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AN ANALYSIS OF THE BEHAVIOUR OF FLUID MUD IN ESTUARIES

Nicholas V M Odd BSc (Eng) FICE

James G Rodger BSc PhD

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Registered Office: Hydraulics Research Limited,
Wallingford, Oxfordshire OX10 8BA.
Telephone: 0491 35381. Telex: 848552

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ABSTRACT

Fluid mud is a dense suspension containing a concentration of mud flocs which is high enough to significantly change the physical properties of the mud/water mixture compared to that of clear water with the same salinity and temperature. Fluid mud is formed under a variety of conditions. Once formed it may flow under the influence of gravity and hydrostatic forces and then settle and dewater in navigation channels and berths, causing siltation over and above that due to settlement directly from suspension. The movement of fluid mud has been cited as the most likely cause of rapid siltation in the majority of the ports in the world located in muddy estuaries. In the past, it has been very difficult to predict or quantify this extra siltation, which may in some cases double the annual maintenance dredging bill for a new port extension.

A theoretical analysis waa made of the behaviour of fluid mud in estuaries, including a review of the literature. Basic theories were devised and modified to describe the formation of fluid mud from a more dilute suspension and its motion down a sloping bed and along a flat bed. The analysis casts a new light on several previously unexplained phenomena observed in estuaries.

It is recommended that fluid mud be defined as being a flocculated suspension of mud particles in the range 60,000 to 120,000ppm, but more typically with a value of about 75,000ppm. For engineering calculations one can probably assume that it has a yield shear strength of about $0.1N/m^2$ and a viscosity about twice that of water.

Theory indicates that one would not expect a significant layer of fluid mud to form from a suspension in estuaries where the peak concentration is less than about 2500ppm. A fluid mud layer will gain mass and thickness at a maximum rate of about $40g/m^2/s$ when the overlying water has a concentration of suspended mud of about 20,000ppm.

Fluid mud will stand on a sloping bed provided the gravitational forces do not exceed its shear strength. Fluid mud which is thicker than this critical value will flow as a body down the slope towards the deepest part of the channel cross section or along the bottom of a sloping channel during the slack water period.

The turbidity current will tend to thicken and become diluted by entrainment of the overlying water. This effect may be offset by scour of an underlying muddy bed. The thicker the layer grows the faster it flows so it is possible that the turbidity current could turn into an avalanche of mud.

One can form an equation to calculate the rate of transport of mud in a turbidity current, but there are one or two empirical coefficients which could only be evaluated by making observations of a mud flow in an estuary such as the Bristol Avon or Parrett Estuary.

A layer of fluid mud will start to flow along the flat bed of an estuary as a result of the imposed hydrostatic head caused by the increase in water surface slope after slack tide. A layer of uniform fluid mud will always shear at its base. The critical depth at which the layer will move varies directly with the Bingham shear strength and inversely with the water surface slope. Thin layers of fluid mud which form at slack water in many estuaries are more resistant to motion than thick layers and tend to be eroded from the surface downwards. This process of re-erosion can be investigated in any recirculating flume or in a circular flume.

To date, it has only been possible to model the formation and instant resuspension of a thick layer of fluid mud in a very turbid estuary. It should be possible to model the formation and movement of fluid mud in both canalised and wide estuaries. However, there is a need to check the theories described in the report with accurate field and laboratory experiments.

The results from the study have clarified our understanding of the behaviour of fluid mud in estuaries and highlighted the need for further experimental and field observations. The results of the research will be incorporated into a mathematical model which will improve our capacity for predicting the effects of proposed port engineering works on siltation in navigation channels and berths.

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1 INTRODUCTION

Fluid mud is a dense suspension containing a concentration of mud flocs which is high enough to significantly change the physical properties of the mud/water mixture compared to that of clear water with the same salinity and temperature.

Fluid mud is formed under a variety of conditions. Once formed it may flow under the influence of gravity and hydrostatic forces and then settle and dewater in navigation channels and berths, causing significant rates of siltation over and above that due to settlement directly from suspension.

The report analyses the behaviour of fluid mud in estuaries. It describes and quantifies the formation of fluid mud from a more dilute suspension and its motion down a sloping bed and along a flat bed. The report does not include an analysis of the generation of fluid mud by wave action or mechanical agitation. However, the theories are applicable to the movement of a layer of fluid mud formed by either of these processes.

2 DEFINITION OF FLUID MUD

The interaction of suspended mud flocs starts to hinder and reduce the rate of settling at concentrations greater than about 10,000ppm. Krone (Ref 1) defined fluid mud as a suspension with a concentration in excess of this value. However, there are estuaries in the United Kingdom where this value is exceeded over the whole cross-section on the main run of the tide and where a well defined layer of fluid mud forms at slack water which has concentrations in the range 60,000 - 100,000ppm. When the concentration of a flocculated mud exceeds about 75,000ppm, a weak interlocking matrix of mud flocs is formed which has a Bingham yield strength of about $0.1N/m^2$. Layers of fluid mud with concentrations of about 70,000 - 100,000ppm have been reported in the Parrett (Fig 1) (Ref 2), Bristol Avon (Ref 3), Severn (Ref 4), Weser (Ref 5), Gironde (Ref 6), and Thames Estuaries (Ref 7) and many other estuaries in different parts of the world (Ref 8).

The physical properties of suspensions of less than about 50,000ppm do not differ significantly from that of water. The only effect of the suspended concentrations is to increase the effective bulk density of the mud water mixture. The Bingham yield strength of muds with concentrations in excess of about 100,000ppm rapidly increases and the mud mixture behaves as a very weak soil. Such a weak soil does not readily flow down the slope with a gradient of less than about 1 in 10.

There is, therefore, only a relatively narrow band of concentrations in the range 50,000 - 120,000ppm when a mud behaves as a non Newtonian fluid. The actual concentration of the fluid mud found in a particular estuary will depend on its particular rheological properties and the shearing rate within the mud layer. A typical value for the concentration of a fluid mud layer in an estuary with low internal shearing rates is about 70,000ppm. The typical value in the boundary layer of a wave with much higher shearing rates is probably about 100,000ppm.

2.1 Properties of fluid mud

Fluid mud in an estuary usually has a dry density in the range 70 to 100kg/m^3 and a viscosity 2 - 10 times higher than seawater and an apparent Bingham shear strength in the range 0.1 - 0.4N/m^2.

A fluid mud suspension consists of a mixture of flocs of different sizes and density which continually touch and interlock with each other hindering the upward flow of water and transmitting shear stresses through interparticular friction and fluid friction. The larger flocs are loose aggregations of smaller denser and stronger flocs. The shearing of the fluid causes a continual disruption and deformation of the mud flocs. Higher shearing rates generally result in smaller and stronger flocs. The non-Newtonian properties of the fluid vary with the floc structure and hence the shearing rate.

3 FORMATION OF FLUID MUD

3.1 Formation of fluid mud from a suspension

Fluid mud may be formed as a result of settlement from a dilute (< 50,000ppm) suspension in either still or flowing water or as a result of a disturbance of a settled bed by wave action or mechanical agitation.

The main factor determining whether fluid mud forms from a dilute suspension is the manner in which the settling velocity of the mud flocs varies as a function of the concentration, (Fig 2), (Ref 9). In-situ measurements of the settling velocity of mud flocs has shown that the settling velocity increases approximately linearly with the concentration, in the range 10 - 4,000ppm. In this range, the flocs settle more or less independently of each other, but their average size increases with the concentration because of the number of collisions between the flocs increases in proportion to the concentration.

The settling velocity reaches a peak and remains constant at about 2 to 4mm/s with concentrations in the range 4,000 to about 20,000ppm. It should be noted that settling velocities in laboratory apparatus will usually be an order of magnitude less than in estuaries due to the short flocculation times. In this range, the flocs are starting to hinder the movement of water being displaced as they settle. Although the flocs touch each other they are too weak to transmit significant forces without deforming. The settling velocity of mud flocs decreases rapidly from about 2mm/s to a minimum value of approximately 0.05mm/s as the concentration rises from 20,000 to about 100,000ppm.

Provided the bed stress is less than about $0.07N/m^2$, fluid mud dewaters at a rate of about 0.05mm/s to form a weak soil with a dry density of about 170 - $300Kg/m^3$ which has a relatively high shear strength and does not flow easily (Figs 3 and 4) (Ref 10 and Ref 11).

A layer of fluid mud will form at the bottom of a channel carrying a dilute suspension of mud if the net flux of mud settling into the layer of water above the bed exceeds the rate at which mud deposits on the bed. In this context, the bed is assumed to be a weak soil with a bulk density of about $1.2t/m^3$. In the absence of a significant amount of vertical turbulent exchange near slack water the net flux of settling particles is the product of the settling velocity and the concentration of mud in suspension. This flux rises to reach a maximum of about $40g/m^2/s$ at a concentration of about 20,000ppm and falls rapidly at higher concentrations as shown in Figure 5. At slack water the mud flocs settle towards the bed. Fluid mud with a concentration of about 70,000ppm dewaters or deposits onto the bed at a rate of about $4g/m^2/s$.

It follows that a layer of fluid mud will only grow in thickness if the flux of mud settling onto the bed exceeds the rate at which the fluid mud dewaters. This flux is equivalent to a concentration of about 2500ppm in the overlying water as shown in Figure 5. It is therefore postulated that fluid mud is unlikely to form at slack water in an estuary unless the concentration of mud in suspension in the bottom of a water column exceeds about 2500ppm. In the author's experience fluid mud is seldom found in estuaries where this concentration is not exceeded. For example, fluid mud will only be found in the Thames estuary in the mud reaches during spring tides.

A knowledge of the settling velocity function maybe used to predict the growth of fluid mud in a mathematical model of an estuary. The main uncertainty is the dry density and rate of dewatering of the fluid mud which probably varies with the rheology and proportion of silt in the mud.

3.2 Formation of fluid mud by wave action

Wave action in shallow water in an estuary can fluidise the surface of a mud bed. Wave action erodes the weakest surface layers of the mud deposits. Experiments with a naturally settled bed of mud has shown that wave action erodes mud of a given dry density at about the same peak shear stress that is required with a steady uni-directional flow. A very high proportion of the eroded mud is contained in the relatively thin boundary layer of the wave in the form of fluid mud with concentrations of about 100,000ppm. This happens because the sharp density gradient damps the vertical turbulent exchange with the water column above.

3.3 Formation of fluid mud by a mechanical agitation

The disturbance by drag heads of dredgers and by propellors of ships with a small underkeel clearance may fluidise a mud bed. The spoil discharged from a suction dredger will often have the consistency of fluid mud.

