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MATHEMATICAL MODELLING OF MOORED SHIPS IN THE TIME DOMAIN

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ABSTRACT

This report describes the initial development of a mathematical model in the time domain of a moored ship in waves. This model, called SHIPMOOR, is necessary in order to represent, correctly, the motions of vessels on conventional moorings. This is because, normally, moorings are highly non-linear making a frequency space description inadequate.

This time domain model complements a frequency space model called UNDERKEEL which is being developed in parallel under the same contract. Hydrodynamic coefficients for describing the motion of a vessel in waves will be obtained from UNDERKEEL for use in SHIPMOOR. Used together in this way these models will allow a partial description of moored vessel movements in waves for the particular case of interest in coastal work, ie where the underkeel clearance is limited. However, further work, requiring an additional contract, will be needed to enable non-linear wave forces, which are now known through random wave physical modelling of harbours to be of crucial significance in controlling berth tenability, to be represented mathematically. Only when this is completed can SHIPMOOR be validated against the motions of actual moored ships since such motions will contain non-linear responses.

In this report SHIPMOOR is applied to two cases of practical interest under the simplifying assumption of frequency independent hydrodynamic coefficients. One application is to a vessel executing subharmonic sway motions against fenders. Such motions are known, through physical modelling, to be caused by fenders being stiffer than mooring lines in conventional mooring systems. Comparisons made with an idealised physical model showed good agreement. In these tests the waves were of single period. Further development of SHIPMOOR is needed to represent random wave forcing.

The second application is to the passing ship problem where the disturbance created by a vessel underway can cause lines to part on vessels moored nearby. Comparisons of movements and mooring line loads from SHIPMOOR are made with physical model results obtained some years ago for an oil tanker berth in Milford Haven. SHIPMOOR was found to provide a good reproduction of maximum mooring line loads caused by passing ships. But results were sensitive to starting conditions in the simulation and further study of this aspect is in progress.

The results of the above applications to problems of practical interest are encouraging and extension of SHIPMOOR to enable frequency dependent hydrodynamic coefficients to be represented is underway under this contract. This will enable simulations to be carried out with linear random wave forcing.

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1 INTRODUCTION

There is an increasing demand from the designers of harbours and vessel moorings for mathematical models of moored ships. Used in conjunction with suitable mathematical wave prediction models, a moored ship model will forecast likely ship movement. This will enable the designer to make a first estimate of such things as the breakwater protection required in a proposed harbour or the likelihood of a ship's loading and unloading being disrupted by excessive ship movement, ie berth "downtime". These factors critically affect a port or berth's viability. Designers can use mathematical models to assess layouts at a feasibility stage and to test, possibly many, alternative proposals, including different sites, until preferred options emerge.

At an early design stage of a project there is usually insufficient funding available to carry out a physical model investigation which, although providing a comprehensive description of both harbour response and moored ship behaviour, becomes time consuming, and therefore expensive, if a significant number of different options have to be considered. On the other hand mathematical models, which can describe to a reasonable accuracy the most important effects already identified in many previous random wave physical models, allow many different parameters to be varied at minimum cost. Such models can be used to good effect to narrow down the number of possible options. This leads naturally to the use of a physical model, at a later stage in design, to provide more detailed hydraulic information on a reduced number of options.

It might be thought that variation in the size of ships would preclude a meaningful description of the motions of a moored ship in any particular harbour because of the impossibility of covering all cases: this is not true. It may have been true twenty or thirty years ago when general cargo vessels of many sizes were the norm. But now berths are usually designed for a particular type of ship, examples being container vessels and roll on/roll off ferries. Thus, the standardisation in vessel size which has already occurred will normally allow a 'design' vessel to be defined for any particular berth within a harbour.

While a revolution has occurred in vessel type over recent decades, the same cannot be said for vessel moorings. However, operators are becoming aware that berthing and mooring techniques will have to improve. Berthing and mooring has now become the most labour intensive part of a vessel's passage since electronic controls in modern ships have reduced the size of crews needed while the vessel is at sea. Better moorings will further reduce the man power needed.

The merchant fleets of the developed world will then be able to make the reductions in crew costs which they need if they are to compete effectively with the lower man power costs in the fleets of developing countries (Ref 1).

Better moorings have the additional advantage that berth tenability in exposed harbours will be improved. And in newly proposed harbour extensions and proposed new harbours it may well be possible to accept less shelter of berths than would be the case with existing mooring practices. Breakwaters are expensive to build in the deep water needed for large modern ships. Reduced need for shelter could well make all the difference between a proposal being feasible or not (Ref 2). Thus, the time is ripe for development of improved moorings and, with such development, standardisation in mooring design will inevitably follow. As part of this process, the modelling of vessel moorings can be expected to assume increased significance with computer simulations in the time domain playing an important role.

To satisfy the requirement for mathematical modelling of waves in harbours, and ships moored in waves, HR is undertaking a 3 year research programme with DoE funding covering the period 1985 to 1988.

In one contract (PIF No HR 2/8) a computer model is being developed which is capable of describing both the main or primary wave activity within a harbour and associated long period disturbances (such as set-down) due to wave grouping. This model will extend HR's present capability of describing the primary wave disturbance within a harbour using a computer model developed under an earlier research contract (Ref 3). Our present model has been much used in project work but it cannot simulate important wave non-linearities identified in random wave physical models; a separate, less accurate, long wave model having to be used for this purpose (Ref 4). Although they cause smaller forces on vessels than the primary waves, long period disturbances excite resonant motions of large vessels on their moorings and thereby cause berth downtime. The model of waves in harbours being developed under the contract PIF No HRL 2/8 will enable both primary and long period wave activity to be evaluated more accurately. This information can then be used as input to appropriate mathematical models of ships moored in waves to get a good first estimate of berth tenability.

A second contract (PIF No HRL 4/8) of which this report forms a part, is concerned with the development of mathematical models of moored ships. The work being done under this contract falls conveniently into two stages:

1. to calculate hydrodynamic coefficients which control the response of the vessel in the six degrees of freedom of movement (see Fig 1);

2. to set-up a time development model so that non-linearities present in conventional mooring systems, eg fenders that are stiffer than mooring lines, can be described.

At the end of the present contract the work done should enable the response of moored ships to the primary waves to be well defined. Thus, dredged levels of berths and navigation channels, taking account of vertical vessel motions in waves, can be defined. In addition, vessel movements and mooring loads at primary wave periods and those long period vessel movements due to subharmonic response, caused by vessel moorings being non-linear, can be described. An extension of the present contract will be needed to calculate long period non-linear wave forces like those due to set-down and radiation stress. Once this is done the full non-linear wave disturbance available from the mathematical model of waves in harbours, at present under development and mentioned above, can be used in the ship models. It will then be possible to describe moored ship response, on non-linear mooring systems, to both primary wave activity and non-linear wave effects. This would complete the amalgamation of harbour and ship models needed to estimate berth tenability.

The above discussion has shown that modelling ship motion mathematically is a three stage process; the third stage being the extension to non-linear wave forces.

The first stage is to calculate how the ship responds to ordinary wind generated waves when it is not moored. This calculation gives as its results the linear forces waves exert on the ship and the ship's hydrodynamic added masses and inertias and damping coefficients. These will be needed again at a later stage. It is feasible to do stage one calculations in the frequency or wave period domain. Responses are calculated by studying the effect that regular waves of a given period have on the ship. Forces, added masses and inertias and damping coefficients are defined as functions of wave period.

The stage one model UNDERKEEL which is under development in the present contract builds upon a mathematical model of pitch and heave (Fig 1) which was developed under a previous research contract (Ref

5). This model has already been used to good effect in project work to determine a safe underkeel allowance for very large vessels (draughts up to 22m) in the Dover Strait (Ref 6). Under the present research contract the remaining four degrees of freedom (roll, surge, sway and yaw) will be described (Fig 1). The model is specifically designed to deal with vessels with small underkeel clearances, ie the typical case in coastal waters.

Unmoored ships respond to wind generated waves of fairly short period (up to about twenty seconds) - these are the waves which are visible to anyone on the seashore. But for most modern commercial ships at conventional moorings, long period oscillations (periods longer than twenty seconds) are more significant. This is because for movements in the horizontal plane (ie surge, sway and yaw) the restraining forces acting on the ship arise from its moorings. Mooring ropes act like springs in a spring/mass system. Commercial vessels have large masses. As a result, the system has a long resonant period - typically between 20 seconds for a 5,000 tonne displacement vessel and over 2 minutes for a very large oil tanker or bulk carrier. Hydrodynamic damping on the ship at these long periods is small. Consequently, even small long period forces acting on a large ship can set up resonances in these horizontal modes of motion.

Long period ship motions are not normally caused directly by wind generated water waves. They are more likely to arise from non-linear effects due to wave grouping (this is illustrated schematically in Fig 2). One of these effects leads to small variations in water surface elevation known as set-down beneath wave groups. Set-down is a depression in the mean water surface under groups of large waves. It is detectable by instruments although not noticeable to the naked eye. And set-down is known to be capable of driving long period ship oscillations both directly (Ref 7) and via harbour resonance (Ref 8).

There are also interactions like radiation stress (Ref 9) which cause forces associated with wave grouping whenever significant diffraction occurs around the moored ship. These forces can be calculated only after wave diffraction around the ship has been computed.

As already mentioned, the third stage in modelling moored ship motions is calculating long period non-linear wave forces like those due to set-down and radiation stress and this third stage will require an extension of the present contract.

This report is concerned with stage two, ie calculating the responses of the moored ship once hydrodynamic coefficients and forces have been defined. This involves adding the forces exerted by the ship's mooring to the wave forces and using the resultant to solve the ship's equations of motion. A model is being developed to do this under the present contract (PIF No HRL 4/8). This report describes a simplified version of that model, which has been called SHIPMOOR, together with its application to two problems of practical interest.

The theoretical difficulty of stage two is that mooring forces are normally non-linear; forces do not increase linearly with rope extension or fender compression. Even if the ropes and fenders can be assumed to be perfectly elastic, the mooring system is bilinear rather than truly linear. This arises in sway where the spring rate of the system will be much larger when the ship is pressed against the fenders, which are normally stiff, than it will be when it moves off the fenders and is only restrained by the relatively softer mooring ropes.

The non-linearity of the mooring system rules out modelling stage two by using the wave period or frequency domain approach used for the unmoored ship in stage one. Frequency domain methods make the assumption that a regular periodic forcing provokes a regular periodic motion in response. Non-linear systems do not necessarily behave like that. For example, the impact oscillator (described later in this report) may be used to model sway in the limit of infinitely stiff fenders and it responds subharmonically to regular forcing. The response is periodic, but it is at a longer period than the forcing and it is not regular.

Instead of the frequency domain, moored ship motions must be calculated in the time domain. The method proceeds in a series of time-steps. At the start of each time-step, the ship's position and velocity are known; and its acceleration is calculable from known forces and mass. Acceleration and velocity are integrated numerically across the time-step to give the new ship's velocity and position at the end of the time-step. This new position and velocity then become the initial position and velocity for the next time-step.

The impact oscillator which is a limiting case of a vessel swaying against fenders illustrates another important aspect of non-linearity apart from its influence on modelling techniques. What tends to happen is that any swaying ship will bounce off its fenders and return after an interval given approximately by the natural sway period of the mooring. In some cases, the net effect of wave forcing during the bounce is to feed energy into the ship's motion; this energy is not dissipated during the ship's relatively brief period of contact with the fenders. So the ship starts on its next bounce with more kinetic energy than it started with on its last one. The result is a build up of energy and hence successively larger bounces. It is a resonant effect. Like a child being pushed on a swing, small forces can produce a large motion if they are applied at the right time.

The ship resonating in sway is an example of subharmonic resonance. The child in the swing is an example of the ordinary type of resonance; the swing moves periodically and force is applied periodically; and the period of the forcing is the same as the period of the motion. A subharmonic resonance also has a periodic motion and a periodic forcing. But in the subharmonic case the period of the motion is given by an integer multiple of the forcing period. This explains how a fairly short period forcing given by wave action can resonate with the long natural sway period of a large moored ship (Ref 10).

Subharmonic resonance is therefore interesting from two points of view. There is a practical interest because it can cause large ship movements which can damage moorings. And there is a theoretical interest because it is an effect due to non-linear moorings.

These two aspects of interest are reflected in the treatment given to subharmonic resonance in this report. This forms the first application of SHIPMOOR which is described in Chapter 3. There is firstly a theoretical investigation which uses a simplified model of the swaying ship (ie the impact oscillator). The impact oscillator represents the case of infinitely stiff fenders; the ship bounces off them instantaneously with the same sway speed at which it hit them. These simplifications are dropped in the practical investigation. This includes the effects of finite fender stiffness and the ship rolling as well as swaying.

In carrying out this work it has been possible to investigate subharmonic responses to random wave forcing. This shows that the fear expressed in some recent work (Ref 11), that subharmonic responses can be missed in simulations if a comprehensive set of initial conditions is not considered, is unfounded. While true with regular wave forcing it appears that random wave forcing ensures that all the possible subharmonic modes occur during the simulation in a statistically meaningful way. In particular, provided simulations of long enough duration are made, a stable estimate of the standard deviation of the time series of vessel motion is obtained irrespective of the initial conditions. In addition, random wave forcing appears to lead to a statistical framework for the description of extreme movements in terms of the standard deviation and average period of the motion. Thus, the chaotic behaviour described in Ref 11 with regular wave forcing does not appear to lead to any particular difficulties in describing the statistics of extreme movements once random wave forcing (the more realistic case) is used.

The second application of SHIPMOOR is to the case of a moored ship being passed by another ship; the well known "passing ship" problem. The disturbance in the water due to the passing ship will cause movement of the moored ship. This movement can be large enough to sometimes cause mooring lines to part. It has been modelled mathematically using data to represent realistic ships and moorings. Results are presented in Chapter 4 of this report where comparisons are presented between SHIPMOOR and a physical model investigation carried out at Hydraulics Research into passing ship problems in Milford Haven (Ref 12).

2 THE TIME DEVELOPMENT MODEL - SHIPMOOR

SHIPMOOR has been developed at HRL to do stage two of the modelling process outlined in the introduction. Given characteristics of a ship's mooring system and the ship itself and wave forces acting on the ship, the model will solve the ship's equations of motion and hence calculate movements and mooring loads.

Because the mooring system will generally be non-linear, this solution has to be done in the time domain. The accelerations and velocities of the ship are integrated successively across a series of time steps to compute the ships motion. The Time Development Model does this using a multi-step, fourth-order, predictor-corrector algorithm (see 2.4 for details).

In this section of the report, the ship's position and orientation will be denoted by a six component vector, $\underline{x}$.

x_1 = surge position of ship
x_2 = sway position of ship
x_3 = heave position of ship
x_4 = roll orientation of ship
x_5 = pitch orientation of ship
x_6 = yaw orientation of ship

We use a time variable, t; a $\cdot$ denotes a time derivative, so, for example:

$\dot{x}_1$ = surge velocity
$\ddot{x}_1$ = surge acceleration.

And we define a 6 x 6 inertia matrix, $\underline{\underline{M}}$, which contains the ship's displacement mass ($M_{11} = M_{22} = M_{33}$), moments of inertia (M_{44}, M_{55}, M_{66}) and products of inertia ($M_{46} = M_{64}$). All other components are zero for a normal ship if the moments and products of inertia are taken about the ship's centre of mass.

2.1 Ship dynamics and its equation of motion

We can define the ship's 6-component momentum vector (3 components of linear momentum plus three components of angular momentum) as a matrix product of $\underline{\underline{M}}$, and $\underline{\dot{x}}$:

$$\underline{H} = \underline{\underline{M}}\,\underline{\dot{x}} \qquad (1)$$

Newton's second law applies. Assuming that the ship's moments and products of inertia have been taken about either a stationary pivot or its centre of mass, we can write the ship's equation of motion. Using $\underline{F}$ to represent the external forces and moments acting on the ship:

$$\underline{\dot{H}} = \underline{F}\,(\underline{x}, \underline{\dot{x}}, t) \qquad (2)$$

In SHIPMOOR it is assumed that the linear motions of the ship (surge, sway, heave) are small compared to the ship's length, beam and draught. Rotations (roll, pitch, yaw) are also assumed to be always small angles. With these assumptions, we can write:

$$\underline{\dot{H}} = \underline{\underline{M}}\,\underline{\ddot{x}} \qquad (3)$$

$$\underline{\underline{M}}\,\underline{\ddot{x}} = \underline{F}\,(\underline{x}, \underline{\dot{x}}, t) \qquad (4)$$

The external force, $\underline{F}$, includes forces due to a variety of hydrodynamic effects. We can identify:

(i) A hydrodynamic inertia force, $\underline{\underline{A}}\,\underline{\ddot{x}}$. $\underline{\underline{A}}$ is a 6 x 6 matrix containing the ship's hydrodynamic added masses and inertias (see 2.2).

