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A STUDY ON RIVER REGIME

By

W R White BSc, PhD
R Bettess BSc, PhD
Wang Shiqiang

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Registered Office: Hydraulics Research Limited,
Wallingford, Oxfordshire OX10 8BA.
Telephone: 0491 35381. Telex: 848552

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ABSTRACT

The purpose of regime theory is to predict the size and shape of stable alluvial channels. The theory was first developed from empirical studies based on extensive field measurements. Recent improvements in our understanding of sediment transport processes, however, have introduced the possibility of relating the size of regime channels to these fundamental sediment transport processes. The general approach is described together with a number of extremal hypotheses which have been suggested to determine regime conditions. These extremal hypotheses assume that the channel dimensions are such to maximise or minimise the value of some appropriate functional. The predictions of channel dimensions using various extremal hypotheses and sediment transport relationships are compared with observed channel data. The application of regime theory to natural rivers requires the definition of a dominant discharge. A number of proposed expressions for dominant discharge are investigated and compared with data from gravel rivers.

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APPENDICES:

NOTATION

A	mean value
B (m)	width
C	sediment concentration
C_1, C_2, C_3	coefficients
D (m)	sediment diameter
d (m)	water depth
Fr	Froude number
f ($m^{\frac{1}{2}}$)	silt factor
g (ms^{-2})	acceleration due to gravity
k	coefficient in sediment transport equation
k_1, k_4	coefficients
L (m)	length of reach
m_1, m_2, m_3	coefficients
n	Mannings n value
P (m)	wetted perimeter
Q (m^3s^{-1})	discharge
Q_i (m^3s^{-1})	maximum monthly discharge
Q_s (m^3s^{-1})	sediment discharge
Q_{si} (m^3s^{-1})	sediment discharge corresponding to maximum monthly discharge
Q_T (m^3s^{-1})	total water and sediment discharge
R (m)	hydraulic radius
S	slope
SD	standard deviation
V (ms^{-1})	velocity
X	sediment concentration by weight
z	side slope of channel

α	parameter
β	parameter
γ	specific weight of water
γ_s	specific weight of water and sediment mixture
ρ (km^{-3})	density of water
ρ_s (km^{-3})	density of sediment
w (ms^{-1})	fall velocity

1 INTRODUCTION

Regime theory is the prediction of the size and shape of stable channels flowing through alluvium. The subject developed out of the desire to design large irrigation channels, particularly in the Indian sub-continent, in the late nineteenth and early twentieth century. Experience had taught engineers that if a canal was constructed through alluvium with no consideration to the hydraulics and sediment transport of the flow then sometimes sediment would be deposited on the bed and the channel aggrade or, in other channels, the bed would be eroded. Similarly in some canals the banks would be eroded while in others material would be deposited at the edges of the canal and the main flow would meander from side to side. It was postulated by the engineers of the period that for a given water discharge there was one stable channel of a given size and slope that would convey the flow. If a channel of a different size or slope was constructed then accretion or erosion would take place. Channels which did not alter appreciably from year to year - though possibly varying during the year - were said to be in 'regime' (Inglis, 1949). The prediction of size and slope of such stable channels became known as regime theory.

The initial approach to the subject was an empirical one. Measurements were taken on existing channels which were demonstrably in regime and equations were sought to relate the observed size, shape and slope of the channel to the discharge it carried (Lindley, 1919). Lacey (1929, 1933) advanced the subject by appreciating that not only the discharge but the type of sediment through which the the channel flowed was significant. The equations he formulated were

$$V = 1.15 \sqrt{(fR)} \qquad (1)$$

$$P = 2.67 \sqrt{Q} \qquad (2)$$

$$S = 0.00055\ f^{\frac{1}{2}}\ Q^{-\frac{1}{2}} \tag{3}$$

where V is the velocity, R hydraulic radius, P wetted perimeter, Q discharge, and S the slope. Note that these equations are in foot, second units. f is a coefficients, termed the silt factor, which depends on the sediment diameter D and is given by

$$f = \sqrt{(2.5D)} \tag{4}$$

The empirical equations that were developed have some disadvantages. There appear to be regional variations so that equations developed from data from canals in the Punjab cannot be confidently applied to alluvial channels in mid-USA. Also the equations can only be applied within the range of the data for which they were derived.

The equations (1) to (3) are typical of the type of relationships developed by empirical regime theory. A channel has effectively three degrees of freedom: it may adjust its width, depth and slope. Three equations are, therefore, required to describe it completely. This implies that if three independent equations linking the appropriate variables and describing three relevant but different physical phenomena are specified then they can be solved to produce a regime theory. It has long been thought that the appropriate phenomena are:

1. alluvial friction
2. sediment transport
3. stability of the banks

Such a regime theory, based on the equations describing the dominant physical processes, has been termed a rational regime theory.

The advantages of such a rational regime theory are its universal nature in that it can be applied with confidence anywhere in the world and that the range of application is only limited by those of the underlying theories describing the physical processes. There is also the advantage that the relationship between the dominant physical processes is more explicit and hence more easily understood.

Regime theory arose out of the need to design irrigation and drainage canals but its use has not been limited to this application. Recently it has been used to study natural rivers, some of the problems of such applications are discussed later. Regime theory has been used in the design of physical models (White, 1982) and in assessing morphological changes in rivers as a result of engineering works (HR, 1983). It has also been used in developing an explanation of the meandering and braiding of streams (Bettess and White, 1984).

2 RATIONAL REGIME THEORY

As explained in the introduction, rational regime theory is based on the belief that a knowledge of the dominant physical processes can be used to determine the channel dimensions and slope. Since the channel has three degrees of freedom; it may alter the width, depth or slope, it follows that three equations are required to make the system soluble. Evidence suggests that sediment transport and alluvial friction are important physical processes whose description must be included. A formulation of the remaining required equation, however, has remained problematical. A number of extremal hypotheses have been suggested to provide this third relationship. Attempts have been made to justify the use of such extremal hypotheses by analogy with extremal principles in Newtonian mechanics but no satisfactory, rigorous justification has yet been provided.

In 1982, White et al considered in detail the regime theory developed using the Ackers and White sediment relations (Ackers and White ; White et al, 1980) together with the principle of maximum sediment transport rate, that is, it is assumed that the width of the channel is such that the sediment transport rate is a maximum. By comparing the results of the regime theory with a wide range of field and flume data they successfully demonstrated that such a rational regime theory could provide valuable predictions of regime channels for a wide range of conditions.

The work of White et al (1982) posed two problems, however, what would be the effect on the regime theory developed if the Ackers and White sediment relations were replaced by other sediment transport and alluvial friction equations and what would be the effect of replacing the principle of maximum sediment transport rate with other extremal principles. Though the White et al regime theory had been shown to be extremely useful it was unknown whether it could be improved by using other sediment relationships.

White et al (1982) had shown that the principle of maximum sediment transport rate was equivalent to the principle of minimum stream power so that the resulting regime theory was the same. It was, therefore, not clear whether the various extremal hypotheses that had been proposed were different formulations of the same principle and would hence result in the same regime theory or whether there were essential differences between the various extremal hypotheses.

Sections 3 and 4 summarise work which has already been published in 'Extremal hypotheses applied to river regime' by Dr R Bettess and Dr W R White, a paper presented at International Workshop on Problems of Sediment transport in gravel-bed rivers, Colorado

State University, USA, August 1985, and in 'A rational approach to river regime' by Wang Shiqiang, Dr W R White and Dr R Bettess, a paper presented at the 3rd International Symposium on River Sedimentation, University of Mississippi, USA, March 1986. Copies of these papers appear in the Appendix.

3 EXTREMAL HYPOTHESES

3.1 Extremal hypotheses

A number of extremal hypotheses have been proposed to provide the equations necessary to formulate regime relations. These are now discussed and, where possible, related to each other.

<u>Minimum Stream Power (Chang, 1980)</u>

This hypothesis is stated as follows: 'For an alluvial channel, the necessary and sufficient condition of equilibrium occurs when the stream power per unit length of channel γQS is a minimum subject to given constraints, where γ is the specific weight of water, Q is discharge and S is slope. Hence, an alluvial channel with water discharge Q and sediment load Q_s as independent variables tends to establish its width, depth and slope such that γQS is a minimum. Since Q is a given parameter, minimum γQS also means minimum channel slope', Chang (1980).

<u>Minimum Unit Stream Power (Yang and Song, 1979)</u>

This hypothesis is stated as follows: '... for subcritical flow in an alluvial channel, the channel will adjust its velocity, slope, roughness and geometry in such a manner that a minimum amount of unit stream power is used to transport a given sediment and water discharge', Yang and Song (1979).

Unit stream power is defined as stream power per unit weight of water

$$\frac{Q\gamma}{\rho g}\frac{LS}{BdL} = VS \tag{5}$$

where L is the length of the reach, B is the width, d is depth, g is acceleration due to gravity and V is velocity.

Maximum Friction Factor (Davies and Sutherland, 1980)

This hypothesis is stated as follows: 'If the flow of a fluid past an originally plane boundary is able to deform the boundary to a non-planar shape, it will do so in such a way that the friction factor increases. The deformation will cease when the shape of the boundary is that which gives rise to a local maximum of friction factor. Thus the equilibrium shape of the non-planar, self-formed flow boundary or channel corresponds to a local maximum of friction factor', Davies and Sutherland (1980).

The friction factor is given by

$$f = \frac{8gdS}{V^2}. \tag{6}$$

Using the continuity equation

$$Q = BVd \tag{7}$$

we have

$$f = \frac{8gB^2d^3S}{Q^2} \tag{8}$$

Minimum Energy Dissipation Rate (Brebner and Wilson, 1969, Yang et al, 1981)

This hypothesis is stated as follows: 'A system is in an equilibrium condition when its rate of energy dissipation is at a minimum value', Yang et al (1981).

The rate of energy dissipation in a reach of a stream of length L is given by

$$(Q\gamma + Q_s\gamma_s)\ LS, \tag{9}$$

where Q and Q_s are the water and sediment discharges, respectively and γ and γ_s are the specific weights of water and sediment, respectively.

Maximum Sediment Transport Rate (Singh, 1961; White et al, 1982)

This hypothesis is stated as follows: '... for a particular water discharge and slope, the width of the channel adjusts to maximise the sediment transport rate.' White et al (1982).

Minimum Froude number

For a particular water discharge and sediment load, the width of the channel adjusts to minimise the Froude number $[Fr_{min}]$.

Minimum total friction resistance

For a given discharge and sediment load the channel adjusts to minimise the total frictional resistance $[FR_{min}]$.

Minimum friction factor

For a given discharge and sediment load the channel adjusts to minimise the friction factor $[FF_{min}]$.

Minimum discharge

For a particular slope and sediment concentration the channel characteristics are those associated with the smallest discharge.

The operation of some of these extremal hypotheses in determining channel width is demonstrated in Figure 1.

3.2 Relationships between extremal hypotheses

Although from the statements of these hypotheses they all look different a number of them can be related to each other.

White et al (1982) showed that maximum sediment transport rate is equivalent to minimum stream power for a fixed discharge Q. This equivalence is independent of the sediment relations used. Davies and Sutherland (1983) point out that when considering minimum energy dissipation rate for sediment concentrations less than 1000 ppm by weight, the error in neglecting the $\gamma_s Q_s$ term is less than 0.1% and so minimum energy dissipation rate is equivalent to minimising γQLS which is equivalent to minimum stream power. The similarity can be further demonstrated (Brebner and Wilson, 1967). If we define Q_T to be the total discharge of water and sediment and C to be the sediment concentration by volume then

$$Q = Q_T (1-C) \text{ and } Q_s = CQ_T. \qquad (10)$$

We have therefore

$$Q\gamma + Q_s \gamma_s = Q_T (1-C)\gamma + CQ_T \gamma_s. \qquad (11)$$

$$= Q_T \left[(1-C)\gamma + C\gamma_s \right] \tag{12}$$

but $\left[(1-C)\gamma + C\gamma_s \right]$ is the specific gravity of the mixture so $Q\gamma + Q_s\ \gamma_s$ becomes $Q_T\ \gamma_T$ where both refer to the combined water and sediment mixture. Minimum energy degradation is thus equivalent to minimising $Q_T \gamma_T$ LS.

Davis and Sutherland (1980) proposed the extremal hypothesis that there should be an extremum in the friction factor. The expression used for the friction factor was

$$f = \frac{8gdS}{V^2} \tag{13}$$

Since the definition of Froude number is

$$Fr = \frac{V}{\sqrt{g}} \tag{14}$$

It follows that

$$f = \frac{8S}{FR^2} \tag{15}$$

We have therefore that maximising the friction factor is equivalent to minimsing the Froude number for a given slope.

4 COMPARISON OF EXTREMAL HYPOTHESES AND SEDIMENT RELATIONSHIPS

At the initiation of the study there was no indication of whether the various proposed extremal hypotheses were essentially the same or whether they would provide different results. Nor was it clear what impact the selection of different sediment relations would have on the regime theory developed.

4.1 Effect of different extremal hypotheses

The first part of the study concentrated on comparing the regime theories obtained using the Ackers and White sediment relationships or the Chang-Parker sediment transport equation together with the Keulegan friction law. The latter sediment relationships were chosen as Griffiths (1984) had indicated that the relationships seemed to demonstrate curious behaviour. For a fixed sediment diameter, discharge and sediment concentration, regime conditions were found for the following extremal hypotheses:

(a) minimum stream power
(b) minimum unit stream power
(c) maximum energy dissipation
(d) maximum friction factor

Since it has previously been shown that maximum sediment transport rate is equivalent to minimum stream power this extremum hypothesis was not considered separately. Under the present formulation no maximum was found in the friction factor. It has been reported (A Bassi, private communication) that using the different formulation of fixed values for sediment diameter, discharge and channel slope there is a maximum in the friction factor but this has yet to be investigated. The results showed that, as indicated above, maximum energy dissipation was for all practical cases equivalent to minimum stream power.

Since the sediment relationships used were derived from laboratory from rectangular channels it was assumed that the initially calculated widths and depths were for a rectangular channel. The values of width and depth were then adjusted to give values corresponding to a trapezoidal section of the same cross-sectional area, where the side slope z (z horizontal to 1 vertical) of the trapezoid was given by Smith's (1974) empirically determined relationship:

$$z = \begin{cases} 0.5 \text{ if } Q < 1\text{m}^3/\text{s} \\ 0.5\, Q^{0.25} \text{ if } Q > 1\text{m}^3/\text{s} \end{cases} \qquad (16)$$

If the width to depth ratio is large these adjustments are small. Since the Chang-Parker sediment relationship was derived on predominantly laboratory data the same procedure of adjustment was applied to results obtained using this equation. Problems did arise in some cases, however, where the width to depth ratio was as low as 1×10^{-5}. In such circumstances the adjustment procedure is totally unrealistic.

The predicted widths for Ackers and White and the extremum hypotheses of minimum stream power and minimum unit stream power for a range of discharges are shown in Figure 2. The results are for a D_{35} size of 0.01m and a sediment concentration of 10 ppm. For comparison purposes various empirically derived regime relationships are also shown. Since the Ackers and White relationships depend upon sediment diameter and sediment concentration the predictions of the Ackers and White theory for gravel rivers should be shown as a region rather than a single curve on this graph, so that a direct comparison is difficult but it can be seen that there is reasonable agreement between the empirically and theoretically derived results. It can further be seen that the differences between the hypotheses of stream power and unit stream power are no larger than the uncertainty in the empirically derived equations and for this parameter range there is no basis for preferring one hypothesis to the other.

The same Figure shows the results using the Yang-Parker transport equation and the Keulegan friction law. It can be seen that using hypotheses of minimum stream power and minimum unit stream power the width is wildly overestimated. This demonstrates that the behaviour the various extremal hypotheses is dependent on the sediment transport relationships with which they are associated and the two cannot be considered independently.

A comparison was also made of the predictions of slope. Figure 3 shows regime slopes predicted by various empirically derived regime equations and from regime equations based on Ackers and White sediment relationships. The Ackers and White results are based on sediment diameters of 0.01m and 0.1m and a sediment concentration of 10 ppm. Appropriate sediment diameters were used in the empirical equations. Again direct comparison is difficult since the Ackers and White results depend upon both sediment diameter and sediment concentration and so are more properly plotted as a region on this Figure. The results using the minimum stream power and minimum unit stream power are indistinguishable on this plot. The results for Parker-Chang sediment transport equation and Keulegan friction equation with a sediment diameter of 0.01m are also shown.

Griffiths (1984) studied regime relationships provided by using the Ackers and White sediment relationships together with the principles of minimum stream power and minimum unit stream power and came up with results somewhat at variance with those of White et al (1982) using the identical sediment relations and extremal hypotheses. The differences leading to the different conclusions were in the details of the sediment relationships. White et al used the hydraulic radius in the expressions for the sediment mobility and shear velocity whereas Griffiths used depth. This apparently minor change leads to major changes in the width dependence of the system. Results using the two different formulations are shown in Figure 4. The Ackers and White results are based on a sediment diameter of 0.01m and a sediment concentration of 10 ppm. The radical differences between the results are partly disguised by the rectangular to trapezoidal transformation described above but it is clear that the use of depth rather than hydraulic radius in both the expression for sediment mobility and shear velocity leads to unsatisfactory results. Tests indicated that the replacement of R by d in the expression for sediment mobility made only a minor change, the major change resulting from the replacement of R by d in the expression for the shear velocity.

Having established that different sediment relationships and extremal hypotheses generate different regime relationships it is important to establish which combination provides the best predictions. To study this the predictions of the various theories were compared with field data.

The observed data consisted of 203 sets of data from sand rivers and canals and 59 sets of data from gravel rivers. The data from sand channels covered the following ranges

$0.34 < Q(m^3/s) < 24{,}300$

$1.8 < B(m) < 1100$

$0.11 < D(mm) < 4.7$

$1 < X(ppm) < 3000$

The data was selected from International Commission on Irrigation and Drainage (1966) and Brownlie (1981).

