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DISPERSAL OF DREDGED MATERIAL
Mathematical model of plume

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CONTRACT

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It is published on behalf of the Department of the Environment, but any opinions expressed in the report are not necessarily those of the Funding Department. The work was carried out by Ms M C Ockenden and Dr E A Delo in Mr T N Burt's section of the Tidal Engineering Department of Hydraulics Research, Wallingford under the management of M F C Thorn.

ABSTRACT

The fate of dredge material disposed of at open water sites by hopper dredgers has significant ecological and engineering importance. The suitability of a site for continued or proposed new disposal of dredged material can only be considered if the processes which occur in both the short-term and long-term with respect to the dispersal of the material are well understood.

Following release from the hopper, the dredged material descends through the water column as a well defined jet. During the descent large volumes of water are entrained in the jet and so the material becomes separated from the jet and remains in the upper portion of the water column. This material may be described as a near surface plume and is advected by the current from the disposal point. The descending jet collapses as a result of impact on the bed and the material which is not deposited on impact will move out radially under its own momentum. When sufficient energy has been dissipated material will begin to settle rapidly on the bed.

A computational model was developed to predict the movement of the plume of material put into suspension during the disposal process. The objective of the work was to create an economic means of predicting the dispersion of disposed dredged material in tidal waters.

The model was based on an analytical solution of a simplified differential equation which described the spread of material from a source. It was assumed that the current velocity, depth of flow and turbulent diffusion remain constant for the length of the plume, the flow was parallel to the x-direction and the material was fully mixed throughout the depth.

A technique of convoluting solutions was implemented to enable the model to represent changes in the current direction and water depth during a tide and the effect of concentration of suspended solids on the settling velocity. During a particular time step all the parameters were held constant but were changed from one time step to the next according to the tidal conditions. The results from the model were presented as contoured plots of suspended solids concentration and deposited mass.

The sensitivity of the model to the time step, diffusion coefficients and settling velocity of the material were investigated. The length of the time step did not affect the solution except for relatively short time steps in which the distance advected was of the same magnitude as the cell size. Longitudinal diffusion was seen to have significant effect on the spreading of the plume. The effect of the lateral diffusion coefficient was less marked. The results were shown to be quite sensitive to the settling velocity of the material.

A practical application of the model was made to demonstrate its function. The Tees Bay Inner Disposal Site was selected as this was the location of previous and current field studies of dispersion of dredged material. One run of the model was made and the pattern of deposited mass on the bed was presented.

CONTENTS

1 INTRODUCTION

Recent years have seen a substantial increase in the size and draught of vessels passing through ports. Many ports require a regular programme of dredging to maintain navigable depths in the docks and in the entrance channels. Such maintenance dredging is costly and methods are continually sought to reduce the input of effort into such maintenance.

Part of the solution is clearly to be found in the appropriate design of ports and their access channels. Their design should minimize the hydraulic conditions that favour settlement of suspended material. In existing ports, however, and those where physical factors are contrary, the maintenance of deep water may be an unavoidably heavy burden.

Historically, maintenance dredging has been carried out using local experience to determine when and where to dispose of dredged material. There is increasingly greater pressure to maximize dredging efficiency. In addition, and perhaps more importantly, dredging exercises in channels which pass through industrial areas involve moving sediments which may have significant accumulations of pollutants. These then either become concentrated in the disposal sites, or thrown into suspension in the mud clouds caused by dredging works. Recently, it has been suggested that disposed dredged materials represent a major input of trace metals to the marine environment. The mass loads could be substantially higher than the total input to the seas around England and Wales from the disposal of industrial wastes and sewerage sludges.

It is clearly of interest then, from an environmental point of view as well as from an engineering standpoint, to gain greater insight into the dispersal of material arising from dredging operations. Mathematical models are increasingly recognised as useful tools in many research programme, and in the context of dispersal of dredged spoil, polluted or otherwise, they attempt to answer the two questions regarding where the material goes to and what happens to it on the way. The answer to the first question is governed by the hydrodynamic processes taking place in the area of dispersal. The answer to the latter question is governed by the physical processes of sediments falling to the bed.

The disposal process may be divided into three distinct transport phases according to the physical forces or processes that dominate during each period. These stages have been described by a number of investigators Clark et al (1971), Koh and Chang (1973), Gordon (1974), Brandsma and Divoky (1976), Johnson and Holliday (1978) and Bokuniewicz et al (1978). The most common terminology for these stages is convective descent, dynamic collapse and pressure diffusion (WES, 1986). A diagramatic representation of the transport processes during open-water disposal is shown in Figure 1.

Following release from the hopper, the dredged material descends though the water column as a well defined jet. During the descent large volumes of water are entrained in the jet and so the material becomes separated from the jet and remains in the upper portion of the water column. This material may be described as a near surface plume and would be advected by the current from the disposal point. The descending jet collapses as a result of impact on the bed and the material which is not deposited on impact will move out radially under its own momentum. When sufficient energy has been dissipated material will begin to settle rapidly on the bed. Diffusive processes will then dominate and any remaining material will be mixed with the lower water column. The concentration of suspended solids will be lower and settling will take place but at a much slower rate.

It is the plume of material put into suspension during the disposal process that is the subject of this report.

The objective of the work was to create an economic means of predicting the dispersion of disposed dredged material in tidal waters.

2 THE MATHEMATICAL MODEL

2.1 General description of physical processes

The dispersion and deposition of suspended solids depends mainly on advection by the current velocity, the settling velocity of the sediment and the diffusion due to natural turbulence in the flow.

Advection

In a turbidity cloud generated from a surface source, the horizontal component of velocity of a suspended solid particle is determined by the current velocity of the water into which it falls. This is known as advection.

Settling velocity

The vertical component of velocity of a suspended solid particle depends both on the characteristics of the flow, such as turbulence, and those of the sediment, such as size, shape and density of the particles and the tendency of the sediment to flocculate. The settling velocity reflects these properties.

Diffusion

The spread of material away from a dense cloud is known as diffusion. Longitudinal diffusion is caused by the difference in velocities of the water at the surface and at the bed. Lateral diffusion determines the rate of spread of the cloud due to natural turbulence in the moving current. In an estuary, the scale of turbulent eddies may be laterally restricted, so the cloud may form a long thin ribbon which spreads sideways only slowly.

2.2 The differential equation

2.2.1 The basic equation

The basic differential equation is:

$$\frac{\partial}{\partial t}(dc) + \frac{\partial}{\partial x}(duc) + \frac{\partial}{\partial y}(dvc) - \frac{\partial}{\partial x}\left(dD_x \frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial y}\left(dD_y \frac{\partial c}{\partial y}\right) + W_s (c - c_e) = 0 \quad (1)$$

where:

c = depth averaged concentration (kg/m^3)

d = water depth (m)

x,y = co-ordinate directions parallel and normal to the flow (m)

u,v = flow velocity in the x and y directions respectively, (m/s)

D_x, D_y = Diffusion coefficients in the x and y directions respectively (m^2/s)

W_s = particle fall velocity (m/s)

c_e = depth averaged background concentration (kg/m^3) ($c > c_e$)

t = time(s)

2.2.2 Simplification of the equation

Equation (1) can be simplified if several assumptions are made. It is assumed that the velocity, depth and turbulent diffusion remain constant for the length of the plume. It is also assumed that the flow is uni-directional, parallel to the x-direction and that the material is fully mixed throughout the depth from the point of release. Concentration (c) is defined as the excess over the background. The basic equation then reduces to:

$$\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} - D_x \frac{\partial^2 c}{\partial x^2} - D_y \frac{\partial^2 c}{\partial y^2} + \frac{W_s}{d} (c - c_e) = 0 \quad (2)$$

This partial differential equation is the continuity equation for the spread of material from a source. The terms represent the rate of change of concentration with time, the rate of decrease of concentration per unit volume by advection, longitudinal diffusion, lateral diffusion, and loss of material from suspension due to settling, respectively.

2.3 Solution Technique

2.3.1 Basic solution for a point release

The methods of Carslaw and Jaeger (1959) can be modified to solve equation 2 for the instantaneous release of a slug of material into a body of water flowing at velocity u (with flow parallel to the x axis). The concentration at time t after release is:

$$c(x,y,t) = \frac{Q}{4\ \pi t d\ (D_x D_y)^{\frac{1}{2}}} \exp \left\{ - \frac{1}{4t} \left[\frac{(x-ut)^2}{D_x} + \frac{y^2}{D_y} \right] - \frac{W_s t}{d} \right\} \qquad (3)$$

where:

Q = mass of substance released (kg)

D_x, D_y = diffusion coefficients in the x and y directions respectively (m^2/s)

boundary conditions are:

$c_{(\infty, \infty, t)}$ = background for $t = 0$

$\frac{\partial c}{\partial x} \rightarrow 0$ as $t \rightarrow \infty$

Equation 3 gives a gaussian concentration profile with the centre moving downstream at velocity u (Fig 2), with a decay term to represent material falling out of suspension to the bed. By multiplying the right hand side of Equation 3 by the depth d, the equation can be expressed in terms of mass, where d multiplied by c(x,y,t) gives the mass of material per unit ara.

This gives a continuous solution for the concentration, but is limited by the constraints of constant depth and current velocity.

2.3.2 Combination of solutions

To develop a model which could predict the dispersion of dredged material in tidal waters the basic solution presented above was repeatedly applied over a number of relatively short time periods throughout the tidal cycle. The flow depth and current velocity were changed each time the solution was applied to represent the tidal conditions. The typical length of time for which the tidal conditions were assumed to be constant was 15 minutes. The dispersion of the plume of dredged material was thereby given by the convolution of discretised analytical solutions.

The area for which a solution is required is divided into a grid of cells each of dimensions XMESH by YMESH. The analytical solution (equation 3) is then solved in terms of mass at the centre of each cell. This mass, multiplied by the dimensions of the cell, is then used as the total mass in the cell. It is assumed that the mass is evenly distributed throughout the depth within the cell.

A combination solution is made up by treating each cell in the grid as a point source at the centre of the cell. Each "source" spreads out independently of all the other "sources" and therefore contributes mass to other cells according to the discretised form of the analytical solution. By adopting a relatively short time interval it is assumed that the current velocity and the depth in the spreading function (the dispersion equation - Equation 3) are constant during the time step, but may change from one time step to the next. The mass in each cell at the end of the time step is given by the sum of all contributions from other cells which could affect it during the time step. The mass at the end of the time step is then used as the magnitude of the point source in that cell for the next time step.

Mathematically, every point of the mass distribution function at the start of a time step is spread out according to the spreading function with parameters of velocity and depth particular to the time step. The convolution of these functions gives the new mass distribution which is used for the next time step. A mathematical justification for the combination of solutions by successive convolutions is given in Appendix A.

Due to discretising the continuous solution, small discrepancies arise in the sum of all masses in suspension after a time step. For conservation of mass, the total mass in suspension at the end of the time step must equal the total mass in suspension at the start of the time step minus the mass which has settled out during that time step. Therefore, during each time step a single multiplicative correction factor is applied to every cell so that the condition of conservation of mass is obeyed.

To choose representative values for the magnitude and direction of the current velocity and for the depth, which are to be held constant during a time step, the centroid of the plume is tracked. The values of velocity magnitude and direction and of depth at the centroid of the plume at the start of each time step are used as representative of the whole grid area during that time step.

2.3.3 Calculation of suspended solids concentration and mass on bed

At the end of each time step n, the concentration of suspended solids in each grid cell $c_e(i,j,n)$ is given by the mass in suspension in the cell at the end of the time step $m_e(i,j,n)$ divided by the depth of the cell at the end of the time step, $d_e(i,j,n)$.

The flux of material settling onto the bed in a cell is proportional to the suspended solids concentration in that cell at any time and the settling velocity of the material. The flux dm/dt may be expressed as

$$\frac{dm}{dt} = c\ W_s$$

The value of the suspended solids concentation in a cell changes from $c_o(i,j,n)$ at the start of a time step n to $c_e(i,j,n)$ at the end of a time step. The mass which is deposited in a cell is calculated based on the average concentration in the cell during a time step and is given by

$$m_d(i,j,n) = \tfrac{1}{2}\left(c_o(i,j,n) + c_e(i,j,n)\right) W_s\ t_n$$

where:

t_n is the length of time step n.

For conservation of mass, the sum over all grid cells of mass falling out during a time step must equal the true mass falling out as given by the decay term in Equation 3. That is:

$$\Sigma m_d(i,j,n) = \Sigma m(i,j,n)\ \left[1 - \exp\ (-W_s t_n/d)\right]$$

where:

d = representative depth of grid area.

2.4 Sensitivity Analysis

A number of model runs were made to test the sensitivity of the model to changes in the length of the time step t_n, the diffusion coefficients, D_x and D_y, and the settling velocity, W_s. Effects on both the suspended solids concentrations and the mass deposited are observed. The results are presented in the form of contour plots of suspended solids concentrations and mass on the bed.

2.4.1 Sensitivity to time step

A set of tests was run with all parameters constant except the length of a time step. Actual conditions were: grid cell size, XMESH = 100m, YMESH = 100m; depth 30m; velocity 0.1m/s on bearing 180°; diffusion coefficients DX = 5.0m ^{2}/s, DY = 0.5m ^{2}/s; settling velocity 0.001m/s.

Figures 3 to 6 show the contour plots of suspended solids concentrations after 2 hours, comparing the single step calculation with multi-step calculations. Figures 3 and 4 show that the length of the time step generally does not affect the final solution significantly, unless the time step is short enough that each cell "source" does not spread over enough other cells to give a gaussian profile. In this case a correction factor to conserve mass is needed which is no longer close to unity, and this has the effect of reducing the spread of the plume. This effect is shown in Figure 6 (8 steps of 15 minutes).

Figures 7 and 8 show the mass deposited on the bed after two hours. Due to the way the deposited mass is calculated (using concentrations from the start and end of a time step) if the centroid moves more than one grid cell during a time step, a peaked effect can appear in the mass on the bed. This is shown in Figure 7, where the centroid has moved 180m each time step with a grid size of 100m.

2.4.2 Sensitivity to diffusion coefficients

Test were run using identical conditions but varying either the longitudinal or lateral diffusion coefficient. Actual conditions were: grid cell size, XMESH=100m, YMESH=100m; depth varying as a plane between 30m and 40m; settling velocity 0.001m/s; time step 30 minutes (6 time steps); velocity varying each time step as follows:

0.05m/s at 190°
0.08m/s at 175°
0.10m/s at 160°
0.13m/s at 145°
0.16m/s at 130°
0.20m/s at 115°

(a) Longitudinal diffusion

Figures 9, 10 and 11 show the effect of varying the longitudinal diffusion coefficient, with D_x=10m^2/s, 5m^2/s, 1m^2/s respectively. The concentration distributions show that the rate of spreading of the plume is significantly modified by these changes with substantially greater longitudinal spreading for runs with higher values of D_x.

(b) Lateral diffusion

Figures 12, 13 and 14 show the effect of varying the lateral diffusion coefficient, with D_y=1.0m^2/s, 0.5m^2/s, 0.1m^2/s respectively. Again, the concentration distributions show that the rate of spreading of the plume is modified, though in this case, changing the diffusion coefficient by a factor of 10 does not have such a marked effect.

2.4.3 Sensitivity to settling velocity

Tests were run using identical conditions but with different settling velocities. Actual conditions were: grid cell size, XMESH = 100m, YMESH = 100m; depth varying as a plane between 30m and 40m; diffusion coefficients D_x=5.0m^2/s, D_y=0.5m^2/s; time step 30 minutes (6 time steps); current velocity varying each time step, as in Section 2.4.2.

In each run, the initial concentration in the starting cell was 1000ppm. Figures 15 to 18 show concentrations in suspension (in ppm) with varying settling velocities. In each case the plume has spread over approximately the same total area, but as the settling velocity increases the number of contour levels decreases and the spacing between them increases, showing that the concentration in the centre of the plume is lower as a result of more material having settled out of suspension. The effect is less marked as the settling velocity changes from 0.0001m/sec to 0.001m/s, but is very noticeable as it changes from 0.001m/s to 0.005m/s (Figures 17 and 18). With a settling velocity of 0.0001 m/s the concentration at the centroid is 30-35 ppm; with a settling velocity of 0.005 m/s the concentration at the centroid is only 5-10 ppm. Figure 19 shows the effect of altering the settling velocity at the start of each time step to reflect the concentration at the centroid of the plume. The actual settling velocity used for each time step was given by

$$W_s = \tfrac{1}{2} C_c .10^{-6}$$

where

W_s is settling velocity in m/s

C_c is concentration of suspended solids at centroid of plume in ppm.

In this case the concentration distribution is very similar to that given by a settling velocity of 0.0001 m/s (Fig 15), with a concentration at the centroid of 30-35 ppm.

Figures 20 to 22 show the distribution of mass settled on the bed after 180 minutes, with settling velocities 0.005m/s, 0.001m/s, 0.0005m/s respectively. The mass distributions each show a pear-shaped area marking the total spread of the plume; this is due to the velocity direction which changes in the six time steps from 190° to 115°. The increased settling velocity causes more mass to be deposited, particularly around the starting point of the plume as shown by the many tightly-packed contours around the starting point in Fig 20, where the settling velocity is 0.005 m/s. As the settling velocity decreases, there are fewer mass contours and the mass is less spread out (Fig 22, settling velocity 0.0005 m/s. Figure 23 shows the effect of making the settling velocity proportional to the concentration at the centroid (as in Fig 21). When Figure 23 is compared with Figure 22, which has the same contour interval, it can be seen that the decreasing settling velocities of Figure 23 cause little mass to be deposited.

3 APPLICATION OF THE MODEL

3.1 Introduction

The model described in the report was developed to predict the dispersion of dredged material which is disposed of at open water sites. Accordingly, as a means of demonstrating the practical application of the model a run was made based as closely as possible on the data obtained from the field work conducted on the disposal of dredged material in Tees Bay (Delo and Burt, 1987). The location of the Tees is shown in Figure 24.

The material dredged from the River Tees estuary as a result of maintenance dredging operations is disposed of at the Tees and Hartlepool Port Authority (THPA) disposal site in Tees Bay (Fig 25). This site, known as the Inner Disposal Site, is some 8km offshore in a water depth of 25-35m and has an area of approximately 4.6km^2. On average about 1.7 million cubic metres of dredged material per annum is disposed of at this site by THPA's three dredgers.

The area of the National Grid covered by the model is shown in Figure 26, with the relative position of the Inner Disposal Site and the area covered by the contour plots of results from the model.

3.2 Input Data

The bathymetry was idealized as a plane varying from 20m close to the SW boundary to 46m near the NE limit of the disposal site. The section of this bathymetry within the area covered by the contour plots of results is shown in Figure 27. Figure 28 shows the typical tidal variation of water level, relative to Lowest Astronomical Tide (LAT).

The magnitude of the velocity for a time step was given by the discharge divided by the depth at the centroid at the start of the time step. The discharge against time relative to high water is shown in Figure 29. The direction of the velocity against time is shown in Figure 30 and was derived from current meter data.

The model was run for an initial concentration of 200ppm starting at point 462100E, 532400N, near the centre of the Inner Disposal Area (see Fig 26). The model was started 2 hours after low water (i.e. 8 hours after high water). The longitudinal diffusion coefficient was taken to be 5.0m^2/s and the lateral diffusion coefficient was set to 0.5m^2/s. The settling velocity used was proportional to the concentration at the centroid of the plume at the beginning of a time step, given by

$$W_s = \frac{1}{2} C_c .10^{-6}$$

where

W_s is settling velocity in m/s
C_c is concentration in ppm

until it decreased to 0.03mm/s, after which the settling velocity was constant at 0.03mm/s.

The length of the time steps was not a constant but was derived in conjunction with the current velocity at the particular period during the tide to give an advection of the centroid of the plume of 200m.

3.3 Results

The model was run until the concentration at the centroid fell below 20ppm. Contour plots of the concentrations in suspension are shown for 33 minutes and 63 minutes after disposal in Figures 31 and 32 respectively and for the final distribution (83 minutes after disposal) when the concentration at the centroid had fallen below 20ppm (Fig 33).