4 STRESS-STRAIN PROPERTIES OF FLUID MUD

The rheological properties of fluid mud with concentrations of about 70,000 - 100,000ppm have been examined by Krone (Ref 12) in a rotating cylinder viscometer at shearing rates in the range 2 - $50s^{-1}$, which is in the range commonly found in the lowest layers of an estuarine channel for bed stresses in excess of that allowing deposition. In 1964, the senior author used the same equipment to test three different mud samples from British estuaries (Fig 6). Laminar conditions prevailed in the 12mm gap between the cylinders. The physical properties of the fluid mud varied after the suspension was stirred and depending on whether the shearing rate was increasing or decreasing. It behaved as an unstable non-Newtonian fluid with a differential viscosity varying between 1 and 20 times that of seawater. The relative viscosity tended to decrease at higher shearing rates and increase during reducing shearing rates in the range 2 - $10s^{-1}$. Krone showed that mud flocs grow when the shearing rates are less than about $6s^{-1}$.

Fluid mud with a concentration between 70,000 and 80,000ppm appears to have an effective Bingham yield strength of about $0.1N/m^2$, (Fig 6). This value was not proved conclusively. Static testing as has been proposed by Dr D J A Williams of University College of Swansea.

At shearing rates in the range 10 - $50s^{-1}$ the laminar fluid mud suspensions were unable to transmit a stress of more than about 0.2 - $0.25N/m^2$. The transmitted shear decreased with increasing shearing rate in some situations. Shear rates in excess of $1,000s^{-1}$ only occur very close to the bed during peak tidal currents in an estuary. The same concentrations of the fluid mud were tested in a capillary viscometer with shearing rates in the range 100 - 1500s^{-1}. In this case, the properties of the fluid mud were independent of shearing rate indicating that the flocs were broken down to simple aggregations of primary clay particles. At concentrations of about 75,000ppm the differential viscosity was only twice that of seawater. However, the effective Bingham shear strength was about 1N/m^2, which is approximately equal to the bed stress exerted by a velocity of 1m/s over a smooth mud bed.

When fluid mud is sheared it transmits the resulting shear stress by two mechanisms, firstly by interparticulate friction and secondly by fluid friction.

In laminar conditions;

$$\tau = \tau_B + \rho_m \nu_m \frac{du}{dz} \qquad (1)$$

where

τ_B is the Bingham yield shear strength (N/m^2)
u is the velocity (m/s)

Laminar conditions prevail when the Reynolds number is less than about 1000. The effect of a ten-fold increase in the differential viscosity is to extend the range of velocities for which laminar conditions prevail and to thicken the laminar sub-layer when smooth turbulent conditions exist. Smooth turbulent conditions probably occur in the thicker moving fluid mud layers found in turbid estuaries. The thickness of the laminar sub-layer for a Newtonian fluid is:

$$\delta = \frac{11.6 \nu_m}{\sqrt{\tau_o / \rho_m}} \qquad (2)$$

where

τ_o is the bed stress (N/m^2)

ν_m is the kinematic viscosity (m^2/s)

ρ_m is the bulk density of the fluid mud (kg/m^3)

5 MOVEMENT OF FLUID MUD DOWN A SLOPING BED

The bed of a typical estuary slopes towards a central channel which itself generally slopes in a seaward direction or towards the centre of deep reaches. At slack tide a layer of fluid mud may form over a wide area of the bed of an estuary. Once the fluid mud exceeds a critical depth it will tend to flow down the bed slope towards the local deep region within the estuary where it will form a pool of fluid mud with a flat surface.

5.1 Initial motion

Fluid mud will stand on a slope in still water provided that the component of the net gravitational force down the slope does not exceed the Bingham yield shear strength of the mud (Fig 7).

$$F = d_m \cdot \Delta\rho \cdot g \cdot \sin\Theta = \tau_B \qquad (3)$$

where

$$\Delta\rho = \rho_m - \rho_w \qquad (4)$$

d_m is the critical thickness (m)

If Θ is small. $\sin\Theta \approx \Theta$

$$d_{m_{crit}} = \frac{\tau_B}{\Delta\rho g \Theta} \qquad (5)$$

Typically $\tau_B = 0.1\ N/m^2$ $\Delta\rho = 107.0 - 1020 \simeq 50 kg/m^3$

The critical thickness of a fluid mud layer with the above properties that will stand on a given slope is as follows:

bedslope θ	$d_{m_{crit}}$ (m)
10^{-1}	0.002
10^{-2}	0.02
10^{-3}	0.2
10^{-4}	2.0
10^{-5}	20.0

The same result is shown graphically in Figure 8.

Bed slopes in estuaries typically vary between about 10^{-1} on the steeper banks and 10^{-4} or less in the longitudinal direction. A layer of fluid mud bigger than about 0.2m will not remain stationary on slopes exceeding about 10^{-3}.

5.2 Movement of fluid mud down a submerged slope

When a layer of fluid mud starts to flow down a slope it behaves as a dense fluid with a bulk density of about 1070kg/m^3 as compared with the density of a typical estuarine water of 1020kg/m^3. A density current of this type (Ref 13) has features in common with free-surface flows, but differs essentially because the buoyancy of the surrounding fluid reduces the gravitational force in proportion to the normalised density difference

$$g^1 = g \left(\frac{\rho_m - \rho_w}{\rho_w}\right) = g \frac{\Delta\rho}{\rho_w} \qquad (6)$$

Initially, the mud will flow as a uniform slug with no differential velocity within the layers. As the internal stress increases the core of the fluid mud without relative motion decreases as shown in Figure 9. The detailed structure of the velocity profile is difficult to determine because the flow below the central slug may be laminar and the flow above may be turbulent and there is no theory readily available to specify the transfer of momentum within and between the various sub-layers. However, assuming a small slope and in the absence of concentration gradients in the mud layer, and assuming no erosion or entrainment, one can formulate the following equations to calculate the mean velocity in the layer:

$$\left(\tau_B + \rho_m \frac{f}{8} U_m^2 (1 + \alpha)\right) = \Delta\rho\, g\, d_m\, \Theta \qquad (7)$$

$$U_m = \left\{\frac{8\,\Delta\rho g d_m \Theta - 8\,\tau_B}{\rho_m (1 + \alpha)\, f}\right\}^{\frac{1}{2}} \qquad (8)$$

where

f is a friction factor
α is the ratio τ_i/τ_o

Harleman (Ref 13) recommends a value of 0.43 for α in turbulent flows.

The resulting relationship between d_m, U_m and Θ is shown in Figure 10.

The rate of mud transport, q_m, down the slope is given by:

$$q_m = C_m\, d_m \left\{\frac{8\,\Delta\rho\, g\, d_m\, \Theta - 8\,\tau_B}{\rho_m (1 + \alpha)\, f}\right\}^{\frac{1}{2}} \qquad (9)$$

q_m is rate of mud transport (kg/m/s)

q_m increases with approximately the three-halves power of the thickness of the mud layer. Neither α or τ_B are absolute constants. τ_B varies with the shearing rate. α tends to zero at low velocities and probably increases with the densimetric Froude number.

The above theory assumes that the bulk Richardson number, R_i, is high enough to inhibit significant entrainment of clear water into the fluid mud layer, that is:

$$R_{i_B} = \frac{\Delta\rho \, g \, d_m}{U_m^2} \geqslant 10 \qquad (10)$$

This condition is probably only satisfied for bed slopes flatter than about 2×10^{-3} (Fig 10). For bed slopes greater than 2×10^{-3}, the entrainment of overlying water is likely to become significant and the mud layer will tend to become thicker and more diluted. The entrainment velocity varies as a function of R_{i_B} as follows (Ref 14):

$$V_e = U_m \frac{0.1}{\left(1 + 63R_{i_B}^2\right)^{3/4}} \qquad (11)$$

For values of R_i less than unity, α increases rapidly from about 0.4 to 0.8. The stress exerted by the fluid mud layer on the underlying bed at these higher flow velocities will tend to erode the exposed layer of mud. A weak mud deposit with a dry density of about 200kg/m^3 would start to be eroded at a bed stress of about 1N/m^2, which is equivalent to a velocity of about 1m/s.

If the erosion of mud at the base of the fluid mud layer was more or less balanced by entrainment of clear water into the surface of the layer it will tend to thicken. As a result, the velocity would increase thereby increasing the rate of erosion and entrainment and under certain conditions this could cause an avalanche of mud.

If entrainment occurs the thickness of the layer varies with distance and one needs to consider the full equations for unsteady turbulent flow which may be expressed in one dimension in the following form:

$$\frac{\partial d_m}{\partial t} + V_e + \frac{\partial}{\partial x}\left(U_m d_m\right) = 0 \qquad (12)$$

$$\frac{\partial U_m}{\partial t} + \frac{\partial}{\partial x}\left(\frac{U_m^2}{2}\right) + g\frac{\Delta\rho}{\rho}\left(\frac{\partial d_m}{\partial x} + \Theta\right) + \frac{\tau_o}{d_m}(1 + \alpha) = 0 \qquad (13)$$

$$\frac{\partial}{\partial t}\left(C_m d_m\right) + \frac{\partial}{\partial x}\left(C_m U_m d_m\right) - H\left[|\tau_o| - \tau_e\right] m_e\left(|\tau_o| - \tau_e\right) = 0 \qquad (14)$$

These equations can only be solved by numerical methods.

6 MOVEMENT OF FLUID MUD ALONG A FLAT BED

For many years there has been disagreement on whether or how a layer of fluid mud might start to move along a channel with a flat bed. Experiments by Krone (Ref 1) in a shallow flume showed that a thin layer of fluid mud was eroded and re-suspended before it began to move. However, observations in many estuaries have shown that siltation rates cannot be accounted for solely by considering the transport of suspended mud and can only be explained by the movement of a thin layer of fluid mud along the flat bed of an estuary. There are no well documented observations of velocity profiles within a fluid mud layer on a flat mud bed known to the author. Approximate observations have been made in the Bristol Avon and Parrett estuaries where the bed of the channel has a slope in the seaward direction.

6.1 Initial motion

Consider a static pool or layer of fluid mud that has formed on the bed of a deep channel in an estuary at slack water. As the water surface slope increases after the change of the tide it will exert a net body force on the layer of fluid mud (Fig 7) which will be resisted by the Bingham shear strength of the mud. If one assumes that the shear stress exerted by the water flowing over the fluid mud is negligible, it follows that at the point of incipient motion:

$$\rho_w \, g \, d_m \frac{\partial \eta}{\partial x} \simeq \tau_B \qquad (15)$$

where

$\frac{\partial \eta}{\partial x}$ is the water surface slope

The thickness of a fluid mud layer that can resist movement by a given imposed water surface slope is shown in Figure 11.

At this stage the maximum shearing force at the bottom of the layer depends on the product of the thickness of the layer and the water surface slope. This means that thin layers are more resistant to motion than thick layers. It also explains why thin layers of fluid mud in a traditional straight flume do not fail in this mode but are eroded from the surface downwards. This mode (Ref 15) of failure does not occur in circular flumes because they do not generate a water surface slope in the circumferential direction. The variation in the effective stress on the base of layers of fluid mud 0.1 and 1.0m thick in the mud reaches of the lower Thames estuary during a spring tide is shown in Figure 12. In reality, the thickness of the fluid mud layer in the Thames estuary is unlikely to exceed a thickness of 0.1m in the absence of agitation by dredgers. The maximum water surface slope was about 4×10^{-4}. The water surface slope in an open mud flume is usually less than this value.

