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NUMERICAL MODEL SIMULATION OF ENTRAINMENT OF SAND BED MATERIAL

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ABSTRACT

The study of sediment transport generally is very difficult but more so in the case of estuaries because:

- the water movements are continually changing with the rise and fall of the tide

- a wide range of sediment exists on the bed and in suspension

- certain sediments are not found in some parts leading to unsaturated loads in the water.

In recent years sediment transport models have been developed and used for making engineering assessments of the impact of works on the sediment regime. At present the full potential of the models cannot be realised because of the lack of calibration and verification data, and gaps in our understanding of the fundamental sedimentation processes.

The basic aim of this research was to improve the representation of sand transporting processes in computer models. It is relevant to the sand transport consequences of civil engineering works on the operation of ports and harbours and on environment aspects, and will ultimately benefit the industry by helping to minimise maintenance dredging of ports and navigation channels.

The first phase of the project, covered in this report, concentrates on finding the best numerical model representation of the exchange of sand between the bed and the flow, based on assessments of the available data and theoretical analyses. The report also contains descriptions of the fundamental physical processes affecting sand transport in estuaries and a brief review of existing numerical models of sand transport.

A sand transport model, based on theoretical and empirical relationships, has been tested to verify that it simulates the relevant physical processes. It was found that the most appropriate formulation for entrainment of sand at the bed was in terms of an entrainment rate. The connection between this entrainment rate and the associated sand transport law has been considered and it was concluded that strictly only one of these should be specified.

The model was compared with some flume data to test its response to a change in the sediment load. It was shown that the model simulation could be calibrated by adjusting the settling velocity and vertical diffusivity parameters.

The implications on the suspended load due to the unsteadiness in accelerating and decelerating flow and the numerical simulation of these effects will be studied in the second phase of the project. It is also intended to study the behaviour of mixtures of different sand sizes and consider how this might be represented in a computer model. These aspects will be described in a later report.

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1 INTRODUCTION

1.1 Estuarine sediments

An estuary is a partly enclosed body of tidal water where river water is mixed with and diluted by sea water. In a general sense the estuarine environment is defined by salinity boundaries rather than by geographical ones, but although the salinity has influence on the clay sediment fractions it is the currents generated by the tidal volume flowing in and out of the estuary which dominate the movement and distribution of sediments. The sediments themselves may have originated from natural erosion inland or from seawards. They consist of materials ranging from the finest clay particles to coarse sand and gravels. A convenient classification of sediments uses a geometric scale of sizes.

	mm	phi units
Very coarse sand	1.0 - 2.0	- 1
Coarse sand	0.5 - 1.0	0
Medium sand	0.25 - 0.5	1
Fine sand	0.125 - 0.25	2
Very fine sand	0.064 - 0.125	3
Coarse silt	0.032 - 0.064	4
Medium silt	0.016 - 0.032	5
Fine silt	0.008 - 0.016	6
Very fine silt	0.004 - 0.008	7
Coarse clay	0.002 - 0.004	8
Medium clay	0.001 - 0.002	9

TABLE 1 SEDIMENT GRADINGS

A significant feature of estuaries is the wide range of sediment sizes found in them. These sediments are sifted and sorted by the tidal currents.

In the main channels bed stresses are usually too high to allow the finer materials to accumulate although they may settle temporarily at slack water. Only coarse sand and gravel can exist as permanent deposits in these high energy regions. Along the shallow margins of the estuary, and further upstream, the tidal currents are too weak to move the sand and either no sand is transported there or it is covered by silt or clay to produce characteristic mud flats. These mud flats are colonised by various forms of marine life and become the feeding grounds of birds. If conditions are suitable the level of the mud flats rises and eventually a salt marsh develops.

1.2 Research Objectives

The study of sediment transport generally is very difficult but more so in the case of estuaries because

- the water movements are continually changing with the rise of the tide
- a wide range of sediments exist on the bed and in suspension
- certain sediments are not found in some parts leading to unsaturated loads in the water

In recent years sediment transport models have been developed and used for making engineering assessments of the impact of works on the sediment regime. At present the full potential of the models cannot be realised because of the lack of calibration and verification data, and gaps in our understanding of the fundamental sedimentation processes.

The basic aim of this research was to improve the representation of sand transporting processes in computer models which involved some work to gain a better understanding of the underlying physics.

The first phase of the project, covered in this report, concentrates on finding the best numerical model representation of the exchange of sand between the bed and the flow, based on assessments of the available data and theoretical analyses. The report also contains descriptions of the fundamental physical processes affecting sand transport in estuaries and a brief review of existing numerical models of sand transport.

The implications on the suspended load due to the unsteadiness in accelerating and decelerating flow and the numerical simulation of these effects will be studied in the second phase of the project. It is also intended to study the behaviour of mixtures of different sand sizes and consider how this might be represented in a computer model. These aspects will be described in a later report.

2 POTENTIAL LOAD MODELS

The simplest type of sediment transport model is essentially a single equation representing conservation of bed material.

$$\frac{\partial M}{\partial t} + \frac{\partial T_s}{\partial x} = 0 \qquad (1)$$

where M (kg/m^2) is the quantity of sediment on the bed and T_s (kg/sec/m width) is a prescribed sand transport formula. The basic assumption for this type of model is that the flow is saturated with sediment, which means that the flow is carrying the maximum sand transport that can be maintained for the given hydraulic and sedimentary conditions. Under saturated conditions the transport can be calculated from one of the many sediment transport laws to be found in the literature. An appraisal of available methods is given by van Rijn (1984). The flow parameters (water depth, mean velocity and shear velocity) required for the transport calculation could be obtained from measurements in a physical model but it is usually quicker and cheaper to generate this data on a regular grid from a separate numerical model of water movements.