(ii) Hydrodynamic damping, $\underline{D}(\underline{\dot{x}})$.

(iii) The force exerted by wave action, $\underline{f}(t)$.

And $\underline{F}$ also includes:

(iv) The buoyancy forces and righting moments, $\underline{h}(\underline{x})$.

(v) Mooring forces and moments, $\underline{g}(\underline{x},\dot{\underline{x}})$.

It is conventional to transfer most of these forces to the left hand side of (4). The basic equation of motion is therefore:

$$(\underline{\underline{M}}+\underline{\underline{A}})\ \ddot{\underline{x}} + \underline{D}(\dot{\underline{x}}) - \underline{g}(\underline{x},\dot{\underline{x}}) - \underline{h}(\underline{x}) = \underline{f}(t) \qquad (5)$$

2.2 Hydrodynamic forces

Hydrodynamic forces are forces associated with the flow of water. In equation (5), therefore, the hydrodynamic forces acting on the ship are represented by the terms involving:

$\underline{f}$ - the wave forcing force
$\underline{D}$ - the damping force
$\underline{\underline{A}}$ - the added inertia matrix.

These hydrodynamic quantities are all calculated in stage one of the three stage moored ship modelling process outlined in the introduction. HR has a model under development to do the necessary calculations but only those for heave and pitch have been completed to date (Ref 5). An alternative approach using a source method has been developed (Ref 13) but this method is costly in computing time and also causes difficulties when applied to cases with small underkeel clearance. HR's model is intended to overcome these problems.

Both methods are based on finding approximate potential flow solutions for regular oscillatory flows around the ship. Whichever method is used, the flow around the ship is found in terms of a potential function, $\phi(\underline{r}, t)$. (The vector $\underline{r}$ is being used here to symbolise positions in space in and around the hull.) Having calculated the potential, Bernoulli's equation gives the pressure, p:

$$\frac{1}{\rho}\, p(\underline{r}, t) = \frac{\partial\phi}{\partial t} - \tfrac{1}{2}\,(\nabla\phi)^2 \qquad (6)$$

Here, we drop the second term on the right hand side which is quadratic in velocity but in the third stage of development, where non-linear wave forces are needed, this term will be retained. For the time being, therefore, SHIPMOOR uses a linearised pressure equation:

$$p(\underline{r}, t) = \rho\,\frac{\partial\phi}{\partial t}(\underline{r}, t) \qquad (7)$$

And since the flow is assumed to be regularly oscillatory, we can show the time dependence of both quantities by an exponential function, $e^{i\omega t}$:

$$p(\underline{r}, t) = P(\underline{r})\, e^{i\omega t}$$

$$\phi(\underline{r}, t) = \Phi(\underline{r})\, e^{i\omega t}$$

(7) becomes:

$$P(\underline{r}) = i\omega\rho\, \Phi(\underline{r}) \tag{8}$$

Integrating this pressure over the hull surface gives the total hydrodynamic forces and moments acting on the ship.

The same basic method works for calculating all the hydrodynamic quantities; $\underline{f}$, $\underline{D}$ and $\underline{\underline{A}}$. Calculations differ only in the flow conditions which are assumed to apply.

Wave forcing is calculated assuming that the ship is stationary and with a known regular wave field propagating past it. The potential flow calculation gives the diffracted wave field near the hull and hence the pressure it exerts. Integrating the pressure gives wave forcing directly. Random wave forcing and forcing by multi-directional waves can be simulated by adding forces due to different frequency and directional components.

Damping and added inertia are found by considering the forces acting on a ship which is in regular oscillatory motion. Far from the ship, the water is assumed to be still. But close to the ship, its motion causes regular water waves radiating away from it in some roughly circular pattern. Potential flow calculation gives the water flow associated with these waves and hence the pressures, forces and moments on the ship.

We can separate the calculated potential flow forces into two phase components. The component in phase with ship acceleration, we call the hydrodynamic inertia force. And the component in phase with velocity, we call the hydrodynamic damping.

Both components are linearly proportional to the motion which causes them. But there is coupling between motions; for example, a surge motion will usually cause a heave force and a pitching moment in addition to a reaction force in surge. We express this coupling and the linear proportionality by matrices. Thus the added inertia matrix, $\underline{\underline{A}}$, expresses the inertia force:

Inertia force = $\underline{\underline{A}}\,\underline{\ddot{x}}$

And the damping coefficient matrix, $\underline{\underline{B}}$, expresses the damping force:

Damping $(= \underline{D}(\underline{\dot{x}})) = \underline{\underline{B}}\,\underline{\dot{x}}$

These matrices are functions of the frequency of the ship's oscillatory motion. But if treated as constants, they will approximate inertial and damping forces for motions close in frequency to the frequency for which they were calculated. They can thus be used whenever the ship's motion has a narrow frequency band. Using the constant matrix approximation, the equation of motion (5) becomes:

$$(\underline{\underline{M}}+\underline{\underline{A}})\,\underline{\ddot{x}} + \underline{\underline{B}}\,\underline{\dot{x}} - \underline{g}(\underline{x}, \underline{\dot{x}}) - \underline{h}(\underline{x}) = \underline{f}(t) \qquad (9)$$

Although it is only an approximation, (9) has the advantage over the alternative exact formulation (Ref 13) of necessitating far less computation. It is therefore more economical to use. For this reason, and because they are adequately accurate for qualitative research, constant matrices were used in both applications of the model described in this report. This simplification will be dropped in future work to be carried out using frequency dependent hydrodynamic coefficients.

It should be noted that the hydrodynamic forces whose calculation is described here are completely linear. Their full analysis would involve a number of quadratic and higher order terms in wave height and ship movement. These have been neglected here. This approximation can be justified by invoking the two assumptions that wave height is much less than wavelength and ship movement is much smaller than ship dimensions. Some of these non-linearities however, have important effects at low frequencies. As explained already these would be covered by stage three of the moored ship modelling process but their description lies beyond the scope of the present contract.

2.3 Mooring forces

Non-linear effects are included in calculating mooring forces. Rotations of the ship in roll, pitch and yaw are assumed small. Hysteresis effects in mooring lines and fenders are neglected. But SHIPMOOR does allow the force in a mooring line or fender to be a non-linear function of its extension or compression.

This function may be expressed within the mathematical model in either of two ways. it may be by an analytical function. Alternatively, it can be via a table of load/extension data read into and stored on the computer.

Calculating mooring forces is therefore simple once rope extensions and fender compressions are known. The main purpose of SHIPMOOR is to calculate these extensions and compressions.

Calculation is complicated by the models using two different co-ordinate systems. The position of the ship in surge, sway and heave is best considered relative to axes fixed in the earth. This is also the obvious frame of reference for locating all mooring anchorages on the quay-side. Mooring attachments to the ship, on the other hand, are obviously better located in terms of their position on the ship. This gives another co-ordinate system which uses axes fixed relative to the ship. We also use this second co-ordinate system to express the ship's rotations in roll, pitch and yaw.

The origin for the axes in the ship is taken to be at the ships centre of rotation. This is usually its centre of mass. Occasional cases have a fixed pivoting point about which the ship rotates; this point can be used instead. Any arbitrary origin can be used for the axes fixed in the earth.

Both co-ordinate systems are assumed to be right-handed. Rotations are taken to be positive when they are in a clockwise direction viewed along the axis of rotation looking towards the origin.

It is often necessary in SHIPMOOR's calculations to express a position which has been specified in one co-ordinate system in terms of the other. For example, one frequently necessary calculation is finding the stretched length of a mooring rope. We know the position of the end of the rope attached to the quay. And we know the position of the other end too. But the quay-end position is in earth-based co-ordinates and the ship-end position is in ship-based co-ordinates. We have to use our knowledge of the ships position to express the ship-end position in the quay-end's co-ordinates. We can then easily find the distance between the two ends of the rope - ie its stretched length.

Transforming between the two co-ordinates systems is greatly simplified by assuming that all the ship's angles of rotation are small. Denoting roll, pitch and yaw by ϕ_1, ϕ_2, ϕ_3 and surge, sway, heave by s_1, s_2, s_3 (see Fig 1), we can express a position on the ship (ξ_1', ξ_2', ξ_3') as a position relative to earth co-ordinates (ξ_1, ξ_2, ξ_3):

$$\xi_1 = \xi_1' - \phi_2\,\xi_3' + \phi_3\,\xi_2' + s_1 \qquad (10a)$$

$$\xi_2 = \xi_2' - \phi_3\,\xi_1' + \phi_1\,\xi_3' + s_2 \qquad (10b)$$

$$\xi_3 = \xi_3' - \phi_1\,\xi_2' + \phi_2\,\xi_1' + s_3 \qquad (10c)$$

The inverse transformation is:

$$\xi_1' = \xi_1 + \phi_2\,\xi_3 - \phi_3\,\xi_2 - s_1 \qquad (11a)$$

$$\xi_2' = \xi_2 + \phi_3\,\xi_1 - \phi_1\,\xi_3 - s_2 \qquad (11b)$$

$$\xi_3' = \xi_3 + \phi_1\,\xi_2 - \phi_2\,\xi_1 - s_3 \qquad (11c)$$

Similar transformations are available to translate other vectors, and forces in particular, between the two co-ordinate systems. Since Coriolis and Centrifugal forces are negligible because ship movements are assumed small, force transformations are just rotations of axes similar to the position transformations above; (10) and (11).

The force which acts along a mooring line is a function of its extension. The stretched length and hence extension can be calculated given the ship's position and orientation. Force is calculated from the known load/extension curve of the mooring line. We resolve the force into x, y, and z components. These are usable immediately to give surge, sway and heave forces. Roll, pitch and yaw moments are calculated from the same force but transformed to be relative to the ship's axes.

Fender forces require a broadly similar treatment to mooring line forces. For a fender mounted on the quay-side, compression is given by the distance between the uncompressed position of the fender face and the plane of the ship's hull-side. Allowance is made for ship rotation. The normal reaction force of the fender is assumed to act normal to the hull-side plane. Dynamic friction forces can be introduced if wanted. They act in the plane of the hull-side in opposition to the local velocity of the hull across the fender.

Naturally, fender forces do not act, and neither do mooring line forces, if calculation shows that there is no hull contact with the fender or that a particular line is slack.

2.4 Numerical solution of the equations of motion

The motion of a moored ship, subject to the simplifying assumptions already discussed, is governed by its equation of motion (9). This equation is generally non-linear because it contains non-linear mooring forces. It is consequently insoluble in the frequency domain. But a numerical solution can be found in the time domain by time stepping.

Time stepping methods give the solution y(t), to a differential equation as a series of values at discrete time steps. Each value in the series has to be calculated in turn. At the start of each step, we know the initial value of y from preceding calculations; the differential equation gives its derivative $\dot{y}$. We integrate the derivative across the time step to give y's value at the end of the time step. This final value will serve as the initial value for the next time step and so the process proceeds through time.

Many different time stepping algorithms are available; SHIPMOOR uses a fourth-order predictor-corrector algorithm developed by Hamming (Ref 14). Predictor-corrector methods have the advantages of being quick to run and having easily controlled accuracy. Some accuracy can be traded off to speed program running if desired. One of their disadvantages is that they can be numerically unstable because of the existence of 'parasitic solutions' in some circumstances. The Hamming scheme was chosen as being less prone to instability than most.

It is not suitable for starting the calculation of the solution time series. For this reason, SHIPMOOR always does the first three time steps in any run using a Runge-Kutta scheme. Thereafter the Hamming scheme is used exclusively.

Hamming's method calculates a time series solution $(\ldots, y_{i-2}, y_{i-1}, y_i, y_{i+1}, \ldots)$ for a given first-order differential equation:

$$\dot{y} = f(y, t) \qquad (12)$$

At the ith time step, all values in the time series up to and including y_i are known and we wish to calculate y_{i+1} next. Calculation is a three stage process.

First, an explicit predictor equation is used to give a first estimate for y_{i+1}:

$$y^*_{i+1} = y_{i-3} + \frac{4}{3}\,\delta t\,(2f(y_i, t_i) - f(y_{i-1}, t_{i-1}) +$$

$$2f(y_{i-2}, t_{i-2})) \qquad (13)$$

Second, an improved estimate is got using an implicit corrector equation:

$$y'_{i+1} = \frac{9}{8} y_i - \frac{1}{8} y_{i-2} + \frac{3}{8} \delta t \, (f(y^*_{i+1}, t_{i+1}) +$$

$$2f(y_i, t_i) - f(y_{i-1}, t_{i-1})) \qquad (14)$$

The corrector expression is used iteratively within the model by inserting successive new values, y^*_{i+1}, into $f(y^*_{i+1}, t_{i+1})$ on the right hand side of the equation until a satisfactory convergence is reached.

Third and final stage; the definitive value for y_{i+1} is fixed by allowing for estimated truncation errors:

$$y_{i+1} = y'_{i+1} - \frac{9}{121} (y'_{i+1} - y^*_{i+1}) \qquad (15)$$

Equations (13), (14), (15) show Hamming's method as it is applicable to first order scalar differential equations. But it can be extended to vector equations in the obvious way by letting both f and y be vectors. Extension to second order systems (such as (9)) is achieved by introducing a further variable, u:

$$u = \frac{dy}{dt} = \dot{y}$$

Then (9) becomes:

$$(\underline{\underline{M}}+\underline{\underline{A}}) \, \dot{\underline{u}} + \underline{\underline{B}} \, \underline{u} - \underline{g}(\underline{x},\underline{u}) - \underline{h}(\underline{x}) = \underline{f}(t)$$

$$\dot{\underline{u}} = (\underline{\underline{M}}+\underline{\underline{A}})^{-1} (\underline{f}(t) + \underline{g}(\underline{x},\underline{u}) + \underline{h}(x) - \underline{\underline{B}} \, \underline{u}) \qquad (16)$$

$$\dot{\underline{x}} = \underline{u} \qquad (17)$$

Equation (9) is thus equivalent to a set of twelve simultaneous first-order differential equations which can be solved by Hamming's method.

3 SUBHARMONIC MOTION OF A SHIP IN SWAY

Observation shows that moored ships can make substantial long period sway movements (Ref 10). These movements can be large enough to break the ropes restraining the vessel. It is of course possible for long period motions to be driven directly by long period waves. But there is also a mechanism by which short period waves can cause long period movement; this is subharmonic response.

For example, a regular ten second wave might excite a moored ship and make it sway subharmonically at a twenty, thirty or forty (or longer) second period. A subharmonic response is a type of periodic response to periodic forcing; but instead of the response period and the forcing period being the same, in subharmonic response the response is at a subharmonic of the forcing frequency, ie the response period is an integer multiple of the forcing period.

Subharmonic responses cannot occur in linear systems because linear systems responses are always at the forcing frequency. But many different types of non-linear systems can have subharmonic responses.

In the particular case of a moored ship, the necessary non-linearity of the mooring system arises from the disparity in stiffness between the fenders (which are relatively stiff) and the restraining ropes (which are relatively soft). Non-linear rope load-extension curves or fender load-compression curves are not a necessary pre-condition for subharmonic responses to occur. They can happen even with linear ropes and fenders.

Physically, the subharmonic motion of a moored ship is a series of bounces against fenders. Consider, for simplicity, the motion of a ship of no wave forces act on it (Fig 3c). After leaving the fender (A), it will move for half a swaying cycle as if it were restrained only by mooring ropes (Fig 3a). Then (B) it returns to the fender and moves for its next half cycle as if it were restrained only by fenders (to C). In reality, cycles would repeat until damping decays the motion away; the decay is not shown on this simplified diagram.

Note that the natural frequency of the motion is approximately twice that of the ship's motion on its ropes. This is a consequence of the fender's being much stiffer than the ropes so that the periods of fender contact are of relatively short duration (Fig 3c).

When an oscillatory force is applied (Fig 3d), the ship does not respond entirely at the forcing frequency as it would if the mooring forces were linear. Instead, the response at the forcing frequency is superimposed on a bouncing motion at or close to the mooring natural frequency. The result is a subharmonic motion. Fig 3d shows a schematically typical example; in this case, the forcing is at three times the natural mooring frequency and the result is a third mode subharmonic motion.

The mathematical theory of subharmonic motions is greatly simplified if we assume that both fenders and mooring lines have linear load characteristics. We also assume that the ship's roll and yaw movements are negligible so sway can be considered in isolation.

