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Wallingford

WAVE PREDICTION IN DEEP WATER AND  
AT THE COASTLINE

A review of recent techniques of predicting,  
analysing and modelling wave conditions

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## SUMMARY

This report is a review of methods of prediction, analysis and modelling of wave conditions in deep water and at inshore locations. It is intended for coastal engineers as an outline of the methods currently available for predicting wave conditions, with emphasis on the most recent techniques.

The report is in two parts. Part 1 contains a description of wave generation at sea and techniques for analysing measured wave data and predicting wave conditions from wind records. Part 2 contains a description of the shallow-water processes affecting waves as they travel from deep water to the coast, together with a review of the computational techniques for modelling these wave transformations.

The main topics covered in Part 1 are:

1. The physical processes involved in transferring energy from the wind to water waves;
2. A description of irregular sea states and methods of analysis of recorded wave traces;
3. Empirical methods and computational models for calculating wave conditions corresponding to given wind conditions;
4. Methods of statistical extrapolation of wind and wave data to predict wave conditions in very severe storms.

The main topics covered in Part 2 are:

1. A description of the main physical processes affecting waves in shallow water;
2. A description of present computational modelling techniques of wave transformations in coastal areas, and an assessment of the range of applicability of each type of computational model;
3. Examples of the use of shallow-water computational models in different types of coastal engineering problem.



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## GENERAL INTRODUCTION



# 1 GENERAL INTRODUCTION

Until quite recently, coastal engineering problems such as the design of sea walls, prediction of beach changes, and siltation of navigation channels, have usually been treated by experience gained at other sites and the use of a few empirical formulae. In the last decade or so it has become possible to adopt a more scientific approach to the treatment of these problems. There are a number of reasons for this change of approach. Since the Second World War, wave-rider buoys and seabed-mounted pressure-sensing wave recorders have been developed and now provide a reliable means of recording wave data. The theoretical understanding of waves, and in particular the statistical analysis of irregular waves, has advanced greatly over the past twenty years. Perhaps most important of all, the development of high-speed digital computers has made possible the analysis of large amounts of wind and wave data, and the computational modelling of complex wave processes.

In addition to these technical developments, the stringent economic climate in the UK in the late nineteen-seventies and eighties has made the provision and maintenance of sea defences a considerable financial burden on water authorities and local councils. The unnecessary capital costs of an over-designed structure, or maintenance costs of an under-designed structure, can no longer be borne easily. It is therefore essential in the present economic climate to undertake detailed and reliable investigations of the design of coastal protection works in order to optimise their cost-effectiveness.

An indispensable part of almost all coastal engineering problems is the prediction of wave conditions at the site. Very often, these wave conditions are required as input to further studies. For instance, in designing a sea wall the predicted wave conditions during a severe storm can be reproduced in a model wave flume to determine the amount of wave overtopping and wall damage for a variety of seawall designs. Similarly, wave conditions determined at points along a navigation channel provide input to a subsequent physical or computational model investigation of the infill rate of suspended sediment.

This report is concerned with the fundamental problem of predicting wave conditions at inshore locations. The method used for this wave prediction depends on the type of coastal engineering problem being

considered. Listed below are the main types of problems commonly encountered by coastal engineers.

- (i) Design of harbour and coastal defence structures such as sea walls, breakwaters, dykes and groynes;
- (ii) Real-time prediction of wave heights and extreme water levels for coastal flood warning;
- (iii) Longshore and onshore-offshore movement of beach material,
- (iv) Prediction of wave conditions at the entrances to harbours, and subsequent wave behaviour inside harbours;
- (v) Manoeuvring of ships in navigation channels;
- (vi) Infill of navigation channels by seabed material;
- (vii) Assessment of the possible effects of offshore dredging.

The type of wave information needed is different for different problems. For the design of a coastal defence scheme or breakwater, it is the wave conditions during extremely severe storms that are of interest. Such structures have to be designed to withstand, with minimal damage, extreme storms which occur on average only, say, once in fifty years. A reliable prediction of wave conditions during such storms is necessary to determine the design strength of the structure and the quantity and type of artificial armouring against wave attack.

The prediction of movement of beach material and sedimentation of channels, on the other hand, requires knowledge of less severe storms and the day-to-day wave climate. The movement of beach material and bed sediment displays considerable sensitivity to the duration, height and period of the waves. For these problems a knowledge of the frequency of occurrence of the full range of wave heights and periods is therefore required.

Harbour design problems can include the choice of overall layout, extensions to breakwaters, structural design of walls and piers, siltation, strength of moorings and fenders, and motion of moored ships. Although harbour engineering is properly considered as a separate subject in its own right, a thorough

investigation of harbour design problems relies on accurate predictions of wave conditions within the harbour, which in turn requires knowledge of waves at the entrance to the harbour. As in the previous examples, the type of wave information required depends on the problem. Structural design problems require an analysis of waves during severe storms, an assessment of 'downtime' would consider more moderate wave conditions, and problems involving ship motions often require an analysis of very long period waves.

In many problems, such as the design of coastal structures and harbours, wave conditions are required at only one site. In others - beach movement, navigation channels - wave conditions are required along considerable stretches of coastline or channel length. In these latter cases, wave predictions have to be made at regular spatial intervals in the sea area being considered.

In the topics discussed so far, the calculation of wave conditions can be carried out at any convenient time. For example, the prediction of extreme waves can be incorporated into the early stages of design of a sea wall. Similarly, the wave conditions which give rise to the infill of a navigation channel can be calculated retrospectively.

In contrast there is sometimes a requirement for forecasting in "real-time" i.e. to predict the wave conditions likely to occur in the following few days (eg, for ship routeing) or the following few hours (in connection with coastal flood warning). Although considerable effort is being spent on real time wave predictions for deep water, they are rarely carried out at present for coastal areas, although there is an obvious value in improving coastal flood warning systems. It is envisaged that with the installation of computer systems and rapid data transmission links, these predictions could be carried out routinely at local water authority offices.

It can be seen, therefore, that the type of wave conditions required at the coastline, and hence the type of wave modelling needed to predict these conditions, depends on the engineering problem. This report describes several different wave models for achieving this objective. In order to predict shallow-water wave conditions, however, it is first necessary to consider wave action in deep water, i.e. where the effects of the seabed on wave propagation are small compared with those of the wind. Part 1 therefore deals with methods currently used to predict waves in deep water.

As waves approach the shore from deep water, the decreasing water depth becomes more important and eventually shallow water effects dominate the behaviour of the waves. These shallow water effects include refraction, shoaling, diffraction, bottom friction, wave breaking, and reflections from structures and deep channels. In Part 2 these processes are described in some detail and a full account is given of how they are modelled computationally.

PART 1

DEEP-WATER WAVE CONDITIONS





## 2 INTRODUCTION TO PART 1

The determination of wave conditions offshore in deep water is a necessary preliminary stage to the subsequent calculation of shallow-water wave conditions at an inshore site. By "deep-water" is meant water of sufficient depth that the effects of the seabed on wave behaviour are negligible. Water whose depth is greater than about half the wavelength is generally accepted as deep water. For some purposes this is a conservative value, and seabed effects will sometimes not become important until the water is considerably shallower. For example, at a depth equal to a quarter of a wavelength there is only an 8% difference in wavelength compared with its value in deep water.

In deep water the main effect influencing wave behaviour is the wind blowing over the sea surface. In Section 2.1, the physical processes involved in transferring energy from the wind to the water waves are discussed. A description of the irregular sea states resulting from these wave generation processes is given in Section 2.2. There follows in Section 2.3 an account of the various empirical methods and computational models for calculating wave conditions corresponding to given wind conditions. Finally, in Section 2.4 it is shown how wind and wave data can be extrapolated to predict wave conditions for very severe storms.

## 3 GENERATION OF WAVES BY WIND

Although there are a number of unusual sources of wave action, such as landslides, earthquakes and heavy rainfall, the only important waves around UK shores are those generated by wind blowing over the sea surface. These waves are not always generated close to the site of interest; waves generated up to thousands of miles out to sea can travel with very little loss of energy to a coastal location.

The mechanism by which wind energy is transferred to the sea to form surface waves is highly complex and as yet not completely understood. However, three broad processes have been identified.

- (1) The flow of air over the sea exerts a tangential stress on the water surface resulting in the transfer of energy to the water and the formation and growth of waves. This process is dominant in the very early stages of wave growth;

- (ii) The air flow is usually turbulent within a few metres of the sea surface. Wind eddies are formed and these create a rapidly varying (in space and time) pressure and shear stress on the water surface. When these changes in pressure and shear stress "match" (i.e. have the same length and velocity) any existing water waves, these waves will be amplified. This process occurs once water waves have been established;
- (iii) When waves of a certain size have been created, the wind can add further energy to the waves by "form drag" i.e. by exerting greater force directly on the rear (upwind) side of the crest than on the front face which is sheltered. The maximum wave growth will be obtained when the average wind speed matches the speed of the water waves.

The sea state during a storm is always short-crested and irregular. This type of sea state can be considered as the superposition of a large number of regular sinusoidal waves with different periods and travelling in different directions. The second of the wave generation processes mentioned above implies that, because of turbulent eddies in the wind, waves with different periods and directions can be created even when the wind has a constant mean speed and direction. Usually, of course, the average wind strength and direction do both vary with time, making the process of wave growth even more complex.

Waves created in this way at or close to the site of interest form an important part of the deep-water wave field. These waves are generally referred to as "wind waves" or "storm waves". Their periods are usually within the range 2s to 16s.

However, as mentioned earlier, when an inshore site faces the open ocean, waves generated in storms at a great distance can travel with very little loss of energy to the coast. These types of waves, referred to as "swell waves", can also form a significant part of the deep-water wave field.

Swell waves typically have quite a different character to storm waves due to the long distances that they have travelled. These types of waves can easily be recognised by their long crests and nearly regular period. Waves resulting from a storm will travel away from the generation area in much the same way that waves radiate from a point where a stone has been thrown into a pond. At a large distance from the

generation area, therefore, the waves will appear to be almost uni-directional and long-crested. These waves will, moreover, usually have only a narrow range of periods. The reason for this is that waves of different period travel at different speeds as they propagate away from the generation area. The longer period waves travel more rapidly than the shorter period ones. Therefore, although originally all wave components of different periods exist together in the generation area, in time they separate from one another with the longest period components reaching a distant site first, followed by the shorter period components up to several days later. The periods of swell waves actually tend to be somewhat longer, say 12s to 25s, than those in locally generated storms. There are two reasons for this. Firstly, there is a certain amount of interaction between wave components as they begin to travel outwards from the generation area. This has the effect of transferring energy from the shorter period components to the longer period ones. Secondly, the shorter period components tend to lose their energy through viscous effects more readily than the longer period waves. This energy loss to short period waves can be quite significant when these waves have travelled large distances.

A good illustration of the characteristics of swell waves is provided by the paper of Barber and Ursell (Ref 1). These authors carried out some pioneer research into the propagation of swell waves which they recorded on the north coast of Cornwall at Pendeen and Perranporth. By observing the reduction in the period of swell waves arriving at the coast over several days they were able to deduce the distance that the waves travelled. In this way they identified wave trains which had been generated from as far away as the South Atlantic, a distance of about 6000 miles.

This was probably an untypical occurrence, and certainly swell from this distance does not contribute significantly to wave conditions on the Atlantic coast of the UK. The major importance of this observation was rather to demonstrate how far long period swell waves can propagate before being destroyed by viscous effects. Swell from the mid and north Atlantic form a far more significant part of the wave climate on the western coasts of the UK, and Barber and Ursell identified these types of swell waves too. Recent observations of waves off the Outer Hebrides carried out by the Institute of Oceanographical Sciences at Taunton indicate that in this area over 60% of the wave energy can be attributed to swell (Ref 2).

#### 4 DESCRIPTION OF AN IRREGULAR SEA STATE

Most early computational models of wave behaviour only considered waves with a regular sinusoidal shape and a single period travelling in a single direction (Fig 1). Until the nineteen-seventies these were also the only types of waves that were designed to be generated by wavemakers in physical models. Although these types of waves can sometimes be a fairly good representation of swell waves, it was indicated in the previous section that storm waves form a quite irregular surface pattern.

Most modern physical and computational wave models make use of a description of such an irregular sea state. Before considering how these models work, it will be necessary to show how an irregular sea state can be measured and described. Figure 2 shows a typical wave trace that a wave measuring device would record. Usually such traces are not obtained continuously throughout the period of deployment since this would provide quantities of data too large to be readily analysed. Instead, wave records are obtained at set intervals for a specified recording period. Typical values are three hours for the interval between recording periods, and twenty minutes for the length of a recording period.

Two main techniques have been developed for analysing such wave traces. The technique of spectral analysis is the more comprehensive of the two but requires considerable computational effort. However, with the rapid development of modern computers, this analysis can now be carried out quickly and routinely (Ref 3). The spectral analysis method is nowadays the preferred and more commonly used technique. However, it is important also to acquire an understanding of the alternative technique, i.e. wave counting analysis. Although this method is being superseded by spectral analysis, it provides a good introduction and some "feel" for the statistical concepts used in both methods. Wave counting analysis is still of use where a computer is not available and quick results are needed.

##### 4.1 Wave counting analysis

To begin with, it is necessary to define what is meant by wave crests and troughs, wave height and wave period for traces of irregular sea states (see Fig 2). A wave crest is any maximum which occurs in the wave trace, and a trough is any minimum. The wave period can be defined in several ways, for example as the

time interval between two wave crests. A more commonly used definition is the time interval between successive crossings of the mean water level by the water surface in a downward direction (Fig 2). These crossings are known as "zero down-crossings". Sometimes a definition of wave period involving "zero up-crossings" is used; this definition will result in a very similar analysis. The zero-crossing wave height is defined as the difference in water surface elevation between the highest crest and lowest trough that occur between successive zero down-crossings. For reasons discussed later in this section this is the most commonly used definition of wave height. It can be seen from Figure 2 that the wave height and period will change from wave to wave (unlike the regular sinusoidal wave, where the wave height and period are constant with time). The aim of the wave counting analysis is to determine a number of average statistical quantities (such as wave height and period) for a particular recording period. For the analysis to be valid, the wave trace must be sufficiently long; a fifteen minute to half-hour wave record, which is usually several hundred times a typical wave period, is an adequate length. These average statistical quantities are assumed to be representative of the sea state during a length of time equal to the interval between successive recording periods.

The procedure involves counting the total number of zero down-crossings and wave crests in the trace. For each zero down-crossing interval the wave height and wave period are recorded. From these data, a number of statistical quantities can be calculated. Among the most important are:

| <u>Symbol</u>      | <u>Name</u>             | <u>Definition</u>                                                     |
|--------------------|-------------------------|-----------------------------------------------------------------------|
| $\bar{H}$          | Mean wave height        | The mean of all the measured wave heights                             |
| $H_{\max}$         | Maximum wave height     | The largest wave height on the record                                 |
| $H_{\text{rms}}$   | RMS wave height         | The root-mean-square of all the measured wave heights                 |
| $H_s$ or $H_{1/3}$ | Significant wave height | The mean height of the largest one-third of all measured wave heights |

| <u>Symbol</u> | <u>Name</u>           | <u>Definition</u>                                                                                  |
|---------------|-----------------------|----------------------------------------------------------------------------------------------------|
| $H_{1/n}$     |                       | The mean height of the largest 1/n of all measured wave heights (n = 10 and 100 are common values) |
| $T_z$         | Zero-crossing Period  | The mean zero down-crossing (or up-crossing) period                                                |
| $T_c$         | Wave Crest Period     | The mean period between adjacent wave crests                                                       |
| $\zeta$       | RMS Surface Elevation | The root-mean-square of all recorded surface elevations                                            |
| $\epsilon$    | Spectral width        | A measure of the range of wave frequencies present                                                 |

$\epsilon$  is related to the parameters  $T_c$  and  $T_z$  by

$$\epsilon^2 = 1 - (T_c/T_z)^2 \quad (1)$$

The significant wave height has been found to be very similar to the estimated visual height that an experienced observer of an irregular sea state would report.

A number of approximations and assumptions can be made in the counting analysis to make it amenable to relatively quick analysis by hand. Draper (Ref 4) and Tucker (Ref 5) describe some methods for achieving this. Some of these approximations stem from the discovery that wave heights on many wave traces correspond to a Rayleigh distribution:

$$P(H) dH = \frac{2H}{H_{rms}^2} \exp \left[ -H^2/H_{rms}^2 \right] dH \quad (2)$$

in which  $P(H)$  is the probability that the wave height exceeds a value  $H$ . The Rayleigh distribution can be used to relate some of the average statistical quantities to each other thereby simplifying the counting analysis. The most important relations are

$$H_{rms} \approx 2.7 \zeta \quad (3)$$

$$H_s \approx 1.41 H_{rms} \approx 3.8 \bar{\zeta} \quad (4)$$

$$H_{1/10} \approx 1.27 H_s \approx 1.80 H_{rms} \quad (5)$$

$$H_{1/100} \approx 1.67 H_s \approx 2.36 H_{rms} \quad (6)$$

$$H_{max} \approx [(\ln N)^{\frac{1}{2}} + 0.289 (\ln N)^{-\frac{1}{2}} - 0.247 (\ln N)^{-3/2}] H_{rms} \quad (7)$$

where  $N$  is the number of zero crossings on the wave trace.

If computer models, rather than hand analysis, are employed there is little to be saved by using these approximate relationships. However, the Rayleigh distribution has another important use in extrapolating wave heights on measured traces to extreme wave height values. The Rayleigh distribution will give the probabilities of occurrence of very large wave heights.

The Rayleigh distribution of wave heights can be derived theoretically assuming that the spread of wave periods is small. An equivalent way of regarding this assumption is that there should be no crests below the mean water level in a wave trace, and no troughs above it. If this is the case,  $T_c$  will be equal to  $T_z$  and the spectral width parameter  $\epsilon$  will be zero (Eq 1). In real wave traces, of course, there will generally be some crests below mean water level and some troughs above, giving an  $\epsilon$  between zero and one. A more general distribution than the Rayleigh has been derived for arbitrary  $\epsilon$ . However this distribution will give information only on the surface elevation and not on the wave heights.

The definition of a wave height from a trace as the zero crossing height is an attempt to define the wave height in such a way that Rayleigh statistics will apply for most real sea states. In effect, this definition attempts to ignore crests below and troughs above mean water level. Using the zero crossing definition of wave height, the theoretical relationships derived from the Rayleigh distribution (such as  $\bar{\zeta}/H_{rms}$  and  $H_s/H_{rms}$ ) agree well with the values determined directly from the trace.

Although most wave traces have been found to obey a Rayleigh distribution of wave heights in deep water, the same is not true for shallow water. The reason is that shallow water effects will alter the distribution of wave height. For instance, waves higher than a certain limit will start to break, thereby truncating the Rayleigh distribution at the high wave height end.

Although the average wave heights and periods provided by counting analysis are sometimes sufficient for structural design problems, in some situations these average quantities can be highly misleading. An example of this is provided by Figure 3 which will be discussed in the following sub-section. In other applications, such as the response of ships to wave activity or the movement of beach and seabed material, it is essential to know the distribution of wave energy with wave frequency. In these applications the spectral analysis technique is needed.

#### 4.2 Wave spectra and spectral analysis

Unlike wave counting analysis, the spectral analysis technique determines the distribution of wave energy with wave frequency (inverse of period) as well as various average statistical quantities. Spectral analysis uses the mathematical technique of Fourier transformations for converting a wave trace of sufficient length (a measure of wave energy against time) into a "wave spectrum" (a measure of wave energy against wave frequency). A graph of wave energy against frequency is known as the "frequency spectrum", and the function describing the variation of wave energy with frequency is known as the "frequency spectral function" and is denoted by  $S(f)$ . It is conventional to express the spectral function in terms of frequency rather than period. Figures 3 and 4 show examples of frequency spectra.

The principal assumption underlying the technique of Fourier analysis is that any irregular sea state can be regarded as a superposition of a (usually large) number of regular sinusoidal waves each with different frequencies. The wave height,  $H$ , associated with a single frequency,  $f$ , can be calculated from the frequency spectrum by the integral

$$H^2 = 4 \int_{f-\delta f/2}^{f+\delta f/2} S(f) df \quad (8)$$



where  $\delta f$  is a small increment in frequency.

The wave energy associated with the same frequency is determined from

$$E = \frac{1}{8} \rho g H^2 \quad (9)$$

where  $\rho$  is the density of water and  $g$  is the gravitational acceleration.  $E$  represents the total energy density (i.e. energy per unit area of sea surface) for all wave frequencies in the narrow range  $f - \delta f/2$  to  $f + \delta f/2$ . This is shown pictorially by the narrow strip in Figure 4.

If the increment  $\delta f$  is sufficiently small, the total energy in the range  $f - \delta f/2$  to  $f + \delta f/2$  can be represented to a good degree of accuracy by regarding this energy as concentrated in a single regular sinusoidal wave with frequency  $f$ . This process of discretisation can be carried out for the remainder of the spectral function, and the full spectral function can be considered as a superposition of all these discretised components.

The computations involved in spectral analysis are sufficiently complex to require a computer. A discretised form of the wave trace is supplied to the computer program, and a frequency spectrum, again in discretised form, is calculated. Very efficient computer programs are available for this (Ref 3) and nowadays this process is carried out routinely.

Statistical analysis shows that the average quantities such as significant wave height, zero crossing period, etc, are related to the "spectral moments",  $m_n$ , defined by

$$m_n = \int_0^{\infty} f^n S(f) df \quad (10)$$

The approximate relations of the principal average quantities are given below

$$H_s \approx 4.0 \sqrt{m_0} \quad (11)$$

$$H_{1/10} \approx 5.1 \sqrt{m_0} \quad (12)$$

$$T_z \approx \sqrt{(m_0/m_2)} \quad (13)$$

$$T_c \approx \sqrt{(m_2/m_4)} \quad (14)$$

$$\bar{\zeta} = \sqrt{m_0} \quad (15)$$

$$\epsilon^2 \approx 1 - m_2^2/(m_0 m_4) \quad (16)$$

The spectral moments are usually calculated computationally during the Fourier analysis procedure. Equations 11 and 12 are approximate relations in which a Rayleigh distribution of wave heights has been assumed.

Often, when only a spectral analysis of wave traces is carried out, the significant wave height is defined as  $4\sqrt{m_0}$ . In practice, this definition produces a wave height slightly in excess of the significant wave heights derived from a counting analysis. Care needs to be taken, therefore, in specifying how a significant wave height has been defined. The same caution is needed for the other parameters mentioned here. For example,  $(m_0/m_2)^{1/2}$  is only an approximation to the average zero crossing period obtained from a counting analysis, so care is needed in defining  $T_z$ . The peak period,  $T_p$ , (i.e. the period at which the largest wave energy occurs) however, can only be derived satisfactorily by spectral analysis. Since it is conventional to use the same symbol for significant wave height and zero-crossing period ( $H_s$  and  $T_z$ ) in both counting analysis and spectral analysis, it is especially important to state which definition is used.

Figure 3 shows an example where knowledge of the full frequency spectrum rather than average statistical quantities is required. The figure shows two frequency spectra taken from wave recordings in Perran Bay, North Cornwall, both having nearly identical values of  $H_s$  and  $T_z$ . The broken line spectrum is for a swell sea with only a small storm sea component. This spectrum shows a single peak with a regular decay of wave energy at frequencies to either side. The storm sea contribution is shown by the slight irregularities in the curve at high frequencies. For this spectrum the  $H_s$  and  $T_z$  values are a good measure of the average wave height and period. For such spectra the  $T_z$  value occurs at a higher frequency (lower period) than the peak period  $T_p$ . The full-line curve shows a quite different spectrum, consisting of a storm sea and swell with roughly equal energies. For this spectrum there are two peaks. These correspond to the frequencies with maximum energy in

the storm waves and swell waves respectively, with the storm wave peak occurring at the higher of the two peak frequencies. Whereas the  $H_g$  value is still a good representation of an average wave height, it can be seen that the  $T_z$  value gives no indication of the frequencies at which the peak energies occur. Indeed for double-peaked spectra it is misleading to consider any type of average frequency, and a full frequency spectrum should always be determined and used in subsequent calculations.

Unlike recorded wave traces, the spectral function does not, in fact, completely define the sea surface at all points in space and time. Some information is discarded after the Fourier analysis. The spectral function only defines the energy associated with each sinusoidal wave component. A complete specification requires, in addition, knowledge of what stage of the wave profile (i.e. a crest or a trough or some point in between) of each component is present at every position and time. This stage of the wave profile is known as the "phase", and information about the phases of the wave components does not appear in the spectral function. The analysis of an irregular sea state based on spectral functions therefore includes a degree of uncertainty or randomness. Indeed, an irregular sea is sometimes called a "random sea" and models (physical and mathematical) of this type of sea state are called "random wave models". This randomness means that the representative wave heights and period for a spectrum are essentially statistical quantities, in other words, they are some kind of average value over a time interval spanning many wave periods or, equivalently, over a large surface area.

#### 4.3 Combined frequency and directional spectra

Both the wave-counting and spectral analysis techniques make use of recorded wave traces of surface elevation against time. No information on the distribution of wave energy with direction is present in such data. The determination of directional properties of a random sea presents a more difficult problem. Many types of wave recorders have been developed recently to record wave directions as well as the surface elevation, but these instruments have yet to be fully tested and their accuracies assessed.

The representation of a spectrum in frequency and direction is a simple extension of the representation of the frequency spectrum alone. It is assumed that the sea state can be regarded as a superposition of a

large number of regular sinusoidal waves each with different frequencies and directions. The energy associated with each frequency and directional component is denoted by a two-dimensional spectral function  $S(f, \theta)$  where  $\theta$  represents wave direction. The wave height of each individual frequency and directional component is then calculated from the integral

$$H^2 = 4 \int_{f-\delta f/2}^{f+\delta f/2} \int_{\theta-\delta\theta/2}^{\theta+\delta\theta/2} S(f, \theta) d\theta df \quad (17)$$

and the energy density  $E$  is given by Equation 9.  $E$  represents the total energy density for components with frequency in the narrow range  $f - \delta f/2$  to  $f + \delta f/2$  and direction in the range  $\theta - \delta\theta/2$  to  $\theta + \delta\theta/2$ . Figure 5 shows a typical two-dimensional spectrum, and the narrow vertical box in the figure indicates one component.

## 5 WAVE GENERATION MODELS

In principle, the best means of determining wave conditions in deep water would be to deploy one or more wave recorders in suitable deep water locations close to the inshore area of interest. Wave data from the recorder(s) could then be Fourier-analysed at various time intervals, and the resulting spectral functions would give a representation of the deep-water conditions. However, there are a number of disadvantages, and such a method is not commonly used. The first drawback is that wave recorders would need to be deployed for at least a year to determine seasonal fluctuations. If sufficient data is required for severe storm analysis, even longer deployment, say ten years, would be needed. In almost all instances it is impossible to wait for this length of time. In some cases, wave recorders may have been deployed in the past at nearby locations for other purposes and their data could be used again for the current engineering problem. Sometimes use can be made of "data banks", collections of wave data specifically assembled for reuse in subsequent studies. Two such data banks for UK coastal regions and other areas have been compiled by the Marine Information and Advisory Service (MIAS) of the Institute of Oceanographic Sciences (Ref 6) and the UK Meteorological Office (Ref 7). Despite these possibilities, wave recorders are unlikely to have been deployed for more than one or two years at any site and very often much less than this. A further disadvantage is that most recorders do not give information on the directional spectrum.