The distribution of mass on the bed at the end of the run (when the centroid concentration was less than 20ppm) is shown in Figure 34. The masses are given as proportions of the original mass. The total mass on the bed is approximately 0.5% of the starting mass. Most of the mass had fallen out within 300m of the point of release.

4 CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

A computational model was developed to predict the dispersion of dredged material at open water sites. The model was based on an analytical solution of a simplified differential equation which described the spread of material from a source. It was assumed that the current velocity, depth of flow and turbulent diffusion remain constant for the length of the plume, the flow was parallel to the x-direction and the material was fully mixed throughout the depth.

The simplified differential equation had terms which represented the rate of change of suspended solids concentration with time, the rate of decrease of concentration per unit volume by advection, longitudinal diffusion, lateral diffusion and loss of material from suspension due to settling.

A technique of convoluting solutions was implemented to enable the model to represent changes in the current direction and water depth during a tide and the effect of concentration of suspended solids on the settling velocity. During a particular time step all the parameters were held constant but were changed from one time step to the next according to the tidal conditions. The results from the model were presented as contoured plots of suspended solids concentration and deposited mass.

The sensitivity of the model to the time step, diffusion coefficients and settling velocity of the material were investigated. The length of the time step did not affect the solution except for relatively short time steps in which the distance advected was of the same magnitude as the cell size. Longitudinal diffusion was seen to have significant effect on the spreading of the plume. The effect of the lateral diffusion coefficient was less marked. The results were shown to be quite sensitive to the settling velocity of the material.

A practical application of the model was made to demonstrate its function. The Tees Bay Inner Disposal Site was selected as this was the location of previous and current field studies of the dispersal of dredged material. One run of the model was made and the pattern of deposited mass on the bed was presented.

4.2 Recommendations

It is recommended that the model is tested against data obtained from different field situations with different hydraulic and sediment characteristics.

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FIGURES.

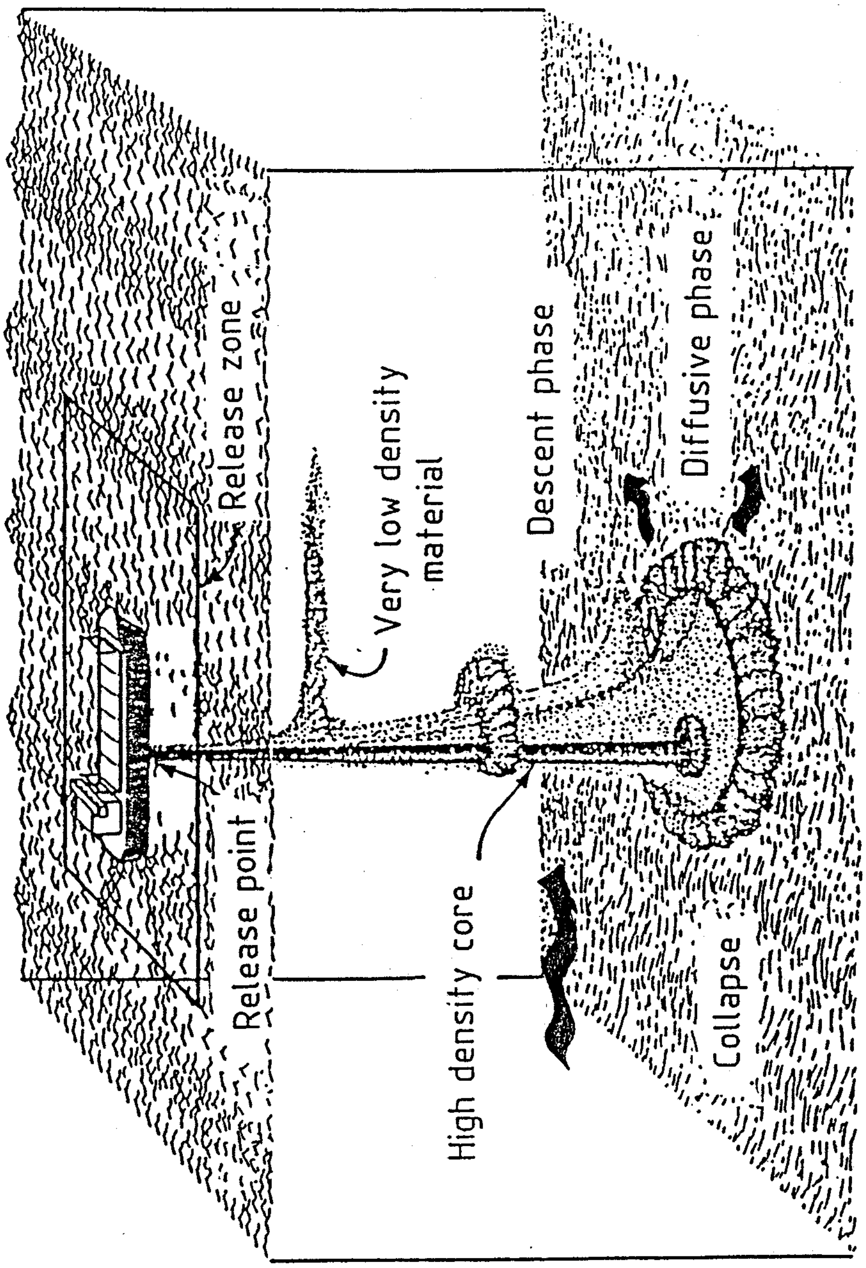


Fig 1 Transport processess during open-water disposal (after WES)

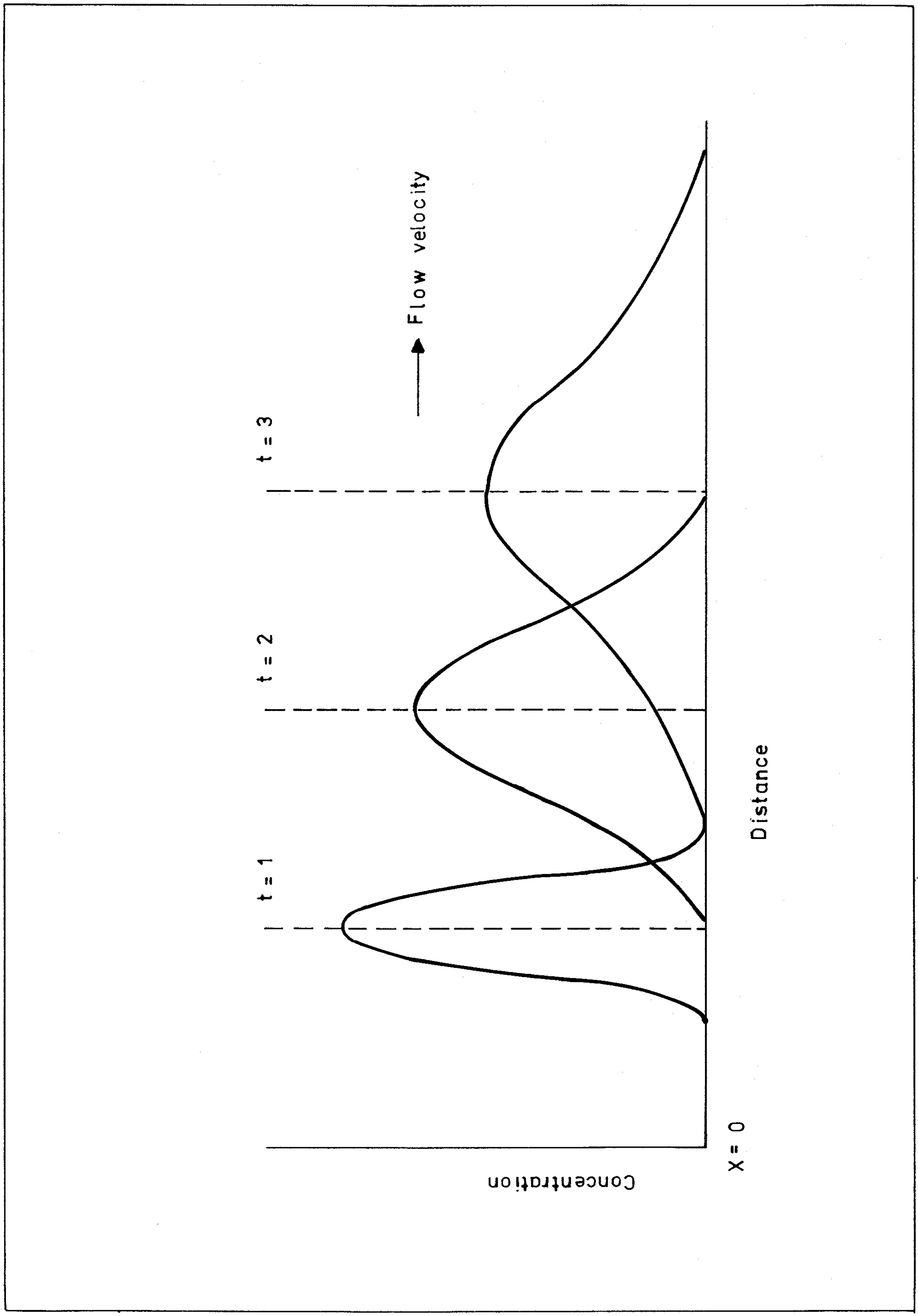


Fig 2 Gaussian concentration profile with centre moving downstream

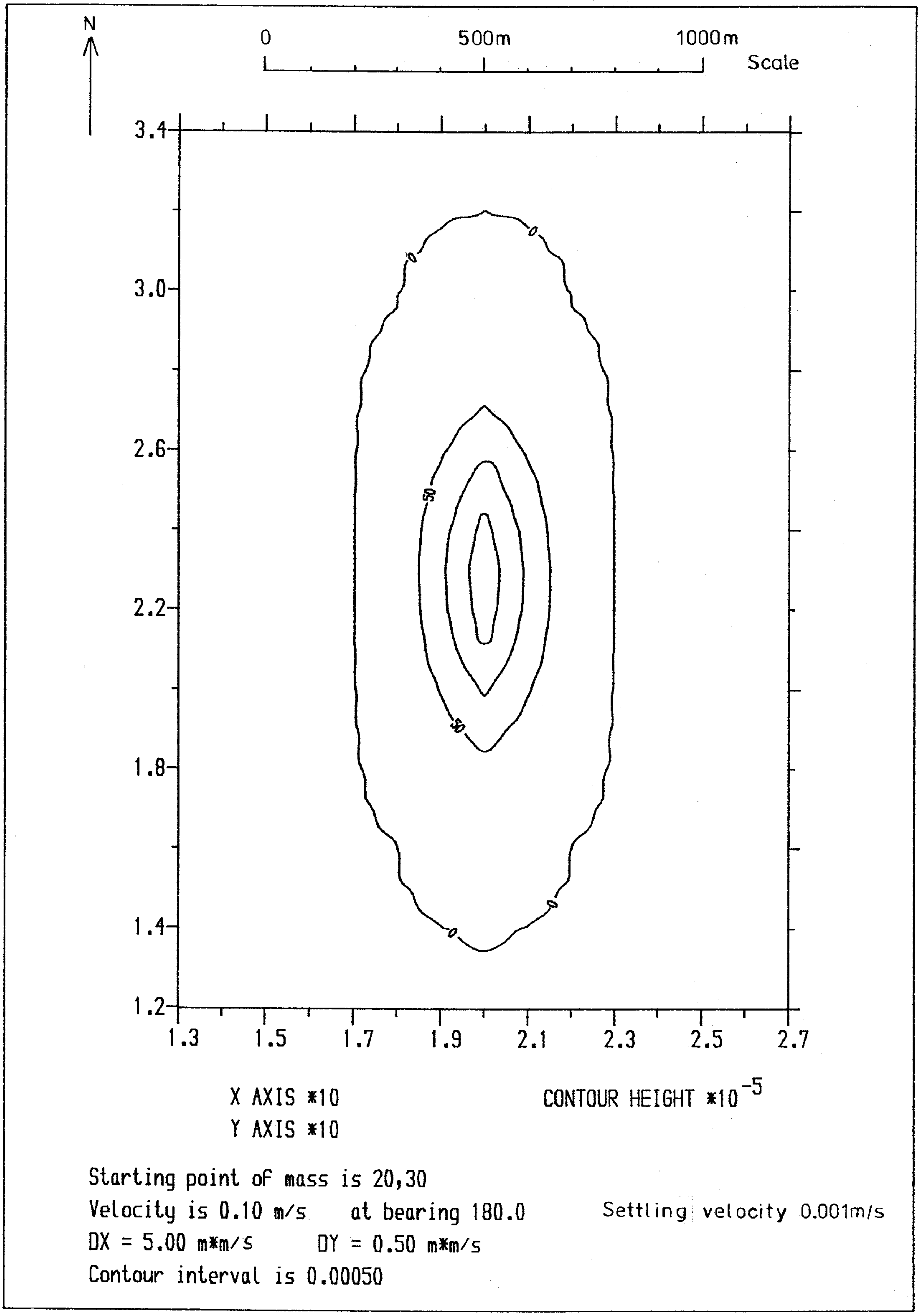


Fig 3 Concentrations in suspension after 120 minutes
One time step of 120 minutes

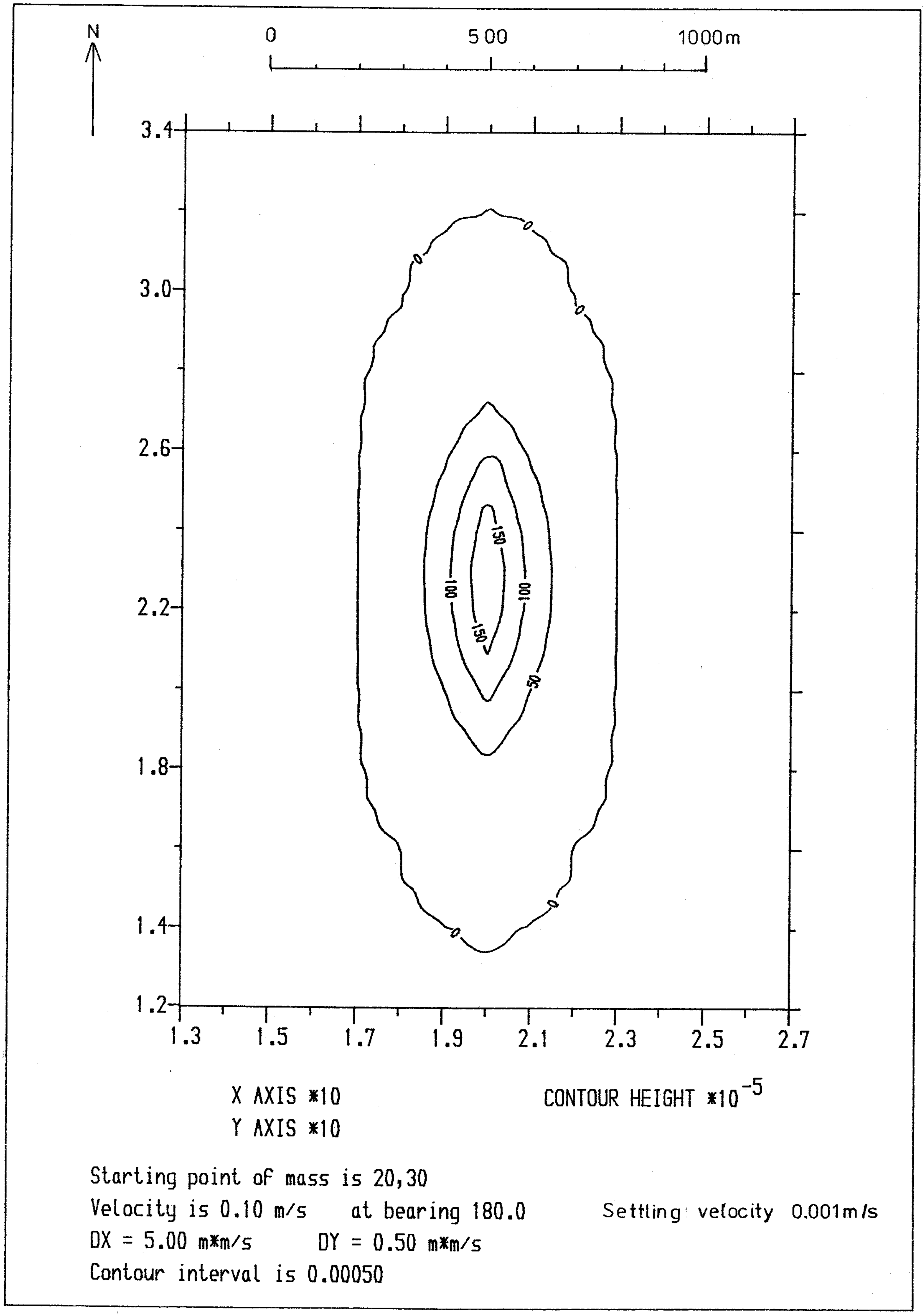


Fig 4 Concentrations in suspension after 120 minutes
Two time steps of 60 minutes

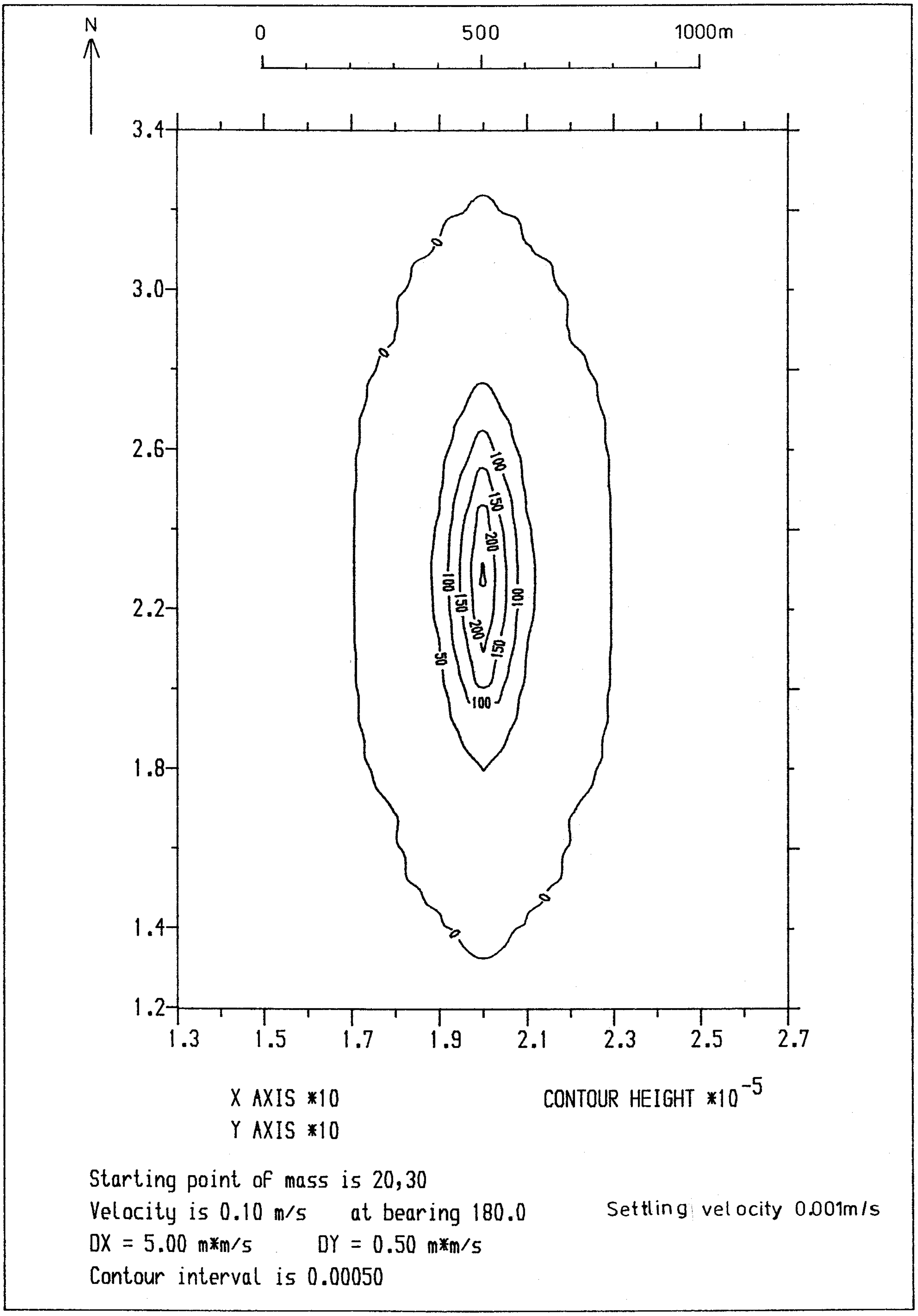


Fig 5 Concentrations in suspension after 120 minutes
Four time steps of 30 minutes

