Scour below marine pipelines due to random waves on mild slopes

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ABSTRACT: This paper provides a practical stochastic method by which the maximum equilibrium scour depth beneath a marine pipeline exposed to random waves on mild slopes can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes including the effect of breaking waves, and using the empirical formulas for the scour depth on the horizontal seabed by Sumer and Fredsøe (1996). Results will be presented and discussed by varying the seabed slope and water depth. An approximate method is also proposed, and comparisons are made with the present stochastic method. Generally, it appears that the approximate method can replace the stochastic method.

1 INTRODUCTION

The present work addresses the scour below a pipeline on a mild-sloped seabed due to random waves alone. In general, for current plus waves, a pipeline resting on the seabed is situated within the boundary layer of the flow close to the bed. In deep water, the flow can be considered as steady, while in shallow and intermediate water depths, there is commonly combined wave-current flow. A typical design condition for a pipeline in the vicinity of the seafloor in, e.g., the North Sea is that the flow is wave-dominated and that the seabed consists of fine sand. When a scour hole develops, this may have considerable effect on the dynamic behaviour and the on-bottom stability of the pipeline. After installation, for example, on a plane or sloped seabed consisting of fine sand, it may experience different seabed conditions, e.g., the seabed may be flat or rippled. This is mainly due to the complicated flow generated by the interaction between the incoming flow, the pipeline, and the seabed. The result will depend on the incoming flow velocity, the geometry of the bed and the bed material, as well as on the ratio between the near-bed oscillatory fluid particle excursion amplitude and the pipeline diameter. Additional details on the background and complexity as well as reviews of the problem are given in, e.g., Whitehouse (1998) and Sumer and Fredsøe (2002). Myrhaug and Ong (2011a) gave a review of the authors' studies on two-dimensional (2D) random wave-induced equilibrium scour characteristics around marine structures including comparison with data from random wave-induced scour experiments. Recently the authors have also provided practical stochastic methods for calculating the maximum equilibrium scour depth around vertical piles (Myrhaug and Ong, 2013 a, b; Ong et al., 2013) and below pipelines (Myrhaug and Ong, 2011b) due to 2D and three-dimensional (3D) nonlinear random waves. To our knowledge, no studies are available in the open literature dealing with random wave-induced scour below pipelines on mild slopes.

The purpose of this study is to provide an engineering approach by which the maximum equilibrium scour depth below a pipeline exposed to random waves alone on mild slopes can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Battjes and Groenendijk (2000) wave height distribution for mild slopes including the effect of breaking waves, and using the empirical formulas for the scour depth by Sumer and Fredsøe (1996). Results are presented and discussed by varying the seabed slope and water depth. An approximate method is proposed and compared with the present stochastic method.

2 SCOUR IN REGULAR WAVES

The 2D scour below a fixed pipeline on a horizontal seabed in regular waves was investigated in laboratory tests by Sumer and Fredsøe (1990). They obtained the following empirical formula for the equilibrium scour depth S below the pipeline with diameter, D (see Fig. 1)

$$\frac{S}{D} = 0.1 KC^{0.5} \qquad (1)$$

where the Keulegan-Carpenter number KC is defined as

$$KC = \frac{UT}{D} \qquad (2)$$

Here U is the undisturbed linear near-bed orbital velocity amplitude, T is the wave period, and Eq. (1) is based on data for which $2 \le KC \le 1000$.

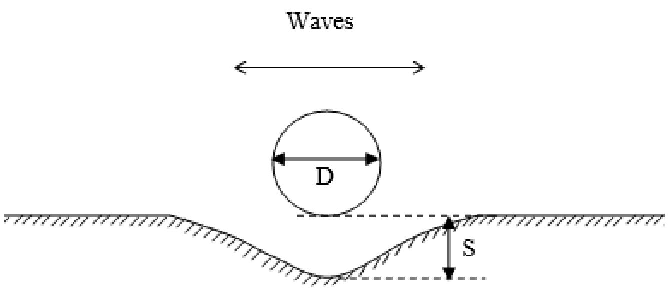

Fig. 1 Definition sketch of the scour depths (S) below a pipeline with diameter (D) on a horizontal bed.