The most recent recorders are designed to record directional information, but these are expensive to deploy, and have not yet been thoroughly tested.

For many deep-water wave predictions, therefore, recorded wave data are not used directly, and a method of hindcasting wave conditions from records of wind data is used. Wind records provide a far more comprehensive data base than wave records and also contain directional information. Reliable wind records going back many years are kept for a large number of weather stations around the UK coast. Very often, wind speeds have to be multiplied by a "mark-up factor" to account for winds generally being stronger at sea than at coastal weather stations. The UK Meteorological Office has drawn up lists of these mark-up factors for sites all around the UK coast. Empirically derived graphs or, better, computational models of wave generation by wind are then used to determine wave conditions corresponding to a given set of wind conditions. Recorded wave data does play an important role in this process since it can be used to calibrate the wave generation models. For this purpose quite short wave records are adequate; data lasting only a few months is often sufficient.

The earliest types of wave prediction methods used a combination of simple formulae and empirically determined factors to give the wave height and period for a variety of steady wind conditions. Usually these methods are presented as graphs from which the wave height and period can be read off for a range of storm conditions. Among these methods can be mentioned:

- (i) The SMB (Sverdrup-Munk-Bretschneider) method (Ref 8). This method uses empirical expressions derived from a comprehensive series of observations at locations in the North Pacific, North Atlantic and Great Lakes.
- (ii) The Darbyshire-Draper method (Ref 9). This method is based on observations around the UK coast, and is only valid for these areas.
- (iii) The JONSWAP method (Refs 10, 11). This method uses a semi-empirical formula derived from specifically designed experimental observations in the southern North Sea during the Joint North Sea Wave Project (hence the acronym JONSWAP). The JONSWAP method has the advantage over the previous two in that it will determine the spectral function in frequency (though not direction). However, if wave prediction graphs

are used, this spectral information will be lost. Typical wave forecasting curves using the JONSWAP method are shown in Figure 6 (for wave heights) and Figure 7 (wave periods).

- (iv) The Pierson-Moskowitz method (Ref 12). This method uses a simpler version of the semi-empirical JONSWAP formula. It is applicable to equilibrium sea states in which the energy transfer to the waves is balanced by various dissipation processes. Such sea states can occur at sites exposed to the open ocean. As with the JONSWAP method, a spectral function in frequency, but not direction, can be obtained.

In many deep-water applications, such as for determining ship motions or the response of offshore platforms, the full spectrum is required in modern design methods, and not simply an average wave height and period. Furthermore, for predicting inshore wave conditions, a directional spectrum is ideally needed as input to the more recently developed shallow-water wave transformation models. The graphical methods of wave prediction are therefore now being superseded by computational models based on semi-empirical formulae, such as the JONSWAP formula, in which wave spectra are determined. There are several different types of such models, which differ in the way the wave generation process is modelled and in the numerical techniques adopted.

## 5.1 JONSEY model and HINDWAVE model

The JONSWAP formula, written below, is a semi-empirical formula relating the frequency components of the spectral function to a variety of wind parameters.

$$F(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp \left[ -\frac{5}{4} \left( \frac{f}{f_m} \right)^4 + \ln \gamma \exp \left[ -\frac{(f/f_m - 1)^2}{2\sigma^2} \right] \right] \quad (18)$$

In this equation

- $\alpha$  = Phillips' constant
- $f_m$  = frequency at the peak of the spectrum
- $\gamma$  = a peak enhancement factor

$\sigma$  = a peak width parameter  
 $g$  = acceleration due to gravity

A typical graph of  $F$  against  $f$  is shown in Figure 4. Note the high, narrow peak, the rapid fall-away at low frequencies (long periods), and the "tail" at high frequencies (short periods).

The quantities  $f_m$ ,  $\gamma$  and  $\sigma$  are functions of the mean wind speed, duration of the storm and length of sea over which the wind blows (this length is usually called the "fetch"). Although the formula is in terms of three wind parameters, there are in fact only two independent quantities. Storms can be "fetch-limited", "duration-limited", or "fully-developed". "Fetch-limited" refers to the case where a storm has grown to its maximum extent, and further wave growth is prevented by the limited fetch over which the wind can blow. The usual reason for a limitation of fetch is the proximity of land. "Duration-limited" storms are those whose maximum wave growth is limited by the finite duration of the storm. In the JONSWAP formula, if a storm is fetch-limited, the wind speed and fetch are used to calculate  $F(f)$ ; if the storm is duration-limited, the wind speed and duration are used. The computer model based on the JONSWAP method can decide automatically whether a given storm is fetch-limited or duration-limited. A "fully-developed" storm occurs when a storm has grown to its maximum possible extent. In this state the energy input to the waves by the wind is balanced by a reverse transfer of energy from the waves to the air and by internal dissipation processes such as wave breaking and (in shallow water) energy dissipation at the seabed. Storms can become fully developed in areas exposed to the open ocean. For such storms a simpler form of the JONSWAP formula, known as the Pierson-Moskowitz formula is used.

$$F(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp \left[ -\frac{5}{4} \left( \frac{f_m}{f} \right)^4 \right] \quad (19)$$

This is just the JONSWAP formula (Equation 18) with  $\gamma = 1$ . In a fully-developed sea,  $f_m$  has been shown to be related to the mean wind speed by

$$f_m = \frac{0.13 g}{u} \quad (20)$$

where  $u$  is the average wind speed at a height of 10m above mean sea level. Figure 4 shows Pierson-Moskowitz and JONSWAP spectra with identical  $f_m$ . Both

spectra show the same behaviour at high and low frequencies, but the JONSWAP spectrum has a higher and narrower peak at  $f_m$ . The ratio of  $S(f)$  for the JONSWAP spectrum to  $S(f)$  for the Pierson-Moskowitz spectrum at the peak is equal to the peak enhancement factor,  $\gamma$ , in Equation 18.

The JONSWAP formula will only give the wave frequency spectrum,  $F(f)$ , for a given wind and fetch direction. However, it was noted previously in Chapter 3 that all winds, even when blowing steadily in one direction, generate waves at a range of angles to the main wind direction. The JONSWAP formula used on its own is therefore incomplete; a method of determining the directional spread of wave energy is needed.

Field observations and measurements of the directional spread of wave energy during a wave generation process are quite rare and generally not accurate. The reason for this is that it is only very recently that wave recording instruments capable of measuring directional spectra have been developed. Instead, a simple functional form for the directional wave spectrum is used in most computational models.

This expression is assumed to be the same at all frequencies. Mathematically, this means the total spectral function can be written as the product of two one-dimensional functions.

$$S(f, \theta) = F(f) D(\theta) \quad (21)$$

The directional spectral function  $D$  is usually chosen to be of the form  $\cos^2(\theta - \theta_0)$  or  $\cos^6(\theta - \theta_0)$  where  $\theta_0$  is the mean wind direction. Both types have a bell shape with the peak at  $\theta_0$ , but the  $\cos^6(\theta - \theta_0)$  spectrum has a narrower width. These directional spectral functions were suggested in the original JONSWAP project. Other researchers such as Mitsuyasu (Ref 13) have put forward directional spectra which depend on frequency. Mitsuyasu's function gives a narrow directional spread at the peak frequency, becoming wider at frequencies further from the peak.

These methods will provide a directional spectrum. However, they do not take into account any fetch limitations that may exist at an angle to the main wind direction. An improvement, taking into account restricted fetches over a range of directions, is provided by the method of Seymour (Ref 14). In this method a series of radial lines are constructed on a suitable chart from the deep-water point of interest until land or the edge of the chart is reached. These lines are drawn at constant angular increments, say



10<sup>0</sup>. Seymour's method involves using the JONSWAP formula to calculate the frequency spectrum  $F(f)$  for each of these fetch lines to a limit of 90<sup>0</sup> either side of the main wind direction. Each of these frequency spectra  $F(f)$  is then weighted by the appropriate value of the directional spectral function (i.e. the value of  $\cos^2(\theta - \theta_0)$  or  $\cos^6(\theta - \theta_0)$ ). Finally, all these frequency spectra are superposed to give the full spectrum in both frequency and direction. Figure 8 shows the constructions of fetch lines from a point just offshore from Perranporth on the North Cornwall coast. A computer model known as JONSEY has been developed at Hydraulics Research which calculates spectra (in frequency and direction) using the JONSWAP formula and Seymour's method.

When the JONSEY model is run it is not possible to predict in advance the value for the duration of the storm that should be used to give the worst storm conditions. If, for instance, a large value for the duration is taken, say 24 hours, the average wind speed during this period of time will be quite low. If, however, a much smaller duration is taken, say 1 hour centred on the storm maximum, there will be a much larger average wind speed. It is not clear which pair of average wind speed/duration values will give the worst wave heights. In the model this is determined by a trial-and-error process. A model recently developed at Hydraulics Research, known as HINDWAVE (Ref 15) carries out these calculations systematically for long records of wind data by repeated use of the JONSEY algorithm. In a recent study at Seaford on the Sussex coast (Ref 16), wave predictions from the HINDWAVE model using wind data from the weather station at Dungeness were compared with wave heights measured by a wave rider buoy in the area. Figure 9 shows a comparison for the month of January 1984 and there is generally good agreement.

Experience of using the JONSEY and HINDWAVE models has shown that they generally give good deep-water wave spectra in locations where there are limited fetches due to nearby land. Such areas include the southern North Sea, the eastern part of the English Channel and the Irish Sea. The HINDWAVE model is quick in computational time and can therefore be used to analyse large quantities of wind data. The main drawbacks of these models are that they assume a constant average wind field and do not predict swell waves. Quite often, the wind direction does change considerably during a storm, and there can be variations in wind strength (a lull during the build-up of a storm is typical). Swell waves generated in nearby storms can be as important, if not

more so, than distantly generated swell. To overcome these drawbacks more sophisticated models have been developed.

## 5.2 BRISTWAVE model

The BRISTWAVE model (Ref 17) was developed at Hydraulics Research specifically for predicting wave conditions in the Bristol Channel and Severn Estuary. As with the JONSEY model, a single deep-water site of interest is selected and radial fetch lines constructed from this point. The JONSWAP formula is used to calculate the frequency spectra along the fetch lines, and the method of Seymour is used to combine these spectra and produce a directional spectrum. The difference between the two models is that BRISTWAVE contains a more sophisticated modelling of the wave generation process.

Swell waves tend to be generated during a period of decreasing wind-wave activity as a result of decreasing wind strength and/or a change of wind direction. When this occurs, the longer period waves decouple from the shorter period waves and begin to propagate separately as swell. If the wind picks up again, these swell waves can be absorbed back into the newly generated storm wave spectrum. For some ranges of wind speed, swell waves can grow and propagate quite independently of the storm waves. There exist various formulae which describe this interaction of storm and swell waves, and these are incorporated in the calculations for each fetch direction (Ref 18).

The BRISTWAVE model is quite complex and takes a considerable amount of computer time. It cannot at present be used to analyse large amounts of wind data, and is generally only used for predicting wave conditions during particular storms. Nevertheless BRISTWAVE is a valuable model in applications where analysis of severe storm events is required, such as in the design of maritime structures.

## 5.3 Finite-difference models

Both the JONSEY and BRISTWAVE models determine wave conditions at a single point. If wave conditions are required over a wide area, a model which uses a finite-difference representation of the area is required. This means that the area is covered with a rectangular mesh, and calculations of the wave generation are made at each intersection of the mesh. An example of this type of model is the NORSWAM model developed jointly by Hydraulics Research, the Institute of Oceanographic Sciences, and the

Meteorological Office (Ref 18). Another example is the Fine Mesh Wave Model currently used by the Meteorological Office for wave forecasting around the UK shores. This is referred to subsequently as the MOFMW model.

In these types of models very large areas are covered. The MOFMW model considers an area covering the whole of the North Sea, English Channel, Irish Sea and a sizeable portion of the North Atlantic Ocean extending as far north as Iceland and several hundred miles out to the west. The seaward boundary conditions are provided by a coarser wave generation model presently being extended to cover the whole of the world's oceans. The modelling of wave growth is similar to BRISTWAVE except that calculations are carried out at each mesh point (rather than just a single point) for a series of time steps. It is clear that this type of model is very time-consuming to run and cannot be used in a project-specific manner for hindcasting predictions involving large amounts of wind data.

An important present use of the MOFMW model is in the real-time forecasting of deep-water wave conditions around the UK coast. Wind data from weather stations and ships throughout the modelled area are constantly input to the model at regular intervals, and predictions of wave activity up to 36 hours in advance are forecast throughout the entire area. Although these wave conditions are not corrected for shallow-water effects (apart from a simple refraction calculation for the swell waves), they do provide useful storm-wave data, and some local authorities use these predictions in their storm warning services.

The MOFMW model has been in operation from December 1977 and in its present form from February 1983. It uses a grid size of 25km and a time step of 3 hours, and results (full spectra, significant wave heights and zero-crossing periods for all mesh points) are stored at 12 hour intervals. In a few years time, the archived wave data from this model will provide a very useful data set for the general wave climate in offshore areas all around the UK coastline. The present coarseness of the time and space increments means that hindcast predictions for particular storms and particular sites will probably not be sufficiently accurate, and it would be more appropriate to use a project-specific model such as BRISTWAVE. However, with improvements in computer speed and storage, these time and space increments may eventually be reduced to such a level (eg, 10km and 1 hour) that archived wave data from the MOFMW model can be used directly to

provide wave spectra for specific storms going back many years at any site around the UK coast.

In the Seaford study (Ref 16), wave predictions from this model, as well as hindcasting from wind data, were used as input to a shallow-water wave model. Predicted wave heights from the MOFMW model at a grid point close to the Seaford wave-rider buoy were compared with the measured wave heights. Figure 10 shows wave traces for the same period (January 1984) as for the comparison with hindcast wave heights (Figure 9) and agreement is again generally good.

Since the MOFMW model is linked to a coarser grid model covering the whole world, distant swell is incorporated. In none of the other models considered, however, are distant swell waves included. If swell is known to be important at a site, it is usual to treat it separately from the storm wave spectrum, and sometimes separate shallow-water analyses are carried out.

## 6 PREDICTION OF EXTREME WAVES

For many coastal engineering problems a knowledge of extreme wave conditions is required. Breakwaters and coastal defences have to be sufficiently strong to resist wave attack with minimal damage in severe storms. The storm severity which these structures are designed to resist is usually specified as a return period (typically fifty or a hundred years). The design height of flood prevention works has to take into account the joint probability of extreme water levels (high spring tides plus storm surges) and wave heights.

Accurate predictions of these extreme wave conditions cannot be made on the basis of measured wave data lasting for one year or so, although because of limited data such attempts are sometimes made. A length of time of one year is not sufficient to make reliable statistical extrapolations to events which occur on average once every fifty or a hundred years. However, wind data going back about ten or twenty years will usually provide information for a sufficiently large period of time for reliable extrapolations to be made.

There are a variety of methods used to predict extreme waves, but the basic process is the same. The first step is to convert the full set of wind data to the corresponding wave conditions. Because of the large amount of data involved, a hindcasting procedure using the HINDWAVE computational model is appropriate. If

measured wave data are available for part of the period covered by the wind data, a calibration of the HINDWAVE model can be carried out. This procedure has recently been used for the Seaford Study (Ref 16). An alternative procedure, not using a computational wave generation model but making use of measured wave data, is to derive an empirical relation between the wind parameters (speeds, durations and directions) and the measured wave heights and periods. This empirical relation would be derived for moderate storms (for example with wind speeds exceeding 10 knots) during the period of deployment of the wave recorder. This relation can then be used to convert the full wind record to the corresponding wave conditions. This procedure has been used for a study in the Severn Estuary (Ref 19). A third possibility, similar to the previous one, is to select about 40 or 50 recorded storms and use a full storm sea/swell model (such as BRISTWAVE or NORSWAM) for each storm.

Once a set of wave data covering many years has been hindcast or estimated by these methods, a probability distribution needs to be fitted to this data. To do this the wave data are collected into bands of significant wave height. The frequencies of occurrence of significant wave heights within each band are then calculated. These values can then be combined to give the frequency of occurrence of significant wave heights greater than any particular value. Graphs of these frequencies of occurrence are known as exceedance curves. When wave data is presented in this form, it has usually been found to fit a standard statistical distribution function with reasonably good accuracy. Two of the most commonly used distribution functions are the Weibull function and the Fisher-Tippett I function.

1) Weibull function

$$P(x) = \exp \left[ - (x - \alpha) / \beta \right] \quad (22)$$

2) Fisher-Tippett I function

$$P(x) = 1 - \exp \left[ - \exp \left\{ -(x - \gamma) / \delta \right\} \right] \quad (23)$$

In these distribution functions  $P(x)$  is the probability of a wave height being greater than  $x$ , and  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are parameters to be determined by fitting with the data. In the Weibull distribution a plot of  $\ln \ln (1/P)$  against  $\ln (x - \alpha)$  will give a straight line, while in the Fisher-Tippett I distribution, a plot of  $-\ln \{-\ln (1-P)\}$  against  $x$  will give a straight line.

Generally, the wave data are plotted using the Weibull, Fisher-Tippett I and if necessary other distributions, and the one which gives the best fit is used. Once the best distribution has been found, it can be used to estimate any extreme significant wave height. Figures 11 and 12 shows respectively Weibull and Fisher-Tippett I distributions fitted to the same set of hindcast wave data. In this example both distributions appear to give an equally good fit to the data. When these distributions are extrapolated to find extreme significant wave heights, the Weibull distribution will usually give higher waves than the Fisher-Tippett I distribution. The correct distribution cannot be judged but the one giving the higher significant wave heights (usually the Weibull) will be the more conservative.

In this method, extreme significant wave heights corresponding to severe storm events are determined. Assuming that a Rayleigh distribution holds for extreme sea states, an extreme maximum wave height corresponding to the extreme significant wave height can be determined. Extreme wave periods can be predicted in an identical manner. However, it is more usual that the wave period, spectrum and storm duration corresponding to a particular extreme wave height are required. These quantities are difficult to predict, and some very broad simplifying assumptions have to be made. For instance, in predicting the wave period, it is assumed that the wave steepness (ratio of wave height to wavelength) is the same in the extreme storm as in the measured storms. In predicting the full wave spectrum, the same spectral shape (in terms of dimensionless parameters) as measured storms is assumed. Storm durations can be estimated if the measured wave data show a correlation between the storm duration and maximum wave height, which can then be extrapolated. In Figure 13 such a correlation is shown for a site in the Eastern Mediterranean.

The problem of predicting extreme water levels and overtopping rates for defences against coastal flooding is somewhat more complex. This is because the probabilities of occurrence of extreme wave heights and extreme water levels need to be considered jointly. The problem can be broken down into three components.

- (i) Determination of probability distribution of astronomical tides.

- (ii) Determination of probability distribution of residual surge levels (ie, total water level minus the astronomical tidal level).
- (iii) Determination of probability distribution of wave heights.

Astronomical tides are regular events and are not influenced by the other two components. Surge levels and wave heights, however, are both meteorological in origin and can be expected to display some correlation with each other. To determine this correlation, the first step is to compare measured residual surge levels with wind speeds and directions (or directly with wave data if sufficient is available). Since it is known from hindcasting models how wind parameters are related to wave heights and periods, the joint probability distribution of surge levels and wave heights can be calculated. This is then combined with the (uncorrelated) probability distribution of astronomical tidal levels to give a joint probability distribution of total water level and wave heights. If necessary, the joint probability can be extended to wave heights from various directional sectors. This sort of analysis has recently been carried out for a study at Whitstable on the North Kent coast (Ref 20). Alcock (Ref 21) describes joint probability techniques in some detail.





**PART 2**

**SHALLOW-WATER WAVE CONDITIONS**



## 7 INTRODUCTION TO PART 2

In Part 1 of this report, methods of predicting offshore wave conditions were described. Part 2 contains a description of the methods currently available for evaluating the effects of shallow water on these deep-water waves as they travel towards the coast, emphasizing the most recent developments and techniques.

Twenty years ago, available modelling techniques consisted of little more than an estimation of refraction and shoaling effects by the hand-tracing of wave rays. These methods were time-consuming and error-prone and at best gave only a qualitative idea of refraction and shoaling. Modern methods rely on the use of computer models, which are both substantially quicker and more accurate than hand or graphical methods. Moreover a considerable number of other physical phenomena in addition to refraction and shoaling can be incorporated into these computer models.

There now exists quite a variety of computational models, each one catering for different types of shallow-water wave problems. Because of the rapid development of these models over the past few years, and their specialised nature, the choice of which model to use and an assessment of its likely accuracy in a particular problem is increasingly difficult for the practising coastal engineer. The aim of Part 2 is to provide coastal engineers with a review of the latest available models, together with an indication of the type of coastal engineering problem for which each model is most suited.

In Chapter 8 a description of the most important shallow-water wave phenomena is given. In Chapter 9 the most widely used types of computational model for coastal wave propagation are described, including the most recent developments. The advantages and disadvantages of each model are discussed, along with the coastal engineering problems to which they are best suited. A summary of incorporated wave phenomena, advantages and limitations, and parameters determining suitable applications for each model is presented in Tables 1 and 2. Finally, in Chapter 10, three examples are given in which these models have been used in a variety of different coastal engineering problems.

## 8 WAVE PHENOMENA IN SHALLOW WATER

In Part 1, the formation of waves by wind blowing across the water surface was described. It was assumed that the water was sufficiently deep that the seabed would have a negligible effect on the character of the waves. This chapter considers how the seabed affects waves as they travel towards a coast through water of decreasing depth. In real situations, of course, these shallow-water effects do not act independently but combine with the wind to affect the waves. However, in many coastal areas, and particularly around the UK, seabed effects are limited to a fairly narrow coastal strip only a few kilometres wide. Compared with the fetches over which the waves are generated, typically hundreds of kilometres in storm conditions, it is a good approximation to ignore wind effects close to the coast and concentrate on seabed effects alone.

There exists a wide range of shallow-water phenomena that affect waves, but they can be grouped into two distinct classes. The first class of phenomena has the effect of altering the spatial distribution of wave energy and the distribution of wave energy between frequency and directional components in a spectrum. These phenomena, however, do not add to or subtract from the total amount of wave energy travelling towards the coast, and are therefore known as 'non-dissipative'. The most important of these phenomena are shoaling, refraction, diffraction and certain types of reflection. The second class of phenomena are known as 'dissipative' and involve a reduction in the total amount of wave energy as the waves travel shorewards by converting the energy into another form such as water turbulence, heat, or the motion of seabed material. Wave breaking, interaction of waves with the seabed (usually known as 'bottom friction') and wave reflections from sloping or rough-faced structures belong to this category.

These two types of phenomena will be considered in turn, starting with non-dissipative phenomena.

### 8.1 Non-dissipative phenomena

#### 8.1.1 Shoaling

In order to understand the phenomenon of shoaling it is necessary to introduce the concepts of wave energy flux and group velocity. The speed of propagation of wave energy is usually known as the 'group velocity'. It is important to distinguish between the group

velocity of a wave and the speed at which the wave crest travels, known as the 'phase velocity' or 'celerity'. In deep water the group velocity is half the value of the wave celerity for a regular sinusoidal wave. As the water depth decreases, the celerity decreases but the group velocity changes in a more complex fashion. In very shallow water the celerity and group velocity become equal. Usually the wave energy and the wave crest travel in the same direction, but significant differences between the two directions will occur if the phenomenon of diffraction is important or if fairly strong currents are present.

The wave energy flux is defined as the product of the energy density (ie. wave energy per unit surface area) and the group velocity, i.e.

$$Q = Ec_g \quad (24)$$

where  $Q$  is the energy flux,  $E$  the energy density and  $c_g$  the group velocity.

From Eq 9 in Part 1, the energy density is related to the height of a wave component in a spectrum by

$$E = \frac{1}{8} \rho g H^2 \quad (25)$$

where  $\rho$  is the density of seawater,  $g$  is the acceleration due to gravity ( $= 9.81 \text{ms}^{-2}$ ), and  $H$  is the wave height. To understand how shoaling affects the wave height, consider a simple straight parallel-contoured depth profile sloping towards a beach. A regular sinusoidal wave with fixed period is travelling in a direction perpendicular to the seabed contours. Assuming that no dissipative effects take place, the wave energy flux is preserved as the wave travels forward.

Combining Eq 24 and Eq 25,

$$\frac{1}{8} \rho g H^2 c_g = \text{Constant} \quad (26)$$

or, since  $1/8 \rho g$  is constant

$$H^2 c_g = \text{Constant} \quad (27)$$

Since the value of  $c_g$  depends on the water depth, it follows from Eq 27 that the wave height will alter as the wave progresses shorewards, although no energy is

being added to or removed from the waves. Eq 27 can be written in terms of values of  $H$  and  $c_g$  at inshore and offshore locations (referred to by the subscripts  $o$  and  $i$ )

$$\frac{H_i}{H_o} = \left( \frac{c_{go}}{c_{gi}} \right)^{\frac{1}{2}} \quad (28)$$

This phenomenon, i.e. the variation in wave height due to changes in group velocity, is known as shoaling, and the ratio  $(c_{go}/c_{gi})^{\frac{1}{2}}$  is the 'shoaling factor'. Fig 14 shows the variation of shoaling factor with depth as a wave travels inshore. Notice how the shoaling factor (and therefore the wave height at that water depth) decreases slightly and then starts to increase rapidly in shallow water. This sudden increase in wave height often causes the waves to break.

### 8.1.2 Refraction by varying water depth

It has been shown that waves approaching a coastline perpendicularly are modified by shoaling. If waves approach at an angle, an additional effect occurs, again caused by varying water depth. This phenomenon is refraction. Unlike shoaling, refraction causes a change in wave direction as well as wave height.

The refraction of waves is caused by differences in the celerities of waves in different depths. As an example consider the simple parallel-contoured seabed that was used when discussing shoaling. This time, envisage a wave coming in at an angle to the coast rather than perpendicularly (Fig 15). As a wave crest approaches the shore at an angle, the part of the wave crest in shallower water will travel more slowly than the part in deeper water. The effect therefore is that as the wave moves forward the crest bends round to align itself more nearly parallel to the depth contours. The change in wave direction is given by Snell's law of refraction.

$$\frac{\cos \theta_i}{\cos \theta_o} = \frac{c_i}{c_o} \quad (29)$$

where  $\theta_i$  and  $\theta_o$  are the wave directions at inshore and offshore points as defined in Fig 15.  $c_i$  and  $c_o$  are the wave celerities at the two points.

As well as altering the direction of the wave crest, refraction also causes the energy density (and therefore the wave height) to alter. This can be illustrated in the example just considered. Imagine two closely spaced lines drawn in such a way that they are always at right-angles to the wave crests, as shown in Fig 15. These lines are known as wave orthogonals or rays, and represent the direction of travel of the wave crest, which is also the direction of propagation of wave energy (a more precise definition of orthogonals and rays is given later when wave refraction due to currents is considered). As these rays are followed inshore, it can be seen that their separation becomes wider. In fact, the theory of refraction shows that the energy density of the waves is inversely proportional to the separation of the rays. In this example it is seen therefore that refraction has the effect of decreasing the energy density (and hence the wave height) as the waves travel inshore.

