gain situating on the top of the soil matrix in non-cohesive soils is investigated.

3 STABILITY OF A GRAIN

Assuming that the soil grain has a spherical shape then the drag force (F_d) of a fluid in accordance to Stokes law in an upward vertical flow is

$$F_D = 6\pi\mu v d, \tag{8}$$

where μ is the viscosity of the fluid, v is the velocity of the sphere relative to the fluid and d is the diameter of the sphere. Investigating the force equilibrium of the sphere it can be shown that the terminal velocity of the falling sphere in a stationary liquid is

$$v_t = \frac{(\rho_s - \rho_w)g}{18 \cdot \mu} d^2, \tag{9}$$

where ρ_s is the density of the falling spherical body, ρ_w is the density of the water, and g is the acceleration of gravity.

If the size of the container is small then the wall of the container is relatively close to the falling sphere resulting in decrease in the terminal velocity. It has been estimated that the effective velocity is smaller than half of the calculated Stokes terminal velocity (Cistin, 1966). In order to take into account this effect a multiplying factor 0.5 is introduced when the terminal velocity of the grain is calculated.

The seepage velocity or true velocity (v_t) of the upward flow between the grains in a soil matrix is calculated from the Darcy seepage velocity or discharge velocity (v_D) as:

$$v_t = v_W = \frac{v_D}{n} = v_D \frac{1+e}{e} \tag{10}$$

where e is the void ratio and n is the porosity. Darcy's seepage velocity (Darcy, 1856) depends on the intrinsic permeability of the soil matrix (k) and the hydraulic gradient (i) as:

$$v_{Di} = k_i i_i \tag{11}$$

where the hydraulic gradient is the ratio of the hydraulic head and the length of the flow. Subscript i refer to the direction of the flow. Substituting the terminal velocity and the true velocity into equation 7 the stability condition of a grain can be defined as:

$$\frac{(\rho_s - \rho_w)g}{36\mu} d^2 > \frac{v_{Dz}}{n} = \frac{k_z i_z}{n} \tag{12}$$

At a given temperature both the density and the viscosity of the water is constant. In homogeneous soil the intrinsic permeability and the void ratio or the porosity is also constant. Thus the un-equilibrium condition in Eq. 12 can be simplified as:

$$i_z < c d^2 , \tag{13}$$

where c is a constant, given as:

$$c = \frac{n(\rho_s - \rho_w)g}{36 k_z \mu}. \tag{14}$$

The size of the grain losing its stability at a given hydraulic gradient can be calculated as:

$$d = \sqrt{\frac{i_z}{c}}. \tag{15}$$

It can be concluded from equations 12-15 that in a given soil and water temperature the stability of a grain depends not only on the hydraulic gradient but also on the grain size.

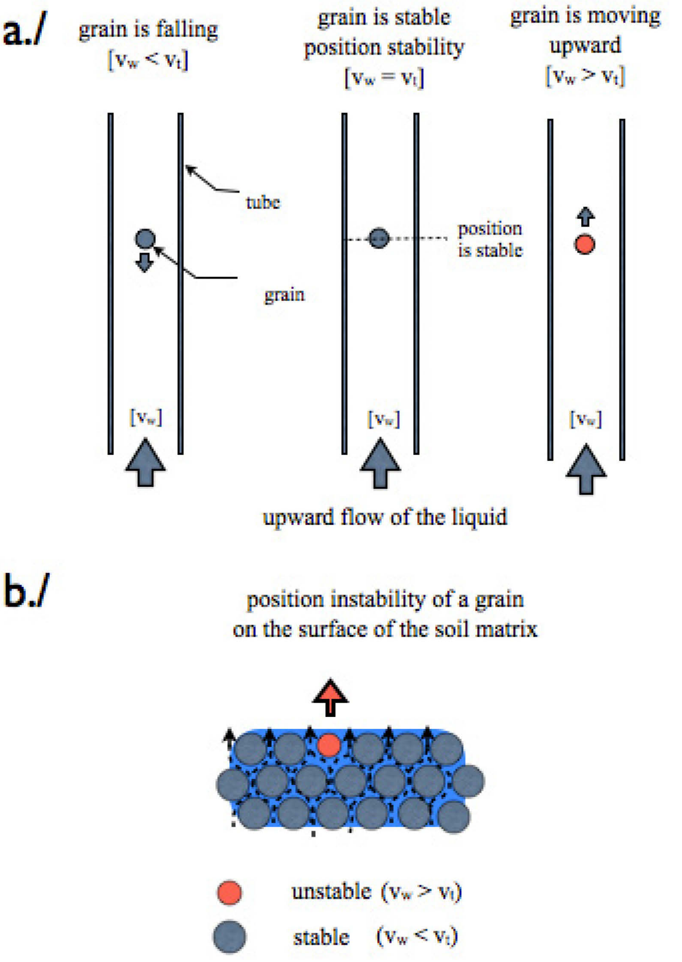

Figure 2 Stability investigation of a single grain.
(a) Spherical grain in an upward vertical flow.
(b) Developing position instability of a grain on the surface of a soil matrix in non-cohesive soil.