The thickness of a fluid mud layer formed at slack water in an estuary with negligible side slopes is limited by the mass of mud in the overlying water column so that:

$$d_m \geqslant \frac{1}{C_m} \int_0^d c\,dz \simeq \frac{\bar{C}d}{C_m} \tag{16}$$

The typical thickness of a layer of fluid mud in the Parrett estuary at slack water is 1.4m as shown in Figure 1. A typical value in the Thames estuary is likely to be less than 0.1m. If the critical shear strength of the fluid mud is about $0.1 N/m^2$, it is evident that a 0.1m thick layer will not fail at the base but erode or be entrained from the surface downwards as assumed in the original mud transport model of the Thames estuary (Ref 7). The fluid mud will continue to dewater during the erosion process as has been observed in the HR circular flume.

A thick layer of fluid mud is more likely to shear at the base of the layer. Initially, the layer will move as a rigid slug of mud with no internal relative motion because the internal shear stress will be less than the shear strength of the mud matrix. The overlying water may or may not be flowing in the same direction. As the water surface slope, velocities and internal stresses increase the thickness of the slug of undeformed mud will diminish and the level of turbulence in the lower part of the layer will increase.

The presence of the density interface will severely limit the level of turbulence within the fluid mud layer and hence the internal Reynolds stresses.

However, the overlying water will tend to erode the surface of the fluid mud layer once the interfacial stress exceeds the critical stress for erosion of the mud at a dry density of about $75kg/m^3$ which is approximately equal to the Bingham yield strength of about $0.1N/m^2$. The sharp density gradient at the interface between the fluid mud (75,000ppm) and the overlying water will inhibit vertical mixing and the dispersal of mud flocs into the upper layers of the flow.

At about this stage smooth turbulent conditions should exist throughout the whole thickness of the fluid mud layer except for a thin laminar sub-layer at the base.

6.2 Velocity profile in the fluid mud layer

The velocity profile in the fluid mud layer may be calculated as follows:

If the thickness of the fluid mud layer is small compared to the total depth of flow and if the local gradient Richardson number at the interface exceeds unity, the mixing length distribution within the smooth turbulent regions of the layer may be defined as follows:

$$l_m(z) = 0.4z \text{ for } 0 < \frac{z}{d_m} < 0.1 \quad (17)$$

$$l_m(z) = 0.04\, d_m \text{ for } 0.1 < \frac{z}{d_m} < 1.0 \quad (18)$$

The above distributions were derived from theories by Odd and Rodger (Ref 16). Provided the interfacial shear exceeds the Bingham shear strength and the Reynolds shear stress distribution is linear.

$$\tau_z = \tau_o - (\tau_o - \tau_i)\frac{z}{d_m} \qquad (19)$$

One can define the velocity profile in three parts (Fig 13).

Firstly, a laminar sub-layer with a thickness of a few millimetres where viscous forces prevail and the shear stress is defined as follows:

$$\tau_z = \tau_B + \rho_m \nu_m \frac{du}{dz} \qquad (20)$$

which, with equation 19, results in the following velocity profile:

$$u_1 (z) = \frac{\alpha z}{2 \nu_m} \left(2 - \frac{\beta z}{\alpha}\right) \qquad (21)$$

where

$$\alpha = \frac{\tau_o - \tau_B}{\rho_m} \qquad (22)$$

and

$$\beta = \frac{\tau_o - \tau_i}{\rho_m d_m} \qquad (23)$$

The approximate thickness of the laminar sub-layer is given by Equation (24):

$$\delta = \frac{11.6 \ \nu_m}{\sqrt{\tau_o / \rho_m}} \qquad (24)$$

The second region of the velocity profile extends over about 10% on the fluid mud layer where the momentum mixing length increases linearly with the distance from the bed. The Reynolds stress in this region are defined by

$$\tau (z) = \rho_m (0.4z)^2 \left|\frac{du}{dz}\right| \frac{du}{dz} + \tau_B \qquad (25)$$

which, with equation 19, results in the following velocity profile for Z << 1

$$U_2 (z) = 5\alpha^{\frac{1}{2}} \left\{ \tfrac{1}{2} \ln \left(\frac{z}{\delta}\right) - \frac{\beta}{2\alpha} (z - \delta) \right\} + u_1(\delta) \qquad (26)$$

These two regions only contribute about 5% of the discharge of water and mud in the layer as a whole.

In the third region, which extends over 90% of the layer, the momentum mixing length is assumed to have a constant value and the shear stress is defined by

$$\tau (z) = \rho_m (0.04\, d_m)^2 \left|\frac{du}{dz}\right| \frac{du}{dz} + \tau_B \qquad (27)$$

giving a velocity profile of:

$$u_3 (z) = \frac{1}{0.06\, \beta d_m} \left\{ (\alpha - 0.1\, \beta d_m)^{3/2} - (\alpha - \beta z)^{3/2} \right\}$$

$$+ u_2 (0.1\, d_m) \qquad (28)$$

The predicted velocity profile in a flowing fluid mud layer where the interfacial stress exceeds the Bingham shear strength of the mud is shown in Figure 14, in terms of a relative depth and a relative velocity defined in Equation (29):

$$\frac{u (z)}{\left[\frac{\tau_o - \tau_B}{\rho_m}\right]^{\frac{1}{2}}} = \frac{u(z)}{\alpha^{\frac{1}{2}}} \qquad (29)$$

The velocity profile is linear in the mid-regions of the layer with a slip zone near the bed. If the theory was confirmed by accurate field observations it could be used to calculate the effective friction factor for the layer in terms of its overall Reynolds number.

The general equation for the motion of a mud layer along a channel is as follows: .

$$W(\eta_i) \left\{\frac{\partial \eta_i}{\partial t} + v_\rho\right\} + \frac{\partial}{\partial x}(U_m A_m) = 0 \qquad (30)$$

$$\frac{\partial U_m}{dt} + \frac{\partial}{\partial x}\left(\frac{U_m^{\,2}}{2} + g\,\eta\right) + \frac{g}{\rho_w}\left((d-d_m)\frac{\partial \rho_w}{\partial x} + 0.5\, d_m \frac{\partial \rho_m}{\partial x}\right)$$

$$+ g\left(\frac{\rho_m - \rho_w}{\rho_w}\right)\left(\frac{\partial \eta_i}{\partial x} - \frac{\partial \eta}{\partial x}\right)$$

$$+ \frac{1}{A_m}\left\{\tau_o\, w(\eta_o) - \tau_i\, w(\eta_i)\right\} = 0 \qquad (31)$$

where

- W is the width of the channel (m)
- η is the water surface level
- η_i is the interface level
- η_b is the bed level
- U_m is the mean velocity (m/s)
- A_m is the cross-sectional area of the layer (m^2)

The rate of mud transport in the layer would be defined by Equation (32):

$$\frac{\partial}{\partial t}(C_m A_m) + \frac{\partial}{\partial x}(C_m U_m A_m) - H\left[|\tau_o| - \tau_e\right]$$

$$m_e\left(|\tau_o| - \tau_e\right) w(\eta_b) = 0 \qquad (32)$$

It is not certain whether the standard mud erosion formula could be used to calculate the erosion of the underlying bed in this case.

For estuaries that are wide in relation to their length it would be necessary to solve for flow in the fluid mud layer in two dimensions in plan.

7 DISCUSSIONS AND CONCLUSIONS

The report is a review of the behaviour of fluid mud in estuaries. The authors have attempted to develop basic theories to quantify the processes governing the formation of fluid mud from more dilute suspensions and its motion down a submerged slope and along a flat bed within an estuary. The report does not include an analysis of the generation of fluid mud by wave action or mechanical agitation. But the theories are applicable to the movement of a fluid mud layer formed by those different processes.

Fluid mud is a relatively common phenomena in turbid estuaries. It is a dense suspension containing a concentration of mud flocs which is high enough to significantly alter its physical properties compared to that of clear water with the same salinity and density. Fluid mud occurs at concentrations in the range of about 60,000 - 100,000ppm in the form of well defined layers in the muddy reaches of turbid estuaries. Suspensions with concentrations in the range 10,000 - 20,000ppm have reduced or hindered settling rates but do not have a significant shear strength or enhanced viscosity. The shear strength of mud rapidly increases at dry densities greater than about $0.1 kg/m^3$. At these higher concentrations the authors consider that mud should be treated as a weak soil because it will only flow down a very steep slope. It is recommended that fluid mud be defined as being a flocculated suspension of mud particles with a concentration in the range 60,000 - 120,000, but more typically with a value of about 75,000ppm.

As defined, fluid mud consists of a mixture of flocs of different sizes which continually touch and interact with each other hindering the upward flow of water and transmitting shear stresses by interparticular friction and fluid friction. The shearing of fluid mud causes a deformation and disruption of the mud flocs causing it to behave as a plastic non-Newtonian fluid with a variable Bingham shear strength and variable viscosity. For engineering calculations one can probably assume that it has a yield shear strength of about $0.1 N/m^2$ and a viscosity about twice that of water. However, these values are based on extrapolation made in a rotating cylinder viscometer and they need to be checked by other means.

A layer of fluid mud forms on the bed of an estuary at slack water if the mass flux of mud settling from suspension above the bed exceeds the rate of dewatering of the fluid mud. The rate of dewatering of fluid mud with a concentration of about 75,000ppm is about 0.05mm/s equivalent to a mass flux of about $4g/m^2/s$. There is a need to confirm this value. The minimum concentration in the overlying water which will match this mass flux is about 2500ppm. Therefore, one would not expect a significant layer of fluid mud to form from a suspension in estuaries where the peak concentration is less than about 2500ppm.

A fluid mud layer will gain mass and thickness at a maximum rate of about $40g/m^2/s$ when the overlying water has a concentration of suspended mud of about 20,000ppm.

Field observations and experiments show that the concentration of mud in a fluid mud layer is usually almost uniform throughout its depth. The maximum thickness of a fluid mud layer formed from suspension is directly proportional to the amount of mud held in suspension, provided the bed shear exceeds $0.1N/m^2$ thereby preventing dewatering at the base of the fluid mud layer. Fluid mud layers are usually only a fraction of a metre in thickness and seldom exceed two metres in the most muddy estuaries.

A fluid mud layer will stand on a sloping bed provided the gravitational forces do not exceed its shear strength. A critical thickness for the onset of motion varies directly with the Bingham yield strength and inversely with the slope. Fluid mud which is thicker than this critical value will flow as a body down the slope towards the deepest part of the channel cross-section or along the bottom of a sloping channel during the slack water period when water surface slopes are at a minimum. The mud flows in the form of a turbidity current with density of about $1.07t/m^2$ compared to a typical estuarine water of $1.02t/m^3$. The flow will be smooth turbulent with a slug of mud near the centre of the layer with no relative motion where the internal stress is less than the Bingham yield strength. The turbidity current will tend to thicken and become diluted by entrainment of the overlying water. This effect may be offset by scour of an underlying muddy bed. The thicker the mud layer grows the faster it flows so it is possible that the flow could turn into an avalanche of mud.