The sediment carrying capacity of flow increases significantly for high water velocities - typically in proportion to the fourth power. This means that the flow will tend to pick up material from the bed when it accelerates and to deposit excess material when it decelerates. If the flow is always saturated with sediment the difference in transporting capacity must define the quantity of material picked up or deposited on the bed. This is the basis for the potential load model.

The potential load model is naturally most suited to situations where the bed material is narrowly graded and where there is an adequate supply of erodible material on the bed to maintain the saturated load. These conditions are more often met in rivers and it is in such situations that potential load models have been found most successful. See for example Cunge and Perdreau (1973), Thomas and Prasuhn (1977) and Bettess and White (1979).

Lepetit and Haguel (1978) extended the modelling approach used in river studies, to simulate 2-dimensional local scour round a jetty in a steady flow. The model is quasi-steady and uses a perturbation technique to feed the changes in depth back into the flow. Transport is calculated from a saturated bed load sediment law and bed changes calculated from the 2-dimensional form of equation 1, for conservation of sediment. The model results were shown to agree qualitatively with scour patterns measured in a mobile bed physical model.

The previously mentioned models have the common feature that the water flow is either steady or varying very slowly. Under those circumstances the potential load model can be applied to total (bed plus suspended) loads. In estuaries, where lag effects are more important, the potential load modelling approach is only appropriate, if at all, to medium and coarse sands which move mainly as a bed load and respond relatively quickly to the changing flow onditions. Odd, et al (1976) describe an application of this type to the River Great Ouse. Using a model of the river the evolution of bed level profiles over a period of two years were reproduced and predictions were made of the changes which would occur following the construction of a tidal barrage and/or extracting fresh water. A second application to a 2-dimensional area in the south east corner of the Wash proved more difficult because there was not enough data to calibrate the model and there was more variability in sediments on the bed. Nevertheless, it was still possible to identify the probable trends of changes for the engineers to use in their assessment of the various reservoir proposals.

Crotogino & Hotz (1984) describe a similar 2-dimensional model study of the Jade Estuary, West Germany. Model results were compared with regions observed to have accreted by over 1m between 1974 and 1976. A fairly good qualitative agreement was obtained but there were some unexplained discrepancies.

Chaloin et al (1985) had similar experiences with a potential sand transport model during a study of morphological evolution in the River Canche estuary. Some qualitative agreement was found with observed evolutions over the previous year and predictions of likely trends were made for the following five years with and without projects. However, the authors conclude that applications are limited to cases where the balance between currents and topography is markedly upset.

3 SUSPENDED SAND TRANSPORT MODELS

Although potential load models have been used successfully in some applications they only have limited value in estuaries where there is not a continuous supply of erodible material on the bed. The reason is that this sort of model cannot take into account how much sediment is actually being carried by the flow.

The consequences of this are:

1. erosion may not in fact occur in a region of potential erosion identified by the model if there is no erodible material on the bed;

2. deposition may not in fact occur in a region of potential deposition identified by the model if

the actual sediment load of the approaching water is insufficient to saturate even the slower flow;

3. erosion may in fact occur in an area of potential deposition if the sediment load of the approaching flow is very low for example after flowing over an area of rock bed.

In order to overcome these limitations a different sort of model is required based on conservation principles which simulates the sediment transport in terms of a suspended solids concentration. The erosion or deposition of material on the bed can then be assumed in the model depending on whether the actual load is less or greater than the saturated load which would obtain under steady, uniform flow conditions at the same values as the instantaneous flow. Under these circumstances the suspended solids concentration, c, (kg/m^3) satisfies, (eg Graf (1971))

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + (w - w_s)\frac{\partial c}{\partial z}$$

$$= \frac{\partial}{\partial x}\left(D_x\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z\frac{\partial c}{\partial z}\right) \qquad (2)$$

where

u, v, w are the velocity components (m/s)
x, y, z are space co-ordinates, with z vertically upwards (m)
w_s is the settling velocity (m/s)
t is time
D_x, D_y, D_z are diffusion coefficients (m^2/s).

3.1 Models of vertical profiles

Most solutions to be found in the literature are for special cases of this equation. The earlier solutions by Schmidt (1925) and Lane et al (1941), and later Hunt (1965) provide insight into the vertical structure of the suspended solids profile. These assume one-dimensional, uniform, steady flow conditions for which equation 2 reduces to

$$w_s\frac{\partial c}{\partial z} + \frac{\partial}{\partial z}\left(D_z\frac{\partial c}{\partial z}\right) = 0 \qquad (3)$$

This equation represents the equilibrium profile obtained as a balance between settling and vertical diffusion due to the turbulence. It is important to appreciate that equilibrium defined in this way does not mean saturation. Indeed the sediment load can be in equilibrium if the bed is not mobile, even when the flow is not saturated with sediment. Integration with respect to z and the application of a boundary condition of zero flux of sediment at the free surface (and implicitly also at the bed) yields the governing equation

$$w_s c + D_z \frac{\partial c}{\partial z} = 0 \tag{4}$$

This can be integrated further if the vertical structure of the diffusivity is prescribed. These profiles have proved valuable in the understanding of sediment transport but they are not relevant to the unsteady, unsaturated flow conditions which are the main concern here, so these special solutions are not considered further. Graf (1971) is a good source of additional information on these solutions.