The refraction of water waves by a varying-depth seabed is very similar to the refraction of light waves through media of different optical densities. Indeed, much of the mathematical theory of water wave refraction has been adapted from theory in optics. The ideas of focussing and scattering of light have their counterpart in water waves. Fig 16 shows an example of the focussing of wave rays caused by refraction over a semi-circular shelf. The shelf acts as a sort of lens. It can be seen from Fig 16 that a smooth line is formed where rays cross; this line is known as a caustic. Two caustics are shown in Fig 16 and these meet at a point (known as the cusp) some distance beyond the shelf. In the vicinity of caustics and ray crossings, refraction theory always breaks down. The reason is that the energy density of waves is inversely proportional to the separation of rays. If this separation is zero, as it is where rays cross, refraction theory will predict infinite wave energy, which obviously does not happen in nature. What actually occurs is that another wave phenomenon is always present at ray crossings and caustics. This phenomenon is diffraction and is considered in Section 8.1.4.

Since both refraction and shoaling are caused by spatial variations in water depth, they are usually considered together as a single phenomenon. Eq 27, which was derived for shoaling alone, can be extended to include refraction as well.

$$H^2 c_g b = \text{Constant between two rays} \quad (30)$$

in which  $b$  is the separation of the rays. This equation again represents the conservation of energy flux. Eq 30 can be written in terms of the values of  $H$ ,  $c_g$  and  $b$  at inshore and offshore locations.

$$\frac{H_i}{H_o} = \left( \frac{c_{go}}{c_{gi}} \right)^{\frac{1}{2}} \left( \frac{b_o}{b_i} \right)^{\frac{1}{2}} \quad (31)$$

As before, the expression  $(c_{go}/c_{gi})^{\frac{1}{2}}$  is the shoaling factor, and  $(b_o/b_i)^{\frac{1}{2}}$  is known as the refraction factor. The change in ray direction is also related to the ray separation.

$$\frac{\sin \theta_i}{\sin \theta_o} = \frac{b_i}{b_o} \quad (32)$$

where  $\theta_o$  and  $\theta_i$  are the offshore and inshore wave directions defined as angles between the wave ray and the depth contours (Fig 15).

Plate 1 shows waves approaching the coastline of Mudeford Sandspit on the English south coast near Bournemouth. The waves approach at a steep angle, and refraction causes the waves to bend more into line with the beach direction. Plate 1 should be compared with Fig 15.

### 8.1.3 Refraction by currents

Depth variations are not the only cause of refraction of waves. If reasonably strong currents are present, these will also cause waves to refract. The refraction of waves by currents is more difficult to visualise than refraction by depth variations because the direction of travel of wave energy in the presence of currents is no longer the same as the direction of travel of the wave crests. When currents are present it is essential to make the distinction between wave orthogonals and wave rays.

Wave Orthogonal A wave orthogonal is a line drawn perpendicular to the wave crests, in the direction of travel of the crests.

Wave Ray A wave ray is a line drawn in the direction of travel of the wave energy.

A further important distinction has to be made when considering the wave period, celerity or group velocity. It is necessary to specify whether these



quantities are measured relative to the seabed (these are referred to as 'absolute' quantities, and their symbols have a subscript 'a') or relative to an observer moving with the current (these are referred to as 'relative' quantities, and their symbols have a subscript 'r')

The theory of current refraction shows that the wave rays, wave orthogonals and current directions are related to each other by

$$\underline{c_{ga}} = \underline{c_{gr}} + \underline{U} \quad (33)$$

in which  $c_{ga}$  is the absolute group velocity (which is directed along rays),  $c_{gr}$  is the relative group velocity (which is directed along orthogonals) and  $U$  is the current velocity (Fig 17). The underlines in Eq 33 denote vector quantities. Eq 33 can be expressed as two equations for the magnitude of the absolute group velocity and ray direction respectively:

$$c_{ga} = (c_{gr}^2 + U^2 + 2Uc_{gr} \cos(\delta - \alpha))^{\frac{1}{2}} \quad (34)$$

$$\tan \mu = \frac{U \sin \delta + c_{gr} \sin \alpha}{U \cos \delta + c_{gr} \cos \alpha} \quad (35)$$

in which  $\delta$  is the current direction,  $\alpha$  is the orthogonal direction and  $\mu$  is the ray direction. For refraction by a combination of currents and depth variations, the conservation of energy flux condition becomes

$$\frac{H^2}{f_r} \frac{c_{ga}}{b} = \text{Constant between two rays} \quad (36)$$

The quantity on the left-hand side of Eq 36 is known as 'wave action'. Comparing Eq 36 with Eq 30 (conservation of energy flux for refraction by depth variations only) we see that there is an extra term, the relative wave frequency  $f_r$  (wave frequency is the inverse of wave period). Note also that the absolute group velocity appears in Eq 36. This equation can be expressed in terms of inshore and offshore quantities

$$\frac{H_i}{H_o} = \left( \frac{c_{gao}}{c_{gai}} \right)^{\frac{1}{2}} \left( \frac{b_o}{b_i} \right)^{\frac{1}{2}} \left( \frac{f_{ri}}{f_{ro}} \right)^{\frac{1}{2}} \quad (37)$$

The first two terms on the right-hand side of Eq 37 are the shoaling and refraction factors. The term  $(f_{ri}/f_{ro})^2$  is known as the 'Doppler factor' since it represents the well-known Doppler effect i.e. that wave frequencies change when they are measured by a moving observer.

The prediction of inshore wave heights and directions when currents are present is considerably more complex than for refraction by depth variations alone. However, combined current and depth refraction can be readily handled by computer models, and it is recommended that these models are employed even for simplified bathymetries and current fields.

#### 8.1.4 Diffraction

The term 'diffraction' refers to wave phenomena which cause energy to travel in a different direction to that of the wave rays. Although there are many sources of such wave behaviour, they can be classified under the headings of 'external' and 'internal' diffraction.

External diffraction occurs whenever the water surface is pierced by a solid obstruction such as a breakwater, oil-rig platform leg or natural headland. The same applies to solid floating objects, a ship will cause diffraction of waves. Instead of leaving a sharply defined shadow region in the lee of the obstacle, wave energy crosses the shadow boundary, thus travelling in a different direction to that of the main wave train. A less obvious effect is internal diffraction, which occurs wherever there are rapid spatial changes of wave height that a pure refraction analysis would predict. Such areas include the caustics and ray crossings mentioned in Section 8.1.2 at which refraction theory predicts an infinite wave height. As with external diffraction, internal diffraction involves the transmission of wave energy in a direction different to the main wave direction.

Both external and internal diffraction have the effect of removing the sharp discontinuities in wave height predicted by refraction theory, and the overall effect is to give a smooth spatial distribution of wave height. As with refraction, much of the theory of water wave diffraction has been carried over from wave theory in other areas of physics such as optics, acoustics and sonar waves.

Plate 2 and Fig 18 show the long groyne at Hengistbury Head on the English south coast between Bournemouth

and the Isle of Wight. Fig 18 is a drawing to the same scale as the aerial photograph in Plate 2. Some interesting wave diffraction effects can be seen in the photograph, with the circular diffracted wave in the lee of the groyne particularly noticeable. Waves also diffract around both ends of Beerpan Rocks, the shallow rocky area just offshore from the groyne which shows up as a dark patch in Plate 2. Because of the fairly narrow gap between the tip of the groyne and the western end of Beerpan Rocks, the waves diffracting around these two points combine to give an elongated circular pattern. This wave pattern extends to almost a full semi-circle in the lees of both obstacles. Waves diffracting around the eastern end of Beerpan Rocks can also be seen in the lee of the rocks.

#### 8.1.5 Reflections

Wave reflections can either involve the total reflection of wave energy or some dissipation of wave energy. In this section we will consider only those reflection processes which do not involve loss of energy from the waves. As with diffraction, reflection of waves can either be by surface-piercing obstacles or due to bathymetric variations, and so again a classification into 'external' and 'internal' processes is possible.

Reflection of waves from seawalls and breakwaters is a familiar sight. Often such processes involve considerable loss of wave energy, particularly if the structures have rough, gently-sloping sides. However, smooth vertical-faced seawalls or quays will reflect waves with very little, if any, loss of energy. When this type of reflection occurs, a standing wave pattern is created by the interference of the direct and reflected waves. The resulting sea surface does not give the appearance of a travelling wave but only of an up-and-down motion of water (hence the name 'standing wave'). The term 'clapotis' is sometimes used for this phenomenon. At regular spatial intervals perpendicular to the seawall the direct and reflected waves add together giving a doubled wave height. Very often in rough conditions a spectacular upward jet with a lot of spray is created at these locations (see Plate 3). At the seawall itself such a jet is often formed, exerting considerable force on the structure and often overtopping the wall.

As well as being reflected from surface-piercing obstacles, waves can also be reflected (in water that is generally shallow) by sudden or large changes in water depth. This reflection can be either partial or

total, but in neither case is any wave energy dissipated. Partial reflection of waves actually occurs whenever there is a bed slope of any size. This effect is negligible for gentle slopes but can become significant for long, steep slopes. The strength of the reflected wave also generally increases for smaller glancing angles (i.e. the angle between the wave ray and depth contours). The situation is analogous to the refraction of light between media of different optical densities in which the majority of energy is transmitted from one medium to the other, but there is a weak reflected wave. If, however, light strikes such a boundary travelling from one medium towards another of lower optical density (such as from glass to air) this reflected wave becomes quite strong if the incident ray angle is close to the 'critical' angle (see Fig 19). For glancing incident angles smaller than the critical angle, the reflection of light from the boundary is total. An analogous phenomenon occurs with water waves, where the equivalent of an 'optically less dense medium' is a region of deeper water. Thus if a wave approaches deep water from shallow water (with a fairly rapid transition of depth between the two regions), total reflection can take place. This is quite a common occurrence when waves encounter the sides of a dredged channel (see Fig 20). This phenomenon has both beneficial and adverse consequences. Much reduced wave activity occurs in the channel itself, with obvious benefits to navigation. However, there is also considerably increased wave activity along the side of the channel, causing greater erosion of the channel side and infill of the channel bottom.

Plate 4 shows total internal reflection at a channel in a model wave tank. The reduced wave height in the channel and the increased wave height at the channel side are clearly seen. The hexagonal wave pattern is caused by the interference of the direct and reflected wave. In Ref 22 a more detailed explanation of this effect in terms of a transition region (known as a 'wave jump') between two wave trains of different properties is given. This phenomenon is related to Mach reflections (section 8.3.1)

## 8.2 Dissipative phenomena

### 8.2.1 Bottom friction

In water which is sufficiently shallow for waves to 'feel' the seabed, wave energy can be lost by interaction with the seabed. This energy is converted into turbulent water motion, heat, and the motion of

seabed material. These dissipation processes are collectively known as 'bottom friction'.

The rate of dissipation of wave energy depends on the water depth, the wave characteristics and the nature of the seabed. It is proportional to the cube of the water velocity at the seabed, which in turn is a function of the wave height, wave period and water depth. Energy losses also depend on the type of seabed material. With coarse material (shingle or gravel) energy losses due to percolation of water between the seabed particles can occur, but usually the voids are filled with finer material. The roughness of the material, measured by the average dimensions of the particles, will also cause loss of wave energy by water turbulence close to the seabed. With finer material such as sand, percolation is negligible. An effect of waves on this finer material is to form ripples of the material on the seabed. Wave energy is again lost by water turbulence close to the seabed, but the roughness of the seabed is determined by the size of the ripples rather than the individual sand particles. If water velocities at the seabed exceed a certain threshold, sandy material can be brought into suspension and transported by the waves, thereby removing energy from the waves. Very fine material (clay or mud) is readily brought into suspension and a layer of fluidised material is created above the seabed. This fluidised mud layer has an extremely strong damping effect on waves.

Generally, waves need to propagate over considerable distances (of the order of kilometres) in shallow water for bottom frictional losses to be significant. In coastal areas where the seabed slopes reasonably steeply to a depth of about 20m frictional losses can usually be neglected to a good approximation. However, in large flat shallow sand bays, bottom friction can be the dominant phenomenon. This is certainly the case with shallow mud flats, although these are not common around U.K. coasts, being limited to large estuarine areas.

#### 8.2.2 Wave breaking

Wave breaking is the most obvious and spectacular of all shallow-water wave phenomena. Although waves can break in deep water by the action of wind on wave crests or when the crests exceed a certain steepness, by far the most important type of breaking waves from the point of view of energy dissipation are those that are caused to break by the shallowing water close to the shore. These depth-limited breaking waves are

such complex phenomena that they are unlikely to be fully understood or adequately represented in numerical models for some years to come. In addition to dissipating wave energy through turbulence, depth-limited breaking waves can exert very strong forces which make them an important consideration in the design of maritime structures.

The distance from the shore at which waves break varies considerably. As a rule of thumb, waves begin to break when the depth becomes less than twice the significant wave height. In the narrow region between the line of breakers and the shore (known as the 'surf zone') complex wave effects occur, including a set-up of the water level, creation of longshore currents, and undertow and rip currents travelling offshore from the beach. Wave phenomena in the surf zone will not be described in this report, and for many coastal engineering applications phenomena shorewards of the breaker line need not be modelled in detail. However, for a detailed understanding of wave run-up and alongshore and onshore-offshore movement of beach material, surf zone effects do need to be considered.

A simplified representation of wave breaking is used in present computational wave models, with the main purpose being to determine the amount of wave energy dissipated. Refs 23 and 24 describe how an expression for energy losses due to breaking can be introduced into these models. Calculation of the breaking energy loss is commonly based on the limiting wave height allowed by the breaking process at a given depth. The method works by calculating a wave height assuming no breaking takes place and comparing this value with the height at which breaking starts to occur at that depth. If the calculated wave height exceeds the breaker height, it is reduced to the value of the breaker height. The method recommended in the American Shore Protection Manual (Ref 8) for calculating the breaker height ( $H_b$ ) for regular waves is to determine  $H_b$  from:

$$H_b = \frac{b d}{1 + \frac{a d}{g T^2}} \quad (38)$$

where  $d$  is the water depth,  $g$  is the acceleration due to gravity and  $T$  is the wave period.  $a$  and  $b$  are functions of the seabed slope,  $m$ , given by

$$a = 43.75 (1 - \exp(-19m)) \quad (39)$$

$$b = \frac{1.56}{1 + \exp(-19.5m)} \quad (40)$$

For wave spectra in shallow water Vincent (Ref 25) has derived the following expression

$$H_b = \frac{1.17 (\alpha g d)^{\frac{1}{2}} T_p}{\pi} \quad (41)$$

where  $\alpha$  is the Phillips constant in the JONSWAP formula for deep-water spectra (usually taken as 0.0081) and  $T_p$  is the period at which maximum wave energy occurs. Note that in Eq 41,  $H_b$  should be compared with the predicted significant wave height,  $H_s$ .

Recently, more sophisticated methods for determining breaking energy losses have been advanced. These are based on the similarity of the wave breaking process with other hydraulic phenomena such as a hydraulic jump (Refs 26 and 27) and a tidal bore (Refs 28 and 29).

### 8.2.3 Reflections

Wave energy is usually dissipated to some extent when waves are reflected by obstacles, either man-made or natural. Generally the dissipation of energy will be greater if the slope of the obstacle is gradual, the surface material is rough, and the roughness layer is thick. The modern design of breakwaters takes these factors into consideration in attempting to reduce wave reflections as much as possible. The wave energy that is reflected can produce a partial standing wave, but with smaller wave heights and less spectacular jets of the sort described in Section 8.1.5. As well as a reduction in the general wave activity, the dissipation of reflected wave energy results in lower overtopping rates and less seabed scour.

Natural beaches are probably the most effective of all absorbers of wave energy, reflecting very little of storm and swell waves. However, both natural beaches and man-made structures do reflect very long period waves such as 'surf-beat' (or 'edge') waves or those associated with set-up and set-down of the sea level between wave groups (see the following section). With all types of obstacles longer period waves are reflected more strongly than those of shorter period.

### 8.3 Other wave phenomena

The shallow-water wave phenomena described in Sections 8.1 and 8.2 are the principal processes affecting waves. These processes can all have very important effects in the appropriate sea conditions and their influence is apparent over large areas.

There are, however, many other phenomena. Although they are of less general importance for coastal engineering problems and often less extensive in sea area, they may be significant at particular locations and for particular problems. They are usually difficult to describe mathematically and incorporate in a general manner into computational models. Among them can be listed:

#### 8.3.1 Mach Reflections

When waves reflect from near-vertical faced structures at a shallow glancing angle, the interference of the direct, reflected and diffracted waves can cause a high crest (or 'stem') perpendicular to the structure and extending a short distance seawards of it. This phenomenon can give wave heights along the stem up to 2.4 times the incident wave height. A complex, local phenomenon such as this is best studied in scaled physical models.

#### 8.3.2 Wave Grouping

It is well known that swell waves do not come inshore as a regular pattern but in groups of several waves which are higher than average. There are a number of causes of this grouping effect. Perhaps the most important is the interference of waves travelling in the same direction but with slightly different frequencies. The effect of the interference of these waves is to create a series of groups of high waves with low waves in between.

#### 8.3.3 Long waves

Storm and swell waves will rarely have periods longer than thirty seconds. However, waves with periods of the order of a few minutes are quite common in coastal regions. Usually these waves have small heights and are noticeable as a general rise and fall of the water level. Moving pressure fronts can create these types of wave by inducing a raising and lowering of the sea level in line with the (spatial) variations in atmospheric pressure. The water level can also be raised and lowered when wave grouping occurs. Under the groups the water pressure is lowered because of the high velocities of the water particles. This will



result in a general depression of the water level under the groups, with a corresponding rise of water level between the groups. When the wave groups reach the shore, the primary wave is dissipated but the associated long wave is not. This is reflected from the beach, giving rise to the phenomenon of surf beats. Long waves are particularly important inside harbours or small enclosed bays where their heights can be amplified by resonance. The periods of these waves are of the same order as the natural periods of resonance of boats and ships and this will cause moored vessels to oscillate, and create manoeuvring difficulties for vessels under way.

## 9 COMPUTATIONAL MODELS OF SHALLOW-WATER WAVE PHENOMENA

### 9.1 Introduction

The sizes of coastal areas to be covered by a model of shallow-water wave phenomena can vary greatly. The smallest would cover an area with dimensions no more than a few hundred metres, in locations where the seabed slopes sharply to deep water. At the other extreme, areas hundreds of kilometres square may need to be considered. In the majority of cases the area to be covered would be too large for scaled physical models, and therefore computational models are almost always used.

The previous section has shown that there exist a wide range of phenomena that affect waves as they approach the coastline, and generally they will all be present to some degree. Although computational models have been developed which describe these effects well individually (with perhaps the exception of wave breaking) it is a much more difficult problem to model their interaction. The main thrust of present-day model development is towards combining wave phenomena. At present, however, the interaction of all physical processes is too complex to be fully modelled and therefore the present generation of models have to omit or simplify some wave processes. To cover the full range of wave processes, therefore, it is necessary to have a variety of different models, each catering for coastal engineering problems in which some wave phenomena are more important than others.

As well as being restricted by the number and type of incorporated wave phenomena, some models are also restricted by their numerical solution procedures. An important numerical restriction is that some models require a certain minimum number of grid points per

wavelength. For large areas and short period waves the number of grid points required by these models becomes so large that the data preparation, computing storage and run time are excessive.

Comparisons of predictions from a computational model with field data is a necessary step in calibrating the model and assessing its accuracy. Unfortunately to date, few systematic field measurement studies have been carried out. The acquisition of good quality field data and the development of appropriate instrumentation is an area of high priority.

The accuracy of a model depends on the incorporated phenomena and numerical procedures. Clearly the more sophisticated models will tend to be more accurate. However, as well as the obvious consideration of accuracy, the usefulness of a computational model depends on its generality and flexibility. These ideas are explained below.

Generality: To be of general use, a model must be able to describe quite general coastal situations. A successful model by this criterion will be able to incorporate any irregular seabed topography and coastline, sea areas of any size, and any type of incident wave condition. As a rule, the more sophisticated a model is, in terms of incorporated wave phenomena, the less general it is. Common among these restrictions in the more sophisticated models are the requirement for a perfectly flat seabed, a parallel-contoured seabed, a perfectly straight coastline, or the limitation to effects in one-dimension (in the horizontal plane) only. In coastal engineering applications, such models, despite their more accurate modelling of wave phenomena, would be more restricted in their use. For a practical wave model the criterion of generality can be of sufficient importance to override the drawback of having a relatively simple representation of wave phenomena.

Flexibility: Very often, coastal engineering problems require a large number of options to be investigated. These may include the position and orientation of breakwaters for harbours or beach protection, the orientation, length and depth of dredged channels, or the amount and extent of offshore dredging for commercial gain. In each case a number of options needs to be investigated, usually combined with a variety of offshore wave conditions and water levels. A useful computational model must therefore be able to introduce variations to a particular scheme quickly and accurately. As seen from the examples just mentioned, these variations can take the form of

changing the bed topography, location and nature of maritime structures, and incident wave conditions. Flexibility of computer models is vital to a thorough investigation of options within a reasonable time and cost schedule.

In the remainder of this section, we shall describe the most recent types of shallow-water computational models that are of general use to coastal engineers. Before these models are used, a preliminary assessment of the coastal engineering problem should be made to determine whether a model investigation is necessary and, if it is, what type (or types) of model should be used. In this choice it is important to bear in mind not only the accuracy, generality and flexibility of the models, but also the accuracy and form of the input data, the required accuracy of the solution to the problem, and the required form of results from the models. The advantages and limitations of these models are summarised in Tables 1 and 2.

## 9.2 Forward-tracking ray model

Ray tracing has been the traditional method used by coastal engineers for evaluating the refraction and shoaling of waves as they approach the coastline. Up to about twenty years ago this was done graphically by hand. Briefly, the method involved using a chart of the coastal area, and drawing on it a line, roughly parallel to the shore, in deep water. A series of rays starting at points at regular intervals along this line would be traced shorewards. The construction of rays was carried out in small spatial steps. At the end of each step the new ray direction, and the refraction and shoaling coefficients were calculated. The whole process was repeated for different incident wave periods and directions. In the late nineteen-sixties this rather tedious process was computerised, so giving the first type of computational coastal wave model.

In common with all computational models of coastal regions, the forward-tracking ray model uses a grid of depth values covering the coastal area of interest (Fig 22). Sometimes more than one grid is used as shown in Fig 22. Each grid is subdivided into rectangular elements (often they are squares) and depth values from charts of the area are recorded at each of the element vertices. The computerised ray tracing process is similar in concept to the graphical method but the computer algorithm allows ray tracing to be done with greater accuracy and far greater speed.

The procedure used at Hydraulics Research, for example, involves dividing each rectangular element into two right-angled triangles. Under the assumption that the wave celerity varies linearly (in space) in each triangle, it can be shown that the ray paths are circular arcs. Knowing the ray position and direction at the entry point to a triangle, the ray path across the triangle and the point of exit from the triangle can be readily determined. The exit point becomes the entry point to the next triangle, and in this way a ray is traced across the grid system until either it leaves the grid system or encounters the coastline, where it is stopped. As with the graphical method, this process is repeated for a set of initially parallel rays in deep water, although the speed of the computational method allows a far greater number of rays to be traced. Ref 30 describes such an algorithm in more detail.

Early computational ray models were limited to the tracing of ray paths and calculation of refraction and shoaling coefficients at various points along each ray path. Modern forward-tracking computer models are more sophisticated, but before describing these more recent developments it is instructive to point out the deficiencies of the early models, using Fig 23 as an illustration. Perhaps the most important limitation concerns the wave behaviour where rays converge and cross forming caustics, and where they diverge leaving 'dead' areas. Important examples of both effects can be seen in Fig 23. As described in section 8.1.4, internal diffraction processes occur in both types of area, and these processes are not included in the model. Therefore the high wave heights that the model will predict in the convergent ray areas and low wave heights in the dead areas will be unrealistic. Further drawbacks of these early ray models are that current refraction, reflections, bottom friction and wave breaking are not included, and the incident wave data is in the form of a single wave period and direction rather than a spectrum.

Some of these drawbacks are overcome in the more recent forward-tracking ray models. These new developments are outlined below.

#### (a) Refraction by currents

Although the theory of combined depth and current refraction is mathematically complex, the incorporation of current refraction into a ray tracing model involves relatively little extra computational effort or data preparation effort. The only additional information to be supplied is gridded

values of current velocities and directions, in the same format as the depth data. Currents from any source can be incorporated provided they are known in advance. It is envisaged that tidal currents would be the main type of current used in these models because tides represent the strongest source of currents in the UK waters and, because of their periodicity, are readily predictable. Ref 31 gives details of computational models of combined current-depth refraction of water waves.

(b) Wave reflections from structures

A reflecting boundary, such as a seawall or breakwater, can be represented in the computational model by a series of straight line segments, each segment lying in one grid element (Fig 21). Rays are reflected from these line segments according to the law of reflection (i.e. angle of incidence = angle of reflection). Each line segment has a reflection coefficient associated with it by which the ray's energy is reduced after reflection. This representation of reflecting boundaries allows an irregular boundary to be incorporated very accurately and also allows different reflection coefficients to be used for different parts of the boundary.

(c) Ray averaging

The usual method of obtaining inshore wave heights is by calculating refraction and shoaling coefficients at intervals along each ray. A better method, however, is to average the effects of rays over each of the grid elements. This method can be easily incorporated into existing computer models (Ref 32) and has a number of advantages. An important advantage is that the ray averaging technique has the effect of 'smoothing' wave heights near caustics. This smoothing effect is similar to the actual diffraction process, although the method is purely numerical and no attempt is made to model the diffraction process. Another advantage is that intersecting wave trains can be taken into account. This situation can occur, for instance, where a direct wave interferes with a reflected wave. A third useful feature is that wave heights are generated in a regular array over the whole sea area. This information can be readily presented as a contour diagram of wave heights using computer graphics.

(d) External diffraction

New means of representing diffraction by breakwaters, combined with subsequent refraction and reflections,

using a forward-tracking ray method have recently been developed. These methods have been designed principally for evaluating diffraction around breakwaters at harbour entrances but they can be used equally well in coastal applications for diffraction around headlands or offshore breakwaters. The reader is referred to Ref 33 for details of these methods.

(e) Bottom Friction and Wave Breaking

Both bottom friction (Ref 34) and wave breaking (Refs 3,4,8) can be incorporated into a forward-tracking ray model, although both processes are simplified to a considerable extent. Because of the strong non-linearity of these wave processes it is not possible to construct inshore wave spectra from a series of runs representing spectral components (see below). A further (but related) limitation is that these processes will not be correctly modelled in locations where there are intersecting wave trains.

(f) Incident Wave Spectra

A single run of a forward-tracking ray model uses a single offshore wave period and direction as input. It is possible to cover a full wave spectrum by performing a number of runs at different combinations of period and direction. Wave heights at inshore points from each run can be weighted by the offshore wave energy associated with that particular frequency and directional component and combined to give a total wave height. However, this superposition of wave components at inshore points is only valid if all the modelled wave processes are treated linearly. This is the case for non-dissipative phenomena, but the representations of bottom friction and breaking (see above) involve non-linear terms.