Therefore, the hydraulic gradient by itself as given in criterion #3 is not sufficient to define the stability of one particular grain in the soil matrix. Thus the “global” equilibrium conditions (#1-3) used to define the stability of a soil matrix in an upward flow is not relevant and applicable to a single grain.

4 PIPING AND SAND BOILING

The diameter of a grain, which losing its stability at the critical hydraulic gradient, can be calculated as:

$$d_c = \sqrt{\frac{i_c}{c}} = \sqrt{\frac{\gamma'}{c\gamma_w}}. \qquad (16)$$

where d_c is the critical diameter of a grain, which looses its stability at the critical hydraulic gradient.

Grains with diameter of d_c or bigger will lose their stability when the hydraulic gradient reaches the critical value. Grains with smaller diameter than d_c lose their stability at lower hydraulic gradient than i_c. Thus for soil matrix, in which the size of the grains exceeds the size of d_c, the “global” equilibrium condition is valid and the criteria (#1-3) or Eqs.1-6 can be used for the stability investigation. On the other hand the conventional or Terzaghi criterion is not valid or not applicable to soil particles which diameter is smaller than d_c. Thus soil matrix containing grain size smaller than the critical diameter can lose its stability at lower hydraulic gradient than the critical one. The critical diameter depends on the permeability coefficient of the soil (Eqs. 12; 16). The relationship between these parameters is shown on Figure 3.

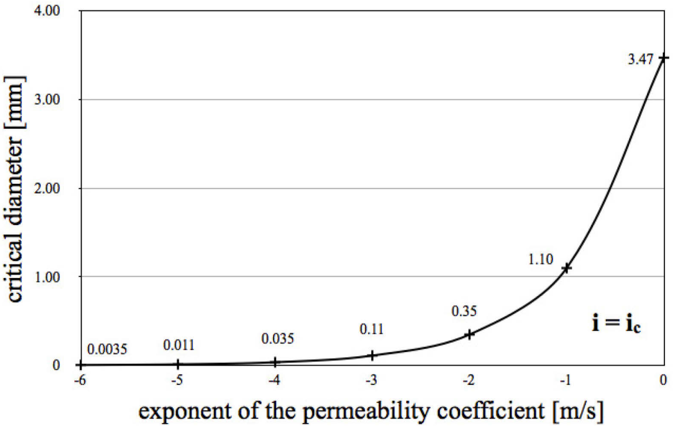

Figure 3 The relationship between permeability and the critical diameter of a soil grain at the critical hydraulic gradient. The exponents of the permeability coefficient are plotted against the critical diameters. In the calculations it is assumed that the water temperature is 15 C. The density of the water is then 9.991286 KN/m^3 and the viscosity is 1.1382x10^{-3} Pas (Korson et al., 1969).

The removal of the first grain from the surface increases both the hydraulic gradient and the permeability of the soil, which could lead to progressive hydraulic failure and eventually to piping and sand boiling (Beek, 2010). It is suggested that this progressive hydraulic failing is the physical process which explains why piping and sand boiling do occurs at lower gradients than the critical one.

5 CONCLUSIONS

Terzaghi heave criterion is derived from the global equilibrium of the soil column from first principles. The initiation of sand boiling or piping starts with the removal of single grain. Thus the equilibrium conditions of a single grain should be investigated instead of the entire soil prism.

The equilibrium of a grain on the top of the soil matrix in a non cohesive soil is lost when the velocity of the upward flow exceeds the terminal velocity of the grain. Assuming that the grains are spherical and employing Stokes and Darcy’s laws the relevant equation can be derived from first principles.

The stability of a grain situating on the top of a given soil matrix depends on the hydraulic gradient of the flow and on the diameter of the grain. Thus the hydraulic gradient by itself is not sufficient to define the stability criterion for a single grain.

Critical diameter, corresponding to grain size losing its stability at the critical hydraulic gradient, is introduced. This diameter separates the grains into two parts. The grains, which size exceeds the critical diameter lose their stability at the critical hydraulic gradient. Thus the “global” equilibrium condition (Terzaghi criterion) is applicable to soil matrix containing grains which diameters are bigger than the critical one. Grains, with smaller size than the critical diameter, lose their stability at lower hydraulic gradient than the critical one. Thus for these grains the “local” equilibrium conditions should be applied.

The removal of a grain from the top of the soil matrix increases both the hydraulic gradient and the permeability of the layer leading to a progressive destabilization of the entire soil matrix. This physical process can explain why hydraulic failure, known as piping and sand boiling, occurs at lower hydraulic gradients than the critical ones.

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