3.2 Evolution of suspended load

A class of solutions which have more relevance to estuaries have been presented by Kalinske (1940), Dobbins (1943), Mei (1969) and Lean (1980), for unsteady and uniform or steady and non-uniform conditions governed respectively by the equations

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z}\right) + w_s \frac{\partial c}{\partial z} \tag{5}$$

$$u_o \frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z}\right) + w_s \frac{\partial c}{\partial z} \tag{6}$$

These equations are mathematically the same when the advection velocity u_o is constant. The first represents a concentration changing with time following a change in the magnitude of a uniform flow, while the second represents the concentration changing as a function of position as might occur for example when clear water flows from an area with an inerodible bed into an area where erosion can commence. Solutions of these equations provide information about the time or distance of travel required for the sediment concentration to adapt to changes in the flow conditions.

The assumption of constant eddy diffusivity permits analytic solution of these equations. The diffusivity normally used is

$$D_z = \frac{1}{6} \kappa u_* d \tag{7}$$

which is the depth averaged value of the parabolic eddy viscosity

$$\nu_\tau = \kappa u_* z(1-z/d) \qquad (8)$$

consistent with the logarithmic velocity profile. κ is the Von Karman constant. Apmann and Rumer (1970) present experimental evidence that supports this assumption. The exact solutions, which involve the use of Laplace Transforms, or similar, may be expressed in the form of infinite series. Mei (1969) recognised that an approximate solution, valid for small times of distances of travel, could be obtained from the expansion of the Laplace Transform for large values of the transform parameter. Under most conditions this expansion is valid for distances of the order of twenty water depths or the equivalent in a time dependent situation. Lean (1980) proposed an alternative bed boundary condition and the present author has reworked Mei's solution for this case. This solution is used in the sediment transport model proposed in the next section.

3.3 Boundary conditions

The solution of the unsteady equation 5 requires an initial condition at say t = 0 and boundary condition z = 0 (the bed) and z = d (the free surface). The surface boundary is clearly zero vertical flux of sediment viz

$$w_s c + D_z \frac{\partial c}{\partial z} = 0 \text{ at } z = d \qquad (9)$$

There are two possible conditions at the bed which admit analytical solution, firstly one could assume (Mei 1969) that the concentration c(o,t) at the bed responds instantaneously to the changing flow conditions. That is

$$c(o,t) = c_s(o,t) = \beta_s \bar{c}_s(t) \qquad (10)$$

where

$c_s(o,t)$ is the concentration of the equilibrium profile at the bed when the flow is saturated with sediment

$\bar{c}_s(t)$ is the depth averaged value of this profile

$$\beta_s = c_s(o,t)/\bar{c}_s(t) \text{ is a profile factor} \qquad (11)$$

This is a much more realistic condition than that implicit in a potential load model which assumes that the full load responds instantaneously. However, the condition still implies an infinite rate of exchange of material at the bed at $t = 0$ (or at $x = 0$ in the non-uniform version).

Lean (1980) assumes that the rate E ($kg/m^2/s$) at which material is entrained into the flow is the quantity which responds most readily to changes in flow. In this case the boundary condition at the bed would be

$$E = -\left(D_z \frac{\partial c}{\partial z}\right)_{z=0} = -\left(D_z \frac{\partial c}{\partial z}s\right)_{z=0} \tag{12}$$

or, from equation 4, the net vertical flux F_z ($kg/m^2/s$) at the bed is prescribed as

$$F_z = w_s(c_s - c)_{z=0} = w_s\beta_s(\bar{c}_s - \bar{c}) \tag{13}$$

This provides an alternative boundary condition to condition 10. The asymptotic form of solutions to equation 5 for initial concentration $\bar{c}_o$ are

$$c(t,z) = \beta_s\bar{c}_o e^{-R\zeta} + \tfrac{1}{2}\beta_s(\bar{c}_s - \bar{c}_o)\left[\operatorname{erfc}(\sigma\zeta + \tau) + e^{-R\zeta}\operatorname{erfc}(\sigma\zeta - \tau)\right] \tag{14}$$

for the bed concentration boundary condition (Mei) and

$$c(t,z) = \beta_s\bar{c}e^{-R\zeta} + \tfrac{1}{2}\beta_s(\bar{c} - \bar{c}_o)\left[\operatorname{erfc}(\sigma\zeta + \tau) + e^{-R\zeta}\operatorname{erfc}(\sigma\zeta - \tau)\right] - \beta_s(\bar{c}_s - \bar{c}_o)\left[(1 + \tfrac{1}{2}R\zeta + 2\tau^2)\operatorname{erfc}(\sigma\zeta + \tau) - \frac{2}{\sqrt{\pi}}\exp\left[-(\sigma\zeta + \tau)^2\right]\right] \tag{15}$$

for the bed entrainment boundary condition (Lean), where

$$R = w_s d/D_z \tag{16}$$

$$\tau = w_s(t/4D_z)^{\frac{1}{2}} \tag{17}$$

$$\sigma = d(4D_z t)^{\frac{1}{2}} \tag{18}$$

$$\zeta = z/d \tag{19}$$

These solutions are valid when $\tau \leqslant 1$, ie for the time needed for the water to flow over a distance equal to about 10 to 100 water depths.