In practice, to cover a spectrum with a series of runs can involve considerable computational effort, particularly if a number of different bathymetries and layouts are investigated. Ref 35 describes some tests to determine if a single run at an average wave period would give similar wave height results to those determined from a full spectral coverage. A run at the median period (the period which bisects the area under an energy versus wave frequency graph) was found to give very close results to those using the full spectrum. Refraction patterns generally alter more markedly with changes in direction than period, and it is recommended that runs from a range of directions should still be performed. However, sometimes mainly mono-directional seas can occur (for instance when a

distant swell is dominant). One run at a single offshore direction can be sufficient in these cases.

The median period tests were carried out for a harbour rather than a coastal application. Since rays travel much further in a coastal model, an irregular ray pattern is more likely to develop. Wave heights from a forward tracking ray model should therefore be treated with some caution, and a computer plot of ray paths is recommended in evaluating the reliability of results. As a common rule, forward-tracking models are often somewhat unreliable in the prediction of actual wave heights, but are more accurate in predicting the wave heights in one scheme relative to another. This is because the effects of unrealistic ray patterns common to both schemes will cancel when the schemes are compared.

Because of the inaccuracies inherent in irregular forward-tracking ray patterns, and the difficulties in representing an offshore wave spectrum, a method of back-tracking (or reverse-tracking) of rays from an inshore point of interest out to deep water has been developed. Forward-tracking ray models are still used, however, where results are required at spatial intervals over large areas (eg. along a beach or channel), where bottom friction and breaking are important, and for inshore points sheltered by headlands, bays or estuaries.

### 9.3 Back-tracking ray model

In the early nineteen-seventies, back-tracking ray models (Ref 36) were developed in order to utilise the spectral representation of deep-water wave conditions and to reduce greatly the influence of caustics and dead areas. The problems of caustics and non-spectral representation were the most serious sources of error in forward-tracking ray models. The back-tracking method is based on the principle of reversibility of ray paths, i.e. the path of a ray traced backwards (opposite to the actual direction of wave travel) is identical to the path traced forwards (in the direction of wave travel). The computational process involves tracing fans of rays at small angular increments from an inshore point of interest until they reach deep water (Fig 25). Some rays will not reach deep water and turn shorewards and strike the coast. These are ignored, and only rays reaching the offshore boundary are considered. The directions of these rays at the offshore boundary are grouped in angular 'boxes' (typically 10° wide) which represent a discretisation of the offshore directional spectrum. A discretisation of the period spectrum is obtained by

constructing a series of ray fans at regular period intervals (1.5 seconds is a typical interval). Fig 25 shows one such fan of rays.

The construction of these fans of rays allows the inshore spectrum to be determined in terms of the offshore spectrum, provided it is known how a spectrum transforms along a ray. Refraction theory applied to wave spectra shows that the quantity  $cc_g S(f, \theta)$  is a constant along a ray, where  $c$  is the wave celerity,  $c_g$  the group velocity, and  $S(f, \theta)$  the spectral function in frequency ( $f$ ) and direction ( $\theta$ ). The spectral function at an inshore point can then be determined in terms of the spectral function offshore as

$$S_i(f, \theta) = \frac{c_o c_{go}}{c_i c_{gi}} S_o(f, \theta) \quad (42)$$

where the subscript  $i$  denotes inshore values and  $o$  denotes offshore values. This method makes the assumption that  $S_o(f, \theta)$  is the same along the whole offshore boundary.

An important feature of the back-tracking ray method is that it removes the problem of caustics and other unrealistic ray patterns where internal diffraction takes place. However, as with the ray averaging method in the forward-tracking model, no attempt is made to describe the diffraction process. In the back-tracking ray model the problem of internal diffraction is tackled by considering the refraction of spectra rather than individual wave components. This can be understood using Fig 5 as an illustration. This figure shows an offshore spectrum with a regular distribution of energy about the peak period and direction, giving a smooth appearance to the figure. A typical inshore spectrum, on the other hand, would be quite irregular with a number of bumps and hollows. Sometimes a few high and very narrow spikes occur. These spikes correspond to caustics, i.e. areas of concentration of wave energy at the inshore site for particular period and direction components. However, the useful inshore wave parameters (significant wave height ( $H_s$ ), zero-crossing period ( $T_z$ ), etc) are statistical quantities determined from the volume under the whole spectrum. Because of the narrowness of any spikes, their contribution to the total volume is small.

The main advantage of the back-tracking ray method is that it determines the full wave spectrum at an inshore site, along with the associated statistical



wave quantities. As with the forward-tracking ray model the effects of current refraction can be included. There are, however, some limitations. The back-tracking ray model is best suited for investigations where wave conditions are required at a small number of inshore points. Such applications would include evaluating wave conditions at entrances to harbours, at the locations of maritime works, or at beaches or dredged channels which are small in length. If wave conditions are required at a large number of inshore points a forward-tracking ray model or finite-difference model would be preferable. A further limitation is that the back-tracking ray model is not appropriate for sea states which are mono-directional (such as those dominated by long-distance swell). It is also restricted in the wave phenomena that can be modelled. Because the back-tracking ray method relies on the linear superposition of wave components to construct the inshore spectrum, non-linear effects such as bottom friction and breaking cannot be incorporated. However, a back-tracking study will give a conservative estimate of wave heights, and such studies are often done as a preliminary to a forward-tracking ray model or finite difference model which can include dissipative effects.

If the back-tracking model is used on its own, it is possible to compare predicted wave heights with the maximum wave height ( $H_b$ ) that is allowed by the depth at the inshore point before breaking takes place.  $H_b$  can be calculated using Eq 41 in Section 8.2.2.

#### 9.4 Finite Difference Refraction Model

An alternative to a ray tracing method, but using the same basic set of equations, is a finite difference method. As with ray methods, the sea area of interest is covered by a grid subdivided into rectangular elements. The solution procedure involves approximating the derivatives in the governing equations by differences between values at neighbouring grid points (hence the name 'finite difference').

The offshore wave conditions are specified at each grid point along the row at the extreme seaward edge of the grid. In this model it is possible both to specify a wave spectrum (in period and direction) and to have different offshore spectra at different points (this latter feature is more general than the back-tracking ray model in which a spatially homogeneous offshore spectrum is required).

The method works by calculating wave conditions successively along grid rows, starting at the seaward end. Using the offshore wave conditions along the extreme seaward row, and the finite difference formulation of the refraction equations, the wave conditions along the second row can be calculated. These wave conditions in turn provide the basis for calculating the wave conditions along the third row. The process is repeated until the final row, furthest inshore, is reached. Because of the way the solution proceeds row by row, this method is known as a 'marching' method. Full details of such a model are given in Ref 37.

The advantages of the finite difference refraction model are that it can treat a full offshore spectrum (unlike the forward-tracking ray model) but is also able to include bottom frictional and breaking dissipation processes (unlike the back-tracking ray model). As with both types of ray model, the finite-difference model can also include the effects of refraction by currents. It does, however, have some drawbacks. In common with the forward-tracking ray model, it will suffer from the difficulties with caustics and dead areas, although the fact that a series of wave components representing a spectrum is being calculated means that these features will be smoothed somewhat. Diffraction effects (internal and external) and reflections (again internal and external) cannot easily be included in such a model. The finite difference method also suffers from a limitation in the type of sea areas that can be modelled. If the coastline changes direction significantly from the main alongshore direction determined by the refraction grid, errors can arise at points near this coastline. This is because wave conditions at a grid point are calculated from known conditions at neighbouring points on the previous row. These latter points should be in the open sea and not on land or very close to land. Although it is possible to represent land points by special boundary conditions or as sea areas of very small depth, the sharp jump that will be caused in wave conditions between neighbouring points will introduce errors. This type of model should therefore be used where there is a reasonably straight coastline and should not, for instance, be used to evaluate wave conditions in a well-sheltered bay or at the sides of an estuary. For such cases a forward-tracking ray model can be employed.

## 9.5 Parabolic Model

Experience has shown that the most important deficiency of forward-tracking ray models and

finite-difference refraction models is the inability to incorporate diffraction. New types of models, known as parabolic models, have recently been developed which use a marching finite difference procedure but with a more complex set of governing equations which incorporate some diffraction effects. This method attempts to model the actual diffraction process rather than using a numerical smoothing technique. Much effort has been devoted to developing parabolic models since the late nineteen-seventies and there is now a considerable technical literature on the subject. Refs 38, 39 give two of the pioneering investigations. Ref 40 describes some of the most recent developments and Ref 41 contains a literature review on the subject.

As with the finite-difference refraction model, parabolic models can in principle incorporate offshore wave spectra, current refraction, bottom friction and wave breaking. The main advantage of parabolic models is the inclusion of diffraction effects, but this greater accuracy is achieved at the expense of limits to its range of applicability, in particular to the size of the sea areas that can be modelled. Parabolic models require a much finer grid mesh than the refraction models because, unlike the refraction models, they require a minimum number of elements per wavelength to resolve the wave profile. In practice this will limit parabolic models to fairly small coastal areas, no more than a few kilometres wide at most, and usually considerably less. A further limitation is that wave directions are limited to an angular sector (typically about  $45^\circ$ ) either side of the grid direction perpendicular to the shoreline. In common with the finite-difference refraction model, backward-reflected waves are difficult to incorporate, and a reasonably straight coastline facing the open sea is required.

Parabolic models can be useful when waves travel over uneven bathymetry such as systems of shoals and channels which are quite common in UK coastal waters. In such situations it may also be desirable to use a combination of models. For instance, a back-tracking ray model could be used to determine wave conditions just offshore from a shoal system and then a parabolic model can be used for the transformation of the waves over the shoals.

## 10 EXAMPLES OF USE OF SHALLOW-WATER COMPUTATIONAL MODELS

In this chapter three examples of the use of shallow-water computational models are described, in order to illustrate how the choice of computational model is made and to show some of the procedures followed in using the models. All three examples are taken from studies contracted recently to Hydraulics Research for sites around the UK coast. In each case the nature of the site and the type of engineering problem are different.

### 10.1 Carrickfergus Harbour, Belfast Lough

In 1983, HR carried out a wave prediction study for a site near Carrickfergus Harbour in Belfast Lough, Northern Ireland. A new small-boat harbour basin was to be built adjoining the existing harbour, and wave conditions corresponding to design offshore storm spectra were required at this site.

The geography of the coastline, and the depth and nature of the seabed in the surrounding coastal area are crucial in choosing which type of computational method to use. The site and the adjacent coastal area are shown in Fig 22. Carrickfergus harbour is well protected at the side of the Lough with a fairly narrow directional 'window' through which waves can arrive at the site. The Lough is reasonably shallow, with the 20m depth contour about 15km offshore from Carrickfergus harbour.

The long shallow sea area suggests that bottom friction could be a significant source of energy dissipation, indicating that one of the forward-tracking types of model should be used. However, because of the sheltered location of the site at the side of the Lough, the finite difference and parabolic models, which require reasonably straight coastlines facing the open sea, are ruled out. A forward-tracking ray model was therefore chosen. A single run of the forward-tracking model using a suitably chosen period and direction was considered sufficient to represent the offshore spectrum. The use of a single direction was considered a reasonable approximation because of the narrowness of the directional 'window' for the site.

Because of the sensitivity that single runs of the forward-tracking ray model display to slight changes in offshore period and direction, it was decided to

'calibrate' the model against results from a back-tracking ray model. The latter does not include bottom friction effects but will take into account an offshore spectrum. A number of runs of the forward-tracking ray model were performed without bottom friction, each run using slightly different periods and directions from their representative values. The period and direction which gave wave heights at the inshore site closest to the back-tracking ray model results were then selected. The forward-tracking ray model was then run again with bottom friction included and using the calibrated offshore period and direction. This calibration of a forward-tracking ray model against the back-tracking ray model is often carried out and gives greater confidence in the accuracy of the forward-tracking model tests. Three runs with different friction factors were subsequently performed (corresponding to different seabed roughnesses) to determine the sensitivity of results to the friction factor.

A system of six grids was used to cover the sea area as shown in Fig 22 and a plot of the forward-tracked ray paths is shown in Fig 23. This ray path plot indicates clearly areas of ray crossings, in particular the formation of two long, prominent caustics separated by a dead area where the rays have diverged. At the top of the figure there are some small offshore islands at which rays are stopped before they reach the main coastline.

Table 3 gives some results from this study for one design offshore storm spectrum, indicating that both refraction/shoaling and bottom friction have a significant influence on wave heights at Carrickfergus harbour.

## 10.2 Shoreham Harbour

Shoreham harbour is on the Sussex coast facing the English Channel. In 1984, HR was commissioned to carry out a study of the infill rates of a proposed dredged channel to be used as an approach to the harbour. Unlike the Carrickfergus harbour study, in which interest was centred on extreme wave conditions, this study required an analysis of moderate storm events and the everyday wave climate.

Fig 24 shows the geography and bathymetry of the area. To the east the depth contours fall away quite sharply but to the west the bed slope is far gentler. Grab-samples from the seabed indicated a sandy bottom close to the shore with a hard immobile bed further out. It is possible that bottom frictional effects could be significant for wave directions from the

west, but it was decided that a back-tracking model (in which frictional effects are excluded) alone was the most appropriate model for the problem. The reasons for this decision are listed below:

1. The wave conditions determined by the chosen shallow-water computational model were to be used subsequently for calculations of sediment infill of the dredged channel. Because the rate of infill is strongly dependent on both wave height and period, and because present-day computational sediment models are accurate only to an order of magnitude, it was considered safer to use conservative wave height and period values without bottom friction.
2. Since the everyday offshore wave climate and moderate storms were being considered, a large number of offshore spectra were required to be transformed inshore (in fact thirty-three spectra were considered). The computational effort involved in calibrating and running a forward-tracking model for each of these cases would have been excessive.
3. The channel length was sufficiently small (about 700m) that only one prediction point needed to be considered.

Before describing how the back-tracking ray model was used, it is of interest to indicate briefly how the offshore wave spectra were determined. Wind data consisting of mean hourly windspeeds and directions going back four years were available from the Coastguard Station at Shoreham. The wind data was multiplied by an appropriate 'mark-up' factor to take account of the fact that winds are generally stronger over the sea than on land. Using HR's HINDWAVE model (see Section 5.1 and Ref 15) this wind data was analysed to give corresponding wave conditions offshore. The wave data thus calculated was then grouped into discrete wave height/wave direction categories, and the frequency of occurrence of each category determined. Thirty-three of the most common and important of these categories were selected, and the corresponding period and directional spectra were calculated using the JONSWAP/Seymour method.

The back-tracking ray model produces a set of 'transfer functions', i.e. a set of values for each period and directional component by which an offshore spectrum is multiplied to give an inshore spectrum. Two such transfer functions were obtained for the water level at Mean High Water Springs and Mean Low

Water Springs respectively. The transfer functions were multiplied by each of the offshore spectra in turn to give the corresponding inshore spectra and values of  $H_s$ ,  $T_z$  and average direction. By assuming that the MHWS and MLWS wave heights each applied for 50% of the time, it was possible to estimate the frequency of occurrence of inshore wave conditions averaged over a tidal cycle.

A plot of ray paths for one period at MHWS is shown in Fig 25. Generally the ray behaviour is quite smooth with relatively few crossings. The overall effect is that waves at the inshore site are reduced in height and have a narrower directional spread than offshore. This is typical of many coastal areas with reasonably regular depth profiles.

### 10.3 Durham Coast

An important type of coastal engineering problem occurs when there are proposals to dredge gravel or other material from the seabed relatively close to the coast. The resulting changes in bed levels will alter the refraction pattern and reduce bottom frictional dissipation thereby altering wave conditions on the nearby coastline with possible adverse consequences for the stability of the beaches. In 1982 an investigation of the effects of dredging coal waste dumped at sea off the Durham coast was carried out by Hydraulics Research.

Fig 26 shows the site, and the line A-B is the extent of the coastline that was considered to be affected by the offshore dredging. In contrast to the two previous examples, where wave conditions were required at only one inshore point, wave conditions are required at regular spatial intervals along a stretch of coast. The finite-difference model was chosen for this study because the coastline was reasonably straight, a representation of the full wave spectrum was required, and bottom frictional effects were considered to be significant.

The procedure involved running the model with the original seabed levels and again with the levels assuming the proposed dredging had taken place. A single offshore spectrum derived from the JONSWAP formula for a severe storm condition was considered adequate for the purposes of this 'before-and-after' exercise.

In the event, only slight changes in wave height were found and were not considered to have a significant effect on the nearby beaches. It is interesting, however, to see the sort of wave behaviour predicted

by the model. RMS wave heights at twenty points between A and B are shown in Fig 27. Bottom friction causes a general reduction in wave height at the shoreline. The fluctuations in wave height from point to point are due to the spatial redistribution of wave energy by refraction. In practice, internal diffraction effects, which are not included in the model, would tend to smooth out these spatial fluctuations in wave height.



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## **Tables**



TABLE 1 SHALLOW-WATER COMPUTATIONAL MODELS. WAVE PROCESSES INCORPORATED

| WAVE<br>PROCESSES<br>MODELLED    | DEPTH<br>REFRACTION<br>AND<br>SHOALING | CURRENT<br>REFRACTION | INTERNAL<br>DIFFRACTION                                                     | EXTERNAL<br>DIFFRACTION                                                                                                                   | REFLECTIONS                                                            | BOTTOM<br>FRICTION                                      | WAVE<br>BREAKING                                                                                                   |
|----------------------------------|----------------------------------------|-----------------------|-----------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------|---------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| COMPUTATIONAL<br>MODEL           |                                        |                       |                                                                             |                                                                                                                                           |                                                                        |                                                         |                                                                                                                    |
| FORWARD<br>TRACKING<br>RAY MODEL | Yes                                    | Yes                   | Not modelled<br>but numerical<br>smoothing by<br>ray averaging<br>process   | Yes for<br>long thin<br>structures<br>(eg break-<br>waters) or<br>long wedge-<br>shaped<br>obstacles<br>(eg some<br>natural<br>headlands) | Yes                                                                    | Yes, but<br>not for<br>intersec-<br>ting wave<br>trains | Yes, but<br>not for<br>intersec-<br>ting wave<br>trains.<br>Approximate energy<br>loss in<br>shallow<br>water only |
| BACK<br>TRACKING<br>RAY MODEL    | Yes                                    | Yes                   | Not modelled<br>but numerical<br>smoothing by<br>spectral<br>representation | No                                                                                                                                        | Yes                                                                    | No                                                      | No, apart from<br>check at<br>inshore point                                                                        |
| FINITE-<br>DIFFERENCE<br>MODEL   | Yes                                    | Yes                   | Not modelled<br>but numerical<br>smoothing if<br>offshore<br>spectrum used  | No                                                                                                                                        | No. Back<br>scattered<br>waves of any<br>type cannot<br>be modelled    | Yes                                                     | Yes.<br>Approximate energy<br>loss in shallow<br>water only                                                        |
| PARABOLIC<br>MODEL               | Yes                                    | Yes                   | Yes                                                                         | Yes, but<br>difficult<br>to incorporate in<br>general<br>manner                                                                           | No. Back<br>scattered<br>waves of<br>any type<br>cannot be<br>modelled | Yes                                                     | Yes.<br>Approximate energy<br>loss in shallow<br>water only                                                        |

Present parabolic models do not combine all the wave processes in this table

TABLE 2 SHALLOW-WATER COMPUTATIONAL MODELS. PARAMETERS DETERMINING MODEL SUITABILITY

| COASTAL<br>PARAMETER                | TYPE OF<br>BATHYMETRY                                                                      | EXTENT OF<br>SEA AREA                                                                                                                                               | TYPE OF<br>COASTLINE                                                                              | NUMBER OF<br>INSHORE POINTS<br>PER RUN        | OFFSHORE<br>WAVE<br>CONDITIONS                                                     |
|-------------------------------------|--------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|-----------------------------------------------|------------------------------------------------------------------------------------|
| COMPUTATIONAL<br>MODEL              |                                                                                            |                                                                                                                                                                     |                                                                                                   |                                               |                                                                                    |
| FORWARD<br>TRACKING<br>RAY<br>MODEL | Reasonably<br>gentle and<br>regular<br>depth varia-<br>tions. Poor<br>for shoal<br>systems | Unlimited.<br>Element<br>size deter-<br>mined by<br>depth<br>variations                                                                                             | Any                                                                                               | Many, covering<br>entire modelled<br>sea area | Single period<br>and direction.<br>Spectrum can be<br>covered by<br>multiple runs. |
| BACK<br>TRACKING<br>RAY MODEL       | Reasonably<br>gentle depth<br>variations                                                   | Unlimited.<br>Element<br>size deter-<br>mined by<br>depth<br>variations                                                                                             | Any, except<br>where exter-<br>nal diffrac-<br>tion import-<br>ant (eg<br>shelter by<br>headland) | One                                           | Homogeneous<br>period and<br>direction<br>spectrum                                 |
| FINITE<br>DIFFERENCE<br>MODEL       | Reasonably<br>gentle and<br>regular<br>depth varia-<br>tions. Poor<br>for shoal<br>systems | Unlimited.<br>Element<br>size deter-<br>mined by<br>depth<br>variations                                                                                             | Reasonably<br>straight<br>coast<br>facing open<br>sea                                             | Many, covering<br>entire modelled<br>sea area | Period and<br>direction spec-<br>trum (need not<br>be homogeneous)                 |
| PARABOLIC<br>MODEL                  | Reasonably<br>gentle depth<br>variations                                                   | Limited to<br>few kilo-<br>metres at<br>most, often<br>considerably<br>less.<br>Element size<br>determined<br>by minimum<br>number of<br>elements per<br>wavelength | Reasonably<br>straight<br>coast facing<br>open sea                                                | Many, covering<br>entire modelled<br>sea area | Period and<br>direction spec-<br>trum (need not<br>be homogeneous)                 |

**TABLE 3    CARRICKFERGUS HARBOUR.    RESULTS USING FORWARD-TRACKING RAY MODEL**

FORWARD TRACKING RAY MODEL

OFFSHORE CONDITIONS

|                         |          |
|-------------------------|----------|
| MEDIAN PERIOD           | 8.80 s   |
| SIGNIFICANT WAVE HEIGHT | 5.68 m   |
| RMS WAVE HEIGHT         | 4.02 m   |
| MEAN DIRECTION          | 100 °N   |
| MEDIAN DIRECTION        | 88.98 °N |

INSHORE CONDITIONS

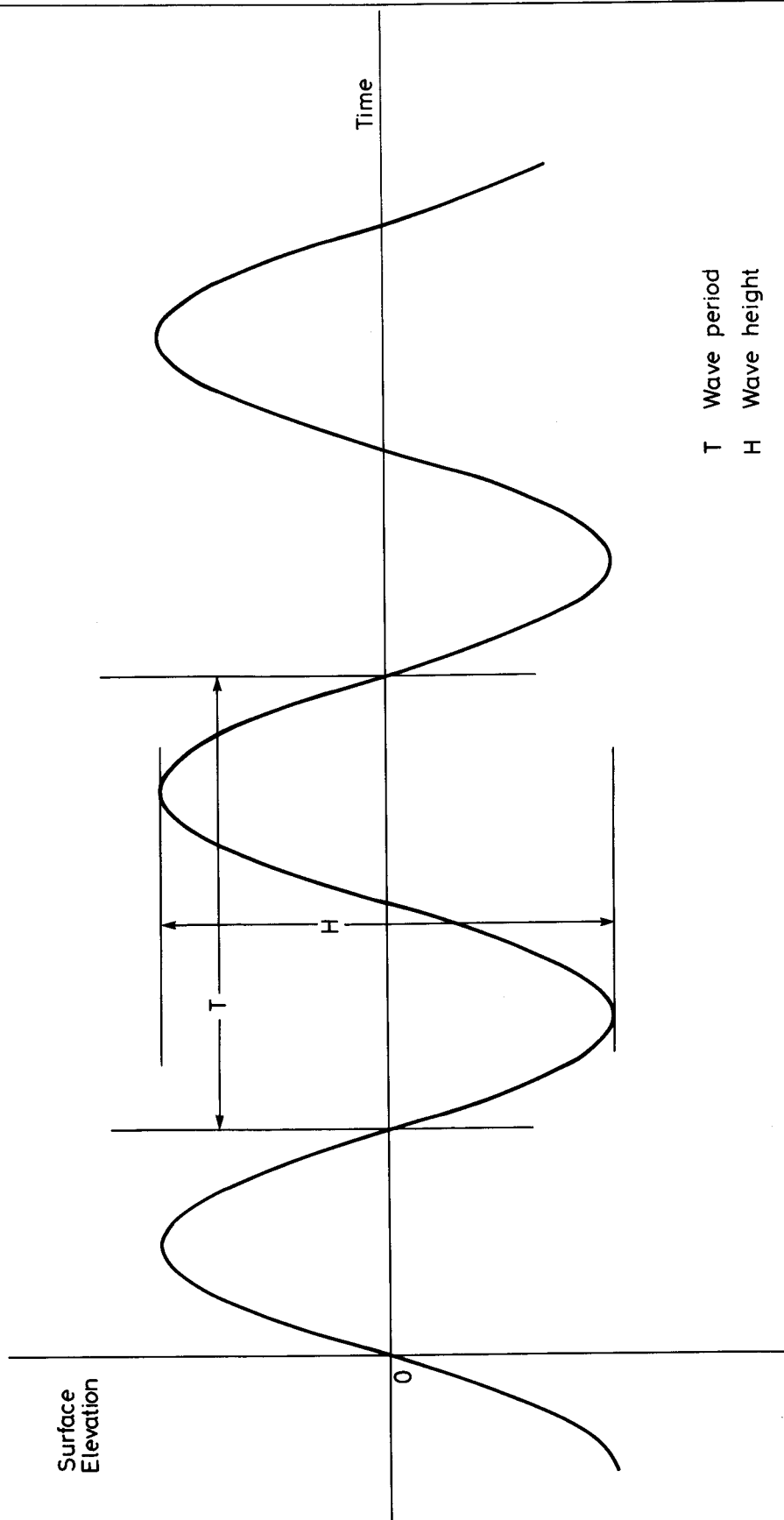
|                                           |          |
|-------------------------------------------|----------|
| MEAN DIRECTION                            | 98.29 °N |
| SIGNIFICANT WAVE HEIGHT (no friction)     | 3.80 m   |
| SIGNIFICANT WAVE HEIGHT (friction = 0.02) | 2.45 m   |
| SIGNIFICANT WAVE HEIGHT (friction = 0.04) | 1.82 m   |
| SIGNIFICANT WAVE HEIGHT (friction = 0.06) | 1.48 m   |





## **Figures**





T Wave period  
H Wave height

Fig 1 Regular sinusoidal wave

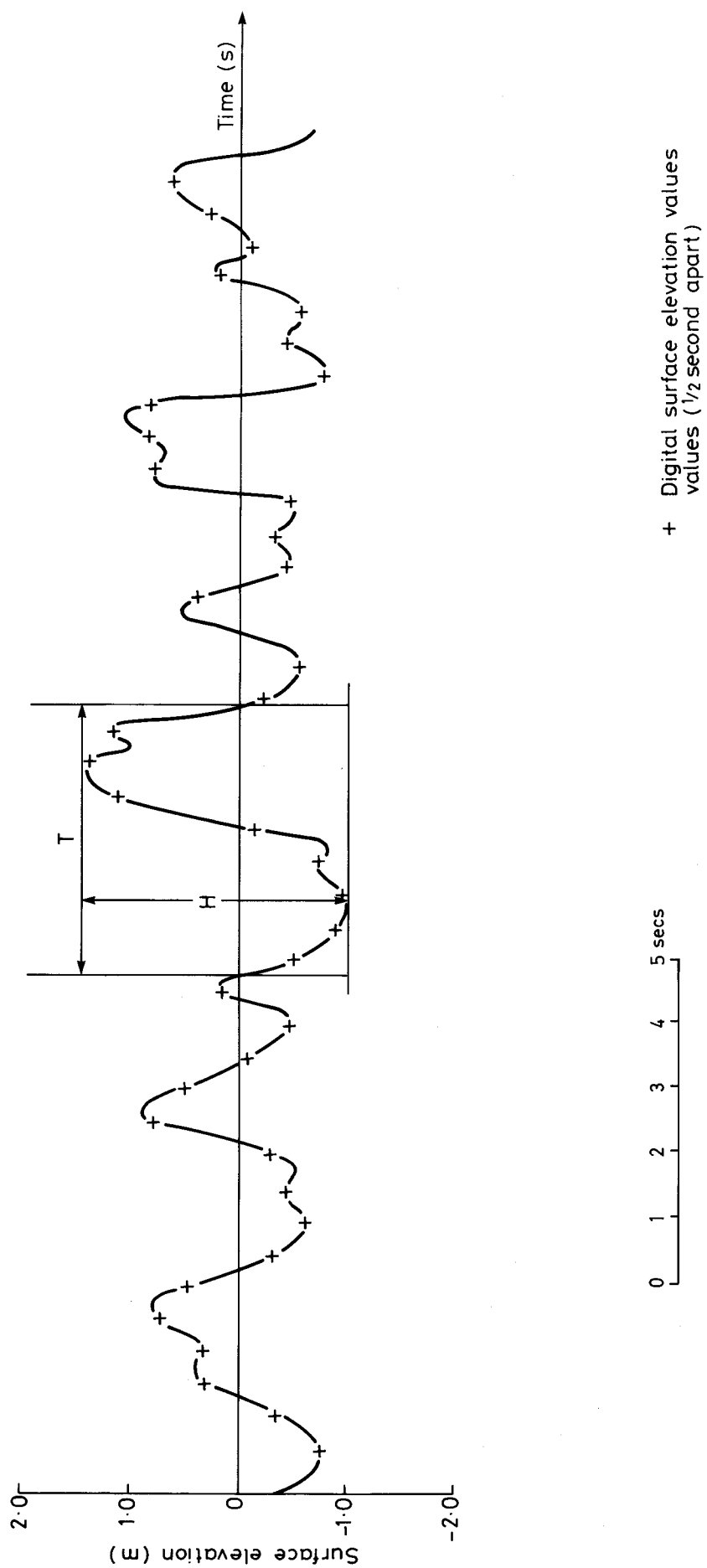


Fig 2 Wave trace of an irregular sea. Taken from measurements made at Perranporth, Cornwall

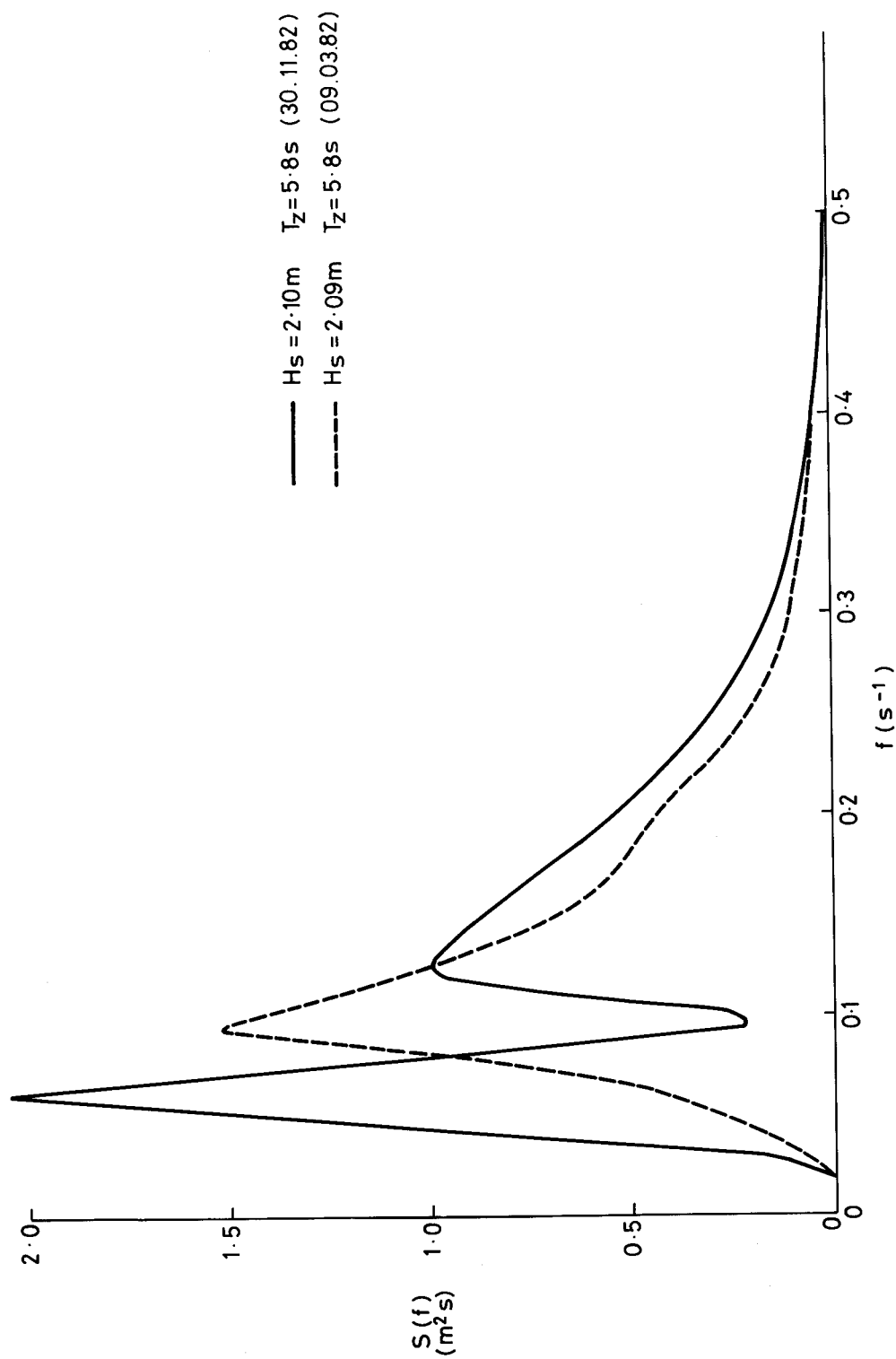


Fig 3 Comparison of two wave spectra with the same  $T_z$ . Taken from measurements made at Perranporth, Cornwall

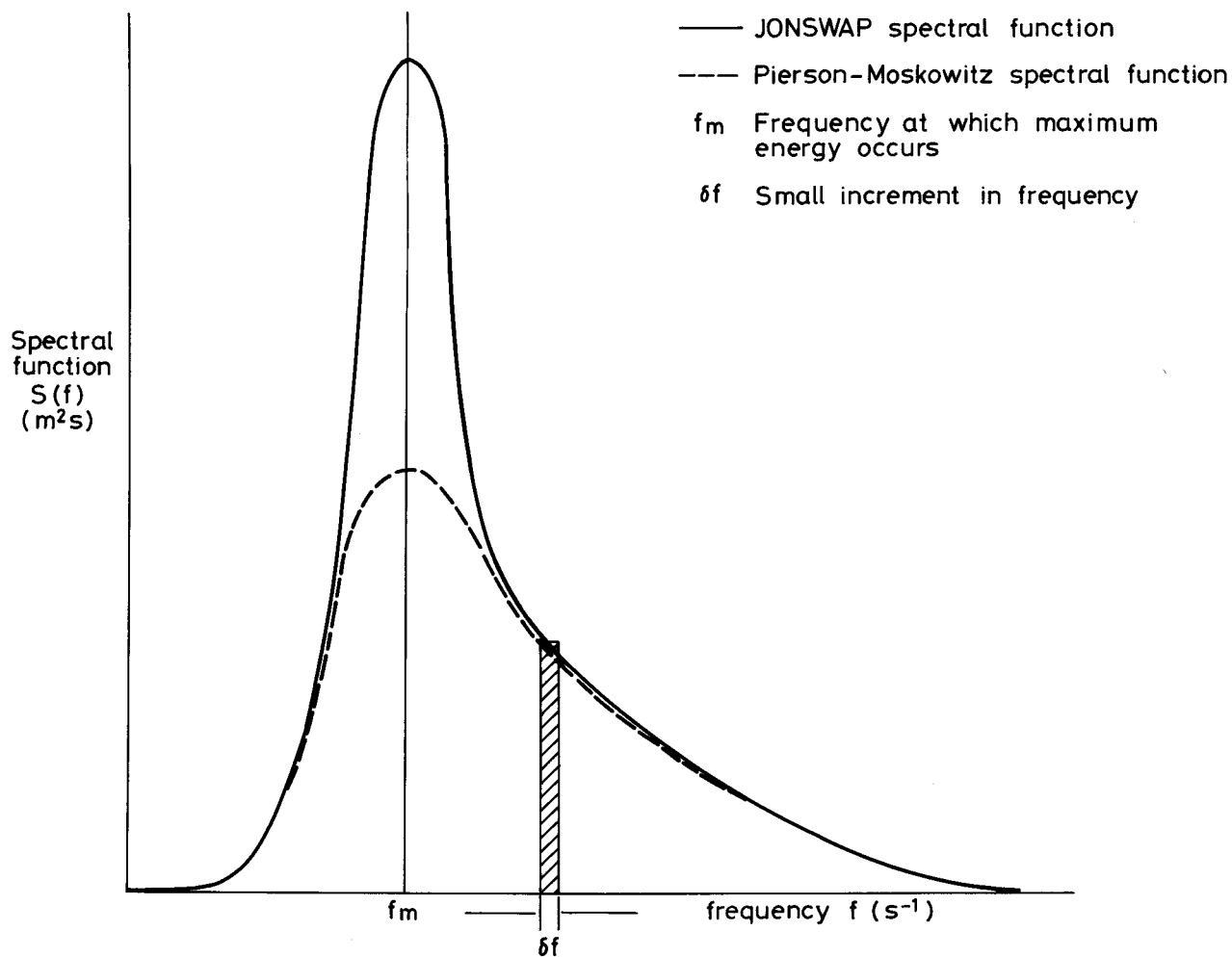


Fig 4 Typical JONSWAP and Pierson-Moskowitz spectra

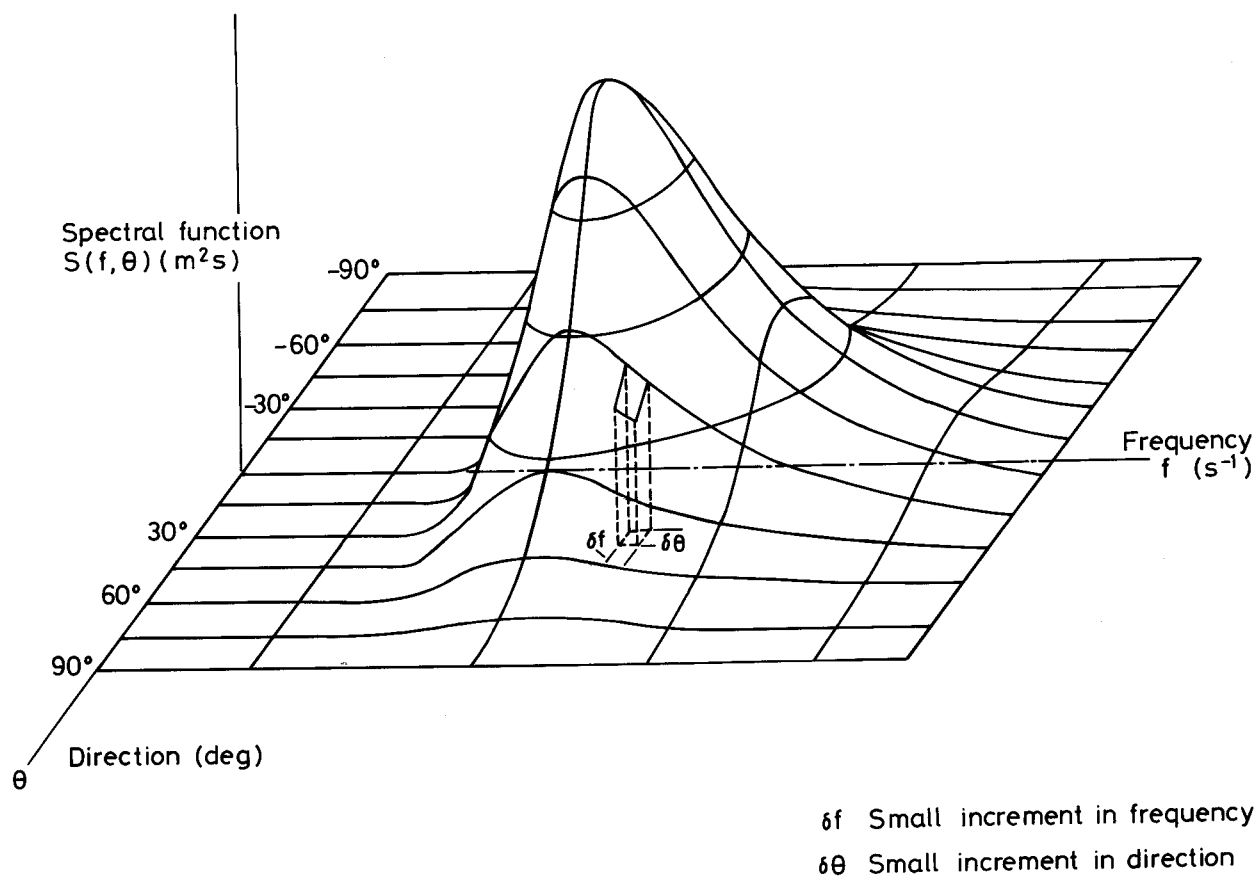


Fig 5 Example of a Two-dimensional spectrum in frequency and direction

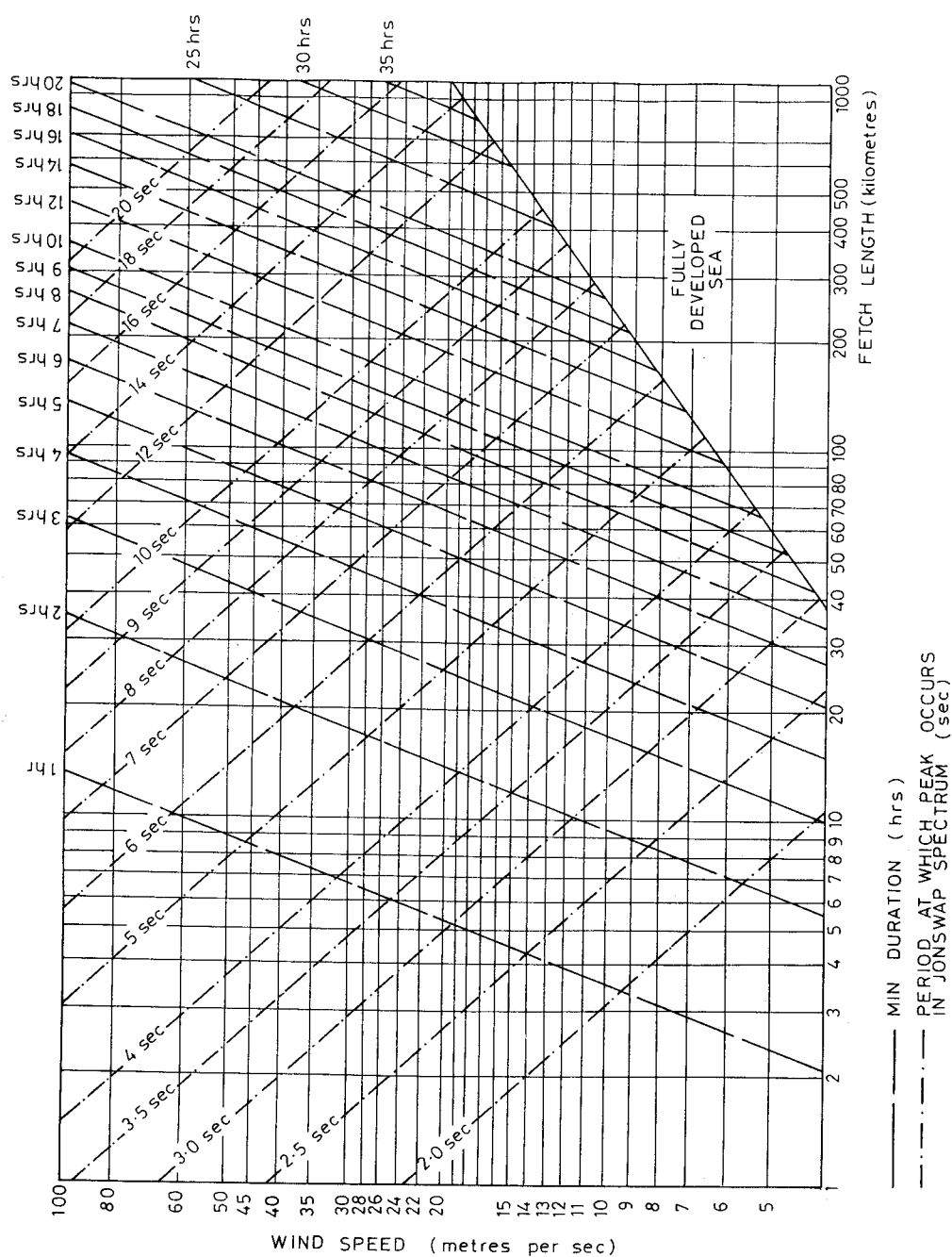


Fig 6 Deep-water wave forecasting curves for JONSWAP spectrum. Peak periods



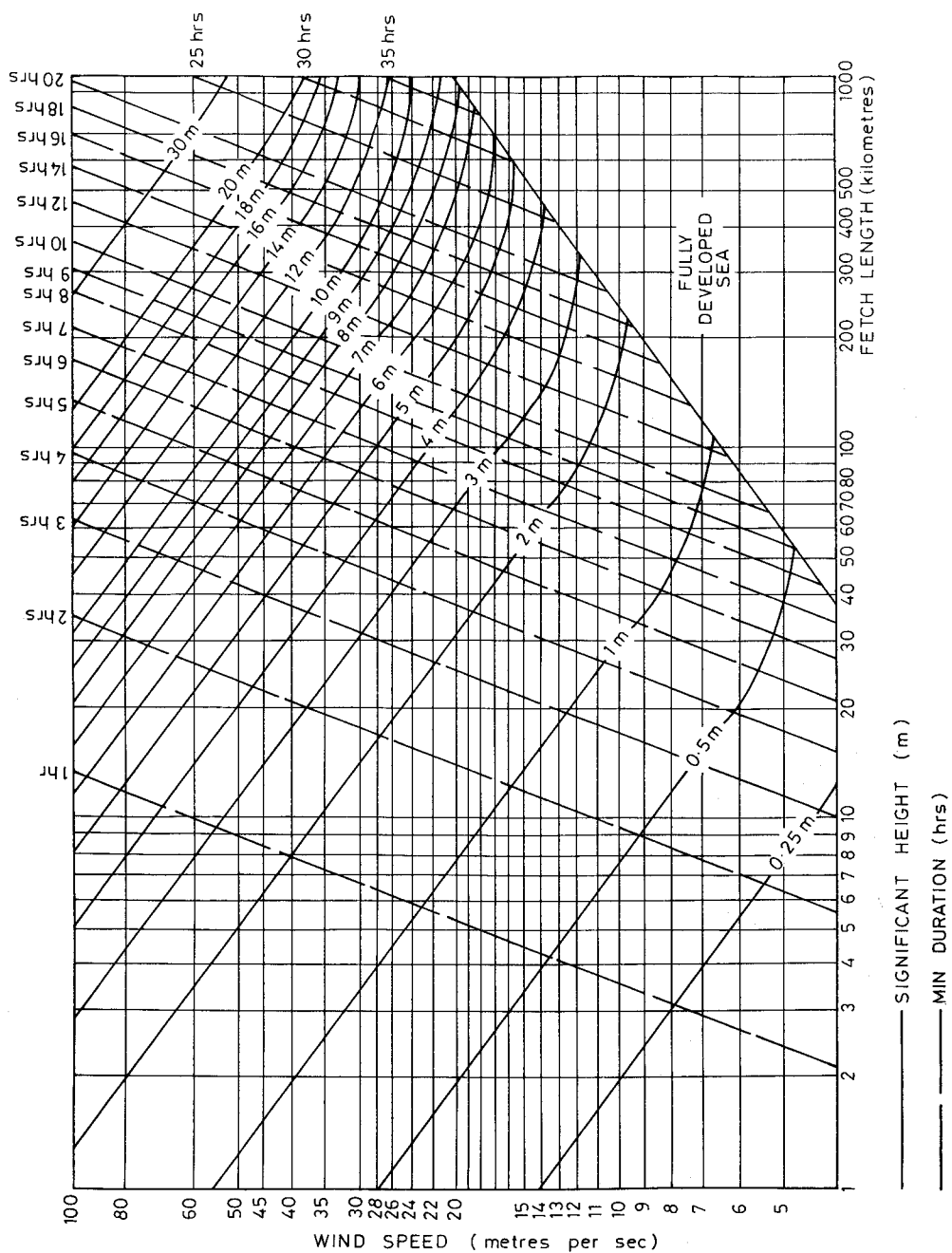


Fig 7 Deep-water wave forecasting curves for JONSWAP spectrum.  
Significant wave heights

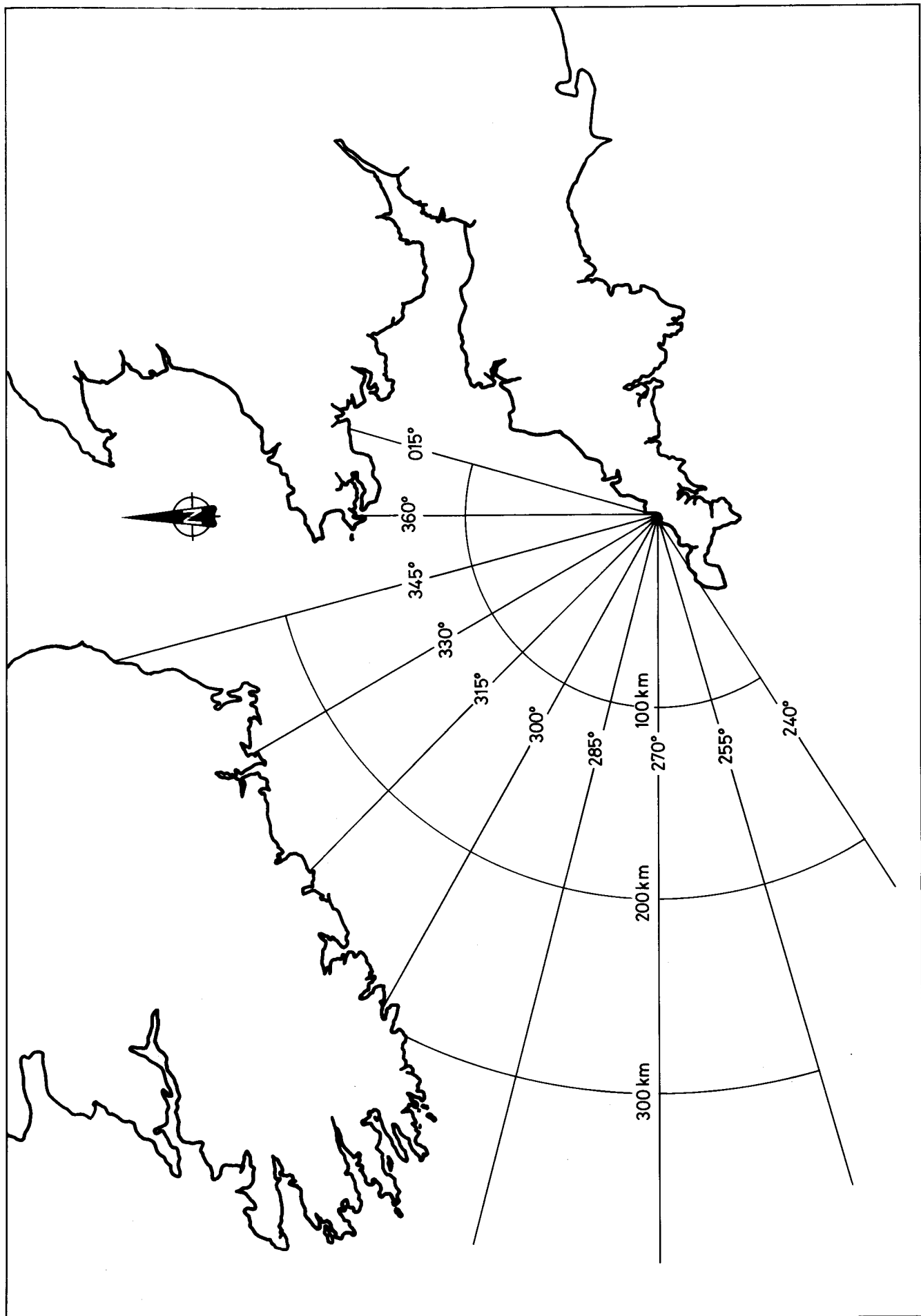


Fig 8 Fetch lines for a location near Perranporth, Cornwall

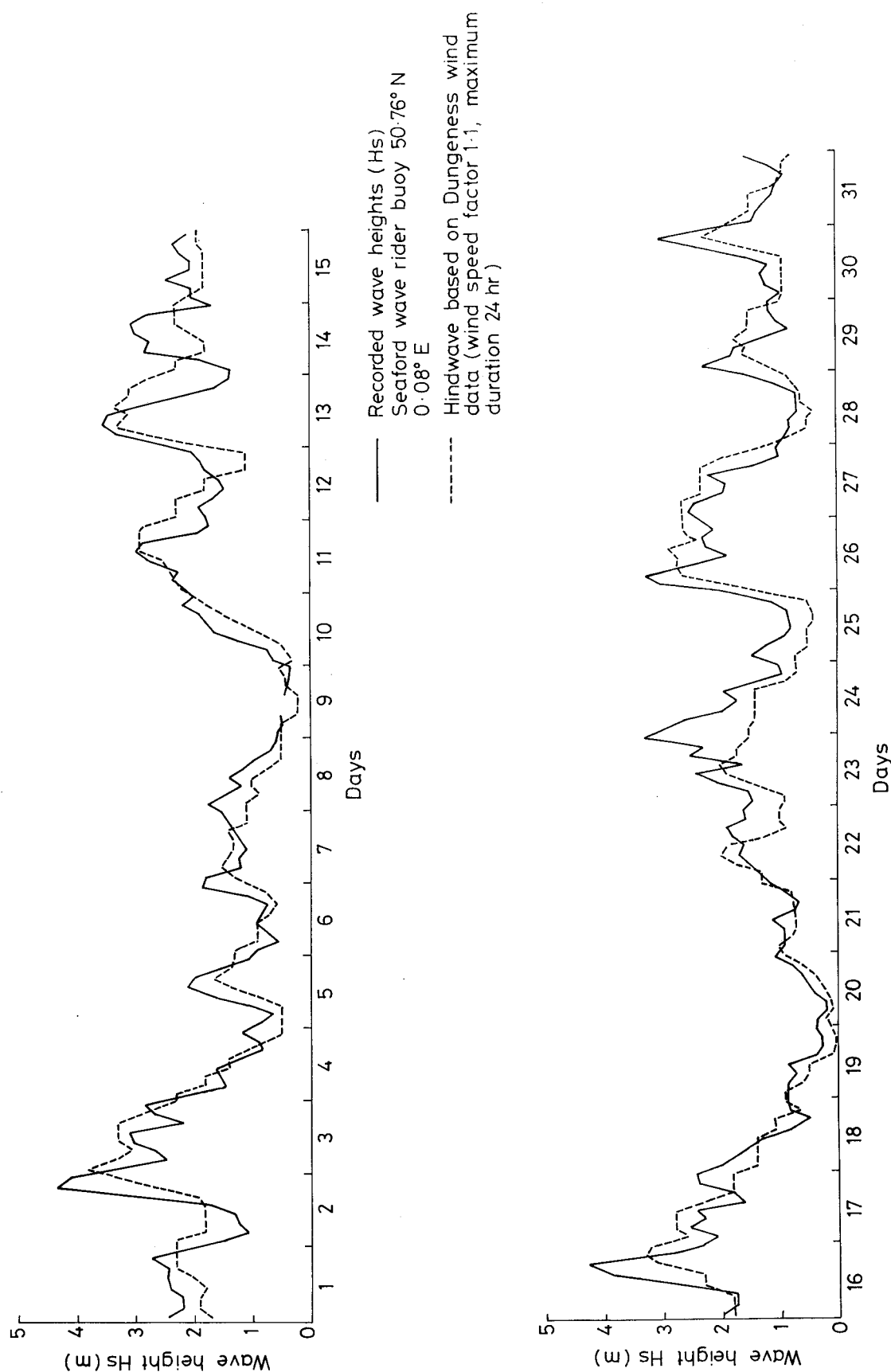


Fig 9 Comparison of recorded wave heights and hindcast wave heights. Seaford, January 1984

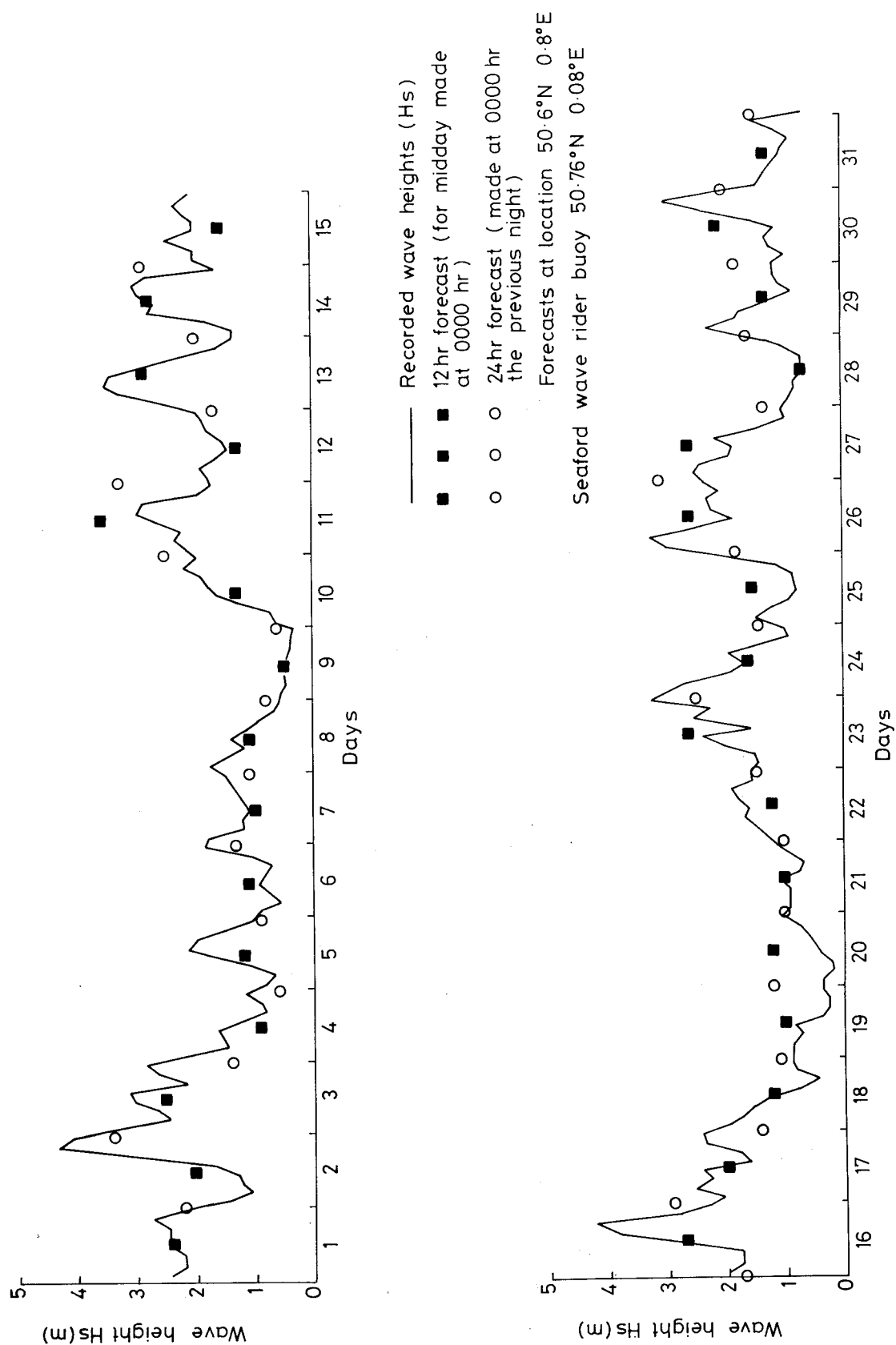


Fig 10 Comparison of recorded wave heights and forecast wave heights using the Met. Office computational model. Seaford, January 1984

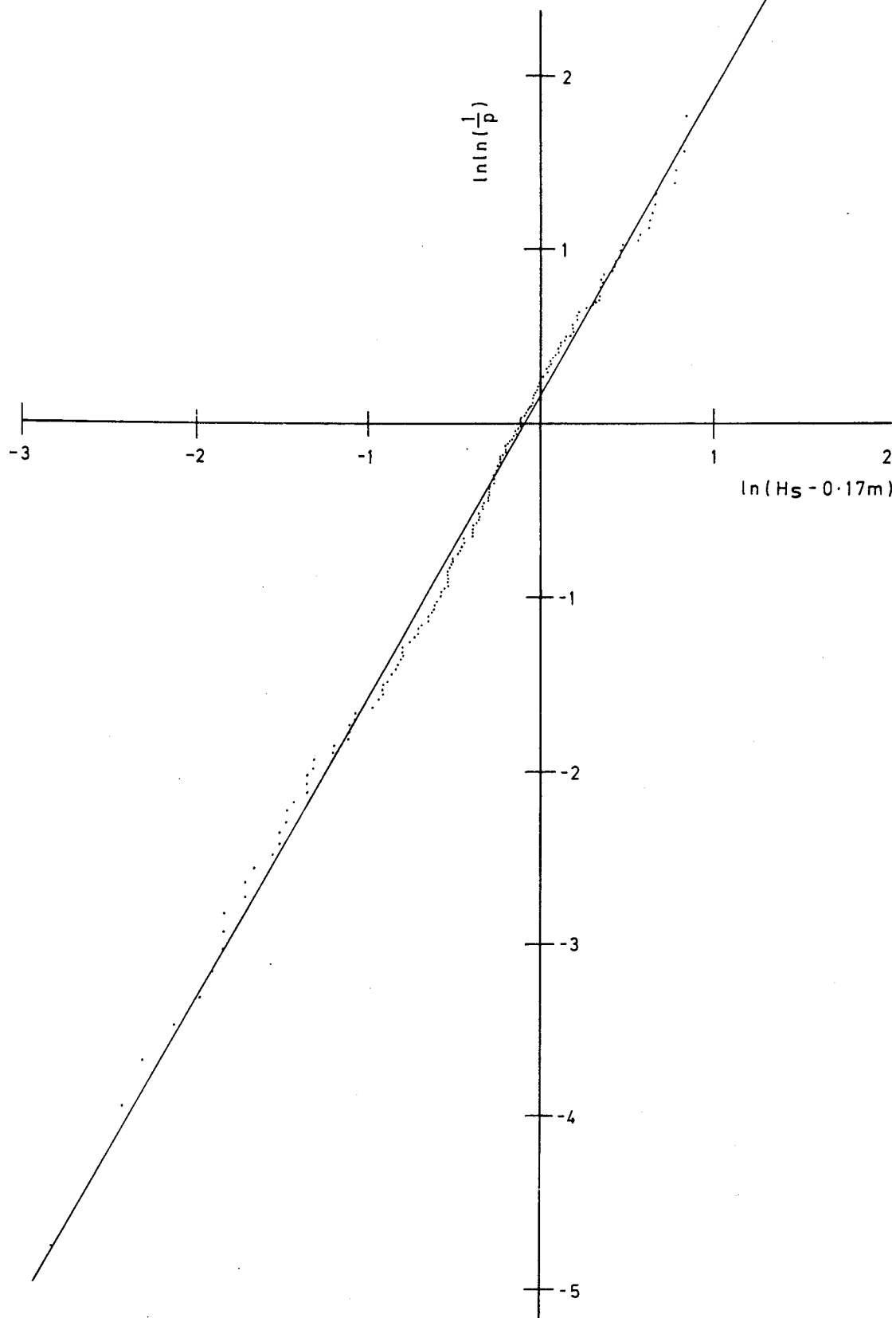


Fig 11 Weibull distribution of  $H_s$ . 184 points (same data as Fig 12)

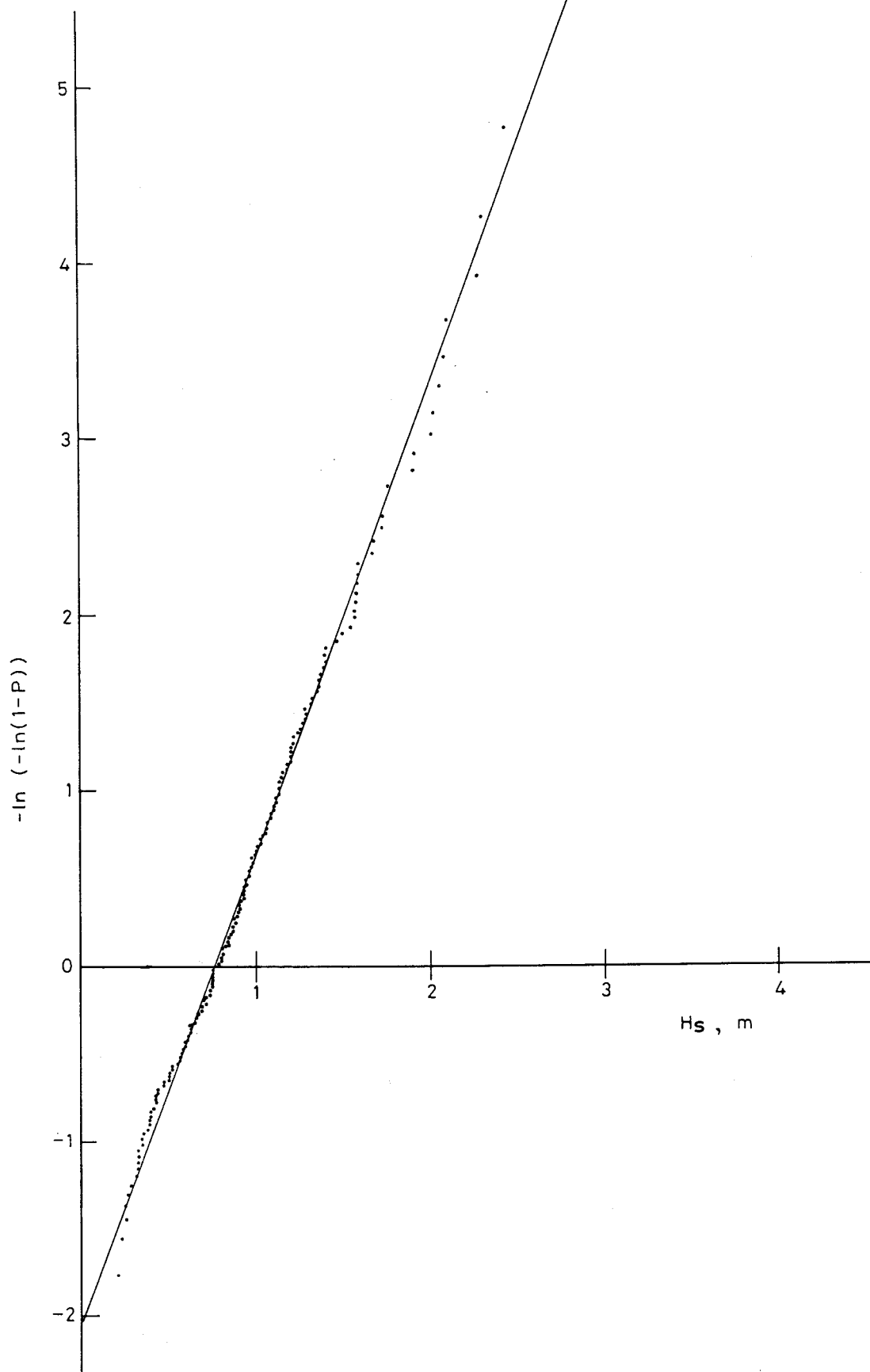


Fig 12 Fisher-Tippett I distribution of  $H_s$ . 184 points (same data as Fig 11)

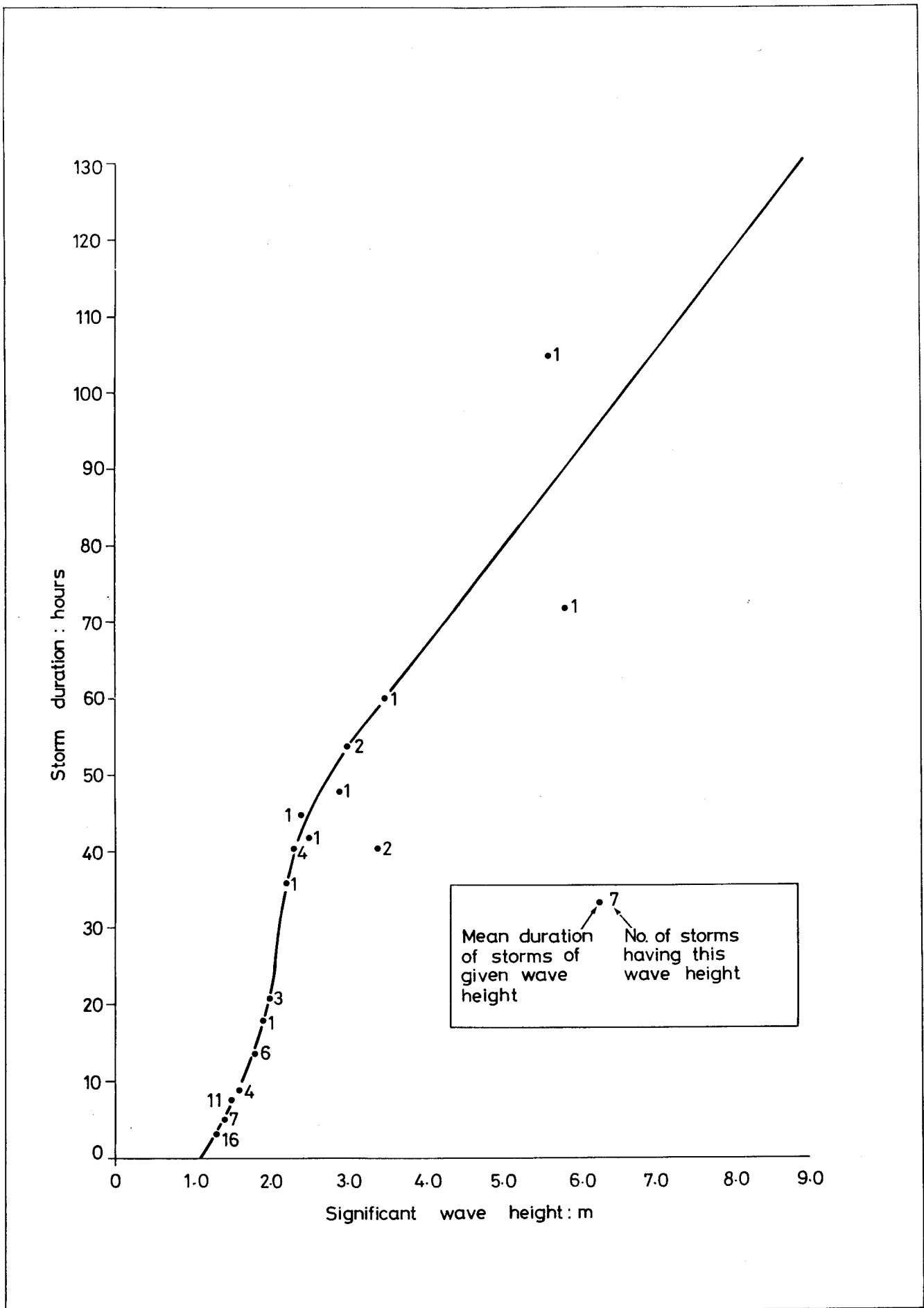


Fig 13 Correlation of storm duration and peak  $H_s$  for storms 1979-80 for a site in the East Mediterranean

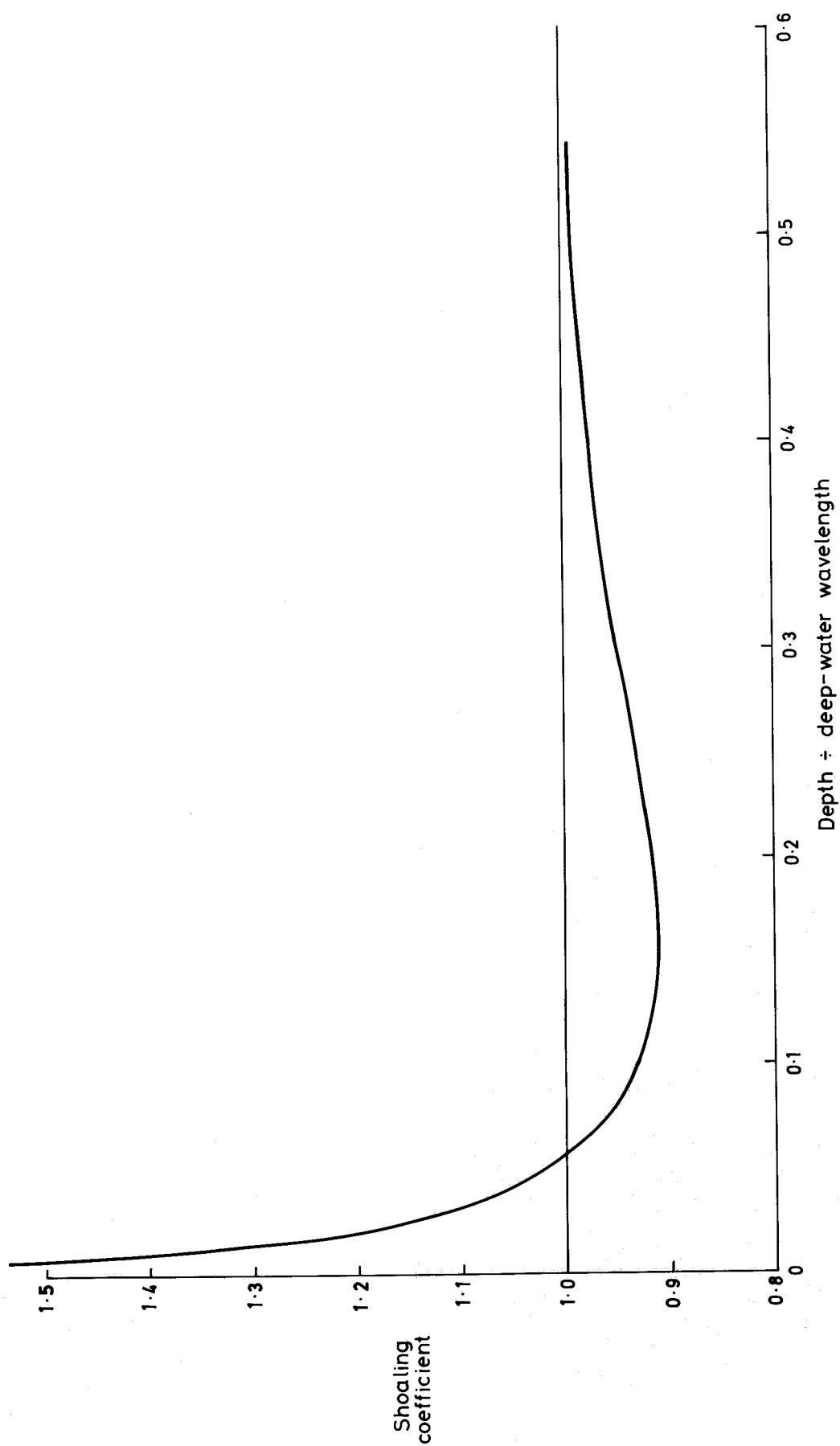


Fig 14 Variation of shoaling coefficient with depth



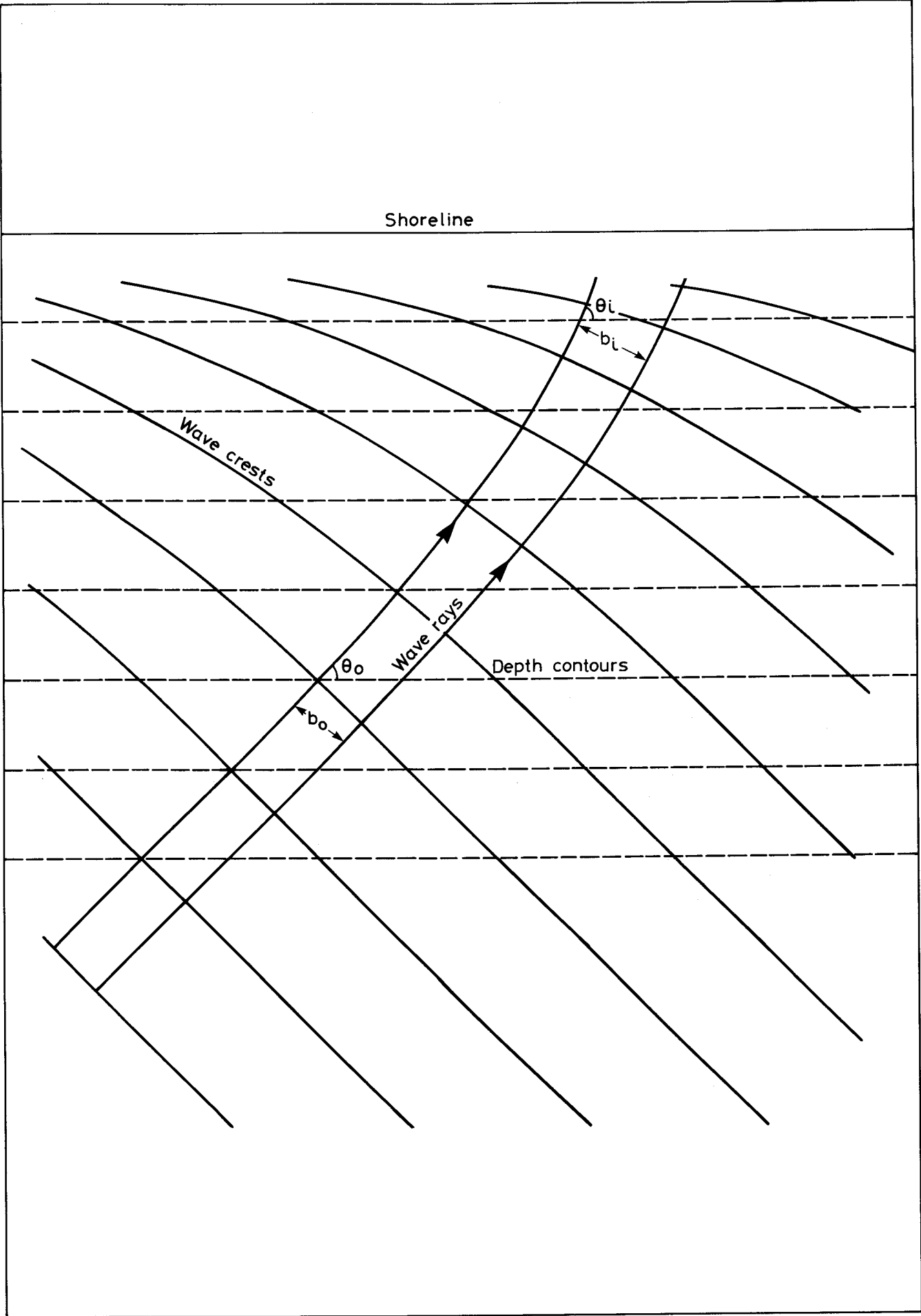


Fig 15    Refraction of waves over a parallel-contoured seabed  
( compare with plate 1 )

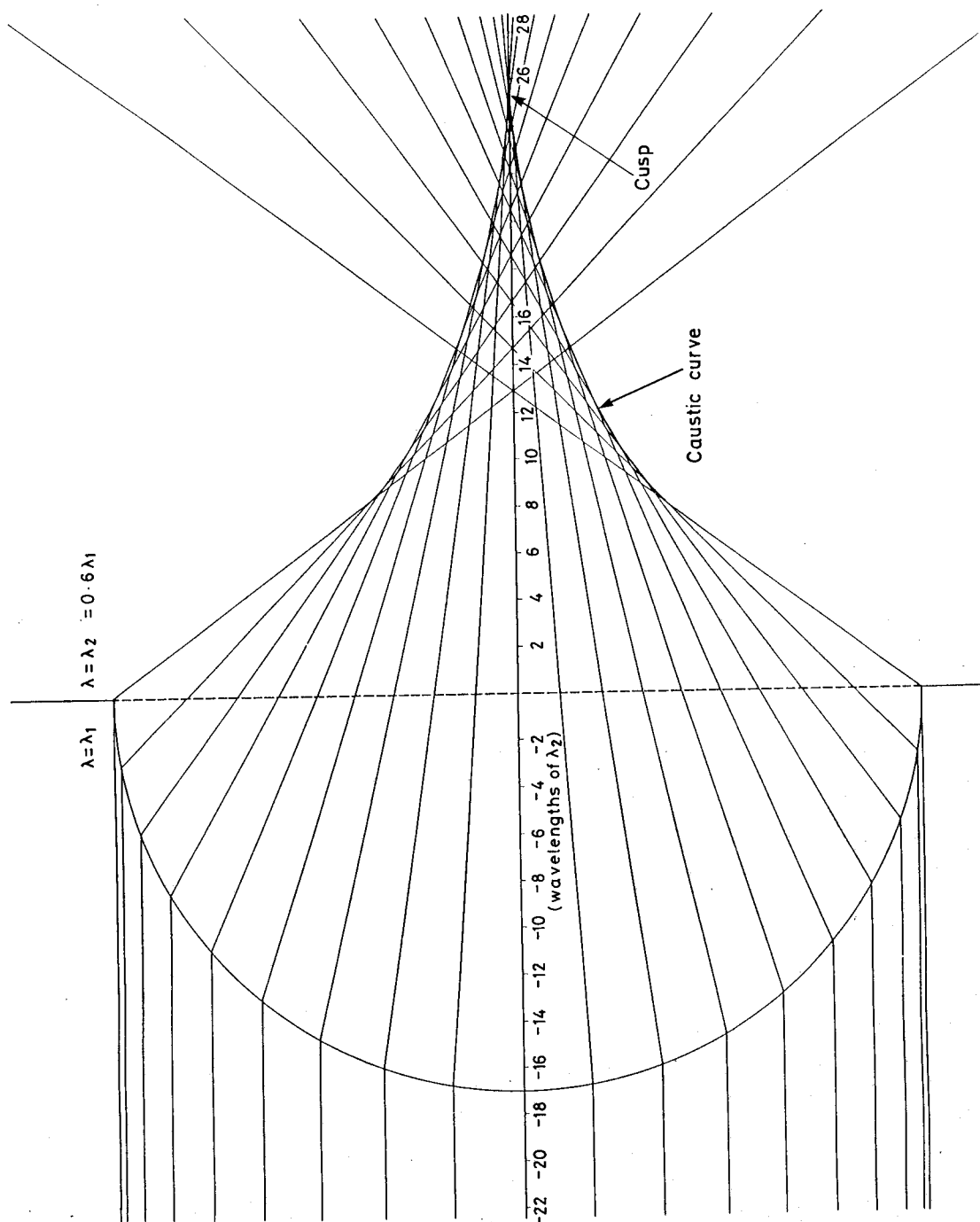


Fig 16 Refraction of waves over a semi-circular shelf showing the formation of caustics ( $\lambda$  = wavelength)

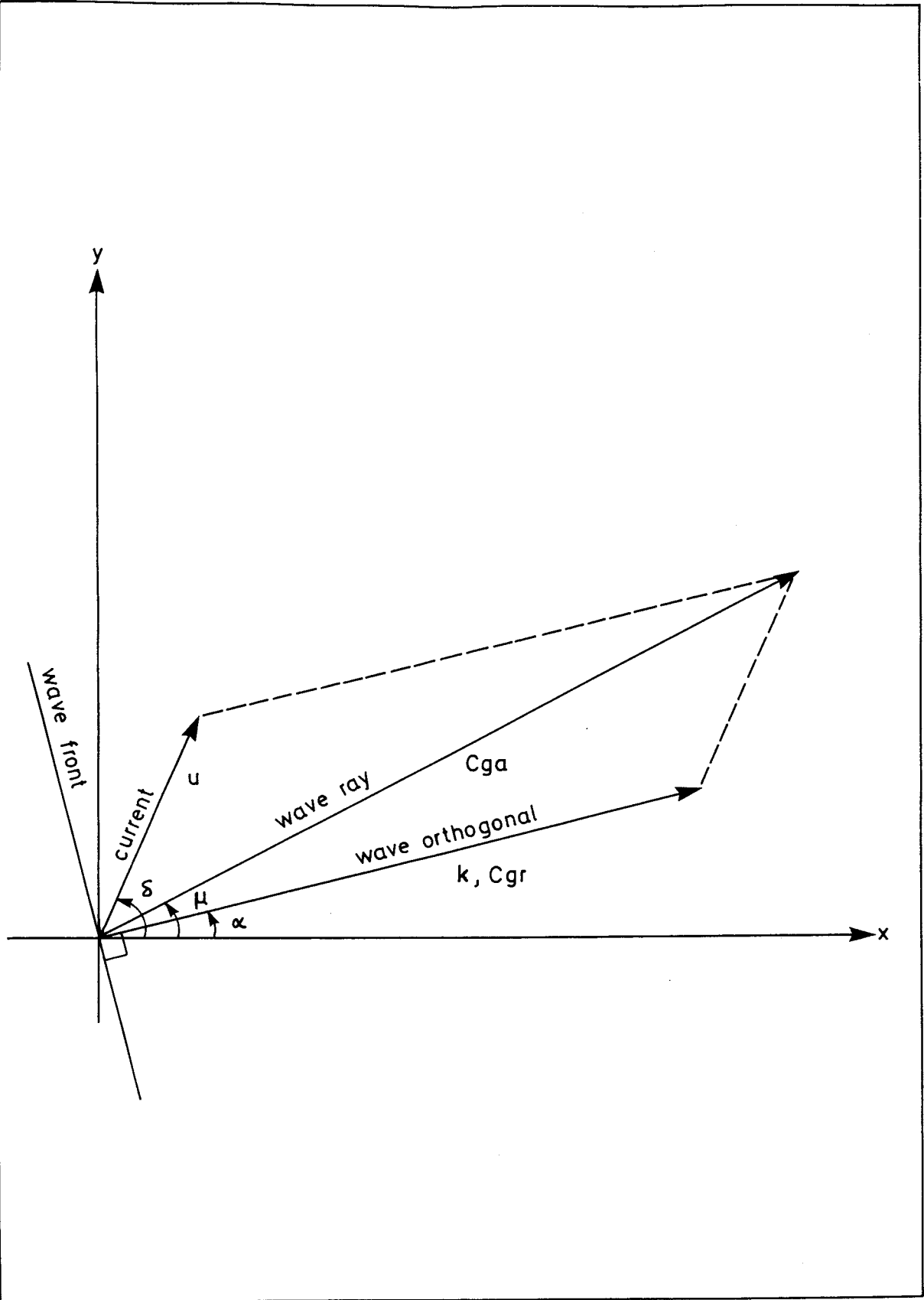


Fig 17    Geometry of currents, wave orthogonals and wave rays

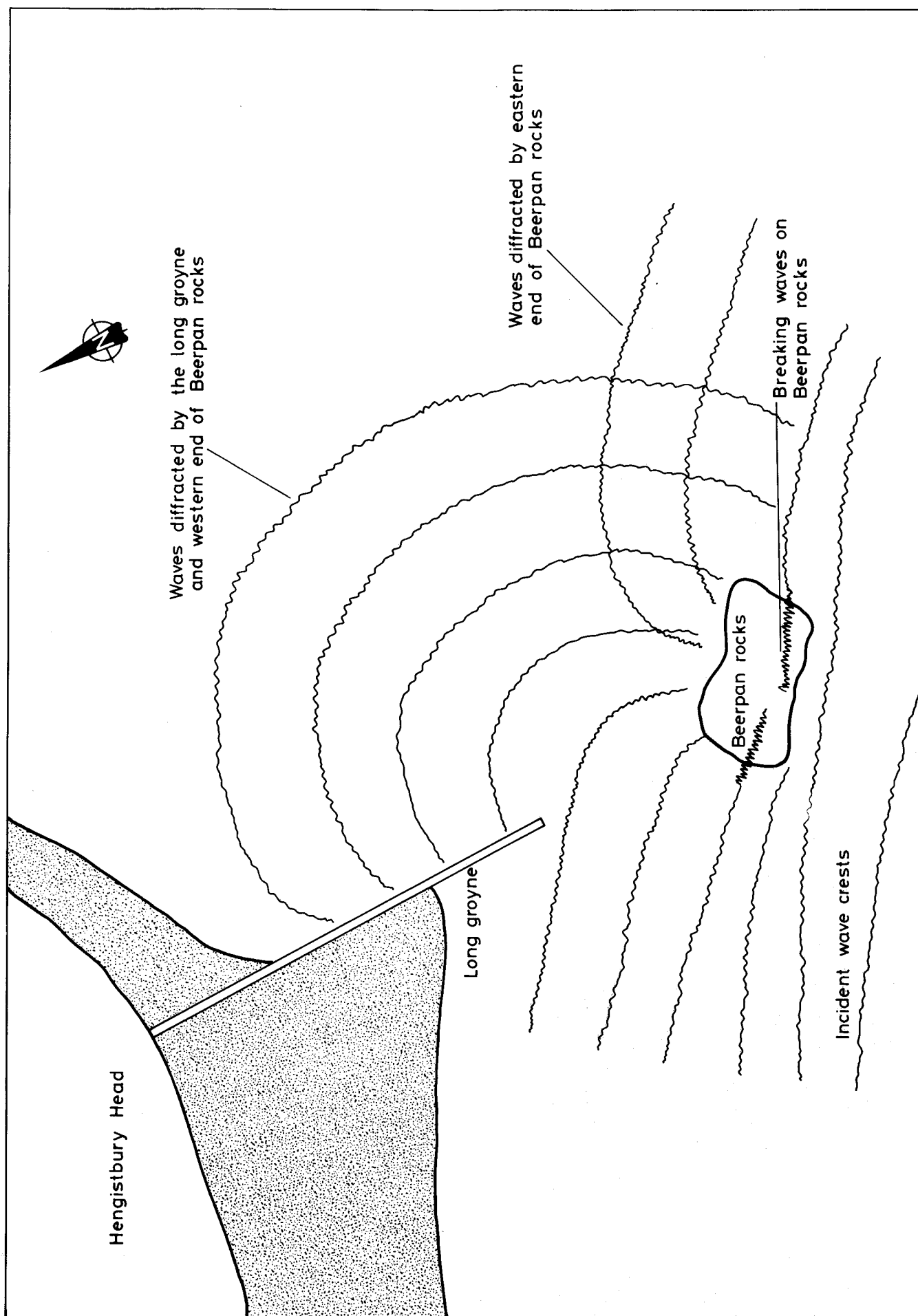


Fig 18 Diffraction of waves around Hengistbury Head and Beerpan rocks on the English south coast. This is a drawing of the aerial photograph in plate 2

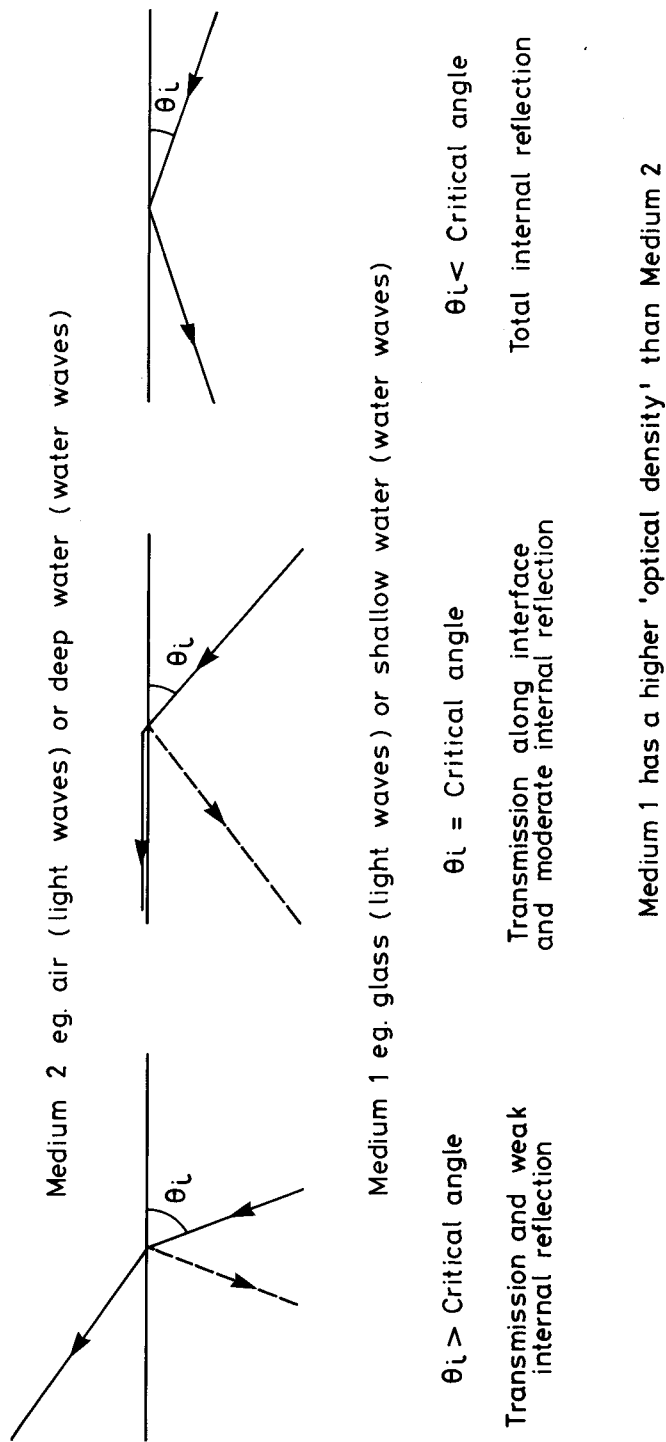


Fig 19 The critical angle and total internal reflection of rays

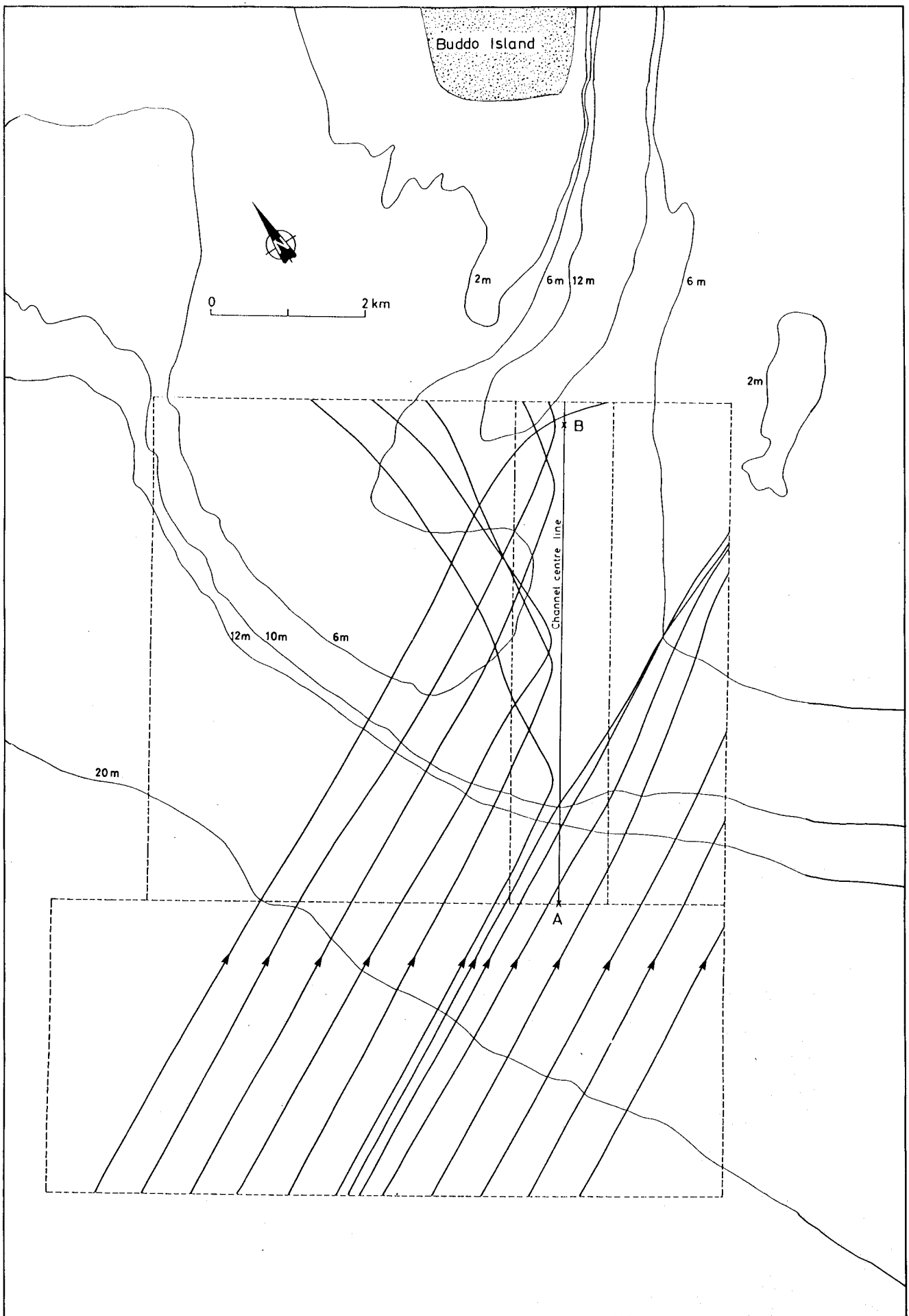
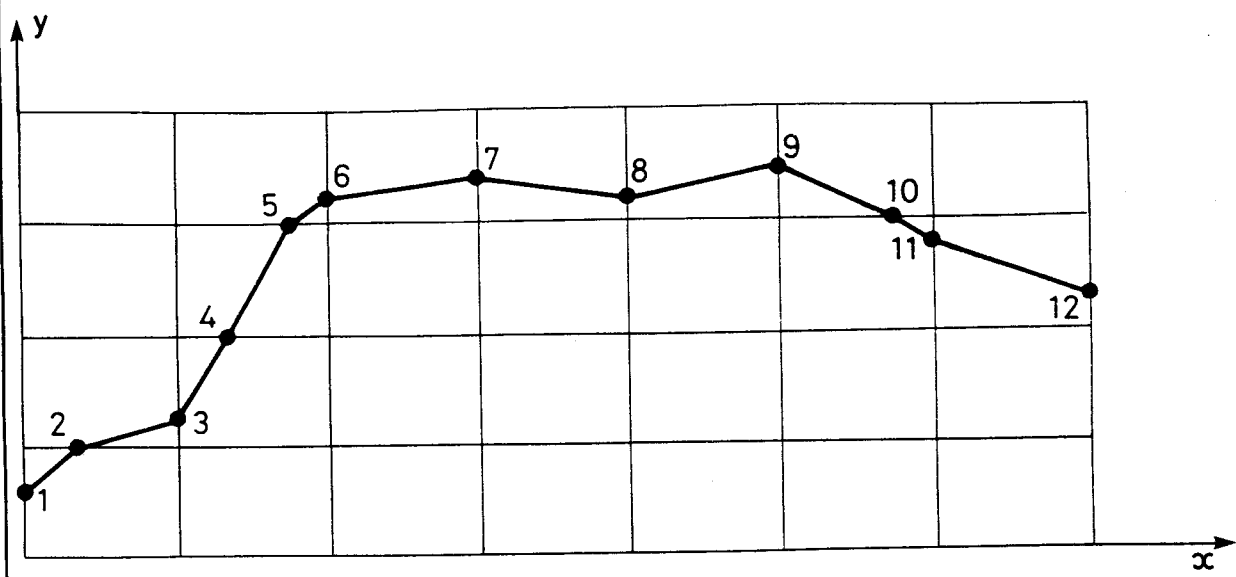


Fig 20 Total internal reflection of rays from the side of a dredged channel at Port Qasim, Pakistan

Part of a Refraction Grid (with subdivision into triangles omitted for clarity)



Input data to forward tracking ray model

| x - coordinate | y - coordinate | Reflection coefficient |
|----------------|----------------|------------------------|
| $x_1$          | $y_1$          | $R_1$                  |
| $x_2$          | $y_2$          | $R_2$                  |
| $x_3$          | $y_3$          | $R_3$                  |
| .....          | .....          | ....                   |
| $x_{12}$       | $y_{12}$       | $R_{12}$               |

Fig 21 Representation of reflecting boundaries in the forward-tracking ray model

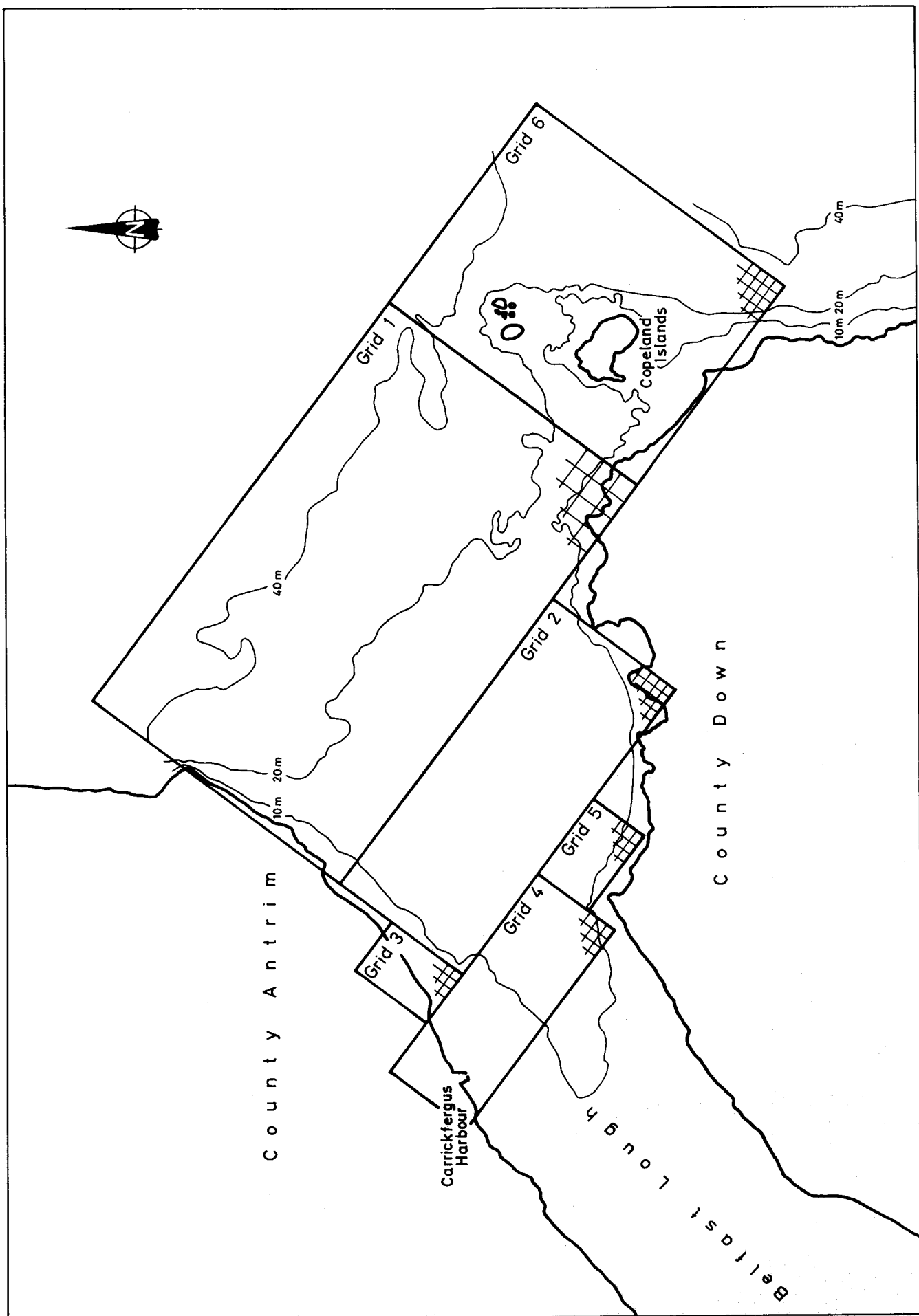


Fig 22 Location map of Carrickfergus harbour showing depth contours and grid system



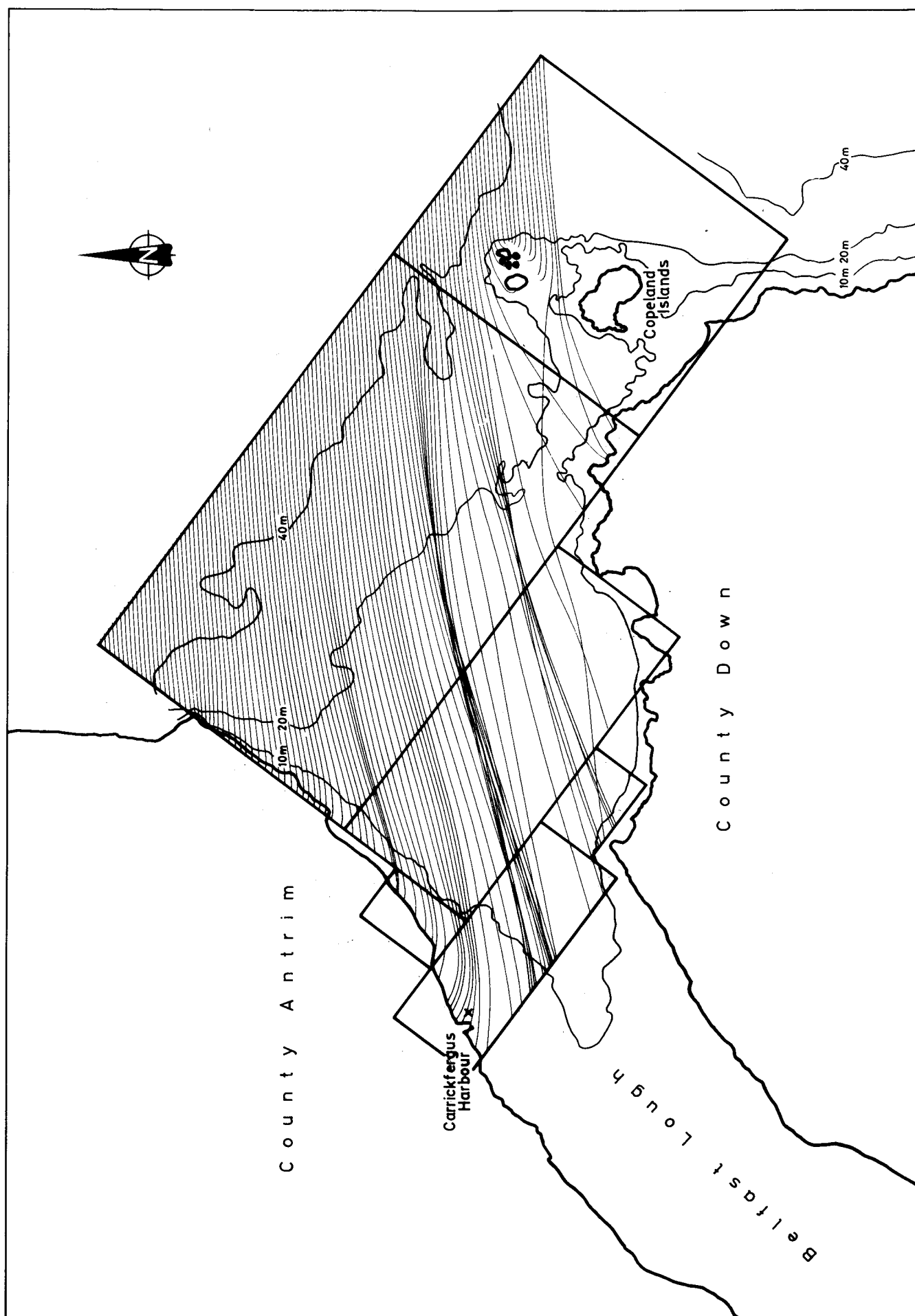


Fig 23 Carrickfergus Harbour study. Ray path diagram using the forward-tracking ray model

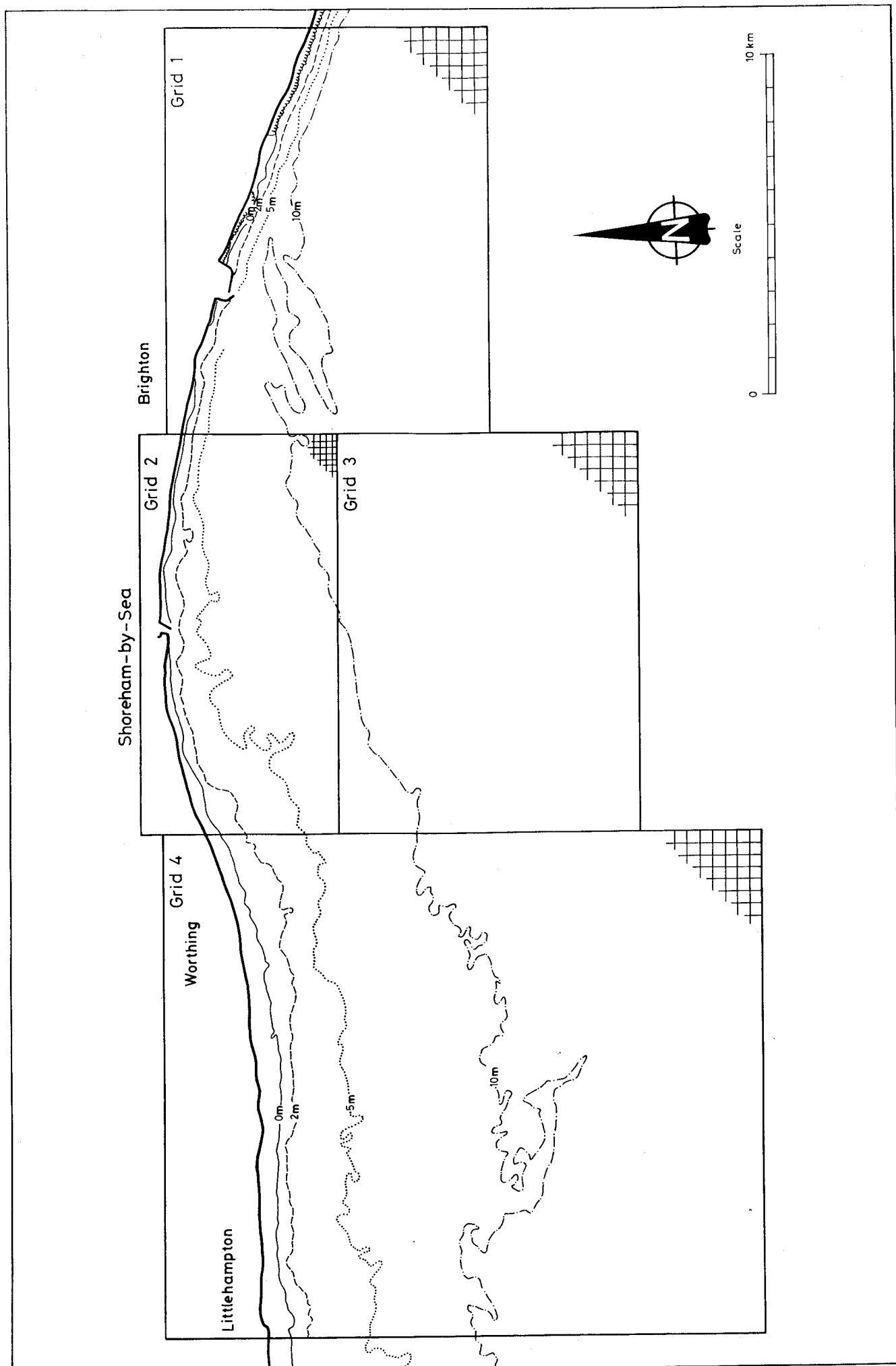


Fig 24 Location map of Shoreham harbour showing depth contours and grid system

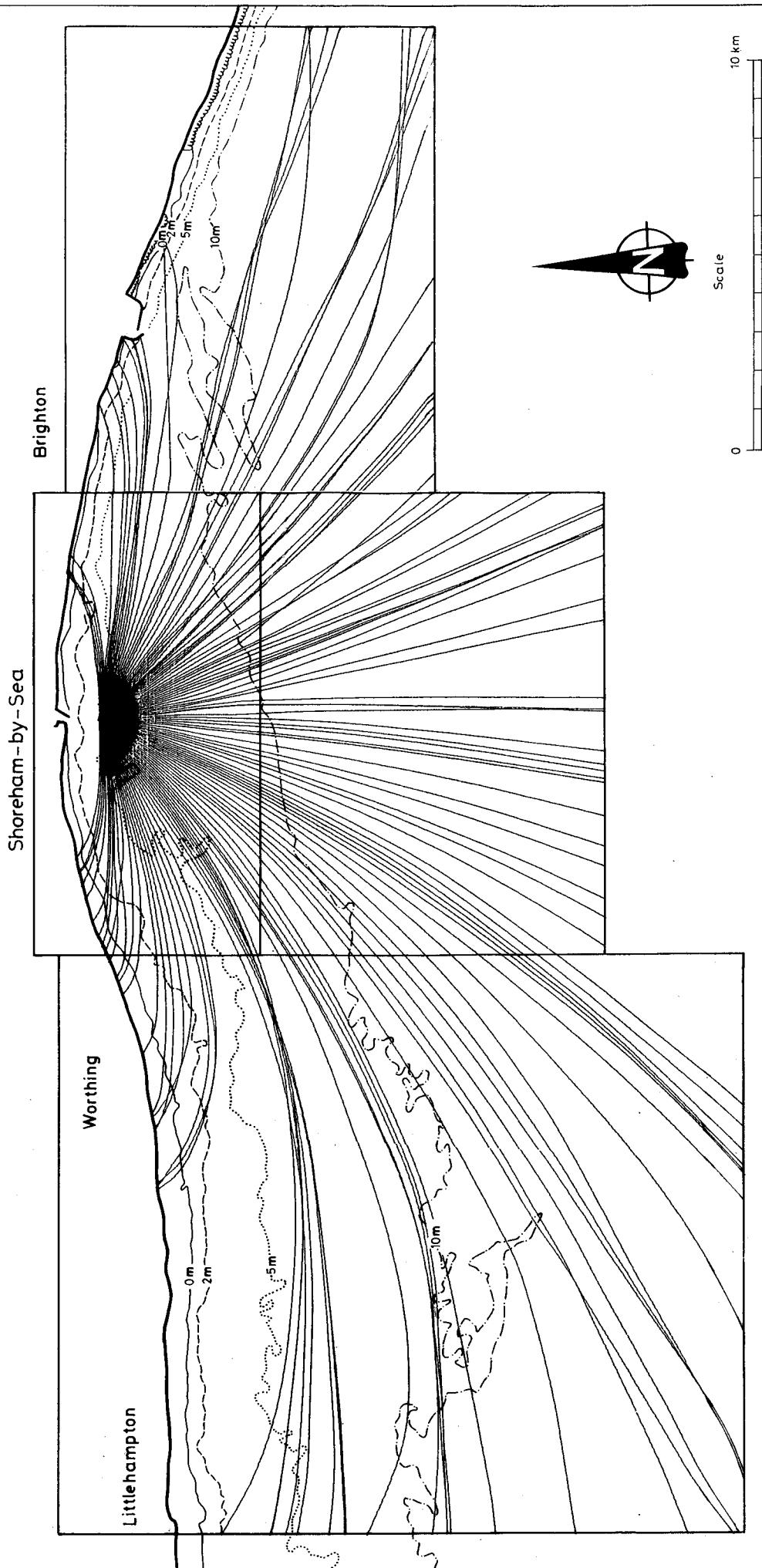


Fig 25 Shoreham harbour study. Ray path diagram using the back-tracking ray model

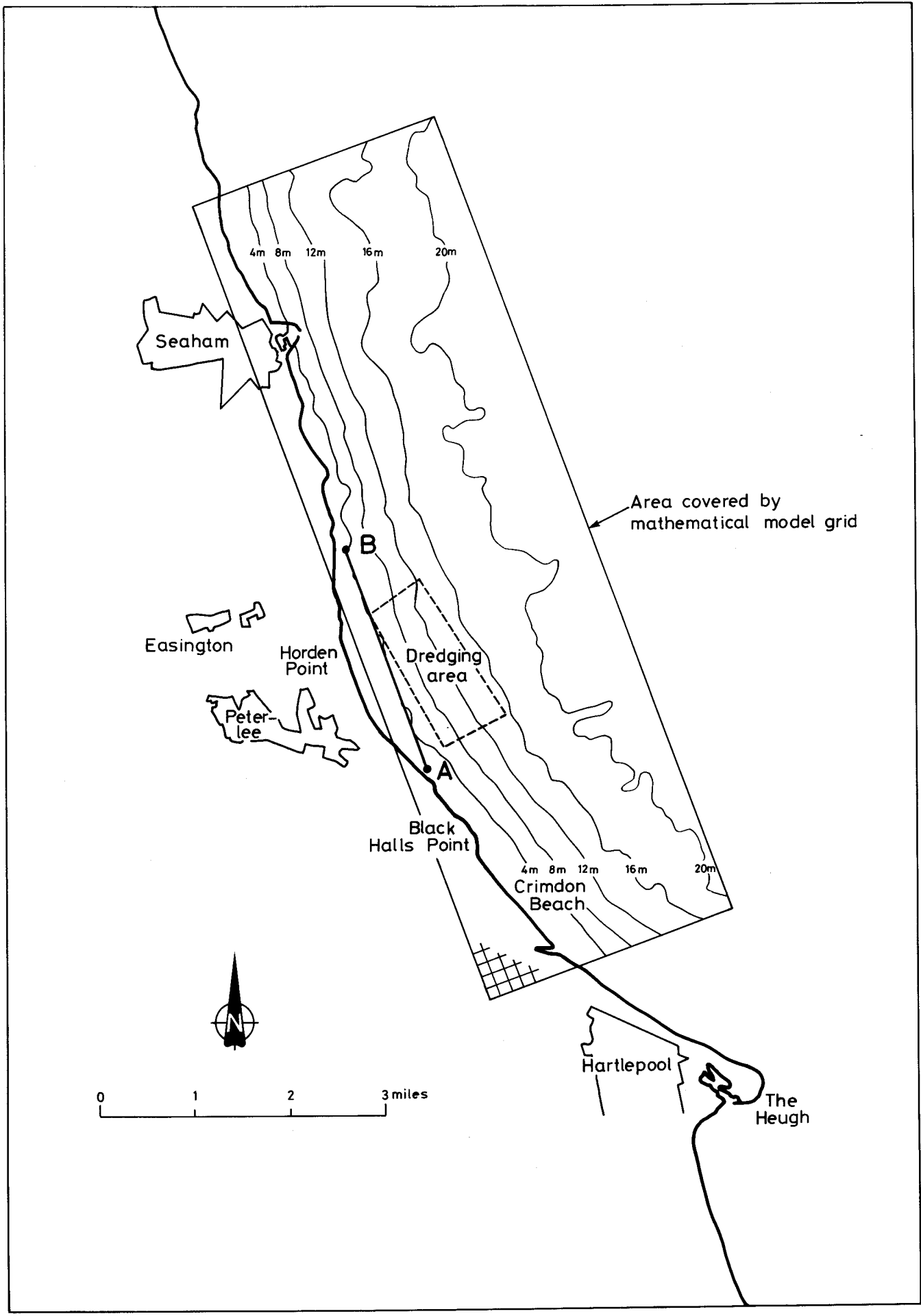


Fig 26 Location map of Durham coast showing depth contours and grid system

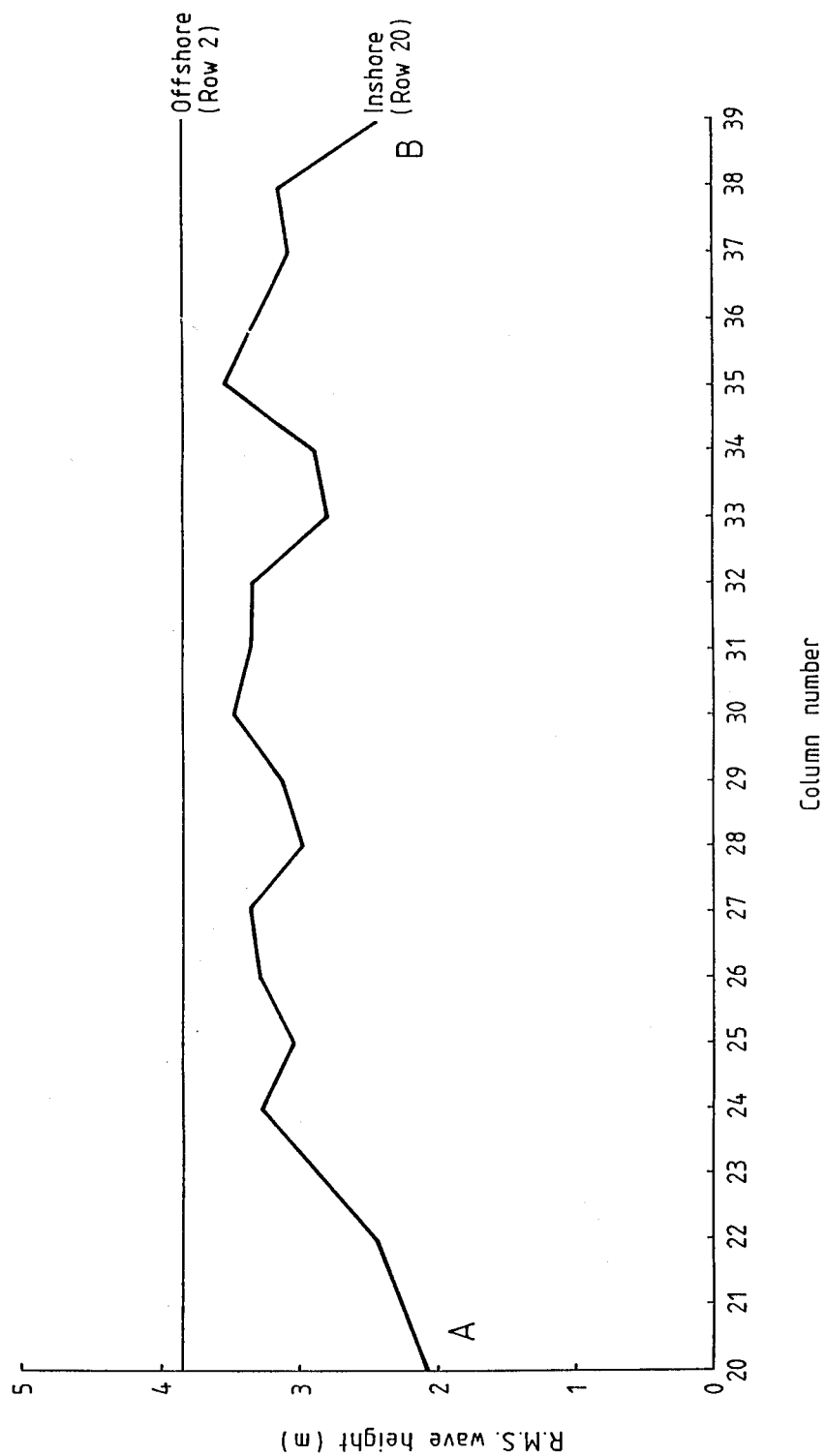


Fig 27 Durham coast study. Inshore R.M.S. wave heights along line A - B



## **Plates**







Plate 1    Coastline at Mudeford Sandspit near Bournemouth, England showing refraction of waves



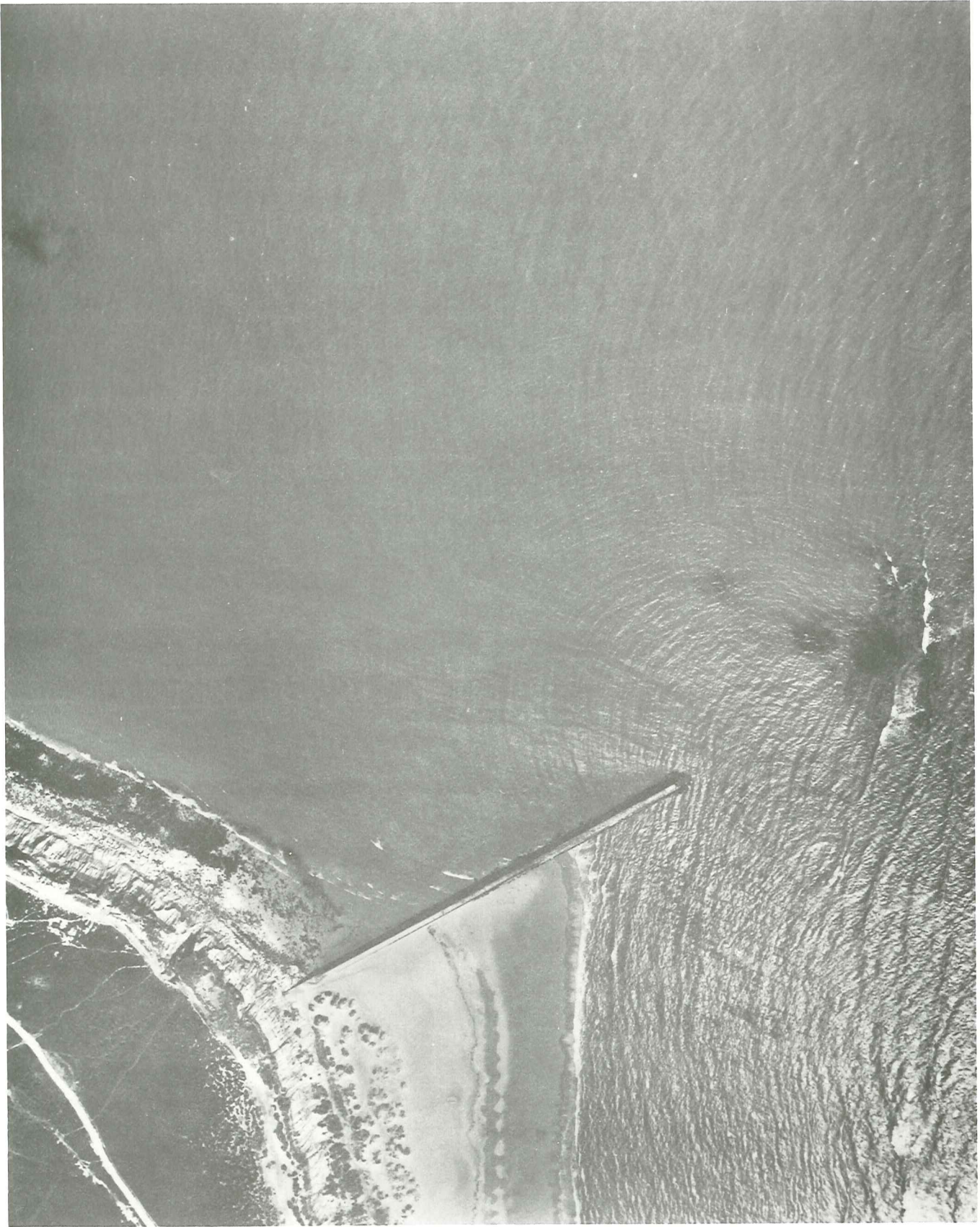


Plate 2    The long groyne at Hengistbury Head near Bournemouth, England  
showing diffraction of waves



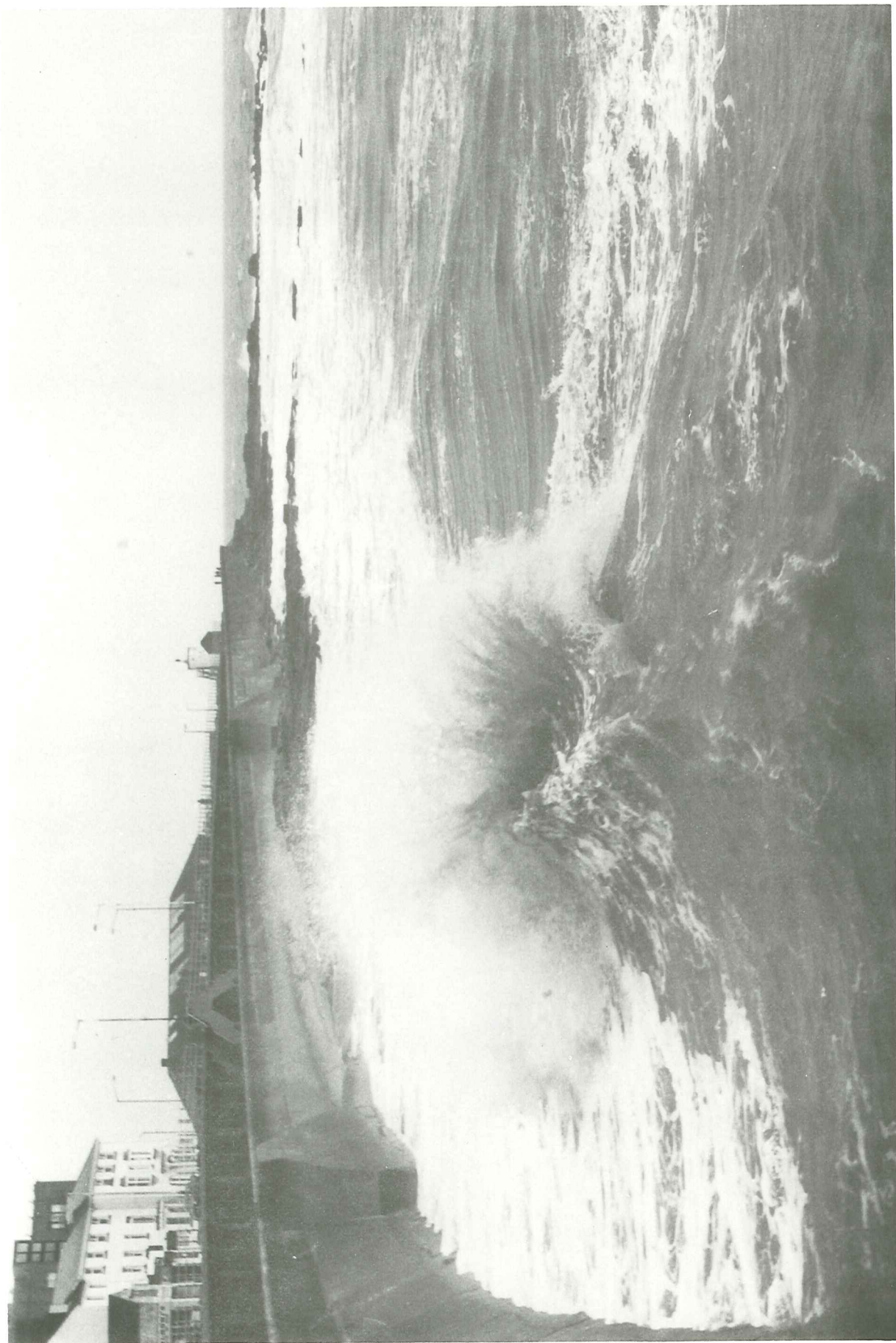


Plate 3    Reflection of waves from a vertical faced seawall at Porthcawl,  
             South Wales

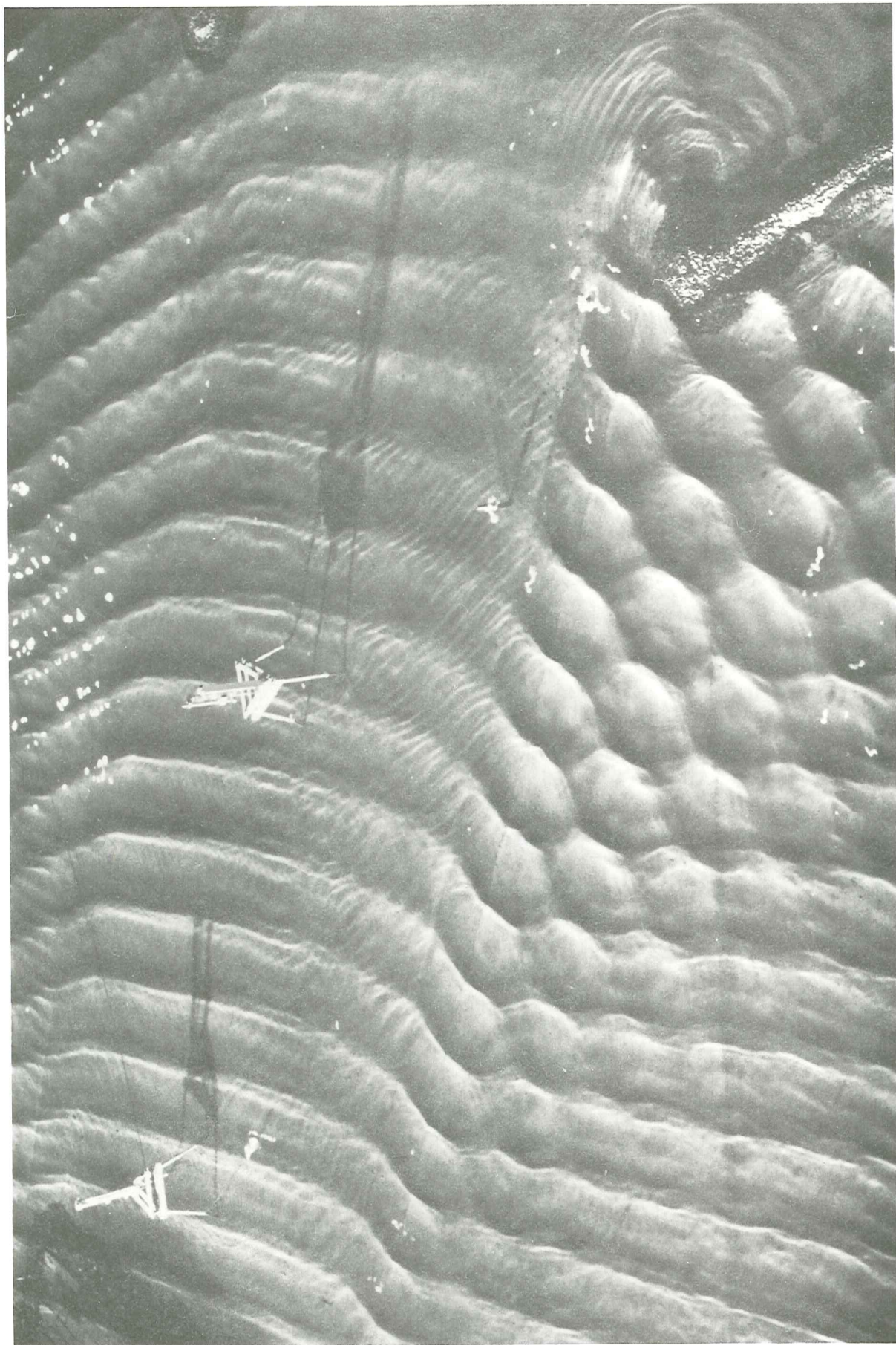


Plate 4 Total reflection of waves from the side of a channel in a model wave tank