The gravel river data covered the range

$2.7 < Q(m^3/s) < 9000$
$5.2 < B(m) < 550$
$20 < D(mm) < 145$

The data was selected from Griffiths (1981), Charlton et al (1978) and Kellerhals et al (1972).

A comparison of the observations with the different predictions with Ackers and White sediment relationships and various extremal hypotheses is shown in Table 1. The comparison is made in terms of the discrepancy ratio, that is, the ratio of the predicted to the observed values. Values are given of the mean discrepancy ratio A, which indicates on average how good were the predictions of each method, and the value of the standard deviation SD which indicates the scatter of individual predictions. Figure 5 shows a comparison of observed and predicted widths using the Ackers and White equations together with minimum stream power for the sand data. A similar comparison for gravel data using maximum sediment concentration is shown in Figure 6. The results using the Engelund and Hansen sediment relationships showed a similar behaviour.

The results show that the principle of minimum stream power or maximum sediment concentration gives the best agreement with field data. Of the remaining extremal hypotheses the principle of minimum Froude number provides the best results. The principles of minimum stream power and maximum sediment concentration while being equivalent provide slightly different predictions since one is using an observed sediment concentration and the other an observed slope.

Discrepancies in the measurements of these quantities will lead to differences in predicted values of width.

The larger deviation of mean discrepancy ratio from the value of 1 and the larger standard deviation for the predictions of slope reflect the greater sensitivity of the slope to the specified values than either the width or the depth. No values are shown under slope for the principle of maximum sediment concentration since under this formulation slope is specified and hence cannot be predicted. For gravel rivers there is no comparison of depth as the data was unavailable. The agreement between predicted and observed results indicate the usefulness of extremal hypotheses in providing realistic predictions of regime conditions in alluvial channels.

4.2 Effect of different sediment relationships

Using the Ackers and White sediment relationships, the principle of minimum slope or maximum sediment concentration provided the best agreement with observed data. It does not follow, however, that these extremal hypotheses will provide the best agreement if other sediment relations are considered. Calculations were, therefore, performed using the Engelund and Hansen equations (1967) and equations due to Yang (1982). Comparisons of predicted with observed data are shown in Table 2. A more detailed analysis of the distribution of discrepancy ratios for the Ackers and White and Engelund and Hansen sediment relationships are given in Figures 7 and 8.

The accuracy using the Ackers and White and Engelund and Hansen relations are comparable, with marginally better predictions by Ackers and White, but both, in general, give better predictions than the Yang or

Chang-Parker-Keulegan sediment relations. Figure 7 shows that the Ackers and White formulation with maximum sediment concentration predicts the width to within ±25%, 66% of the time. The corresponding figure using the Engelund and Hansen equations is 48%. Figure 8 shows that the slope prediction using the Ackers and White equations are within a factor of two 79% of the time.

Both the Yang and the Chang, Parker, Keulegan formulations exhibit systematic over or under prediction under certain circumstances.

In empirical regime theory variables such as the channel width or slope are related to the discharge, sediment diameter and other variables using equations of the form

$$B = a\, Q^{b_1} D^{b_2} \ldots, \tag{17}$$

For example,

$$B = 2.67 Q^{0.5} \qquad \text{(Lacey, 1929)} \tag{18}$$

It is possible to approximate the results predicted by rational regime theory by equations of the form

$$B = k_1\, Q^{m_1} X^{m_2} D^{m_3} \tag{19}$$

and

$$S = k_4\, Q^{c_1} X^{c_2} D^{c_3} \tag{20}$$

Values of the exponents derived for different ranges of conditions using the Ackers and White and the Engelund and Hansen sediment relations are given in Table 3.

The exponent m_1 in equation (19) is approximately 0.57 using the Ackers and White sediment relationships or approximately 0.52 using the Engelund and Hansen sediment relationships. The exponent m_2 varies with the values of both the discharge and the sediment diameter. Using the Ackers and White relationships m_2 is slightly less than zero for fine sediments and is slightly greater than zero for coarse sediments. This is in agreement with field and laboratory data. The values of the exponents m_1, m_2, m_3, c_1, c_2 and c_3 are all in qualitative agreement with observations, see Table 3.

It should be observed that not all sediment relationships can be combined with an extremal hypotheses to derive a regime theory. For example if the following sediment transport (Bogardi, 1974) and friction equations are used

$$X = K \frac{V^3}{g d \omega} \qquad (21)$$

and

$$V = \frac{1}{n} d^{2/3} S^{\frac{1}{2}} \qquad (22)$$

then for given values of Q and X no minimum exists for the slope.

It is of interest to note that providing that one had confidence in the applicability of an extremum hypothesis a study of the regime predictions from a set of sediment relationships gives a quick indication of the validity of the relationships over a wide range of conditions. For those sceptical of extremum hypotheses, however, it only provides an indication of the range of validity of extremal hypotheses.

5 DOMINANT DISCHARGE

The original application of regime theory was to irrigation canals. A characteristic of such canals is that the range of discharge is limited so that there is little inherent difficulty in deciding the discharge to be used in the regime relations. More recently regime theory has been applied to natural rivers. By contrast natural rivers have a wide range of discharges varying throughout the year and from year to year. It is thus more difficult to know which is the discharge that should be used in the regime theory.

It has been assumed that the dimensions of a river channel can be related to a particular discharge, referred to as the dominant discharge. Inglis suggested that 'there is a dominant discharge and its associated charge and gradient, to which a channel returns annually. At this discharge, equilibrium is most closely approached and the tendency to change is least. This condition may be regarded as the integrated effect of all varying conditions over a long period of time'. Unfortunately there is no universally agreed method of determining the dominant discharge.

5.1 Proposed definitions of dominant discharge

Model tests carried out by Inglis at Poona suggested that the dominant discharge was a little higher than bankfull discharge and was of the order of 60% of the maximum discharge. It was suggested that for flasher rivers the dominant discharge was 50% of the maximum.

For British rivers Nixon (1959) showed that the ratio of maximum discharge to bankfull discharge could vary from 1.23 to 6.85, see Table 4. Since the ratio of maximum discharge to bankfull discharge shows such considerable variation, the identification of dominant discharge with both a discharge slightly higher than bankfull and 60% of the maximum discharge is contradictory. The notion of dominant discharge being a fixed proportion of the maximum discharge is also open to criticism. For channels taking a more or less constant flow the dominant discharge must be approximately 100% of the maximum discharge. It can thus be seen that if such a relationship applies the percentage cannot be a constant but must be related to the variability of the flow.

To avoid the problems associated with using maximum or bankfull discharges it has been suggested that the dominant discharge has a fixed frequency. Blench (1957) suggested that the dominant discharge was given by the median annual flood. Nixon (1959) postulated that the dominant discharge was that flow that was exceeded 0.6% of the time. Undoubtedly for a particular location on a particular river the dominant discharge will have a fixed frequency or probability of exceedance, the problem is ascertaining whether this value is constant for all locations on all rivers. Under such an approach, however, the dominant discharge can only depend upon the distribution of flows and specifically the nature of the flow exceedance curve at a particular probability. Such definitions completely ignore both rarer and more frequent flow events. Also no account has been taken of the nature of the sediment in the channel and the fact that the size and shape of the channel is determined by sediment transport phenomena.

To include the details of the sediment in the determination of the dominant discharge Gandolfo (1955) and Terrell and Borland (1958), apparently independently following the work of Schaffernak (1916, 1922) suggested that the dominant discharge was that flow that transports the greatest sediment load. The determination of this discharge is illustrated in Fig 9. The idea is an attractive one. It can be criticized, however, in that the dominant discharge only depends on the nature of the flow exceedance and sediment transport rating curve in a local neighbourhood and not on the form of the curves for all discharges exceeding the threshold of motion for the sediment.

This criticism is met by the form of the dominant discharge suggested by Komura (1969) who suggested that the dominant discharge was given by the expression

$$\frac{\sum_{i=1}^{n} Q_i \, Qs_i}{\sum_{i=1}^{n} Qs_i} \tag{23}$$

where Q_i is the maximum monthly discharge Qs_i is the corresponding total sediment load and n is the total number of data. The form of the expression for the dominant discharge is attractive though it would seem to be an approximation of

$$\frac{\int Q \, Q_s dt}{\int Q_s dt} \tag{24}$$

Komura further assumed that Qs_i is equal to αQ_i^{β}, which simplifies his expression for dominant discharge to

$$\frac{\sum_{i=1}^{n} Q_i^{(1 + \beta)}}{\sum_{i=1}^{n} Q_i^{\beta}} \tag{25}$$

On the basis of field data and sediment equations he gave the value of β as 1.0 for rivers where bed load is predominant, 2.0 where suspended load is predominant and 1.5 where bed load and suspended load are equally balanced. A value of β equal to 1.0 implies that

$$Qs_i = \alpha Q_i \tag{26}$$

$$\text{or } \frac{Qs_i}{Q_i} = \alpha \tag{27}$$

i.e. that the concentration is independent of the flow. A value of β equal to 2.0 implies that

$$\frac{Qs_i}{Q_i} = \alpha Q_i \tag{28}$$

i.e. that the concentration increases linearly with discharge.

Much of the discussion of dominant discharge has been performed in the abstract and apart from the work of Nixon (1959), little attempt has been made to relate the concepts to more than one or two examples from real rivers. As the relationship that is being sought is, of its nature, an empirical one and not a theoretical one, this is somewhat surprising. In an attempt to redress this balance recourse was made to an extensive set of data for gravel rivers in Alberta, Canada (Kellerhals, Neil and Bray, 1972). For a number of sites, the bankfull dimensions of the channel, the discharges corresponding to different return periods and sediment data are given. The subject of the study was to determine the dominant discharge from the channel characteristics, using Inglis' concept of the dominant discharge being that steady discharge which would produce the observed channel form and then to compare this dominant discharge with the suggested definitions.

5.2 Determination of dominant discharge

Initially the observed channel width and slope were taken and the Ackers and White regime theory (White et al 1982) used to determine the corresponding discharge and sediment concentration. These discharges were then regarded as the dominant discharges. The original channel characteristics are given in Table 5 and the calculated dominant discharges in Table 6. To determine whether these represented a fixed return period, the return period of each discharge was determined.

A histogram of the return periods is shown in Fig 10. To determine if there was any trend of return period with size of river a plot was made of return period against dominant discharge, Fig 11. No relationship is discernible.

5.3 Comparison of expressions for dominant discharge

The calculated dominant discharge was then compared with those discharges determined from the various proposals for dominant discharge. To evaluate the expression for the dominant discharge expressed in terms of that flow which transports the greatest sediment load it is necessary to know the sediment transport rate for various discharges. This was calculated using the observed width and the Ackers and White sediment relationships for both sediment transport and frictional resistance (Ackers and White, 1973; White et al, 1980). The dominant discharges and the corresponding discrepancy ratios, ratios of predicted to real values, are shown in Table 6. The closer the discrepancy ratio is to one the better are the predictions. The results suggest that the expressions proposed for the dominant discharge give values which are too low for the gravel rivers considered. The major discrepancy between the discharges predicted and those calculated suggest that more work needs to be done to elucidate the definition of dominant discharge. The best predictive expressions are those based on a flow frequency. The discharge which is exceeded 0.6% of the time gave the best predictions.

It may be that a different result would have been obtained if data from sand channels has been used. It is conceivable that since gravel rivers are characterised by less frequent sediment movement than sand channels so the dominant discharge is larger and less frequent than for sand channels.

6 CONCLUSIONS

Instead of being faced with an array of different empirically derived regime theories developed for different ranges of conditions in a variety of countries, the engineer can now confidently use a rational regime theory which has a wide range of applicability and is universal.

1. The use of different extremal hypotheses lead to different rational regime theories.

2. The use of different sediment relationships lead to different rational regime theories.

3. In comparisons with observed channel data the best predictions were produced by the Ackers and White sediment relationships together with either of the two equivalent extremal hypotheses of maximum sediment concentration or minimum slope.

4. Significant differences result in the rational regime theory developed depending upon whether depth or hydraulic radius is used in the sediment relationships. The use of hydraulic radius leads to better results.

5. To apply regime theories to natural rivers an expression is required for the dominant discharge. Of the various proposed expressions investigated all gave discharges that were too low when applied to Alberta gravel river data.

6. The best predictor of dominant discharge for the gravel river data investigated was that discharge which is exceeded 0.6% of the time.

7. The present definitions of dominant discharge are unsatisfactory and further work is required to investigate the problem.

7 RECOMMENDATIONS FOR FUTURE WORK

1. The physics behind the workings of extremal hypotheses is unknown. Implicit in any extremal hypothesis is an assumption about the distribution of shear over the bed and banks of a channel. To improve rational regime theory work must be done to make this assumption explicit. It will then be possible to take into account the effect of varying composition and stability of bed and bank material. This would also enable the investigation of the role of bank vegetation in determining channel width.

2. There is at present no satisfactory definition of dominant discharge to enable regime theory to be confidently applied to natural rivers. In view of the usefulness of regime theory as a quick, easy method to assess channel behaviour this is a major shortcoming and work is required to rectify this deficiency.

8 ACKNOWLEDGEMENTS

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TABLES

TABLE 1 **Comparison of discrepancy ratios for different extremal hypotheses**

Type of channel	Extremal principle	Width A	Width SD	Depth A	Depth SD	Slope A	Slope SD
Sand	X_{max}	1.01	0.38	1.08	0.34		
	S_{min}	1.03	0.41	1.05	0.41	1.26	1.01
	Fr_{min}	0.98	0.34	1.10	0.37	1.29	1.01
	FR_{min}	0.84	0.30	1.18	0.45	1.31	1.10
	FF_{min}	1.03	0.40	1.04	0.42	1.14	0.69
	VS_{min}	1.33	0.68	0.90	0.31	1.19	0.78
Gravel	X_{max}	1.06	0.45				
	S_{min}	0.95	0.40			0.93	0.76
	Fr_{min}	0.74	0.27			0.97	0.80
	FR_{min}	0.78	0.28			1.01	0.84
	FF_{min}	1.00	0.54			0.33	0.25
	VS_{min}	1.38	0.88			0.97	0.81

TABLE 2 Discrepancy ratios using various sediment relationships

Type of channel	Extremal hypotheses	Sediment relation	Width A	Width SD	Depth A	Depth SD	Slope A	Slope SD
Sand	S_{min}	A-W	1.03	0.41	1.05	0.41	1.26	1.01
		E-H	0.92	0.40	0.93	0.36	1.50	1.00
		Yang	1.01	0.49	1.96	1.82	0.01	0.00
Sand	X_{max}	A-W	1.01	0.38	1.08	0.34		
		E-H	0.97	0.36	0.97	0.30		
		Yang	0.69	0.26	1.16	0.39		
Gravel	X_{max}	A-W	1.06	0.45				
		E-H	0.80	0.28				
		Yang	0.63	0.23				
		CPK	0.56	0.20				

TABLE 3 Exponents in regime equations

1. $B = k_1 Q^{m_1} X^{m_2} D^{m_3}$

Range	Parameter	Ackers and White	Engelund and Hansen
$0.2 < D(mm) < 0.5$	m_1	0.57	0.53
$50 < X(ppm) < 200$	m_2	-0.05	-0.15
$Q(m^3/s) < 1000$	m_3	0.15	-0.10
	k_1	5.6	7.6
$50 < D(mm) < 200$	m_1	0.56	0.52
$10 < X(ppm) < 50$	m_2	0.08	0
$Q(m^3/s) < 1000$	m_3	-0.3	0.15
	k_1	4.5	4.25

2. $S = k_4 Q^{c_1} X^{c_2} D^{c_3}$

Range	Parameter	Ackers and White	Engelund and Hansen
$0.2 < D(mm) < 0.3$	c_1	-0.24	-0.17
$50 < X(ppm) < 200$	c_2	0.41	0.61
$100 < Q(m^3/s) < 500$	c_3	1.27	0.53
	k_4	0.0003	0.00004
$5 < D(mm) < 200$	c_1	-0.26	-0.17
$10 < X(ppm) < 50$	c_2	0.28	0.65
$100 < Q(m^3/s) < 500$	c_3	0.36	0.53
	k_4	0.0007	0.00004

TABLE 4 Maximum and bankfull discharges, after Nixon

Serial No	River Basin	River	Gauging Station	Maximum discharge cusecs	Bankfull discharge: cusecs	Ratio bankfull/ maximum discharge
1	Thames	Thames	Teddington	27,777	9,090	3.05
2		Thames	Day's Weir	12,300	3,704	3.31
3		Wey	Tilford	1,840	742	2.48
4		Blackwater	Swallowfield	1,100	450	2.45
5	Severn	Severn	Bewdley	23,500	10,000	2.35
6		Severn	Montford	18,000	7,000	2.57
7		Avon	Evesham	8,000	3,500	2.29
8		Stour	Kidderminster	2,900	500	5.8
9	Avon (Bristol)	Avon	Bath	12,063	4,200	2.87
10		Avon	Melksham	4,000	1,500	2.67
11		Semington Brook	Semington	600	450	1.33
12	Mersey	Mersey	Iriam Weir	9,400	5,000	1 88
13		Irwell	Alelphi Weir	21,500	9,000	2.4
14	Wye	Wye	Cadora	32,000	18,020	1.78
15		Wye	Belmont	27,800	10,896	2.55
16		Wye	The Nyth, Erwood	28,300	15,568	1.82
17		Wye	Rhayader	7,010	2,169	3.24
18	Cheshire	Weaver	Ashbrook	7,500	2,500	3.0
19	Northumberland	Derwent	Eddy's Bridge	4,450	650	6.85
20		Tyne	Barrasford	31,900	14,630	2.19
21	Hampshire	Wallington	North Fareham	400	325	1.23
22	Trent	Trent	Nottingham	36,000	11,750	3.08
23		Derwent	Derby	7,270	2,500	2.9
24		Dove	Rocester	3,590	1,400	2.56
25	Great Ouse	Ouse	Brounshill	11,000	2,300	4.8
26		Ouse	Bedford	9,821	2,620	3.75
27		Cam	Bottisham	2,570	1,240	2.1
28		Lark	Isleham	678	300	2.2
29	Dee and Clwyd	Dee	Erbistock	16,000	6,000	2.67