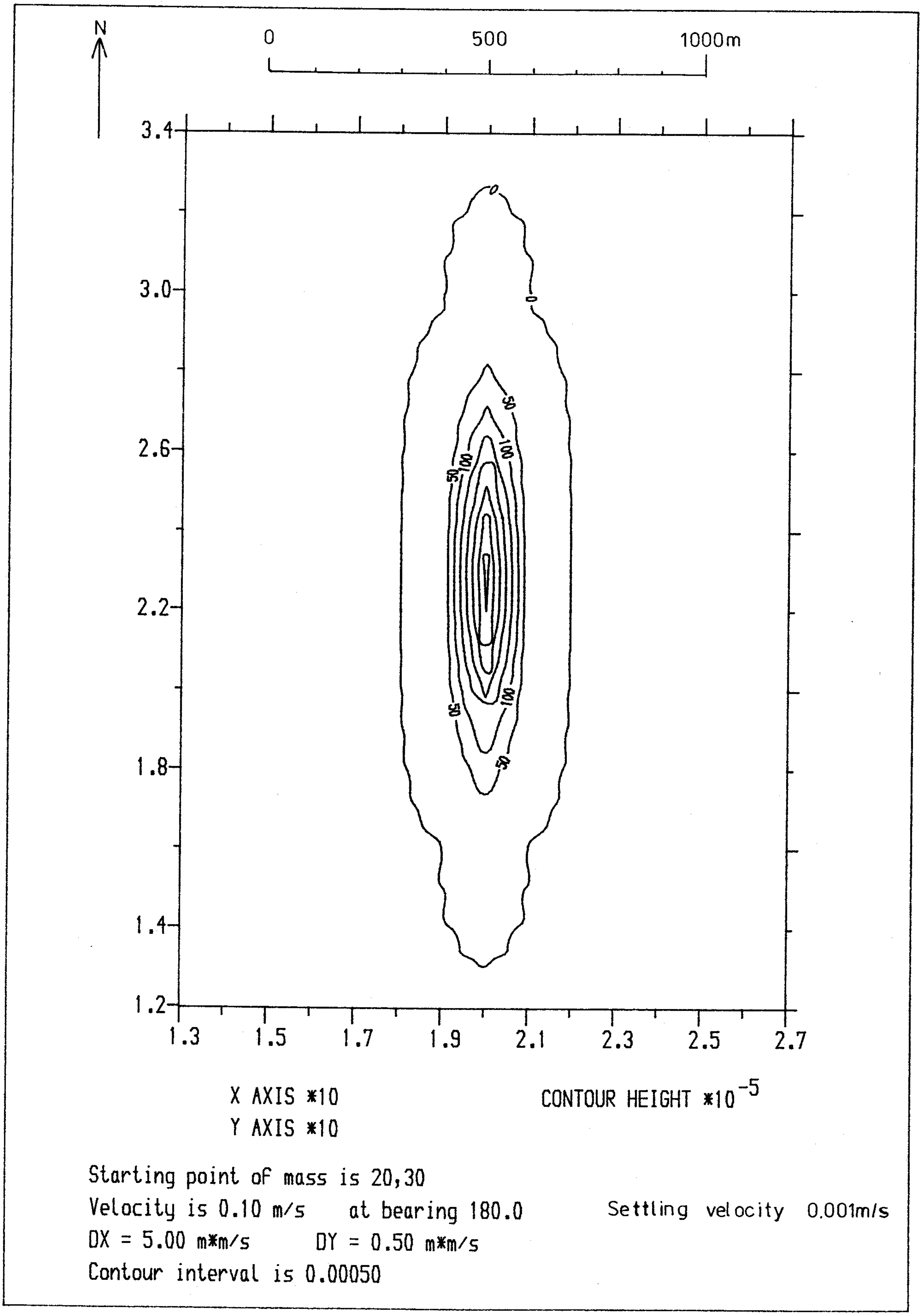


Fig 6 Concentrations in suspension after 120 minutes
Eight time steps of 15 minutes

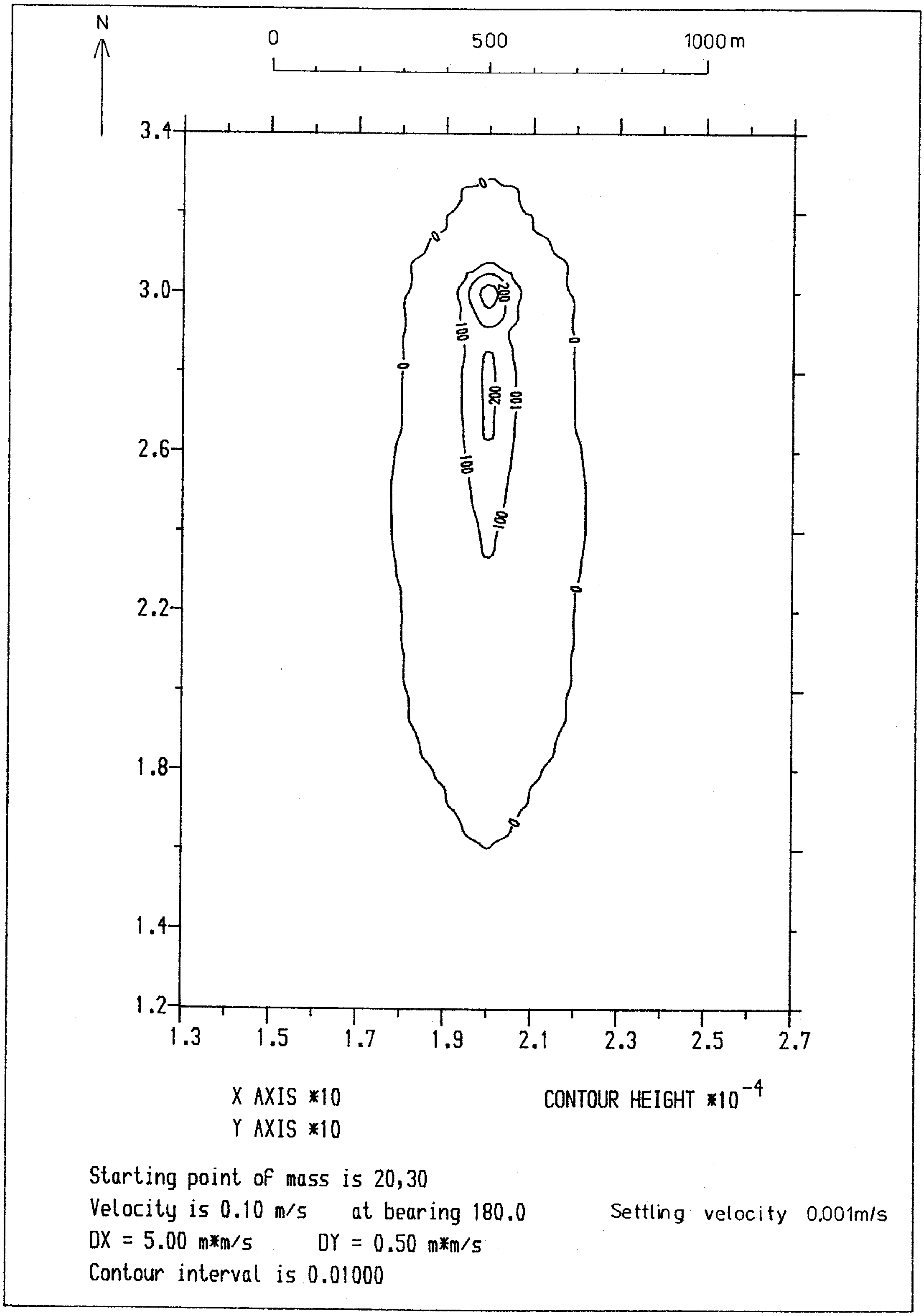
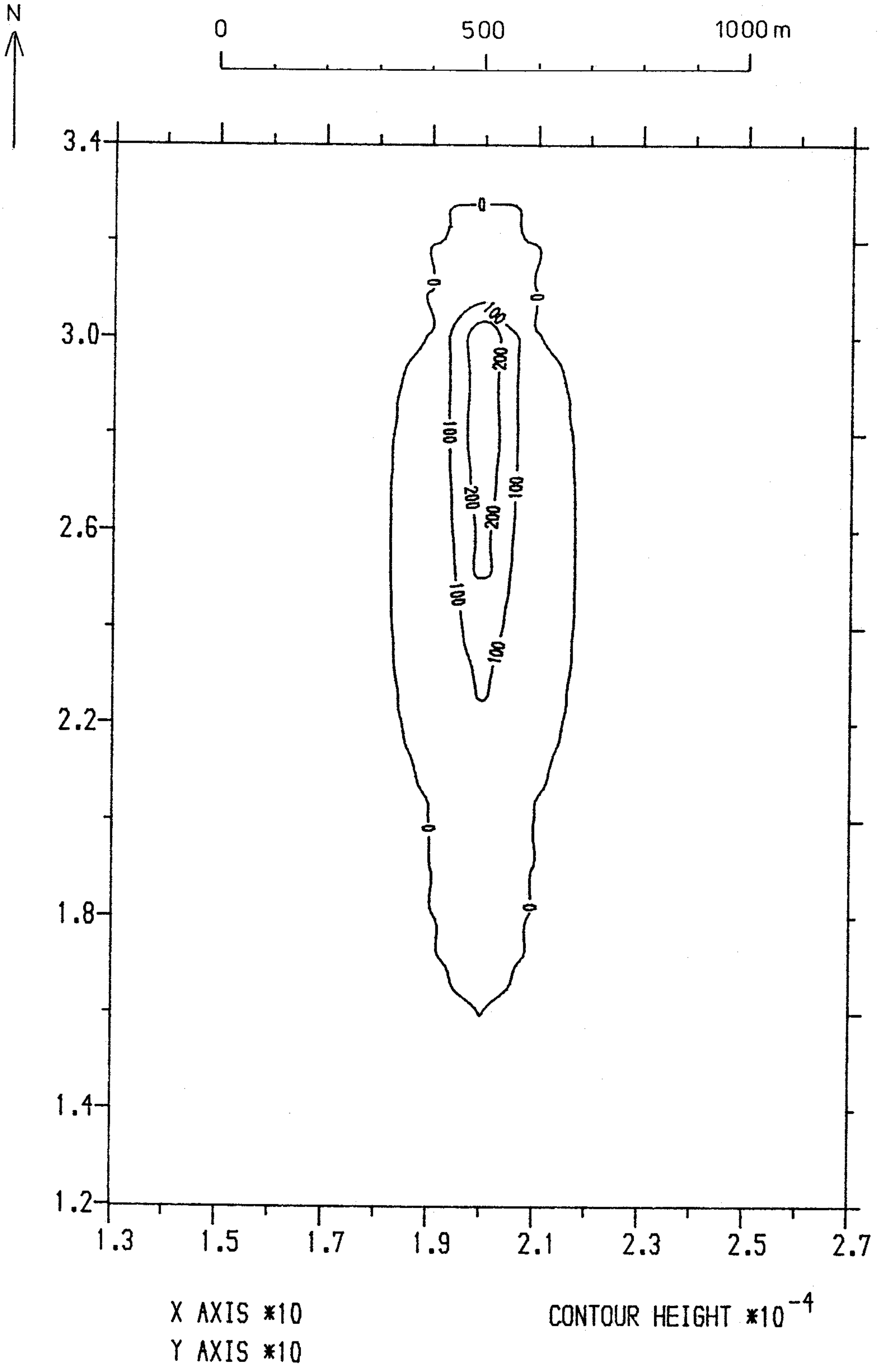


Fig 7 Distribution of mass on the bed after 120 minutes
Four time steps of 30 minutes



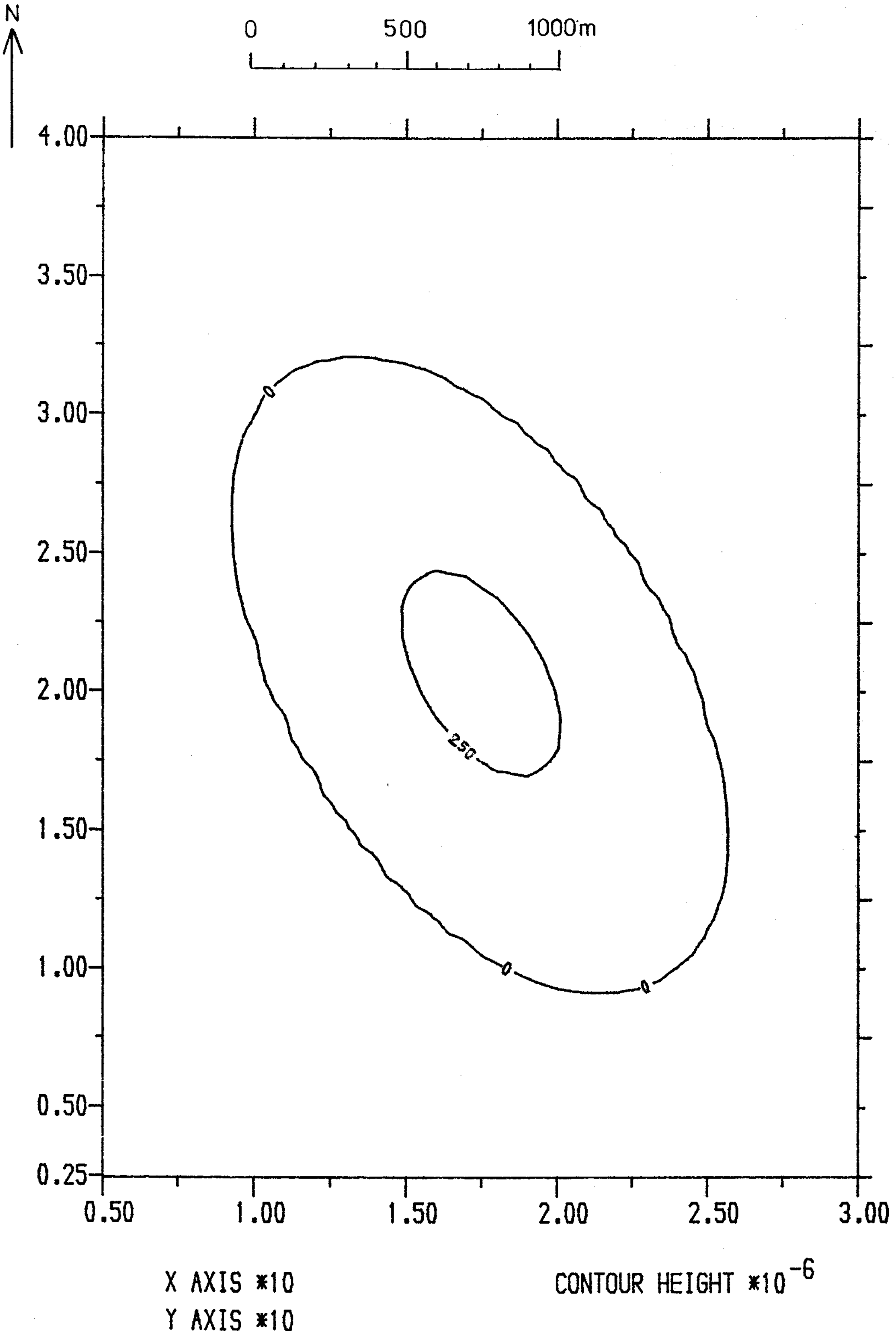
Starting point of mass is 20,30

Velocity is 0.10 m/s at bearing 180.0 Settling velocity 0.001m/s

DX = 5.00 m*m/s DY = 0.50 m*m/s

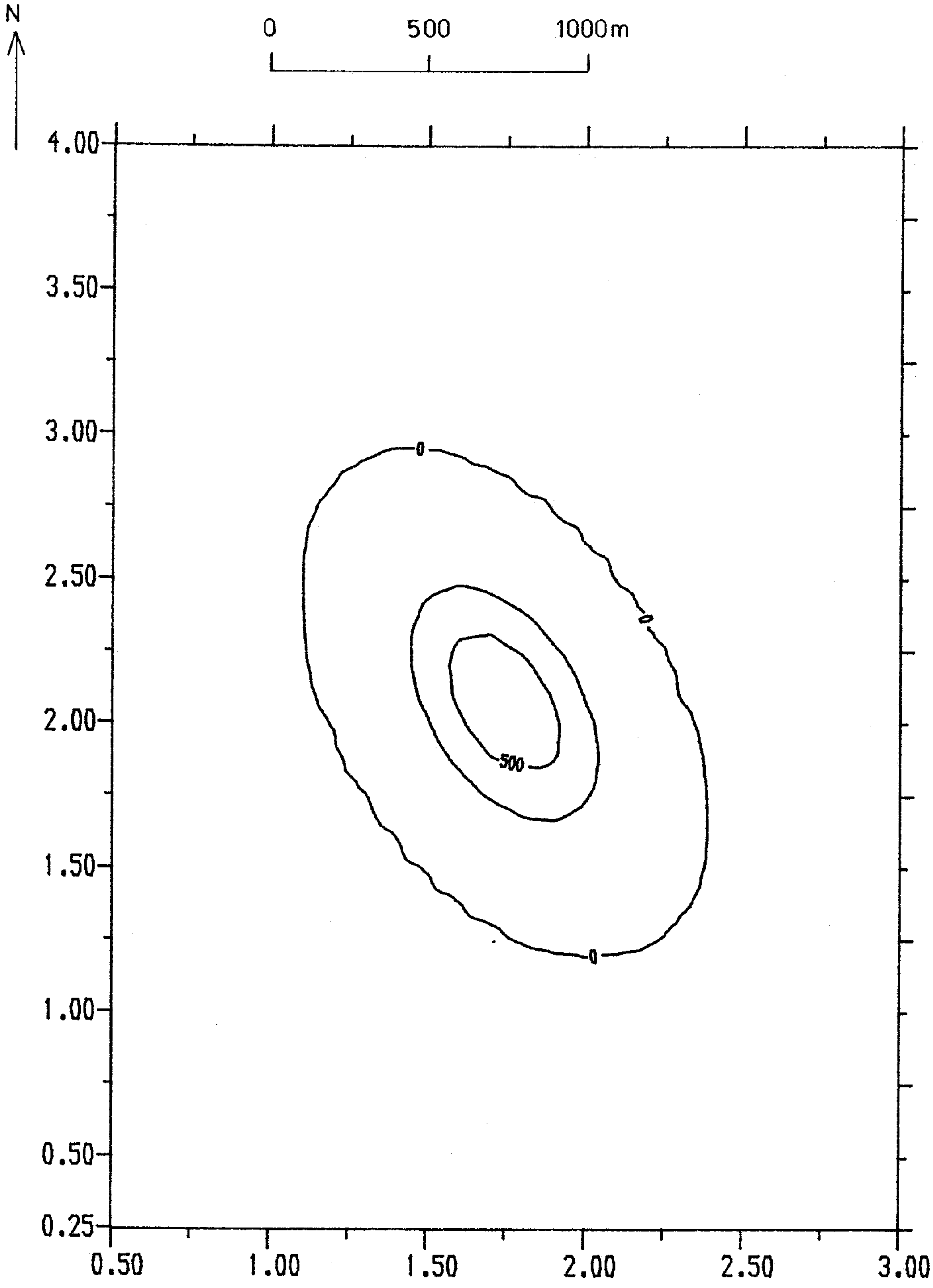
Contour interval is 0.01000

Fig 8 Distribution of mass on the bed after 120 minutes
Eight time steps of 15 minutes



Starting point of mass is 10,30
Settling velocity is 0.0010 m/s
DX = 10.00 m*m/s DY = 0.50 m*m/s
Contour interval is 0.00025