One can form an equation for the rate of transport of mud in the turbidity current, but there are one or two empirical coefficients which could only be evaluated by making observations of a mud flow in an estuary such as the Bristol Avon or Parrett estuary.

The turbidity current ceases to flow when either the bed slopes falls below a critical value or it joins a pool of fluid mud in a deep section of the estuary.

A layer of fluid mud will start to flow along the flat bed of an estuary as a result of the imposed hydrostatic head caused by an increasing water surface slope after slack tide. A layer of uniform fluid mud will always shear at its base. The critical depth at which the layer will move varies directly with the Bingham shear strength and inversely with the water surface slope. Thin layers of fluid mud, which form at slack water in many estuaries, are more resistent to motion than thick layers and tend to be eroded from the surface downwards. This process can be investigated in a recirculating flume or a circular flume.

Once a thick layer of fluid mud flows along a flat bed it behaves as a dense lower layer in a two layer flow.

Initially, the layer will move as a slug with no relative internal motion. Gradually, a velocity profile will form near the bed and the slug will completely disappear when the interfacial shear stress at the top of the layer exceeds the Bingham yield shear strength.

The sharp density gradient at the top of a fluid mud layer damps vertical turbulent exchange within the layer and limits the vertical turbulent exchange of momentum for a given velocity gradient to a value of about one hundredth of that for clear water. Theory developed by the authors indicates that this will tend to produce linear velocity profiles within the layer with a corresponding modification of the friction factor for the layer as a whole. The theory of a velocity profile within a fluid mud layer, and hence the rate of mud transport, needs to be checked by making accurate observations in an estuary.

To date it has only been possible to model the formation and instant resuspension of a thick layer of fluid mud in a very turbid estuary (Ref 2). It should be possible to simulate the formation of movement of fluid mud in both canalised and wide estuaries. However, there is a need to check the theories described in this report with field and laboratory observations.

Existing observations on the movement of fluid mud were generally made using standard equipment for measuring suspended concentrations and velocity profiles. The sensitivity and methods of locating the probes relative to the plane of no horizontal motion have not been accurate enough to define the growth, movement and resuspension of fluid mud. There is a need for an accurate set of observations to be made in either the Bristol Avon or the Parrett estuaries. There is also a need to make repeatable laboratory measurements of the rate of dewatering of flocculated fluid mud suspensions in a settling tube.

8 ACKNOWLEDGEMENTS

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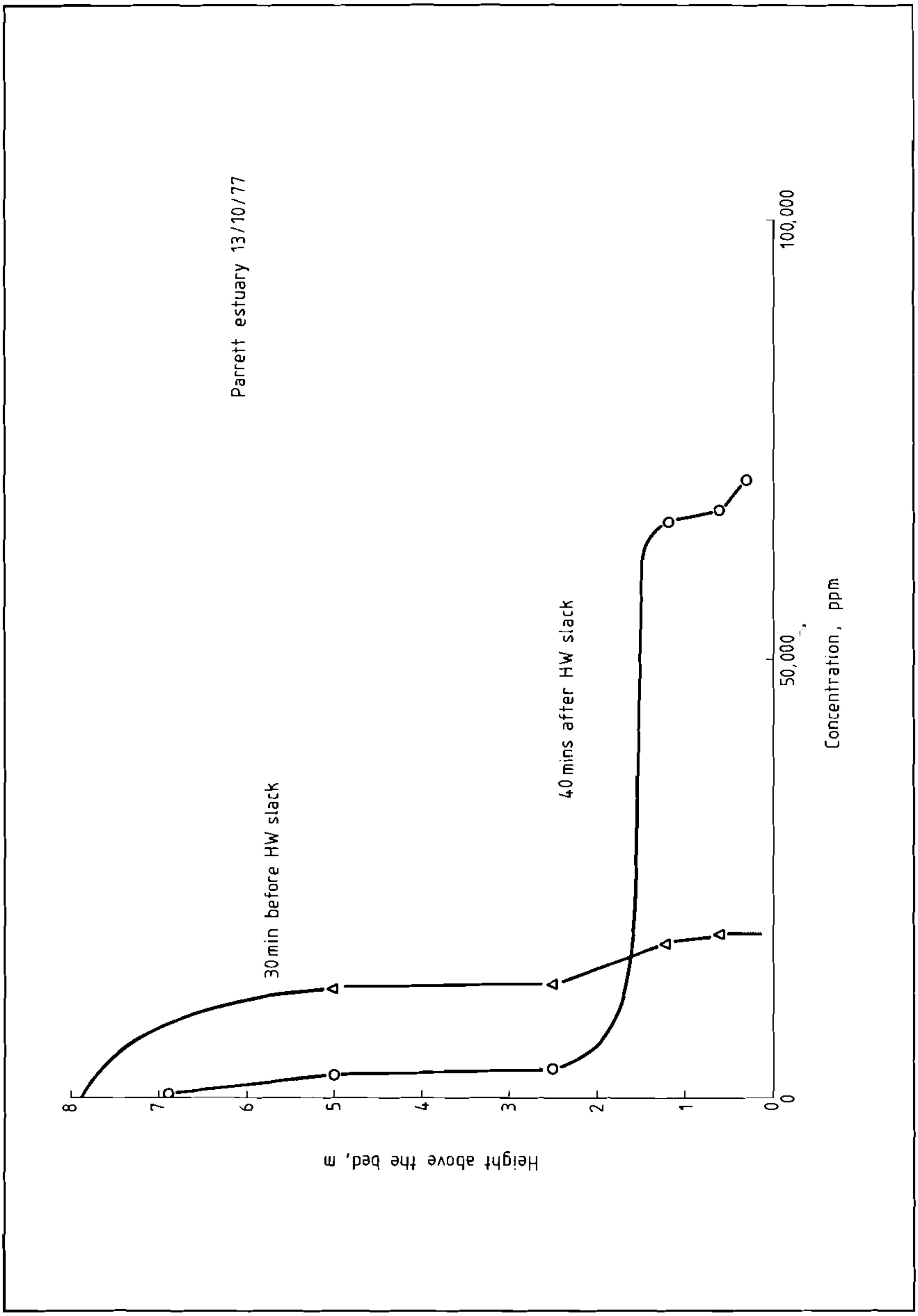