TABLE 5 Gravel channel characteristics

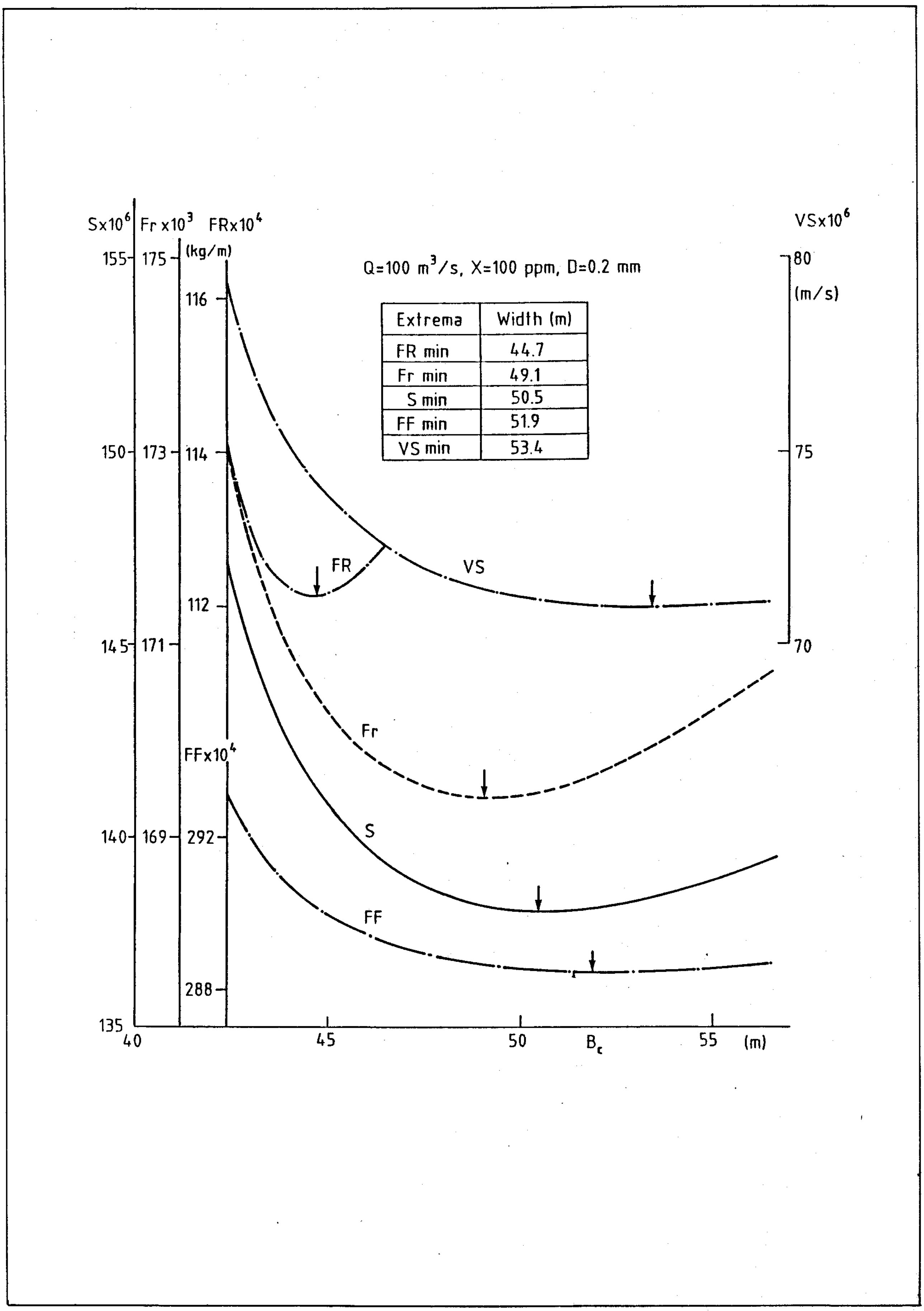
Reach No	Sediment Diameter (mm)	Width (ft)	Depth (ft)	Slope
1	19.0	1570.0	19.1	0.00074
2	12.0	1800.0	24.8	0.00069
3	18.0	1560.0	36.6	0.00022
8	24.0	920.0	22.8	0.00052
10	23.0	365.0	12.6	0.00094
12	19.0	480.0	7.4	0.0030
18	16.0	168.0	3.8	0.0052
19	25.0	270.0	16.3	0.0012
20	16.0	440.0	15.0	0.00084
21	20.0	125.0	6.7	0.00055
22	28.0	262.0	4.5	0.0033
26	19.0	75.0	6.7	0.0039
36	23.0	659.0	11.4	0.0025
37	11.0	800.0	25.0	0.00035
40	8.0	162.0	3.6	0.0057
41	11.0	279.0	10.8	0.0012
42	17.0	102.0	4.0	0.0036
50	13.0	405.0	12.6	0.00035
52	24.0	136.0	5.5	0.0036
53	18.0	195.0	4.2	0.0021
57	15.0	93.0	11.2	0.0012
64	16.0	488.0	11.3	0.0018
65	8.0	551.0	8.1	0.0012
67	45.0	115.0	2.5	0.015
83	13.0	225.0	4.1	0.0059
87	17.0	415.0	7.8	0.0016
88	18.0	396.0	6.5	0.0017
90	13.0	473.0	14.1	0.00094
92	16.0	100.0	3.1	0.0024
93	37.0	102.0	4.1	0.0032
95	22.0	252.0	4.9	0.0037
97	8.0	114.0	6.7	0.00080
101	12.0	250.0	5.6	0.0019
106	10.0	39.0	1.9	0.011
108	16.0	389.0	7.6	0.0020
110	6.0	173.0	7.3	0.00059
113	31.0	144.0	2.6	0.0035
116	26.0	96.0	4.8	0.0014

TABLE 6 Calculated dominant discharges

Reach No	Dominant discharge	Q_{MT}	Q_{MT}/Q_D	Q_M	Q_M/Q_D	$Q_{0.6\%}$	$Q_{0.6\%}/Q_D$	Q_{MAF}	Q_{MAF}/Q_D
1	489750	131580	0.27	39600	0.08	290530	0.59	220000	0.45
2	402230	139810	0.35	51700	0.13	330140	0.82	261000	0.65
3	737650			56000	0.08	339480	0.46	300000	0.41
8	297740	69750	0.23	13400	0.04	89580	0.30	91000	0.31
10	52410	16720	0.32	1980	0.03	23090	0.44	25200	0.48
12	27740	7370	0.26	3190	0.11	16120	0.58	16600	0.60
18	4948	828	0.17	283	0.06	1966	0.40	1910	0.39
19	31450			721	0.02	7009	0.22	6240	0.20
20	56670	9814	0.17	1370	0.02	12450	0.22	12300	0.22
21	9966			166	0.01	1826	0.18	1150	0.12
22	20510			302	0.01	4071	0.20	3270	0.16
26	1940			79	0.04	1833	0.95	1200	0.62
36	80600			5080	0.06	49390	0.61	26000	0.32
37	156850	35280	0.22	7770	0.05	46110	0.29	45500	0.30
40	2071	929	0.45	502	0.24	2655	1.28	3400	1.64
41	17290	2815	0.16	920	0.05	6294	0.36	5100	0.29
42	2963	601	0.20	172	0.06	1363	0.46	1170	0.39
50	59170	17650	0.30	2120	0.04	21400	0.36	14200	0.24
52	6268			88	0.01	1123	0.18	1170	0.19
53	111320	1722	0.15	172	0.01	2090	0.18	2040	0.18
57	3906			60	0.01	452	0.12	413	0.11
64	44860	7579	0.17	3270	0.07	16230	0.36	16300	0.36
65	34930			3520	0.10	20270	0.58	18800	0.54
67	3750	672	0.18	203	0.05	1352	0.36	1530	0.41
83	5354	797	0.15	220	0.04	1746	0.33	1930	0.36
87	39780	6748	0.17	1100	0.03	9497	0.24	8900	0.22
88	37630	7384	0.20	1350	0.04	12220	0.32	12200	0.32
90	50750	10960	0.22	3230	0.06	26332	0.52	23100	0.46
92	3380	903	0.27	188	0.06	1319	0.39	1230	0.36
93	5890			254	0.04	1408	0.24	1320	0.22
95	14670	2987	0.20	684	0.05	4931	0.34	4920	0.34
97	3974	1106	0.28	123	0.03	1483	0.37	1220	0.31
101	12270	1840	0.15	339	0.03	2303	0.19	2600	0.21
106	389	84	0.21	23	0.06	183	0.47	211	0.54
108	30050	3984	0.13	583	0.02	5392	0.18	3910	0.13
110	7009	1301	0.18	303	0.04	1637	0.23	1800	0.26
113	8551	1614	0.19	402	0.05	2224	0.26	2060	0.24
116	5721			137	0.02	1147	0.20	730	0.13
Mean			0.22		0.05		0.39		0.36

Q_{MT} Discharge at which sediment transport is a maximum
Q_D Dominant discharge
Q_M Mean discharge
$Q_{0.6\%}$ Discharge which is exceeded 0.6% of time
Q_{MAF} Mean annual flood