Eqs. (1) and (2) are valid for live-bed scour, for which $\theta > \theta_{cr}$ where θ is the undisturbed Shields parameter defined by

$$\theta = \frac{\tau_w}{\rho g(s-1)d_{50}} \qquad (3)$$

where τ_w is the maximum bottom shear stress under the waves, ρ is the density of the fluid, g is the acceleration due to gravity, s is the sediment density to fluid density ratio, d_{50} is the median grain size diameter, and θ_{cr} is the critical value of the Shields parameter corresponding to the initiation of motion at the bed, i.e. $\theta_{cr} \approx 0.05$ for sand. One should note that the scour process attains its equilibrium stage through a transition period. Thus, the approach is valid when it is assumed that the storm generating random waves has lasted longer than the time-scale of the scour. It should be noted that this is the case for a rigid pipeline in a 2D problem, which will be handled in the present study. However, in reality, after this length of time the pipeline may have lowered into the scour hole, altering the scour depth and potentially leading to sedimentation. Further details on the time-scale of the scour are given in Sumer and Fredsøe (1996).

The maximum bottom shear stress within a wave cycle is taken as

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \qquad (4)$$

where f_w is the friction factor, which here is taken from Myrhaug et al. (2001), and is valid for waves plus current for wave-dominated situations (see Myrhaug et al. (2001), Table 3)

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \qquad (5)$$

$$(c,d) = (18, 1) \text{ for } 20 \le A/z_0 < 200 \qquad (6)$$

$$(c,d) = (1.39, 0.52) \text{ for } 200 \le A/z_0 < 11000 \qquad (7)$$

$$(c,d) = (0.112, 0.25) \text{ for } 11000 \le A/z_0 \qquad (8)$$

where $A = U/\omega$ is the near-bed orbital displacement amplitude, $\omega = 2\pi/T$ is the angular wave frequency, and $z_0 = d_{50}/12$ is the bed roughness (see e.g. Soulsby (1997)). The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically.

It should be noted that the KC number can alternatively be expressed as

$$KC = \frac{2\pi A}{D} \qquad (9)$$

Moreover, A is related to the linear wave amplitude a by

$$A = \frac{a}{\sinh kh} \qquad (10)$$

where h is the water depth, and k is the wave number determined from the dispersion relationship $\omega^2 = gk \tanh kh$.

It should be noted that since Eq. (1) appears to be physically sound for $KC = 0$, i.e. S equals zero for $KC = 0$, the formula can be taken to be valid from $KC = 0$. This extension of Eq. (1) relies on the threshold of motion near to the pipeline being exceeded, which for small values of KC may not be the case.

3 SCOUR IN RANDOM WAVES

Here a tentative stochastic approach will be outlined following the approach presented in Myrhaug et al. (2009) and Myrhaug and Ong (2011b), except for the modification performed by adopting the Battjes and Groenendijk (2000) wave height distribution. As a first approximation, it is assumed that the scour formulas for the case of a horizontal bed described in Section 2 can be applied for the case of mild slopes as well. Figure 2 shows the definition sketch of the scour below a pipeline on a mild slope.

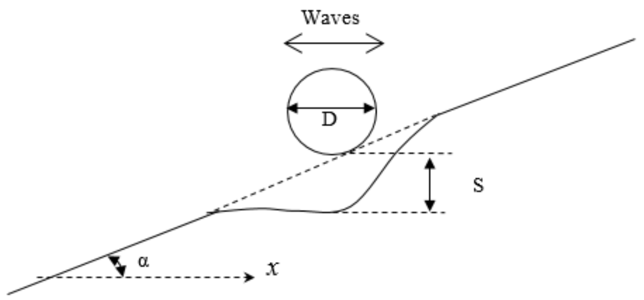

Fig. 2 Definition sketch of the scour depth (*S*) around a circular vertical pile with diameter (*D*) on a mild slope (*α*).

3.1 *Theoretical background*

At a fixed point in a sea state with stationary narrow-band random waves consistent with regular linear waves in finite water depth h and wave height H = $2a$, the near-bed orbital displacement amplitude, A, and the near-bed horizontal orbital velocity amplitude, U, can be taken as, respectively,

$$A = \frac{H}{2\sinh k_p h} \tag{11}$$

$$U = \omega_p A = \frac{\omega_p H}{2\sinh k_p h} \tag{12}$$

where $\omega_p = 2\pi / T_p$ is the spectral peak frequency, T_p is the spectral peak period, and k_p is the wave number corresponding to ω_p determined from the dispersion relationship

$$\omega_p^2 = g k_p \tanh k_p h \tag{13}$$

Moreover, A and U are made dimensionless by taking $\hat{A} = A / A_{rms}$ and $\hat{U} = U / U_{rms}$, respectively, where