We take sway to be in the x direction. The ship will have an equilibrium position resting on the fenders; this can be arbitrarily assigned as x = 0. We assume constant (sway) added mass and damping coefficients, A and B; and a monochromatic beam sea drives the motion at a frequency ω. Ropes have a combined stiffness λ_1, and fenders λ_2. The sway motion equation is:

$$(M+A)\ddot{x} + B\dot{x} + \lambda_1 x = F \cos \omega t \qquad x > 0 \qquad (18a)$$

$$(M+A)\ddot{x} + B\dot{x} + \lambda_2 x = F \cos \omega t \qquad x < 0 \qquad (18b)$$

We define the ship's natural frequency moving on its mooring ropes, which we have already seen is approximately half its natural bouncing frequency of sway motion with fenders:

$$\omega_o = \sqrt{\frac{\lambda_1}{M+A}} \qquad (19)$$

Three dimensionless parameters fully describe the system:

$$\eta = \frac{\omega}{2\omega_o}, \quad \xi = \frac{B}{2(M+A)\omega_o}, \quad \Lambda = \frac{\lambda_1}{\lambda_2} \qquad (20)$$

We will refer to the first parameter as the frequency ratio in this report. Its physical significance is that it gives (approximately) the number of forcing cycles in each natural period between fender contacts. Subharmonic response, if it occurs, normally does so at the integer multiple of the forcing period which is closest to the natural period of the ship motion. The frequency ratio is therefore a good guide to which subharmonic mode is likely to be excited.

It is also an indicator of how close to resonance the swaying ship is. Other factors affect results, but in general terms the closer the frequency ratio is to being an integer the closer the ship is to a subharmonic resonance. Through its resonant effect, the frequency ratio is important for determining the amplitude of subharmonic responses.

The second parameter in (20) is a measure of damping in the system. It was confirmed during this investigation that subharmonic responses are sensitive to damping.

The third parameter in (20) is the ratio of total mooring line to fender stiffness. It is these stiffnesses being unequal which makes the system non-linear and hence makes subharmonic responses possible. With very stiff fenders ($\frac{\lambda_1}{\lambda_2} \ll 1$) the ship can be considered to bounce off the fenders instantaneously. This was a simplifying assumption used in Refs 10 and 11, and it will be used in section 3.1 of this report. In practical cases of ship movement, although fender stiffnesses are significantly greater than line stiffnesses, the difference may not be great enough for the simple bouncing model to apply and the ship might spend a significant proportion of its time in fender contact. The parameter therefore gives a measure:

(i) of the applicability of the bounding model
(ii) of the likely proportion of fender contact time
(iii) (in a sense) of the non-linearity of the mooring system.

We can use the three parameters (20) to re-write (18) in a dimensionless form.

Defining dimensionless time and space variables:

$$\tau = \omega t \qquad (21a)$$

$$X = \frac{(M+A)\,\omega^2}{F}\,x \qquad (21b)$$

(18) becomes:

$$\frac{d^2X}{d\tau^2} + \frac{\xi}{\eta}\frac{dX}{d\tau} + \frac{1}{4\eta^2}X = \cos\tau \qquad X > 0 \qquad (22a)$$

$$\frac{d^2X}{d\tau^2} + \frac{\xi}{\eta}\frac{dX}{d\tau} + \frac{1}{4\Lambda\eta^2}X = \cos\tau \qquad X < 0 \qquad (22b)$$

3.1 Impact oscillators

The limiting case of infinitely stiff fenders ($\Lambda = 0$) is in many respects easier to analyse than the general case discussed so far. And it approximates to the behaviour of ships on stiff fenders.

Infinitely stiff fenders cannot be compressed. Because of this, the part of equation (22) which deals with ship motion on the fender ($X < 0$) is irrelevant; the ship rebounds from the fenders immediately after contact is made. We assume that fenders are perfectly elastic as well as infinitely stiff so the ship rebound without loss of kinetic energy - it leaves the fenders at the same speed as it hit them. Fender contact instantly reverses the ship's sway velocity. When off the fenders, the ship's equation of motion remains the same:

$$\frac{d^2X}{d\tau^2} + \frac{\xi}{\eta}\frac{dX}{d\tau} + \frac{1}{4\eta^2} X = \cos\tau \qquad X > 0 \qquad (23)$$

Systems like this one with instantaneous reversals of motion on impact are sometimes called impact oscillators.

Impact oscillators arise in many contexts as an idealisation of a physical process. They often have subharmonic motions associated with them.

Thompson et al (Ref 11) did an extensive investigation of subharmonic motions of a single point mooring using an impact oscillator model. Another impact oscillator investigation was done earlier by Lean (Ref 10). The following subsections of this report describe an investigation done at HR. Thompson's findings were confirmed. And additional results were obtained, in particular we calculate responses to random wave forcing which, to our knowledge, is something no previous authors have done for impact oscillators.

3.1.1 Impact oscillator model

The investigation used a time-stepping numerical model of the impact oscillator to solve (23). Because of the impact oscillator's singular behaviour at fender impact, SHIPMOOR could not be used; a simpler model was used instead. It proceeded in a series of time steps (of length δ), integrating across each in turn by the Runge-Kutta-Merson method to give an approximate series solution $\{X_1, X_2, X_3, \ldots, X_i, \ldots\}$ at times $\{\delta, 2\delta, 3\delta, \ldots, i\delta, \ldots\}$. The sign of the solution X_i was checked at each step. Impact oscillator rules require X to be positive. If it had gone negative, a fender contact had occurred at some time in the previous time step. The exact time of contact and velocity at that time was found. Calculation of the solution then resumed from that contact time but with the velocity reversed (ie positive) so reproducing the impact oscillator.

This method differs fundamentally from the method used in Refs 10 and 11. Lean (Ref 10) used a technique based on harmonic analysis of the response. Thompson's method (Ref 11) relied on an extrapolation of the known analytic solution to (22) from one impact to the next. Both their methods are designed for use with regular sinusoidal forcing, neither will cope easily with a random forcing term. We wished to use random forces. Hence the use here of a time-stepping procedure which is versatile enough to include a randomised force.

Random force time series were created for use in the model by Fourier transforming a set of complex Fourier coefficients of random phase but conforming to known spectral amplitudes. Gaussian spectra were used:

$$S(f) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(f-f_o)^2}{2\sigma^2}\right) \tag{24}$$

Spectral peak frequency, f_o, was fixed at one cycle per sixteen time-steps. Spectral width, σ, was chosen to give a spread of frequencies similar to that found in natural sea waves:

$$\frac{\sigma}{f_o} = 0.26 \tag{25a}$$

$$\sigma = 0.01625 \text{ cycles/time step} \tag{25b}$$

Fourier transforming these derived coefficients gave a pseudo-random force time series which repeated itself every 4096 time steps. It could be used, with interpolation whenever necessary, for the forcing term on the right hand side of (23).

The original definition of the frequency ratio, η, (Eq 20) is strictly speaking only applicable to a regularly forced subharmonic motion because it includes the regular forcing frequency. It needs adaptation for use in random forcing. The simplest re-definition available is to define it by the mean frequency of the forcing spectrum, ie f_o in (24). Frequency ratio thereby becomes the ratio of f_o to the natural frequency of the mooring system in random seas.

$$\bar{\eta} = \frac{\pi f_o}{\omega_o}$$

3.1.2 Responses to regular forcing

Thompson's detailed investigation (Ref 11) produced many results using regular forcing of an impact oscillator. Some are reported here partly for their interest and partly because we reproduced them using our impact oscillator model as part of its proving process.

Thompson's investigation showed that regular subharmonic motions are at their largest at near integral values of η. This result might be expected; the peaks are presumably due to subharmonic resonance.

He then did further investigation to illuminate response behaviour at non-integer η values. Attention concentrated on frequency ratios in the range between four and five using a damping coefficient $\xi = 0.2$.

At a frequency ratio of four, the response was regular fourth mode; ie (after an initial settling period) the motion was a succession of identical bounces each with a period of four forcing cycles (Fig 4a). This type of response continued for all η values up to 4.260 (Fig 4b). Near this value, there was a bifurcation. At $\eta = 4.262$, a different response pattern had set in; the motion was still a series of bounces with periods of about four forcing cycles, but only alternative bounces were the same (Fig 4c). The repeating period of the response had become eight rather than four forcing cycles; the motion was therefore eighth rather than fourth mode. Another bifurcation to a sixteenth mode response (Fig 4d) occurred at $\eta \approx 4.296$; and another (thirty-second mode) at $\eta \approx 4.305$.

Coming down from $\eta = 5$, only one bifurcation was identified. At $\eta \approx 4.555$, the response changed from fifth to tenth mode.

Responses around $\eta = 4.5$ were chaotic. This term is used (Ref 11) when there is no periodic repetition of the response (Fig 4e) and different solutions starting from slightly different initial conditions would always diverge so that small initial differences would become larger after a time (Ref 11).

Thompson also noted that, even if a subharmonic response solution did exist, it would not necessarily be found by starting from any initial condition. Some initial conditions lead to other patterns of motion. These are likely to cause smaller movements than the subharmonic motion would. Thompson, therefore, expressed concern that experimental workers could miss potentially dangerous large resonant responses by not trying a comprehensive range of initial conditions. We will show this concern is unfounded when random wave forcing (the realistic case) is used.

3.1.3 Responses to random forcing

In the real world, swaying ships are driven by the forces of random not regular waves. Responses to regular forcing therefore, although they might be of theoretical interest, are not necessarily of practical importance. We wanted to investigate the practically more relevant responses to random forces.

Our procedure for generating random forces has been outlined in 3.1.1. Several force time series were generated, each one was different (in phasing of components) but all had the same spectrum (Eq (24) and (25)). Using these force time series, two sets of experimental runs were calculated; one at $\bar{\eta} = 4.0$, the other at $\bar{\eta} = 4.5$. Both these runs used the damping parameter of $\xi = 0.2$. A third set of runs was made with $\bar{\eta} = 4.5$ and double the damping $\xi = 0.4$.

Each forcing sequence was 4096 time steps long. We found that, after running through a forcing sequence a few times, the response sequence eventually settled into a limit cycle of length 4096 or 8192. Responses would thereafter repeat ad infinitum, ie for as long as the forcing sequence was repeated.

The same limit cycle was reached for a given forcing sequence regardless of initial conditions. Thus, the chaotic responses found in regular waves (Fig 4e) do not appear to disrupt repeating cycles of random wave forcing.

The pressure of the impact oscillator greatly magnified response amplitudes; responses to the unit rms forcing would have had rms values of about 6 or 7 units without it but actual rms responses averaged 113.6 for $\bar{\eta} = 4.0$ and 129.9 for $\bar{\eta} = 4.5$ (both with damping of $\xi = 0.2$).

Despite the response at $\bar{\eta} = 4.5$ being slightly larger, the responses at $\bar{\eta} = 4.0$ and $\bar{\eta} = 4.5$ appear qualitatively similar (Fig 5). There is certainly no clear-cut distinction like their difference under regular forcing where $\eta = 4.0$ gives a resonant, regular fourth mode response (Fig 4a) and $\eta = 4.5$ gives a chaotic one (Fig 4e). Under irregular forcing, both give responses which appear to be similarly random. Given that the force spectrum is broad enough that, for a nominal value of $\bar{\eta} = 4.0$, it will contain sizeable components at values out to $\eta = 3$ and $\eta = 5$ and beyond, this is perhaps not surprising. The influence of any one frequency component will be small in the overall effect. An $\eta = 4.5$ component will be present in the force spectrum in runs at a mean $\bar{\eta} = 4.0$ and vice-versa.

Physically, setting $\bar{\eta} = 4.5$ rather than $\bar{\eta} = 4.0$ for the same forcing spectrum corresponds to using less stiff ropes. The softer ropes will allow the ship to move further out for the same force and simultaneously lower the mooring's natural frequency. These effects are detectable in the impact oscillator results.

Larger values of η cause a lowering of the response frequency and they allow larger movements.

But it is the extremely large sway movements of a ship which endanger its moorings, not the averaged sized ones. We want to be able to predict how large a movement we are likely to get in a given span of time. For this, we have to derive the probability distribution of the ship's extremely large movements.

A possible distribution, often used to predict such things as maximum wave heights, is the Gumbel distribution:

$$P(h) = 1 - \exp\left(-n \exp\left(-\frac{h^2}{2\sigma^2}\right)\right) \tag{26}$$

The distribution can be interpreted in the context of our swaying ship as; if σ^2 is the ship's mean square distance from the fenders, then P(h) is the probability of sway ever exceeding a distance h from the fender in a time which includes n fender contacts.

The applicability of the theory was checked by making cumulative probability plots (Figs 6 and 7). Overall response maxima over 2048 successive time-steps are plotted against a logarithmic scale, $-\ln(-\ln(1-P))$. The plots should theoretically fall in a straight line if the Gumbel distribution applies. The theoretical line shown in each plot is that defined by equation (26) using the average values of σ^2 and n from each set of runs. As can be seen, a least squares fit curve through the data does indeed correspond closely to the theoretical line for both $\bar{\eta} = 4.0$ and $\bar{\eta} = 4.5$. One can therefore use the Gumbel distribution (26) to predict extreme sway motions in irregular seas once σ and n are defined.

This result is of considerable significance as it implies that the usual Gaussian statistics apply to the case of a ship swaying against fenders. This is in spite of the non-linearity displayed by the impact oscillator under regular wave forcing. In particular, the fear expressed by Thompson (Ref 11) that simulations (computer and tank tests) could miss subharmonic responses, if a comprehensive set of initial conditions is not considered, only applies if regular wave forcing is used. With random wave forcing we have shown that a meaningful value of the rms of the response is obtained in that this value together with the average number of fender impacts in a given time interval allows the probability distribution of extreme vessel movements off the fenders to be described.

The final point that can be made concerns the uncertainties that occur in the rms response. The average error (measured deviation from the mean rms obtained by averaging over the set of rms) in the rms values for the runs with $\bar{\eta} = 4$ and 4.5 and with a damping parameter $\xi = 0.2$ was 9% to 10%. It is emphasized that this is the average error in the rms value from a single run (the error in the average rms value calculated from the set of runs can be expected to be nearer 2 to 3%). The relatively high uncertainty in rms values from single runs reflects the relatively low damping parameter used in those cases. The effect of this is to give a relatively narrow response function which in turn means that long random sequences of waves are required for forcing if good coverage of the response function is to be obtained in frequency space which itself will yield a stable rms value. A theoretical estimate of the average error that can be expected in a single run is about 7% which is not too different from the observed error of 9 to 10%. To test this idea further a third set of runs was carried out with $\bar{\eta} = 4.5$ and with double the amount of damping, $\xi = 0.4$. This larger damping parameter is more representative of measured damping coefficients obtained from physical models of vessels, with small underkeel clearances, swaying against fenders. This set of runs produced on average rms value of 89.14 with an average error in the rms value from a single run of 5%. This lower error is in accordance with increased damping producing a broader response in frequency space which leads to a better estimate of the rms value from the same length of run. In this case the theoretical estimate of the average error expected in a single run is also 5%. These results are important in that the length of random wave simulation tests must be sufficient to obtain a good estimate of the rms response value if accurate predictions of maxima are to be made via equation (26).

3.2 Simulations using SHIPMOOR

The impact oscillator is an interesting model of a swaying moored ship which highlights fundamental features of subharmonic motions such as changes in the response mode as the frequency ratio changes. But it does not include all the features of real ship motion. In particular, it cannot show the effects of finite and varying fender stiffness. Nor does it include interactions between the ships roll and sway motions. To simulate the subharmonic sway motion of a ship which is also rolling against finite stiffness fenders, we used SHIPMOOR.

3.2.1 Experimental conditions

The frequency ratio, η, is defined in equation (20). It is the ratio of forcing frequency to a moored ship's natural frequency of sway movement. The impact oscillator shows how important the frequency ratio is in determining the type of subharmonic response which occurs. It remains important with finite stiffness fenders.

We did a series of tests to simulate frequency ratio effects on subharmonic moored ship motions. For a fixed forcing frequency, the frequency ratio can be changed by altering the natural frequency of the ship's mooring system. In our tests we used a constant frequency of 0.1Hz for the forcing wave. The natural frequency of sway was changed by taking different stiffnesses for the ship's mooring ropes.

The model ship we used represented a moored 145000 tonne displacement bulk carrier. It had a longitudinally symmetric hull, a flat bottom, pointed ends, a 250m length, a 40m beam and a 16m draught. Radii of gyration were 59m in pitch and yaw, and 14m in roll. It was restrained by a simple mooring system with four fenders and two lateral lines at the bow and stern.