Fig 9 Concentrations in suspension
Longitudinal diffusion coefficient $10 m^2/s$



X AXIS *10
Y AXIS *10

CONTOUR HEIGHT $*10^{-6}$

Starting point of mass is 10,30
Settling velocity is 0.0010 m/s
DX = 5.00 m*m/s   DY = 0.50 m*m/s
Contour interval is 0.00025

Fig 10 Concentrations in suspension
Longitudinal diffusion coefficient 5m $^2/s$

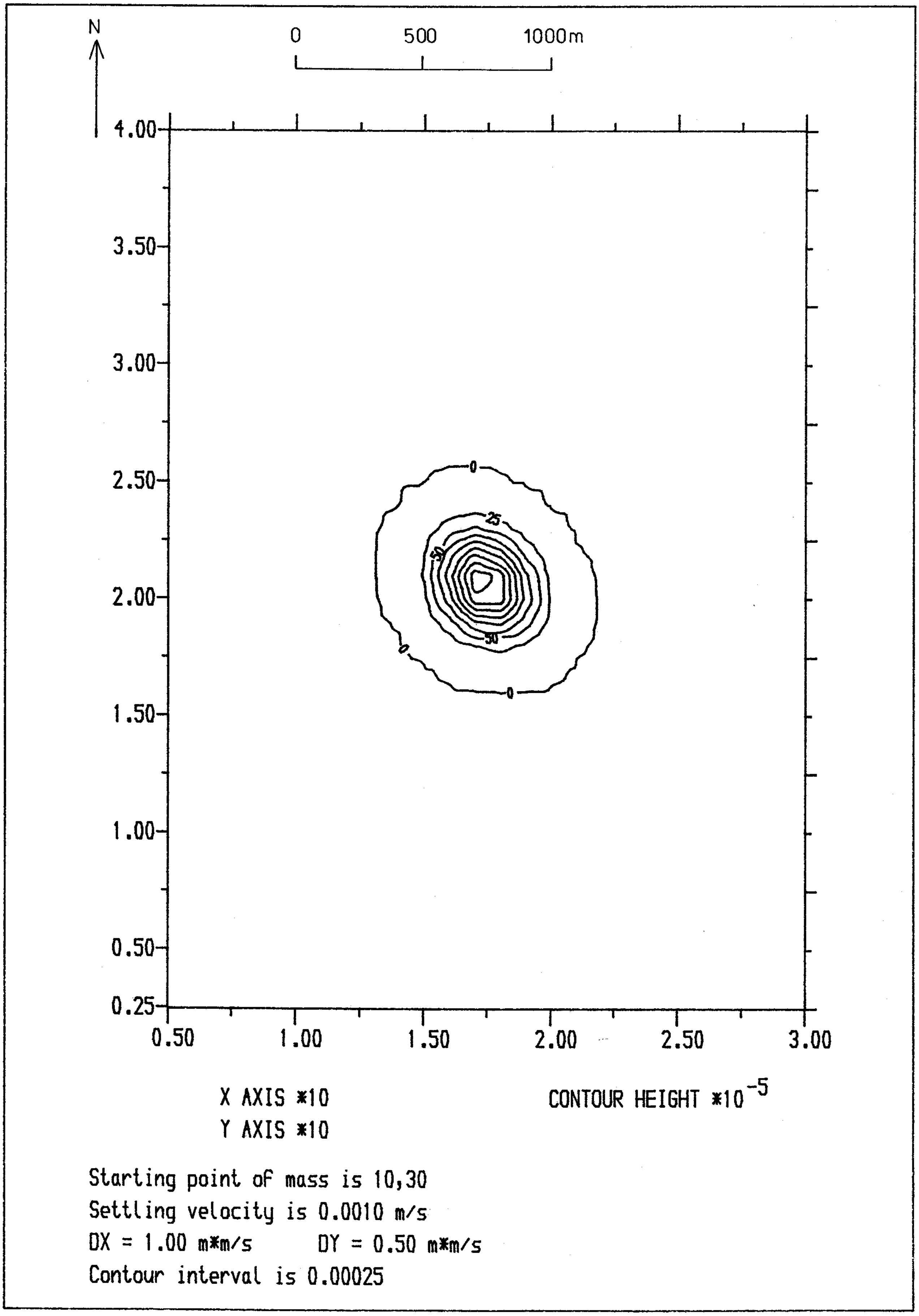


Fig 11 Concentrations in suspension
Longitudinal diffusion coefficient $1 m^2/s$

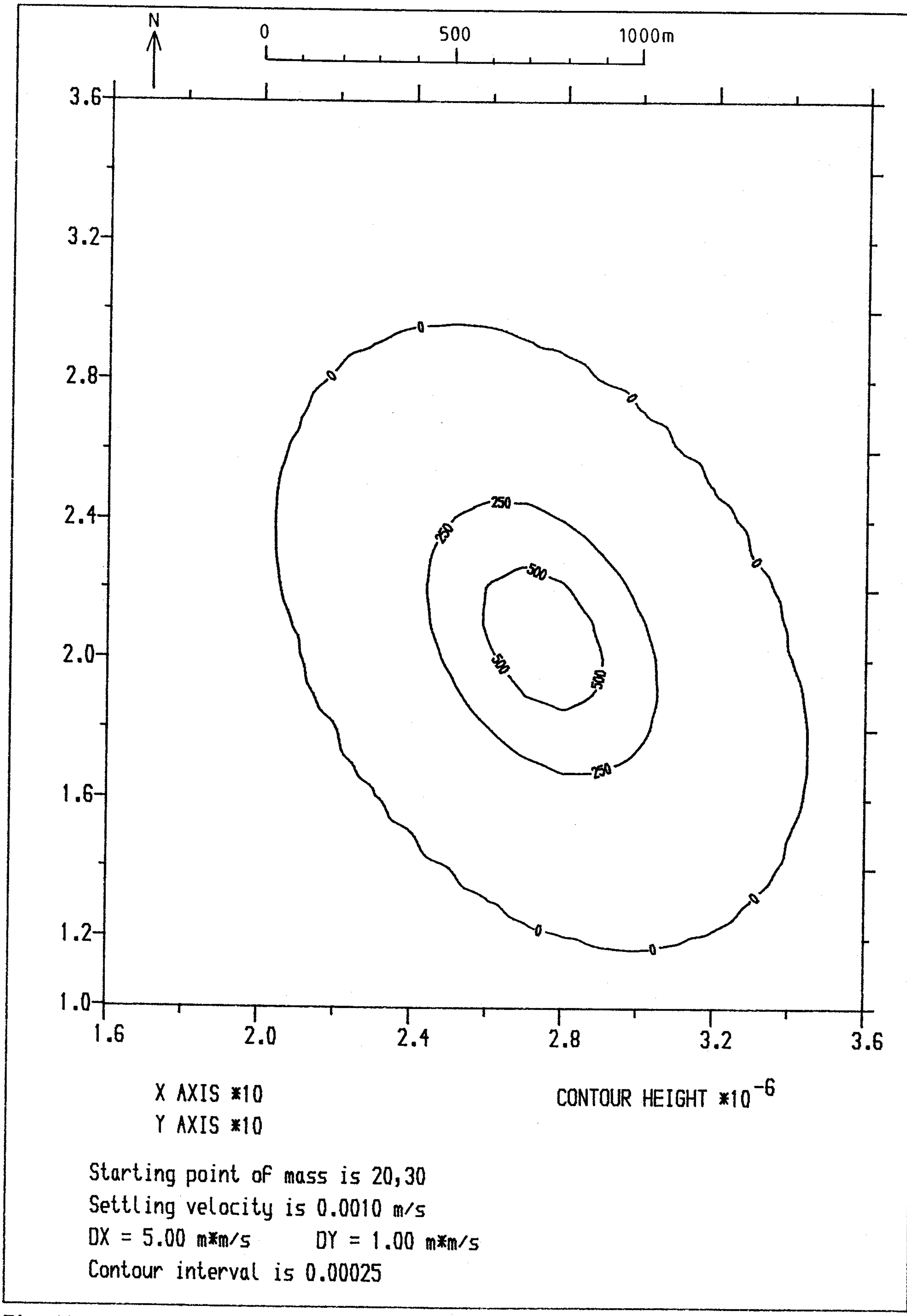
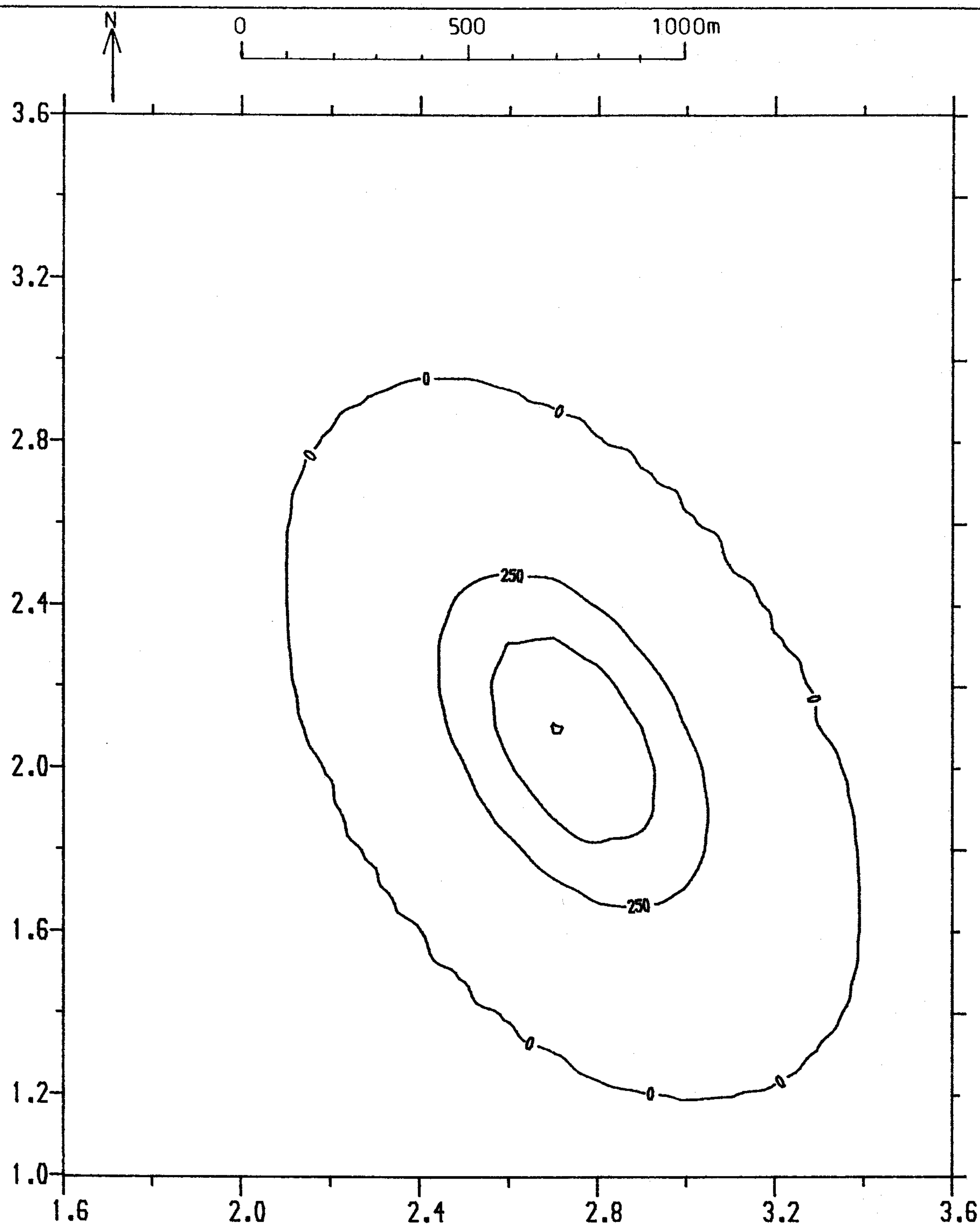


Fig 12 Concentrations in suspension
Lateral diffusion coefficient 1.0m²/s

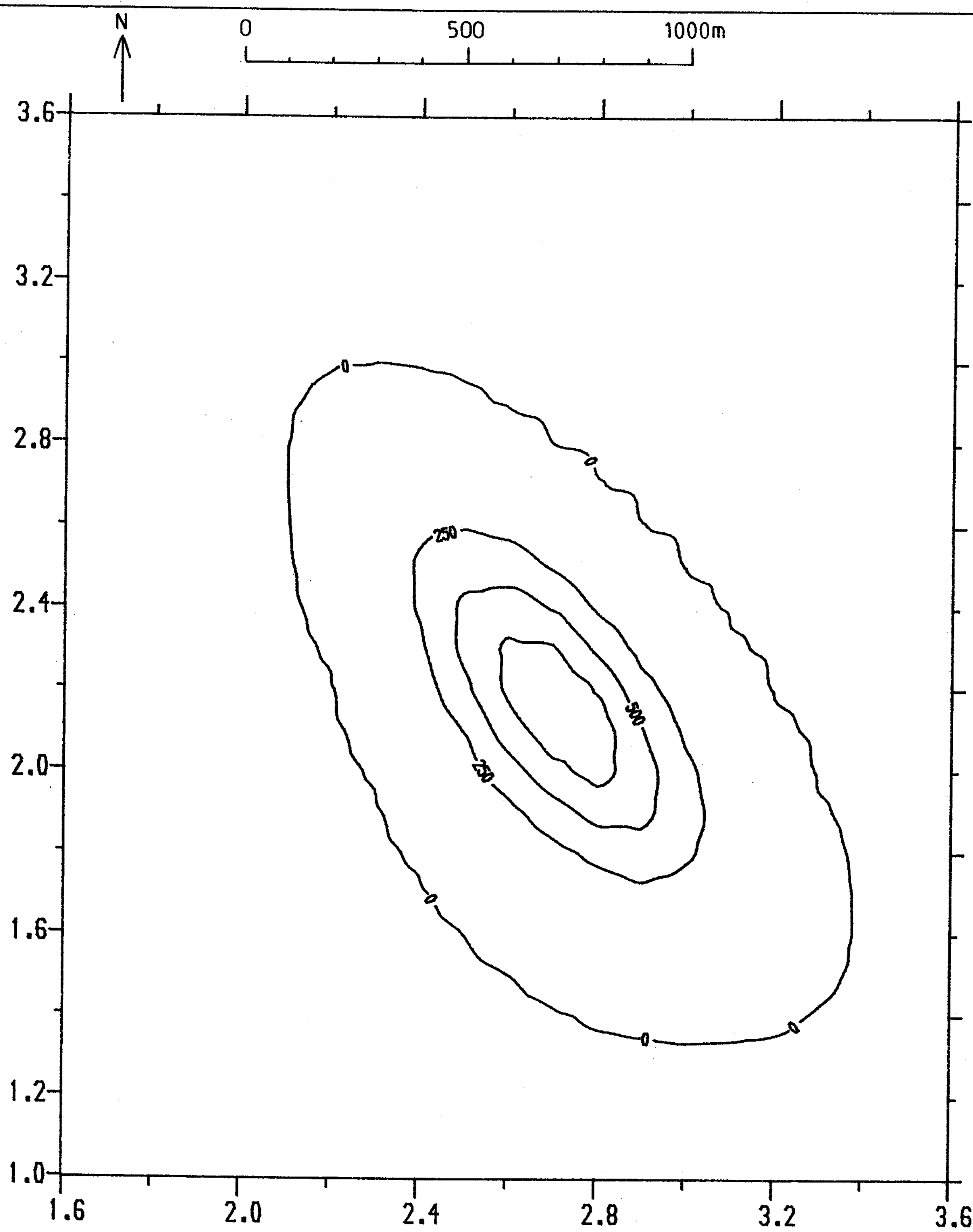


X AXIS *10
Y AXIS *10

CONTOUR HEIGHT *10^{-6}

Starting point of mass is 20,30
Settling velocity is 0.0010 m/s
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 0.00025

Fig 13 Concentrations in suspension
Lateral diffusion coefficient 0.5m^2/s

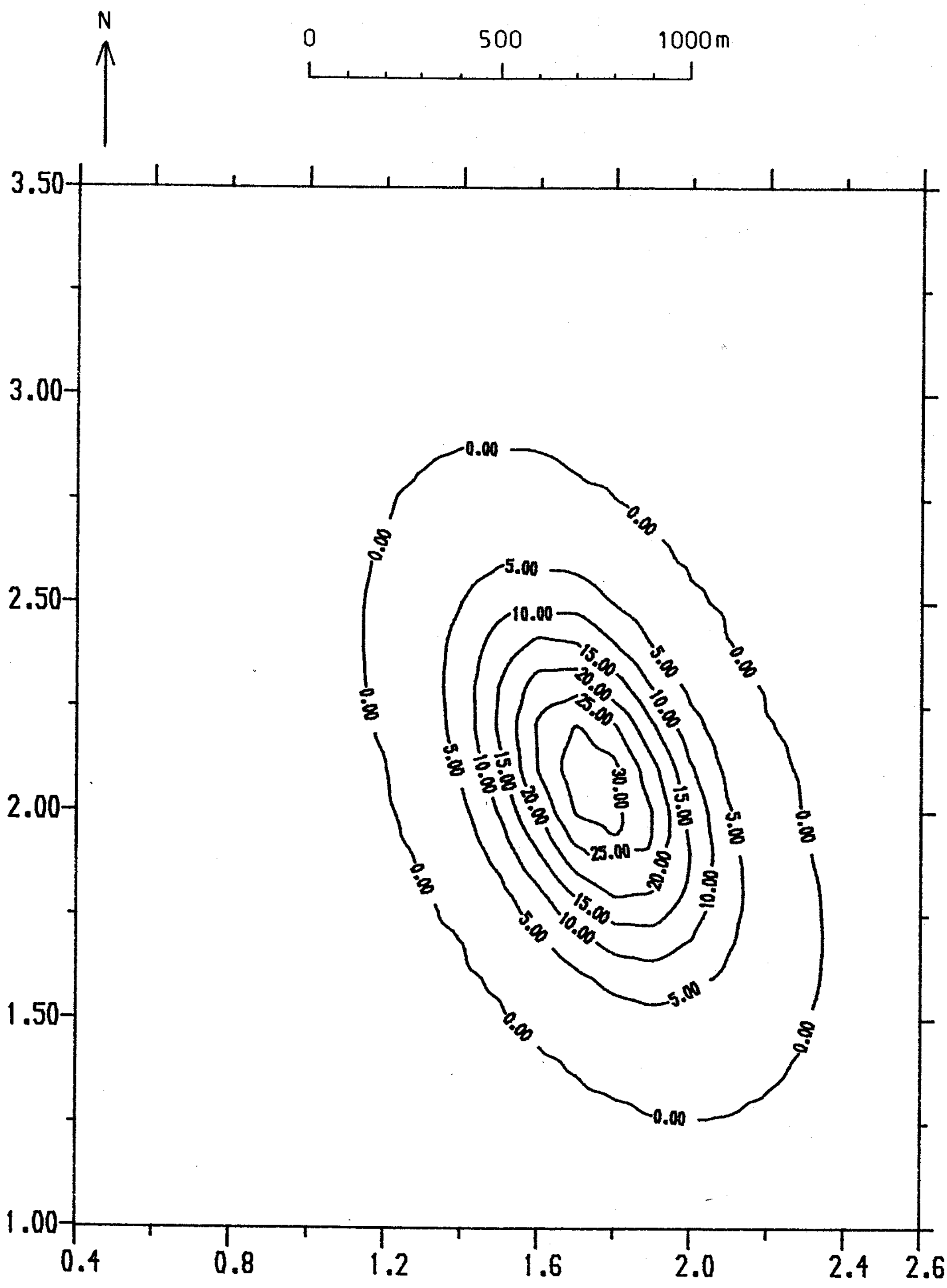


X AXIS *10
Y AXIS *10

CONTOUR HEIGHT *10^{-6}

Starting point of mass is 20,30
Settling velocity is 0.0010 m/s
DX = 5.00 m*m/s DY = 0.10 m*m/s
Contour interval is 0.00025

