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MATHEMATICAL SIMULATIONS OF A SHIP MOORED IN WAVES AND OF THE EFFECTS OF A PASSING VESSEL ON A MOORED SHIP

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Report No SR 145 January 1988

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ABSTRACT

This report follows an earlier report "Mathematical modelling of moored ships in the time domain" which described the initial development of a computer model of a moored ship. Modelling in the time domain is necessary to represent the non-linear characteristics of typical mooring systems.

We have further developed our time domain model, SHIPMOOR, to be able to treat wide frequency band responses accurately. This report describes those developments and their use in simulating a ship moored in waves. Comparison with experimental results gives valuable confirmation of SHIPMOOR's accuracy.

We have also developed a separate mathematical model of forces induced on a moored ship by a passing ship. This has also been verified by comparison with experimental and the results are described in this report.

These models form part of a suite of computer programs HR is currently developing to predict ship responses in ports, harbours and navigation channels. The intention is that they be used in harbour design to determine realistic first estimates of berth tenability as well as safe, optimum dredged levels for berths and navigation channels.

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1 INTRODUCTION

In a previous report "Mathematical modelling of moored ships in the time domain" (Ref 1) we described the development and use at HR of a mathematical model of a moored ship called SHIPMOOR. This report describes the further developments of that model.

SHIPMOOR is one component of an integrated package of computer models being created at HR for modelling waves in ports and harbours and their effects on ships. We do not anticipate this approach supplanting the well established methods of physical modelling. Rather, mathematical models have advantages in cost and quickness of setting up, which make them complementary to physical models. In particular, realistic mathematical models can be useful at a preliminary stage in port design, when funds for a physical model are often not available. A mathematical model may identify some of a number of prospective harbour layouts as being inferior in some respects to the rest. Those others can then be tested using a comprehensive physical model to identify the optimal design. In this way the harbour designer gets some of the advantages of both the physical and the mathematical approaches; he benefits from the relative cheapness of the mathematical method because he avoids physical model testing of his less good designs and at the same time his best design is thoroughly tested in a physical model which automatically incudes all moored ship and harbour responses.

Within our package of mathematical models, SHIPMOOR's place comes after other models have calculated the wave loads on, and hyrodynamic properties of a ship moored at a given berth in a harbour. We also need to know the characteristics of the ship's mooring ropes and fenders. SHIPMOOR will then compute the ship's movement and the mooring loads.

Many fenders and ropes have non-linear characteristics: their compression or extension is not linearly proportional to applied load. And there are always possibilities of ropes going slack or ships leaving fenders during their movement; another form of non-linearity.

Since ship moorings are generally non-linear, we cannot expect accurate results from a method which uses an assumption of linearity to compute moored ship motion. In particular, the frequency domain method of superposing responses at different frequencies which we use for unmoored ships (eg Refs 2 and 3) is inappropriate. SHIPMOOR works instead in the time domain by integrating the ship's equations of motion (see Ref 1 for a description of the method).

This report describes a further development of SHIPMOOR. The shortcoming of the version of SHIPMOOR in Reference 1 was that it used constant hydrodynamic coefficients, whereas damping, for example, is in reality frequency dependent. This meant the earlier version of SHIPMOOR was only applicable to ship motions with narrow frequency bands. The developed version reproduces the frequency dependence in the time domain by means of impulse response functions. It can thereby model almost any kind of moored ship response, including broad-banded ones.

As an example of a broad-banded motion, we have used SHIPMOOR in this report to model a moored ship's response to broad-banded random wave forcing (section 3).

We have also modelled a moored ship's movement in response to another vessel sailing close past it (section 4). This is a problem we first tackled in Reference 1. That simulation was fairly successful

but it relied on limited force data derived from experiment (Ref 7). This data had to be extrapolated or interpolated using ad hoc scaling laws to describe circumstances not covered experimentally. We had doubts about the accuracy of this procedure. So a mathematical model has been developed to compute flow around a moving hull approximately and hence to calculate moving ship induced forces. Section 4 of this report describes that model and its use to simulate the same series of tests described in Reference 1. These tests related to a physical model of passing ships in Milford Haven (Ref 6). Results from the new model are therefore compared with both our earlier model based on limited force data and with what happened in physical model experiments.

2 EQUATIONS OF MOTION OF A SHIP

2.1 Method using constant damping and added mass coefficients

> In this report, we shall continue wherever possible to use the same notation as in Reference 1. Thus, we represent the ship's position and orientation by a six component vector \underline{x} where \underline{x}_1 through to \underline{x}_6 represent surge, sway, heave, roll, pitch and yaw, respectively (see Fig 1). Velocity and acceleration are represented by $\underline{\dot{x}}$ and $\underline{\ddot{x}}$. A 6x6 inertia matrix $\underline{\underline{M}}$ contains the ship's displacement M ($\underline{M}_{11}=\underline{M}_{22}=\underline{M}_{33}=\underline{M}$), moments of inertia (\underline{M}_{44} , \underline{M}_{55} , \underline{M}_{66}), and products of inertia ($\underline{M}_{46}=\underline{M}_{64}$). All other components of $\underline{\underline{M}}$ are zero for the conventional laterally symmetric ship provided moments are taken about the ship's centre of mass.

Forces and moments on the ship are also represented by six component vectors. These include:

 $\underline{f}(t)$ - wave forcing, $\underline{g}(\underline{x},\underline{\dot{x}})$ - mooring forces and moments h - buoyancy forces and righting moments

There is also an important force caused by the ship generating waves as it moves about in the water. This is conventionally split into two components: an inertial one in phase with the ship's acceleration, and a damping component in phase its velocity. In Reference 1 we introduced a frequency independent added inertia matrix \underline{A} and damping coefficient matrix \underline{B} to represent the two forces. Using these, the equation of motion takes the form:

$$(\underline{\underline{M}} + \underline{\underline{A}}) \times \underline{\underline{x}} + \underline{\underline{B}} \times \underline{\underline{x}} - \underline{\underline{g}}(\underline{x}, \underline{\underline{x}}) - \underline{\underline{h}}(\underline{x}) = \underline{f}(t)$$
(1)

This equation was used in Reference 1 to simulate a number of moored ship situations. In the next sub section we extend the model to allow for the fact that hydrodynamic coefficients in matrices \underline{A} and \underline{B} depend on frequency.