Fig. 1 Formation of fluid mud layer at slack water

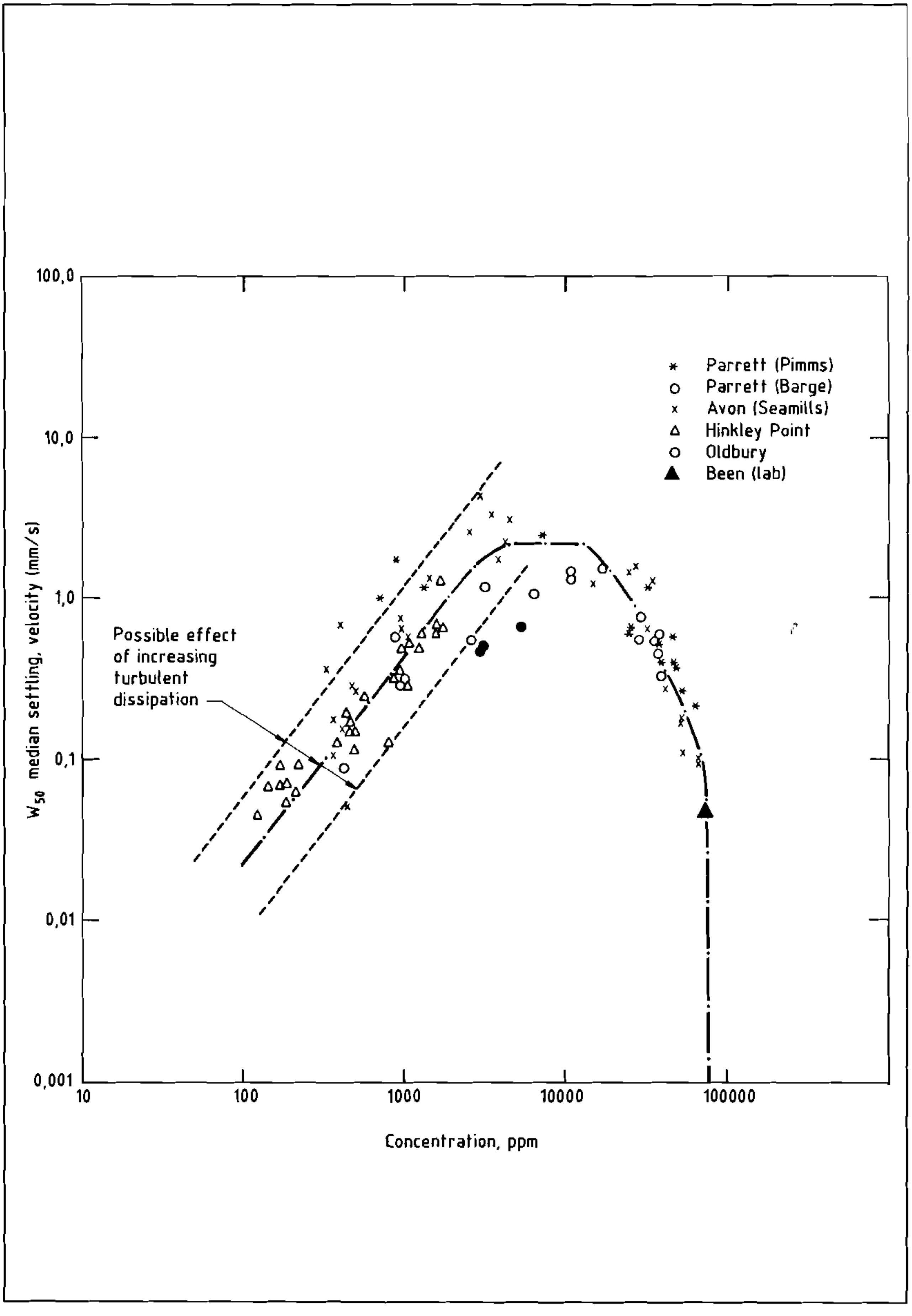

Fig 2 Settling velocity of Severn mud as a function of concentration

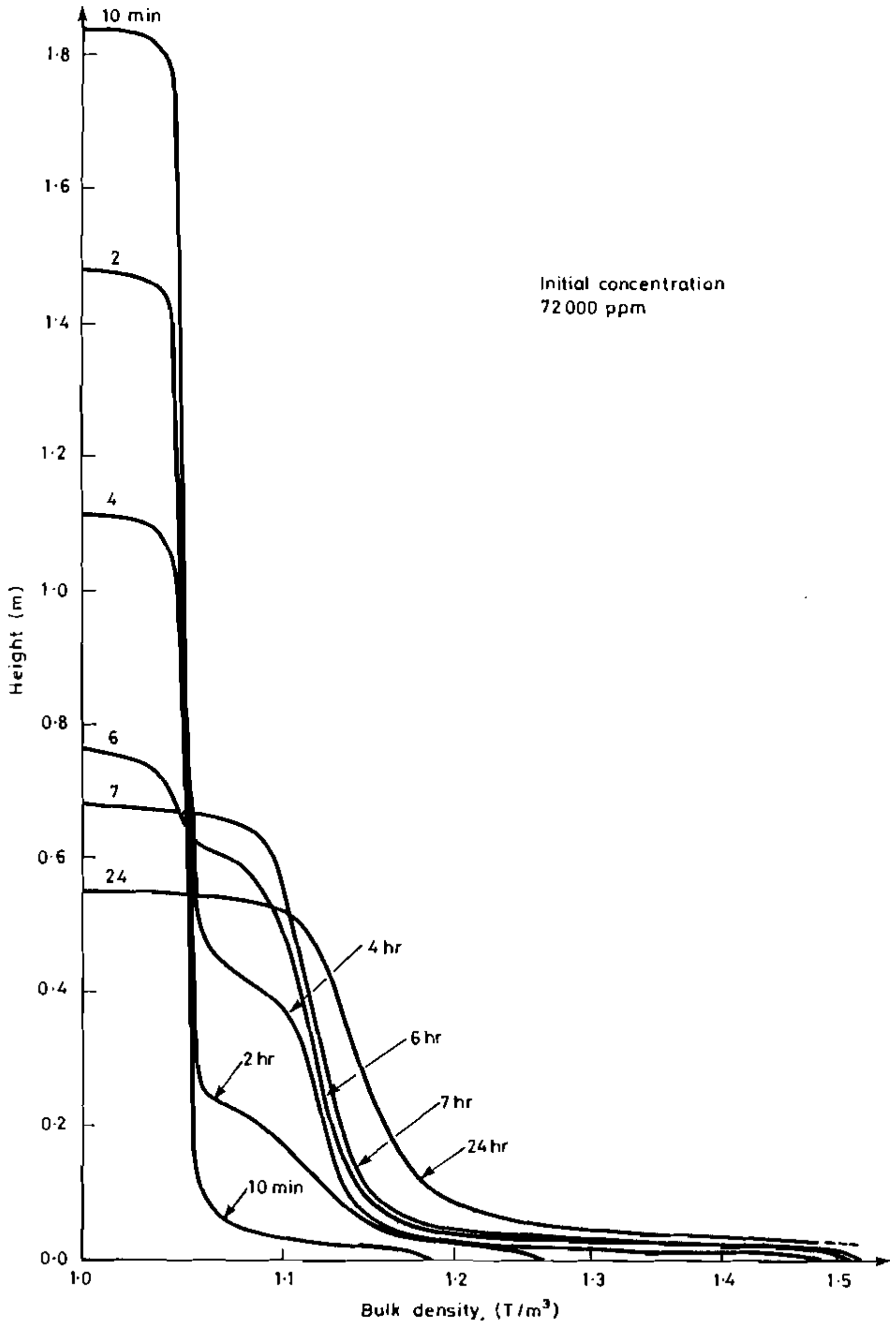

Fig 3 Dewatering of fluid Severn mud: density profiles (K. Been)

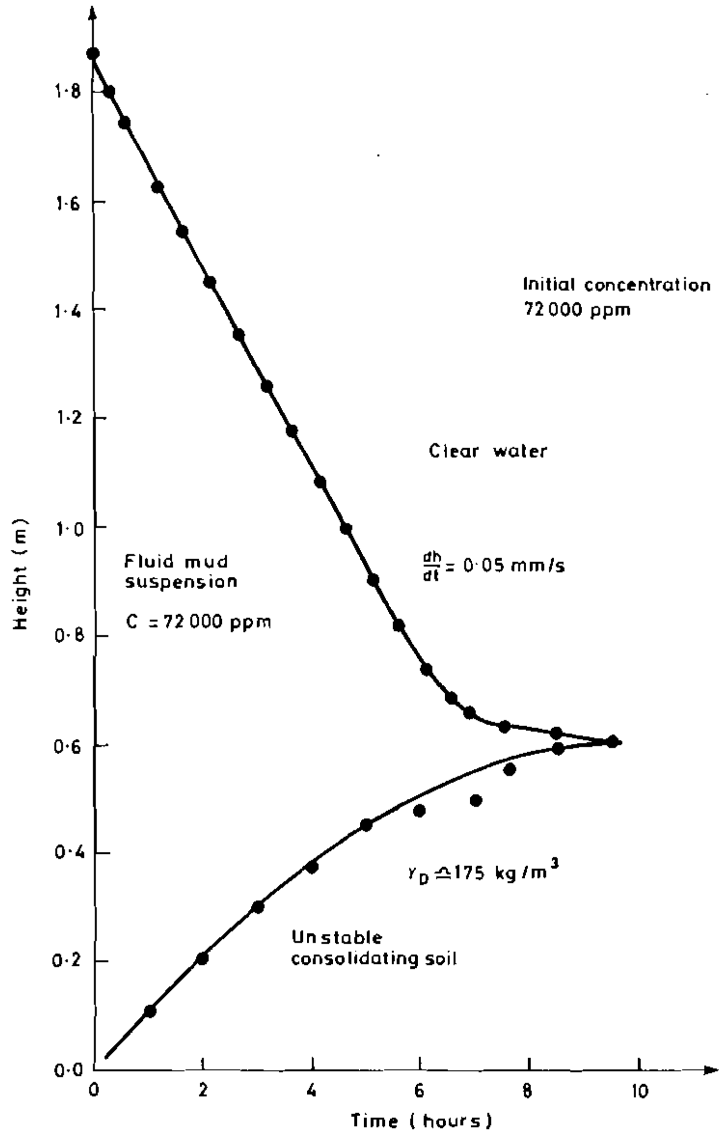

Fig 4 Dewatering of fluid Severn mud: movement of interfaces (K. Been)