3.4 Numerical sand transport models

Although the special solutions described above provide insight into the sediment transport processes they still lack many of the factors which are important in estuaries, namely

1. the combination of non uniformity with unsteadiness

2. variable supply of erodible material

3. lateral as well as longitudinal variation.

The inclusion of these factors completely precludes of analytic solution and leads to the need for numerical models.

The simplest form of numerical sediment transport model takes the form of finite difference or finite element solutions of the approximate equations 5 and 6. Apmann and Rumer (1970) and Yalin and Finlayson (1973) present models of this type. The advantage of seeking numerical solutions is that more realistic eddy diffusivities and velocity profiles can be incorporated. Yalin employs an eddy diffusivity equal to the parabolic eddy viscosity (eq 8) consistent with the logarithmic flow profile. The model was tested against experimental measurements (Fig 5). Though the model is not immediately relevant to estuaries there is no inherent difficulty in extending the numerical techniques to non-uniform, unsteady conditions.

Kerrsens et al (1979) have developed a multi-layer, 1-D model of this type which allows non-uniform cross-sections to be considered but it has apparently only been applied under steady flow conditions. A logarithmic profile is assumed for the vertical structure of the flow. The suspended solids concentration is computed from the 1-dimensional form of the sediment concentration equation 2 using non-uniform vertical grid to give greater accuracy near the bed. Kerrsens et al (1979) assumed the turbulent diffusivity, D_z to equal the parabolic eddy viscosity (eq 8) appropriate for logarithmic flow. The boundary condition at the bed is taken to be the equilibrium concentration, equation 10, which would occur at the instantaneous flow conditions. The solution simulates the transient evolution of the equilibrium profile from a non-equilibrium condition.

The model was tested against infill rates measured in a gas pipeline trench in the Western Scheldt. This model has been developed further to include agitation by waves (van Rijn (1985)).

Galappatti and Vredgdenhil (1985) approached the problem in a different way. They formulated their model on a series expansion in which the vertical dimension is eliminated by means of an asymptotic solution. The resulting depth-averaged model was tested for a steady, unidirectional flow case using a prescribed concentration for the boundary condition at the bed. It is not clear whether this type of boundary condition was used in preference or dictated by the nature of the method. Although the model had limited applicability, the authors concluded that the technique is a step towards bridging the gap between 2D and 3D models.

Although none of the models described so far have been applied to real estuary conditions there is no technical reason why this should not be done. The main problems preventing this at present seem to be the high expense of running 3-dimensional models and deficiencies in our knowledge of sediment transport processes in estuaries. In an attempt to gain an understanding of the consequences of unsaturated flow in estuaries Miles et al (1980) proposed a 2-dimensional, depth-averaged model. This type of model requires special provision to take into account the vertical profile effects of the sediment concentration.

The depth-averaged concentration $\bar{c}(x,y,t)$ satisfies the depth-integrated form of equation 2 which may be written

$$\frac{\partial}{\partial t}(\bar{c}d) + \alpha\left(\frac{\partial}{\partial x}(d\bar{c}\bar{u}) + \frac{\partial}{\partial y}(d\bar{v}\bar{c})\right) = \frac{\partial}{\partial s}\left(dD_s\frac{\partial\bar{c}}{\partial s}\right) + \frac{\partial}{\partial n}\left(dD_n\frac{\partial\bar{c}}{\partial n}\right) + \beta_s w_s(\bar{c}_s - \bar{c}) \qquad (20)$$

where

D_s is a longitudinal dispersion coefficient due to the vertical profile

D_n is the lateral (turbulent) diffusion coefficient

(s,n) are natural co-ordinates in the direction and normal to the flow

The parameters α and β_s are introduced to account for the vertical concentration and velocity profiles. For example

$$\alpha = \frac{1}{\bar{q}\bar{c}d} \int_o^d qc\,dz = T/\bar{q}\bar{c}d \qquad (21)$$

represents the factor required to recover the true transport of sediment from the product of depth averaged quantities, where

$q = (\bar{u}^2 + \bar{v}^2)^{\frac{1}{2}}$ is the horizontal water speed, and
T = the sand transport (kg/m width/s)

Since high concentrations occur near the bed it follows that $\alpha \leqslant 1$, and $\beta_s \geqslant 1$.

Miles et al (1980) analysed sediment transport measurements from the Conwy estuary in terms of α and β_s. The mean value for α was found to be in good agreement with theoretical values from Sumer (1977) and the mean value for β was in good agreement with theoretical values

$$\beta_s = R\{1 - \exp(-R)\}^{-1} \qquad (22)$$

from the exponential equilibrium profile of Lane et al (1941) for conditions typical of the flow in the Conwy Estuary.

The results show that it is possible to obtain sensible profile factors from field measurements and, although, the relations found are valid only for the Conwy, the techniques involved could be applied to any site. With this empirical method of prescribing α and β_s this model has the basic features for studying sediment transport in estuaries. It has advection by currents, dispersion in the direction of flow due to the vertical profile and lateral diffusion by turbulence. Deposition or erosion takes place depending as to whether the instantaneous sediment load exceeds or falls short of the saturated load, and, if required, erosion may be prevented if there is no sediment of the appropriate size available on the bed. A shortage of material on the bed would be reflected in a low concentration of suspended solids being advected away by the flow.