2.2 Method using the

impulse response

function

A time domain simulation using equation (1) is strictly only correct for a ship in (single frequency) simple harmonic motion where \underline{A} and \underline{B} can be evaluated at the relevant frequency.

In realistic situations it is at best only approximately right: the ship's motion will contain components at many different frequencies and added inertia and damping will be different for each

component. Pretending they are the same will lead to inaccurate results. Frequency dependent added mass and damping must be allowed for if we are to model moored ships accurately in the time domain.

Allowing for frequency dependence is straight forward if the system of equations to be solved is linear. Then we can solve in the frequency domain and superpose solutions at different frequencies to get an answer for any desired forcing signal, f(t). This is essentially the method we use for modelling a free ship (Refs 2 and 3).

But for a moored ship, the equation of motion is usually non-linear because of non-linear mooring forces. This non-linearity can have important consequences such as subharmonic resonance (Refs 1, 4,5 and section 3 of this report) so it cannot be ignored. It also makes the frequency domain superposition approach invalid. We have to solve in the time domain instead. This necessitates another way of allowing frequency dependent damping.

The conventional method is to use the impulse response function; we follow this approach.

Various descriptions of an impulse response function can be given. We derive it in one way from Fourier Transform theory in Appendix I. An alternative derivation based on physical arguments is given in Reference 8.

Physically, the impulse response function is a matrix function of time, $\underline{K}(t)$, which represents the force history on a ship after an impulse movement. An impulse movement is simply one in which the ship is stationary at all times except for a brief period δt in which it has a velocity \underline{V} ; i.e.

$$\dot{\mathbf{x}}(t) = 0 \qquad t < 0$$
$$= \underline{\mathbf{v}} \qquad 0 < t < \delta t$$
$$= 0 \qquad t > \delta t$$

The ship's movement causes waves. These in turn cause a force on the ship. The impulse response function describes that wave generated force. At a time t after the impulse, the force is;

$$\mathbf{F}(t) = -\mathbf{K}(t) \cdot \mathbf{V} \, \delta t \,. \qquad (t > \delta t) \tag{2}$$

If we next consider a succession of impulse motions at times $\{0, \delta t, 2\delta t, 3\delta t, \ldots, n \ \delta t\}$ the total force due to all these movements will be;

$$\underline{F}(t) = \sum_{i=0}^{n} -\underline{K}(t-i.\delta t) \cdot \underline{V}_{i} \delta t$$
(3)

Then if we let δt tend to zero, we can consider our succession of impulses as a continuous movement with velocity $\dot{x}(t)$. Equation (3) becomes;

$$\underline{F}(t) = \int_{\tau=0}^{\infty} -\underline{\underline{K}}(\tau) \cdot \underline{\underline{\dot{x}}}(t-\tau) d\tau \qquad (4)$$

Thus, the ship's past movements cause a wave making force in the present which is calculable using the impulse response function.

But we must be able to calculate the impulse response function first. It could be found experimentally for a model ship by applying equation (2). Alternatively, the Fourier Transform derivation (Appendix I) gives an equation for computing it from the ship's damping coefficients \underline{B} :

$$\underline{K}(t) = \frac{2}{\pi} \int_{0}^{\infty} \underline{B}(\omega) \cos \omega t \, d\omega$$
(5)

There is also a wave making force component which equation (4) does not include. $\underline{F}(t)$ includes all the wave damping force, $\underline{B}(\omega) \cdot \underline{x}$, and some of the inertial force, $\underline{A}(\omega) \cdot \underline{x}$, but there is a residual frequency independent added inertia (\underline{A} ') that is not included:

$$\underline{A}' = \lim_{\omega \to \infty} \lim_{\omega \to \infty} \underline{A}(\omega) \tag{6}$$

or equivalently (see Appendix I):

$$\underline{\underline{A}}' = \underline{\underline{A}}(\omega) + \frac{1}{\omega} \int_{\Omega}^{\infty} \underline{\underline{K}}(t) \sin \omega t \, dt$$
 (7)

(NB \underline{A}' is independent of frequency, and equation (7) is true for any frequency, ω .)

So, if a ship's added mass and damping coefficients are known for a wide enough range of frequencies, the impulse response function and residual added inertia are calculable. Knowing them, we can compute the wave making force, which we can include in the ship's equation of motion. It becomes (cf equation (1)):

$$(\underline{\underline{W}} + \underline{\underline{A}}') \xrightarrow{\infty} \underline{\underline{x}}(t) + \int_{\tau=0}^{\infty} \underline{\underline{K}}(\tau) \cdot \underline{\underline{\dot{x}}}(t-\tau) d\tau - \underline{\underline{g}}(\underline{x}, \underline{\dot{x}}) - \underline{\underline{h}}(\underline{x}) = \underline{\underline{f}}(t)$$
(8)

Equation (8) takes into account the frequency dependence of hydrodynamic coefficients and it can be used to describe a moored ship's motion in the time domain.

3 SIMULATING A SHIP MOORED IN A BEAM SEA

3.1 Introduction

We wanted to verify the accuracy of the version of SHIPMOOR with the impulse response function by testing it against experimental data for a realistic moored ship. This requires an impulse response function matrix to be calculated. The calculation can be done (Eq 5) but it needs damping coefficients at many frequencies. Computing that many damping coefficients by a conventional source method takes a lot of computer time and such a method is not well suited to vessels with a small underkeel clearance. We are working on a quicker method for ships in shallow water (Ref 2 describes the model for pitch and heave motions), but it is incomplete for sway, roll and yaw. To save computer time, we preferred to use an impulse response function which had already been calculated.

The chosen function is that given by Van Oortmerssen in Reference 8. He calculated it to represent a loaded 200,000 dwt tanker moored at an open jetty in a water depth of 1.2 times the vessel draught (see Table 1 for dimensions of the vessel and Figure 3 for the impulse response function).

In addition to his mathematical calculation of added mass and damping coefficients, from which the impulse response function was found, Van Oortmerssen also measured coefficients by testing with a 1:82.5 scale model. He used a similar duplication on his wave forcing coefficients. Mathematical and physical model test results agreed well enough for the more important parameters for us to be able to use his impulse response functions.