FIGURES



Extrema	Width (m)
FR min	44.7
Fr min	49.1
S min	50.5
FF min	51.9
VS min	53.4

Fig 1 Fr, S, FF, VS, FR against B_c

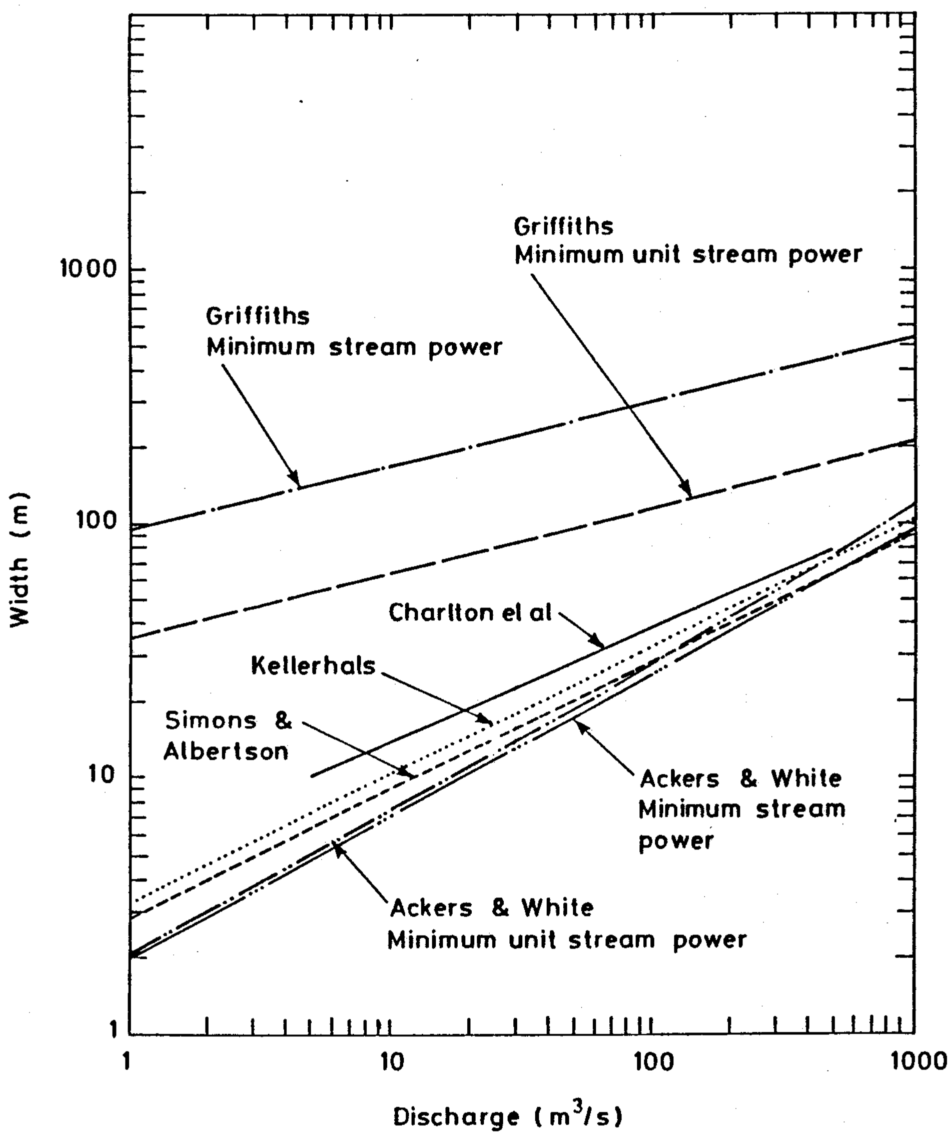


Fig 2 Regime widths for gravel rivers

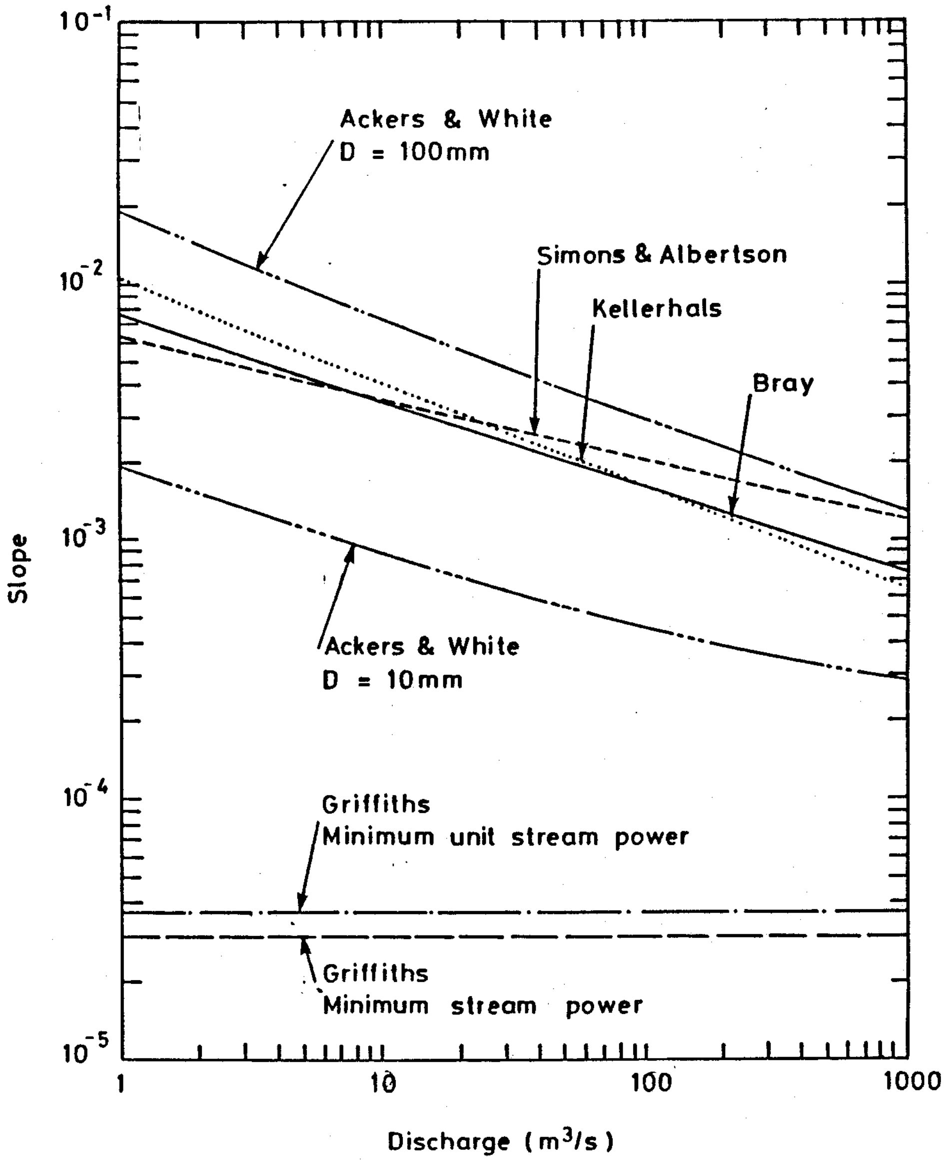


Fig 3 Regime slopes for gravel rivers

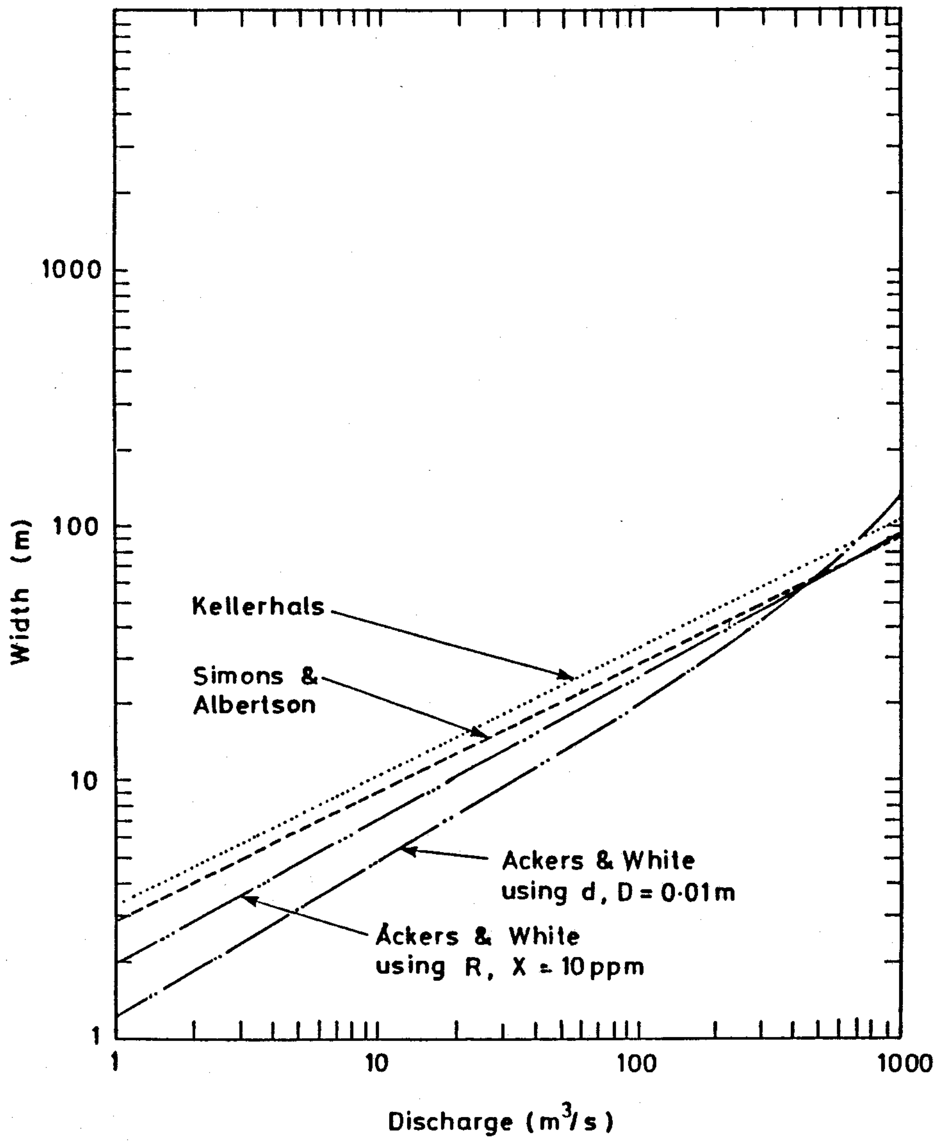


Fig 4 Effect of d or R on regime widths

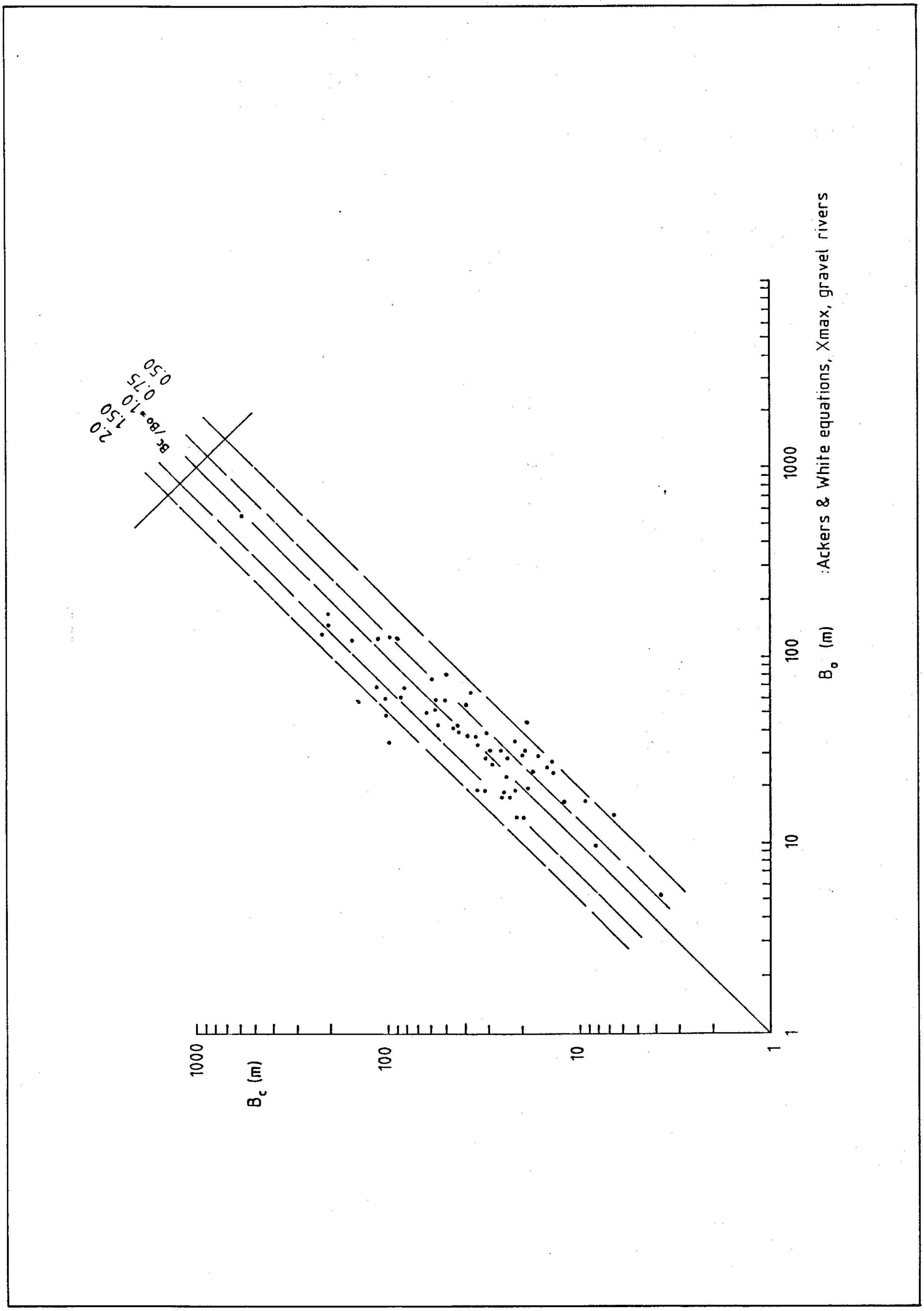


Fig 5 Calculated against observed width

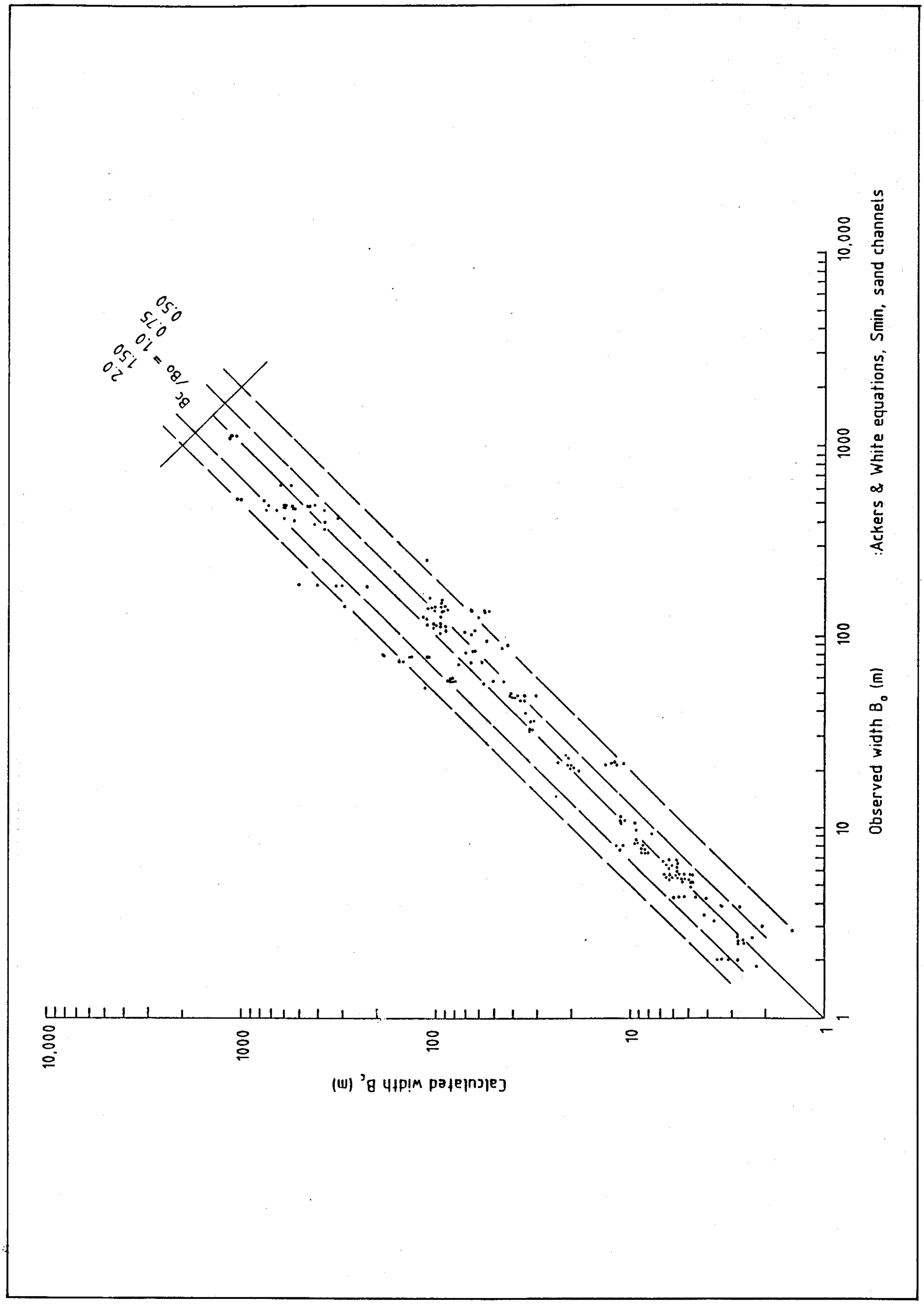


Fig 6 Calculated against observed width

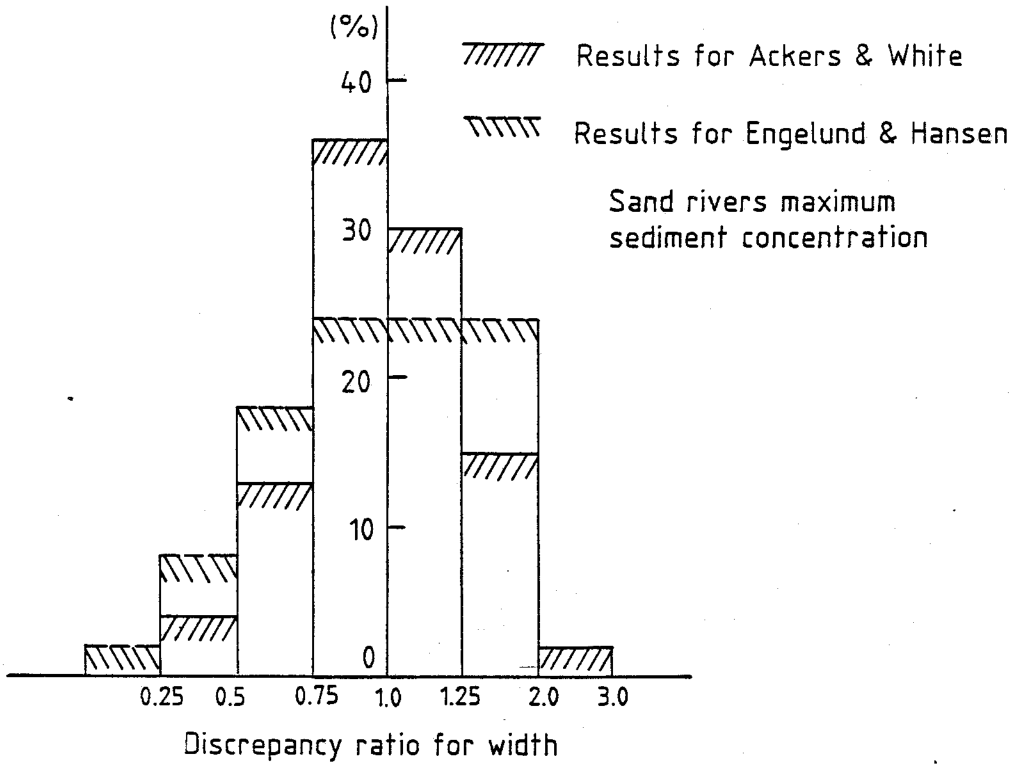


Fig 7 Comparison of discrepancy ratios

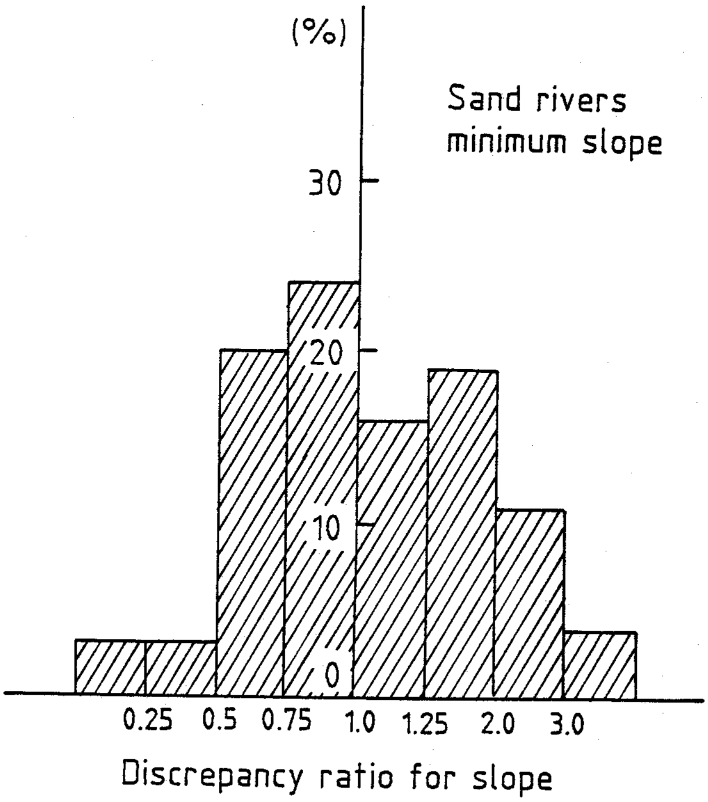


Fig 8 Discrepancy ratios : Ackers & White equations

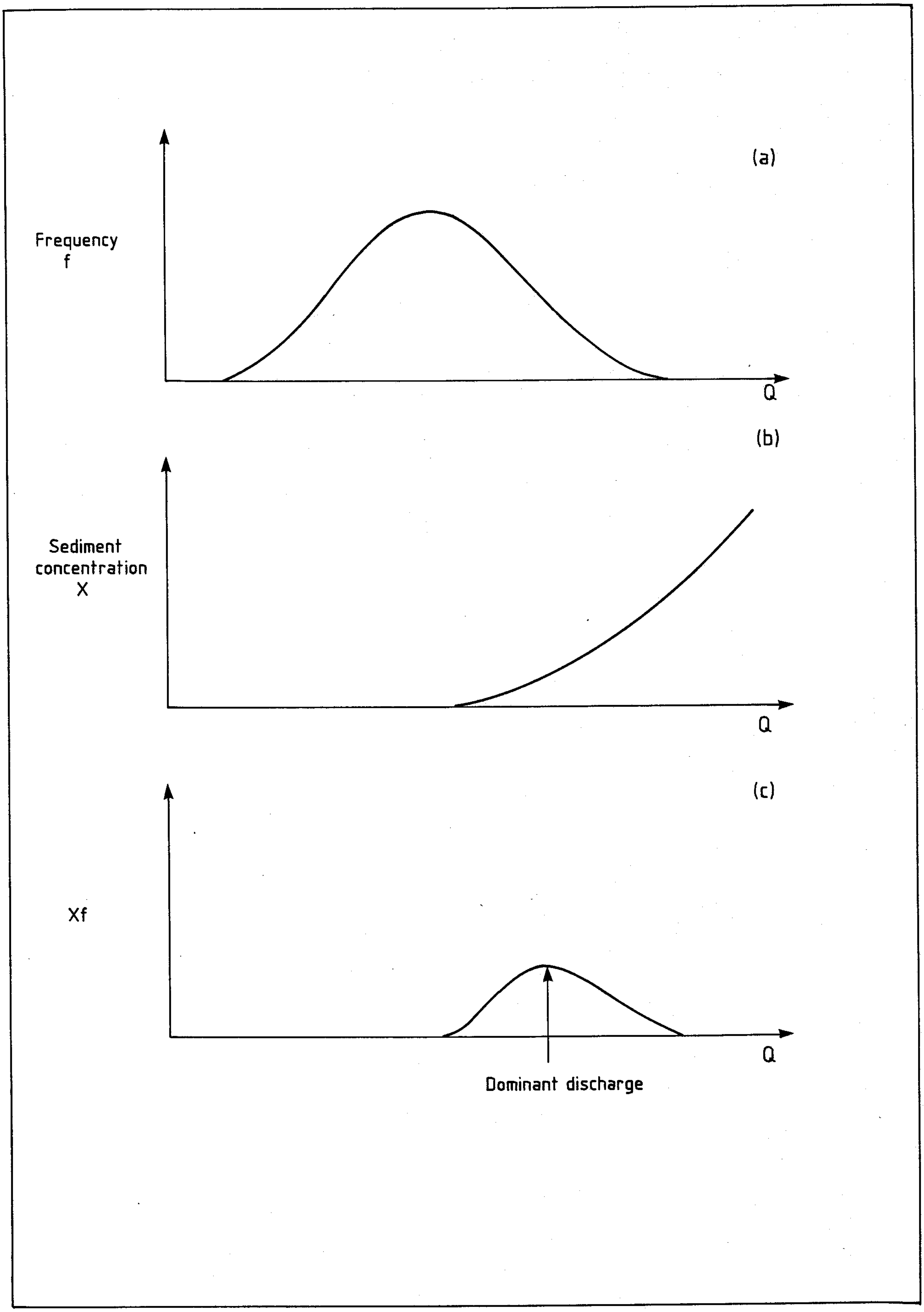


Fig 9 Determination of flow that transport greatest sediment load

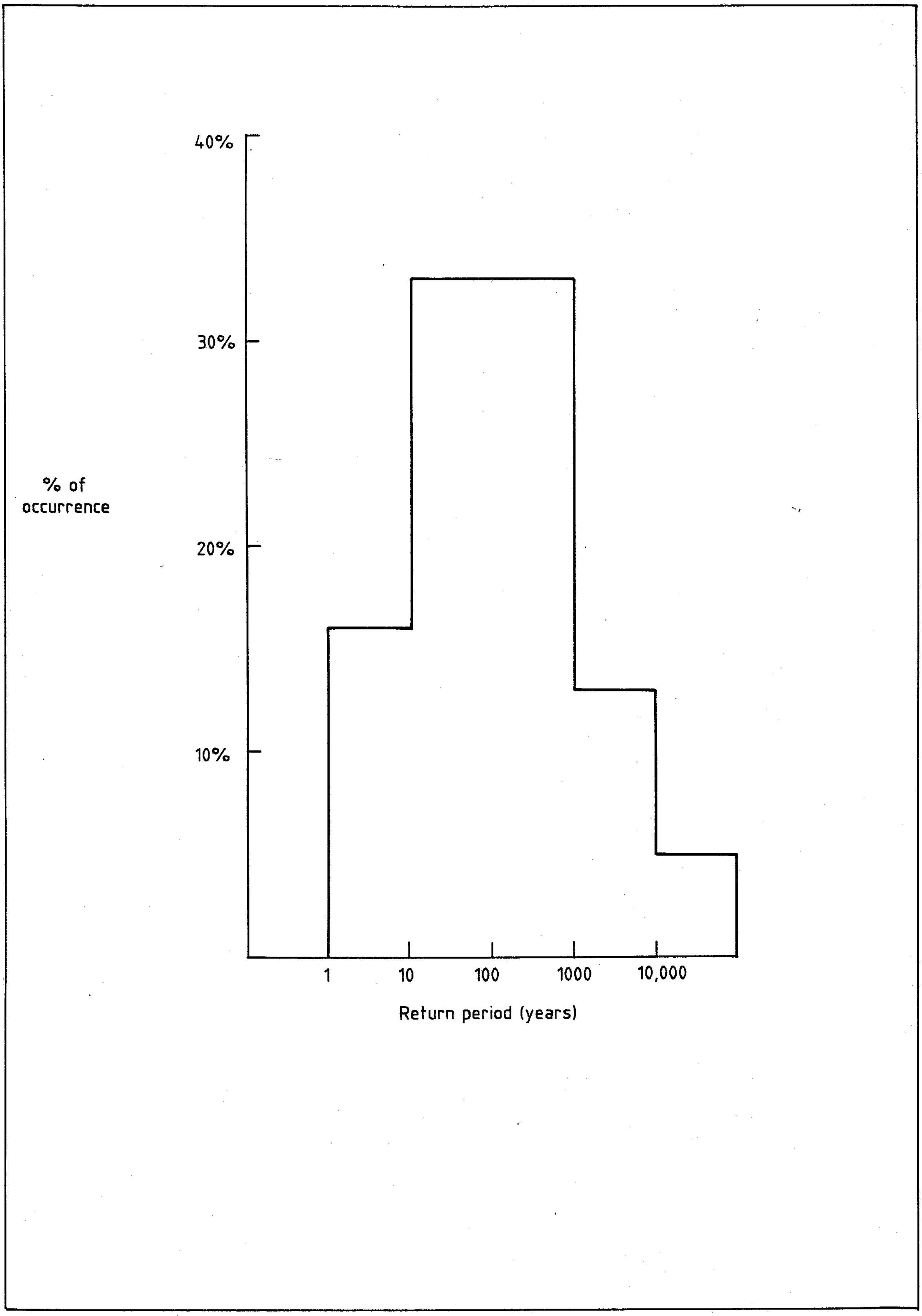


Fig 10 Return periods of dominant discharge

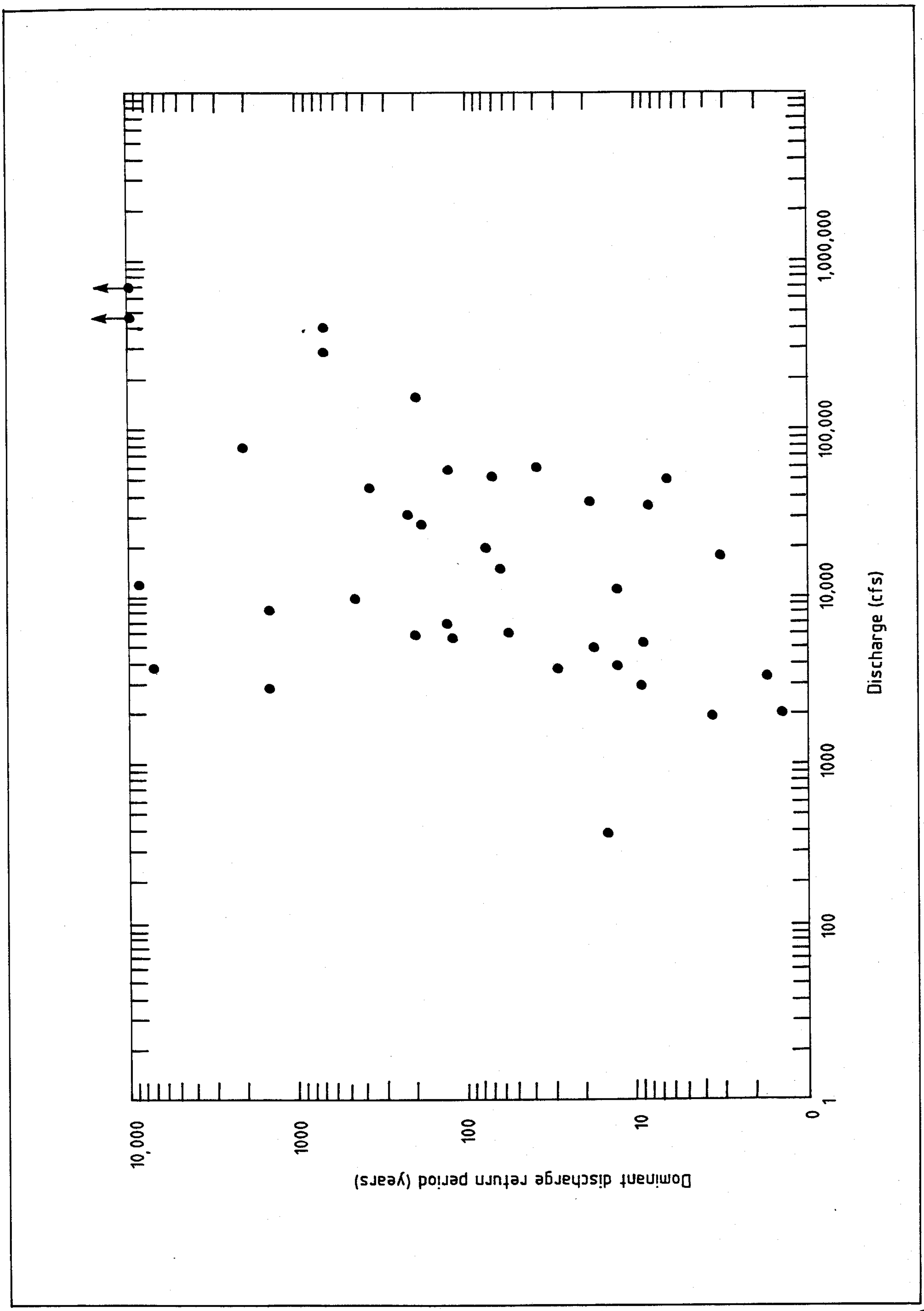


Fig 11 Dominant discharge return period against discharge

APPENDICES

EXTREMAL HYPOTHESES APPLIED TO RIVER REGIME

by Dr R Bettess and Dr W R White

Hydraulics Research Ltd, Wallingford, UK

ABSTRACT

Regime theories are used to predict the shape of stable alluvial channels. The first such theories were entirely empirically based on extensive field measurements. Recent developments in our knowledge of sediment transport processes in alluvial channels, however, have introduced the possibility of developing regime theories based on equations describing these fundamental processes. Frequently an extremal hypothesis, such as minimum stream power or maximum sediment concentration, is invoked to enable the complete system to be determined. It is assumed that the channel dimensions adjust to maximise or minimise the value of some appropriate functional. Various proposed extremal hypotheses are discussed and their predictions in terms of channel shape are compared. The effect of using the various hypotheses with different sediment transport relationships is also considered.

1 INTRODUCTION

The aim of regime theory is to predict the size, shape and slope of a stable alluvial channel under given conditions. It has been the subject of considerable research for over eighty years and continues to be a topic for active research (Lacey, 1929; Blench, 1957; Ackers, 1983). Ignoring plan geometry a channel can be characterised by its width, depth and slope and the object of regime theory is to relate these to the water and sediment discharge conveyed by the channel.

In its earliest phase the subject was dominated by an empirical approach. Extensive measurements were taken on channels and attempts were made to fit empirical equations to the observed data. The channel characteristics were related primarily to the discharge but allowance was also made for variations in other variables such as the sediment size. This method met with some success provided that the derived equations where applied to similar channels from the same geographical area with parameters contained within the parameter range of the data from which the equations were obtained. Any extrapolation beyond the parameter range of the data or to other geographical areas was less successful. It thus became apparent that though there was a discernable relationship between the variables involved there must also be other factors controlling the system which were not being considered.

More recently, as the understanding of the processes of sediment transport and alluvial friction improved, it became possible to contemplate the development of regime relationships utilising equations of sediment dynamics. This held out the prospect of elucidating the significance of some of the factors previously ignored in the empirical analysis and also of enabling the derivation of equations of wider applicability (Ackers, 1983).

For a given water and sediment discharge the alluvial channel that is developed can be characterised by its width, depth and slope. Thus the system has three degrees of freedom. The relevant variables are related by a sediment transport equation and a relationship for alluvial friction. To make the system soluble, however, a third relationship is required. There have been a number of suggestions for a third relationship to close the system (Ackers, 1983). The methods so far yielding the most success have been based on some form of extremal hypothesis.

A bewildering array of extremal hypotheses have been proposed, some of which are related, so that it remains unclear whether all these hypotheses are more or less equivalent or whether there are fundamental differences between them. The behaviour of an extremal hypothesis, however, cannot be divorced from the equations of sediment transport and alluvial friction with which it is associated. In an effort to clarify the situation this paper describes the initial steps of a study in which a number of these extremal hypotheses were tested using different sediment transport relationships.

2 EXTREMAL HYPOTHESES

A number of extremal hypotheses have been proposed to provide the equations necessary to formulate regime relations. These are now discussed and, where possible, related to each other.

$(Q\gamma + Q_s \gamma_s)$ LS, (5)

where Q and Q_s are the water and sediment discharges, respectively and γ and γ_s are the specific weights of water and sediment, respectively.

2.5 Maximum Sediment Transport Rate (Singh, 1961; White et al, 1982)

This hypothesis is stated as follows: '... for a particular water discharge and slope, the width of the channel adjusts to maximise the sediment transport rate.' White et al (1982).

2.6 Relationships Between Extremal Hypotheses

Although from the statements of these hypotheses they all look different a number of them can be related to each other.

White et al (1982) showed that maximum sediment transport rate is equivalent to minimum stream power for a fixed discharge Q. This equivalence is independent of the sediment relations used. Davies and Sutherland (1983) point out that when considering minimum energy dissipation rate for sediment concentrations less than 1000 ppm by weight, the error in neglecting the $\gamma_s Q_s$ term is less than 0.1% and so minimum energy dissipation rate is equivalent to minimising γQLS which is equivalent to minimum stream power. The similarity can be further demonstrated (Brebner and Wilson, 1967). If we define Q_T to be the total discharge of water and sediment and C to be the sediment concentration by volume then