McAnally et al (1984) follow a similar approach but they use a different formulation for deposition and erosion. The deposition rate is based on a representative settling velocity and the erosion rate includes a response time coefficient but no details of its form are given.

The most unsatisfactory aspect of these models is the use of the settling velocity as the main scaling factor for the exchange rate of material between the flow and the bed. A better approximation can be obtained from the analytical solutions (14) or (15) for transient conditions. The rates of exchange of sediment at the bed

$$F_z(\tau) = -\left(D_z\frac{\partial c}{\partial z} + w_s c\right)_{z=0} \qquad (23)$$

are

$$F_z(\tau) = \tfrac{1}{2}\beta_s w_s(\bar{c}_s - \bar{c}_o)\left(\frac{1}{\tau\sqrt{\pi}}e^{-\tau^2} - \mathrm{erfc}(\tau)\right) \qquad (24)$$

$$F_z(\tau) = \beta_s w_s(\bar{c}_s - \bar{c}_o)\left((1 + 2\tau^2)\mathrm{erfc}(\tau) - \frac{2\tau}{\sqrt{\pi}}e^{-\tau^2}\right) \qquad (25)$$

for the boundary conditions of Mei and Lean respectively. The nature of these relations is shown in Fig 1a for a typical flow condition. Since relation 24 implies an infinite flux of sediment at t = 0 the second formulation is favoured in the following. If preferred, the analysis could be repeated for the other case.

Fig 1a shows that the rate of exchange of sediment at the bed varies considerably over times of the order of timesteps normally used in numerical models. Accordingly it is advisable to integrate equation 25 over a model timestep. That is the vertical flux of sediment, $S_s(\tau)$, during the interval (o, t) is

$$S(\tau) = w_s\beta(\tau)(\bar{c} - \bar{c}_o) \qquad (26)$$

where

$$\beta(\tau) = \beta_s D_z w_s^{-2}\left(4\tau^2(1 + \tau^2)\mathrm{erfc}(\tau) + \mathrm{erf}(\tau) - \frac{2\tau}{\sqrt{\pi}}(1 + 2\tau^2)e^{-\tau^2}\right) \qquad (27)$$

is the bed exchange scaling factor that incorporates the effects of the vertical structure and the lag time for the concentration profile to adjust to the changing flow conditions. The nature of $\beta(s)$ is shown in Fig 1b for a typical sediment fraction and timestep.

A new sand transport model has been developed by the author (Miles (1981)) using $S(\tau)$ given in equation 26 to replace the source/sink term on the right hand side of equation 20. This is equivalent to assuming that the sediment load is in equilibrium with the flow, which is reasonable provided the flow is slowly varying in space and time. However it follows that the new method cannot be absolutely precise because the theoretical solutions 14 and 15 have a gradual transition while the approximate computation is calculated from the assumed equilibrium profile at the start of each timestep. However, results under uniform flow conditions shown in Fig 1b indicate that the errors involved are less than the differences which result from applying the two alternative boundary conditions 10 and 12.

4 ENTRAINMENT

4.1 Entrainment rate

Van Rijn (Ref 23) has devised a novel experiment for measuring the entrainment rate. A sediment lift construction was designed whereby the sediment particles could be moved upwards at a constant rate through a circular opening in the flume bottom. The upward speed of the lift could be set at different (constant) values by use of a mechanical drive system. To establish a uniform flow, the flume was equipped with a false floor which was given a pre-set slope equal to the expected water surface slope in each experiment. The false floor was covered with pre-fabricated wooden plates of which sediment particles of a size equal to those in the sediment lift were glued.

In all, five types of almost uniform sand material were used with d_{50} values of 130, 190, 360, 790 and 1500 μm. The flow depth was about 0.25m. The flow velocities were varied in the range of 0.5-1m/s.

The most well-known sediment pick-up functions, as reported in the literature, were summarised and compared with the experimental results, and an empirical sediment pick-up function representing the present experimental results was used to compute the transport rate of the bed-load and the suspended load particles.

The main conclusions of the study were:

1. The experimentally determine pick-up rate for a flat bed could be represented by a simple

empirical pick-up function for particles in the range 130-1500 μm.

2. The predictive ability of the theoretical pick-up functions of Einstein and Yalin was rather poor; the theory of De Ruiter produced the best results for the large particles sizes ($d \geqslant 1000$ μm); while the theories of Nagakawa-Tsujimoto and Fernandez Luque yielded the best results for the smaller particle sizes ($\leqslant 200$ μm).

3. Defining the bed-load transport as the product of the pick-up rate and the saltation or jump length, the bed-load transport was computed for 553 flume and field data, resulting in a score of 60% of the predicted values in the range 0.5 to 2 times the measured values; the bed-load formulas of Meyer-Peter-Müller and Frijlink produced similar results.

4. Applying the proposed empirical sediment pick-up function as a bed-boundary condition in a mathematical model for suspended sediment transport, the adjustment of concentration profiles in a uniform flow without initial sediment load was computed and when compared with experimental results, showed a reasonable agreement.

4.2 Boundary conditions at the bed

As discussed earlier prescribing the entrainment rate of sediment from the bed is considered to be superior to specifying the near bed concentration. The magnitude of the sediment entrainment rate E (kg/unit area/sec) essentially determines the magnitude of the associated sediment transport T_s (kg/m width/sec) and, vice versa, the use of a given sediment transport relations T_s implies a corresponding entrainment E. That is both relations cannot be prescribed independently in a model.