He conducted an extensive programme of tests with his physical model moored in different wave conditions. We have chosen to simulate, using SHIPMOOR, some of those tests which gave interesting results.

Van Oortmerssen's mooring arrangement is shown in Figure 2. The model ship was moored by four lines, each representing two or three wires with nylon tails on the full-sized ship. A twenty tonne pre-tension was applied in each of the four model lines.

Reference 8 gives non-linear load-extension curves for these lines in terms of percentage extension without giving any indication of a reference length. We assumed the extension would apply to the lines' tails which we further assumed to be 25m long. Results obtained using these assumptions turned out to be consistent with Van Oortmerssen's.

The ship had two fenders. In the experiments they were represented using rocking arms to which spring's were attacked; vertical wheels minimised friction in heave pitch and roll. It was, however, found to be necessary to add friction for horizontal motions in our mathematical simulation in order to obtain sensible results.

3.2 Simulation with

regular waves

Van Oortmerssen did a series of tests forcing the ship using regular beam seas with periods (at full scale) between 9 and 41 seconds. Results from only a few of them are given in Reference 8. We simulated just one.

That test represented waves 0.9m high with a frequency of 0.212 rads/s (about 30s period). Under these conditions, Van Oortmerssen reported that the ship

responded subharmonically; its response at the 30s period was superimposed on a larger sway motion at period close to its much longer natural period. In this case the natural sway frequency was apparently about 0.07 rads/s (because of the non-linearity of the moorings, one cannot be precise), so the larger long period motion was a third mode subharmonic motion.

We had previously observed subharmonic sway responses with the constant coefficient version of SHIPMOOR (Ref 1, which also contains a fuller description of subharmonics and subharmonic responses). This test therefore seemed a suitably interesting case to use to see whether we could get subharmonic response using the impulse response function.

We could and did. As Table 2 shows, we got a close agreement with Van Oortmerssen's experimental subharmonic sway - closer than he got himself with his mathematical model. Agreement on line 2 and 3 forces was also good. But our forces in lines 1 and 4 are far too large.

A minor discrepancy between Van Oortmerssen's findings and ours is that he got a subharmonic response starting runs with the ship at rest against the fenders whereas we found we had to start our runs with the ship already moving. Thompson et al (Ref 5) report that subharmonic responses are sensitive to initial conditions. We have previously found the same (Ref 1). Perhaps some otherwise insignificant difference in test conditions allows subharmonic response starting from rest for Van Oortmerssen but not for us?

Yaw behaviour in our simulation was different from Van Oortmerssen's experiment. No yaw values are given in Reference 8, but a diagram (Ref 8, Fig 5.6) indicates

yaw was small and not periodic. Our yaw as larger and dominated by the wave frequency although there were also third and sixth mode subharmonic components present.

At these very low frequencies yaw damping is small. Some subharmonic yaw response is, perhaps, to be expected although Van Oortmerssen did not get any.

Effective damping coefficient can be re-formed from the impulse response function by taking the inverse Fourier Transform of (5), viz:

$$\underline{\underline{B}}(\omega) = \int_{0}^{\infty} \underline{\underline{K}}(t) \cos \omega t \, dt$$
(9)

Applying (9) to Van Oortmerssen's impulse response function, we found that yaw damping coefficients became negative for a small band of frequencies at about 0.08 rads/s. This erroneous result is presumably a consequence of having to approximate $\underline{K}(t)$ by truncating it after 25 seconds which is not really enough to guarantee that long period oscillations are The error is only small and there correctly damped. is no reason to suppose it has a significant effect on the results we have reported; the frequencies concerned are well away from any at which we got any significant yaw response. But it had interesting consequencies on occasion. Among our early tests we had some with moorings with resonant yaw periods of about 80 seconds. Damping was slightly negative; the resultant exponentially growing yaw resonance was most unrealistic but did demonstrate one possible pitfall in using impulse response functions.

In conclusion, there are differences between our simulation and Van Oortmerssen's but bearing in mind that we cannot be certain that we have duplicated the

mooring conditions, the differences are not significant: the important result is the similarity of our sway responses. Sway is the largest and most important component of the ship's motion and we modelled it correctly. This demonstrates that SHIPMOOR is accurate in its impulse response function form.

3.3 Simulation with random waves

Van Oortmerssen did tests using long crested random as well as regular waves. The random wave tests all used the same wave spectrum with a significant wave height of 2.6m and a mean period of 8.9 seconds. Each was 2100 seconds long. They differed in that the angle of wave attack was different for each tests: directions varied between 90 degrees (beam sea) and 180 degrees (bow sea).

We simulated just one of the Van Oortmerssen's tests; the beam sea one. A synthesiser was used to generate random force time series with similar spectral characteristics to those used by Van Oortmerssen. The necessary wave excitation force and moment transfer functions are given in Reference 8 but there is no information given on the relative phasing of the forces; this we had to guess. And, as in the regular wave test, we were uncertain of the details of the moorings.

Allowing for these uncertainties, our results agree acceptably well with Van Oortmerssen's computational results. We did a series of ten tests all using the same force spectra but with different time series. Each was 2048 seconds long - similar to Van Oortmersen's test. Table 3 shows that our results are comparable with his computer results.

The agreement with his experimental results is less good (Table 3). In part, this may be because we have over-estimated roll motion. Roll damping on real ships is mostly caused by viscous effects and vortex shedding, not by wave generation. Our theory includes only wave generation damping, so it generally under-damps roll. Van Oortmerssen subsequently got a closer agreement between his computer model and experimental by raising the roll damping in his computations to a (larger) value derived from still water oscillation tests (Ref 8). We did not try this. Perhaps if we had, we would have improved our results too.

Another important difference is that the vessel in the physical model will experience non-linear wave forces at wave group periods: effects not represented in these computer simulations. Van Oortmerssen found particularly poor agreement for head seas where most of the long period response occurred due to wave grouping effects. These non-linear forces are now being studied as an extension of the present work.