$$Q = Q_T (1-C) \text{ and } Q_s = CQ_T. \qquad (6)$$

increases. The deformation will cease when the shape of the boundary is that which gives rise to a local maximum of friction factor. Thus the equilibrium shape of the non-planar, self-formed flow boundary or channel corresponds to a local maximum of friction factor', Davies and Sutherland (1980).

The friction factor is given by

$$f = \frac{8gdS}{V^2}. \qquad (2)$$

Using the continuity equation

$$Q = bVd \qquad (3)$$

we have

$$f = \frac{8gb^2d^3S}{Q^2} \qquad (4)$$

2.4 Minimum Energy Dissipation Rate (Brebner and Wilson, 1969; Yang et al, 1981)

This hypothesis is stated as follows: 'A system is in an equilibrium condition when its rate of energy dissipation is at a minimum value', Yang et al (1981).

The rate of energy dissipation in a reach of a stream of length L is given by

2.1 Minimum Stream Power (Chang, 1980)

This hypothesis is stated as follows: 'For an alluvial channel, the necessary and sufficient condition of equilibrium occurs when the stream power per unit length of channel γQS is a minimum subject to given constraints, where γ is the specific weight of water, Q is discharge and S is slope. Hence, an alluvial channel with water discharge Q and sediment load Q_s as independent variables tends to establish its width, depth and slope such that γQS is a minimum. Since Q is a given parameter, minimum γQS also means minimum channel slope', Chang (1980).

2.2 Minimum Unit Stream Power (Yang and Song, 1979)

This hypothesis is stated as follows: '... for subcritical flow in an alluvial channel, the channel will adjust its velocity, slope, roughness and geometry in such a manner that a minimum amount of unit stream power is used to transport a given sediment and water discharge', Yang and Song (1979). Unit stream power is defined as stream power per unit weight of water

$$\frac{Q\gamma}{\rho g}\frac{LS}{bdL} = VS \tag{1}$$

where L is the length of the reach, b is the width, d is depth, g is acceleration due to gravity and V is velocity.

2.3 Maximum Friction Factor (Davies and Sutherland, 1980)

This hypothesis is stated as follows: 'If the flow of a fluid past an originally plane boundary is able to deform the boundary to a non-planar shape, it will do so in such a way that the friction factor

We have therefore

$$Q\gamma + Q_s \gamma_s = Q_T (1-C)\gamma + CQ_T \gamma_s \tag{7}$$

$$= Q_T \left[(1-C)\gamma + C\gamma_s\right] \tag{8}$$

but $\left[(1-C)\gamma + C\gamma_s\right]$ is the specific gravity of the mixture so $Q\gamma + Q_s \gamma_s$ becomes $Q_T \gamma_T$ where both refer to the combined water and sediment mixture. Minimum energy degradation is thus equivalent to minimising $Q_T \gamma_T LS$. The extremal hypotheses and their relationships may thus be summarised: there are three independent hypotheses.

(a) Minimum stream power (minimise γQS)

Maximum sediment transport rate (maximise X)

Minimum energy dissipation rate (minimise $(\gamma Q + \gamma_s Q_s) LS$) (approx)

(b) Minimum unit stream power (minimise VS)

(c) Maximum friction factor (maximise f)

3 IMPLEMENTATION OF EXTREMAL HYPOTHESES

To formulate a regime theory an extremal hypothesis has to be combined with appropriate equations for sediment transport and alluvial friction. In formulating such regime theories authors select their favourite sediment relationships. White et al selected the Ackers and White sediment transport theory and the White et al alluvial friction relationships since these have been shown to provide good predictions of both sediment transport and alluvial friction over a wide range of conditions. Other authors, however, have selected many different theories, for example Chang (1980) used Parker's (1978) sediment transport relationship for gravel rivers and the Bray friction relationship; Yang et al (1981) used Yang's sediment transport relationship and the Manning-Strickler roughness relationship. Griffiths (1984) considered the regime relationships derived from the Parker (1978) - Chang (1980) bed load formula and the Keulegan resistance formula.

Griffiths (1984) considered the Parker-Chang bed load formula and the Keulegan resistance formula in the light of a number of extremum hypotheses and concluded that all provided regime channels with an unrealistically restricted range of values of the Shields' entrainment function. Griffiths analysis of the Ackers and White sediment relationships lead him to a similar conclusion. There is, however, a significant difference in the Ackers and White relations used by Griffiths and those used by White et al. In the equations for sediment mobility and shear velocity the latter use hydraulic radius rather than the depth used by Griffiths. With this seemingly small change in the equations Griffiths analysis fails to go through and the results are significantly altered as will be shown later. Thus care must be taken not only in the selection of the sediment transport relationships but also the details of how they are implemented.

4 COMPARISON OF EXTREMAL HYPOTHESES AND SEDIMENT RELATIONSHIPS

We now compare the predictions of a number of combinations of sediment transport relationships and extremal hypotheses. The comparisons are based primarily on the prediction of width since firstly this is a significant parameter associated with a channel and is of interest to engineers and secondly there are established empirical relationships for channel width. A comparison of slopes is also made.

The sediment transport relationships used are the Ackers and White relationships and those used by Griffiths, that is, the Yang-Parker transport relationship and the Keulegan friction law. The former relationships were selected since they have been shown to perform well over a wide range of conditions and the necessary software was easily available. The latter was chosen as Griffiths had indicated that the relationships seemed to demonstrate curious behaviour. Both sets of equations were used to predict widths and depths.