Further insight into the connection between E and T_s can be obtained by considering the requirements of various models. At present our limited understanding of the underlying physics has resulted in two basically different approaches being used. On one hand we have methods tied to sediment transport relations, as frequently used in unidirectional flow and increasingly used in estuaries in the form of potential sand transport models. Models formulated in this way do not need the sediment entrainment rate to be prescribed. On the contrary the exchange of material at the bed is obtained explicitly from the spatial variations in transport by assuming saturated flow conditions and using conservation of sediment.

At the other extreme a model containing a full description of the physical processes in the vertical dimension would only need sediment entrainment and shear stress relations at the bed. No sediment transport relation would be required because the flow and suspended sediment profiles would be generated from the basic equations and together these define the sediment transport. The model of Kerrsens et al (Ref 13) falls into this second category.

The depth-averaged model presented in this report is interesting in that the actual relation between E and T_s can be derived. Thus from equations 12, 21 and 22

$$E = -D_z \ (dc_s/dz)_{z=0} = w_s^2 \ T_s/(\alpha u D_z(1 - \exp(-w_s d/D_z)) \quad (28)$$

It should be noted that this relation applies exclusively to the HR Sandflow-2D model formulation. Other models have different relations between E and T_s and in many cases it is not possible to extract the exact form of it from the equations.

4.3 Calibration of entrainment and profile evolution

Computed sand transports will differ even from fluxes observed under controlled laboratory conditions because of approximations in the models and uncertainties in the prescribed entrainment rate (or sand transport relation). The normal procedure would be to adjust the sediment entrainment rate (or sand transport relation) during calibration to tune the model to agree with observed transport rates. However, due to uncertainties in the form of the turbulent diffusivity, (D_z) and settling velocity (w_s) relations in sediment loaded flow, some adjustment of these parameters will also be necessary to obtain realistic evolutions of te sediment load under unsteady of nonuniform flow conditions.

5 SIMULATION OF SEDIMENT TRANSPORT

5.1 Evolution of sediment load

The mathematical model results were compared with the results of a laboratory experiment performed in a flume with a length of 30m, a width of 0.5m and a depth of 0.7m at the Delft Hydraulics Laboratory (Ref 6). The discharge was measured by a circular weir. The mean flow depth was 0.25m and the mean flow velocity was 0.67m/s. The bed material had a d_{50} = 230 μm and a d_{90} = 320 μm. The median diameter of the particles in suspension was estimated to be about 200 μm, resulting in a representative fall velocity of 0.22m/s (water temperatures 9° C). The stream bed was covered with bed forms having a length of about 0.1m and a height of about 0.015m. Small Pitot tubes were used to determine the vertical distribution of flow velocity. Water samples were collected simultaneously by means of a siphon method at four locations to determine the spatial distribution of the sand concentrations. At each location (profile) five samples were collected at a height of about 0.015, 0.025, 0.05 and 0.22m above the average bed level and these were integrated to give the suspended load transport. The HR Sandflow-2D model was run assuming the overall shear velocity 0.477m/s estimated in Ref 6 and the results are presented in Fig 2a. Note that the Delft results have been transformed into a time basis using $x = ut$. The HR solution evolves in a very similar manner to the Delft solution, which is a very encouraging result considering that the Delft model was more sophisticated in the sense that it was a direct solution of the vertical sediment balance (Eq 6). However, the Delft model is at present limited to steady flow in one horizontal direction. Although the Delft model could in theory be extended to unsteady, two dimensional areas there would be practical difficulties implementing this on the present generation of computers. Consequently the unsteady, depth-averaged, two dimensional Sandflow-2D model is a significant improvement over the other technique.

Both the Delft and HR model results deviate slightly from the observed evolution of the suspended sediment load. In particular the initial rate of increase is too fast in both models. This may be partly due to the formation of the scour hole and/or armouring of the bed slowing down the entrainment of bed material in the flume or because the model parameters were not set correctly. The relevant parameters are the particle settling velocity and the vertical diffusivity. In the HR model the latter is essentially defined by the shear velocity which is particularly difficult to estimate or extract from measurements. Further research on these aspects would obviously help to resolve these difficulties in the future. In the meantime the uncertainties can be overcome by tuning the model parameters during calibration to obtain better agreement with observations (Fig 2b).

6 CONCLUSIONS

A sand transport model, based on theoretical and empirical relationships, has been tested to verify that it simulates the relevant physical processes.

It was found that the best formulation for entrainment of sand at the bed was in terms of an entrainment rate. The connection between this entrainment rate and the associated sand transport law has been considered and it was concluded that strictly only one of these should be specified. Physically the entrainment rate is more fundamental because it is this, together with the physics of the system, which defines the transport. However, since it is easier to measure sand transport, this is often prescribed in practical problems.

The model was compared with some flume data to test its response to a change in the sediment load. It was shown that the model simulation could be calibrated by adjusting the settling velocity and vertical diffusivity parameters. This procedure is justified for practical applications because in nature these parameters are not well defined. For example there is no unique settling velocity because the suspended load would contain a range of sediment sizes and the true nature of the vertical diffusivity profile is not fully understood. These aspects will be considered further in later stages of the project.