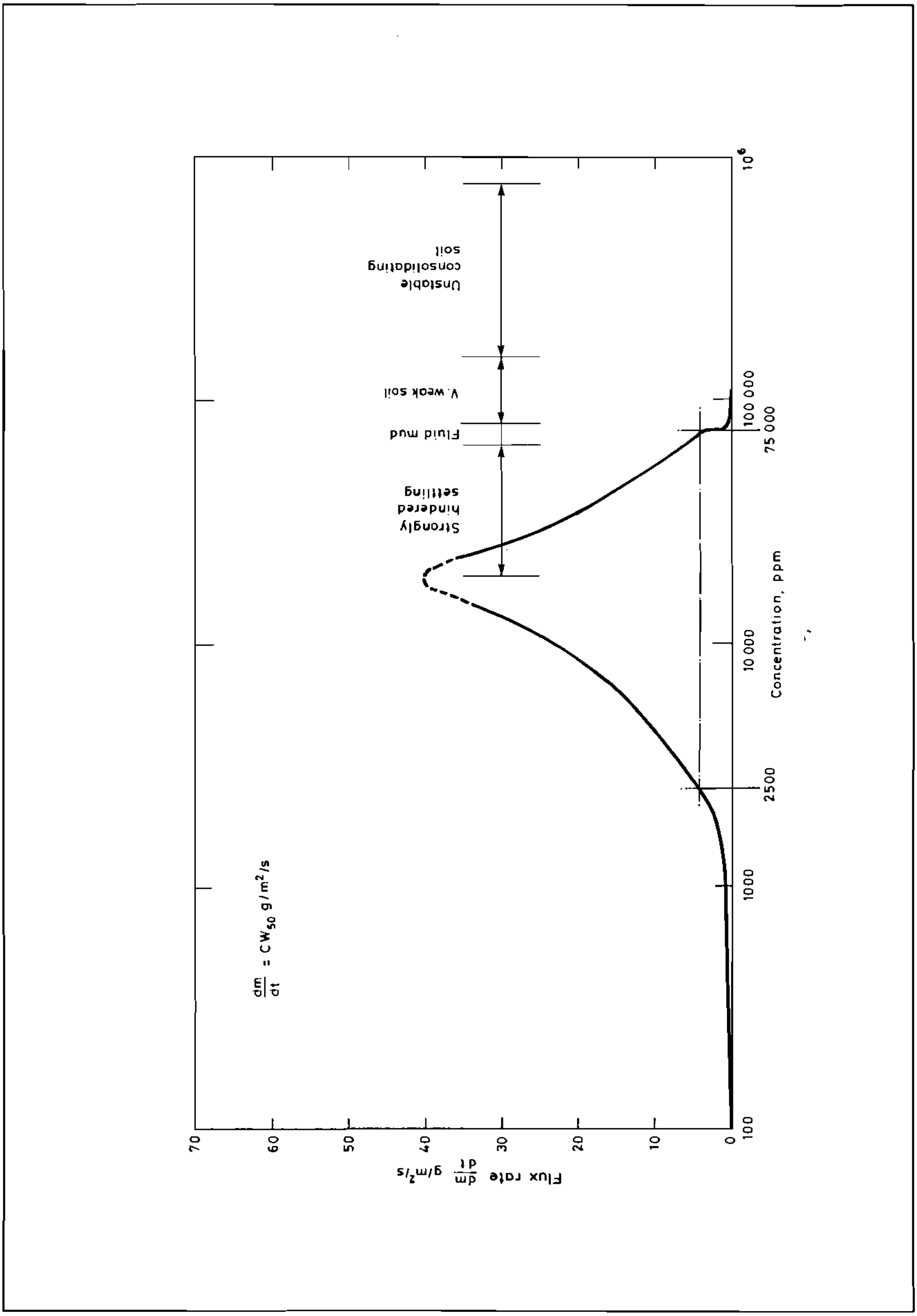

Fig 5 Flux of settling mud as a function of concentration

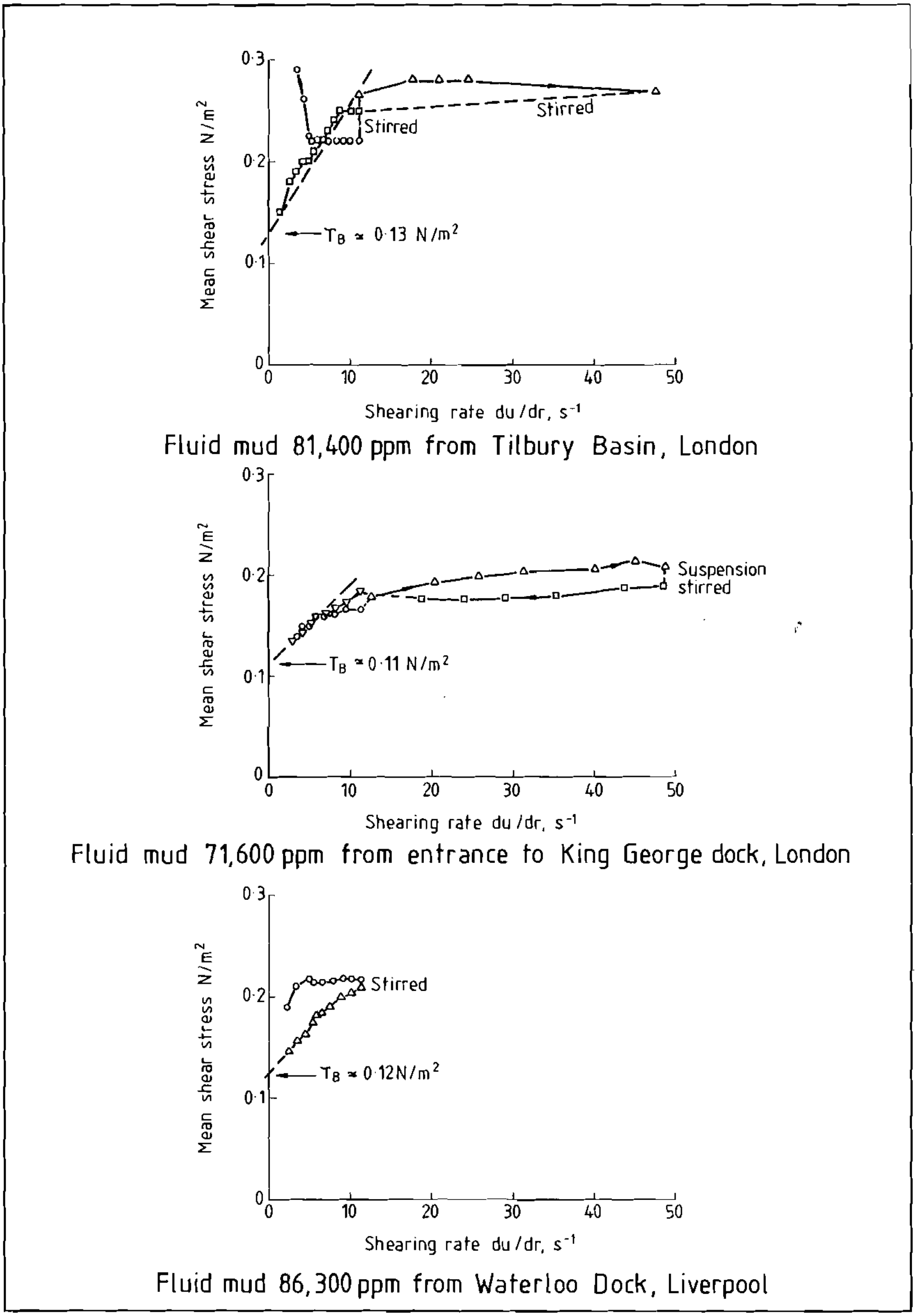

Fig. 6 Behaviour of fluid mud in a rotating cylinder viscometer (laminar conditions)