For a fixed sediment diameter, discharge and sediment concentration, regime conditions were found for the following extremal hypotheses: minimum stream power, minimum unit stream power, maximum energy dissipation and maximum friction factor. Since it has previously been shown that maximum sediment transport rate is equivalent to minimum stream power this extremum hypothesis was not considered separately. Under the present formulation no maximum was found in the friction factor. It has been reported (A Bassi, private communication) that using the different formulation of fixed values for sediment diameter, discharge and channel slope there is a maximum in the friction factor but this has yet to be investigated. The results showed that, as indicated above, maximum energy dissipation was for all practical cases equivalent to minimum stream power.

Since the sediment relationships used were derived from laboratory data from rectangular channels it was assumed that the initially calculated widths and depths were for a rectangular channel. The values of width and depth were then adjusted to give values corresponding to a trapezoidal section of the same cross-sectional area, where the side slope z (z horizontal to 1 vertical) of the trapezoid was given by Smith's (1974) empirically determined relationship:

$$z = \begin{cases} 0.5 \text{ if } Q < 1\text{m}^3/\text{s} \\ 0.5\, Q^{0.25} \text{ if } Q > 1\text{m}^3/\text{s} \end{cases} \qquad (9)$$

If the width to depth ratio is large these adjustments are small. Since the Chang-Parker sediment relationship was derived on predominantly laboratory data the same procedure of adjustment was applied to results obtained using this equation. Problems did arise in some cases, however, where the width to depth ratio was as low as 1×10^{-5}. In such circumstances the adjustment procedure is totally unrealistic.

The predicted widths for Ackers and White and the extremum hypotheses of minimum stream power and minimum unit stream power for a range of discharges are shown in Fig 1. The results are for a D_{35} size of 0.01m and a sediment concentration of 10 ppm. For comparison purposes various empirically derived regime relationships are also shown. Since the Ackers and White relationships depend upon sediment diameter and sediment concentration the predictions of the Ackers and White theory for gravel rivers should be shown as a region rather than a single curve on this graph, so that a direct comparison is difficult but it can be seen that there is reasonable agreement between the empirically and theoretically derived results. It can further be seen that the differences between the hypotheses of stream power and unit stream power are no larger than the uncertainty in the empirically derived equations and for this parameter range there is no basis for preferring one hypothesis to the other.

The same Figure shows the results using the Chang-Parker transport equation and the Keulegan friction law. It can be seen that using both hypotheses of minimum stream power and minimum unit stream power the width is wildly overestimated. This demonstrates that the behaviour of the various extremal hypotheses is dependent on the sediment transport relationships with which they are associated and the two cannot be considered independently.

A comparison was also made of the predictions of slope. Figure 2 shows regime slopes predicted by various empirically derived regime equations and from regime equations based on Ackers and White sediment relationships. The Ackers and White results are based on sediment diameters of 0.01m and 0.1m and a sediment concentration of 10 ppm. Appropriate sediment diameters were used in the empirical equations. Again direct comparison is difficult since the Ackers and White results depend upon both sediment diameter and sediment concentration and so are more properly plotted as a region on this Figure. The results using the minimum stream power and minimum unit stream power are indistinguishable on this plot. The results for Parker-Chang sediment transport equation and Keulegan friction equation with a sediment diameter of 0.01m are also shown.

Griffiths (1984) studied regime relationships provided by using the Ackers and White sediment relationships together with the principles of minimum stream power and minimum unit stream power and came up with results somewhat at variance with those of White et al (1982) using the identical sediment relations and extremal hypotheses. The differences leading to the different conclusions were in the details of the sediment relationships. White et al used the hydraulic radius in the expressions for the sediment mobility and shear velocity whereas Griffiths used depth. This apparently minor change leads to major changes in the width dependence of the system. Results using the two different formulations are shown in Fig 3. The Ackers and White results are based on a sediment diameter of 0.01m and a sediment concentration of 10 ppm. The radical differences between the results are partly disguised by the rectangular to trapezoidal transformation described above but it is clear that the use of depth rather than hydraulic radius in both the expression for sediment mobility and shear velocity leads to unsatisfactory results. Tests indicated that the replacement of R by d in the expression for sediment mobility made only a minor change, the major change resulting from the replacement of R by d in the expression for the shear velocity.

It is of interest to note that providing that one has confidence in the applicability of an extremum hypothesis a study of the regime predictions derived from a set of sediment relationships gives a quick indication of the validity of the relationships over a wide range of conditions. For those sceptical of extremum hypotheses, however, it only provides an indication of the range of validity of extremal hypotheses.

5 CONCLUSIONS

The results from the equations used by Griffiths together with the principle of minimum stream power produce unrealistic values for regime channels. It has already been shown that the Ackers and White sediment and friction relationships together with the same extremal principle produce realistic dimensions for regime channels (White et al, 1982). It can therefore, be concluded that an arbitrary selection of sediment and friction relationships combined with an extremal hypothesis may not provide a satisfactory regime theory. If one believes that it is fortuitous that an extremum principle works in conjunciton with the Ackers and White sediment relationships then the failure of the Griffiths formulation may be regarded as evidence that extremal hypotheses are not universally applicable. If, however, one regards the Ackers and White results as indicating the validity of the underlying extremum hypothesis then one may conclude that the shortcomings of the Griffiths formulation reflect the inadequacies of either the Chang-Parker transport relationships or the Keulegan friction law.

The results using the Ackers and White sediment and friction relationships indicate that though the principles of minimum stream power and minimum unit stream power give differing results for width the differences on a practical range of parameters are such that the authors cannot regard one as being preferable to the other. The predictions of regime slope are virtually identical for the parameter range considered here. The results show, as the theory indicates, that for practical purposes minimum energy dissipation is effectively indistinguishable from minimum stream power. Under the present formulation considered there was no maximum in the friction factor.

The work of Griffiths, in conjunction with the results presented here indicate the care that is required in formulating the equations. The apparently minor adjustment of replacing the hydraulic radius by the depth in the various equations has a major impact upon the results as it radically affects the dependence of the equations on the width of the channel.

6 ACKNOWLEDGEMENTS

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8 NOTATION

b	m	channel width
C		sediment concentration by volume
d	m	depth of flow
f		friction factor
g	ms^{-2}	acceleration due to gravity
L	m	length of reach
Q	m^3s^{-1}	discharge
Q_s	m^3s^{-1}	sediment discharge
Q_T	m^3s^{-1}	water and sediment discharge
R	m	hydraulic radius
S		slope
V	ms^{-1}	velocity
γ		specific weight of water
γ_s		specific weight of sediment
γ_T		specific weight of water and sediment mixture
ρ	kgm^{-3}	density

TABLE 4 Exponents in regime equations

1. $B = k_1 Q^{m_1} X^{m_2} D^{m_3}$

Range	Parameter	Ackers and White	Engelund and Hansen
0.2 < D(mm) < 0.5	m_1	0.57	0.53
50 < X(ppm) < 200	m_2	-0.05	-0.15
Q9m³/s) < 1000	m_3	0.15	0.10
	k_1	5.6	7.6
50 < D(mm) < 200	m_1	0.56	0.52
10 < X(ppm) < 50	m_2	0.08	0
Q(m³/s) < 1000	m_3	-0.3	0.15

2. $S = k_4 Q^{c_1} X^{c_2} D^{c_3}$

Range	Parameter	Ackers and White	Engelund and Hansen
0.2 < D(mm) < 0.3	c_1	-0.14	-0.17
50 < X(ppm) < 200	c_2	0.41	0.61
100 < Q(m³/s) < 500	c_3	1.27	0.53
	k_4	0.0003	0.00004
5 < D(mm) < 200	c_1	-0.26	-0.17
10 < X(ppm) < 50	c_2	0.28	0.65
100 < Q(m³/s) < 500	c_3	0.36	0.53
	k_4	0.0007	0.00004

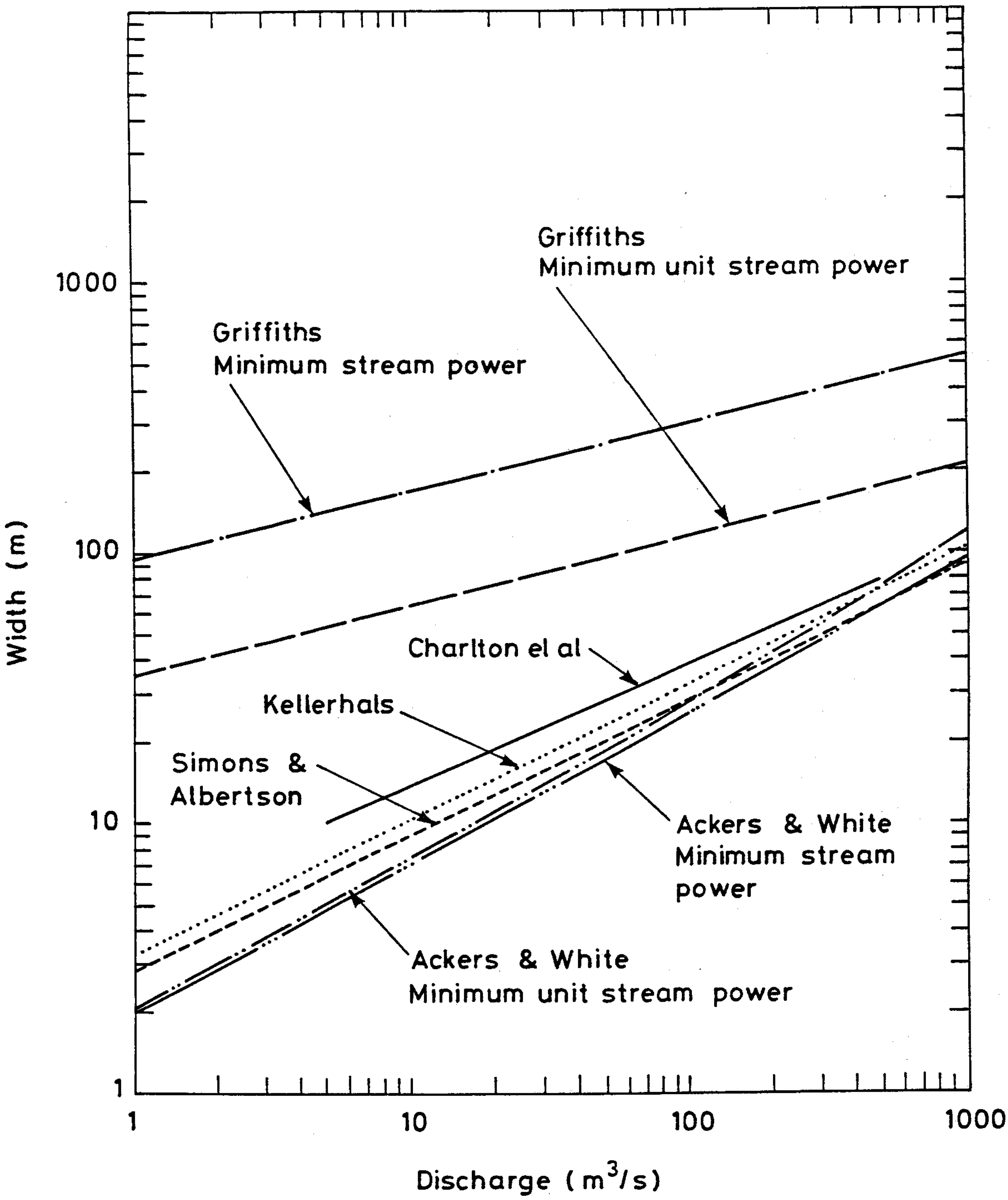

Fig 1 Regime widths for gravel rivers

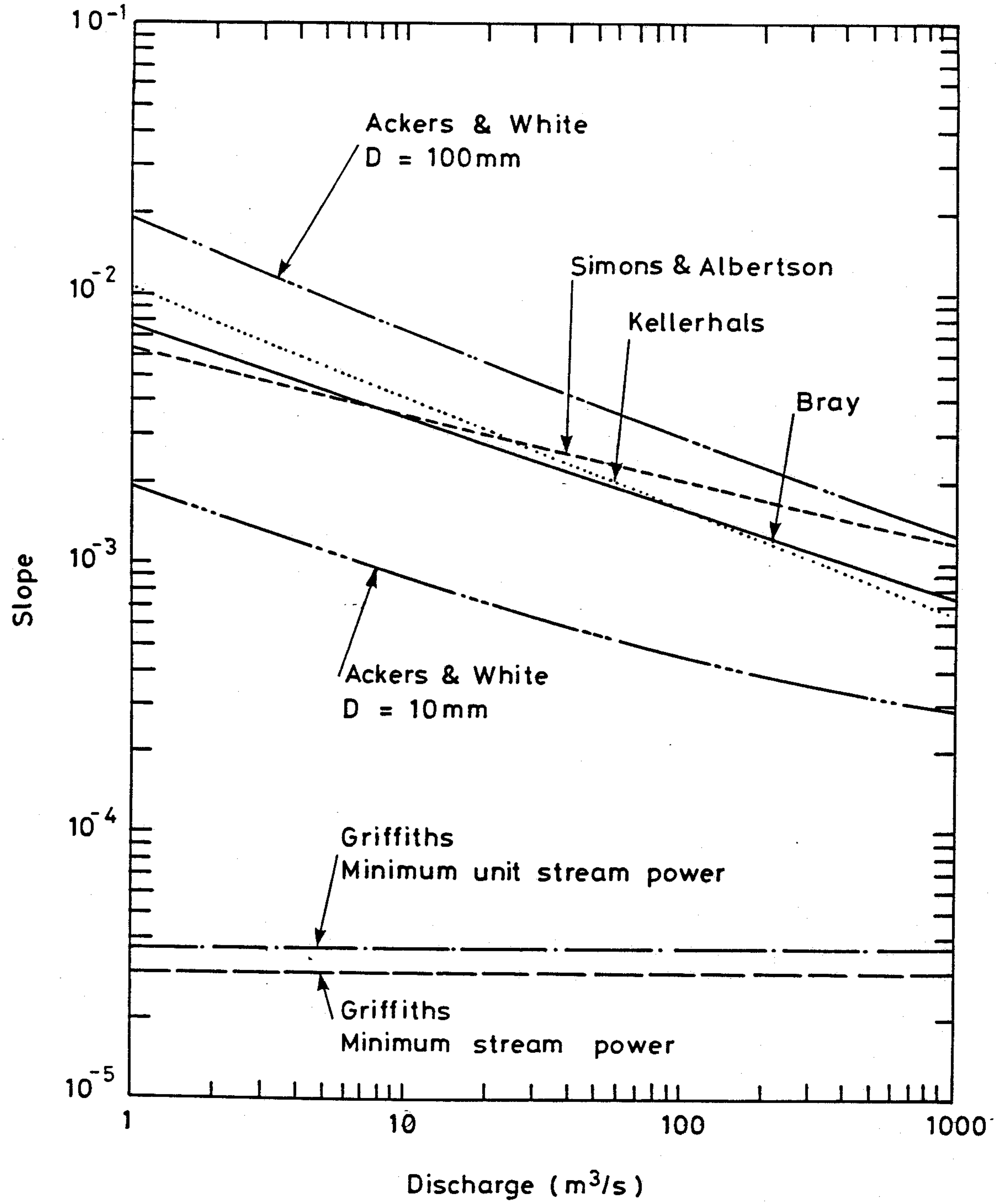

Fig 2 Regime slopes for gravel rivers

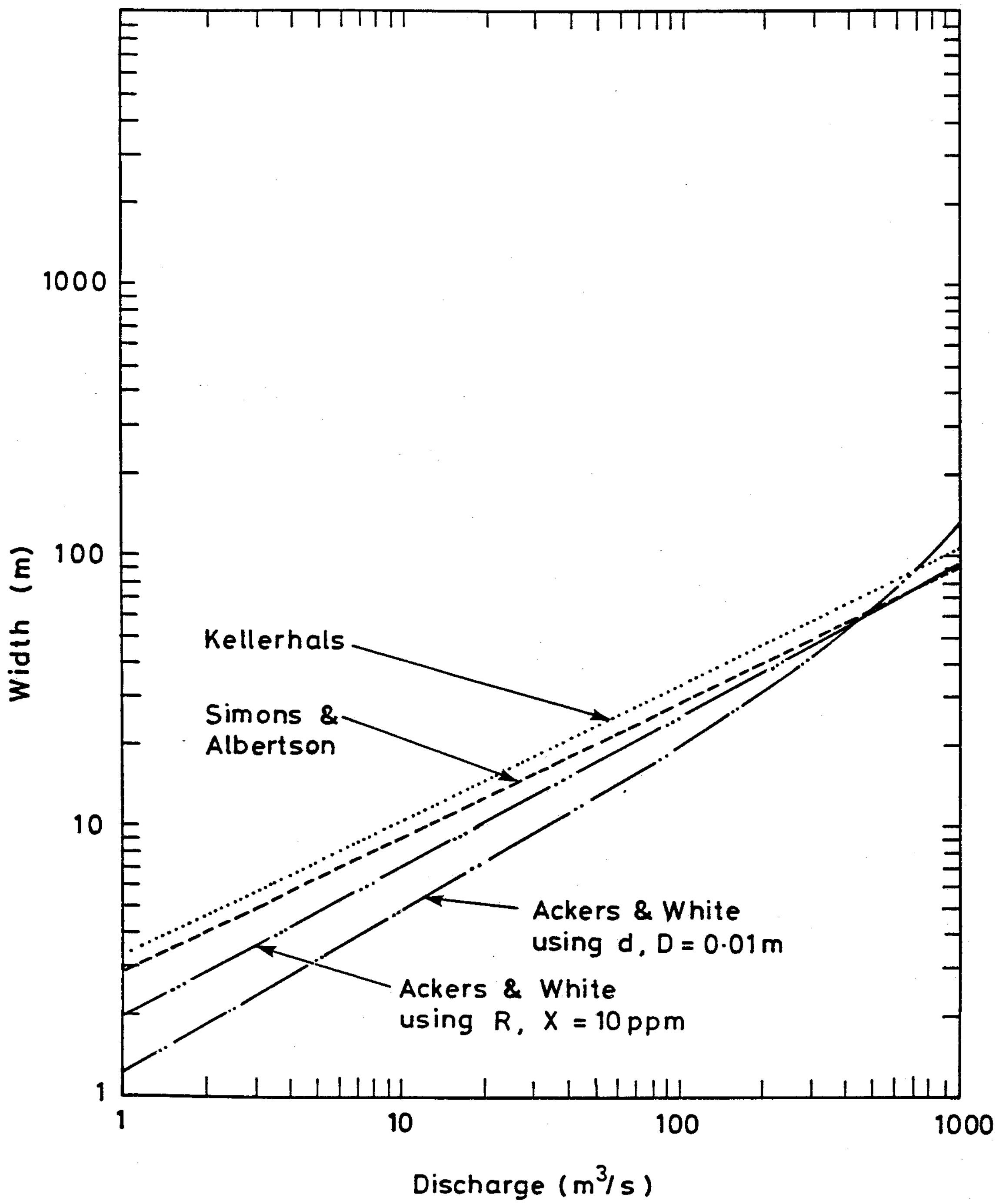

Fig 3 Effect of d or R on regime widths

A RATIONAL APPROACH TO RIVER REGIME

WANG SHIQIANG [1], DR W R WHITE [2] and DR R BETTESS [3]

The purpose of regime theory is to predict the size and shape of stable alluvial channels. The theory was first developed from empirical studies based on extensive field measurements. Recent improvements in our understanding of sediment transport processes, however, have introduced the possibility of relating the size of regime channels to these fundamental sediment transport processes. The general approach is described together with a number of extremal hypotheses which have been suggested to determine regime conditions. These extremal hypotheses assume that the channel dimensions are such to maximise or minimise the value of some appropriate functional. The predictions of channel dimensions using various extremal hypotheses and sediment transport relationships are compared with observed channel data.