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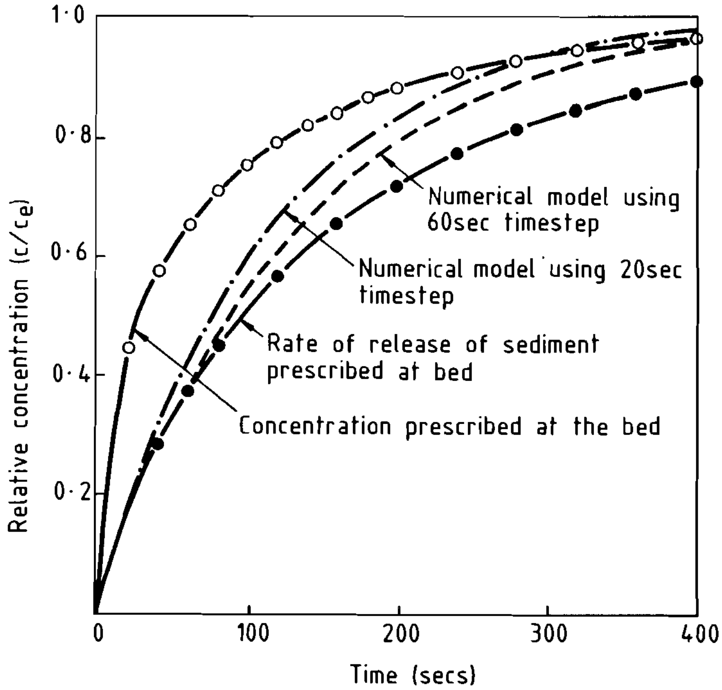
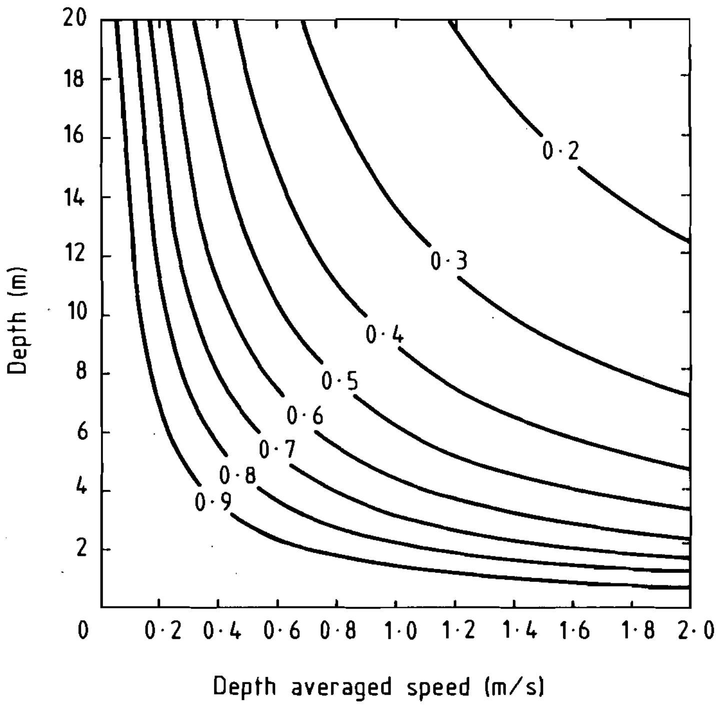
8 LIST OF SYMBOLS

$c(x,y,z,t)$	concentration of suspended solids (kg m^{-3})
$\bar{c}(x,y,t)$	depth-averaged concentration (kg m^{-3})
$\bar{c}_o$	initial concentration (kg m^{-3})
c_s	concentration under saturated conditions
$\bar{c}_s$	depth-averaged saturated conditions
d	water depth (m)
D	sediment grain size (mm)
D_n	lateral diffusion coefficient ($m^2 s^{-1}$)
D_s	longitudinal dispersion coefficient ($m^2 s^{-1}$)
D_x, D_y, D_z	diffusion coefficients in 3-D axes ($m^2 s^{-1}$)
E	sediment pick up rate (kg $m^{-2} s^{-1}$)
F_z	net vertical flux of sand (kg $m^{-2} s^{-1}$)
g	acceleration of gravity (ms^{-2})
M	quantity of mobile bed material (kg m^{-2})
q	current speed, $(\bar{u}^2 + \bar{v}^2)^{\frac{1}{2}}$ (ms^{-1})
Q	discharge per unit width ($m^2 s^{-1}$)
R	settling velocity Reynolds Number, $w_s d/D_z$
(s,n)	intrinsic coordinates
S	source/sink term at the bed (kg $m^{-2} s^{-1}$)
t	time (s)
T_s	sand transport (kg $m^{-2} s^{-1}$)
(u,v,w)	components of velocity vector (ms^{-1})
$(\bar{u},\bar{v})$	depth-averaged velocity components (ms^{-1})
$\bar{u}_{crit}$	threshold velocity for sand transport (ms^{-1})
u_o	uniform velocity (ms^{-1})
w_s	settling velocity (ms^{-1})
(x,y,z)	cartesian co-ordinates (m)
α	profile parameter, $\int ucdz/\overline{duc}$
$\beta(\tau)$	bed exchange factor
β_s	profile parameter, $c(x,y,o,t)/\bar{c}(x,y,t)$
κ	Von Karman constant
ν_τ	eddy viscosity ($m^2 s^{-1}$)
u_*	shear velocity (ms^{-1})
σ	dimensionless variable, $d/(4D_z t)^{\frac{1}{2}}$