Forecasting maximum values, eg the maximum movement and mooring force which can be expected within a given duration in a given storm, is a crucial aspect of modelling for many practical applications. One predictor of extreme values which we have found useful in the past is the Gumbel probability distribution. This estimates that, for a wave-like random variable, the probability of it exceeding any value x depends on its standard deviation, σ , and the number of zero up-crossings, N, in the period of time under consideration:

$$p(x) = 1 - \exp\left[-N \exp\left(-\frac{x^2}{2\sigma^2}\right)\right]$$
 (10)

The Gumbel distribution properly only applies to Gaussian processes. Our ship's mooring forces being non-linear means it is not really applicable to sway, yaw and mooring forces in our tests. Nevertheless, we reasoned that the mooring lines were not very non-linear over the relevant range of extensions, so it would be worth trying.

Cumulative probabilities of the maximum values from each of our tests were plotted (Fig 4). Points should lie in a straight line for a Gaussian random variable. Our yaw response points do not (Fig 4.2); the points fall in a definitely curved line. Other graphs do show sway, fender force and mooring line force points lying in more or less straight lines, but the lines along which they lie do not always agree with the Gumbel theory. Most notably, the slope of the fender 2 points disagrees with theory badly. Line 4's slope is much closer to the theoretical ideal, but the agreement with theory is only fair. Good agreement is achieved in sway. Fender 1 and line 1 are fairly good. On lines 2 and 3, the theoretical lines and the plotted points are displaced but roughly parallel. So although errors are significant at the moderate probabilities plotted here, there are grounds to think that if the lines are extrapolated to predict extreme forces at very low probability the predictions may be tolerably accurate.

In summary, the Gumbel distribution is of uncertain accuracy here; it predicts extreme values of some quantities tolerably well, but maximum fender 2 forces are not well described.

The SHIPMOOR model itself seems to work to the extent that it agrees with Van Oortmerssen's results. But Van Oortmerssen's published results constitute only a small set of data and so they cannot be considered a

conclusive test. Results so far are encouraging but more detailed comparisons with both physical model results and full scale data are necessary to verify our model.

- 4 SIMULATING MOTION INDUCED BY A PASSING SHIP
- 4.1 Summary of previous

simulation

In the earlier report on SHIPMOOR (Ref 1), we described how we had used the model to simulate the motion of a moored ship being passed by another close by. Our simulations attempted to reproduce physical model tests carried out at HR to investigate motions of oil tankers unloading at Milford Haven (Ref 6).

Five out of a long series of the original tests were chosen for mathematical simulation. Dimensions of the various ships involved are given in Table 4. Test conditions like passing ship speed and proximity, and mooring line pre-tensions are listed in Table 5.

The moving ship's direction of travel was different for test 32B from the other tests. In 32B a ship in ballast went down-river, the rest of the tests represented a loaded ship entering harbour. Thus, in most tests the moving ship approached the moored ship's stern, whereas in 32B it approached its bow.

The mooring arrangement is shown in Figure 5. For simplicity, we represented lines 7 and 8 together as one line in our model and likewise lines 9 and 10. This meant all lines in the mathematical simulation represented a double line in nature (see Fig 5). Ropes were steel wires with nylon tails and had

load/extension curves given in Reference 6. Fenders were low friction.

In our earlier attempt at mathematical simulation, we had succeeded in modelling the moored ship's movement and forces in its mooring lines. But there were differences: yaw was often over-estimated and the moored vessel started to surge too early as the passing ship approached. The latter problem may well have been caused by not representing the effects static friction in SHIPMOOR. But it was also possible that the force the passing ship generated was being wrongly estimated.

Passing ship forces were estimated from a series of tests carried out by Remery (Ref 7). This is the only source of experimental force data of which we know, but it has limitations. Remery only tested one size and shape of moored ship in one water depth although he did test three passing ships and a range of passing distances. One has to resort to ad hoc scaling rules to extrapolate and interpolate Remery's results if one is to model other sized ships in other depths of water. Some of these rules have no sound theoretical basis. And so the further one departs from Remery's test conditions, the less confidence one can have in applying his data to predict forces. The Milford Haven tankers were significantly bigger than Remery's test ships, and the passing ship was in much deeper water (39% underkeel clearance against 19.5%) so we were not confident that we were getting our forcing right.

4.2 Passing ship force model

We needed a theory to predict forces on our moored ship. But an exact theory, taking account of the complete flow around both the moving and the stationary ships' hulls, would be excessively complex.

Instead, we have produced an approximate theory which incorporates simplifying assumptions. Irrotational flow is assumed, so we can use potential theory. Free surface effects are neglected. We reduce the flow to two dimensions by taking depth-averaged values. The present version of the model assumes simplified lines for the ships with vertical sides and flat bottoms, but this is not an essential feature of the model and we hope to introduce a more sophisticated treatment in the future.

A finite difference method is used; the area of water around and including the ships is divided into a rectangular grid of cells, and the model computes a potential $\phi(\underline{x})$ which satisfies the continuity condition that net flow into every cell should be zero. The only exceptions to the continuity rule are cells at the bow and stern of the moving ship; the rule here is that net flow should balance water displaced by the hull as it moves in or out of the cell.

Outer boundary conditions are not usually important provided that boundaries are taken far enough away from the ships. The model can use either a no cross-boundary flow condition or a specified far-field potential. The no flow condition can also be used for simulating the effect of a solid quay or canal bank.

Flow under ships' hulls and varying depths of water are allowed for. But the model, being depth-averaged, does not include vertical components of fluid flow.

Pressures are calculated by applying Bermoulli's equation after first computing potentials at a succession of time steps as the passing ship moves forward: the rate of change of potential providing part of the pressure. Summing pressures over the moored ship's hull gives surge and sway forces and yaw moment. Repeated time-stepping builds up a time-history of forces on the moored ship.

Figure 6 shows a comparison of force-histories we have calculated using this model with data found experimentally by Remery (Ref 7). Despite the approximations involved in the mathematical model, agreement is mostly good, the one notable discrepancy being that the shape of the surge force curve is different for small separations.

4.3 Results of

simulations

We applied the Passing Ship Force Model to the Milford Haven tests. Force time histories were calculated for each of our five earlier test conditions. These forces were then fed into SHIPMOOR to estimate how the moored ship responded and to determine the mooring forces.