Introduction

Alluvial channels are continually in a state of adjustment, responding to changes in discharge, sediment load, slope or interference by man. Despite these changes there are underlying equilibrium conditions towards which the system tends even if these equilibrium conditions are never attained. An attempt to predict the equilibrium size, shape and slope of an alluvial channel was first made using empirical methods (Lindley, 1919; Lacey, 1929). Numerous measurements were taken on alluvial channels thought to be in equilibrium. The dimensions and velocity in these channels were related, using empirical equations, primarily to the discharge, though Lacey introduced the notion of a dependence on the sediment characteristics by using a silt factor.

Later the same approach was used to study natural river channels. The chief difference here being the variability of the discharge which is usually much larger in natural rivers than in canals controlled by man. This variability in discharge leads to problems in the definition of the discharge to which the characteristics of the channel should be related. The term dominant discharge was coined for the appropriate discharge though opinion is still divided as to how such a discharge should be determined.

The empirical approach to river regime has met with some success provided that the derived equations are applied to similar channels from the same geographical areas with parameters contained within the parameter range of the data for which the equations were derived. Any extrapolation beyond the range of the data or to different geographical regions is usually less successful. This suggests that though there are discernible relationships between the variables involved the form of the equations used in the empirical approach are not adequate to describe the complete system.

As our understanding of the important sediment transport processes has improved one may consider the development of regime equations which utilise this understanding of sediment dynamics. Such an approach should help in elucidating the significance of the various factors involved and help in the derivation of equations of an appropriate form and hopefully of wide applicability (Hou Hui Chang, 1982).

1. Lecturer, Tsinghua University, Beijing, China.
2. Head, River Engineering Department, Hydraulics Research, Wallingford, UK.
3. Project Manager, River Engineering Department, Hydraulics Research, Wallingford, UK.

If plan geometry is ignored then regime conditions in channels may be described by the following seven variables; discharge, width, depth, velocity, slope, sediment concentration and sediment diameter. There are a number of relationships connecting these variables. The simplest is that relating discharge to the width, velocity and depth but a sediment transport equation and an alluvial friction relationship provide two further equations. It is normally assumed that the sediment diameter, discharge and either slope or sediment concentration are imposed on the system. We have, therefore, six relationships between the original seven variables. To make the system soluble a further relationship is required. A number of different equations have been suggested, most of them based on some extremal hypothesis, and it is these extremal hypotheses which we will consider in greater detail. The advantage of such an approach is that it provides a system in which all the relevant variables are included and which has a wide range of applicability since it is based on theories of sediment mechanics which are valid over a wide range of conditions. This improves on the empirical approaches which have a limited range of applicability.

Extremal hypotheses

A number of extremal hypotheses have been proposed for the development of a regime theory. Nine are discussed below but the list is not exhaustive.

1. Minimum stream power (Chang, 1980) 'For an alluvial channel, the necessary and sufficient condition of equilibrium occurs when the stream power per unit length of channel γQS is a minimum subject to the given constraints, where γ is the specific weight of water, Q is discharge and S is slope. Hence, an alluvial channel with water discharge Q and sediment load Q_s as independent variables tends to establish its width, depth and slope such that γQS is a minimum. Since Q is a given parameter, minimum γQS also means minimum channel slope', Chang (1980). $[S_{min}]$

2. Minimum Energy Dissipation Rate (Brebner and Wilson, 1969; Yang et al, 1981) 'A system is in an equilibrium conditions when its rate of energy dissipation is at a minimum value', Yang et al (1981).

3. Maximum sediment concentration (Singh, 1961; White et al, 1982) '... for a particular water discharge and slope, the width of the channel adjusts to maximise the sediment transport rate', White et al (1982). $[X_{max}]$

4. Maximum Friction Factor (Davies and Sutherland, 1980) 'If the flow of a fluid past an originally plane boundary is able to deform the boundary to a non-planar shape, it will do so in such a way that the friction factor increases. The deformation will cease when the shape of the boundary is that which gives rise to a local maximum of friction factor. Thus the equilibrium shape of the non-planar, self-formed flow boundary or channel corresponds to a local maximum of friction factor', Davies and Sutherland (1980). $[FF_{max}]$

5. Minimum Froude number. For a particular water discharge and sediment load, the width of the channel adjusts to minimise the Froude number. $[Fr_{min}]$

6. Minimum total friction resistance. For a given discharge and sediment load the channel adjusts to minimise the total frictional resistance. $[FR_{min}]$

7. Minimum friction factor. For a given discharge and sediment load the channel adjusts to minimise the friction factor. $[FF_{min}]$

8. Minimum discharge. For a particular slope and sediment concentration the channel characteristics are those associated with the smallest discharge.

9. Minimum Unit Stream Power. This has been included for completeness though doubt has been cast on its applicability in the present situation $[VS_{min}]$.

   The operations of some of these extremal hypotheses in determining channel width is demonstrated in Figure 1.

When considering extremal hypotheses it is not sufficient to consider them in isolation from the sediment relationships also used because the behaviour of a particular extremal hypothesis may be affected by the sediment relationships involved. It is also highly unlikely that a satisfactory regime relationship will result from using poor predictors of sediment mechanics, irrespective of the extremal hypothesis that is used.

Since the only theories we have are empirical and there are no other accepted theories for predicting river regime against which comparisons can be made, the only way of establishing the usefulness of these rational regime theories and their underlying extremal hypotheses is by comparing the predictions that they provide with observations. In this study, therefore, extensive comparisons have been made with data collected from sand and gravel rivers and canals. We provide no other justification for an extremal hypothesis approach to regime theory other than that such rational regime theories provide predictions which are in good agreement with observations.

Relationships between extremal hypotheses

We have described a number of different variational principles and will now investigate the relationships between them in an attempt to impose some order on the bewildering array of suggestions.

White et al (1982) showed that minimising stream power subject to given values of discharge and sediment concentration is equivalent to maximising the sediment concentration subject to given values of discharge and slope. If the effect of sediment transport on the energy dissipation rate is regarded as small then minimum energy dissipation rate is equivalent to minimum stream power for given values of discharge and sediment concentration. If energy dissipation due to sediment transport is included then the two principles are still equivalent for all practical purposes, provided the sediment concentration is not too large. It can also be shown that minimum stream power and maximum sediment concentration are both equivalent to minimum discharge for fixed sediment concentration and slope.

ie	min S	max X	min Q
	Q,X fixed	Q,S fixed	X,S fixed

Davies and Sutherland (1980) proposed the extremal hypothesis that there should be an extremum in the friction factor. The expression used for the friction factor was

$$f = \frac{8gdS}{V^2}. \qquad (1)$$

Since the definition of Froude number is

$$Fr = \frac{V}{\sqrt{gd}} \qquad (2)$$

it follows that $f = \frac{8S}{Fr^2}$. (3)

We have therefore that maximising the friction factor is equivalent to minimising the Froude number for a given slope.

An extremal hypothesis must satisfy two conditions to be of value in regime theory. Firstly the extremal values must exist under a wide range of conditions and secondly the resulting regime predictions should agree with observed channel data. These properties, however, will also depend upon the sediment relationships with which they are associated. The sediment transport relationships considered in this present work are the Ackers and White sediment relationships (Ackers and White 1973; White et al, 1980) [A-W], the Engelund and Hansen relationships (1967) [E-H], the Yang sediment transport equation (1982) together with the Engelund and Hansen resistance equation [Yang] and the Parker bed load formula (1978) together with the Keulegan resistance equation [YPK].

Comparison of different formulations

1. Effect of different extremum hypothesis.

We will first consider the differences that arise from using the same sediment transport relationships with different extremal hypotheses. Table 1 shows calculated widths obtained using the Ackers and White sediment relationships and five different extremal hypotheses for different values of D,Q and X. In none of the cases could a maximum be found for the friction factor.

Table 1 Widths predicted by different extremal hypotheses.

Q m^3/s	D mm	X ppm	S_{min}	Fr_{min}	FF_{min}	FR_{min}	VS_{min}
10	0.5	50	14.5	12.5	-	12.2	18.9
100	0.2	100	47.7	47	51.9	44.7	53.4
100	0.2	500	43.8	43.5	44.5	40.1	50.6
100	1.0	100	65.8	53.9	-	43.7	-
100	100	100	20.6	17.7	29.0	19.1	24.6
1000	0.2	1000	160.3	161.4	155.5	146.0	-

The omissions occur since for those values of D, Q and X no minimum could be found. The results show that the differences between the various extremal hypotheses vary depending upon the particular values of D, Q and X.

Once it is established that different extremal hypotheses may give significantly different predictions one must then consider which extremal hypothesis gives the most useful predictions. This was judged by comparing the predictions using the different extremal hypotheses with observed data. The observed data consisted of 203 sets of data from sand rivers and canals and 59 sets of data from gravel rivers. The data from sand channels covered the following ranges

$0.34 < Q(m^3/s) < 24,300$

$1.8 < B(m) < 1100$

$0.11 < D(mm) < 4.7$

$1 < X(ppm) < 3000$

The gravel river data covered the range

$2.7 < Q(m^3/s) < 9000$

$5.2 < B(m) < 550$

$20 < D(mm) < 145$

A comparison of the observations with the different predictions with Ackers and Whit sediment relationships and various extremal hypotheses is shown in Table 2. The comparison is made in terms of the discrepancy ratio, that is, the ratio of the predicted to the observed values. Values are given of the mean discrepancy ratio A, which indicate on average how good were the predictions of each method, and the value of the standard deviation SD which indicates the scatter of individual predictions. Figure 2 shows a comparison of observed and predicted widths using the Ackers and White equations together with minimum stream power for the sand data. A similar comparison for gravel data using maximum sediment concentration is shown in Fig 3. The results using the Engelund and Hansen sediment relationships showed a similar behaviour.

Table 2 Comparison of discrepancy ratios for different extremal hypotheses.

Type of channel	Extremal principle	Width A	Width SD	Depth A	Depth SD	Slope A	Slope SD
Sand	X_{max}	1.01	0.38	1.08	0.34		
	S_{min}	1.03	0.41	1.05	0.41	1.26	1.01
	Fr_{min}	0.98	0.34	1.10	0.37	1.29	1.01
	FR_{min}	0.84	0.30	1.18	0.45	1.31	1.10
	FF_{min}	1.03	0.40	1.04	0.42	1.14	0.69
	VS_{min}	1.33	0.68	0.90	0.31	1.19	0.78
Gravel	X_{max}	1.06	0.45				
	S_{min}	0.95	0.40			0.93	0.76
	Fr_{min}	0.74	0.27			0.97	0.80
	FR_{min}	0.78	0.28			1.01	0.84
	FF_{min}	1.00	0.54			0.33	0.25
	VS_{min}	1.38	0.88			0.97	0.81

The results show that the principle of minimum stream power or maximum sediment concentration gives the best agreement with field data. Of the remaining extremal hypotheses the principle of minimum Froude number provides the best results. The principles of minimum stream power and maximum sediment concentration while being equivalent provide slightly different predictions since one is using an observed sediment concentration and the other an observed slope. Discrepancies in the measurements of these quantities will lead to differences in predicted values of width.

The larger deviation of mean discrepancy ratio from the value of 1 and the larger standard deviation for the predictions of slope reflect the greater sensitivity of the slope to the specified values than either the width or the depth. No values are shown under slope for the principle of maximum sediment concentration since under this formulation slope is specified and hence cannot be predicted. For gravel rivers there is no comparison of depth as the data was unavailable. The agreement between predicted and observed results indicate the usefulness of extremal hypotheses in providing realistic predictions of regime conditions in alluvial channels.

2. Effect of different sediment transport relationships.

Using the Ackers and White sediment relationships, the principle of minimum slope or maximum sediment concentration provided the best agreement with observed data. It does not follow, however, that these extremal hypotheses will provide the best agreement if other sediment relations are considered. Calculations were, therefore, performed using the Engelund and Hansen equations (1967) and equations due to Yang (1982). Comparisons of predicted with observed data are shown in Table 3. A more detailed analysis of the distribution of discrepancy ratios for the Ackers and White and Engelund and Hansen sediment relationships are given in Figs 4 and 5.

The accuracy using the Ackers and White and Engelund and Hansen relations are comparable, with marginally better predictions by Ackers and White, but both, in general, give better predictions than the Yang or Yang-Parker-Keulegan sediment relations. Figure 4 shows that the Ackers and White formulation with maximum sediment concentration predicts the width to within ±25%, 66% of the time. The corresponding figure using the Engelund and Hansen equations is 48%. Figure 5 shows that the slope prediction using the Ackers and White equations are within a factor of two 79% of the time.

Both the Yang and the Parker and Keulegan formulations exhibit systematic over or under prediction under certain circumstances. Depending upon one's point of view this reflects shortcomings in an extremal approach or in the sediment relationships themselves.

Table 3 Discrepancy ratios using various sediment relationships.

Type of channel	Extremal hypothesis	Sediment relation	Width A	Width SD	Depth A	Depth SD	Slope A	Slope SD
Sand	S_{min}	A-W	1.03	0.41	1.05	0.41	1.26	1.01
		E-H	0.92	0.40	0.93	0.36	1.50	1.00
		Yang	1.01	0.49	1.96	1.82	0.01	0.00
Sand	X_{max}	A-W	1.01	0.38	1.08	0.34		
		E-H	0.97	0.36	0.97	0.30		
		Yang	0.69	0.26	1.16	0.39		
Gravel	X_{max}	A-W	1.06	0.45				
		E-H	0.80	0.28				
		Yang	0.63	0.23				
		YPK	0.56	0.20				

In empirical regime theory variables such as the channel width or slope are related to the discharge sediment diameter and other variables using equations of the form

$$B = a\,Q^{b_1}\,D^{b_2}\,\ldots, \tag{4}$$

for example,

$$B = 2.67\,Q^{0.5} \qquad \text{(Lacey, 1929)} \tag{5}$$

It is possible to approximate the results predicted by rational regime theory by equations of the form

$$B = k_1\,Q^{m_1}\,X^{m_2}\,D^{m_3} \tag{6}$$

and $$S = k_4\,Q^{c_1}\,X^{c_2}\,D^{c_3} \tag{7}$$

Values of the exponents derived for different ranges of conditions using the Ackers and White and the Engelund and Hansen sediment relations are given in Table 4.

The exponent m_1 in equation (2) is approximately 0.57 using the Ackers and White sediment relationships or approximately 0.52 using the Engelund and Hansen sediment relationships. The exponent m_2 varies with the values of both the discharge and the sediment diameter. Using the Ackers and White relationships m_2 is slightly less than zero for fine sediments and is slightly greater than zero for coarse sediments. This is in agreement with field and laboratory data. The values of the exponents m_1, m_2, m_3, c_1, c_2 and c_3 are all in qualitative agreement with observations, see Table 4.

It should be observed that not all sediment relationships can be combined with an extremal hypothesis to derive a regime theory. For example if the following sediment transport (Bogardi, 1974) and friction equations are used

$$X = K\,\frac{V^3}{gd\omega}$$

and $$V = \frac{1}{n}\,d^{2/3}\,S^{\frac{1}{2}}$$

then for given values of Q and X no minimum exists for the slope.

'able 4 Exponents in regime equations.