τ	dimensionless variable, $w_s(t/4D_z)^{\frac{1}{2}}$
τ_b	bed stress (Nm^{-2})
ζ	dimensionless vertical co-ordinate, z/d

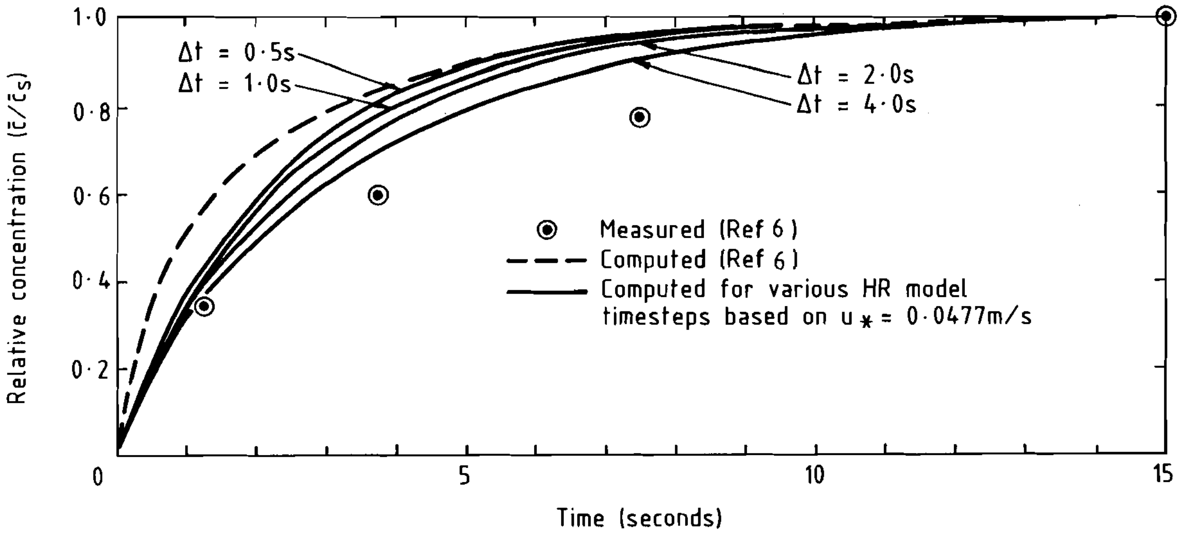


(a) Net sediment exchanges at the bed (F_z) for 0·2mm sand (0·02m/s settling velocity). Water depth = 10m, depth-averaged velocity = 1m/s and shear velocity = 0·05m/s.

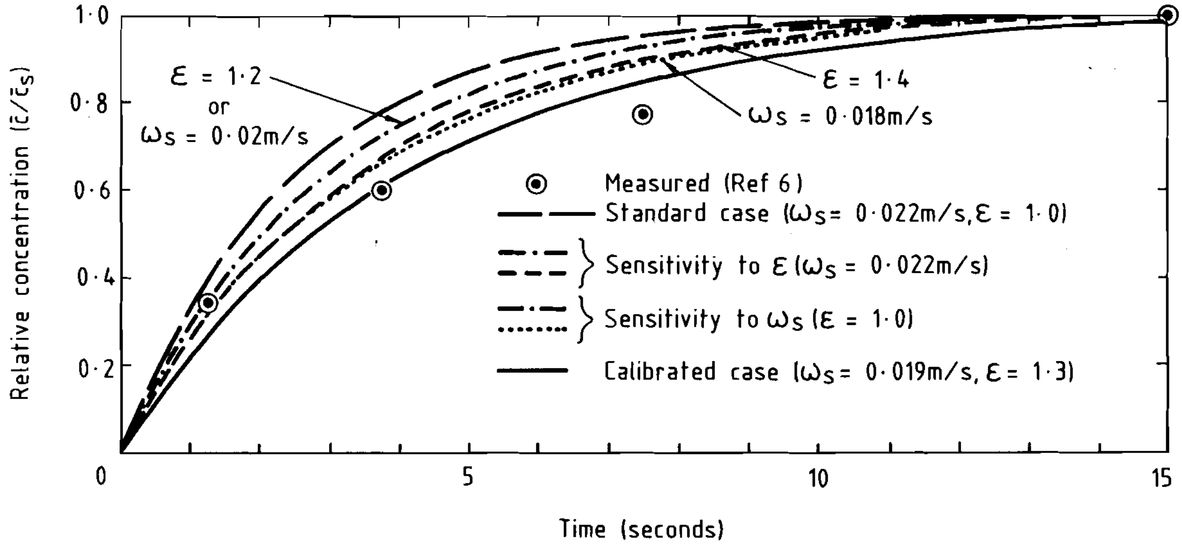


(b) Bed exchange factor (β) for 0·2mm sand (0·02m/s settling velocity) using Darcy-Weisbach friction factor = 0·02

Fig. 1 Sediment exchange relations at the bed.



(a) Sensitivity to model timestep



(b) Sensitivity to settling velocity and vertical diffusion

Flume data and test conditions

Mean flow depth 0·25m, mean flow velocity 0·67m/s
Particle diameters (d_{50}) 230μm bed material, 200μm suspended
Settling velocity 0·022m/s, water temperature 9°C
Sediment density 2650kg/m³, fluid density 1000kg/m³
Overall bed shear velocity 0·0477m/s

Fig. 2 Computed and measured evolutions of sediment load