As we reported previously (Ref 1), there is a difficulty involved in setting initial conditions. The obvious thing to do is to start the moored ship from rest in its equilibrium position, with the passing ship approaching from far away. This method was tried before (called 'starting from rest' in Ref 1). But it does not work well. Because static friction is not represented in SHIPMOOR (there are

difficulties in doing this) the ship tends to creep along the berth in response to small surge forces early in the simulation whereas in reality static friction would hold it steady for a time. Consequently, when the important swaying and yawing motions start, the ship starts from the wrong position in the simulation.

So we adopted an alternative starting procedure. The original experiments (Ref 6) had indicated that the moored ship first swayed away from its fenders at about the time the passing ship's bow drew level with the midship position of the moored ship. We started our tests from this time (called 'starting from centre point' in Ref 1). And using published initial positions and by adjusting initial velocities we were able to get good agreement with the experimental results from Reference 6.

4.3.1 Tests started from rest

All the previous work was based on Remery's experimental force data. Here we use the Passing Ship Force Model forces and a modified start-up procedure. We reasoned that, until the passing ship force pulling the moored ship off its fenders exceeded the total pre-tension pulling it back on, the moored ship would not move in sway. And it would probably not move much in surge either as static friction would hold it until then. The point when the sway force overcomes pre-tension was easily determined by inspection of the Passing Ship Force Model output. We started our tests from that point with the moored ship stationary.

Initially, we tried using the above start condition for the simulation with the ship starting from the equilibrium position indicated in the original experiments. Maximum mooring line forces and

movements are listed in Table 6 with corresponding experimental results. Agreement is poor although it is no worse than that obtained starting from rest using Remery's data (Table 5, Ref 1).

In an attempt to improve things, we changed our moored ship's initial position slightly. The earliest positions recorded in Reference 6 for the experimental moored ship do not coincide with its equilibrium position before the start of the test; presumably the ship creeps along its berth in response to forces as the passing ship approaches. We repeated our tests, starting the moored ship from rest again, but this time using the position given in Reference 6 when the passing ship's bow reached midships of the moored ship. Results (shown in Table 7) are very similar to those in Table 6. It seems that maximum forces are not sensitive to the initial position of the ship.

Pre-tension effects

An extra test was done to check the effects of pre-tension. This test used the same conditions as run 22B except that pre-tension in all lines was increased from 5 tonnes f. per line to 20. Increasing pre-tension was reported in Reference 6 to reduce mooring loads because tauter lines are more effective at restricting vessel motion. Our results (Table 9) corroborate this. Run 50B is similar to 22B but with slightly slower passing ship and slack lines; it has a largest loading of 121 tonnes. Five tonnes pre-tension reduces that to 104 tonnes in 22B, but with twenty tonnes pre-tension, the peak load is 62 tonnes - which is about half the slack-line figure.

Moored ship movement

Moored ship movements are inter-connected with mooring forces: mooring line extensions depend on ship position on the one hand, and these extensions determine mooring forces which partly control ship motion on the other hand. Modelling mooring forces correctly and modelling movements correctly are equivalent. Given that we got forces wrong in our first two series of tests, we expect at least some motions to be wrong too.

Maxima and minima of surge, sway and yaw are listed in Tables 6 and 7 along with the forces. The two test series have similar results.

Sway is under-estimated in most tests and maximum yaw is over-estimated. The two effects counter-balance each other in one important way. Many runs have large mooring forces in lines 7/8. In the experiments, these forces were produced by large sway motions combined with small yaws taking the ship far from the quay. In the mathematical simulations, a larger yaw combines with a small sway (relative to experimental values) to take the bow of the ship far out and give similarly large forces in line 7/8, the forward breasting lines.

At the same time, the large yaw means that the aft end of the ship will be closer to the quay in the simulation than it was in the experiment. Extensions and forces will consequently be low. This does not show on line 2 (which is 7/8's counterpart at the stern) because its maximum force occurs at another time in the motion. But line 1, the stern line (Fig 5), is affected and maximum forces are drastically under-estimated as a consequence.

In surge, the moored ship's motion in most runs is first a move in the stern direction followed by a movement to bow. Run 32B is the exception to this pattern; with the passing ship coming from the bow rather than the stern, the moored ship's directions of motion are reversed. The extreme surge motions listed in Tables 6 and 7 show a tendency for the mathematical model to under-estimate the initial (generally towards the stern) surge and to over-estimate the second surge (generally towards the bow).

These differences in turn affect mooring forces in the spring lines (4 and 5): line 4 restrains the ship against moving backwards and is under-estimated, while line 5 which prevents forward movement is over-estimated.

4.3.2 Dynamic start tests

We had experienced similar under-estimates of sway and over-estimates of yaw in our earlier passing ship study (Ref 1) when 'starting from rest'. Then we had been able to increase sway and decrease yaw by giving the ship initial surge, sway and yaw velocities in the 'starting from centre point' condition. Trial and error using different start velocities gave results which agreed well with the original experiments.

We tried something similar again, but using the Passing Ship Force Model instead of Remery's experimental force data. Each run started with the moored ship in the position it had in the experiments (Ref 6) when the passing ship's bow reached its midships. We assumed this time roughly equalled the time when passing ship forces first exceeded pre-tension. Results given in Reference 6 were sufficient to obtain a first estimate of moored ship velocities but it was necessary to experiment with

different starting velocities, based on the estimated values, to improve the accuracy of the simulations.

These tests' results are shown in Table 8. Surge, sway and yaw are all simulated better this way than before. There are still some mooring line 1 forces which are grossly under-estimated. But mooring line 1 never experiences the largest forces in any run, so the problem is not critical. Overall, forces are simulated more accurately than previously and the results are as good as the 'starting from centre point' ones in Reference 1.

4.4 Conclusions on passing ship model and simulation

> The passing ship force model presents us with the same dilemma as we had using Remery's force data (Ref 1): we can simulate the effects of a passing ship on a moored one, but the simulation will only be accurate if we can set appropriate initial conditions early in the moored ship's motion. We could do that in these simulations using the experimental data in Reference 6 but in the normal type of tests, where the result is not known in advance, initial conditions cannot be set in the same way.

> We would normally have to start tests with the moored ship stationary in its equilibrium position. This was the starting condition for the results shown in Table 6 and those mathematical model results correlate fairly poorly with the experimental ones.

> But the correlation is no worse than that obtained using Remery's forces (Table 5, Ref 1) and the 'started from rest' initial condition. Similarly, our

Table 8 results are about as good as the earlier 'started from centre point' ones (Table 6, Ref 1).