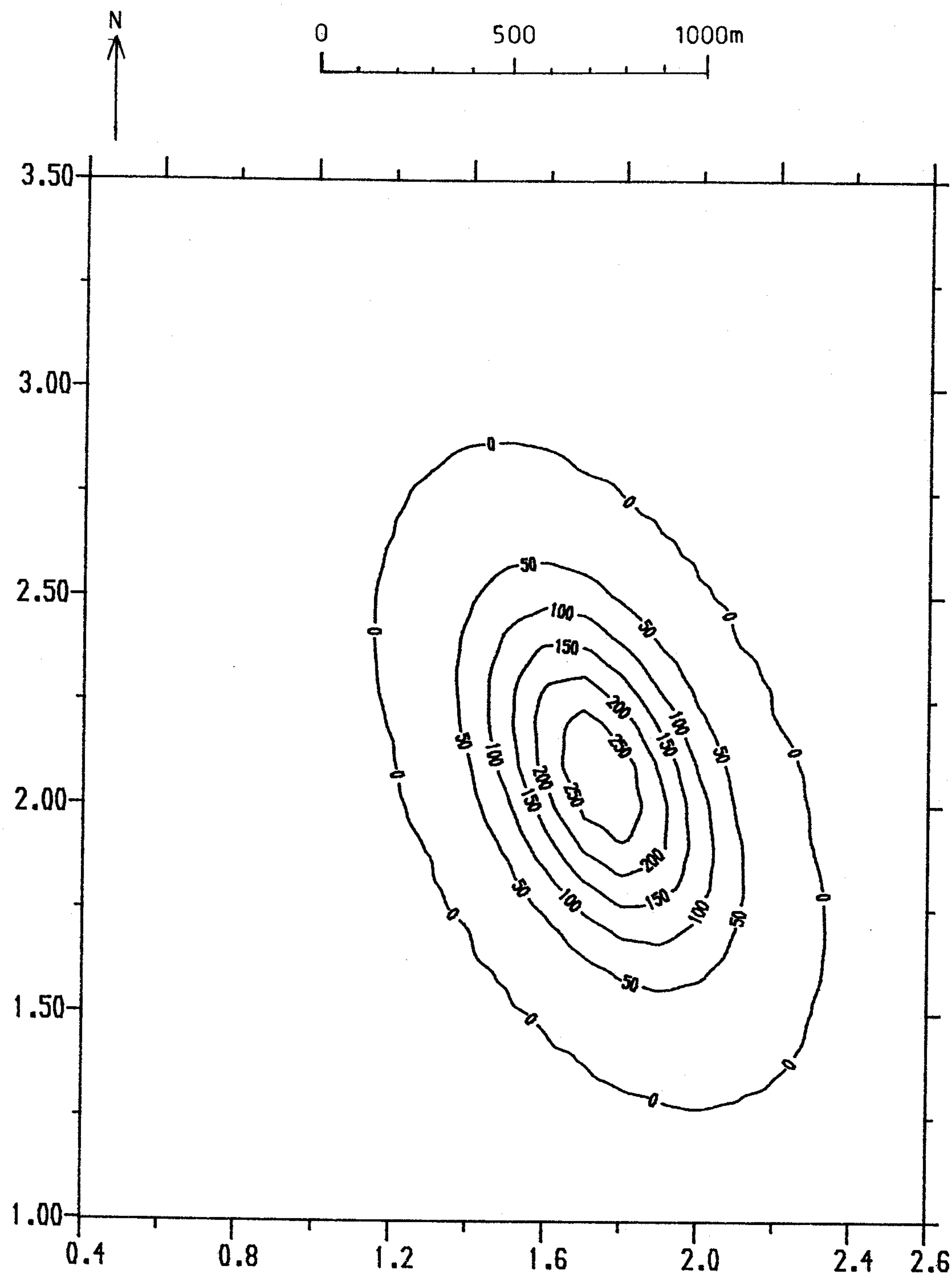
Fig 14 Concentrations in suspension
Lateral diffusion coefficient 0.1m^2/s



X AXIS *10
Y AXIS *10

Starting point of mass is 10,30; initial concentration is 1000ppm
Settling velocity is 0.0001 m/s
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 5.000

Fig 15 Concentrations in suspension (in ppm)
Settling velocity = 0 0001m/s



X AXIS *10
Y AXIS *10

CONTOUR HEIGHT $*10^{-1}$

Starting point of mass is 10,30; initial concentration is 1000ppm
Settling velocity is 0.0005 m/s
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 5.000

Fig 16 Concentrations in suspension (in ppm)
Settling velocity = 0 0005m/s

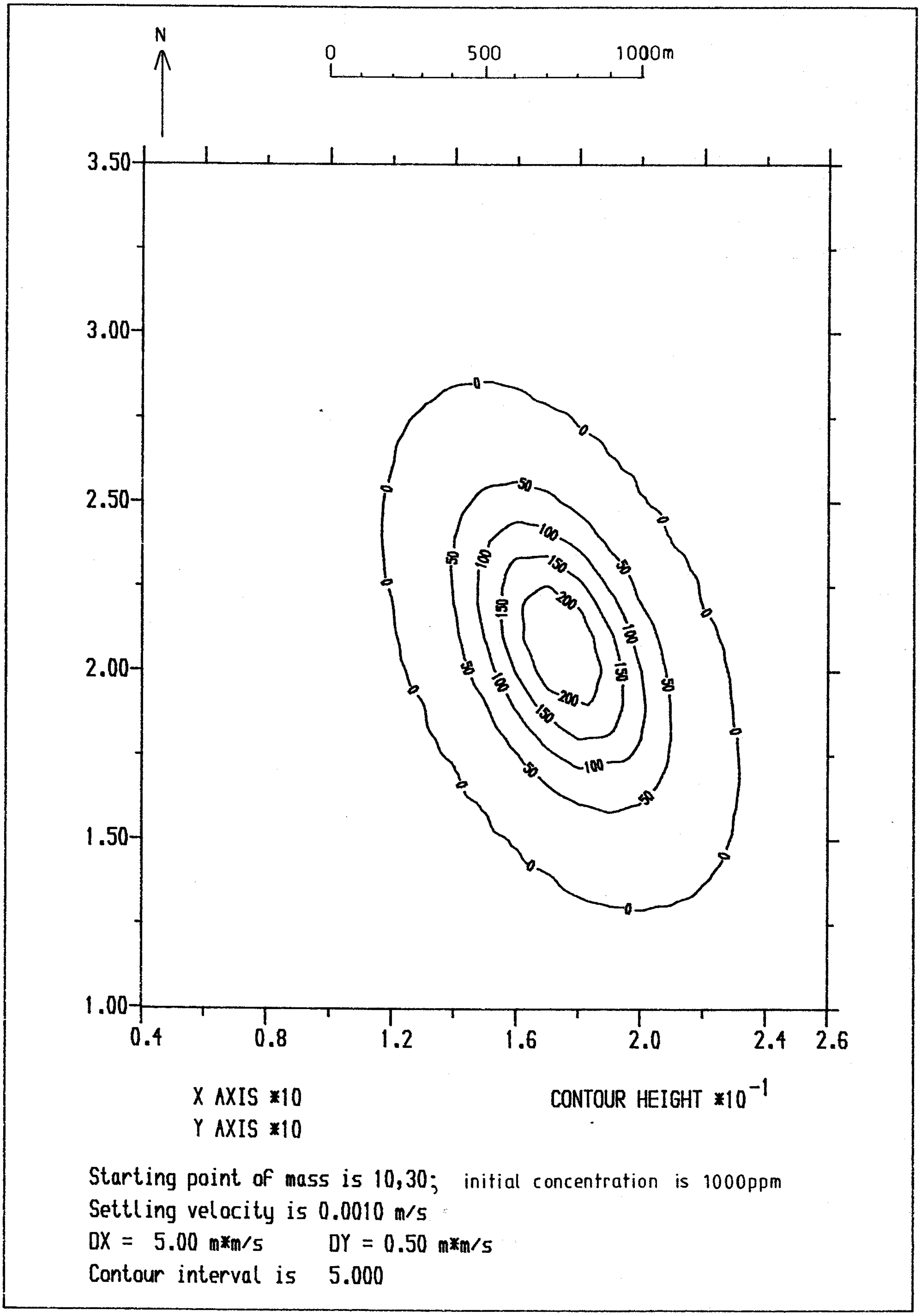


Fig 17 Concentrations in suspension (in ppm)
Settling velocity = 0.001m/s

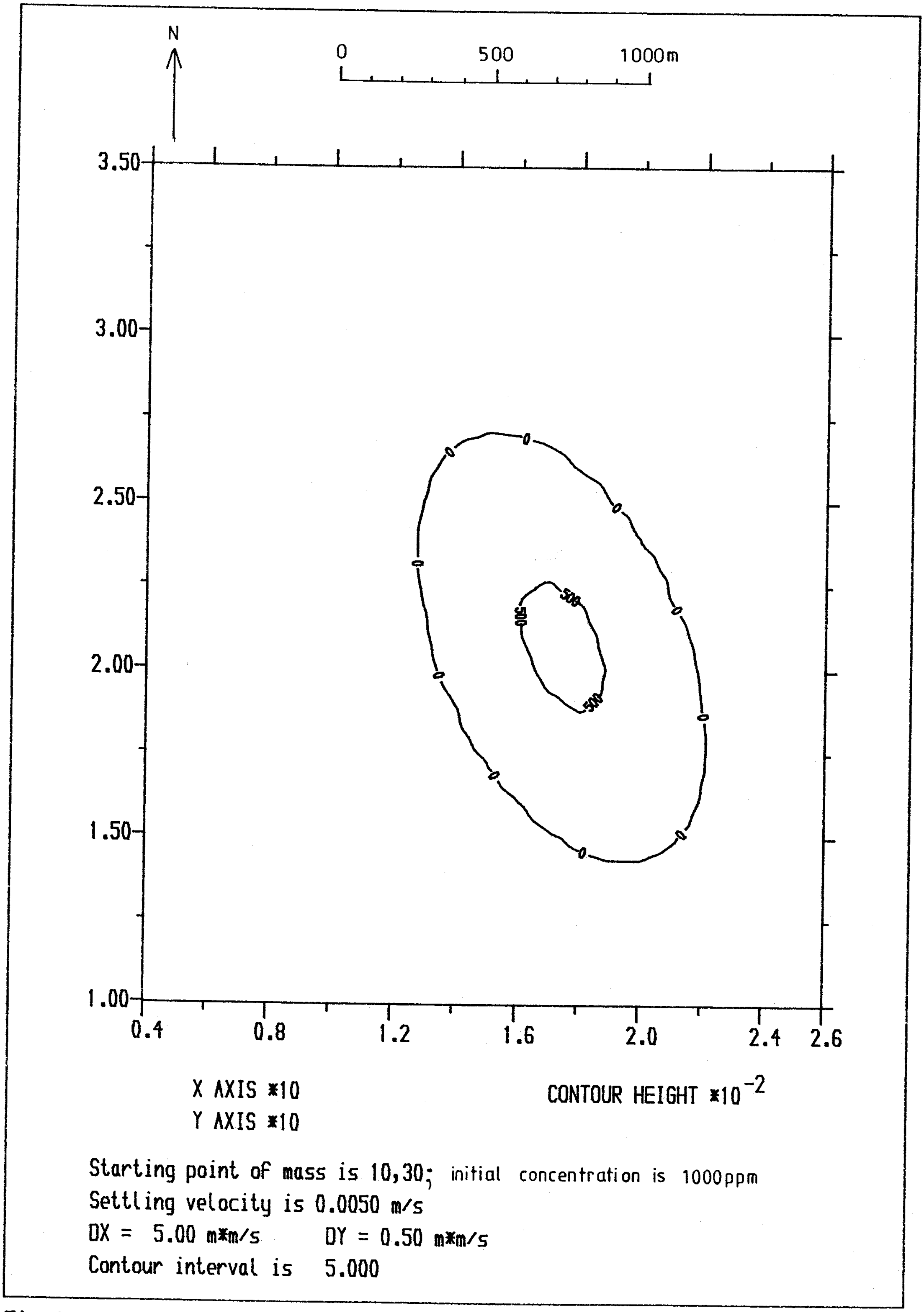


Fig 18 Concentrations in suspension (in ppm)
Settling velocity = 0.005m/s

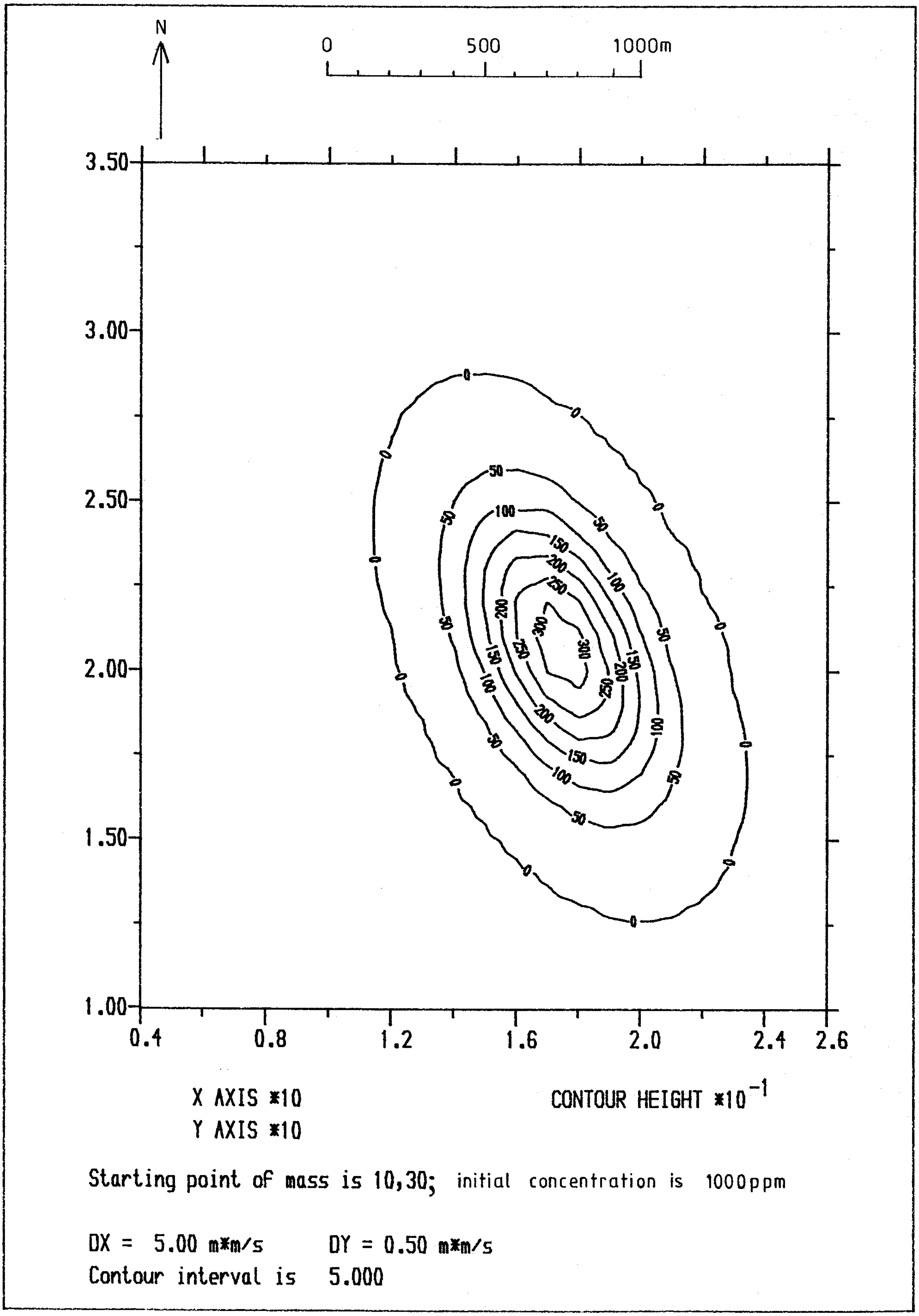


Fig 19 Concentrations in suspension (in ppm)
Settling velocity proportional to concentration at centroid of plume