$$A_{rms} = \frac{H_{rms}}{2\sinh k_p h} \tag{14}$$

$$U_{rms} = \omega_p A_{rms} = \frac{\omega_p H_{rms}}{2\sinh k_p h} \tag{15}$$

and H_{rms} is the root-mean-square (rms) value of H. The definition of U_{rms} in Eq. (15) corresponds to U_m defined in Sumer and Fredsøe (2002) Eq. (2.25). By combining Eqs. (11), (12), (14) and (15) it follows that

$$\omega_p = \frac{U}{A} = \frac{U_{rms}}{A_{rms}} \tag{16}$$

and consequently

$$\frac{U}{U_{rms}} = \frac{A}{A_{rms}} = \frac{H}{H_{rms}} \tag{17}$$

Here the Battjes and Groenendijk (2000) parametric wave height distribution based on laboratory experiments on shallow foreshores is adopted. This cumulative distribution function (*cdf*) is composed of two two-parameter Weibull distributions of the non-dimensional wave height $\hat{H} = H / H_{rms}$:

$$P(\hat{H}) = \begin{cases} P_1(\hat{H}) = 1 - \exp[-(\dfrac{\hat{H}}{\hat{H}_1})^{k_1}] \; ; \; \hat{H} < \hat{H}_{tr} \\ P_2(\hat{H}) = 1 - \exp[-(\dfrac{\hat{H}}{\hat{H}_2})^{k_2}] \; ; \; \hat{H} \geq \hat{H}_{tr} \end{cases} \tag{18}$$

where $k_1 = 2$, $k_2 = 3.6$, $\hat{H}_1 = H_1 / H_{rms}$, $\hat{H}_2 = H_2 / H_{rms}$, $\hat{H}_{tr} = H_{tr} / H_{rms}$. Here H_{tr} is the transitional wave height corresponding to the change of wave height where there is a change of the distribution associated with depth-induced wave breaking, given by

$$H_{tr} = (0.35 + 5.8 \tan\alpha) h \tag{19}$$

where H_{rms} is related to the zeroth spectral moment m_0 by

$$H_{rms} = (2.69 + 3.24\sqrt{m_0} / h)\sqrt{m_0} \tag{20}$$

The values of H_1 and H_2 can either be read from Table 2 in Battjes and Groenendijk (2000), or they can be solved by an iteration procedure solving two equations (see Eqs. (25) and (26)). The model is a so-called point model, i.e. depending on local parameters regardless of the history of the waves in deeper water. It should be noted that the effect of the bottom slope is of a secondary nature compared to the effect of water depth (see Battjes and Groenendijk (2000) for more details).

The zeroth spectral moment, m_0, is obtained as

$$m_0 = \int_0^\infty S(\omega, h) d\omega \tag{21}$$

where $S(\omega,h)$ is the wave spectrum in finite water depth, which can be obtained by multiplying the deep water wave spectrum $S(\omega)$ with a depth correction factor $\psi(\omega,h)$ as

$$S(\omega,h) = \psi(\omega,h) S(\omega) \tag{22}$$

where according to Young (1999)

$$\psi(\omega,h) = \frac{[k(\omega,h)]^{-3} \partial k(\omega,h)/\partial\omega}{\{[k(\omega,h)]^{-3} \partial k(\omega,h)/\partial\omega\}_{kh\to\infty}} \tag{23}$$

ensuring that the frequency part of the wave spectrum becomes proportional to k^{-3} irrespectively of the water depth (see Young (1999) for more details). From Eq. (23) it follows that (see Jensen, 2002)

$$\psi(\omega,h) = \frac{\omega^6}{(gk)^3[\tanh kh + kh(1-\tanh^2 kh)]} \tag{24}$$

For given h, α, and m_0, the values of $\hat{H}_1$ and $\hat{H}_2$ can be either read from Table 2 in Battjes and Groenendijk (2000), or they can be determined by solving the following two equations:

The distribution function has to be continuous, i.e.

$$P_1(\hat{H}) = P_2(\hat{H}) \tag{25}$$

The mean square normalized wave height, or the second moment of the probability density function (*pdf*) of $\hat{H}$, has to equal unity, i.e.,

$$\hat{H}_{rms}^2 = \int_0^{\hat{H}_{tr}} \hat{H}^2 p_1(\hat{H}) d\hat{H} + \int_{\hat{H}_{tr}}^\infty \hat{H}^2 p_2(\hat{H}) d\hat{H} = 1 \tag{26}$$

where P_1 and P_2 are the *pdfs* of $\hat{H}$ and defined as $p_1 = dP_1/d\hat{H}$ and $p_2 = dP_2/d\hat{H}$, respectively.