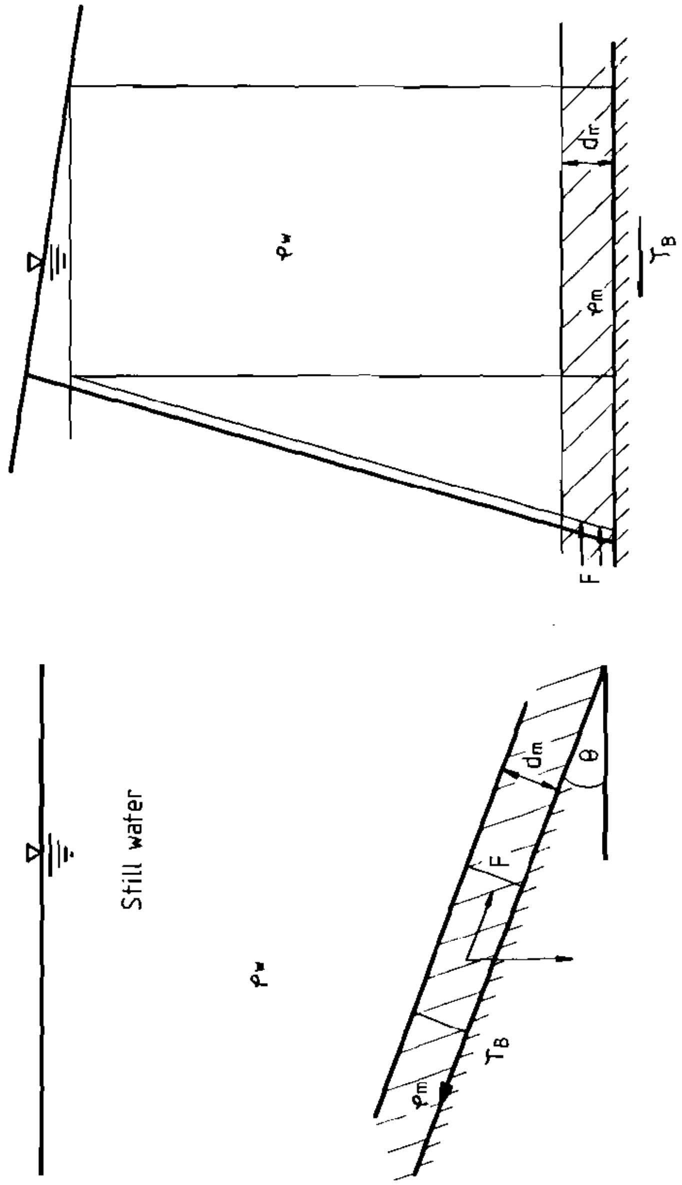

Fig. 7 Main body forces causing the motion of a fluid mud layer

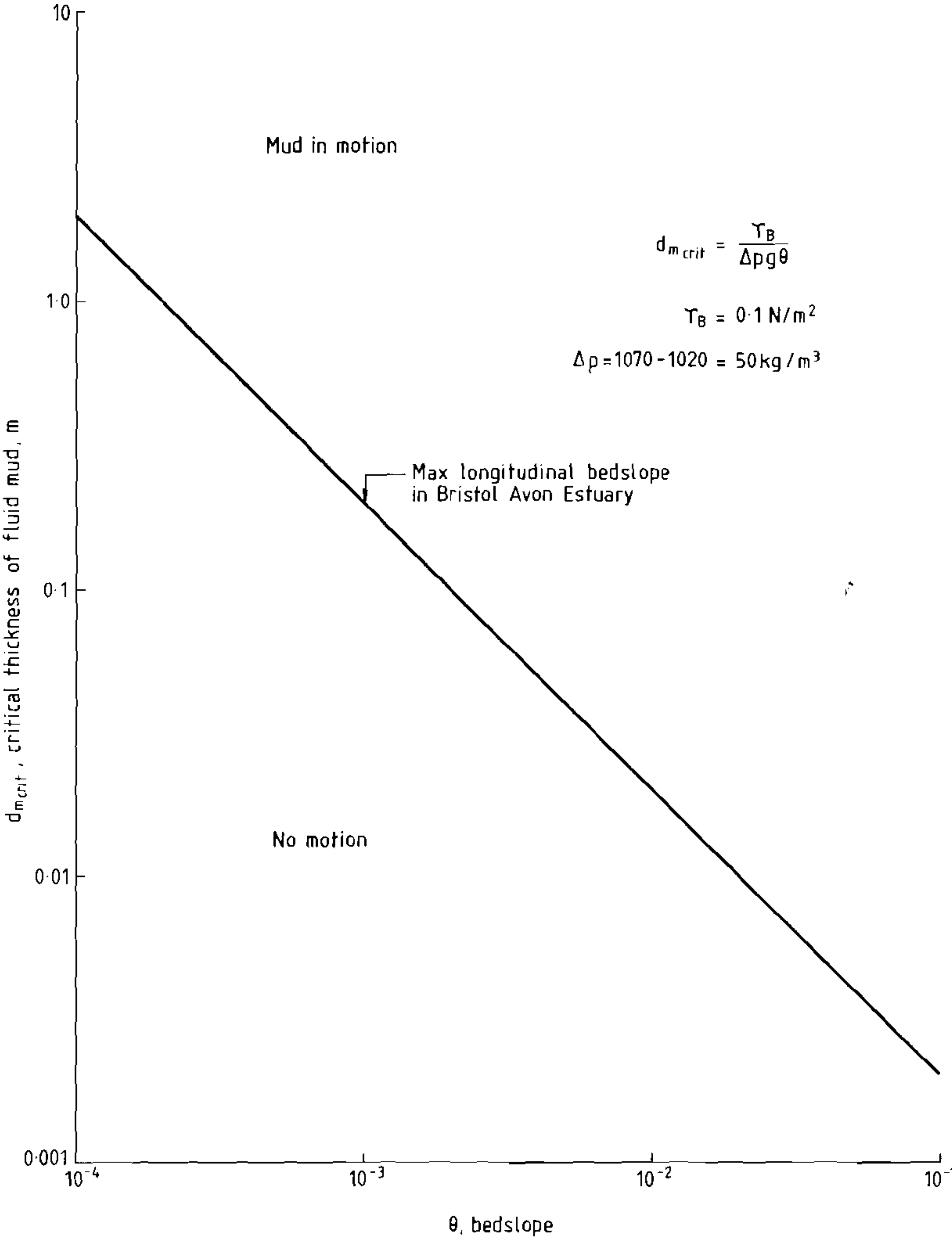

Fig. 8 Thickness of a fluid mud layer that can stand on a given bed slope

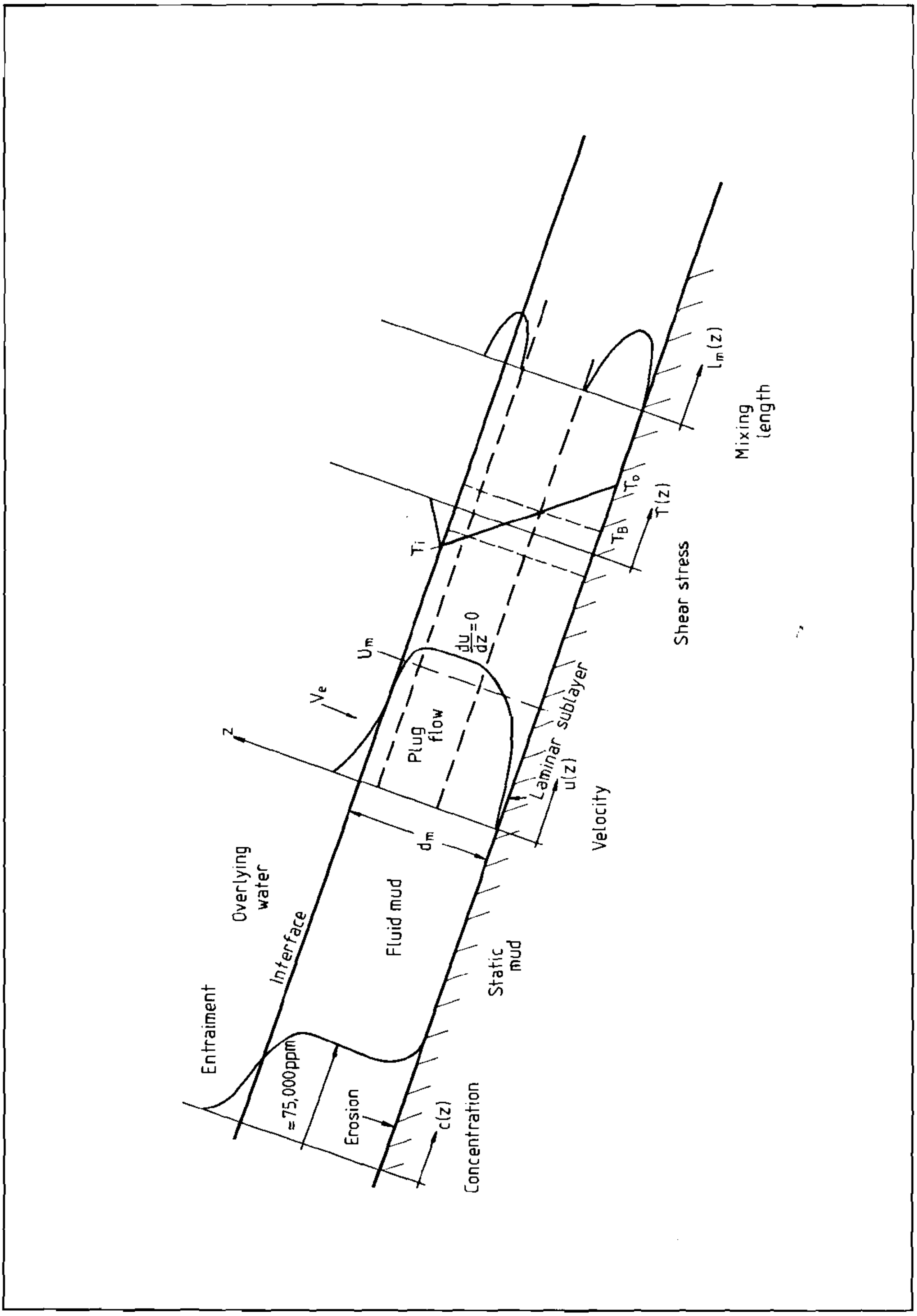

Fig 9 A layer of fluid mud flowing down a slope in still water

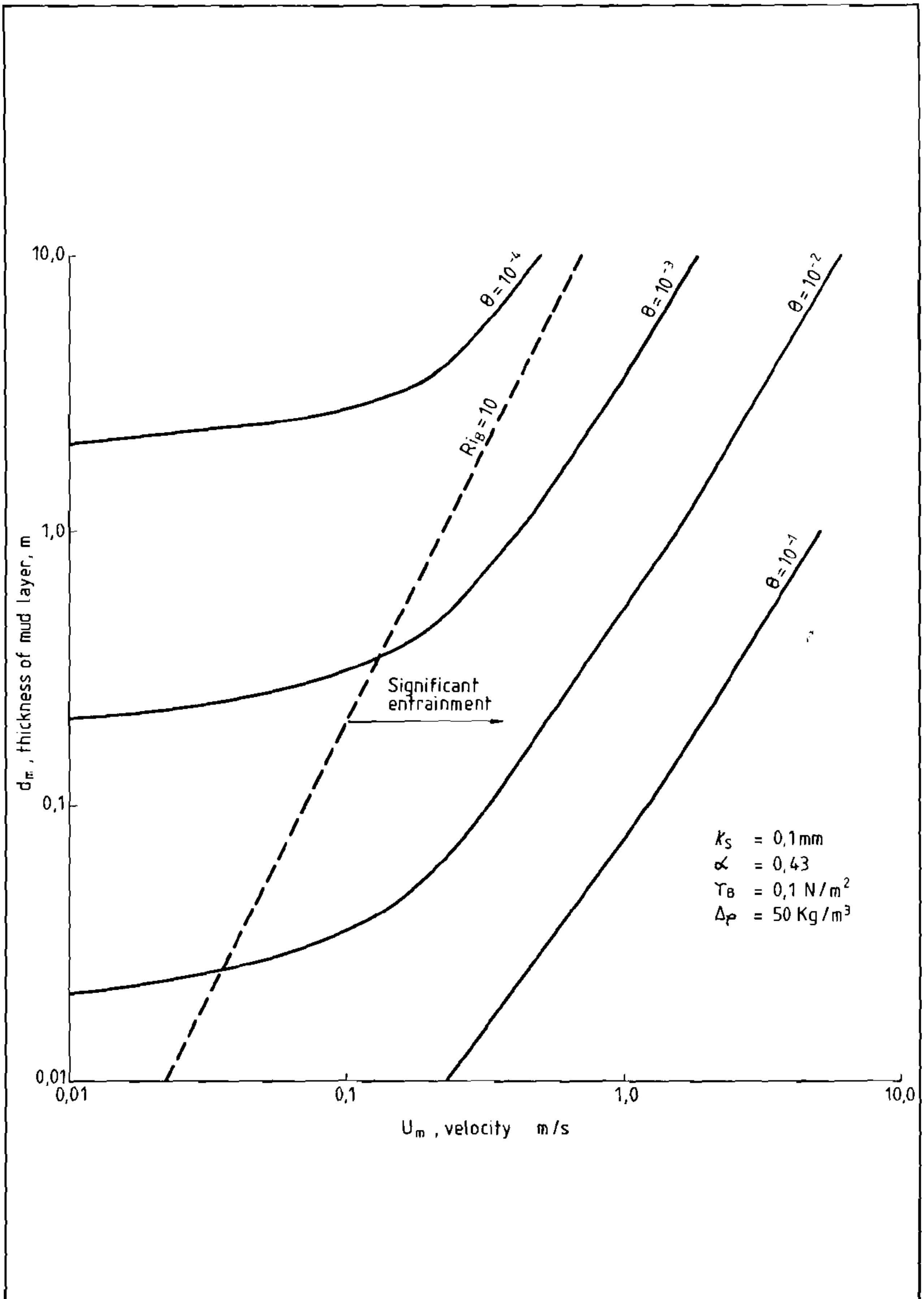

Fig. 10 Motion of fluid mud down a sloping bed in still water

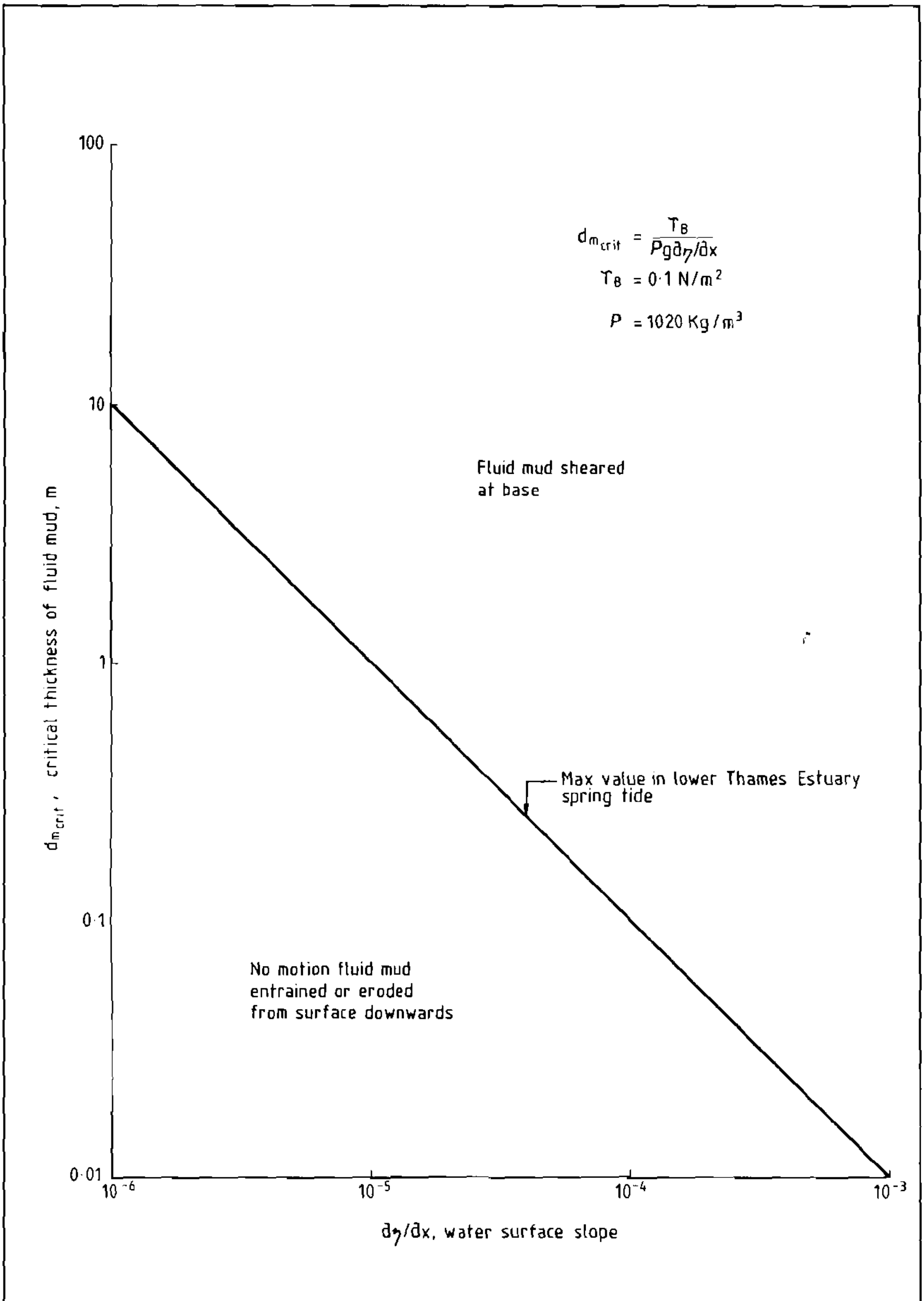

Fig 11 Thickness of a fluid mud layer that can resist movement by a given imposed water surface slope