Predictions using the passing ship force model are therefore no worse than those obtained using Remery's force data. Up until now, Remery's data have been the best information available to us. The force model has the advantage of being more adaptable: it can easily model different sizes of ships in different water depths at different distances, and it can model solid quays, canals, narrow navigation channels and other cases where obstacles restrict current flows. The model is therefore already useful.

Further development is possible and desirable. The current version does not take underwater hull shapes into account: ships are assumed to be flat bottomed. This seems to be acceptable for oil tankers like those described in this report because they have rectangular hull cross-sections and constant draught over most of their length. But a more sophisticated treatment may be needed for finer-lined ships. To date, however, this has not been tried.

So far as harbour and ship operations are concerned, our results demonstrate two of the main conclusions reached in Reference 6. One is the obvious point, shown by comparison of results for runs 17 and 22B, that closer passing ships induce larger moored ship movements and mooring forces.

The other point concerns pre-tension. Our tests show that increasing pre-tension ultimately leads to a reduction in mooring loads. Further, the percentage decrease in maximum mooring line load calculated by SHIPMOOR is similar to that found experimentally.

5 OVERALL

CONCLUSIONS

This report splits naturally into two parts relating to two different applications: a ship moored in waves, and the effect of a passing ship on a moored ship. In this final section we shall bring out the important conclusions from the two parts of the report.

Our tests using the impulse response function form of SHIPMOOR to simulate Van Oortmerssen's mathematical and physical model tests of a ship moored in waves (Ref 8) were successful in modelling important aspects of sway behaviour. In particular, subharmonic response in regular waves as well as responses in irregular waves were well modelled. There were problems with roll response caused by not including viscous effects and drag-type damping forces. A quirk in the impulse response function derived in Reference 8 gave difficulties in yaw. Sway, however, was the motion we were most interested in and that was well represented.

We had mixed success fitting the Gumbel distribution to our maximum forces and movement in random waves. Some graphs fitted to the theoretical curves fairly well, but one force, in particular, fitted badly. The Gumbel distribution is not intended to describe non-linear processes like those we were trying to model here. We conclude that its accuracy for predicting extreme moored ship movement and mooring force probabilities is uncertain.

Our passing ship tests were done using the old version of SHIPMOOR which used constant damping coefficients rather than impulse response functions. In these tests, therefore, we were not testing SHIPMOOR but the separate passing ship force model.

Tests showed that it led to predictions which were as good as those obtained with Remery's force data (Ref 7). That data had previously been our preferred basis for calculating passing ship forces. But the separate force model has greater adaptability; it is applicable to a greater range of ship sizes, variable water depth, solid quays and narrow canals.

Whether using the separate force model or Remery's force data, SHIPMOOR can simulate moored ship motion and mooring forces successfully once motion has been initiated correctly. The trouble is that, in practice, the initial behaviour will not be known; the moored ship will have to be started from rest. For some reason, this start condition does not give accurate results (Tables 6,7).

We can only speculate on why this should be so. It may be connected with difficulties in modelling static fender friction early in each test.

Another possibility is that the hydrodynamic coefficients of the moored ship we used were at fault. These passing ship tests used constant added masses and damping coefficients which are strictly only accurate for regular sinusoidal motions. The moored ship's motion was neither regular nor sinusoidal.

If we had been able to use the impulse response function to calculate motion, results might have been different. In particular, soon after starting the moored ship from rest, equation (8) shows that the ship's effective inertia would be approximately $\underline{M} + \underline{A}'$. We actually used $\underline{M} + \underline{A}(\omega_0)$ (where ω_0 is a fixed frequency approximating to the anticipated return period of the moored ship against the fenders). Sway is the critical motion here, which SHIPMOOR has not modelled particularly well. Added masses are

generally large at low frequencies (Ref 8). Our frequency ω_0 was low, so $A_{22}(\omega_0)$ will be greater than A'_{22} . We therefore over-estimate sway inertia early in every test. Over-estimating inertia, we would consequently under-estimate initial sway acceleration. Hence we might under-estimate the whole sway motion.

This example illustrates another potential benefit of using the impulse response function: more accurate modelling of a ship started from rest (eg in the passing ship problem) may be possible using it. We have already seen its advantages for modelling broad frequency banded motions and motions (such as subharmonic motions) in which components at two or more disparate frequencies are present. There are few moored ship problems to which impulse response functions cannot usefully be applied. Results of our tests using the impulse response function are imperfect and some further development may be necessary, but the potential advantages of using it are so great that we are convinced the impulse response function is the route to follow.

6 ACKNOWLEDGEMENTS

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Dr A J Cooper and Mr G Gilbert devised and developed, for use in another context, the basic method behind the passing ship force model. I am in addition grateful to Mr Gilbert for a number of useful discussions about impulse response functions.

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TABLES.

· . .
Table 1 Dimensions of ship for tests in waves

· · ·

Length between perpendiculars	310m
Breadth	47.20m
Draught	18.90m
Displacement	235,000m ³
Transverse radius of gyration	17.00m
Longitudinal radius of gyration	77.47m
Yaw radius of gyration	79.30m
Distance of centre of gravity forward of mid point	6.61m
Roll metacentric height	5.78m

	Wave freq'y rads s-1	Wave height m	Max mo Line 1	oring l tonn Line 2	ines fo es f Line 3	rces Line 4	Max fend tonn Fender 1	er force es f Fender 2	Sway X _{max} -x _{mín}	Remarks
Van Oortmerssen expt	0.212	6•0	406	629	521	306	4029	3421	9.46	3rd mode subharmoníc
Van Oortmerssen computation	0.212	6•0	208	400	501	305	3366	3771	7.84	3rd mode subharmoníc
SHIPMOOR computation	0.212	6•0	849	574	638	956	Not co	mputed	9.67	3rd mode subharmoníc

Table 2 Responses in regular waves

Table 3 Ship responses in random waves

					Van Oortmen Expt	rssen Comp'n	SHIPMOOR Comp'n
Mooring line	1	r.m.s.	tonnes	f.	18	25	20
Mooring line	2	11	11		28	46	38
Mooring line	3	11	н		30	45	32
Mooring line	4	11	"		14	30	18
Fender 1		11	**		241	257	249
Fender 2		11	"		177	255	183
Sway		п	m		0.47	0.79	0.69
Yaw		11	degrees	н —	0.11	0.06	0.28

Table 4 Ships' dimensions for passing ship tests

	Overall Length	Breadth	Draught	Displacement
	m	m	m	m ³
Moored Ship	384	51.8	19.8	269000
Passing Ship 1	159.4	20.1	9.8	22270 (laden)
Passing Ship 2	244.4	30.5	14.0	80900 (laden)
Passing Ship 3	304.8	48.7	20.1	240000 (laden)
	łr	"	. 10.7	120000 (in ballast)

.

Test No	Passing Ship	Condition	Speed (knots)	Clearance Between Ships (m)	Mooring Line Pretension (tonnes f)
17	3	Laden	4.2	69	5
22B		"		46	**
32B	**	Ballasted	3.9	46	U
47B	2	Laden	3.6	46	5
50B	3	"	4.0	46	30 cm slack

Table 5 Details of passing ship tests

M. Line	1 M.M	7 Expt	2 M•M	2B Expt	3 M•M	2B Expt	47 M•M	7B Expt	5 M•M	OB Expt
1	17	48	21	57	16	30	15	20	16	86
2	39	28	49	34	30	30	12	21	73	64
4	39	40	49	47	33	14	9	17	72	76
5	88	63	104	66	18	14	25	35	121	90+
7/8	· 69	60	85	72	18	22	27	18	107	86
9/10	26	30	34	38	14	23	15	25	35	45
Surge	1.1	1.1	1.4	1.3	-0.57	-0.45	0.25	0.6	2.6	3.2
burge	-2.1	-1.7	-2.6	-1.6	1.0	0.8	-0.82	-1.3	-3.8	-3.3
Sway	0.81	1.2	1.0	1.5	0.41	0.85	0.30	0.25	1.7	2.5
Vau	0.35	0.36	0.42	0.38	-0.19	-0.14	0.13	0.28	0.83	0.90
Iaw	-0.60	-0.25	-0.67	-0.26	0.33	0.18	-0.36	-0.22	-1.11	-0.44

Table 6 Passing ship results, started from equilibrium position

M. Line	1 M•M	7 Expt	2 M•M	2B Expt	3 M•M	2B Expt	47 M•M	'B Expt	5 M•M	OB Expt
1	19	48	25	57	18	30	15	20	18	86
2	38	28	48	34	31	30	13	21	72	64
4	34	40	40	47	29	14	9	17	65	76
5	85	63	98	66	15	14	25	35	117	90+
7/8	-69	60	85	72	19	22	28	18	107	86
9/10	27	30	39	38	14	23	16	25	36	45
Surgo	1.0	1.1	1.1	1.3	-0.47	-0.45	0.22	0.6	2.5	3.2
burge	-2.1	-1.7	-2.4	-1.6	0.91	0.8	-0.78	-1.3	-3.7	-3.3
Sway	0.81	1.2	1.0	1.5	0.41	0.85	0.31	0.25	1.7	2.5
Vou	0.35	0.36	0.43	0.38	-0.20	-0.14	0.13	0.28	0.83	0.90
Iaw	-0.60	-0.25	-0.66	-0.26	0.33	0.18	-0.37	-0.22	-1.12	-0.44

Table 7 Passing ship results, started from centre point position

M. Line	1 M.M	7 Expt	2 M•M	2B Expt	3 M.M	2B Expt	4 M•M	7B Expt	M•M	50B Expt
1	19	48	25	57	35	30	23	20	19	86
2	35	28	53	34	30	30	27	21	64	64
4	37	40	48	47	17	14	15	17	73	76
5	69	63	96	66	13	14	32	35	101	90+
7/8	49	60	85	72	34	22	28	18	90	86
9/10	27	30	35	38	27	23	24	25	24	45
Surge	1.2	1.1	1.3	1.3	-0.48	-0.45	0.52	0.6	2.7	3.2
	-1.8	-1.7	-2.4	-1.6	0.51	0.8	-1.10	-1.3	-3.4	-3.3
Sway	0.92	1.2	0.99	1.5	0.92	0.85	0.37	0.25	1.9	2.5
Yaw	0.24	0.36	0.47	0.38	-0.20	-0.14	0.33	0.28	0.69	0.90
	-0.40	-0.25	-0.67	-0.26	0.33	0.18	-0.35	-0.22	-0.75	-0.44

Table 8 Passing ship results, dynamic start

Table	9	Effects	of	pre-tension
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M. Line	50B Slack lines	22B 5T. Pre-tension	20T pre-tension
1	16	21	29
2	73	49	30
4	72	49	31
5	121	104	62
7/8	107	85	51
9/10	35	34	30
Surce	2.6	1.4	0.26
Jurge	-3.8	-2.6	-0.91
Sway	1.7	1.0	0.34
Ver	0.83	0.42	0.06
TAM	-1.11	-0.67	-0.26

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FIGURES.

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Fig 3.1 Impulse response functions for surge, sway and heave



Fig 3.2 Impulse response functions for roll, pitch and yaw

A house and house and house and house







Fig 4.2 Yaw response exceedance probability in random waves



Fig 4.3 Fender 1 force exceedance probability in random waves



Fig 4.4 Fender 2 force exceedance probability in random waves

Least squares Theoretical 16 15 14 x² 13 kN² x10⁵ 12 11 10 q 7 6 5 4 -+ t Т -1.5 -1.0 -0.5 0.0 0.5 1.5 2.0 2.5 3.0 3.5 4.0 4.5 -2.0 1.0 I 1 I I I I. 1 I 1 L I Exceedance prob. 0.9 0.8 0.7 0.6 0.5 0.4 0.1 0.05 0.02 0.3 0.2

. Fig 4.5 Line 1 force exceedance probability in random waves



Fig 4.6 Line 2 force exceedance probability in random waves

Least squares Theoretical 6.5 · 6.0 -+ 5.5 x² 5.0 kN ² x10⁴ 4.5 4.0 3,5 +3.0 2.5 -2.0 1.5 ┥ -0.5 0.5 2.0 2.5 3.0 3.5 4.0 1.5 4.5 -1.5 -1.0 0.0 1.0 -2.0 I L I I 1 I I 1 1 I I 0.9 0.8 0.7 0.6 0.5 0.4 0.1 Exceedance prob. 0.05 0.02 0.3 0.2

Fig 4.7 Line 3 force exceedance probability in random waves



Fig 4.8 Line 4 force exceedance probability in random waves



Fig 5 Mooring arrangement for passing ship tests



Fig 6.1 Comparison of passing ship force model results and results obtained by Remery (Ref. 7): 30m separation



Fig 6.2 Comparison of passing ship force model results and results obtained by Remery (Ref. 7): 60m separation



Fig 6.3 Comparison of passing ship force model results and results obtained by Remery (Ref. 7): 120m separation



Fig 6.4 Comparison of passing ship force model results and results obtained by Remery (Ref. 7): 200m separation

APPENDIX.

APPENDIX I

A derivation of the impulse response function

Consider frequency dependent added mass and damping coefficients, $A(\omega)$ and $B(\omega)$. Define

$$A' = \lim_{\omega \to \infty} A(\omega)$$
 (A1)

Kramers-Kronig relationships (also known as Hilbert Transforms) state:

$$\omega A(\omega) = \omega A' + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{B(\sigma)}{\sigma - \omega} d\sigma$$
 (A2a)

$$B(\omega) = \frac{-1}{\pi} \int_{-\infty}^{\infty} \frac{\sigma (A(\sigma) - A')}{\sigma - \omega} d\sigma \qquad (A2b)$$

Where principal values of integrals are understood.

In regular motion, $x(t) = Xe^{-i\omega t}$, the force on the ship is:

 $F(t) = -(-i\omega A(\omega) + B(\omega)).-i\omega Xe^{-i\omega t}$

$$= -(-i\omega A' - \frac{i}{\pi} \int \frac{B(\sigma)}{\sigma - \omega} d\sigma + B(\omega)) - i\omega Xe^{-i\omega t}$$
(A3)

Define:

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(-i\omega A' \frac{i}{\pi} \int \frac{B(\sigma)}{\sigma - \omega} d\sigma + B(\omega) \right) e^{-i\omega t} d\omega \quad (A4)$$

$$= A' \delta'(t) + K(t)$$
 (A5)

Where:

 $\boldsymbol{\delta}$ ' is the derivation of the Dirac delta function.

$$K(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{i}{\pi} \int \frac{B(\sigma)}{\sigma - \omega} d\sigma + B(\omega) \right) e^{-i\omega t} d\omega$$
 (A6)

It can be shown, assuming principal values, that:

$$-\frac{i}{\pi}\int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{\sigma - \omega} d\omega = e^{-i\sigma t} \quad (t \ge 0)$$
$$= -e^{-i\sigma t} \quad (t \le 0)$$

So, substituting into (A6), for $t \ge 0$

$$K(t) = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} B(\sigma) e^{-i\sigma t} d\sigma + \int_{-\infty}^{\infty} B(\omega) e^{-i\omega t} d\omega \right\}$$
$$= \frac{1}{\pi} \int_{-\infty}^{\infty} B(\omega) e^{-i\omega t} d\omega$$

And, since $B(\omega) = B(-\omega)$

$$K(t) = \frac{2}{\pi} \int_{0}^{\infty} B(\omega) \cos \omega t \, d\omega, \text{ for } t \ge 0$$
 (A7a)

Similarly, for $t \leq 0$

$$K(t) = \frac{1}{2\pi} \left\{ -\int_{-\infty}^{\infty} B(\sigma) e^{-i\sigma t} d\sigma + \int_{-\infty}^{\infty} B(\omega) e^{-i\omega t} d\omega \right\}$$
$$= 0 \qquad , \text{ for } t \leq 0 \qquad (A7b)$$

Now return to (A3), the expression for force on a ship in regular motion:

$$F(t) = -(-i\omega A' \frac{i}{\pi} \int \frac{B(\sigma)}{\sigma - \omega} d\sigma + B(\omega)) \cdot -i\omega X e^{-i\omega t}$$

This expression is similar to that for a Fourier component of force in an irregular motion x(t) with:

$$-i\omega X(\omega) = \int_{-\infty}^{\infty} \dot{x}(t) e^{i\omega t} dt$$
 (A8)

So we can invoke the convolution theorem for Fourier Transforms to calculate force for any ship motion, noting (A3) and (A4) and (A8):

$$F(t) = -G(t) * \dot{x}(t)$$

$$= -\int_{-\infty}^{\infty} G(\tau) \cdot \dot{x}(t-\tau) d\tau$$

Substituting for $G(\tau)$ from (A5):

$$F(t) = -\int_{-\infty}^{\infty} (A' \delta'(\tau) + K(\tau)) \cdot \dot{x}(t-\tau) d\tau$$
$$= -A' \ddot{x}(t) - \int_{-\infty}^{\infty} K(\tau) \cdot \dot{x}(t-\tau) d\tau$$

But, from (A7b), K(t) = 0 for $t \le 0$

$$F(t) = -A' x(t) - \int_{0}^{\infty} K(\tau) \dot{x}(t-\tau) d\tau$$
 (A9)

Which gives the hydrodynamic force terms in the ship's equation of motion ((8) in this report). Note, sign changes when F(t) is shifted to L.H.S. of equation.

We identify K(t) as the impulse response function. Note that this definition (A7) is the same as (5).

The constant added mass, A', is defined by (A1) which is (6) in the report; the alternative expression (7) can be derived from the Fourier Transform of (A6).

$$B(\omega) - \frac{i}{\pi} \int \frac{B(\sigma)}{\sigma - \omega} d\sigma = \int_{-\infty}^{\infty} K(t) e^{i\omega t} dt$$
$$= \int_{0}^{\infty} K(t) e^{i\omega t} dt$$

Taking the imaginary part:

$$\frac{1}{\pi}\int \frac{B(\sigma)}{\sigma-\omega} d\sigma = \int_{0}^{\infty} K(t) \sin \omega t dt$$

And substituting for the integral from (A2a):

$$\omega (A' - A(\omega)) = \int_{0}^{\infty} K(t) \sin \omega t \, dt$$

$$A' = A(\omega) + \frac{1}{\omega} \int_{0}^{\infty} K(t) \sin \omega t \, dt \qquad (A10)$$