. $B = k_1 Q^{m_1} X^{m_2} D^{m_3}$

Range	Parameter	Ackers and White	Engelund and Hansen
0.2 < D(mm) < 0.5	m_1	0.57	0.53
50 < X(ppm) < 200	m_2	-0.05	-0.15
$Q(m^3/s)$ < 1000	m_3	0.15	-0.10
	k_1	5.6	7.6
50 < D(mm) < 200	m_1	0.56	0.52
10 < X(ppm) < 50	m_2	0.08	0
$Q(m^3/s)$ < 1000	m_3	-0.3	-0.15
	k_1	4.5	4.25

. $S = k_4 Q^{c_1} X^{c_2} D^{c_3}$

Range	Parameter	Ackers and White	Engelund and Hansen
0.2 < D(mm) < 0.3	c_1	-0.24	-0.17
50 < X(ppm) < 200	c_2	0.41	0.61
100 < $Q(m^3/s)$ < 500	c_3	1.27	0.53
	k_4	0.0003	0.00004
50 < D(mm) < 200	c_1	-0.26	-0.17
10 < X(ppm) < 50	c_2	0.28	0.65
100 < $Q(m^3/s)$ < 500	c_3	0.36	0.53
	k_4	0.0007	0.00004

:onclusions

1. The following extremal hypotheses are equivalent

min S	max X	min Q
Q,X fixed	Q,S fixed	X,S fixed

   Two other extremal hypotheses are equivalent

min Froude number	maximum Friction Factor
Q,S fixed	Q,S fixed

2. Extremal hypotheses together appropriate sediment transport relationships can be combined to provide a rational regime theory. Comparisons with observations show that such regime theories have a wide range of applicability.

3. The use of different hypotheses may lead to different predictions. On the data used in this study the principle of minimum slope or maximum sediment concentration provided the best agreement.

4. The use of different sediment relationships may lead to different predictions. Of the sediment relationships considered the Ackers and White and Engelund and Hansen sediment relationships provided the best agreement with observed data.

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Notation

A	mean discrepancy ratio
a	constant
B	(m) width
c_1, c_2, c_3	parameters in equation for S
D	(m) sediment diameter
d	(m) depth
Fr	Froude number
f	friction factor
g	acceleration due to gravity
K	constant in sediment transport relationship
k_1, k_4	constants in equations for B and S
m_1, m_2, m_3	parameters in equation for B
n	Manning's n
Q	(m^3/s) discharge
Q_s	(m^3/s) sediment discharge
S	slope
SD	standard deviation of discrepancy ratios
V	(m/s) velocity
X	sediment concentration by weight
γ	specific weight of water
ω	(m/s) fall velocity

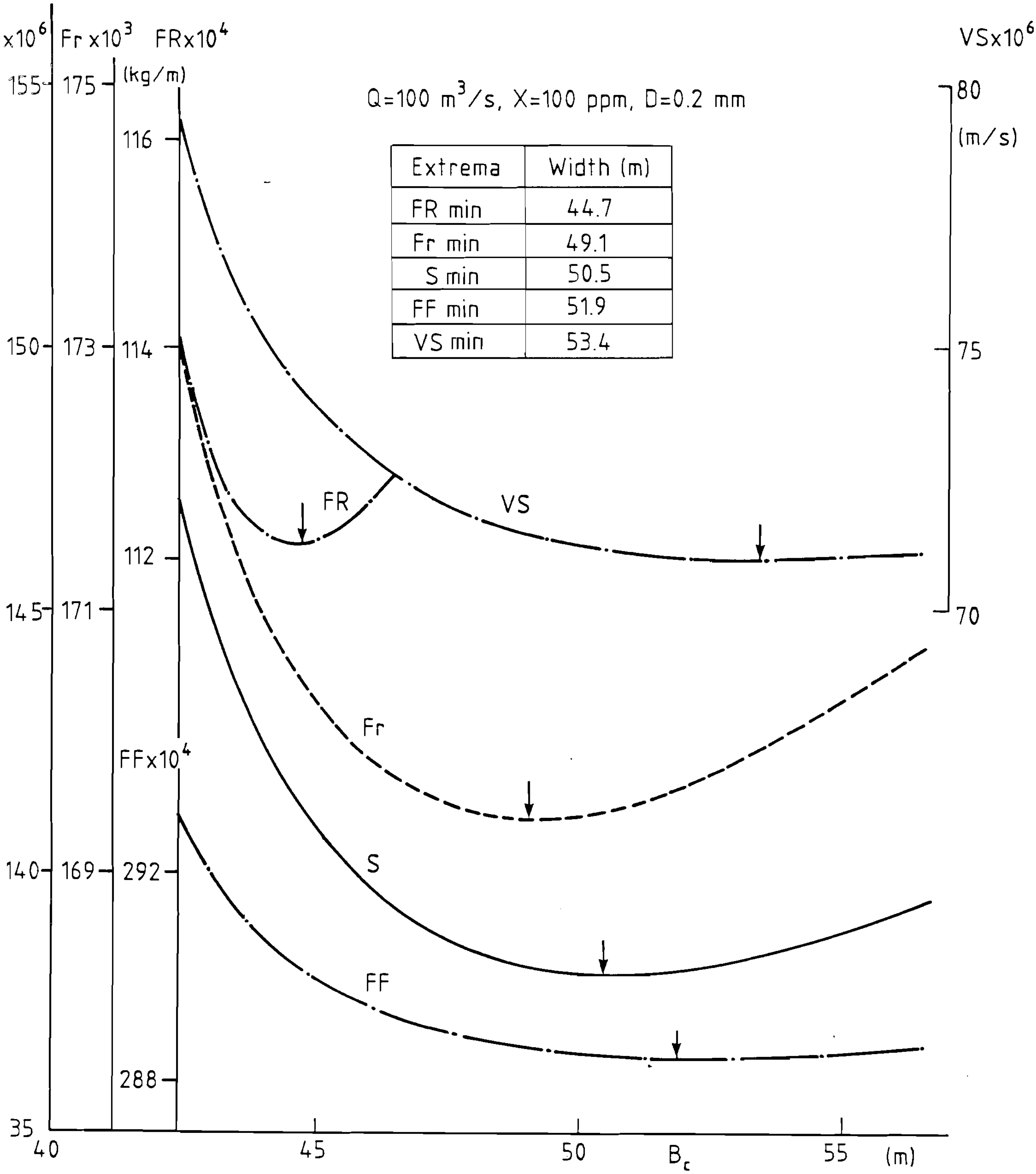

Fig 1 Fr, S, FF, VS, FR against B_c

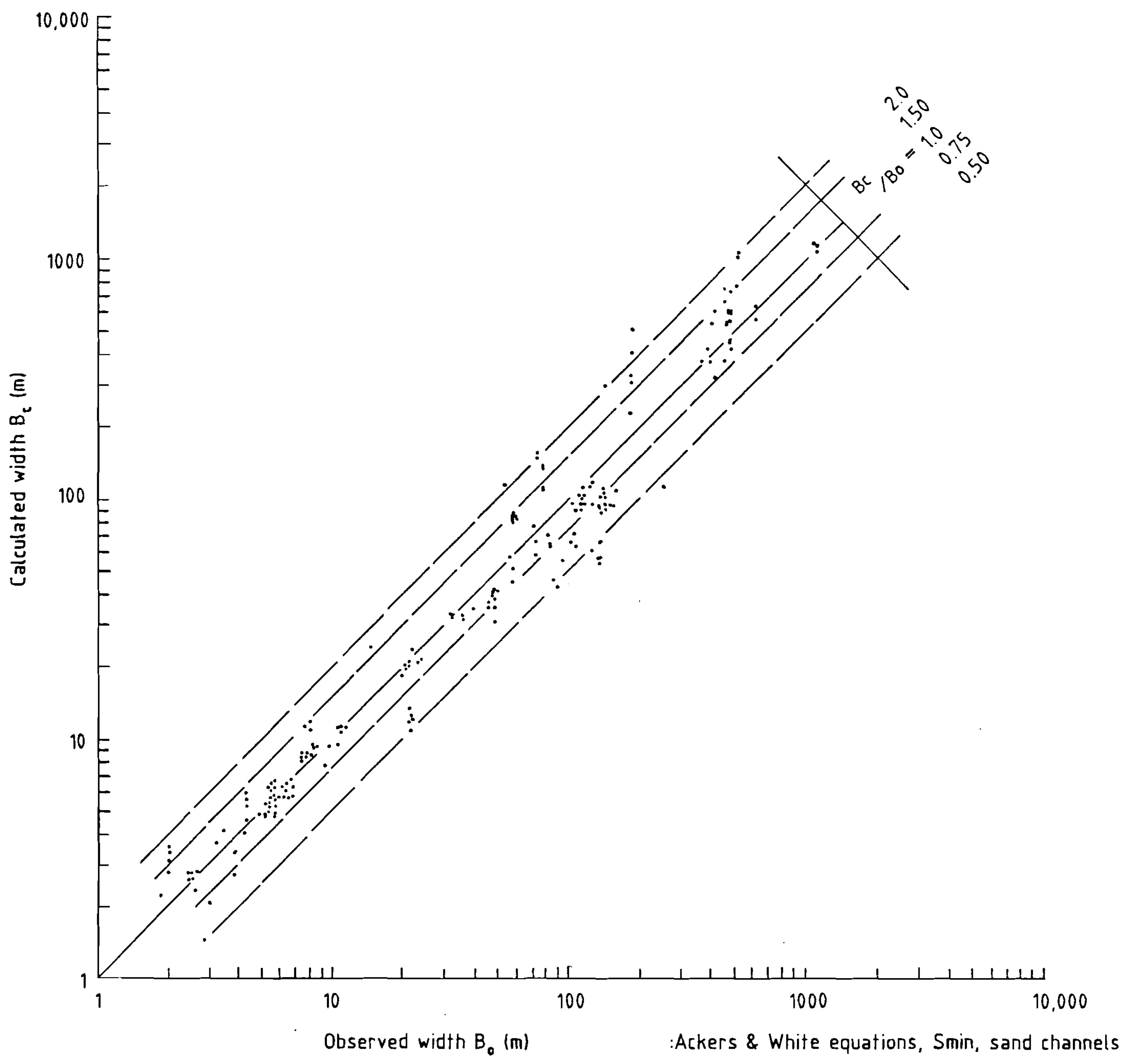

Fig 2 Calculated against observed width

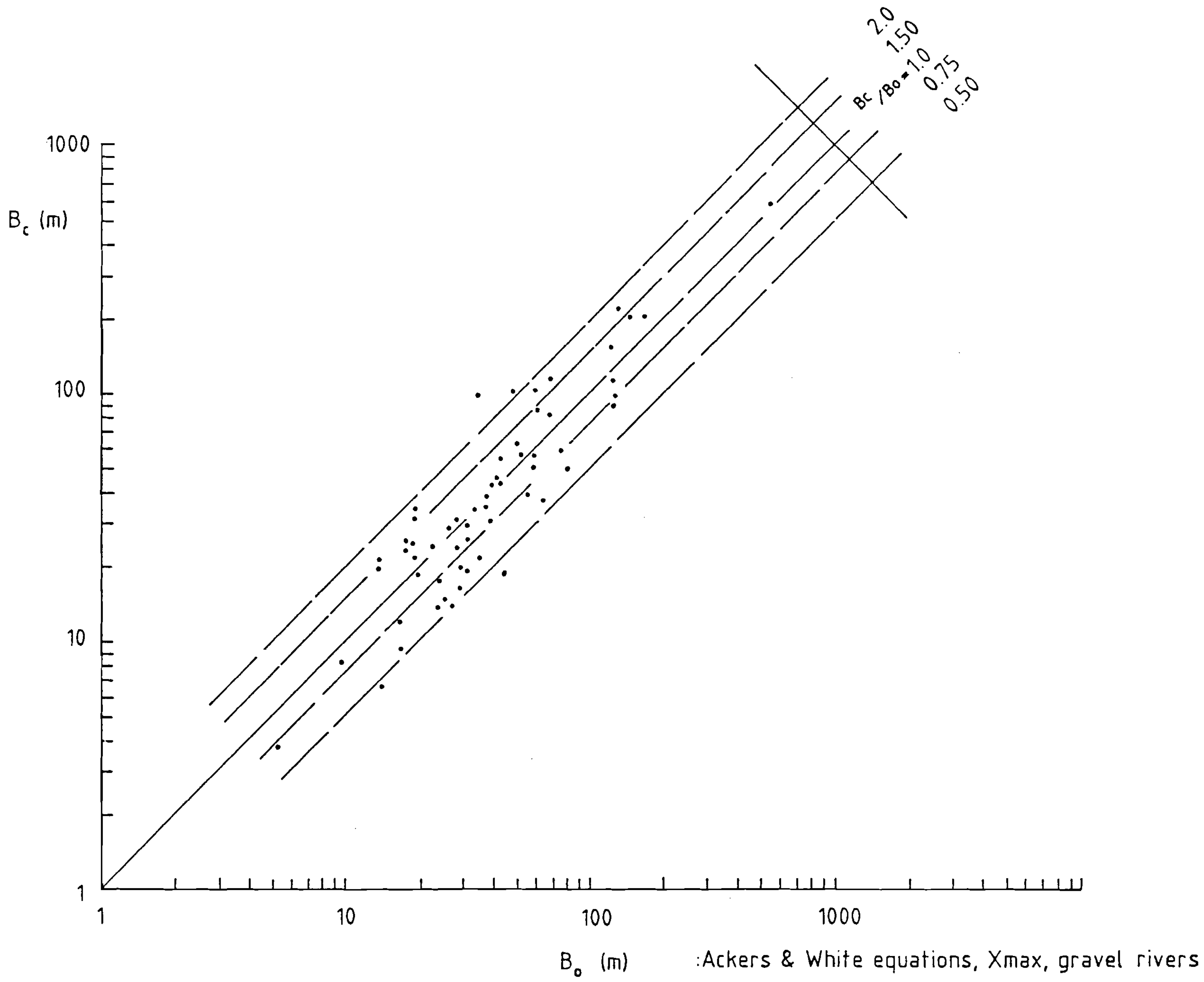

Fig 3 Calculated against observed width

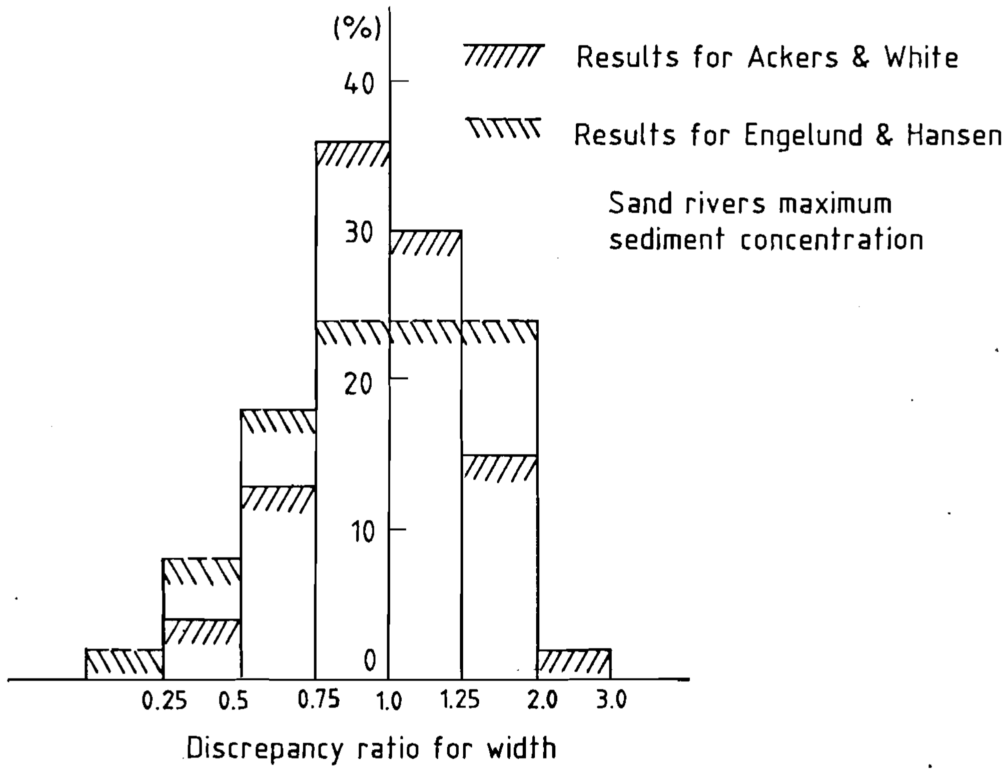

Fig 4 Comparison of discrepancy ratios

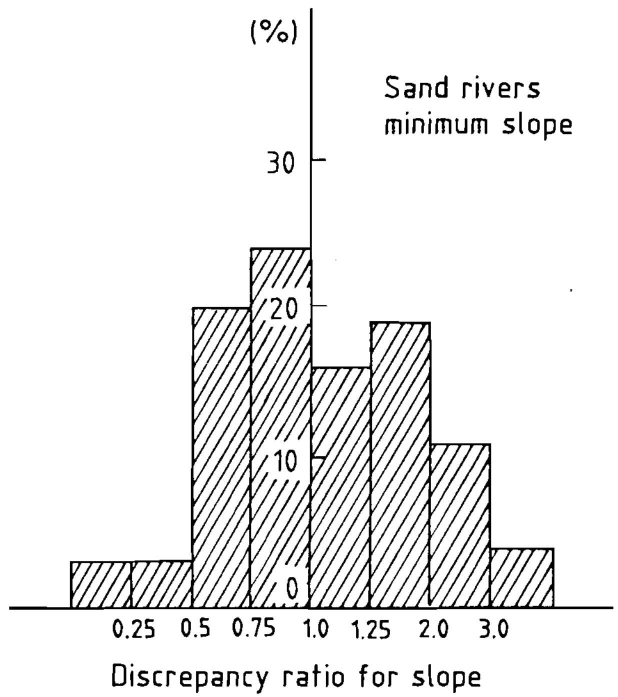

Fig 5 Discrepancy ratios : Ackers & White equations