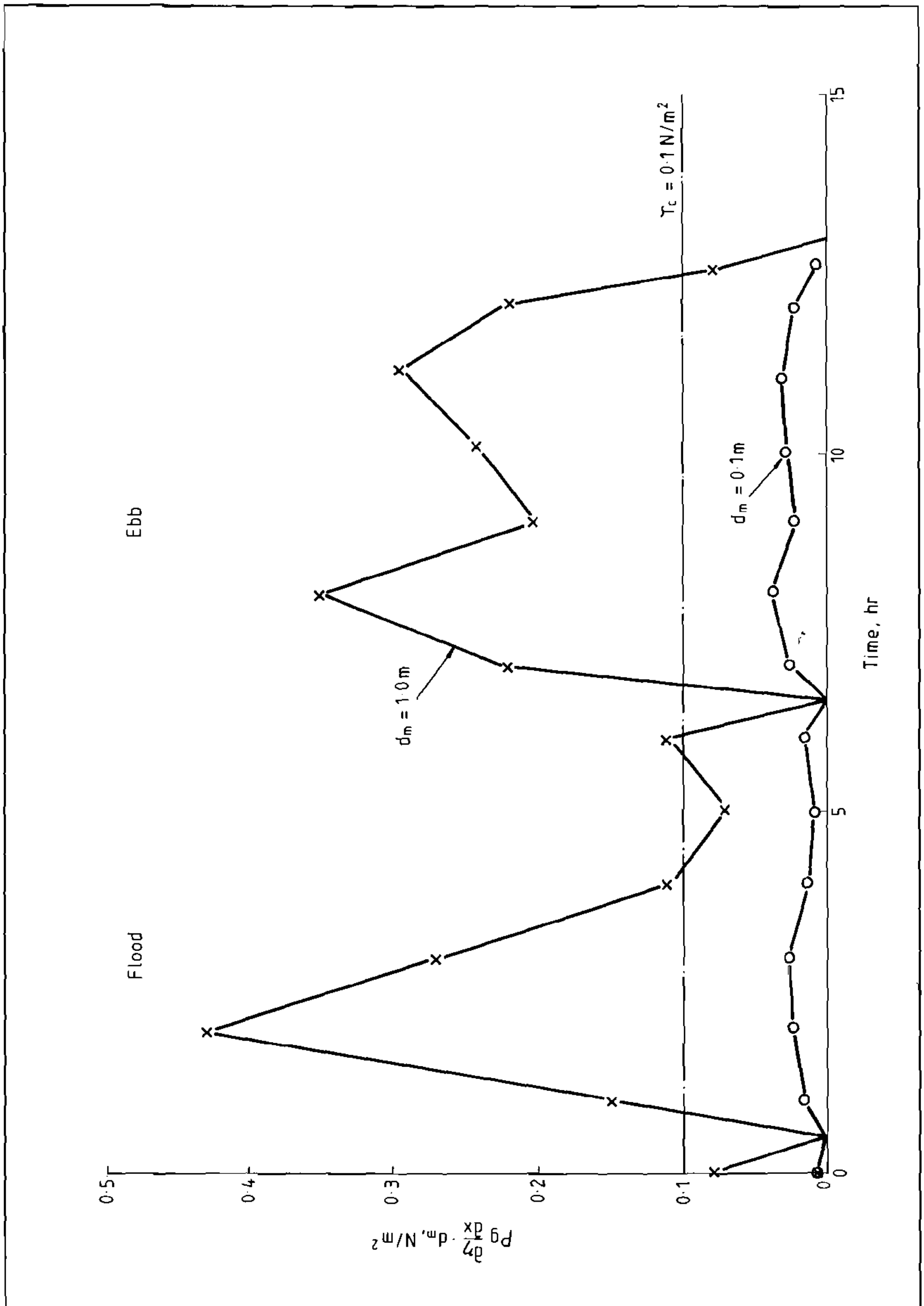

Fig 12 Effective shearing stress at the base of a fluid mud layer in the Thames Estuary

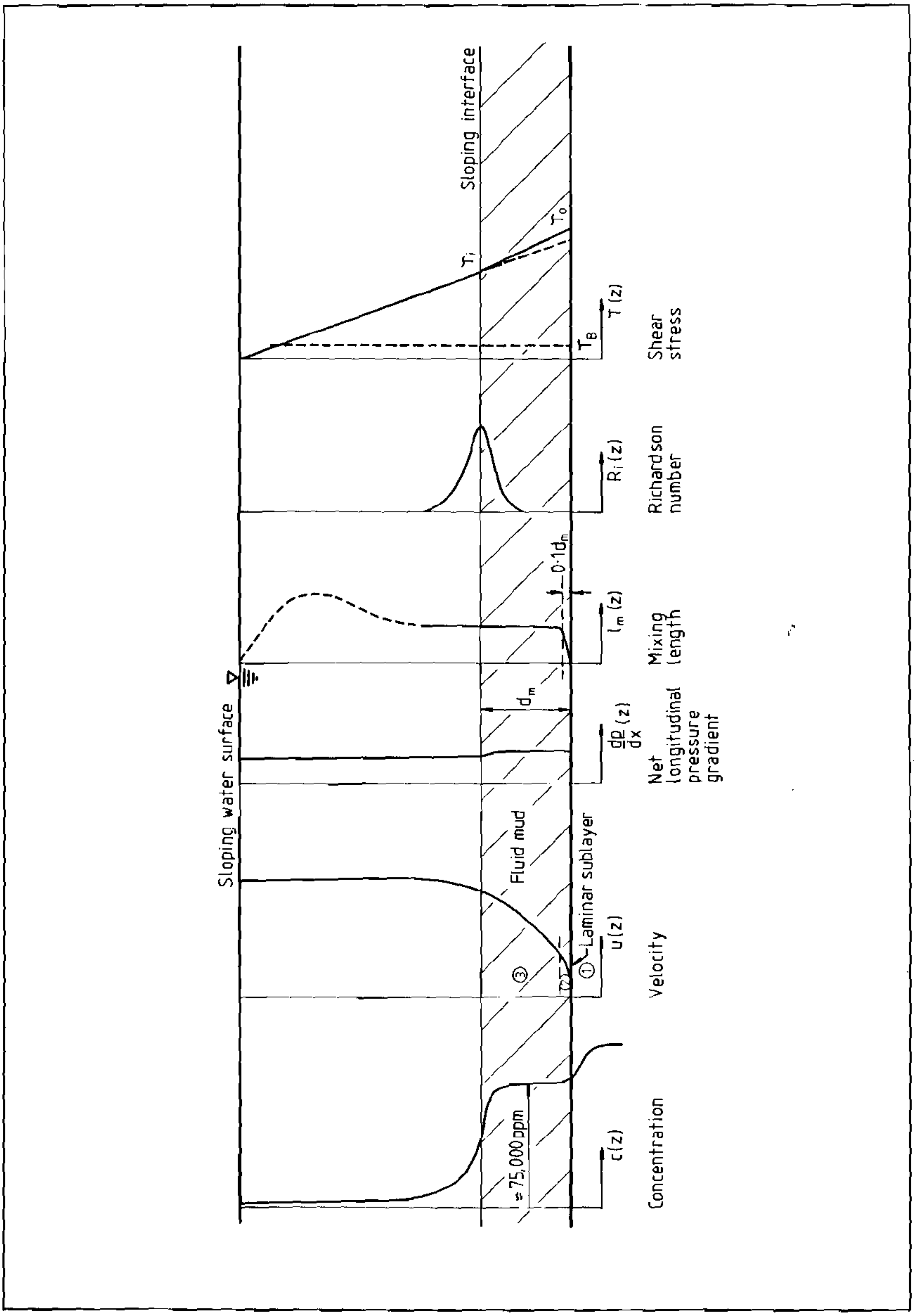

Fig 13 A layer of fluid mud flowing along a channel with a flat bed $\tau_i > \tau_B$

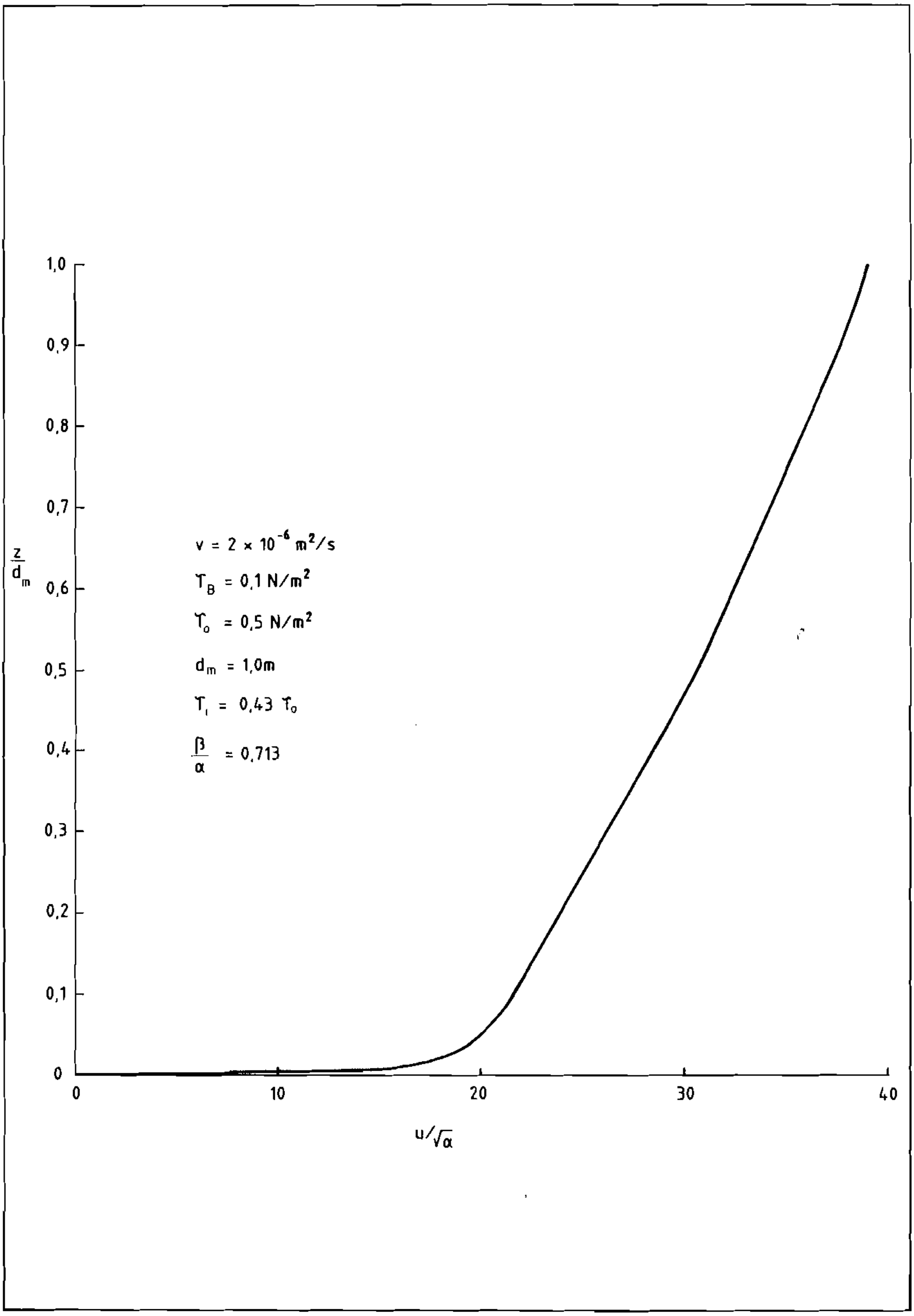

Fig 14 Non-dimensional velocity profile in a fluid mud layer