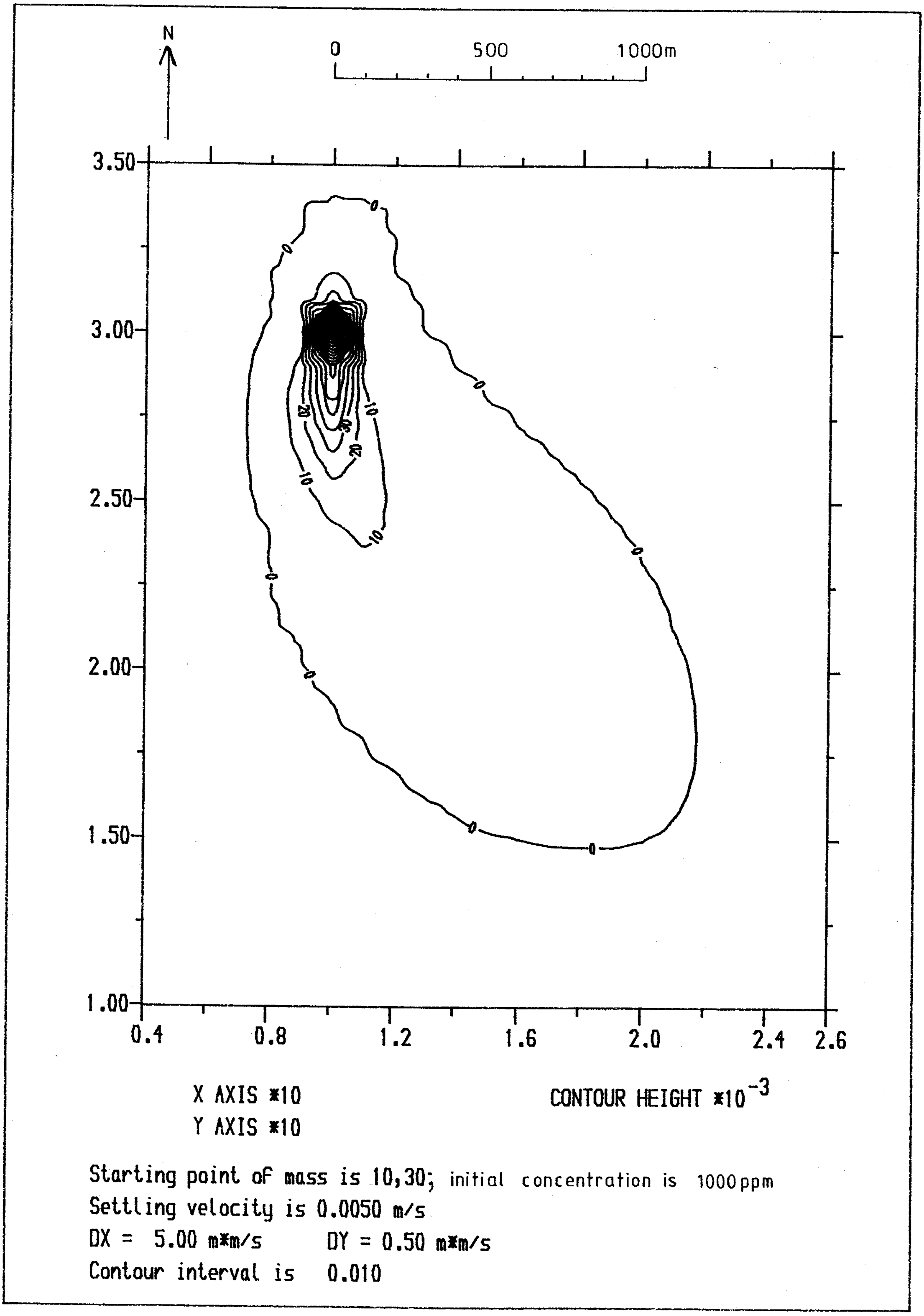


Fig 20 Distribution of mass on bed
Settling velocity is 0.005m/s

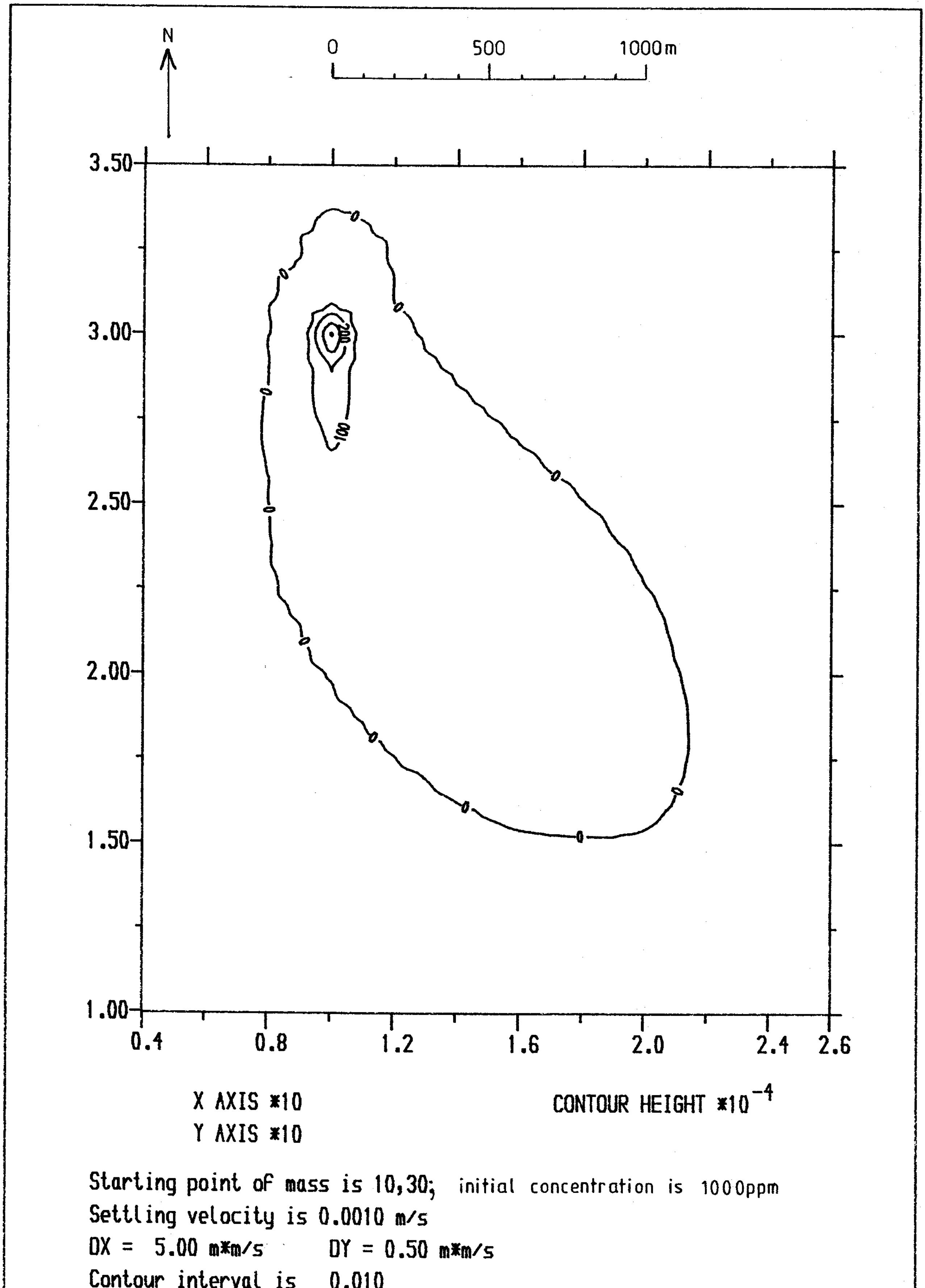
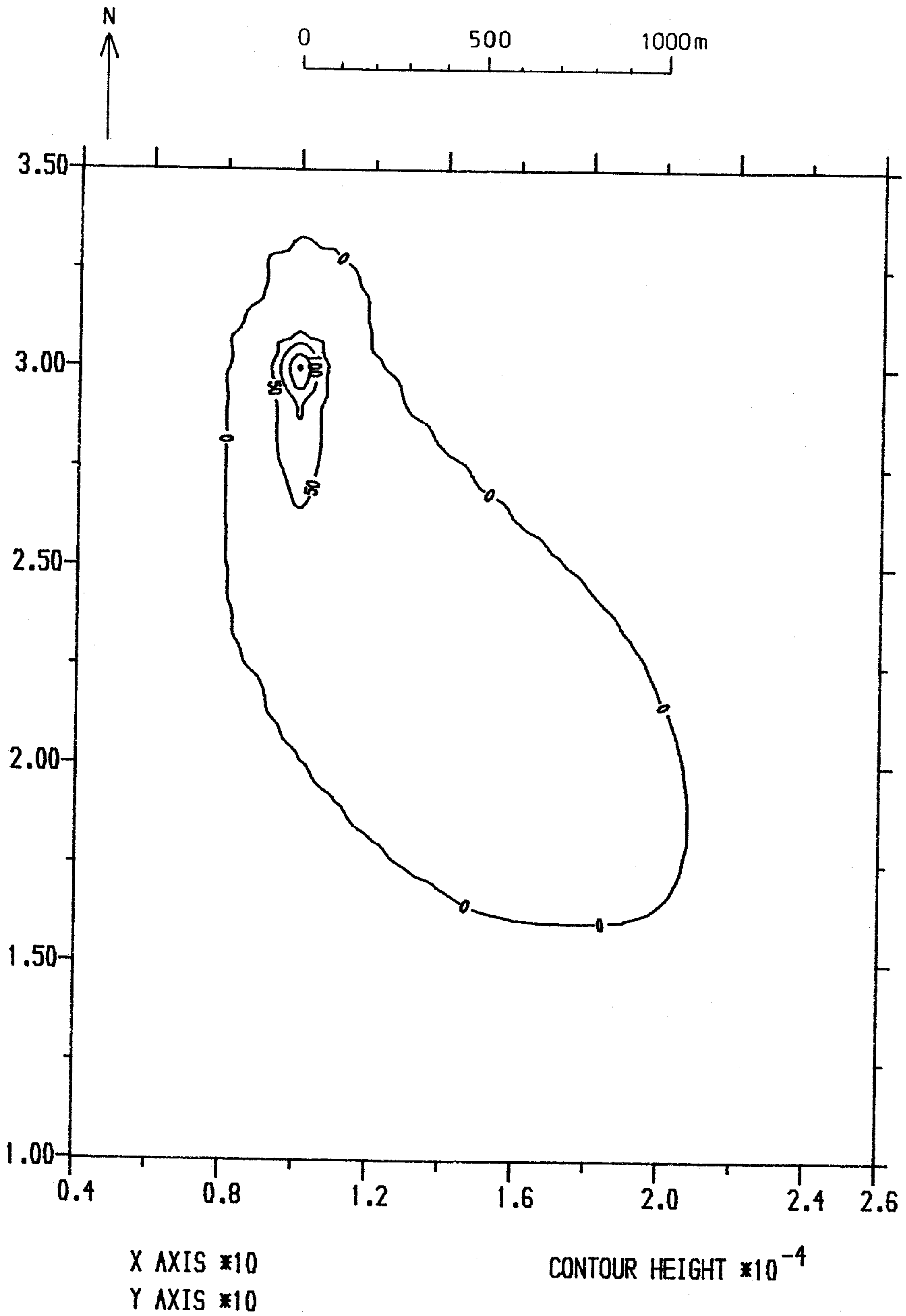


Fig 21 Distribution of mass on bed
Settling velocity is 0.001m/s



Starting point of mass is 10,30; initial concentration is 1000ppm
Settling velocity is 0.0005 m/s
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 0.005

Fig 22 Distribution of mass on bed
Settling velocity is 0.0005m/s

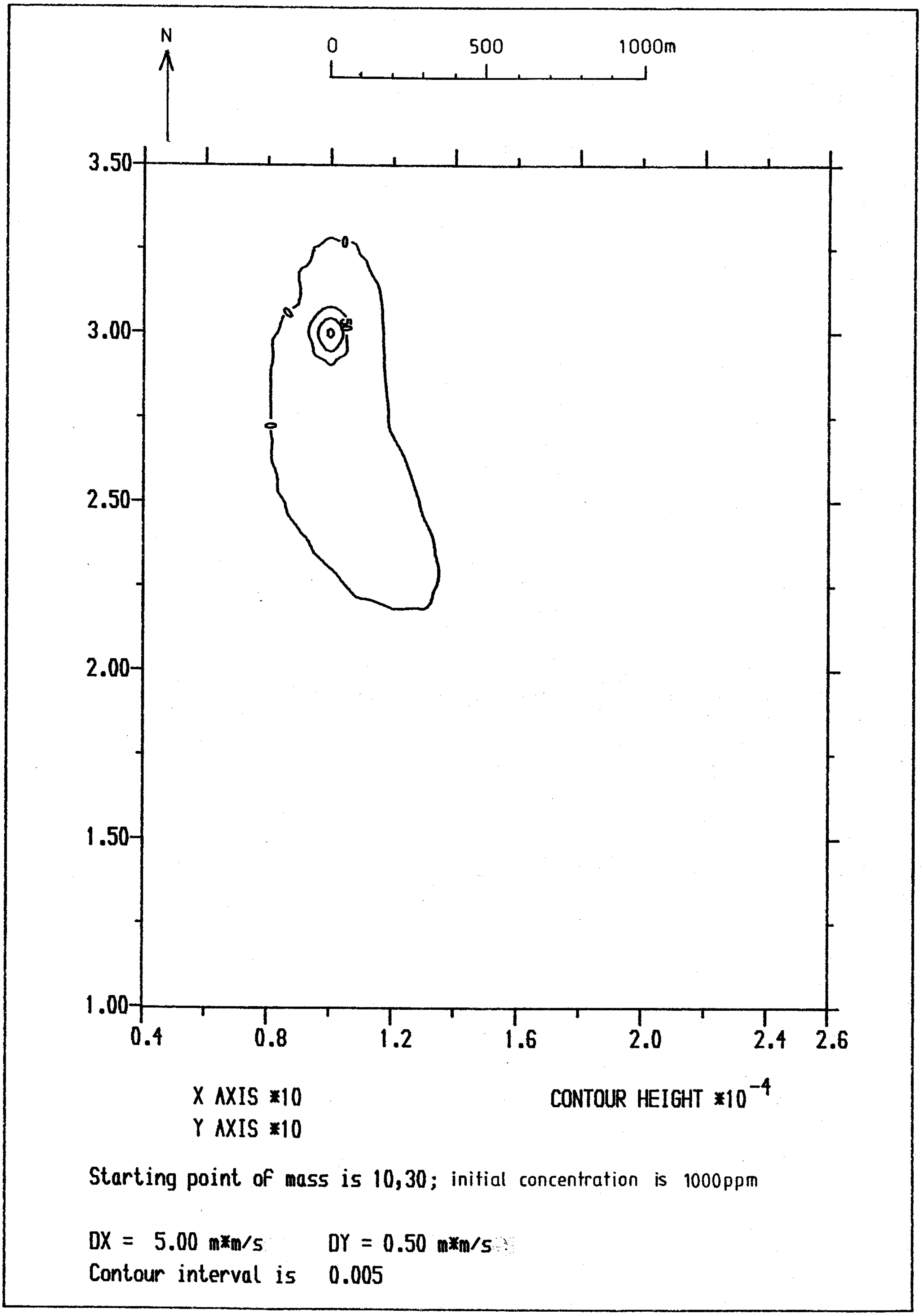


Fig 23 Distribution of mass on bed
Settling velocity proportional to concentration at centroid

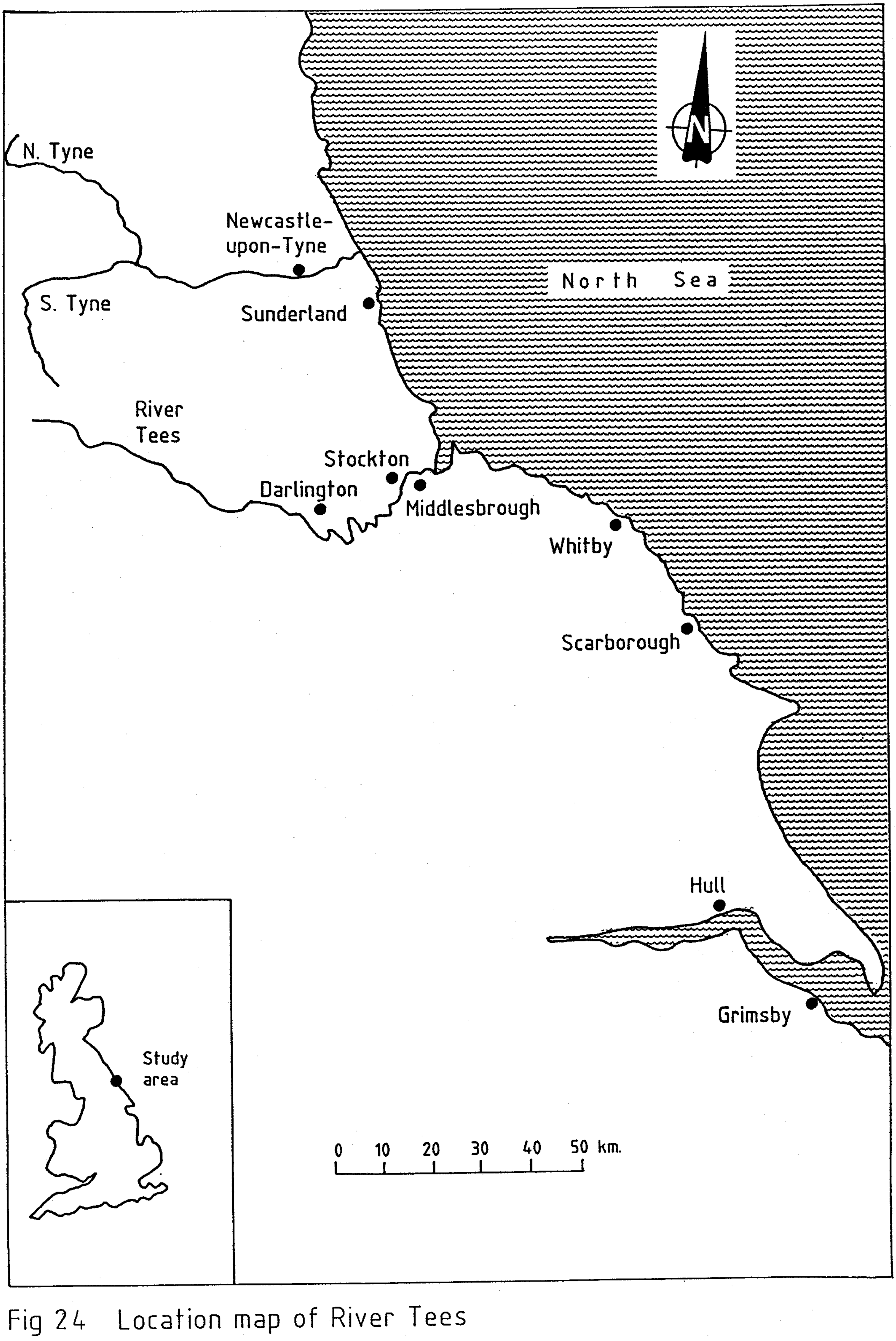


Fig 24 Location map of River Tees

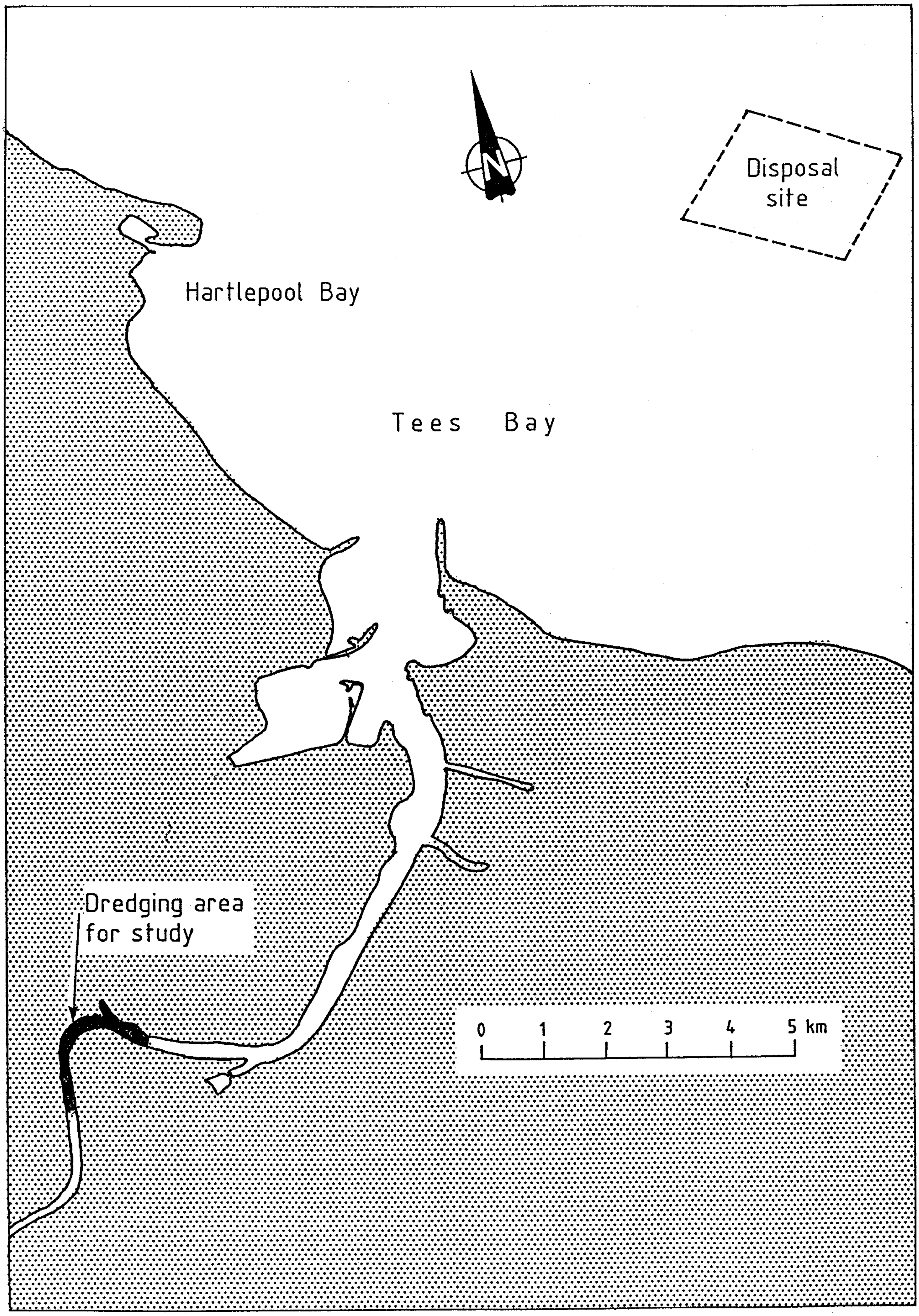


Fig 25 Location map of disposal site

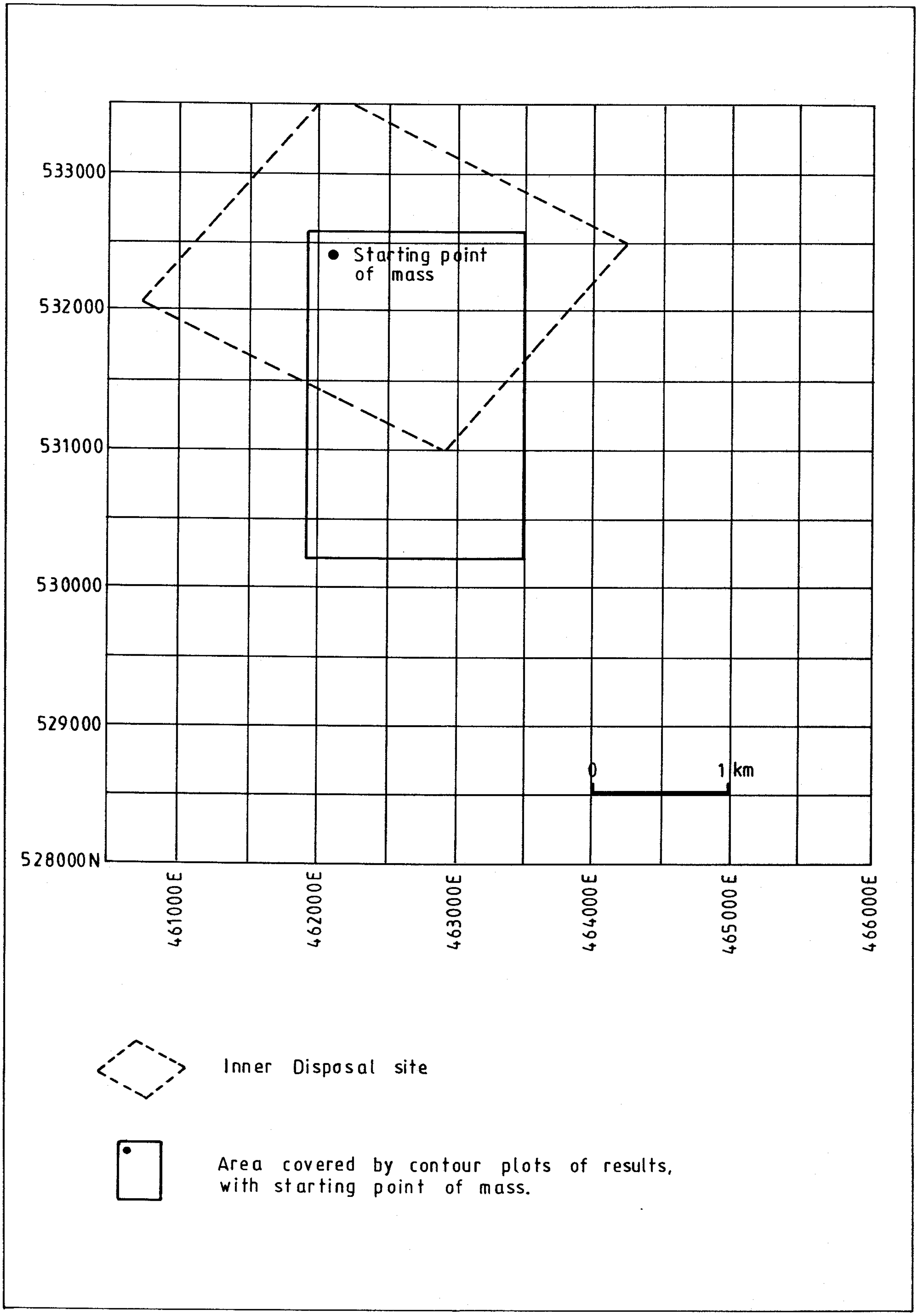


Fig 26 Total area on National Grid covered by model, showing position of Disposal site and area covered by contour plots of results