A few tests were done using non-regular wave forcing (see 3.2.4). All the past were driven by regular sinusoidal waves at a period of ten seconds.

Waves acted beam onto the ship. Since it and its moorings were both longitudinally symmetric, the ship's pitch, yaw and surge motions were not excited; the ship moved in heave, roll and sway only.

The ship's hydrodynamic characteristics (added inertia, damping and wave force) are frequency dependent, so we had to choose values which were correct for appropriate frequencies. In most cases, the appropriate frequency was obviously the forcing frequency (0.1Hz). But there was one important exception to this general rule.

We found that the low frequency subharmonic sway motion was supercritically damped when we used the damping coefficient found at the (higher) forcing frequency. Subharmonic response was consequently very small or absent. This would have defeated the object of the study. A more appropriate damping coefficient is that occurring at the expected subharmonic response frequency. This gave much lighter damping and a realistic reproduction of the low frequency response.

We aimed to produce regular seventh mode subharmonic responses, ie responses at a seventy second period, in our experiments. This period was chosen as being fairly typical of the long period motions experienced with large ships.

But the subharmonic motion we calculated is correct only in the sense of its being correctly proportional to the ship's motion at forcing frequencies; it is not correctly scaled with the height of the waves driving the motion. This is because the subharmonic motion is driven by waves via the primary motion at forcing frequencies. And this primary motion is incorrectly calculated because we have assumed the damping coefficient for the (low) subharmonic response frequency rather than for the (higher) forcing frequency. For maximum possible generality, therefore, we calculate subharmonic response amplitudes relative to the response at the primary wave period.

$$\frac{y_{max}}{y_o} = \frac{\text{Observed subharmonic response amplitude}}{\text{Observed forcing frequency response amplitude}} \tag{27a}$$

The dimensionless response above (27) cannot be used with responses to irregular forcing because response amplitudes are then not constant. Instead, we used the ship's time averaged distance off its fenders as a measure of the subharmonic response, and its rms response at forced frequencies as a measure of primary response. Both quantities can be easily calculated from spectral analyses:

$$R = \frac{\pi}{2} \frac{s - \sqrt{2}\ \sigma}{\sqrt{2}\ \sigma} \tag{27b}$$

Where R = dimensionless response
s = mean distance off fender
σ = rms primary response.

In terms of the parameters defined in equation (20), damping parameters used with SHIPMOOR were within a few percent of $\xi = 0.095$ for most runs (but see 3.2.2) although they varied slightly with mooring line stiffness. Frequency ratios were in the range $6.1 < \eta < 7.0$. These gave seventh mode responses. Mooring line/fender stiffness ratios were typically about $\Lambda = 0.03$. But in the runs simulating the impact oscillator (3.2.2) we used extra hard fenders with ratios around $\Lambda = 0.003$.

3.2.2 Simulating the impact oscillator

Although work has been done and results published for subharmonic response of impact oscillators (Refs 10 and 11), there are few published results which relate to subharmonic response of ships against fenders with finite stiffness. SHIPMOOR on the other hand is designed for simulating realistic conditions and cannot cope with mathematical abstractions such as infinitely stiff fenders. We used extra hard fenders in an attempt to get as close as possible to impact oscillator conditions with our model. This gave us a set of results we could compare with published values. Hence our model could be verified.

The results we chose for comparison were those obtained by Lean (Ref 10). He had used a damping parameter of $\xi = 0.45$ in his study. This was based on the results of physical model experiments, but it is much larger than our computed value (of $\xi = 0.095$). The discrepancy shows the effect of underkeel clearance on hydrodynamic coefficients. Lean's ship had a 10% underkeel clearance compared with 40% for the case considered here.

Reduced underkeel clearance is known to cause increased damping (Ref 13). For compatability with Lean's results, we used his damping value in our simulation.

Under these conditions, our simulated ship rapidly settled into a regular seventh mode subharmonic response at frequency ratios in the range $6.83 < \eta < 6.46$. Response amplitudes are plotted in Fig 8 together with a trend curve taken from calculated results given in Ref 10. Bearing in mind the differences in assumed conditions and the uncertainties in reproducing Lean's curve, the agreement between the two sets of results is very good over this range of frequencies.

We also made calculations using SHIPMOOR at lower frequency ratios in the range $6.45 > \eta > 6.1$. Results are not plotted in Fig 8 because the calculations did not give regular seventh mode subharmonic responses like the others. At $\eta = 6.1$, the response was clearly sixth mode. Responses in the intermediate range were too irregular for a maximum sway value to be fixed.

The latter finding fits in with Thompson's impact oscillator result (Ref 11, see also 3.1.2 and Fig 5 of this report) that fourth and fifth mode responses become chaotic at about $\eta = 4.5$. Judging from that, a chaotic response to the impact oscillator would be expected near $\eta = 6.5$, which is approximately where our regular response broke down.

But it is just as possible that we failed to find a regular seventh mode solution which does exist in the range 6.45 > η > 6.1. Subharmonic responses can be extremely sensitive to initial conditions. If the wrong conditions are chosen, the desired response will not be there. The absence of a regular response may just reflect our not finding appropriate start-up positions and velocities for the ship. However, from the work already carried out with random wave forcing (see 3.1.3) we know such problems do not occur when random waves are used.

3.2.3 Finite stiffness fenders and regular forcing

The preceding section describes conditions which are unlikely even to occur with large modern ships. The fenders were much stiffer than normal. This section describes work done reproducing movement with more realistic fenders which at a total stiffness in sway of 40,000kN/m are only a tenth as stiff as the previous ones.

As before, we investigated the ship's seventh mode response to ten second period regular wave. The seventy second response period was chosen as being a typical natural sway period for a large modern ship.

Both Lean (Ref 10) and Thompson (Ref 11) mention the possibility of co-existing steady state solutions of systems like this one. Several different periodic solutions (with different periods) can exist for the same mooring line and fender stiffnesses. Which of these solutions the ship's motion will adopt depends on the ship's position and velocity at the start of calculation.

We found several solutions co-existing with the regular seventh mode motion in the course of this investigation. they did not cause such large movements as the seventh mode did, so they have not been recorded in the results table (Table 1) and relevant figures (Figs 9, 10). But they sometimes seemed to be more stable and easier to find! Great care was needed in specifying initial conditions to get the desired regular response.

Sometimes what appeared at first to be a large and regular seventh mode response decayed fairly rapidly away to something else (Fig 9d). Here we seem to have chosen marginally unstable initial conditions. We did eventually get a non-decaying response for similar fender and line stiffnesses by assuming a slightly larger sway displacement and velocity at the start. But we cannot entirely discount the possibility that some of the seventh mode responses we assumed to be stable might not also have decayed, like this particular example, if the runs we made had been continued for longer times. It is therefore possible that Fig 10 might include some false results in that not all the seventh mode responses found may be stable. On the other hand it is possible that the stable range of seventh mode responses extends beyond the frequency ratios plotted in Fig 10. We show the range of results tested here but no attempt was made to discover the whole range in view of the lack of realism of regular wave forcing.

3.2.4 Finite stiffness fenders and irregular forcing

All of the experiments using SHIPMOOR up to this point have used a regular ten second wave to force ship motion in roll and sway. But sea waves are generally thought to be random. The following section outlines some tests which we did using random or irregular wave forcing.

It has been mentioned in 3.2.1 that we are using constant hydrodynamic added inertia and damping coefficients in the simulation model whereas these coefficients really vary with frequency. Therefore, if the motion is to be modelled correctly, it will have to be predominantly at or close to the frequency for which the hydrodynamic coefficients are calculated - otherwise the damping and inertial forces on the ship will be wrong. This now requires that we use a forcing signal which has a narrow frequency band.

Narrow band signals typically look like modulated sine waves. They are 'groupy' in the sense that they consist of relatively long groups of larger waves interspersed with relatively calm sections. This can be seen by looking at the heave signal in Fig 11; heave follows forcing fairly closely.

This modulation was sufficient to disrupt and randomise the ship's sway response. No tests showed anything similar to the long period regular response (Fig 9) observed with periodic forcing.

Some long period sway response was always discernible in the ship's motion, but it varied considerably in magnitude. Most of the time the long period response appeared to correlate with the heights of the wave group passing the ship at the time; large waves directly producing large responses.

Only one run produced a response akin to regular subharmonic resonance. During the run with a spectral peak at 0.095Hz there was a period of some 200 seconds of fairly stable wave conditions (see Table 2). This was about three natural periods for the sway motion. In this time the response built-up to a larger than usual level which persisted for a time into a succeeding calm spell.

It seems, therefore, that large subharmonic responses can build-up in irregular seas if a period of relatively steady wave conditions occurs. The response is unlikely to persist for long, but there are likely to be large movements while it lasts which could cause mooring lines to part.

3.3 Conclusions on subharmonic motion

The tests using SHIPMOOR were of limited scope, intended more to demonstrate and verify the model than as an extensive investigation into subharmonic response. The tests with irregular forcing in particular were too few and short in duration to give anything more than anecedotal evidence, but comparison with Lean's impact oscillator results (Fig 8) shows that SHIPMOOR performed satisfactorily.

Our impact oscillator results with regular forcing verified results found by Thompson et al (Ref 11), namely, the period doubling bifurcations (Fig 4), chaotic response (Fig 4e) and the importance of initial conditions. Response was also shown to be sensitive to initial conditions for finite stiffness fenders with regular forcing (Fig 9).

With irregular forcing, however, the impact oscillator response was not sensitive to initial conditions. Extreme sway motions of the impact oscillator are well predicted by the Gumbel probability distribution, Equation (26).

Only a very limited investigation of responses to narrow band irregular forcing on finite stiffness fenders was undertaken, but there seems no reason not to expect the Gumbel distribution to predict extreme motions for this case as well as for the impact oscillator. Once SHIPMOOR has been adapted to cope with wide frequency band movements, i.e. frequency dependent hydrodynamic coefficients, we will be able to model subharmonic responses to irregular forcing including yaw as well as sway motions.

4 MOTION CAUSED BY A PASSING SHIP

Moored ships can be forced to move by the action of other ships passing by. This movement can be considerable if the ship's moorings are badly arranged or allowed to go slack or if the passing ship moves past excessively quickly or closely. Large movements of the moored vessel will cause large loads on its restraining ropes with a consequent risk of them breaking. In the case of loading or unloading oil tankers there is the additional risk of oil spillage. This necessitates a cut-off valve which stops the flow of oil whenever the tanker's movement exceeds some pre-determined threshold value. Interruptions caused this way are inconvenient, time-consuming and costs the operators money whenever they occur. Motions induced by passing ships are therefore of considerable practical importance.

Mathematical models can be used to optimise harbour designs and mooring arrangements by finding configurations which minimise the effects of passing ships. This section of the report describes the use of SHIPMOOR in simulating motion due to a passing ship.

4.1 Theory

The moored ship's motion is caused by the pressure field which the passing ship generates. It is important to realise that this pressure field is not associated with the familiar and visible bow (and stern) waves which fan out from a moving ship. Those waves have too short a wavelength to affect any but the smallest ships and speed limitations in coastal waters generally mean bow waves are small. Large ship's motions are caused by the long wavelength components of the pressure field. This consists in general terms of high pressure areas at the bow and stern and low pressure abeam of the ship (Fig 12). Pressure gradients decrease with increasing distance so that the effect of a passing ship decreases as the separation distance between it and the moored ship, increase.

The general features of this pressure distribution are similar to those which potential theory predicts for two-dimensional flow around a cylinder. This is not surprising since, if the passing ship's underkeel clearance is small, it will approximate to a thin cylinder extending vertically through the water.

We can use potential flow as a model to explain the pressure distribution about the vessel. If we consider flow relative to the moving ship, there must be stagnation points at the ship's bow and stern and accelerated flow close to the ship at midships. Bernoulli's equation (6) predicts high pressure where flow is slow, ie near the stagnation points, and low pressure where it is fast, ie midships.

The moving ship's pressure causes a characteristic series of movements in surge, sway and yaw by the moored ship. High pressure in front of the moving ship initially tends to push the ships apart. This usually causes a small, barely detectable surge of the moored ship in the moving ship's direction of travel. But after the bow of the passing ship draws alongside the moored ship, the moored ship feels the increasing influence of the midships low pressure, and the net force changes to one of attraction. The moored ship is sucked off its fenders and surges, sways and yaws towards the other; sway forcing reaching a maximum when the two ships are alongside. The nett attractive force causes the moored ship to try to follow the passing ship as it moves ahead. The largest movements of the moored ship and loads on its mooring lines often occur in this phase of the motion as the mooring line tensions restrain the moored ship not only against the passing ship forcing but also against the sway and yaw momentums accumulated in earlier stages. But, as the passing ship moves on farther, the moored ship comes more under the influence of the stern high pressure region. This pushes the ships apart again. In the final phase of the motion, the moored ship returns to something like its original position under the combined force of the departing moving ship and loads in its mooring lines.

Surging and swaying forces and yaw moments on the moored ship have been investigated by Remery (Ref 15). In a series of tests using models in a towing tank, he produced a series of graphs to show the forcing time histories for three different sized passing ships at different distances from an instrumented moored ship. Remery's ship was free to pitch, roll and heave, but it was stationary in surge, sway and yaw. Experimental conditions therefore did not duplicate reality. However, this should not affect the validity of his results. Hydrodynamic forces are interdependent with the pattern of water flow around the two ships. This depends in turn on the geometry of the ships' positioning. Normal moored ship movement will not affect the geometry significantly. Moored ship movement is therefore expected to affect forces only slightly.

We have used Remery's results as the basis for passing ship forcing in SHIPMOOR. They need to be scaled to take account of ship's dimensions, separation distance and passing ship speed before we can use them. Dimensional arguments and Bernoulli's equation suggest that force should be proportional to speed squared. Unfortunately, no similarly simple reasoning can be applied to the other scaling laws. Instead, we have attempted a necessarily subjective best fit to the available data. The following scaling laws resulted:

For passing speed, V:

$$F \propto V^2$$

For separation, s:

$$F \; \alpha \; s^{-0.7} \qquad s < 60m$$
$$F \; \alpha \; 0.62 \, s^{-1.12} \qquad 60m < s < 120m$$
$$F \; \alpha \; 0.28 \, s^{-1.38} \qquad s > 120m$$

For ship dimensions:

(surge) $F_1 \; \alpha \; B_o D_o B_p D_p$

(sway) $F_2 \; \alpha \; L_o D_o B_p D_p$

(yaw) $F_6 \; \alpha \; L^2_o D_o B_p D_p$

Where:

L_o = moored ship's waterline length
B_o = moored ship's beam
D_o = moored ship's draught
B_p = passing ship's beam
D_p = passing ship's draught

4.2 Passing ship test conditions

Various investigators have tried to reproduce passing ship induced movements in the past. Dand (Ref 16) used a mathematical model of a similar sort to SHIPMOOR to simulate both model tests (Ref 12) and some full-scale field measurements made with moored ships in narrow channels. Comparison with the model tests showed some very good agreement between measured and computed values. Unfortunately, operational difficulties in the field trials made it difficult to judge the validity of the field measurements. The results showed poor agreement between measured and computed mooring force time histories with the forces being greatly over-estimated by the mathematical model.

In 1971 HRS carried out a series of physical model trials to investigate the behaviour of a moored 250,000dwt tanker when passed by another vessel (Ref 12). There were many tests in the series. The effects of mooring line pre-tension, passing ship speed and separation between the ships were all measured for two alternative mooring arrangements and three different passing ships of which one was used in both loaded and ballasted states.

We have chosen to simulate five of the tests which were particularly well documented. Three of the same tests were also simulated by David (Ref 16).

The mooring arrangements for all these tests is shown in Fig 13. We took our rope load/extension curves from data for the model ropes published in Ref 12.

Dimensions of the ships involved in the five tests modelled for this report are given in Table 5. The 210,000dwt passing ship was tested in both loaded and ballasted conditions; dimensions are given for both.

Only one of our five tests involved the ballasted ship (although the full set of tests in Ref 12 includes many tests with it). This was test 32B. This test was unique among our five in that the passing ship was presumed to be heading down-river and it approached the moored ship from astern.

Table 4 gives further details of the five tests. Rope pre-tensions of 5 tonnes were applied equally per line. This makes the effective pretension 10 tonnes for a double line; all lines were twinned in the original tests.

4.3 Passing ship results

Our first test was done starting the moored ship from rest with the moving ship far away. Initial conditions were thus similar to those in the original physical model tests. Unfortunately, comparing maximum loads in mooring lines from these tests with those from the physical model (Table 5) one can see that in some cases agreement is poor. These tests also did not mimic the dynamic behaviour of the model moored ship well. Subjectively, the centre of the ship followed a figure of eight track, quite unlike the loop the model took (Fig 14). Although it does not show in the figure, yaw was greater in the mathematical model the ship started to move early, away from the passing ship in response to the initial high pressure bow wave, which the model ship was not recorded as doing in the experiments.

This early movement probably shows the effects of friction. Static friction against the fenders would tend to hold the model ship stationary. SHIPMOOR does model dynamic friction but not static friction; there is consequently not an effective mechanism for friction to hold a stationary ship in place. Without static friction our ship was able to move in response to the weak force of the bow pressure.

Published results from the original tests indicate that the moored ship started to move at about the time the passing ship's bow reached the moored ship's centre point. We accordingly decided to do a series of tests which started from this position. This circumvented the problem of the moored ship moving prematurely.

But it introduced another difficulty. We needed an initial velocity as well as the moored ship's position at the start of calculation. Reference 12 gives the ship's position and yaw orientation at our 'new' start time, but it does not tell us the initial velocity. Judging from the ship's position twenty seconds later (also published in Ref 12), it seems unlikely that the ship could have started from rest in every case. We were obliged to experiment with a range of feasible initial velocities for each run to find that which gave the best fit to the published data. Results presented here therefore represent our best approximations to the original results obtained by a process of trial and error.

We also tried to reduce the previously noted excess yaw motion by experimenting with reduced yaw forcing. This had relatively little effect; yaw was affected more by our choice of initial velocity.

In particular, we found that giving the ship an initial sway velocity away from the fenders inhibited yawing. Doing this made the stern mooring lines (ropes 1 and 2, Fig 13) tauter earlier in the motion than they would otherwise have been. They would then pull to straighten the ship up at a time when yaw forcing on the ship was pulling the stern outwards. Yaw rotational velocity was thereby reduced or even reversed early in the motion, and yawing was consequently reduced overall.

Giving the ship an initial sway velocity also helped to change the 'figure of eight' trajectories (Fig 14) into loops like those found in the original experiments (Fig 15). Again, the effect of mooring lines is important. The extra sway velocity early on causes all ropes to become tauter earlier. There will therefore be an increased tension in the ropes early in the motion pulling the ship back onto the fenders. This force causes the sway velocity to reverse early. Since our original figure of eight trajectories occur only if sway velocity reverses after a reversal in surge, earlier sway reversal can change the trajectory from a figure of eight to a simple loop.

And starting the motion from the time when the passing ship reached the moored ship's midships point also improved our simulation of mooring forces. Table 6 shows that most of the maximum mooring forces for individual ropes agree closely with the original data and few show large discrepancies.

4.4 Conclusions on passing ship tests

The results show that, given suitable initial conditions, SHIPMOOR can simulate well the motion of a moored ship excited by a passing ship.

In practice, however, the model will be required to act as a predictive tool. In this role, the only known initial conditions are likely to be with the moored ship at rest and the passing ship approaching from far away. These are the conditions of the first set of tests (Fig 14, Table 5) which gave only moderately good agreement between simulations and original experiments. SHIPMOOR cannot therefore be considered to be an absolutely reliable predictor of movement due to passing ships at this stage.

This unreliability appears to arise from an inability to model the ship motion in its very early stages, before the passing ship reaches midships on the moored ship. The reasons for this are uncertain.

The lack of any simulation of static friction has already been noted and is probably important but another possibility is that the forcing time history was wrong in some respects. Few passing ship forcing data have been published. We used data collected by Remery (Ref 15). But these data refer to different sized ships from those in our tests; we have to allow for the differences by scaling the forces. Our simplified scaling laws can only be approximately correct, and their validity is hard to check because of the shortage of data on passing ship forces. More work needs to be done before we can be sure that the passing ship forcing is right in SHIPMOOR.

5 CONCLUSIONS ON SHIPMOOR

We know that SHIPMOOR is not perfect. Nothing ever is. Mathematical models certainly never are. We continue to work to increase and improve its capabilities. Nevertheless, we believe that it is a useful and adaptable computational tool even at its present stage of development.

We were able to use it to reproduce subharmonic sway responses. Comparison with Lean's impact oscillator results (Fig 8) shows that the model is accurate. We would like to be able to model subharmonic responses to random as well as regular forcing. This was only possible for narrow band forcing for this report; improvements are in hand which will enable us to calculate general responses later.

Very good reproduction of motion due to a passing ship was possible if suitable initial conditions were chosen (Table 6). Reproduction of the motion starting from rest with the ship's far apart was less good. The latter is likely to be required starting condition in practical use of the model. But even in this less favourable case, SHIPMOOR is fairly accurate in predicting mooring forces (Table 5). Its accuracy is comparable to similar results presented in Ref 16. With further development it should become even better.

Work is in hand under the present research contract to extend SHIPMOOR to the case of linear random wave forcing. But further work, requiring an additional contract, will be needed to represent non-linear wave forces. These forces are known from random wave physical modelling of harbours to be of crucial importance in controlling berth tenability. Only when this is completed can SHIPMOOR be validated against the motions of actual moored ships since such motions will contain non-linear responses.

6 ACKNOWLEDGEMENTS

The authors are grateful to Mr G Gilbert and Mr G Lean for many helpful discussions.

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TABLES

Table 1: Responses to regular forcing - finite stiffness fenders

η	ξ	$\frac{y_{max}}{y_o}$
6.09	0.089	21.6
6.13	0.090	20.4
6.15	0.090	19.6
6.17	0.090	18.8
6.20	0.091	18.1
6.21	0.091	18.3
6.22	0.091	17.7
6.23	0.091	17.4
6.27	0.092	16.3
6.31	0.093	15.3
6.35	0.093	14.4
6.38	0.094	13.7
6.42	0.094	12.9
6.46	0.095	12.4
6.50	0.095	12.5
6.55	0.096	12.2
6.59	0.097	11.6
6.63	0.097	11.2
6.69	0.098	10.6
6.75	0.099	10.0
6.81	0.100	9.5
6.91	0.101	8.8
6.96	0.102	8.5
7.01	0.103	8.2

Table 2: Responses to irregular forcing - finite fender stiffness

Forcing spectrum was: $S(f) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(f-f_o)^2}{2\sigma^2}\right)$

f_o Hz	σ Band width Hz	η	Dimensionless response	Comments
0.095	0.01	6.11	7.25	Large subharmonic resonance observed.
0.100	0.01	6.43	4.00	
0.105	0.01	6.75	4.40 }	Same force spectrum
			}	Different force
0.105	0.01	6.75	3.17 }	time-series.
0.110	0.01	7.07	3.80	

Table 3: Ship's dimensions for passing ship tests

	Overall Length	Breadth	Draught	Displacement	
	m	m	m	m^3	
Moored Ship	384	51.8	19.8	269000	
Passing Ship 1	159.4	20.1	9.8	22270	(laden)
Passing Ship 2	244.4	30.5	14.0	80900	(laden)
Passing Ship 3	304.8	48.7	20.1	240000	(laden)
	"	"	10.7	120000	(in ballast)

Table 4: Details of passing ship tests

Test No	Passing Ship	Condition	Speed (knots)	Clearance Between Ships (m)	Mooring Line Pretension (tonnes f)
17	3	Laden	4.2	69	5
22B	"	"	"	46	"
32B	"	Ballasted	3.9	46	"
47B	2	Laden	3.6	46	5
50B	3	"	4.0	46	30 cm slack

Table 5: Passing ship mooring forces - started from rest

Maximum mooring line forces in tonnes force

Run No	17		22B		32B		47B		50B	
Mooring Line	Math Model	Expt	Math Model	Expt	Math Model	Expt	Math Model	Expt	Math Model	Expt
1	36	48	56	57	25	30	16	20	36	86
2	41	28	65	34	42	30	22	21	77	64
4	36	40	51	47	29	14	17	17	80	76
5	64	63	87	66	21	14	39	35	107	90+
7/8	63	60	94	72	30	22	38	18	114	86
9/10	43	30	41	38	23	23	15	25	27	45

Table 6: Passing ship mooring forces - started from centre point

Maximum mooring line forces in tonnes force

Run No	17		22B		32B		47B		50B	
Mooring Line	Math Model	Expt	Math Model	Expt	Math Model	Expt	Math Model	Expt	Math Model	Expt
1	45	48	46	57	17	30	19	20	86	86
2	29	28	42	34	31	30	18	21	72	64
4	31	40	48	47	25	14	18	17	66	76
5	66	63	86	66	22	14	39	35	106	90+
7/8	54	60	94	72	19	22	24	18	93	86
9/10	30	30	39	38	21	23	16	25	44	45

FIGURES

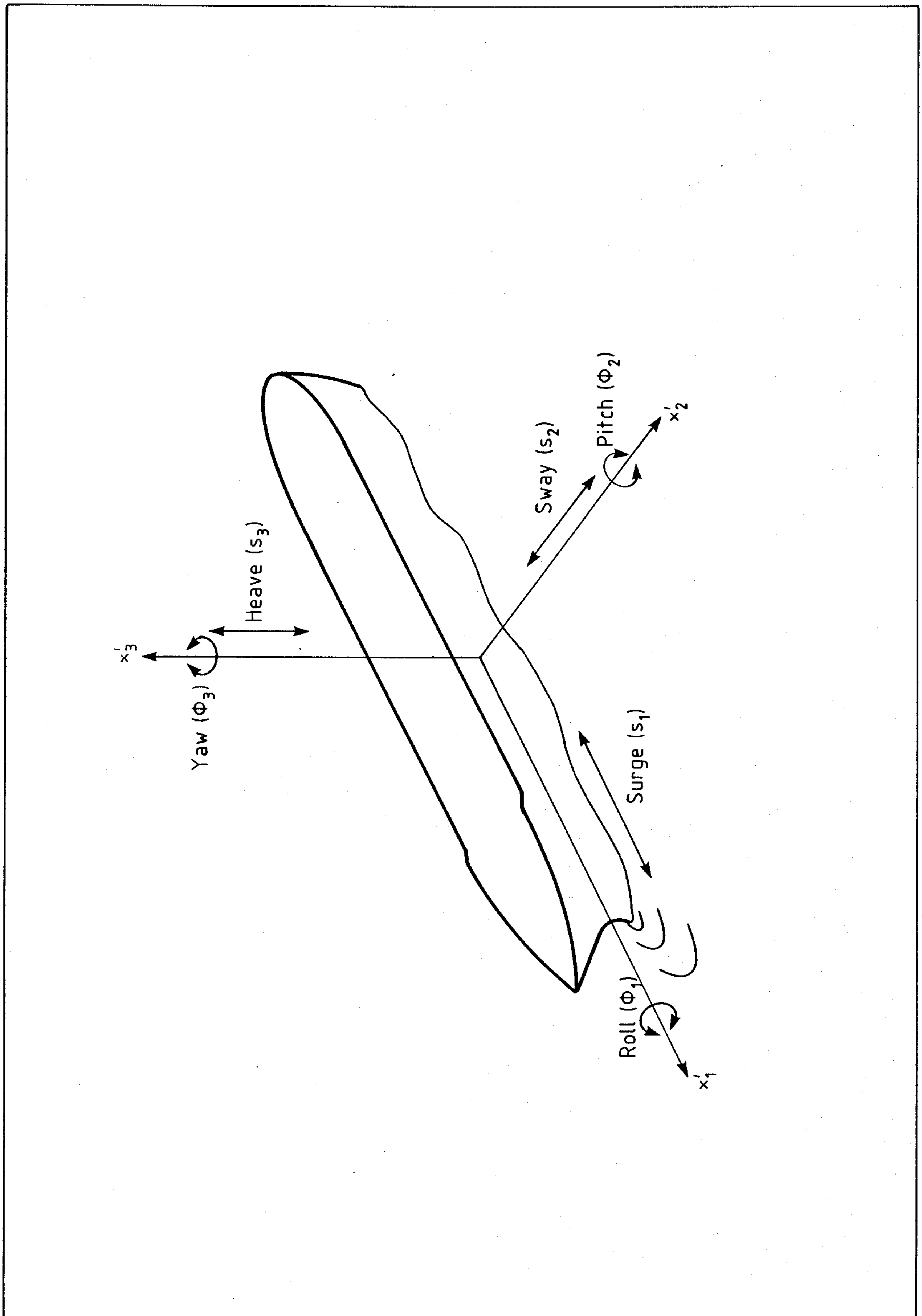

Fig 1 The six degrees of freedom of vessel movement

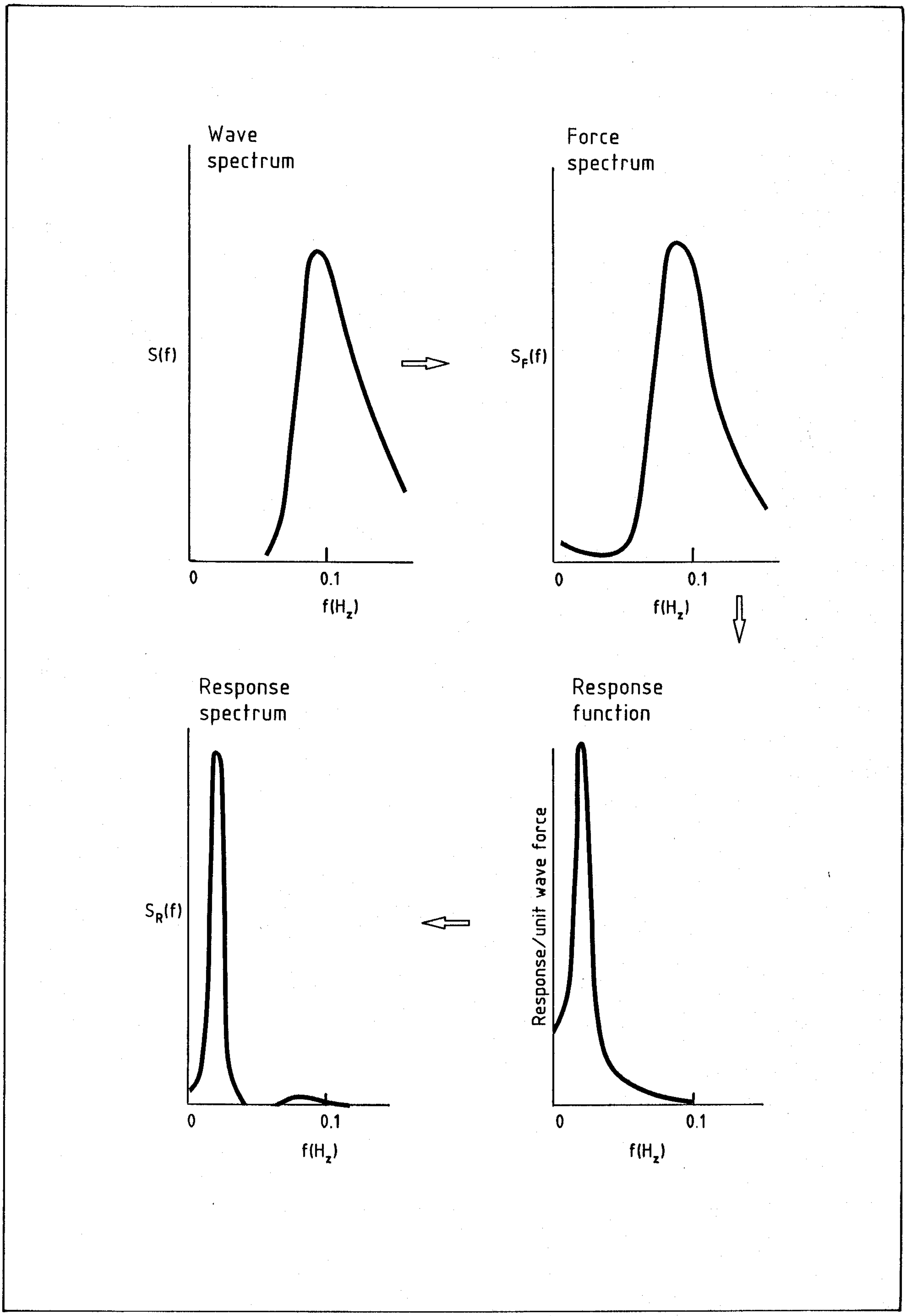

Fig 2 Schematic illustration of horizontal moored ship motion in waves

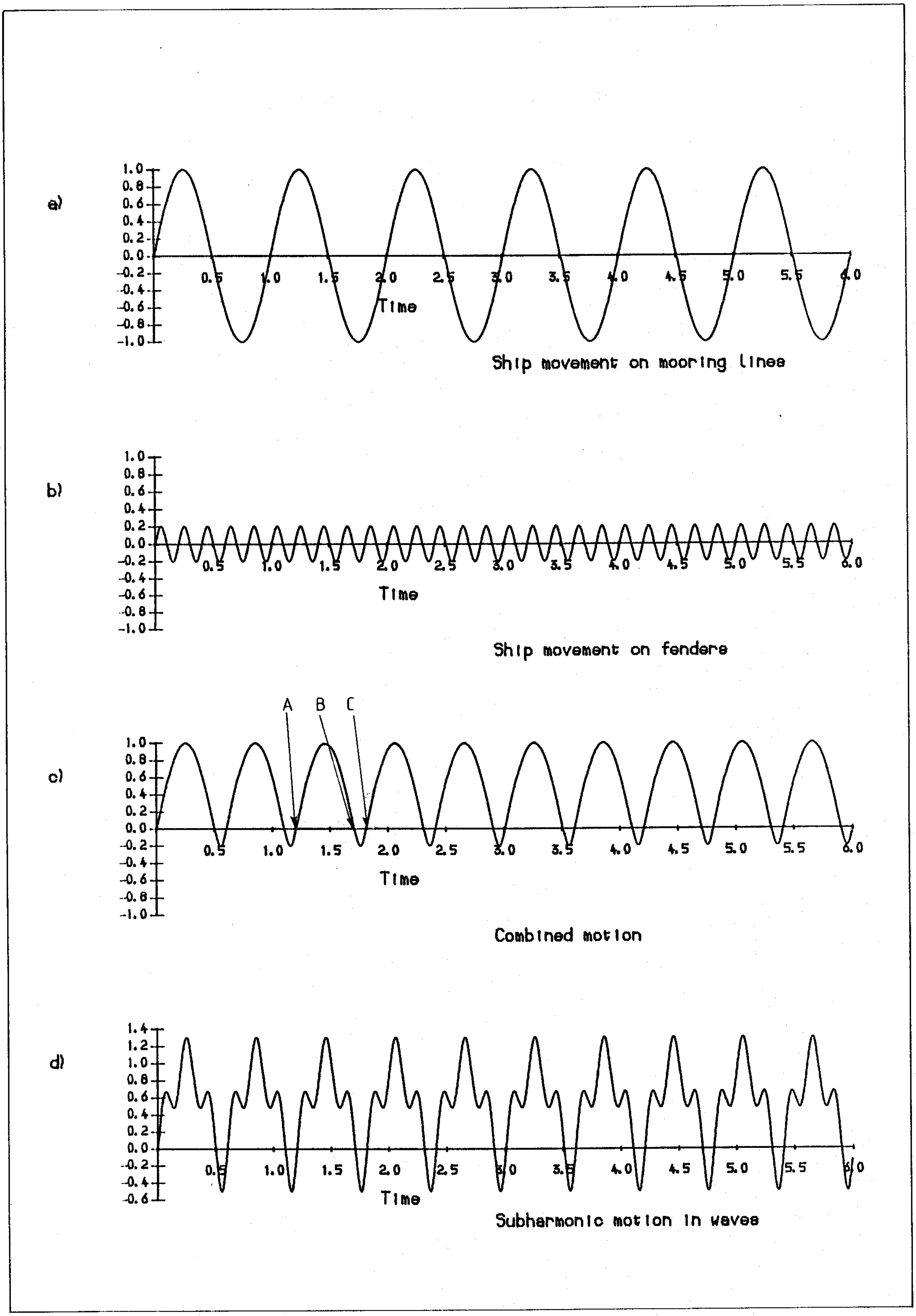

Fig 3 Schematic diagram of subharmonic motion

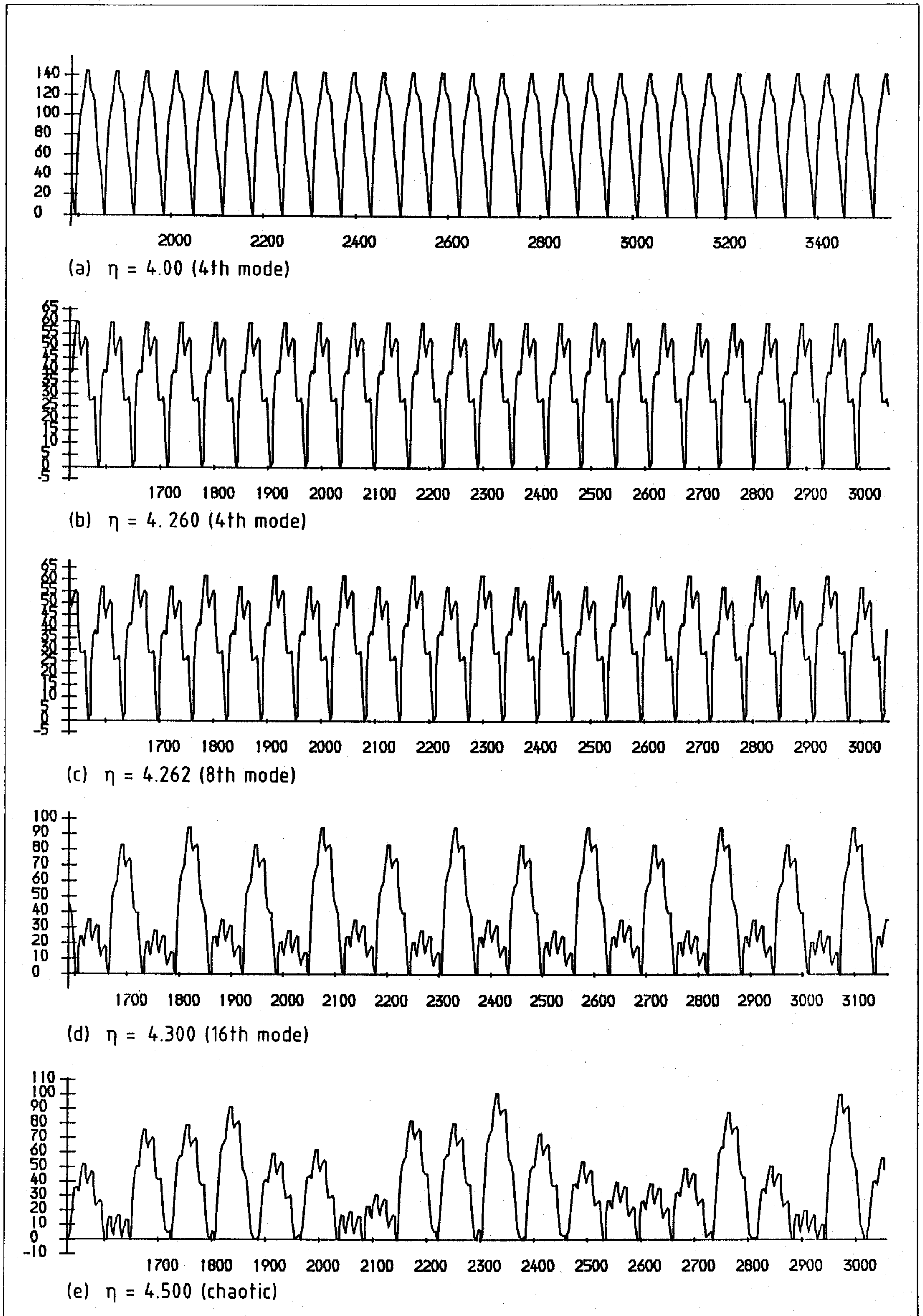

Fig 4 Examples of responses to regular forcing - Impact oscillator

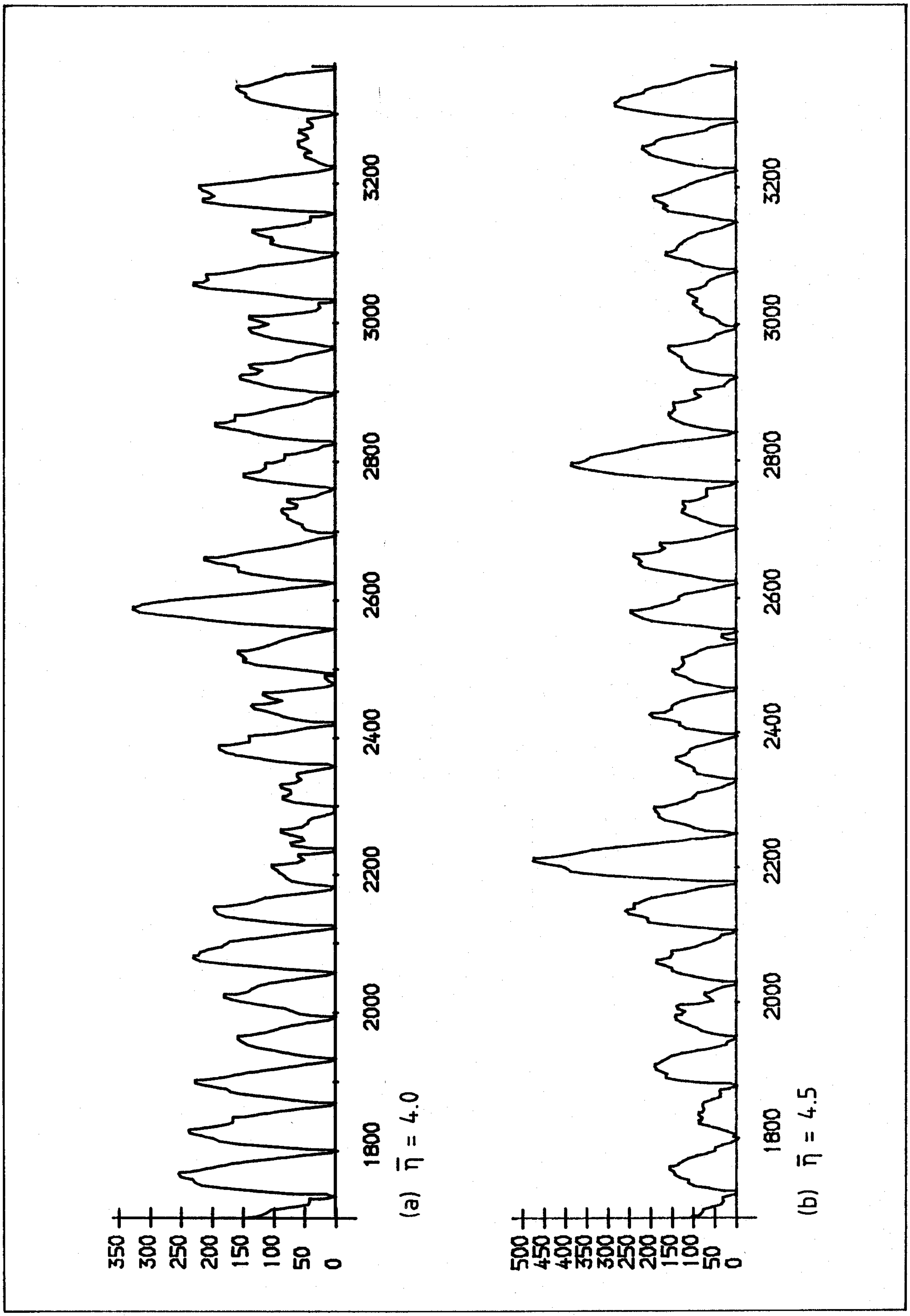

Fig 5 Examples of response to random forcing - Impact oscillator

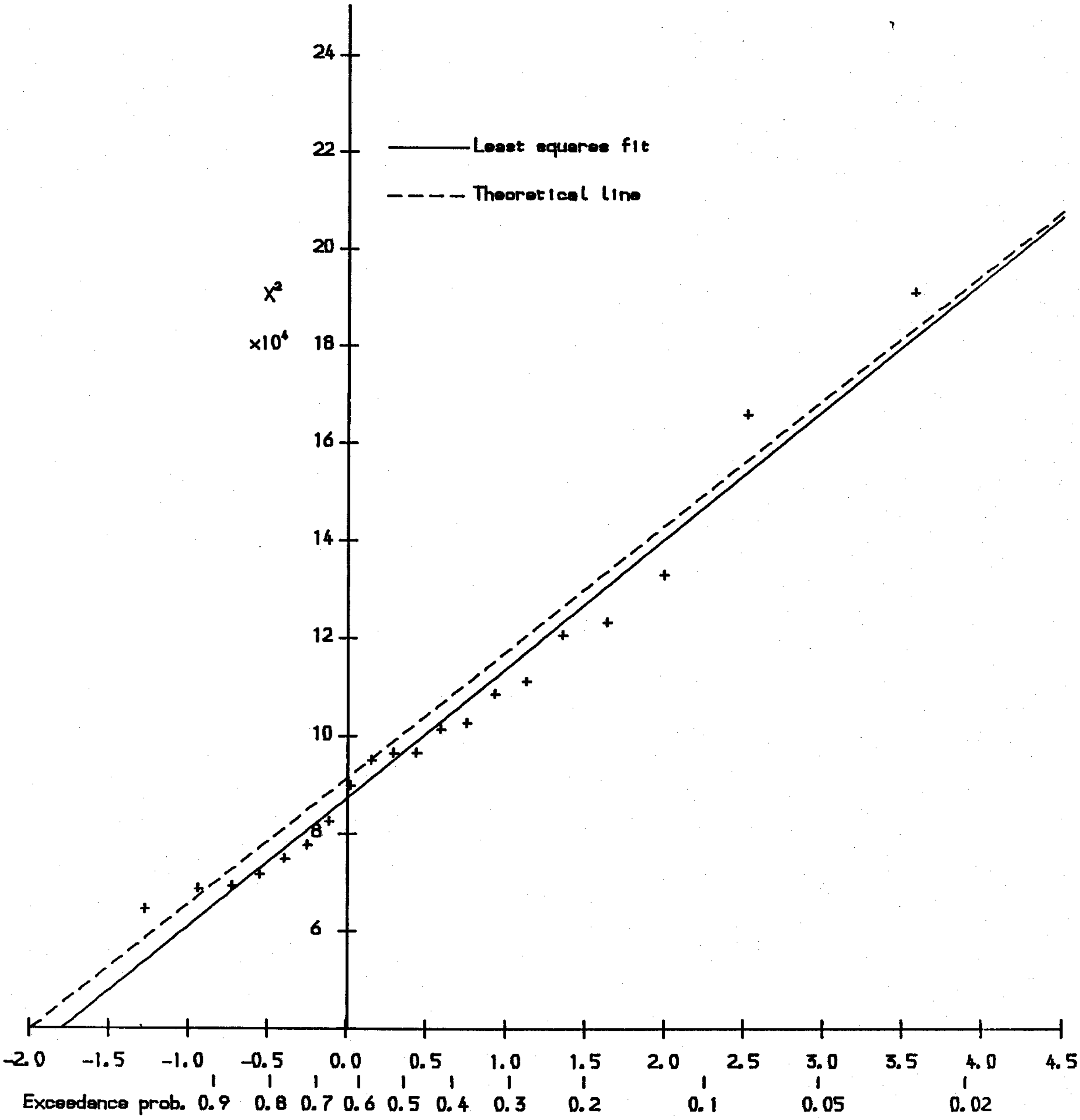

Fig 6 Cumulative probability - Random response to impact oscillator $\bar{\eta} = 4.0$

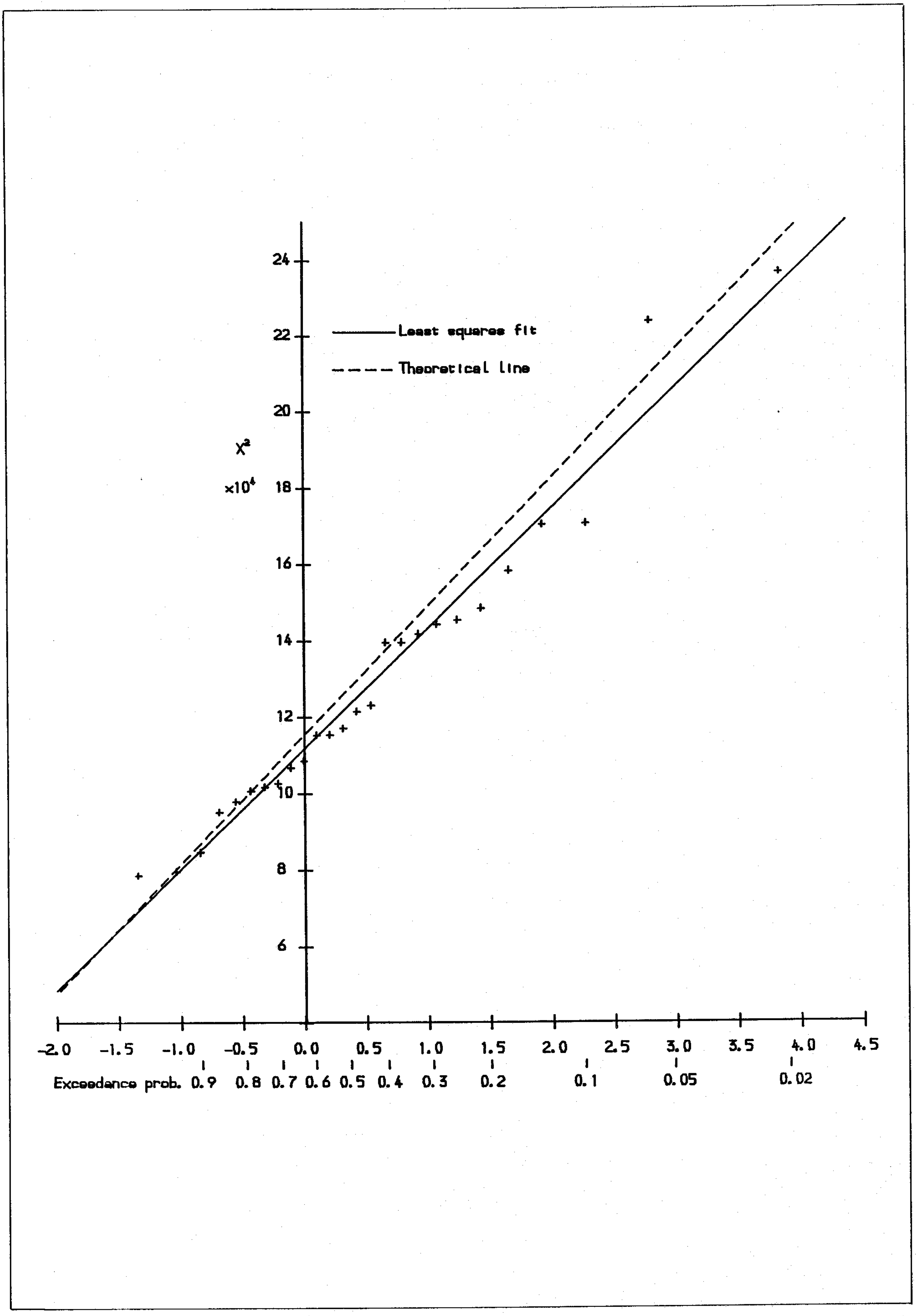

Fig 7 Cumulative probability - Random response to impact oscillator, $\overline{\eta} = 4.5$

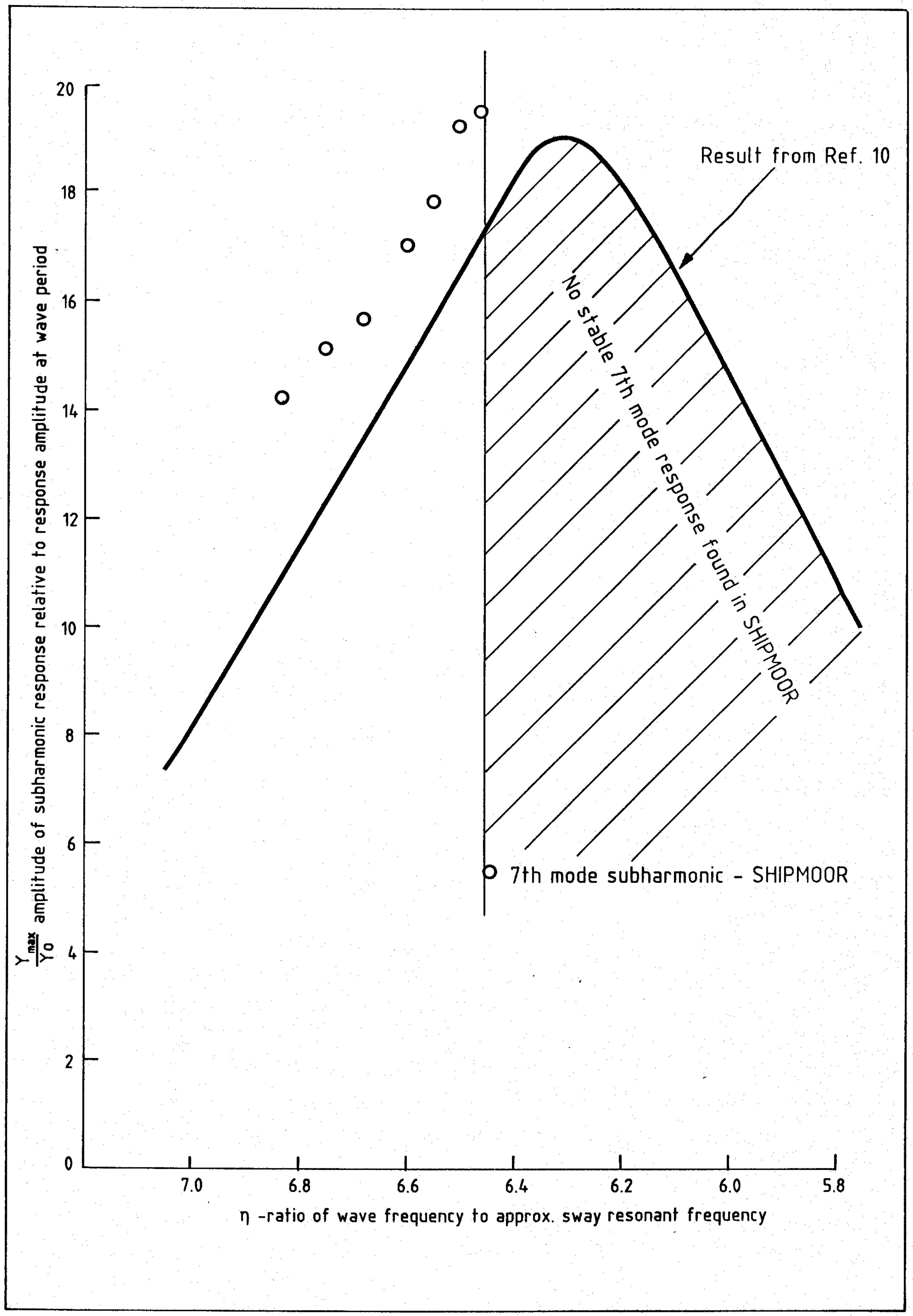

Fig 8 Response amplitudes - Simulated impact oscillator

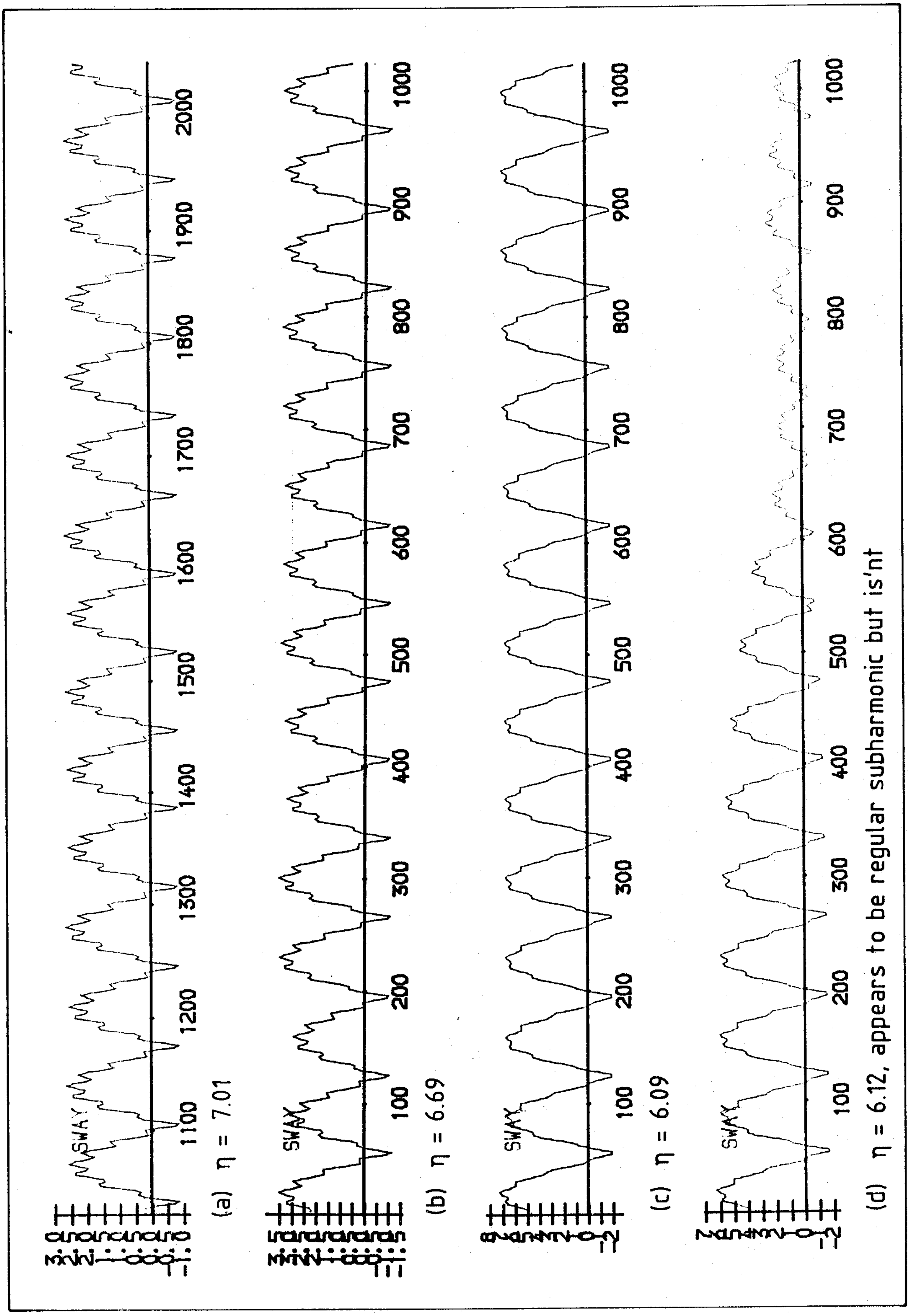

Fig 9 Examples of responses to regular forcing - Finite stiffness fenders

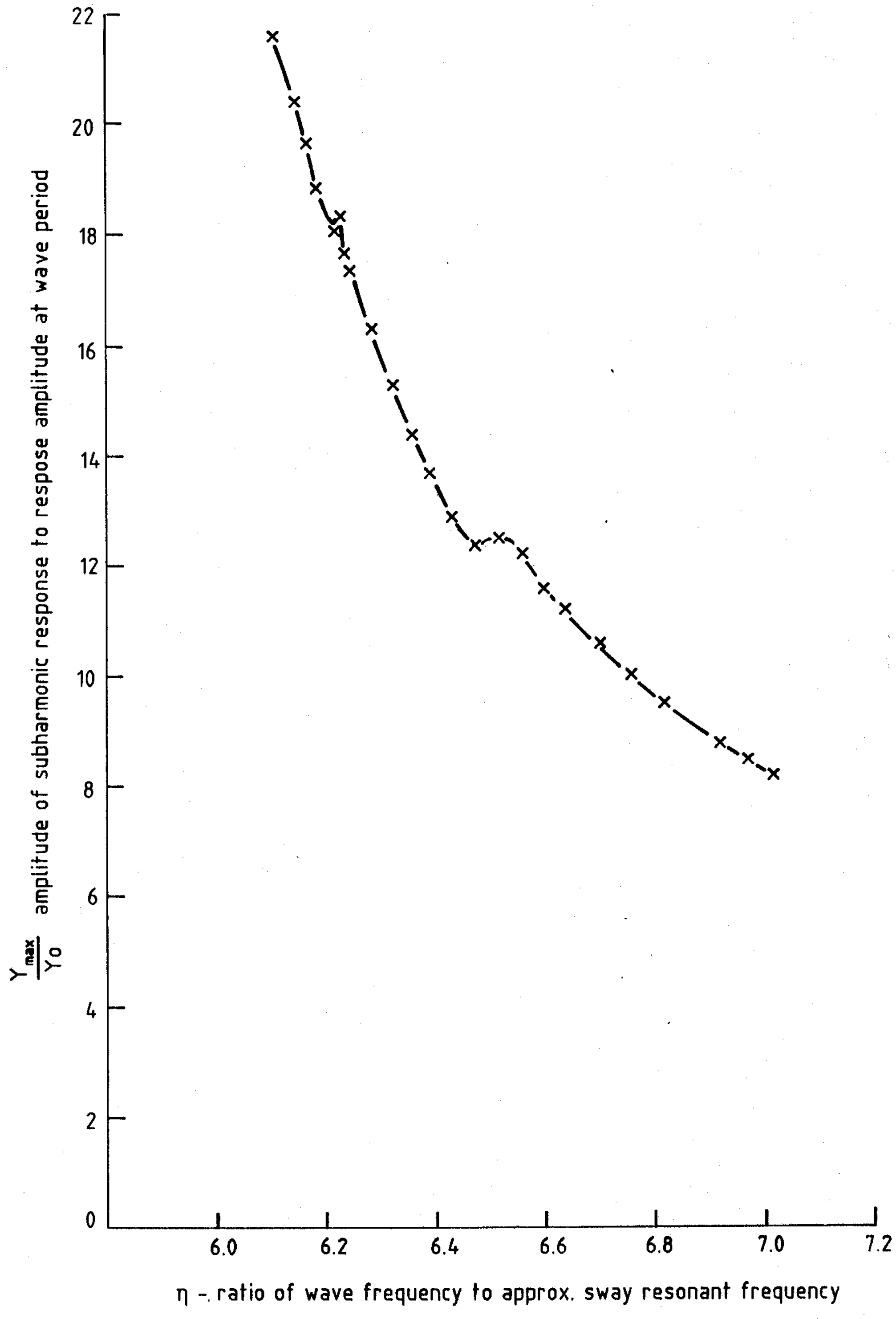

Fig 10 Regular 7th mode response amplitudes - Finite stiffness fenders

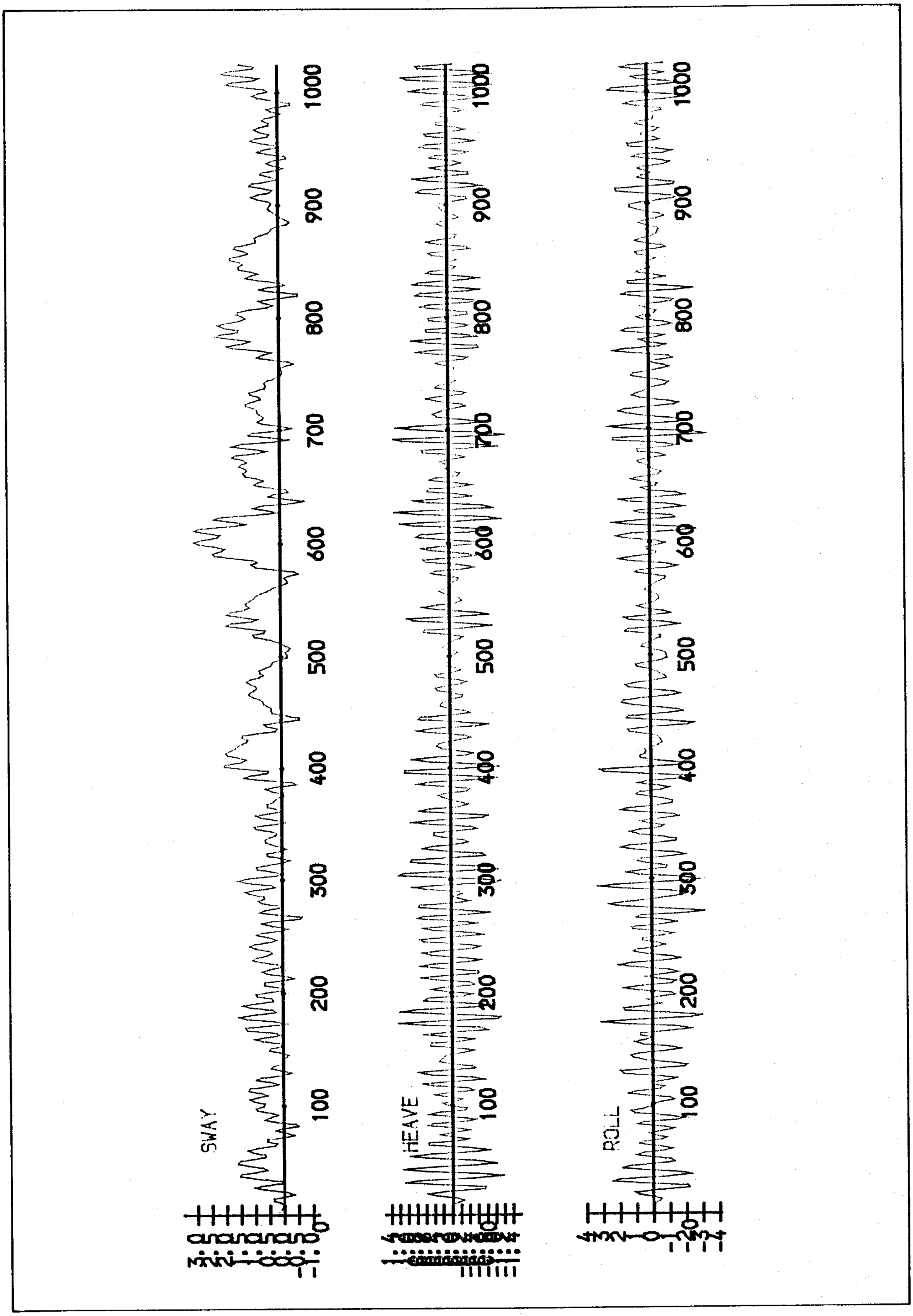

Fig 11   Example of response to irregular forcing - Finite stiffness fenders

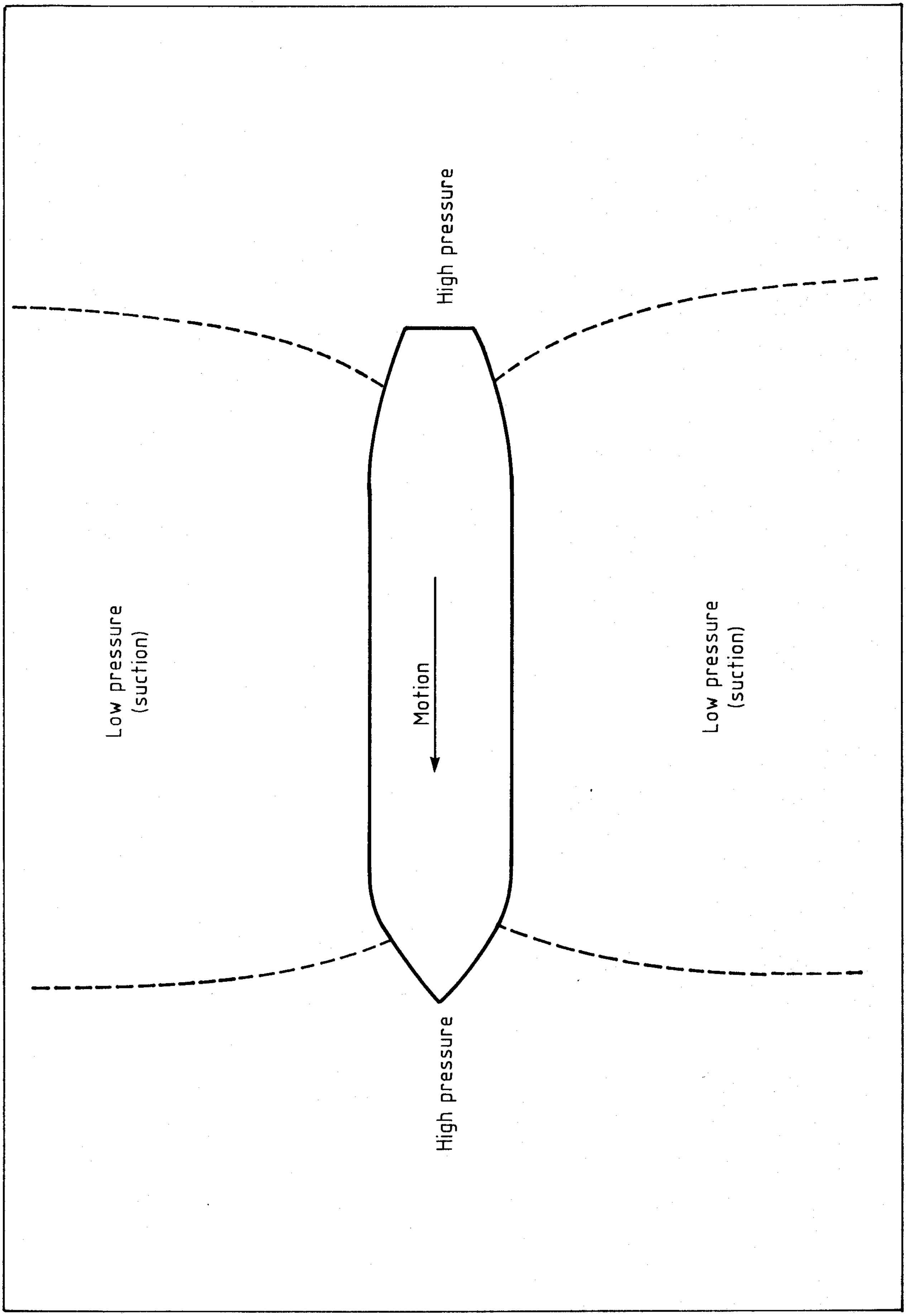

Fig 12 Schematic diagram of pressure around a moving ship

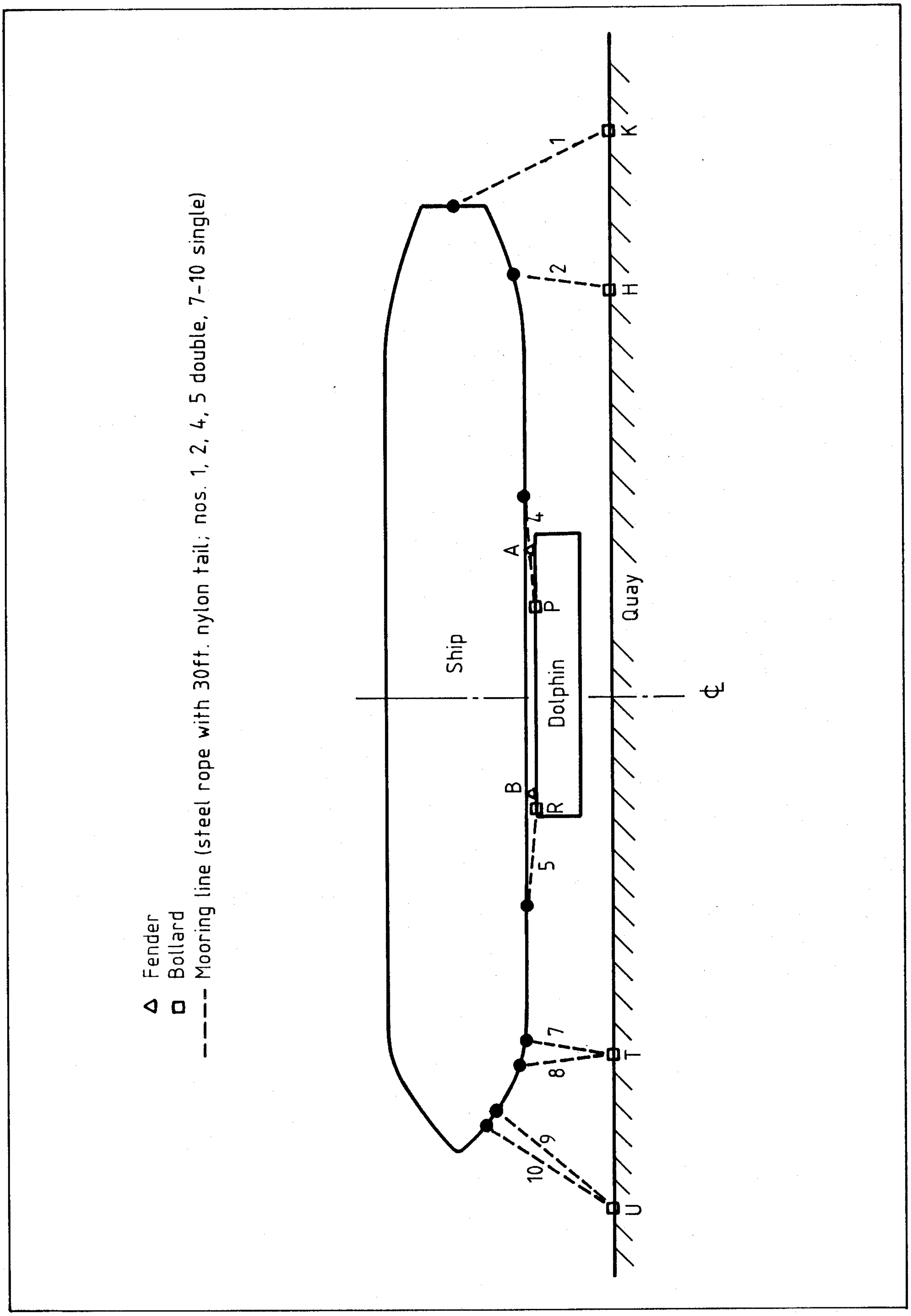

Fig 13 Mooring arrangement for passing ship tests

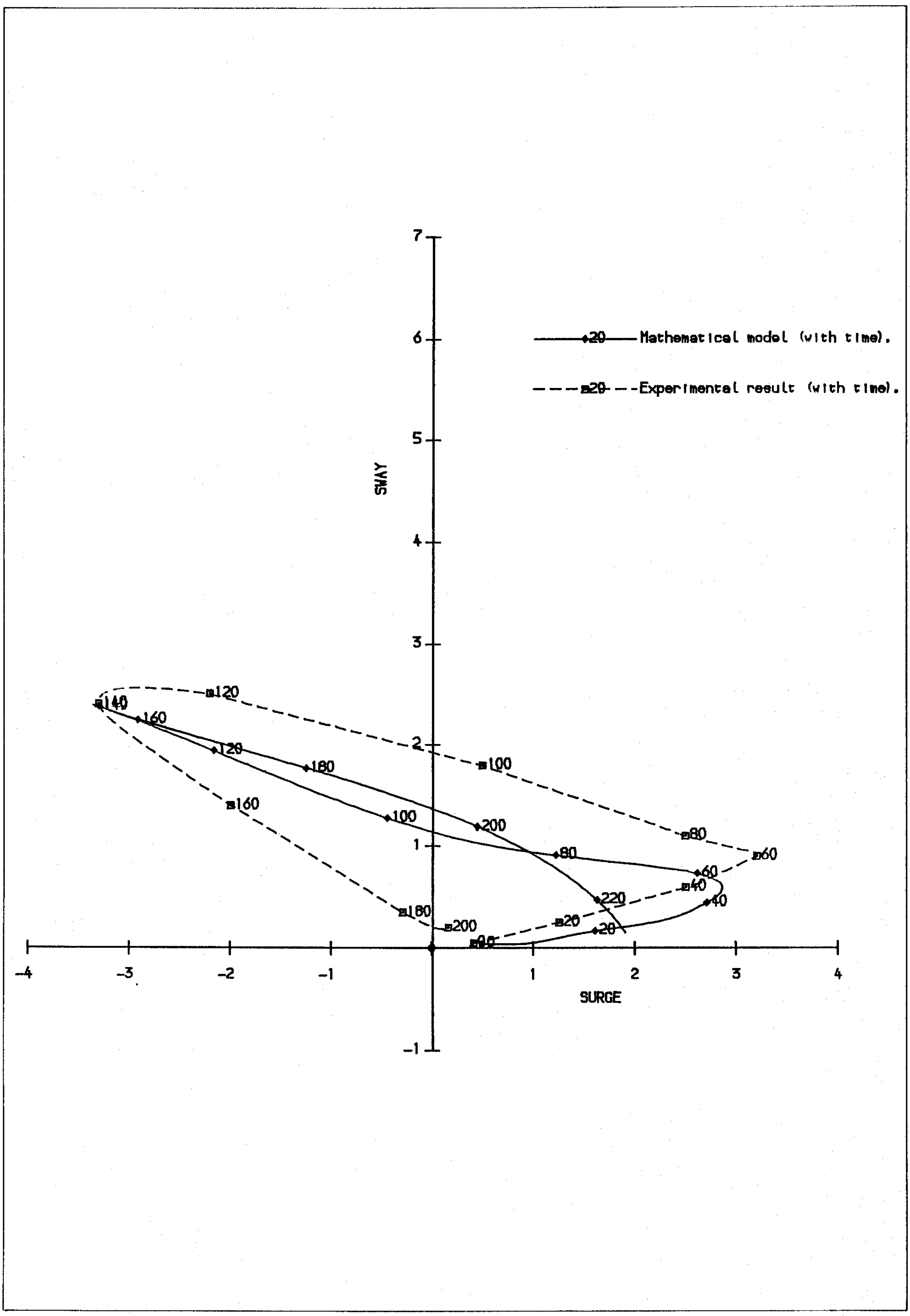

Fig 14 Passing ship motion started from rest (Test 50B)

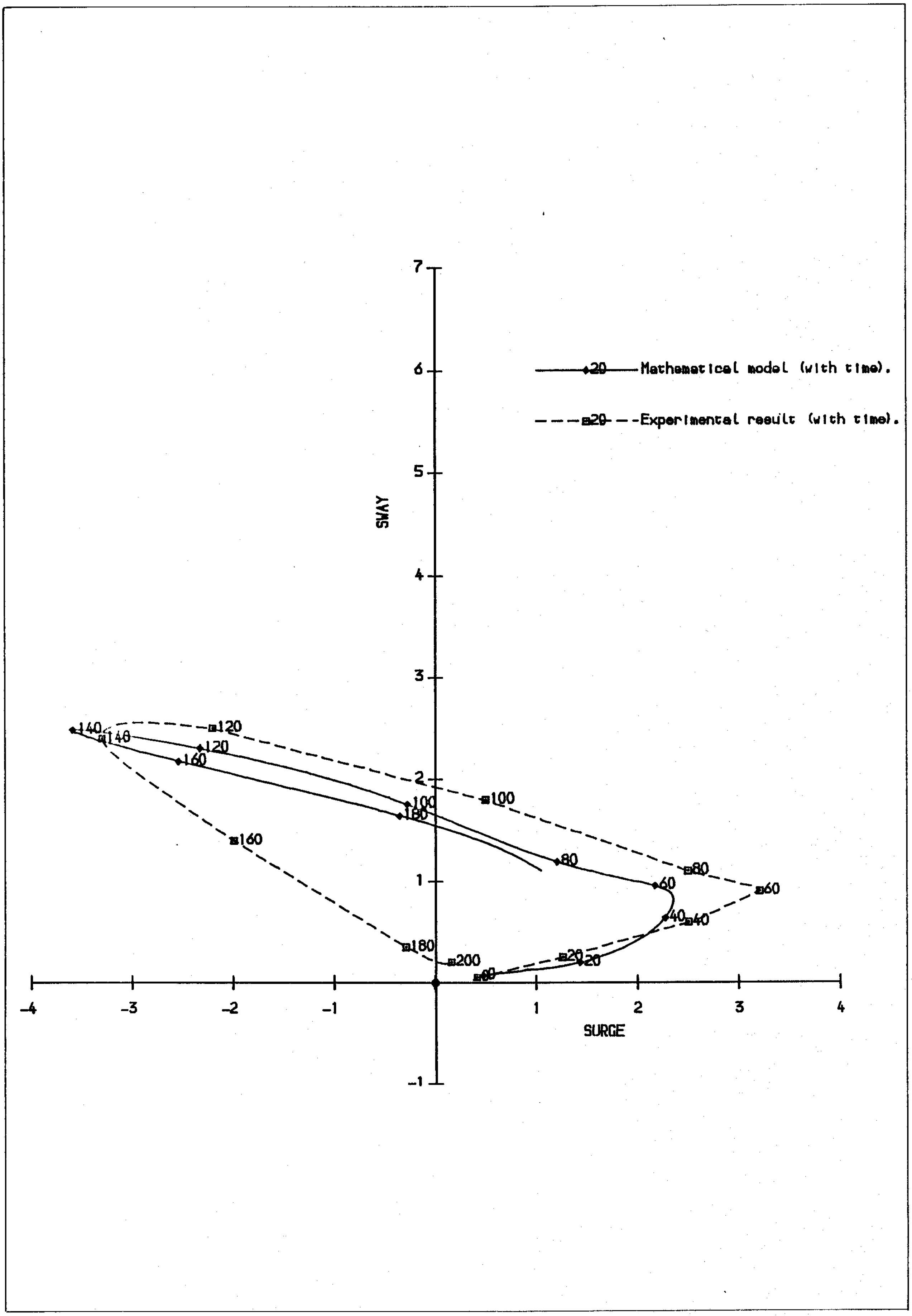

Fig 15 Passing ship motion started from centre-point (Test 50B)