3.2 *Outline of stochastic method*

The highest among random waves in a stationary narrow-band sea-state is considered, as it is reasonable to assume that it is mainly the highest waves which are responsible for the scour response. It is also assumed that the sea state has lasted long enough to develop the equilibrium scour depth. The highest waves considered here are those exceeding the probability $1/n$, $\hat{H}_{1/n}$ (i.e., $1 - P(\hat{H}_{1/n}) = 1/n$). The parameter of interest is the expected (mean) value of the maximum equilibrium scour characteristics caused by the $(1/n)^{\text{th}}$ highest waves, which is given as

$$E[S(\hat{H}) \mid \hat{H} > \hat{H}_{1/n}] = n \int_{\hat{H}_{1/n}}^\infty S(\hat{H}) p(\hat{H}) d\hat{H} \tag{27}$$

where $S(\hat{H})$ represents the scour characteristics, and $p(\hat{H})$ is the *pdf* of $\hat{H}$. More specifically, the present approach is based on the following assumptions: (1) the free surface elevation is a stationary narrow-band process with zero expectation, and (2) the scour response formula for regular waves in the previous section (see Eq. (1)), is valid for irregular waves as well. These assumptions are essentially the same as those given in e.g., Myrhaug et al. (2009), where further details are found.

For a narrow-band process $T = T_p$ where $T_p = 2\pi/\omega_p = 2\pi A_{rms}/U_{rms}$ and $k = k_p$. Then by referring to Eq. (17) it follows that

$$\hat{U} = \frac{U}{U_{rms}} = \frac{A}{A_{rms}} = \frac{H}{H_{rms}} = \hat{H} \tag{28}$$

By substituting Eq. (28) in Eq. (1), Eq. (1) can be re-arranged to

$$\hat{S} \equiv \frac{S}{D} = 0.1\left(KC_{rms}\hat{H}\right)^{0.5} \tag{29}$$

Let S denote $\hat{S}$ given in Eqs. (29) and $KC_{rms} = U_{rms}T_p/D = 2\pi A_{rms}/D$. Then the mean of the maximum equilibrium scour depth caused by the $(1/n)^{\text{th}}$ highest waves follows from Eq. (27) as (i.e. by neglecting any memory effects in the scouring process from the previous waves)

$$E\left[\hat{S}\left(\hat{H}\right)\middle|\hat{H} > \hat{H}_{1/n}\right] = n \int_{\hat{H}_{1/n}}^\infty \hat{S}\left(\hat{H}\right) p\left(\hat{H}\right) d\hat{H} \tag{30}$$

where the *cdf* of $\hat{H}$ is given in Eq. (18), and $p\left(\hat{H}\right)$ is the *pdf* of $\hat{H}$, i.e., $p_1 = dP_1/d\hat{H}$ and $p_2 = dP_2/d\hat{H}$, given as follows:

$$p_1(\hat{H}) = \frac{k_1}{\hat{H}}(\frac{\hat{H}}{\hat{H}_1})^{k_1} \exp[-(\frac{\hat{H}}{\hat{H}_1})^{k_1}] \; ; \; \hat{H} < \hat{H}_{tr} \tag{31}$$

$$p_2(\hat{H}) = \frac{k_2}{\hat{H}}(\frac{\hat{H}}{\hat{H}_1})^{k_2} \exp[-(\frac{\hat{H}}{\hat{H}_2})^{k_2}] \; ; \; \hat{H} \geq \hat{H}_{tr} \tag{32}$$

Moreover, $\hat{H}_{1/n}$ is obtained by solving the equation $1 - P(\hat{H}_{1/n}) = 1/n$.

4 RESULTS AND DISCUSSION

To the authors' knowledge no data exist in the open literature for random wave-induced scour below pipelines on mild slopes. It should be noted that the formulation in Section 3 is general, i.e., valid for a finite water depth.

4.1 *Prediction of parameters*

In the present study, the effect of mild slopes on scour below pipelines in random waves alone is investigated. Four bed slopes α = 1/50, 1/100, 1/150 and 1/250, are considered for this purpose.

The case with the bed slope α = 1/100 is exemplified to show the procedure of calculating all the required parameters. Figure 3 shows the seabed conditions with α =1/100. The water depth at the seaward location (x = 0 m) is 15 m; the horizontal length of the sloping seabed is 600 m; the diameter of the pipeline D is set to be 1m for all the cases.

The wave spectrum in finite water depth *S(ω, h)* can be obtained from the spectrum in deep water *S(ω)*, see Eq.(22). Hence, the random waves with a standard JONSWAP spectrum (γ = 3.3) and significant wave height H_{m0} = 8 m are assumed to describe the sea state in deep water. Figure 4 shows some results of the wave spectra at the four locations transformed from the deep water according to Eqs. (21) - (24). It is clearly seen in Fig. 4 that the wave energy decreases as the water depth decreases. Consequently, Fig. 5 shows that m_0 (Eq. (21)) decreases as the water depth decreases. With the values of m_0 along x, H_{rms} can be determined by Eq. (26) and subsequently KC_{rms} can be computed.

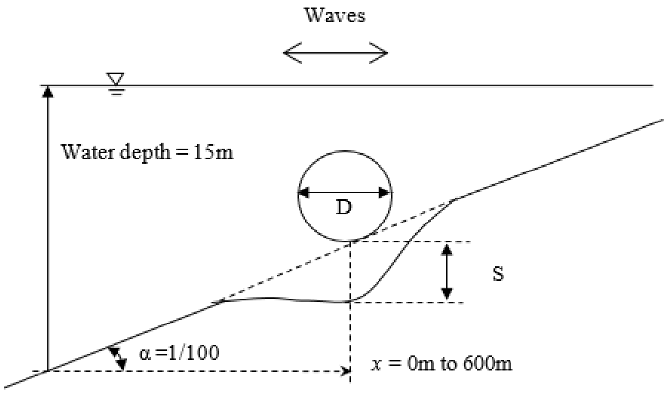

Fig. 3 Definition sketch of the seabed conditions with $\alpha = 1/100$.

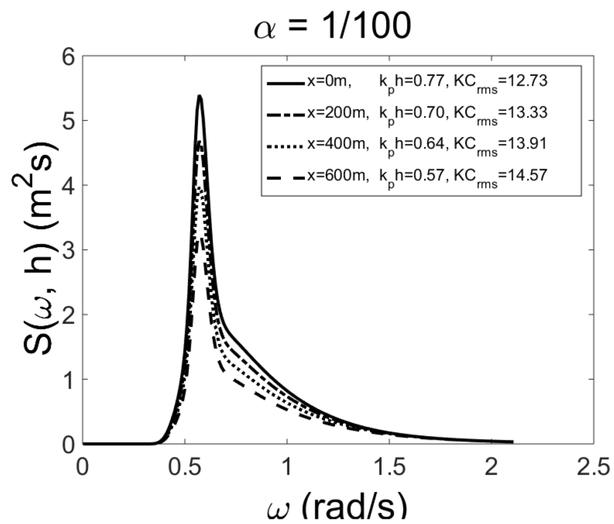

Fig. 4 The transitional wave spectra in finite water depth $S(\omega, h)$ versus ω at four locations for α = 1/100.

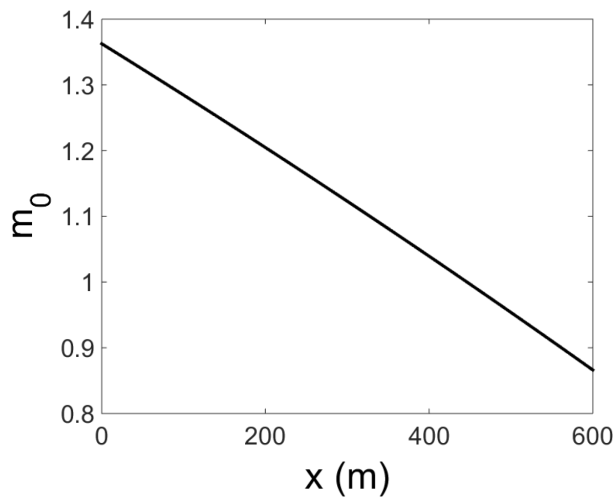

Fig. 5 Zeroth spectral moment m_0 versus x in finite water depth for α = 1/100.

4.2 *Random waves alone*

The *pdf* and *cdf* of the Battjes and Groenendijk (2000) wave height distribution of $\hat{H}$ at the four locations are shown in Figs. 6 and 7, respectively. The discontinuous points in Fig. 6 are due to the transitional wave height $\hat{H}_{tr}$, representing the limiting wave height for non-breaking waves. The figure shows that from location 1 to location 4 (as x increasing from 0 m to 600 m), $\hat{H}_{tr}$ decreases from 1.78 to 1.30, reflecting that the influence of breaking waves on the distributions becomes more significant as the water depth decreases. It should be noted that the area under each *pdf* curve must be equal to one, and this is validated in Fig. 7where the *cdf* curves are shown.

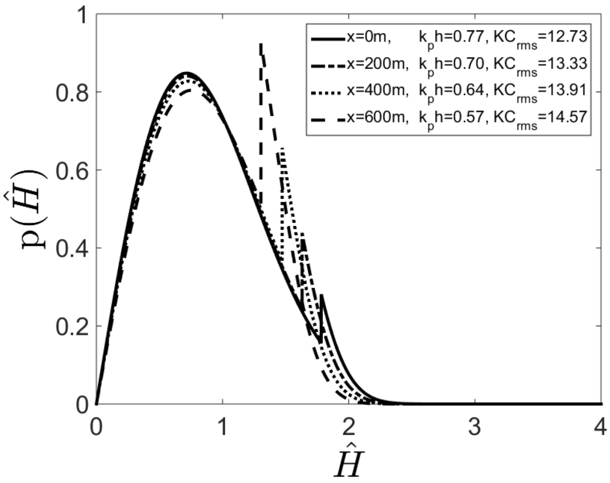

Fig. 6 $p\left(\hat{H}\right)$ at four locations for $\alpha = 1/100$.

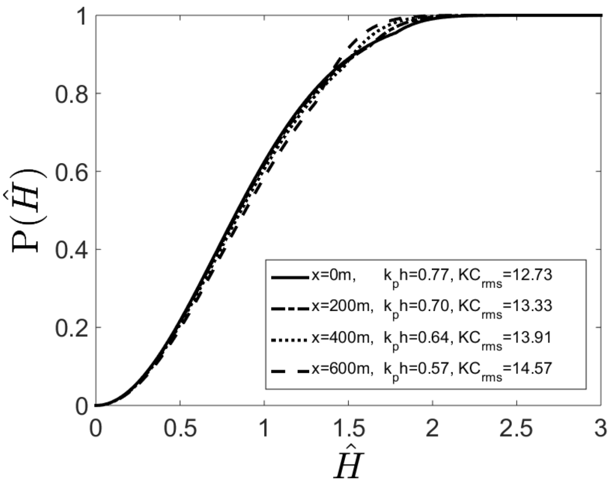

Fig. 7 $P\left(\hat{H}\right)$ at four locations for $\alpha = 1/100$.

Four different bed slopes (α = 1/50, 1/100, 1/150, 1/250) are considered in the present study. Figure 8 shows KC_{rms} versus x for the four slopes; KC_{rms} increases as the water depth decreases for all the slopes. Furthermore, it appears that KC_{rms} increases as the slope increases at a given location x.

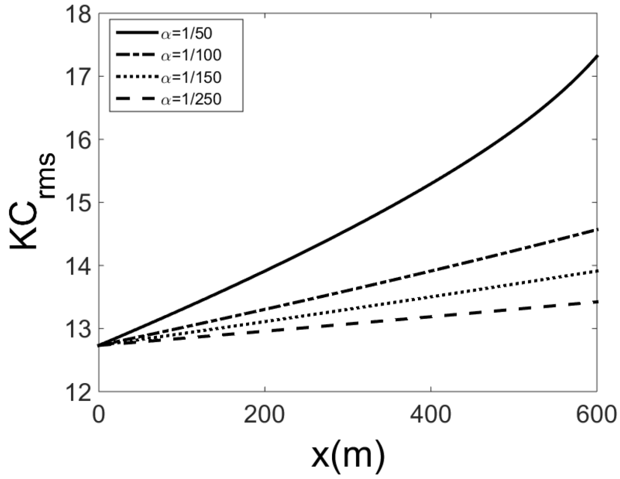

Fig. 8 KC_{rms} versus x for α = 1/50, 1/100, 1/150, 1/250.

Figure 9 shows $S/D_{1/10}$ for the different slopes. The reason for choosing n = 10 is based on earlier comparison between the stochastic method (Myrhaug et al. (2009)) and the corresponding experimental data for random wave-induced scour around pipelines reported by Sumer and Fredsøe (1996), where the stochastic method predictions for n = 10 overall give upper bound values compared with experimental data. Deeper scour hole is encountered when the slope becomes steeper. For all slopes, $S/D_{1/10}$ increases as x approaches to 600 m. At a given location x, it appears that $S/D_{1/10}$ increases as the slope increases. These results are physically sound and consistent with those observed in Fig. 10. It should be noted that the effect of water depth is included by changing the location x.

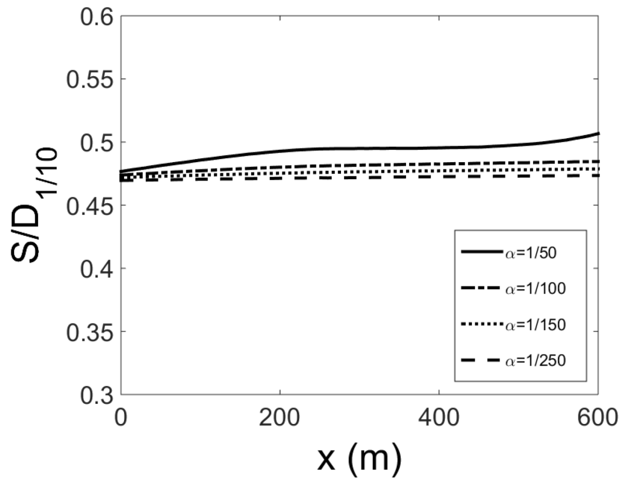

Fig. 9 $S/D_{1/10}$ versus x for α = 1/50, 1/100, 1/150, 1/250.

4.3 *Alternative view: Approximate method*

An alternative pragmatic view of the scour process below pipelines and around a single vertical pile under random waves is that of Sumer and Fredsøe (1996, 2001). They looked for which parameters of the random waves to represent the scour variable, finding by trial and error that the use of H_{rms} and T_p in an otherwise deterministic approach gave the best agreement with data.

This alternative view of the scour process will now be considered using the results of the present stochastic method. The question is how well the mean scour depth caused by the $(1/n)^{\text{th}}$ highest waves, $E\left[S(H)\,|\,H > H_{1/n}\right]$ (see Eq. (30)), can be represented by using the mean of the $(1/n)^{\text{th}}$ highest waves in the scour depth formula for regular waves, i.e. $S\left(E\left[H_{1/n}\right]\right)$.

An alternative *KC* number for random waves in the approximate method can be defined as

$$KC_{1/n} = \frac{E\left[U_{1/n}\right]T_p}{D} = \frac{2\pi E\left[A_{1/n}\right]}{D} \tag{33}$$

Based on the narrow-band assumption, $E\left[U_{1/n}\right]$ and $E\left[A_{1/n}\right]$ can be defined as

$$E\left[A_{1/n}\right] = \frac{E\left[H_{1/n}\right]}{2\sinh k_p h} \tag{34}$$

$$E\left[U_{1/n}\right] = \omega_p E\left[A_{1/n}\right] = \frac{\omega_p E\left[H_{1/n}\right]}{2\sinh k_p h} \tag{35}$$

where $E\left[A_{1/n}\right]$, $E\left[U_{1/n}\right]$ and $E\left[H_{1/n}\right]$ are the mean values of the $(1/n)^{\text{th}}$ largest values of the near-bed orbital displacement amplitude, velocity and wave height, respectively.

The scour depth below a pipeline for random waves alone can be obtained by replacing *KC* with $KC_{1/n}$ in Eq. (1), given by

$$\frac{S}{D} = 0.1 KC_{1/n}^{0.5} \tag{36}$$

The results of the stochastic to approximate method ratio of the scour depth for the four slopes are shown in Fig. 10, denoted by $R_{1/10}$ for $n = 10$. It is interesting to note that the approximate method gives almost the same values as that of the stochastic method for all slopes. Thus, it appears that the approximate method can replace the stochastic method for random waves alone.

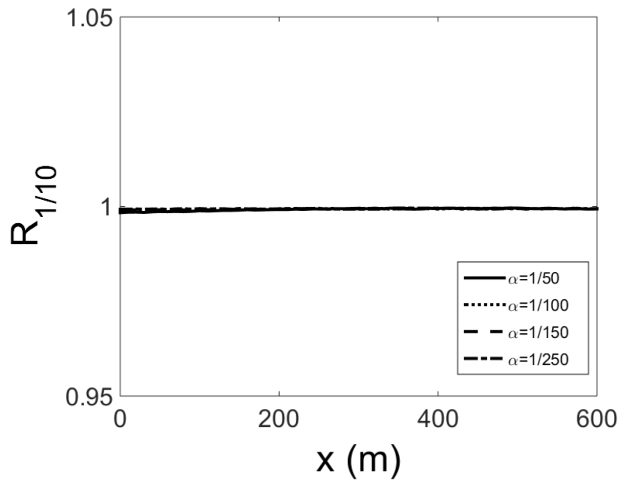

Fig. 10 Random waves alone: The stochastic to approximate method ratio $R_{1/10}$ versus x for four slopes α = 1/50, 1/100, 1/150, 1/250.

4.4 *Shields parameter*

As described in Section 2, the scour prediction model in Eq. (1) is valid for live-bed scour, for which $\theta > \theta_{cr}$, where θ is the undisturbed Shields parameter defined in Eq. (3).

When the bed is sloping, the gravity gives a force component on the grain which may increase or decrease the threshold shear stress required from the flow. The threshold Shields parameter, $\theta_{\alpha cr}$, for initiation of motion of the grains at a bed sloping at an angle α to the horizontal in upsloping flows is related to the value θ_{cr} for the same grains on a horizontal bed by (see e.g. Soulsby (1997, Section 6.4))

$$\frac{\theta_{\alpha cr}}{\theta_{cr}} = \cos\alpha\left(1 + \frac{\tan\alpha}{\tan\phi_i}\right) \tag{37}$$

where ϕ_i is the angle of repose of the sediment.

Following Myrhaug (1995) and Myrhaug and Holmedal (2002), the non-dimensional maximum Shields parameter for individual narrow-band random waves near a horizontal bed, $\hat{\theta} = \theta / \theta_{rms}$, is equal to the non-dimensional maximum bottom shear stress for individual narrow-band random waves, $\hat{\tau} = \tau_w / \tau_{wrms}$. Here θ_{rms} is defined as

$$\theta_{rms} = \frac{\tau_{wrms} / \rho}{g(s-1)d_{50}} \tag{38}$$

where, by definition

$$\frac{\tau_{wrms}}{\rho} = \frac{1}{2} c \left(\frac{A_{rms}}{z_0}\right)^{-d} U_{rms}^2 \tag{39}$$

and θ is defined in Eq. (3). By using this and following Myrhaug and Holmedal (2002, Eq. (21)), $\hat{\theta}$ is given as

$$\hat{\theta} = \hat{H}^{2-d} \tag{40}$$

For random waves it is not obvious which value of the Shields parameter to use to determine the conditions corresponding to live-bed scour. However, it seems to be consistent to use corresponding statistical values of the scour depth and the Shields parameter, e.g., given by

$$E\left[\hat{\theta}\left(\hat{H}\right)\middle|\hat{H} > \hat{H}_{1/n}\right] = n\Gamma\left(2-\frac{d}{2}, \ln n\right) \tag{41}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function (see Abramowitz and Stegun (1972, Ch. 6.5, Eq. (6.5.3)). This is used in conjunction with Eq. (37) when the bed is sloping.

5 SUMMARY AND CONCLUSIONS

A practical method for estimating the scour depth below pipelines exposed to random waves is provided. The main conclusions are:

1. The Battjes and Groenendijk (2000) wave height distribution for mild slopes is applied to describe the random wave condition on mild slopes including the effect of breaking waves. A method for transformation of the wave spectrum from deep water to finite water depth is presented. Then a method is derived for calculating the random wave-induced scour below a pipeline based on assuming the waves to be a stationary narrow-band random process.
2. The present results reveal that the effect of a mild slope increases the scour depth compared with that at the seaward location. Moreover, a larger bed slope causes more scour at a fixed location.
3. The results suggest that the approximate method can replace the stochastic method.

Although the methodology is simple, it should be useful as a first approximation to represent the stochastic properties of the scour depth around pipelines under random waves alone on mild slopes. However, comparisons with data are required before a conclusion regarding the validity of this method can be given. In the meantime the method should be useful as an engineering tool for the assessment of scour and in scour protection work of pipelines on mild slopes.

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