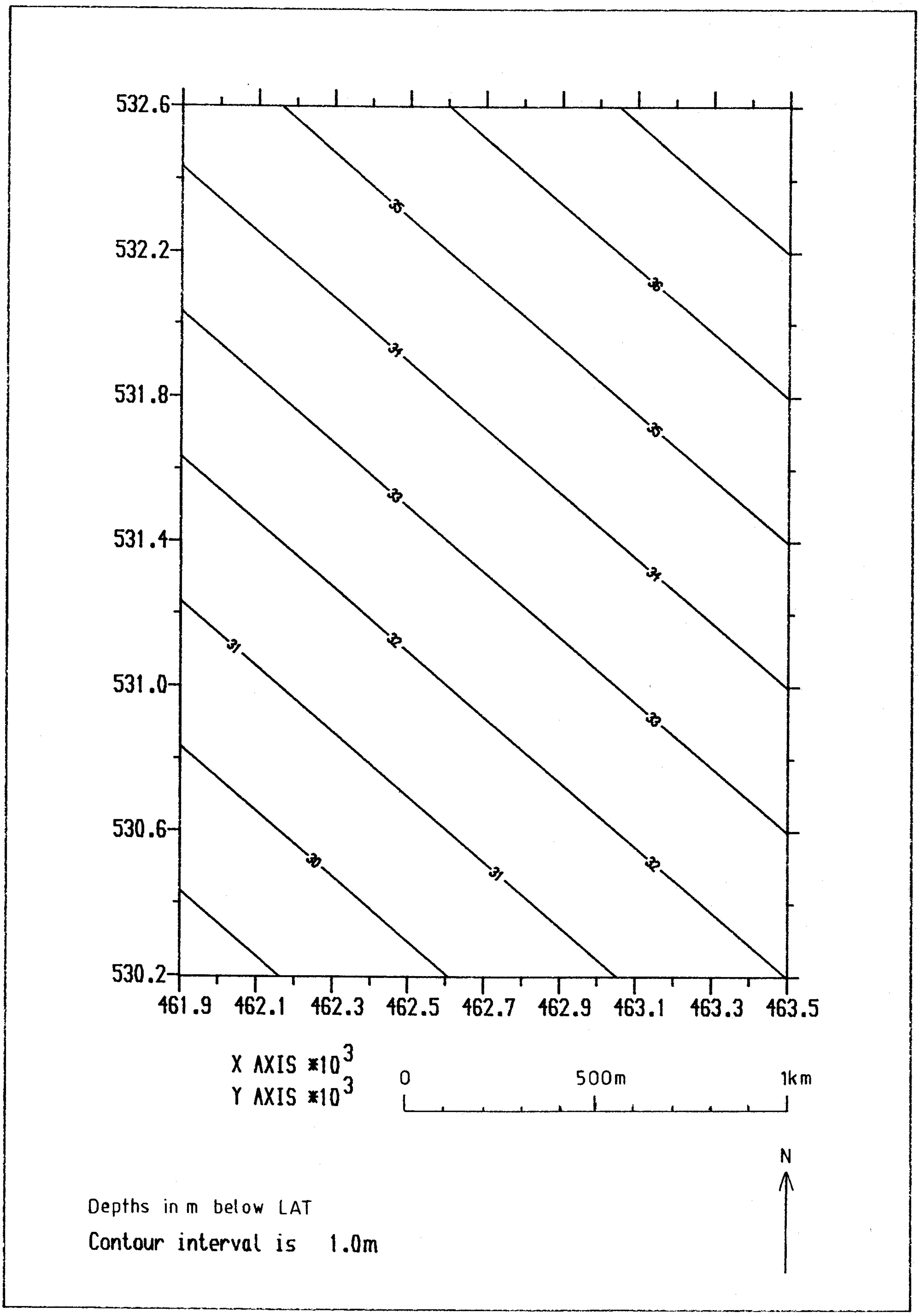


Fig 27 Bathymetry of results area used in model

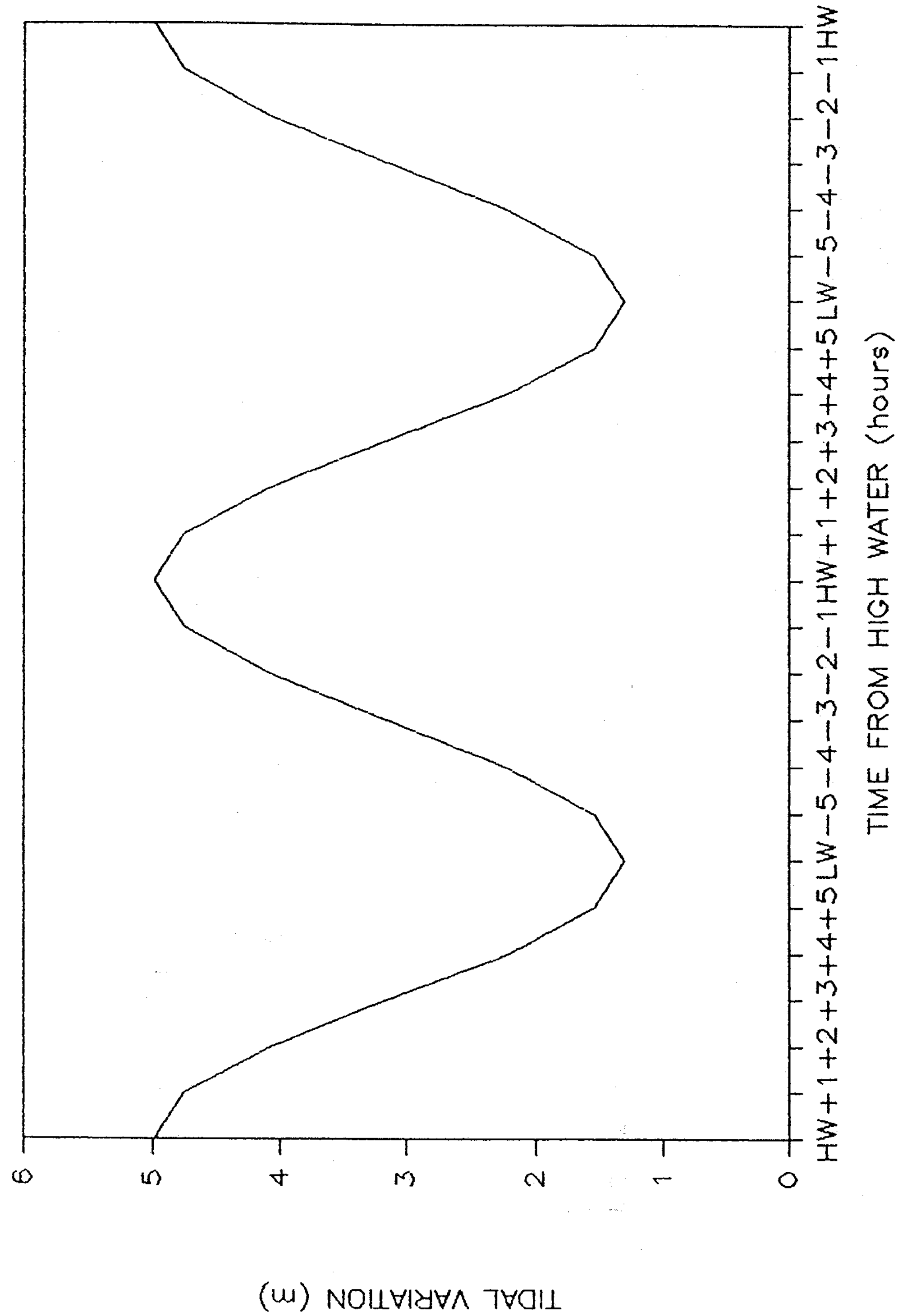


Fig 28 Data from Tees used in model : Tidal variation

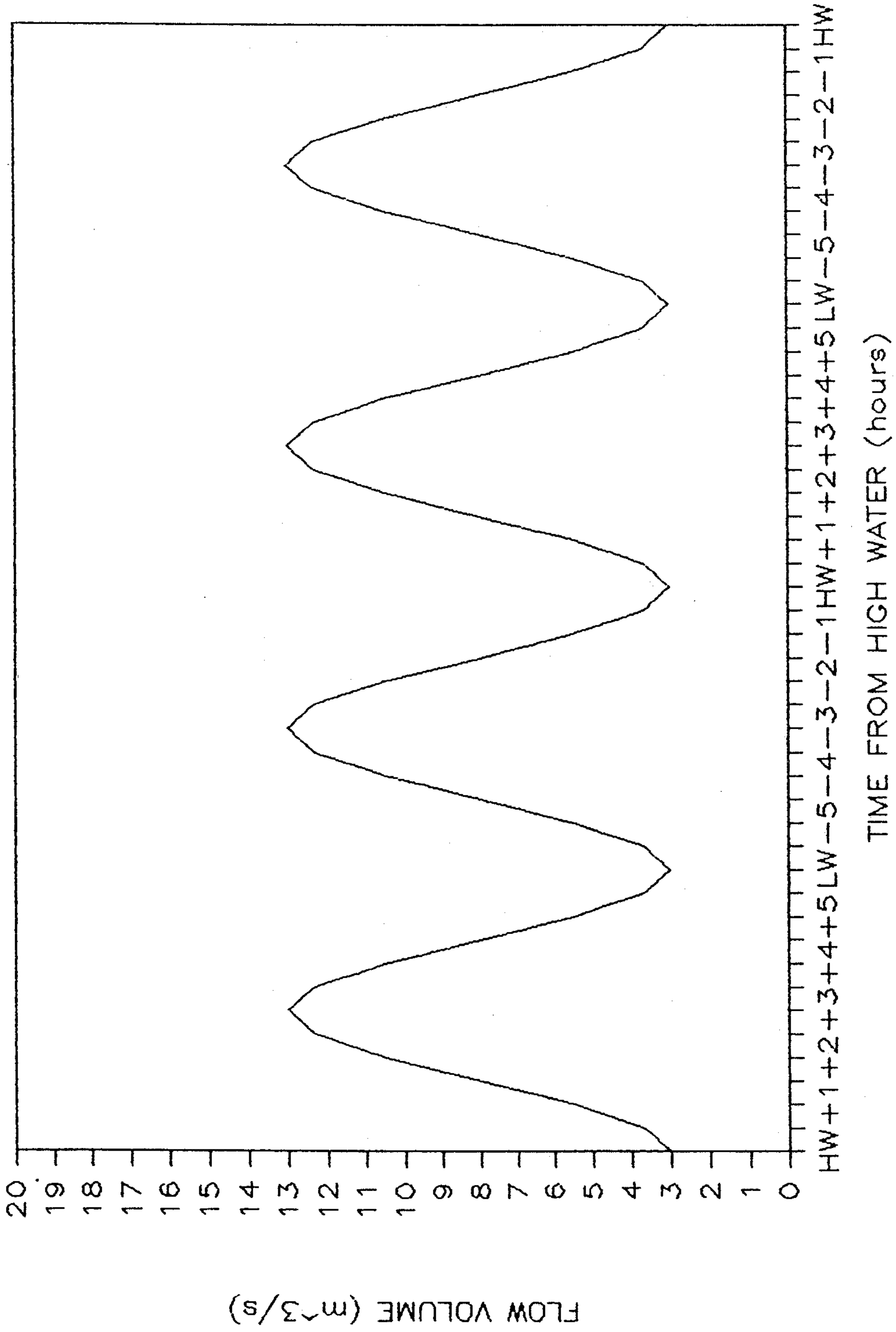


Fig 29 Data from Tees used in model: Flow volume

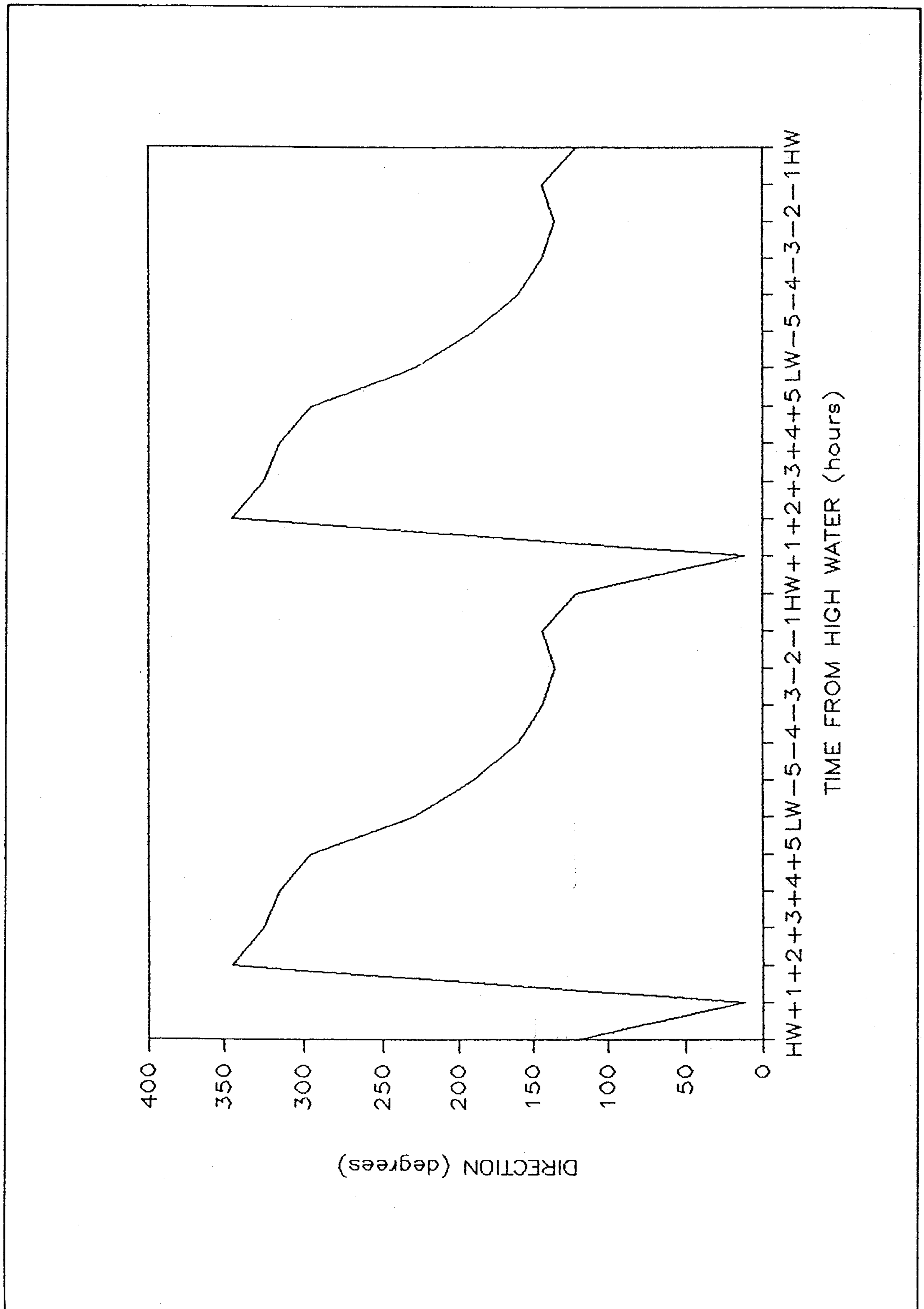
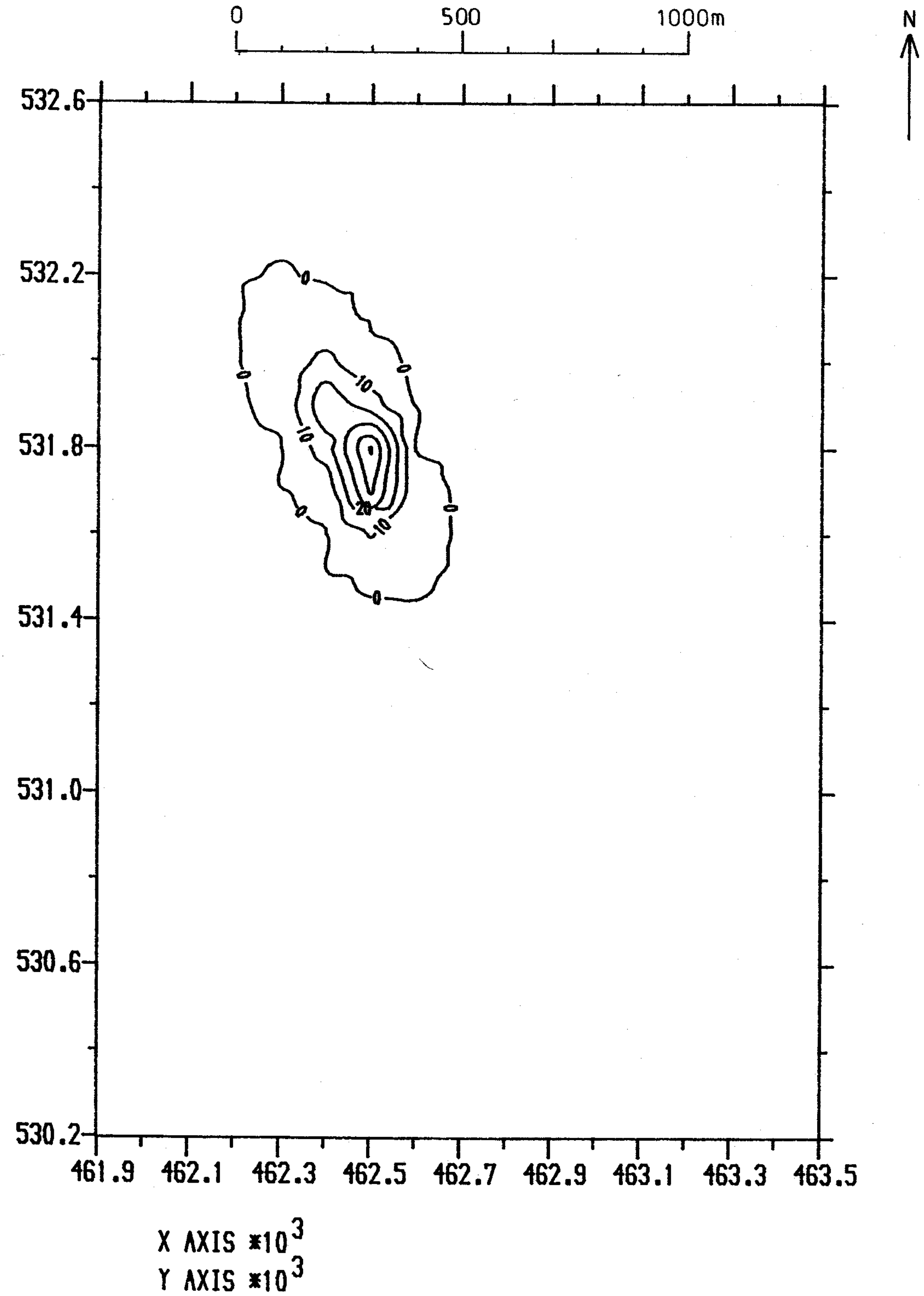


Fig 30 Data from Tees used in model : Direction of velocity



Mass was dumped 8.0 hours after high water, at point 462100,532400

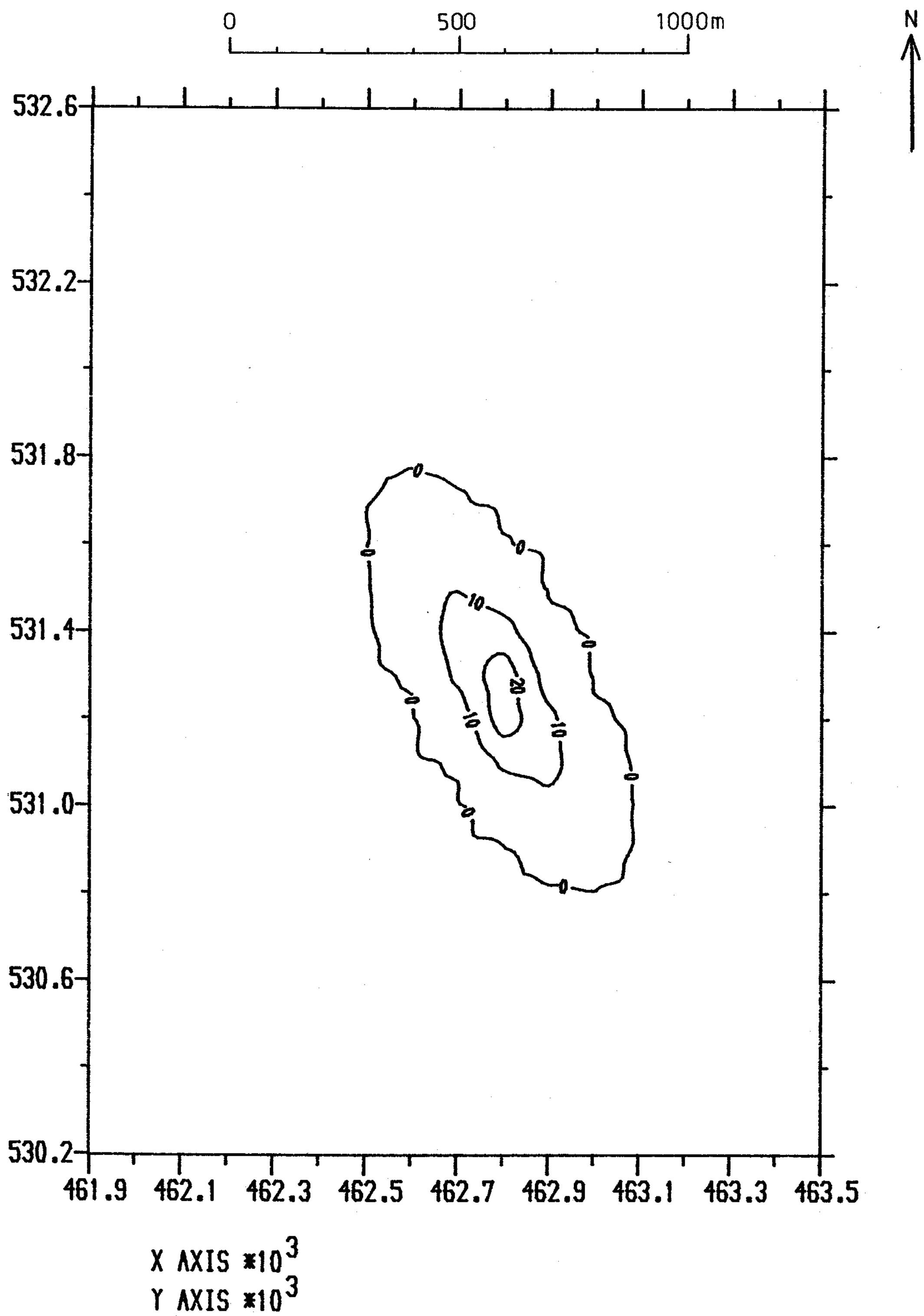
Initial concentration at starting point was 200.0 ppm

Time since dumping is 0 hours, 33 minutes

DX = 5.00 m*m/s DY = 0.50 m*m/s

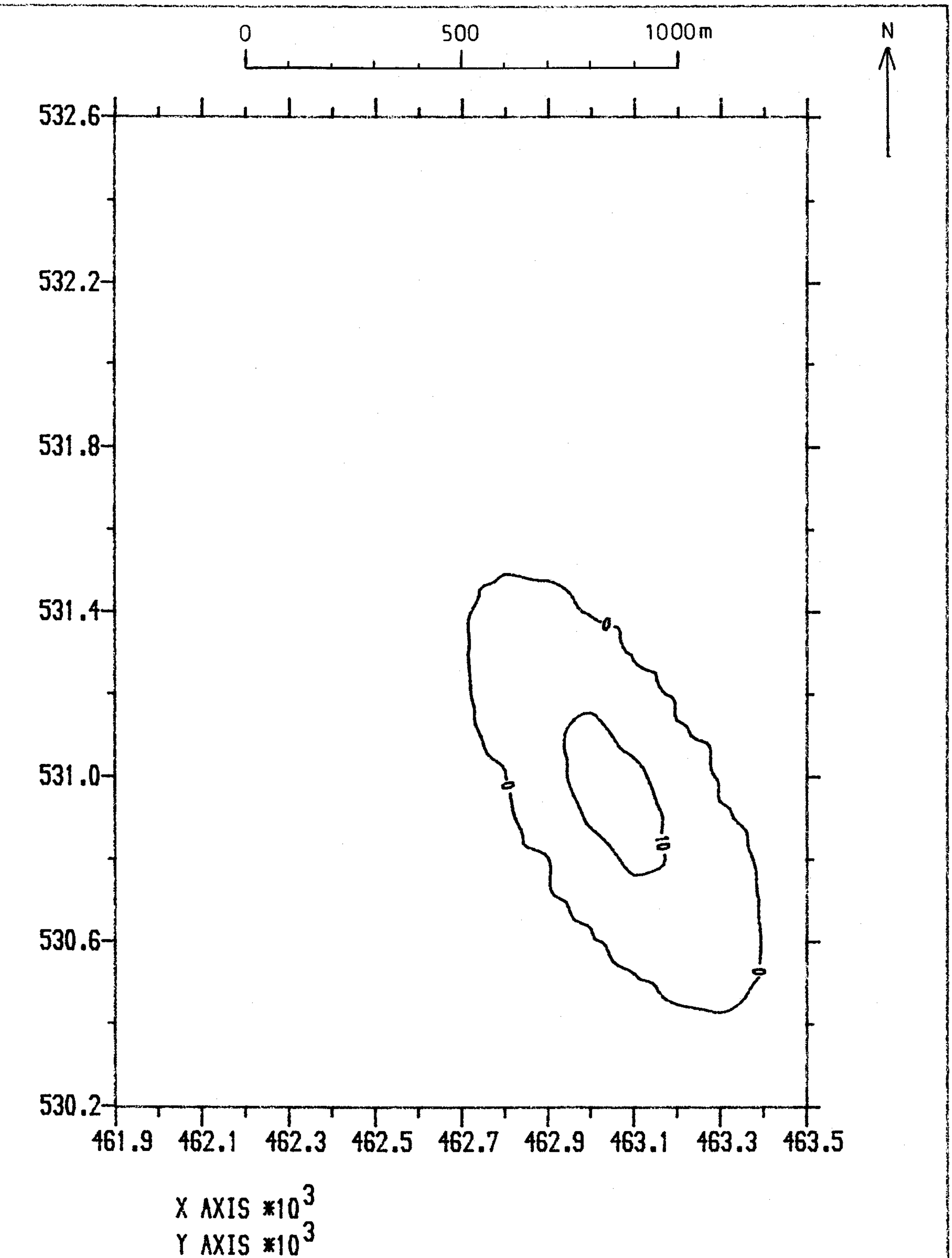
Contour interval is 10.0

Fig 31 Plume model with input data from the Tees estuary
Concentrations in suspension (in ppm) 33 minutes after disposal



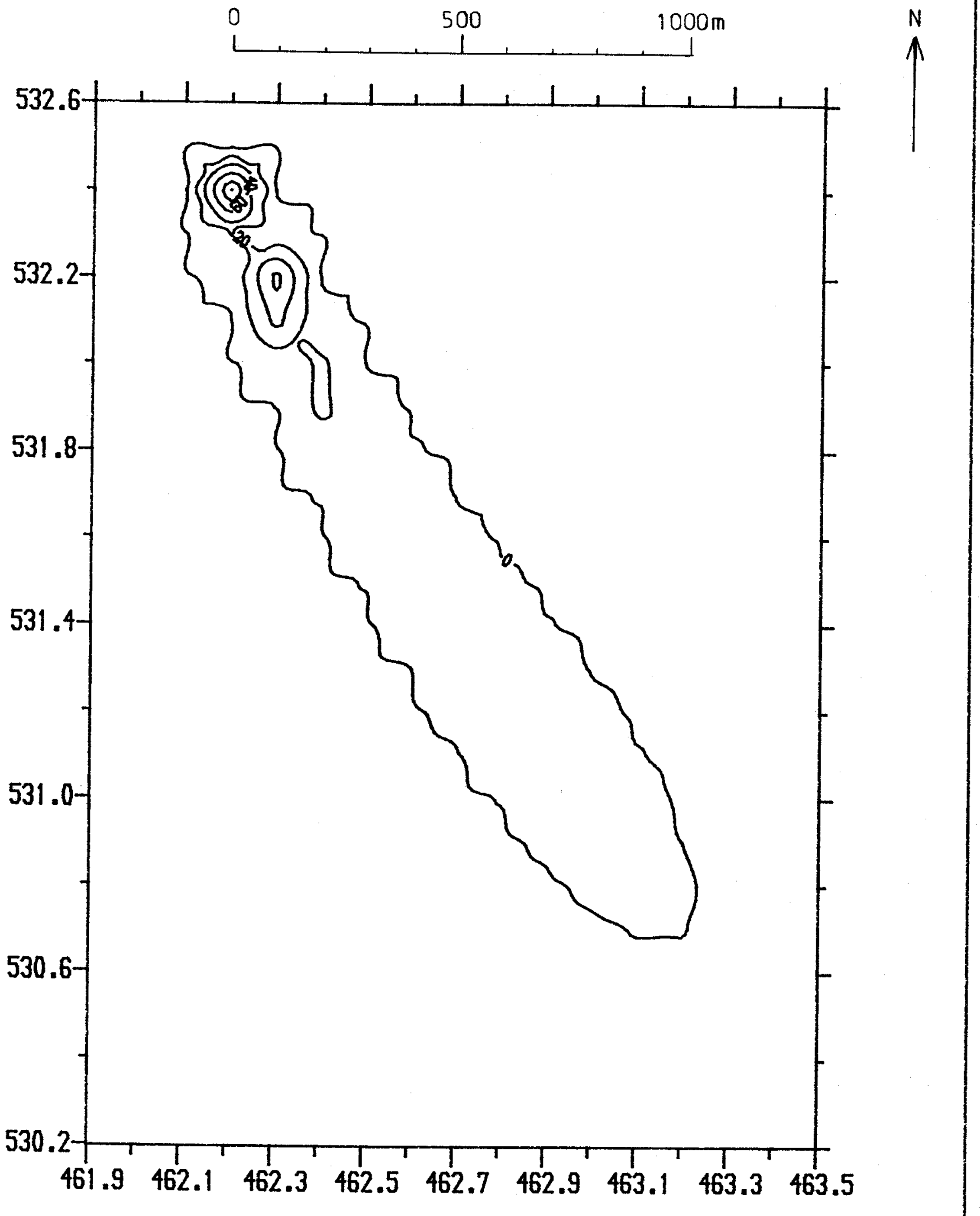
Mass was dumped 8.0 hours after high water, at point 462100,532400
Initial concentration at starting point was 200.0 ppm
Time since dumping is 1 hour , 3 minutes
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 10.0

Fig 32 Plume model with input data from the Tees estuary Concentrations in suspension (in ppm), 1hour 3minutes after disposal



Mass was dumped 8.0 hours after high water, at point 462100,532400
Initial concentration at starting point was 200.0 ppm
Time since dumping is 1 hour , 23 minutes
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 10.0

Fig 33 Plume model with input data from the Tees estuary
Concentrations in suspension (in ppm) 1 hour 23 minutes after disposal



X AXIS *10^3 CONTOUR HEIGHT *10^{-5}
Y AXIS *10^3

Mass was dumped 8.0 hours after high water, at point 462100,532400
Initial concentration at starting point was 200.0 ppm
Time since dumping is 1 hour , 23 minutes
DX = 5.00 m*m/s DY = 0.50 m*m/s
Contour interval is 0.0002

Fig 34 Plume model with input data from the Tees estuary
Distribution of mass on bed after 1 hour 23 minutes

APPENDIX.

APPENDIX A

A general convolution which spreads out a function $f_o(x,y)$ can be written:

$$f_1(a,b,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_o(x,y)S(a-x,b-y,t)\ dx\ dy \qquad \text{(A1)}$$

where S is the spreading function which is convolved with the old distribution f_o to obtain the new distribution f_1

For this particular model, the spreading function is

$$S(a,b,t) = \frac{1}{4\pi t(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-(a-ut)^2}{4tD_x} - \frac{b^2}{4tD_y}\right\}$$

$$\exp\left\{\frac{-W_s t}{d}\right\} \qquad \text{(A2)}$$

This is equation 3 of section 2.3.1, written in terms of mass. The gaussian parts are normalised so that the overall change in total suspended mass depends only on the decay term.

$$\text{ie} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S(a,b,t)\ da\ db = \exp\left\{\frac{-W_s t}{d}\right\}$$

The convolution is now

$f_1(a,b,t)=$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[f_o(x,y) \frac{1}{4\pi t(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-(a-x-ut)^2}{4tD_x} - \frac{(b-y)^2}{4tD_y}\right\}\right.$$

$$\left.\exp\left\{\frac{-W_s t}{d}\right\}\right] dx\ dy \qquad \text{(A3)}$$

In order to justify the combination of solutions in Section 2.3.2 we need to show that f_2 obtained by putting t=2T and performing one convolution is the same as f_{11} obtained by putting t=T and convolving to obtain f_1, then performing a second convolution on f_1 to obtain f_{11}.

i.e $f_2(a,b,2T) = f_{11}(a,b,T)$

The initial distribution of mass $f_o(x,y)$ is a point release which can be represented by a delta function $\delta(x,y)$, which has the value 0 except at (a,b)=(0,0) where it has the value 1.

First consider the single convolution with t=2T:

$$f_2(a,b,2T)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta(x,y)\frac{1}{4\pi(2T)(D_xD_y)^{\frac{1}{2}}}$$

$$\exp\left\{\frac{-(a-x-2uT)^2}{4(2T)D_x}\quad\frac{-(b-y)^2}{4(2T)D_y}\right\}\exp\left\{\frac{-W_s 2T}{d}\right\}dx\,dy \quad (A5)$$

Because of the properties of the delta function, this is just

$$f_2(a,b,2T)=\frac{1}{8\pi T(D_xD_y)^{\frac{1}{2}}}\exp\left\{\frac{-(a-2uT)^2}{8TD_x}-\frac{b^2}{8TD_y}\right\}$$

$$\exp\left\{\frac{-2W_sT}{d}\right\} \quad (A6)$$

Next consider the successive convolutions with t=T:

The first convolution, with x and y integrated from $-\infty$ to $+\infty$ is:

$$f_1(a,b,T) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta(x,y)\, \frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}}$$

$$\exp\left\{\frac{-(a-x-uT)^2}{4TD_x} \quad \frac{-(b-y)^2}{4TD_y}\right\} \exp\left\{\frac{W_s t}{d}\right\} dx\, dy \qquad \text{(A7)}$$

$$= \frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-(a-uT)^2}{4TD_x} - \frac{b^2}{4TD_y}\right\} \exp\left\{\frac{-W_s T}{d}\right\} \qquad \text{(A8)}$$

as $\delta(x,y)=0$ except at $x=0$, $y=0$.

The second convolution is

$$f_{11}(a,b,T) = \iint f_1(x,y,T)\, S(a-x,b-y,T)\, dx\, dy$$

$$= \iint\left[\frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-(x-uT)^2}{4TD_x} \quad \frac{-y^2}{4TD_y}\right\}\right.$$

$$\exp\left\{\frac{-W_s T}{d}\right\} \quad \frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-(a-x-uT)^2}{4TD_x} \quad \frac{-(b-y)^2}{4TD_y}\right\}$$

$$\left.\exp\left\{\frac{-W_s T}{d}\right\}\right] dx\, dy \qquad \text{(A9)}$$

Rearranging the right hand side gives

$$f_{11}(a,b,T) = \iint\left[\frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-2W_s T}{d}\right\}\right.$$

$$\left.\frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-(x-uT)^2}{4TD_x} - \frac{(a-x-uT)^2}{4TD_x} \quad \frac{-y^2}{4TD_y} \quad \frac{-(b-y)^2}{4TD_y}\right\}\right]$$

$$dx\, dy \qquad \text{(A10)}$$

Make the substitutions:

$$p = \sqrt{2}x - \frac{a}{\sqrt{2}}, \quad q = \sqrt{2}\,y - \frac{b}{\sqrt{2}}$$

$$\text{Then } \frac{dp}{dx} = \sqrt{2} \quad , \quad \frac{dq}{dy} = \sqrt{2} \qquad \text{(A11)}$$

The integral limits are unchanged as integrating x from $-\infty$ to $+\infty$ means that p is integrated over the same range. Similarly for q.

$$\text{Now } p^2 = 2x^2 - 2ax + \frac{a^2}{2}$$

and with some rearrangement

$$p^2 + \frac{(a-2uT)^2}{2} = (x-uT)^2 + (a-x-uT)^2 \qquad \text{(A12)}$$

$$\text{Similarly } q^2 = 2y^2 - 2by + \frac{b^2}{2}$$

$$\text{and } q^2 + \frac{b^2}{2} = y^2 + (b-y)^2 \qquad \text{(A13)}$$

Using Equations A12 and A13 in the integrand of Equation A10 and making the substitution, we get

$$f_{11}(a,b,T) = \int_{q=-\infty}^{\infty} \int_{p=-\infty}^{\infty} \left[\frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-2W_s T}{d}\right\} \right.$$

$$\left. \frac{1}{4\pi T(D_x D_y)^{\frac{1}{2}}} \exp\left\{\frac{-p^2}{4TD_x} - \frac{(a-2uT)^2}{4TD_x} - \frac{q^2}{4TD_y} - \frac{b^2}{2(4TD_y)}\right\} \right.$$

$$\frac{dp}{\sqrt{2}} \frac{dq}{\sqrt{2}} \qquad \text{(A14)}$$

Taking the constant factors outside the integral and re-arranging this equation to separate the variables gives: