

یک می کود می

> UNDERKEEL: A computer model for vertical ship movement in waves when the underkeel clearance is small

G	H	Lean	BSc	ARCS DIC
Ε	С	Bowers	BSc	PhD DIC
J	M	A Spencer	MA	

Report No SR 160 February 1988

Registered Office: Hydraulics Research Limited, Wallingford, Oxfordshire OX10 8BA. Telephone: 0491 35381. Telex: 848552 This report describes work funded by the Department of the Environment under Research Contract PECD 7/6/055 (PIF No HRL 4/85) for which the DoE nominated officer was Dr R P Thorogood. It is published on behalf of the Department of the Environment but any opinions expressed in this report are not necessarily those of the funding Department. The work was carried out in the Maritime Engineering Department of Hydraulics Research, Wallingford, under the Management of Dr S W Huntington.

© Crown copyright 1988

Published by permission of the Controller of Her Majesty's Stationery Office.

UNDERKEEL: A Computer model for Vertical Ship Movement in Waves When the Underkeel Clearance is Small

> G H Lean BSc ARCS DIC E C Bowers BSc PhD DIC J M A Spencer MA

Report No SR 160

February 1988

ABSTRACT

Using a simplified representation of flow beneath the hull, it has been possible to extend an earlier mathematical model for heave and pitch to describe all 6 modes of motion of a free ship in waves. The model, called Its UNDERKEEL, has been developed specifically for coastal applications. use is two-fold. It can be used directly to provide a realistic first estimate of safe underkeel allowances for vessels in navigation channels, or Its second use is in defining the hydrodynamic coefficients at berths. needed for a separate computer model called SHIPMOOR which, when fully developed, will be capable of providing a realistic first estimate of berth tenability for feasibility studies of port developments. These computer models complement more accurate, but more expensive, physical models because they can be used at an early stage in design to investigate a wide range of parameters without excessive cost, leaving the way open to use of a physical model for detailed design of favoured schemes.

Results from UNDERKEEL have been compared with results from a separate, but more expensive, mathematical model employing sources on the submerged part of the hull. A ship oscillating in waves, but without forward speed, was considered. The agreement between the models was very good which justifies the more direct theoretical approach taken in UNDERKEEL because it minimises computer costs.

UNDERKEEL has also been compared with a physical model of a supertanker underway in random waves. The physical model investigation was originally carried out as part of a project study to define safe underkeel allowances for supertankers negotiating the Dover Strait. The changing pattern of vessel response observed in the physical model as the wave spectrum, wave direction and underkeel allowance was changed, was very well described by UNDERKEEL. Vertical motions in stern, quartering and bow seas (the most common wave directions for navigation channels) were, generally, slightly overpredicted by UNDERKEEL. This means a realistic first estimate of a safe underkeel allowance can be provided by the mathematical model in this situation. With quartering to beam seas, it was found that vertical motions were overpredicted to a greater degree due to overestimates of vessel roll. This was not unexpected in that non-linear damping effects, not represented in UNDERKEEL, are expected to limit resonant roll motions. It is intended to add these effects subsequently.

Taken overall, the comparisons described here encourage one to use UNDERKEEL, for feasibility studies of port developments, in the manner described above.



CONTENTS

		1050
1.	INTRODUCTION	1
2.	EQUATIONS OF MOTION	5
3.	SOLUTION OF EQUATIONS	9
4.	APPLICATION TO A STATIONARY SHIP	14
5.	APPLICATION TO A SHIP UNDERWAY	16
	5.1 Physical model 5.2 Physical model test programme 5.3 Comparison of physical model with UNDERKEEL	17 20 22
6.	CONCLUSIONS	29
7.	ACKNOWLEDGEMENTS	34
8.	REFERENCES	35

Page

TABLES

- 1. Details of the hulk used for stationary ship tests
- 2. Details of the LANISTES (fully laden)
- 3. Physical model and calibrated mathematical model (earlier version of UNDERKEEL) predictions for change in response in going from 4m to 6m underkeel allowance
- 4. Critical movements of the LANISTES at 12 knots in following, quartering and head seas
- 5. Critical movements of the LANISTES at 12 knots in quartering to beam seas

FIGURES

- 1. The six degrees of freedom of vessel movement
- 2. Definition sketch
- 3. Surge response of stationary ship
- 4. Sway response of stationary ship
- 5. Heave response of stationary ship
- 6. Roll response of stationary ship
- 7. Pitch response of stationary ship
- 8. Yaw response of stationary ship
- 9. Line drawing of tanker
- 10. Swell with 19s spectral peak
- 11. Swell with 14.5s spectral peak
- 12. Storm wave with 13s spectral peak

CONTENTS (CONT/D)

FIGURES (CONT/D)

13. Storm wave with 11s spectral peak
14. Distribution of maximum downward movements in bow sea
15. Heave at 12 kts
16. Roll at 12 kts
17. Pitch at 12 kts

PLATES

1. Overall view of wave basin layout

APPENDIX 1

1. Effect of vessel speed on transfer coefficients

1 INTRODUCTION

Engineering feasibility studies for the development of new ports, and the extension of existing ones to accommodate the large vessels of today, are increasingly taking on a common form. Initially, computer models are required to investigate a wide range of parameters. Having established favoured schemes, a physical model is then used for final optimisation.

A suite of computer models is presently under development at Hydraulics Research (HR) to satisfy the requirement for a realistic first estimate of harbour and ship response to wave action. The model described in this report is one of the vital elements in that package.

The role of UNDERKEEL is two-fold. It can be used directly to describe the vertical motions in waves of vessels in navigation channels and at berths in a harbour. Its second use is in defining the hydrodynamic coefficients needed to describe moored ship motions. These coefficients are used in a separate model called SHIPMOOR which takes account of the non-linear characteristics of conventional mooring systems by solving equations of motion for the moored ship in the time domain (Refs 1 and 2). Ultimately, SHIPMOOR will be used to obtain a realistic first estimate of berth tenability during feasibility studies of port developments while UNDERKEEL will provide the necessary coefficients for SHIPMOOR as well as being used directly to fix a safe dredged depth for navigation channels and berths after allowing for vertical vessel motions in waves.

Although the vertical movement of vessels is well defined once heave, pitch and roll are known (Fig 1) it is still necessary to consider surge, sway and yaw

movements of the vessel to provide a complete description of vertical movement. This is because of coupling of surge into heave and pitch and another set of coupling effects between sway, yaw and roll. Thus, it is necessary to consider all six degrees of freedom of vessel movement even when interest centres on defining vertical motion.

The work described in this report is an extension of an earlier model developed by Lean et al (Ref 3) where heave and pitch motions were considered. In the earlier work, the diffraction of waves around the vessel and the coupling of surge into heave and pitch were ignored as a first approximation. Nevertheless, the earlier model proved very useful in establishing safe underkeel allowances for supertankers negotiating the Dover Strait (Ref 4). Through use of a physical model it was possible to calibrate this approximate computer model and go on to use it to consider the very large number of parameters that entered into that study. This avoided an excessive programme of tests in the physical model. The whole study was a good example of how computer models can prove complementary to physical models and how the combination can provide a more complete description of the physics involved in the study.

In this report we extend the earlier work to include wave diffraction around the vessel, all the coupling terms and roll motions as well as heave and pitch. This will enable a more complete mathematical description to be given for the vertical motions of a free ship in waves as well as leading to the definition of the necessary hydrodynamic coefficients for SHIPMOOR.

To a first approximation sea waves can be considered to be a superposition of wave components. If vessel

response is linear, then vessel movement can also be considered to be superposition of components. In this case each movement component is determined by calculation of vessel response to regular sinusoidal waves with given period and direction. Fortunately, the motions of a free ship are largely linear. The main difficulty occurs with roll damping which is controlled by eddy shedding from the hull and viscous damping in the boundary layer over the hull. But it is common practice to linearise roll damping, making use of physical model and full scale data where available to allow for these non-linearities. All the hydrostatic restoring forces and the remaining inertia and damping forces arising through flow associated with vessel movement in heave, pitch and roll, can be accurately described by linear potential theory. Thus, it is sensible to take the motions of a free ship to be linear and superpose components to represent responses in random waves. This means vertical movements in random waves can be defined by solving the equations of motion separately for each wave component in the full spectrum and then summing component responses.

There are a number of theories that have been used to describe vessel movements in waves. For deep water the well-known "strip theory" presented by Korvin-Kroukovsky (Ref 5) is much used in vessel design by naval architects. The difficulty with this approach in shallow water is that flow around the ends of the ship becomes significant due to the smaller underkeel clearance and this runs contrary to the two dimensional flow idea used in strip theory: it being assumed that flow occurs transverse to the vessel in strips that are independent of one another. This is reasonable for relatively short period responses in deep water where "leakage" of flow around the ends of

the vessel is small but the assumption breaks down when the underkeel clearance is small.

A different approach is provided by the source method where the submerged area is replaced by oscillating sources placed on surface elements that cover the The source strengths are chosen to satisfy the hull. boundary condition on flow normal to the boundary. This method is much used in offshore engineering to describe the motions of oil rigs. Oortmerssen (Ref 6) was the first to apply the method to moored ships but it is expensive on computer time, with, typically, 160 surface elements required to approximate the hull shape. Also, difficulties are experienced with the method when the underkeel clearance is small. It is natural, therefore, to consider more direct methods of calculating responses that are particularly suited to the case of a limited underkeel clearance.

A more direct approach has been taken by Beck and Tuck (Ref 7) for the case of long waves (shallow water theory). They have shown that flow in the far field is similar to that given by a ribbon of sources for surge, heave and pitch while sway, yaw and roll can be represented by doublets. The strengths of the sources and doublets, and hence the inertia and damping coefficients, are obtained by matching the far field solution to the flow deduced in the immediate neighbourhood of the vessel. For long waves the inertia and damping coefficients tend to infinity and zero, respectively in contrast to strip theory where the coefficients tend to a finite value. However, an important omission in their work is the effect of small underkeel clearance where the gradients in velocity and pressure underneath the vessel become large and cause significant increases in the inertia coefficients.

Here, we use a simplified representation of flow beneath the hull to allow evaluation of diffraction, inertia and damping coefficients in the equations of motion when the underkeel clearance is small. The equations are defined in Section 2 and the method of solution is described in Section 3. We then go on to apply these equations to a stationary ship, where a comparison is given with another mathematical model, and a supertanker underway in waves where a comparison is made with physical model results. The conclusions appear in Section 6.

2 EQUATIONS OF MOTION

The notation used to describe vessel motion is consistent with that already given in References 1 and 2. The ship's position and orientation is denoted by a 6 component vector s where:

\mathbf{s}_1	is	surge movement,
s ₂	11	sway movement,
s ₃	**	heave movement,
s ₄	11	roll angle,
s 5	11	pitch angle,
s 6	11	yaw angle.

These motions are defined in Figure 1 where the origin of the right handed co-ordinate system Oxyz is taken to be at the centre of gravity of the ship (Fig 2). The six movements define oscillations of the vessel about its equilibrium position and the motion of any point on the vessel can be obtained in terms of the six variables. Velocity and acceleration are denoted by \underline{s} and \underline{s} , respectively.

The equation of motion takes the form:

$$(\underline{M} + \underline{A}) \cdot \underline{s} + \underline{B} \cdot \underline{\dot{s}} - \underline{h}(\underline{s}) = \underline{f}(t)$$
(1)

This equation has the same form as Equation (1) in Reference 2 except that the function $\underline{g}(\underline{s}, \underline{\dot{s}})$ in Reference 2 is put to zero. This is because \underline{g} represents restraining forces arising from a mooring system: forces that do not apply here as we are considering a free ship.

 $\underline{\underline{M}}$ and $\underline{\underline{A}}$ are 6x6 matrices representing the inertia of the ship. $\underline{\underline{M}}$ denotes the inertia out of water while $\underline{\underline{A}}$ represents the added inertia due to flows created when the ship oscillates in the water. For conventional ships with lateral symmetry these inertias take the following form:

$$\underline{\mathbf{M}} + \underline{\mathbf{A}} = \begin{pmatrix} \mathbf{M} + \mathbf{A}_{11} & 0 & \mathbf{A}_{13} & 0 & \mathbf{A}_{15} & 0 \\ 0 & \mathbf{M} + \mathbf{A}_{22} & 0 & \mathbf{A}_{24} & 0 & \mathbf{A}_{26} \\ \mathbf{A}_{13} & 0 & \mathbf{M} + \mathbf{A}_{33} & 0 & \mathbf{A}_{35} & 0 \\ 0 & \mathbf{A}_{24} & 0 & \mathbf{M}_{44} + \mathbf{A}_{44} & 0 & \mathbf{M}_{46} + \mathbf{A}_{46} \\ \mathbf{A}_{15} & 0 & \mathbf{A}_{35} & 0 & \mathbf{M}_{55} + \mathbf{A}_{55} & 0 \\ 0 & \mathbf{A}_{26} & 0 & \mathbf{M}_{46} + \mathbf{A}_{46} & 0 & \mathbf{M}_{66} + \mathbf{A}_{66} \\ \end{pmatrix}$$

where,

M = displacement mass of the vessel, $M_{44} = MK^{2}_{44},$ $M_{55} = MK^{2}_{55},$ $M_{66} = MK^{2}_{66},$

and K_{44} , K_{55} and K_{66} are the radii of gyration in roll, pitch and yaw. The inertia M_{46} is generally small and we ignore its effect in what follows.

 $\underline{\underline{B}}$ is again a 6x6 matrix. It represents hydrodynamic damping of vessel oscillations in the water. Most of this damping occurs as waves carry energy away from the oscillating ship: the larger the waves created, the greater the damping.

Thus, heave and pitch are heavily damped due to the significant disturbance created by such motions but wave-making due to roll is very much smaller with the result that eddy shedding and viscous damping in the boundary layer become important. As we are using linear potential theory to calculate added inertias and damping we can only find the damping due to wave-making but, as mentioned in the Introduction, it is customary to linearise the important non-linear contributions once estimates have been made of their magnitude. We hope to do this in a subsequent contract and so, in this report, we concentrate on calculating damping due to wave-making. The matrix B takes the form:

$$\underline{\underline{B}} = \begin{pmatrix} B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ B_{13} & 0 & B_{33} & 0 & B_{35} & 0 \\ 0 & B_{24} & 0 & B_{44} & 0 & B_{46} \\ B_{15} & 0 & B_{35} & 0 & B_{55} & 0 \\ 0 & B_{26} & 0 & B_{46} & 0 & B_{66} \end{pmatrix}$$

The final term on the left hand side of Equation (1) represents hydrostatic restoring forces for the vertical motions, heave, roll and pitch. This can be expressed in the form:

$$\underline{h}(\underline{s}) = -\underline{C} \cdot \underline{s}$$

where

	1					
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	C ₃₃	0	C ₃₅	0
=	0	0	0	C ₄₄	0	0
	0	0	C ₃₅	0	C ₅₅	0
	0	0	0	0	0	0
	=	= (0 0 0 0 0 0	$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{33} \\ 0 & 0 & 0 \\ 0 & 0 & C_{35} \\ 0 & 0 & 0 \end{pmatrix}$	$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$	$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$

$$C_{33} = \rho g \int_{L} Bdx,$$

$$C_{35} = -\rho g \int_{L} x Bdx,$$

$$C_{55} = \rho g \int_{L} x^{2} Bdx,$$

$$C_{44} = Mg.Gm,$$

with

- = metacentric height above the centre of Gm gravity,
- B(x) = ship's beam at distance x from the origin (the integrals involving B are taken over the length of the ship).

The term on the right hand side of Equation (1), f(t), is a six component vector representing forces on the vessel due to the incident and diffracted waves.

Inspection of the form of the matrices in Equation (1) shows that they separate the component equations in (1) into two sets. One set couples together surge, heave and pitch, ie motion components s_1 , s_3 and s_5 . These can be called symmetric modes as they involve symmetric flows on either side of the ship. The second set couples sway, roll and yaw, ie components s_2 , s_4 and s_6 . These can be called asymmetric as the associated flows are asymmetric on either side. This feature is useful in calculating added inertias, damping and forcing terms.

The other point to be made about Equation (1) is that the inertia, damping and force coefficients do, of course, vary with wave frequency but by solving the equation separately for each frequency component we effectively treat the coefficients as constants, with different constants for each frequency component. This process is sometimes called solving in the

frequency domain and it is emphasized that this is made possible here by the motions of a free ship being largely linear. Once moored, vessel motions become non-linear and solutions must be sought in the time domain (Refs 1 and 2).

The method of solution of Equation (1) is discussed in the next section.

3 SOLUTION OF EQUATIONS

We consider the response of the vessel to a wave component with radian frequency ω and amplitude a propagating at an angle β with the x axis (Fig 2). In this case the velocity potential of the incident wave takes the form:

$$\Phi_0 = \frac{\text{ag}}{\omega} \frac{\cosh K}{\cosh Kd} \sin (\omega t - Kx \cos \beta - Ky \sin \beta)$$

where the usual dispersion relation is satisfied:

$$\omega^2$$
 = KgtanhKd

Denoting the unit vector normal to the surface of the hull by \underline{n} we can define the force on the vessel due to the incident wave in terms of the pressure P, ie

$$\underline{f}_{0} = \int_{S} \underline{n} P dS$$

where linear potential theory gives,

$$P = \rho \frac{\partial \phi_0}{\partial t} ,$$

and the integral is taken over the submerged surface area S of the hull. Substituting for ϕ_0 in the expression for pressure we find the kth component of

9 ·

the force on the vessel due to the incident wave takes the form:

$$f_{ko} = Re \{T_{ko} a e^{-i\omega t}\}$$

where Re denotes the real part of a complex expression and,

$$T_{ko} = \rho g \int_{S} n_{k} \frac{\cosh K(z+c)}{\cosh Kd} e^{i(Kx \cos\beta + Ky \sin\beta)} dS \quad (2)$$

To allow for a moving ship we must introduce the encounter frequency ω_{\perp} defined by:

 $\omega_{\rho} = \omega - KU\cos\beta$,

where the vessel has speed U along Ox (Fig 2). Thus, the ship will encounter the waves at frequency ω_e instead of the wave frequency ω , which is measured in a stationary frame of reference.

Thus, the force due to the incident wave becomes:

$$f_{ko} = \operatorname{Re} \left\{ T_{ko} = e^{-i\omega} e^{t} \right\}$$
(3)

From now on it will be understood that the real part must be taken of complex expressions and, as the ship oscillates at the encounter frequency, the diffracted wave and flows created by the vessel in the water must also occur at the encounter frequency.

Dealing first with the diffracted wave, we denote its velocity potential by ϕ_7 . This is obtained by solving Laplace's equation,

 $\nabla^2 \phi_7 = 0$

(4)

subject to the following boundary conditions:

$$\omega_{e}^{2} \phi_{7} - g_{\overline{\partial z}}^{2} = 0 \quad \text{on free surface } z = d-c, \qquad (5)$$

$$\frac{\partial \phi_7}{\partial z} = 0$$
 on the seabed $z = -c$, (6)

$$\frac{\partial}{\partial n} (\phi_7 + \phi_0) = 0$$
 on the surface of the hull, (7)

(ie the velocity normal to the hull due to the diffracted wave cancels that due to the incident wave) and

$$\phi_7 \neq 0$$
 at large distances from the ship (8)

This last condition requires the disturbance to represent outgoing waves satisfying the usual "radiation condition" at infinity. Denoting the force due to the diffracted wave by f_7 we can evaluate it in terms of the velocity potential,

$$\underline{f}_7 = \int_{\mathbf{S}} \mathbf{p} \ \rho \ \frac{\partial \phi_7}{\partial t} \ \mathrm{dS}$$

Thus, we find the kth component of the force on the vessel due to the diffracted wave takes the form:

$$f_{k7} = T_{k7} = e$$
(9)

and, in component form, the total force acting on the velocity due to incident and diffracted waves, ie the right hand side of Equation (1), is given by

$$f_{k} = f_{k0} + f_{k7}$$
 (10)

where (3) and (9) define the incident and diffracted components, respectively.

It now remains to determine expressions for the flows created in the water by the oscillations of the vessel, ie the matrices \underline{A} and \underline{B} on the left hand side of (1). Let the amplitude of the jth component of vessel movement be ζ_{i} ie

$$s_{j} = \zeta_{j} e^{-i\omega} e^{t} \qquad j = 1....6$$
 (11)

Thus, the normal velocity of the hull surface is given by:

$$\mathbf{v}_{n} = \underline{n} \cdot \underline{\dot{s}} = -i\omega_{e} \overline{e}^{i\omega_{e}t} \mathbf{e}^{t} \sum_{j=1}^{6} n_{j} \zeta_{j}$$

Denoting the velocity potential due to the 6 oscillations by a linear sum,

$$\phi = -i\omega_{e}e^{-i\omega_{e}t} \sum_{\substack{j=1\\j=1}}^{6} \phi_{j}\zeta_{j}$$

we find the boundary condition on normal velocity,

$$\frac{\partial \phi}{\partial n} = \mathbf{v}_n$$
 on the hull surface

can be satisfied provided,

$$\frac{\partial \phi_j}{\partial n} = n_j \qquad j = 1....6$$
(12)

Thus, the flows created by oscillations of the vessel are described by solving Laplace's Equation (4) subject to the boundary conditions (5), (6), (8) and (12) but with ϕ_7 replaced by ϕ_j for j = 1 to 6. Once solved, the forces on the vessel due to these flows can be obtained in component form, ie

$$F_{kj} = \int_{S} n_{k} \zeta_{j} \rho \frac{\partial}{\partial t} (-i\omega_{e} e^{-i\omega_{e}t} \phi_{j}) dS$$
$$= T_{kj} \zeta_{j} e^{-i\omega_{e}t} , say,$$

where

$$T_{kj} = -\rho \omega_{e}^{2} \int_{S} n_{k} \phi_{j} dS$$
(13)

After substituting for \underline{s} and $\underline{\dot{s}}$ from (11) into (1) we find in component form

$$-\omega_{e}^{2} A_{kj} - i\omega_{e}^{B} B_{kj} = -T_{kj}$$
(14)

where a minus sign appears in front of the transfer coefficient T_{kj} because in (1) these forces have been taken across to the left hand side of the equation of motion.

Taken together, Equations (3), (9), (10) and (14) define all the coefficients of the hydrodynamic forces appearing in Equation (1) in terms of transfer functions T_{kj} (see (2), (9) and (13)). These transfer functions are, in turn, obtained from the form of the incident wave and by solving Laplace's equation for the diffracted wave and flows created by oscillations of the vessel. Then, by solving the following system of (complex) simultaneous equations

we can determine the (complex) amplitudes of vessel movement ζ_{i} .

To allow for a moving vessel Salvesen et al (Ref 8) have shown that the transfer functions representing flows created by vessel oscillations and by diffraction must be further modified (Appendix I).

Having described the method of solution of the equations of motion we go on in subsequent sections to describe their application to two cases. Firstly, a stationary ship and secondly, a supertanker underway in waves.

4 APPLICATION TO A STATIONARY SHIP

Here, we compare the results from UNDERKEEL with results from a separate mathematical model that is also based on linear potential theory. This separate model makes use of sources on the hull (Ref 9) but, as explained in the Introduction, the technique is expensive on computer time and difficulties occur in application as the underkeel clearance is reduced.

Both mathematical models are applied to a large flat-bottomed hulk with pointed ends which is assumed to be freely floating in waves. Although oscillating in the wave motion, the vessel is assumed stationary in the sense that it has no forward speed. The underkeel clearance is taken to be 20% of the draught which is at the upper end of the range of clearances normally required in coastal situations. With this clearance, the source method is expected to be reasonably accurate. The details of the hulk are listed in Table 1.

The results of the comparison are shown in Figures 3 to 8 for surge, sway, heave, roll, pitch and yaw, respectively. The response is displayed as a function of wave direction relative to the ship where $\beta = 0^{\circ}$ represents a following sea and $\beta = 180^{\circ}$ represents a

head sea (Fig 2). For surge, sway and heave (Figs 3 to 5) the response function is simply the amplitude of ship movement divided by the wave amplitude. For the angular movements roll, pitch and yaw (Figs 6 to 8) the response is given in degrees per metre of wave amplitude.

For each movement, plots (a) to (d) are given for the four frequencies 0.04, 0.06, 0.08 and 0.10 Hertz corresponding to wave periods of approximately 25, 16.7, 12.5 and 10 seconds, respectively. This covers a range extending from the longest wave periods likely to occur in nature to the shortest periods able to produce a significant response. Comparison of the responses at 25 and 10 seconds shows how small all the movements become even for 10 second waves. This is a result of the large size of the ship, with more and more cancellation of wave pressures occurring over the hull as wave periods and wavelengths decrease.

Generally speaking, the source method and UNDERKEEL show close agreement. The largest differences occur in roll at frequencies of 0.04 and 0.06 Hertz ie at periods of 25 and 16.7 seconds. These periods straddle the resonant roll period of the vessel which explains why responses are much higher at these periods than at the shorter periods (Figs 6a to 6d). It is unclear at this stage which model represents linear potential theory more accurately. Tn application to the case of small underkeel clearance it has been found that with the source method, the roll response is sensitive to the number of sources distributed over the hull which indicates the possibility of numerical errors. However, the differences between the two models are somewhat academic since it is known that non-linearities due to eddy shedding and viscous damping are more important in damping resonant roll oscillations than the

wave-making term present in potential theory. Thus, in nature, the roll responses at 25 and 16.7 seconds can be expected to be less than those predicted by both the source method and UNDERKEEL.

The results from the earlier UNDERKEEL model (Ref 3) for surge, heave and pitch are displayed in Figures 3, 5 and 7 as the small dashed line. This earlier model only described these three movements. In addition. diffraction around the vessel was ignored as well as coupling effects of surge into heave and pitch. Nevertheless, the responses from the earlier model follow quite closely the responses obtained with the latest version of UNDERKEEL which includes diffraction and all coupling terms. This illustrates that the additional effects now incorporated into UNDERKEEL are relatively unimportant for these three movements. However this is not the case for sway, roll and yaw where diffraction forces in particular become significant.

The realistic description of heave and pitch by the earlier version of the UNDERKEEL explains why the model proved so valuable in the definition of safe underkeel allowances for vessels negotiating the Dover Strait (Ref 4). Encouraging agreement was obtained between results from the earlier version of UNDERKEEL and results from physical model tests carried out specifically for the Dover Strait study. In the next section we pursue this further by comparing responses obtained using the latest version of UNDERKEEL with those physical model responses.

5 APPLICATION TO A SHIP UNDERWAY

A comprehensive study of safe underkeel allowances for supertankers negotiating the Dover Strait was carried out recently (Ref 4). In that study, an earlier

version of UNDERKEEL for heave and pitch was proved against physical model tests carried out using long crested random waves. Vertical movements of the bow and stern of the vessel eventually proved to be the most critical movements in that study because the largest waves in the Dover Strait tended to give bow quartering to bow seas or stern quartering to stern seas and it was found that heave and pitch dominated over roll. This meant the earlier version of UNDERKEEL was proved against physical model results for bow and stern movements directly and not for heave and pitch. Fortunately, the method of measurement used in the physical model required heave, pitch and roll responses first before vertical movements of various points on the bottom of the ship could be calculated. Thus, we are able to use the physical model data collected in the earlier study to directly check predictions for heave, pitch and roll from the latest version of UNDERKEEL.

The following sub-sections give some details of the Dover Strait investigation before the comparison with the latest version of UNDERKEEL is described. This will help to give insight into the method of application of combined physical and mathematical modelling to the problem of defining safe underkeel allowances.

5.1 Physical model

A preliminary study (Ref 10) of the Dover Strait problem was made using the earlier version of UNDERKEEL well before the physical model investigation was carried out. This established that in the physical model we needed to consider only one hull shape and that it could be run at a single speed of 12 knots, a typical value of full ahead manoeuvring speed for supertankers. The tests were carried out with a 1 to 100 scale model of the LANISTES (Table 2 and Fig 9) owned by Shell International Marine Limited.

Froude scaling was used throughout which meant the time scale was 1 to 10 and events occurred in the model ten times faster than in nature.

The wave basin was flat- bottomed and measured 37m by 50m. Two 15m long random wave-makers were used side by side to generate a (long crested) wave front with a width equivalent to 3 kilometers at full scale (Plate 1). This provided sufficient wave front for the vessel to be tested underway for a reasonable length of time. Clearly, in random waves, sufficient tests have to be performed to obtain sensible statistical data and the longer the wave front the fewer the number of tests needed in a given wave condition.

Wave conditions were long crested (uni-directional) but with energy spread over a range of wave periods. Based on Pierson-Moskowitz spectra, but with scaling factors applied to give appropriate significant wave heights, the conditions chosen gave a representative sample of severe Dover Strait conditions. Two spectra, with peaks at 19 and 14.5 seconds, represented swell (Figs 10 and 11) and two more spectra, with peaks at 13 and 11 seconds, represented storm waves (Figs 12 and 13).

Tests were carried out with waves at angles to the ship of 0° (following sea), 30° , 60° , 75° , 90° (beam sea) 105° , 120° , 150° and 180° (head sea).

The model ship was radio controlled and propelled by an electric motor. Initial runs served to verify that the Gumbel probability distribution could be used to predict the risk of exceedence of extreme downward movements in the random waves. An example is provided in Figures 14. In a head sea, the stern of the vessel experienced the largest movements and the maximum

value in the model equivalent of 3km of travel was recorded for each of 20 separate runs of the model vessel. When these 20 maxima were plotted, as the square of downward movement against the risk of exceedence, it was found that the Gumbel distribution gave a very good fit to the data (Fig 14). This in turn means that just the standard deviation and the zero crossing period the movement are needed to predict extreme values in a random sea. This feature of maximum ship movements was verified for other wave directions. It is an important point because it means the risk of a large movement being exceeded or, in other words, the risk of a given vertical motion allowance being exceeded, can be well defined once we have accurate estimates of the standard deviation and zero crossing period of the movement. It is a straightforward calculation to superpose responses from UNDERKEEL for individual wave components in order to build-up the standard deviation and zero crossing period of movement in particular wave spectra. Thus. provided the responses from UNDERKEEL are accurate, we can use the mathematical model together with the Gumbel distribution to predict extreme movements.

Vertical motions of the ship were measured by a system of accelerometers. This gave the ship's heave, pitch and roll which, in turn, allowed responses of various points on the bottom of the vessel to be calculated. Before testing on a moving ship we had confirmed the accuracy of our system by comparing it with independent measurements of the ship's movement when stationary. Good agreement was found between the movements calculated from the accelerometer data and direct records of movement made with a completely different measurement system.

5.2 Physical model

test programme

As explained at the beginning of 5.1, a preliminary study of the Dover Strait problem was made using the earlier version of UNDERKEEL (Ref 10). This helped to narrow down the number of ships to be tested to one, the LANISTES, which was representative of the range of supertankers with draughts between 20.5m and 22m. It was also established using UNDERKEEL that vessel movements at 12 knots were representative of those at 15 knots (of the order of maximum vessel speed) and at speeds down to 8 knots. Thus, it was only necessary to represent the LANISTES at 12 knots in the physical model.

The preliminary study also established the sensitivity of these large vessels to swell approaching the Dover Strait at the western end from the Atlantic, and approaching the Strait on the eastern side from the North Sea. This led to a subsequent study to define the magnitude of such swells in the Strait. This was in addition to definition of the locally generated storm wave climate which had already been carried out as part of the preliminary study.

UNDERKEEL also demonstrated the importance of the directional spread of wave energy present in storm waves. For the particular storm waves expected in the Strait it was found that heave and pitch were enhanced by a directional spread of wave energy. Therefore, to cover the range of storm wave spreads, mean directions and vessel headings that could occur in the Strait, it became necessary to investigate a full range of wave directions from following seas, through beam seas to head seas. Added to this was the fact that UNDERKEEL showed vertical movement to be sensitive to the underkeel allowance.

In view of the huge amount of testing that would have been required were only the physical model used to define the final underkeel allowances, it was decided to carry out enough tests to establish whether UNDERKEEL was sufficiently accurate to be used subsequently in calibrated form when obtaining the final allowances. Therefore, the 9 discrete wave directions already mentioned in 5.1 were chosen for testing in the physical model with underkeel allowances of 4m, 6m and 8m. These clearances encompassed the range of safe allowances finally expected. Although, with the 4 wave spectra chosen to represent swell and storm waves, the number of conditions to be tested still totalled 9x3x4 = 108. And, with about 8 separate runs of the vessel needed for each condition to establish stable estimates of the quantities used to define extreme movements, some 860 separate runs of the vessel appeared necessary. In the event, UNDERKEEL produced such encouraging agreement with physical model results for cases where heave and pitch dominated, that only 62 different conditions had to be investigated.

For example, the physical model showed that bow and stern points on the bottom of the vessel experienced the largest vertical movements for wave directions varying from stern seas around to stern quartering seas and for bow seas around to bow quartering seas. These movements are controlled solely by heave and pitch of the vessel and, after some relatively minor calibration of UNDERKEEL, it was found that the mathematical model not only described well the changing responses with changing wave spectra, it also described the change in response in going from a 4m to a 6m underkeel allowance (Table 3). As a result, it was not necessary to test an 8m allowance for this range of directions and a considerable saving on physical model testing was obtained.

In Deamier seas, the physical model showed that quarter or shoulder points on the bottom of the vessel experienced more vertical movement than the bow or This was, of course, expected as roll is stern. greater for these wave directions. The earlier version of UNDERKEEL did not describe roll and so it was not possible to give mathematical model prediction for movements at the vessel's shoulders. Therefore. the physical model results were used to build-up responses for the shoulder positions. However when evaluating the final responses to the particular multi-directional wave spectra predicted in the Dover Strait, it was found that the bow and stern would experience the largest movements: the amount of wave energy with beamier component directions being insufficient to cause other points on the vessel to dominate.

Thus, the final outcome was that the earlier version of UNDERKEEL was used to predict safe allowances for vertical ship motion in the Strait, after being calibrated against the physical model.

5.3 Comparison of physical model with UNDERKEEL

> Here, we apply the latest version of UNDERKEEL to the cases tested in the Dover Strait physical model and compare results from the two models. Firstly we compare response functions for heave, roll and pitch and secondly, we compare standard deviations of movement for critical points on the vessel.

Response functions for LANISTES

These functions snow how the magnitude of response varies with wave frequency where the latter is defined in a stationary frame of reference. Thus, although the ship responds at the encounter period experienced by an observer on the moving vessel, responses are

presented here in terms of the wave period seen by a stationary observer. This makes comparison easier between responses for different wave directions.

We consider the LANISTES underway at 12 knots with an underkeel allowance of 4m, or just under 20% of the draught. The heave response is shown in Figures 15a to 15i for the 9 wave directions tested in the physical model ranging from stern seas (0°) around to bow seas (180°). In this case the response function is the ratio of the amplitude of vessel movement to the wave amplitude. For roll (Fig 16) and pitch (Fig 17) the response function is the ratio of the amplitude of angular movement in degrees to the wave amplitude in metres.

For all 3 responses, the physical model data is obtained by taking the square root of the ratio of spectral density of ship movement (converted to the stationary trame of reference) to spectral density of the waves. For a particular wave spectrum there is a unique value for this quantity at each frequency But as the spectra chosen (see Fig 10 for component. the swell spectrum with T_{D} = 19s and Fig 12 for the storm wave spectrum with $T_p = 13s$) both possess energy for some of the frequencies, there is overlap in the experimental data. Thus, both spectra provide estimates of the response at some frequencies and, as the amount of energy at these frequencies is different in the two spectra, the similarity in these experimental responses is a measure of the linearity of vessel response. On the whole, the similarity in experimental responses suggests a high degree of linearity, particularly when it is remembered that the spectral values used to form the response function possess statistical uncertainty. This also appears true for roll which is perhaps surprising as non-linear damping due to eddy snedding is expected to

be important. Thus, the basic assumption of linearity made in UNDERKEEL is well justified.

The experimental data for waves at 30° to the stern have been omitted because responses were, to some degree, affected by unrealistic wave reflections between the vessel and the guides used to maintain energy across the wave front.

Taking heave first we see UNDERKEEL predicts responses very well for stern (Fig 15a) bow quartering (Fig 15h) and bow seas (15i) and although there is some overprediction for directions in between, with the most noticeable differences occurring for beam seas (Fig 15e) the mathematical response function forms an envelope for the experimental responses. The reason for some overprediction in beamier seas is unclear at present although sensitivity of the results to the way the shape of the hull is represented in UNDERKEEL has become apparent.

A similar picture emerges for pitch (Fig 17) in that there is very good agreement between UNDERKEEL and experimental results for stern (Fig 17a) bow quartering (Fig 17h) and bow seas (Fig 17i) with less good agreement for directions in between. In the case of pitch there is a noticeable overprediction by UNDERKEEL for waves with frequencies between 0.06 and 0.09 Hertz approaching the stern at an angle of 60° (Fig 17c). There is also an underprediction for very long waves with periods greater than 20 seconds (frequencies lower than 0.05 Hertz) which is apparent for directions from 60° to the stern around to beam seas (Figs 17c to 17e). In application to specific problems, the latter underprediction is not expected to be important because there is generally little energy present in sea spectra at wave periods longer than 20 seconds. The overall error in UNDERKEEL will

be one of overprediction (see under next heading for movement of bow and stern).

Roll responses appear in Figures 16a and 16i. Considering the important role expected for non-linearities like eddy shedding in controlling these responses, the agreement between experiment and UNDERKEEL, which has no non-linearities represented must be considered encouraging.

For stern (Fig 16a) and bow seas (Fig 16i) the theoretical roll response is zero whereas a small amount of roll occurred in the experiments. The experimental roll developed because it was impossible to keep the model vessel travelling in exactly a straight line: corrections to the direction of travel had to be made continually via the radio-controlled rudder as the vessel veered from side to side. This meant that even in bow and stern seas the waves were angled to the ship from time to time and rolling occurred.

Apart from the expected tendency for UNDERKEEL to overestimate roll responses, due to the absence of non-linear damping mechanisms in the mathematical model, there is also an indication that the resonant roll period is too short in UNDERKEEL, ie that it occurs at too high a frequency. This would explain why the peak in the mathematical response occurs to the right of the peak in the experimental results in Figures 16c to 16h. Although a heavier damping coefficient would have the effect of moving the peak in the mathematical response to lower frequencies, it is also possible that the roll added inertia may be slightly too small in UNDERKEEL. This latter effect would make the resonant roll period slightly shorter than the experimental value. Again, once the complete response to a given

spectrum is obtained, UNDERKEEL tends to overpredict rather than underpredict movements (see under next heading for movements of quarter positions on the vessel).

Taken as a whole, the agreement obtained here between experiment and UNDERKEEL is very encouraging. The change in the pattern of response with wave direction is well represented for a vessel underway and this indicates that the complex flows around the vessel controlling added inertia, damping and wave diffraction, are being well described in UNDERKEEL. In addition, the high degree of linearity observed in the experimental results justifies the idea of superposition of component responses to describe vertical ship movements in random waves.

Movement of critical points on LANISTES

When a vessel heaves, rolls and pitches in random waves it is likely to experience the largest vertical movement at one of six positions on the flat part of its keel. For convenience they are listed below in partly abbreviated form:

bow	-	bow,	
starboard bow quarter	-	sd. bow,	
port bow quarter	-	pt. bow,	
starboard stern quarter	-	sd. stern	
port stern quarter	-	pt. stern	
stern	-	stern.	

The bow and stern movements are defined completely in terms of heave and pitch whereas movements of the quarter or shoulder positions depend on roll as well as heave and pitch.

As explained in 5.1, once the standard deviation and zero crossing period of vessel movement in a given wave spectrum has been estimated, it is a straightforward calculation to obtain the risk of exceeding an allowance for vessel movement. And this. in turn, enables a safe allowance with an acceptable risk of exceedence, to be defined. Here, therefore, we compare standard deviations of responses to the various wave spectra tested (the most important parameter for defining safe allowances) with predictions from UNDERKEEL. For each wave spectrum and direction we list experimental and mathematical standard deviations in metres for the two positions on the vessel with the largest movements. The results appear for following, quartering and head seas (0° to 30° and 150° to 180°) in Table 4 and for quartering to beam seas (60° to 120°) in Table 5.

For the directions listed in Table 4 the bow and stern moved through the greatest distances with bow movements exceeding stern movements in stern (0°) and stern quartering seas (30°) and with the opposite generally applying for bow quartering (150°) and bow seas (180°). It can be seen that UNDERKEEL describes this pattern of behaviour well with particularly good agreement on standard deviation values for the swell spectra (T_p = 19s and 14.5s) approaching the stern (0°).

Another feature of the results in Table 4 is the variation in response with underkeel allowance. For 0° and 30° wave directions there is little change in response in going from a 4m to a 6m allowance but for 150° and 180° directions the responses increase significantly. Again, this pattern of behaviour is well described by UNDERKEEL (see also Table 3 for results obtained with the earlier version of UNDERKEEL).

Taken together, the results in Table 4 show that UNDERKEEL is able to describe the changing pattern in response as the wave spectrum, wave direction and underkeel allowance are altered. In addition, UNDERKEEL responses are generally conservative but not excessively so. Both these features are important in application to specific problems. In comparing different proposals in feasibility studies it is important that UNDERKEEL be able to describe the changing pattern in response in the various proposals to enable the most cost effective ones to be chosen. It is also important that UNDERKEEL be sufficiently accurate to provide realistic first estimates of the final safe allowance. In this regard it is useful that UNDERKEEL is slightly conservative.

Turning now to Table 5 for the remaining wave directions, we are able to make comparisons for 4m, 6m and 8m allowances. The full range of allowances were tested because it was accepted in the Dover Strait study that physical model results would have to be used for beamier seas if quarter or shoulder point movements became more critical than movements of the bow and stern (the earlier version of UNDERKEEL not describing roll made mathematical predictions of quarter point movements impossible). In the final application to the Dover Strait, however, it was found that bow and stern movements were the most critical and, after calibration against the physical model, the earlier version of UNDERKEEL was used to define safe allowances. It was during this calibration that it was realised testing an 8m allowance in the physical model was not necessary for the wave directions listed in Table 4 because of the accuracy of UNDERKEEL in predicting the effect of changing from 4m to a 6m allowance.

Table 5 shows that even for waves approaching the stern at 60°, heave and pitch dominate over roll with the result that bow and stern move the most. Bow movements are noticeably larger than stern movements: an effect well described by UNDERKEEL. As indicated by the pitch response function for this direction (Fig 17c) there is some overprediction by UNDERKEEL for waves with periods between 17 and 11 seconds and this is the cause of the overprediction for the spectrum with its peak at 13 seconds. The agreeement for the other spectrum is much closer.

For the remaining directions covered in Table 5, quarter positions generally move the most and a comparison is given between the two largest movements from each model. These positions do not always coincide in the two models because the vessel is executing a complicated corkscrewing motion as it heaves, pitches and rolls and, unless magnitudes and phases closely agree for component motions, the resultants will differ. We expect the roll response to be improved by incorporating non-linear damping into UNDERKEEL and this may well improve the correlation between the two models as far as quarter point movements are concerned. As expected, UNDERKEEL generally overpredicts quarter point movements at present although there is some underprediction for 75° with the 19 second swell spectrum at 6m and 8m underkeel allowances. This underprediction can be explained by the peak of the mathematical roll response being "to the right" of the experimental one (see Fig 16d) thereby producing underprediction for lower frequencies (longer periods). Increasing the damping will also tend to correct this tendency in UNDERKEEL.

6 CONCLUSIONS

- Using a simplified representation of flow beneath the hull, it has been possible to calculate diffraction, inertia and damping coefficients in the equations of motion of a free ship when the underkeel clearance is limited. The resulting mathematical model is called UNDERKEEL.
- 2. This work forms an extension of an earlier mathematical model which described heave and pitch without taking surge coupling and wave diffraction into account (Ref 3). In the latest version of UNDERKEEL, roll motions are also considered along with wave diffraction around the vessel and coupling of surge into heave and pitch as well as coupling between sway, roll and yaw.
- 3. The role of UNDERKEEL is two-fold. It can be used directly to provide a realistic first estimate of safe underkeel allowances for vessels in navigation channels, or at berths, once the wave climate has been defined. Its second use is in defining the hydrodynamic coefficients needed to describe moored ship motions. These coefficients are used in a separate mathematical model called SHIPMOOR (Ref 1 and 2) which takes account of the non-linear characteristics of conventional mooring systems. Ultimately, it is the aim that SHIPMOOR be used to obtain a realistic first estimate of berth tenability during feasibility studies of port developments.
- 4. Results from UNDERKEEL have been compared with results from a separate mathematical model that makes use of sources on the submerged part of the hull. Both models were applied to a large vessel (Table 1) freely oscillating in waves but with no forward speed. The source method uses the same
basic linear potential theory assumed in UNDERKEEL, but it is expensive on computer time and some difficulties are experienced in application when the underkeel clearance is small. For the purposes of comparison, a relatively large clearance for coastal applications of 20% of the draught was chosen to minimise problems with the source method. The comparison (Figs 3 to 8) shows very close agreement between the two mathematical models for surge, sway, heave, pitch and yaw. Away from resonance, the roll responses also agree (Figs 6c and 6d) but near resonance (about 0.05Hz) the source method gives a larger response than It is unclear which model is the more UNDERKEEL. accurate here but the differences are of academic interest. Additional non-linear damping is known to occur on roll resonance and this is likely to cause the magnitude of a real vessel's response to be even smaller than UNDERKEEL's (see also 6(b) below).

- 5. The close agreement obtained between the source method and UNDERKEEL justifies the more direct method of calculation used in the latter model which has been specifically developed for the case of a limited underkeel clearance. This more direct approach is less expensive on computer time and it is likely to be more accurate for clearances of less than 20% of the draught.
- 6. Results from UNDERKEEL have also been compared with results obtained from a physical model of a supertanker (Table 2 and Fig 9) underway at 12 knots in long crested random waves. The physical model investigation was carried out as part of a project study of safe underkeel allowances for supertankers negotiating the Dover

Strait. In the project study an earlier version of UNDERKEEL (Ref 3) proved to be of immense value. The earlier mathematical model was used, initially, to define critical parameters for the problem (Ref 10) and subsequently, to define safe underkeel allowances after calibration against the physical model (Ref 4).

The comparison between the latest version of UNDERKEEL and the physical model indicates the following:

- (a) UNDERKEEL predicts heave and pitch responses as a function of wave frequency very well for stern (Figs 15a and 17a) bow quartering (Figs 15h and 17h) and bow seas (Figs 15i and 17i). There is some overprediction by UNDERKEEL for other wave directions (Figs 15c to 15g and 17c to 17g) but the differences are not excessive.
- (b) Non-linearities like eddy shedding are expected to be important in limiting roll responses. Although these effects are not represented at present in UNDERKEEL, the agreement obtained with experimental values of roll (Figs 16b to 16h) is considered encouraging enough to pursue the addition of non-linear damping to UNDERKEEL, which in turn should reduce the observed overprediction. For stern (Fig 16a) and bow seas (Fig 16i) the theoretical roll response is zero whereas a small amount of rolling occurred in the physical model as the vessel veered slightly from side to side during The differences are therefore testing. unimportant for these wave directions.

- (c) Generally speaking, heave and pitch response functions are well described by UNDERKEEL with a slight overprediction in some cases. The overprediction of roll was expected due to the absence of non-linear damping mechanisms in UNDERKEEL.
- (d) Taken as a whole, the experimental results indicate a largely linear responses to waves and this justifies the superposition of responses to single period, single direction waves when using UNDERKEEL to describe vessel motions in random waves.
- (e) The above conclusions, based on a comparison of response functions, carry over when comparing model predictions for the standard deviation of vertical movements of critical points on the flat part of the supertanker's keel. Bow and stern of the vessel experience the largest movements in stern (0°) stern quartering (30°) bow quartering (150°) and bow seas (180°) . This behaviour observed in the physical model was well described by UNDERKEEL for the full range of wave spectra tested and for the two underkeel allowances of 4m and 6m (Table 4). As these movements are controlled by heave and pitch of the vessel, this agreement parallels the good agreement described above in 6(a). The additional information provided by comparing movements for critical points on the vessel is that phasing of responses, as well as magnitudes, can be checked. The results in Table 4 show that UNDERKEEL follows well the higher bow movements in following seas and the generally higher stern movements in head

seas: features that depend on the relative phasing of heave and pitch.

- (f) For quartering to beam seas, roll becomes more important although even for waves approaching the stern at an angle of 60°, heave and pitch still dominate making the bow and stern experience the largest movements (Table 5). For the remaining wave directions (75° to 120°) quarter or shoulder positions undergo the largest vertical movements with the vessel "corkscrewing" in the water as it heaves, roll and pitches. The overprediction of roll by UNDERKEEL as described above in 6(b) comes into play here causing quarter position movements to be, in the main, overpredicted.
- (g) Experimental results (Fig 14) show that the risk of a large movement being exceeded within a given length of the vessel's track. can be defined once the standard deviation and zero crossing period of the movement is known: the most sensitive parameters here being the standard deviation. This feature allows definition of the risk of a given vertical motion allowance being exceeded. and hence the definition of a safe allowance. Thus, good agreement between the physical model and UNDERKEEL for standard deviations of movements of critical points on the vessel, translates into accurate estimates for safe underkeel allowances. The results described above in 6(e) and 6(f)show that UNDERKEEL will provide a good first estimate for a safe allowance in cases where heave and pitch dominate (Table 4) but that non-linear damping of roll will have to

be considered to limit the overpediction of the safe allowances in situations where beamier seas are dominant.

7 ACKNOWLEDGEMENTS

The authors are grateful to the Department of Transport for permission to use the results of the physical model investigation carried out as part of the Dover Strait Study.

The authors are also grateful to Mr P Beresford for his help in carrying out the physical model work.

8 REFERENCES

- Spencer J M A and Bowers E C. Mathematical Modelling of Moored Ships in the Time Domain. HR Report SR 103, August 1986.
- Spencer J M A. Mathematical Simulations of a Ship Moored in Waves and of the Effects of a passing vessel on a Moored Ship. HR Report SR 145, January 1988.
- 3. Lean G H, Bowers E C, Matsoukis P F and Beresford P J. A Mathematical Model of Vertical Ship Movement in Waves with a Small Underkeel Clearance and its Validation Against Physical Model Results. HR Report IT 277. July 1984.
- 4. Underkeel Allowance for Deep Draughted Vessels in the Dover Strait. HR Report EX 1432, May 1986.
- Korvin-Kroukovsky B V. Theory of Sea Keeping. SNAME, New York, 1961.
- Oortmerssen G van. The motions of a moored ship in waves. Netherlands Ship Model Basin Pub. No. 510, Wageningen, Holland.
- Beck R F and Tuck E O. Computation of shallow water ship motions. 9th Symposium on Naval Hydrodynamics, Paris p1543, 1972.
- Salvesen N, Tuck E O and Faltinsen O M. Ship motions and Sea Loads. Trans. SNAME, Vol 19, p 250, 1970.
- Standing R G. Use of potential flow theory in evaluating wave forces on offshore structures. Proc. Conf. Power from Sea Waves. Inst. Math. Appls. Edinburgh, 1979.

10. Underkeel Allowance for Deep Draughted Vessels in the Dover Strait. Phase 1 - Preliminary study to establish critical parameters. HR Report EX 1309, June 1985.



TABLES.



Length	320m
Breadth	60 m
Draught	24m
Displacement	365 000m ³
Distance of centre of gravity from bow	120m
Height of centre of gravity above keel	17m
Pitch radius of gyration	80m
Roll radius of gyration	14m
Metacentric height	4.6m
Roll period (approximate)	19s
Heave period (approximate)	21s
Pitch period (approximate)	19s

TABLE 1 Details of the hulk used for stationary ship tests

TABLE 2Details of the LANISTES (fully laden)

	LANISTES EQU	IVALENT PHYSICAL MODEL DETAILS (at full scale)
Length overall (m)	343.6	343.6
Length between perpendiculars (m)	330.0	330.0
Beam (m)	56.0	56.0
Draught (loaded) (m)	22.3	22.3
Depth moulded (m)	28.7	28.7
Distance of centre of gravity aft of forward perpendicular (m)	157.7	157.7
Height of centre of gravity above keel (m)	15.05	15.0
Radius of gyration for pitch (m)	84.5	85.0
Metacentre height for roll (m)	7.9 (GM. FLUID) 8.2
Radius of gyration for roll (m)	14.4*	14.7
Roll period (s)	13.5*(approxima	ite) 13.1

* These figures are for BRITISH RESPECT, a similar vessel

TABLE 3 Physical model and calibrated mathematical model (earlier version of UNDERKEEL) predictions for change in response in going from 4m to 6m underkeel allowance

Wave conditon	Sea direction	% change	in response
		Physical model	Mathematical model
	Stern	-5	-5
$H_{s} = 1.5m$	30° to stern	-6	-7
$T_{p} = 19.0s$	30° to bow	+15	+22
r	bow	+33	+33
	Stern	-16	-9
$H_{s} = 2.8m$	30° to stern	-2	0
$T_{p} = 14.5s$	30° to bow	+33	+45
r	bow	+38	+33
	Stern	-11	-5
$H_s = 5.0m$	30° to stern	-6	-0
$T_{p} = 13.0s$	30° to bow	+40	+45
F	bow	+25	+33
· · ·	Stern	-12	-4
$H_{s} = 4.8m$	30° to stern	+6	-2
$T_{p} = 11.0s$	30° to bow	+25	+38
Г	bow	+31	+40

TABLE 4 Critical movements of the LANISTES at 12 knots in following, quatering and head seas

Wave Spectrum Standard deviation (m) of vessel movement for:

0°	30°	150°	180°
(following seas	3)		(head seas)

(a) <u>4m underkeel allowance</u>

T _s (m)	$T_p(s)$		Bow	Stern	Bow	Stern	Bow	Stern	Bow	Stern
1.5	19.0	Experiment UNDERKEEL	0.37, 0.42,	0.32 0.33	0.47, 0.56,	0.39 0.43	0.38, 0.56,	0.47 0.67	0.36, 0.48,	0.40
2.8	14.5	Experiment UNDERKEEL	0.43, 0.47,	0.41 0.37	0.50, 0.61,	0.45 0.54	0.30, 0.37,	0.36 0.42	0.20, 0.24,	0.24 0.26
5.0	13.0	Experiment UNDERKEEL	0.61, 0.73,	0.60 0.57	0.70, 0.81,	0.66 0.80	0.35, 0.34,	0.40 0.37	0.27, 0.21,	0.23
4.8	11.0	Experiment UNDERKEEL	0.43, 0.59,	0.42 0.44	0.51, 0.55,	0.49 0.60	0.14, 0.11,	0.16 0.10	0.086	0.099 0.070
(b) <u>6m</u>	underke	el allowance			4		4	-		
1.5	19.0	Experiment	0.35,	0.34	0.44.	0.42	0.49.	0.54	0.53.	0.53

		UNDERKEEL	0.40, 0.33	0.44, 0.42	0.49, 0.34	0.53, 0.53
2.8	14.5	Experiment UNDERKEEL	0.36, 0.35 0.43, 0.36	0.49, 0.45 0.56, 0.51	0.44, 0.48 0.54, 0.61	0.32, 0.33 0.37, 0.39
5.0	13.0	Experiment UNDERKEEL	0.53, 0.54 0.65, 0.53	0.65, 0.66 0.76, 0.74	0.51, 0.56 0.55, 0.66	0.35, 0.35 0.34, 0.34
4.8	11.0	Experiment UNDERKEEL	0.38, 0.38 0.52, 0.41	0.54, 0.54 0.51, 0.52	0.19, 0.20 0.18, 0.17	0.12, 0.13

(a) 4m und	ierkeel allowance															
Н _s (т) Т ⁽	(s)	60 •			7	ູ້ນ			90 (beam)• seas)			105°		120•	
1.5 19.	.0 Experiment UNDERKEEL	bow 0.98, st. " 0.96, '	ern 0.70 " 0.74	pt. bow sd. stern	0.77, 0.78,	bow (stern (0.78	pt. bow sd. stern	0.95, 1.30,	pt. stei sd. bow	rn 0.86 1.19	sd. bow 0.76, " 0.97,	pt. stern 0.88 sd. stern 1.20	sd. bow 0.79 pt. bow 0.83,	pt. stern 0. stern 1.	.79 .08
2.3 13.	.0 Experiment UNDERKEEL	" 0.66, ' " 1.06, '	. 0.62	pt. bow sd. stern	0.86, 1.28,	sd. stern (sd. bow]	0.74	pt. bow sd. bow	0.92, 1.82,	sd. ste	rn 0.95 1.91	" 0.71, " 0.93,	" 0.69 " 1.08	sd. bow 0.47, pt. bow 0.62,	stern 0. " 0.	.47 .48
pun m9 (q)	erkeel allowance	- -														
1.5 19.	0 Experiment UNDERKEEL	bow 0.91, st: " 0.87, '	ern 0.70 1 0.69	pt. bow	0.91, 0.89,	pow	0.93	pt. bow pt. stern	1.08, 1.06,	sd. stei pt. bow	rn 0.95 0.94	sd. bow 0.90, "0.98,	pt. stern 1.01 " 1.17	sd. bow 0.87, " 1.22,	pt. stern 0. " 1.	.85 .31
2.3 13.	.0 Experiment UNDERKEEL	" 0.88, " " 1.12, '	" 0.68 " 0.96	= =	1.18, 1.43,	" (sd. stern]	0.95	pt. bow sd. bow	1.36, 1.47,	sd. ste pt. ste	rn 1.35 rn 1.55	" 1.01, " 1.30,	" 0.99 " 1.26	" 0.68, " 1.06,	sd. stern () pt. stern 1	.04
(c) 8m und	lerkeel allowance															
1.5 19.	.00 Experiment UNDERKEEL	bow 0.89, st. " 0.80, '	ern 0.69 " 0.64	pt. bow "	1.06, 0.86,	n n	1.07	pt. bow pt. stern	1.23, 0.96,	pt. ste pt. bow	rn 1.07 0.90	sd. bow 0.93, " 1.28,	pt. stern 1.00 " 1.26	sd. bow 0.79, stern 0.90,	pt. stern 0 " 0	.74 .86
2.3 13.	.0 Experiment UNDERKEEL	" 0.95, " 1.09, '	" 0.75 " 0.92	z z .	1.29, 1.51,	n 8d. stern	1.11 1.25	pt. bow pt. stern	1.30, 1.51,	sd. ste "	rn 1.36 1.48	" 1.15, " 1.97	sd. stern 1.15 " 1.74	sd. bow 0.68, " 0.82,	stern 0	.69
													_			

TABLE 5 Critical movements of the LANISTES at 12 knots in quartering to beam seas

Wave spectrum Standard deviation (m) of

Standard deviation (m) of vessel movement for the wave directions:



FIGURES.





Fig. 1 The six degrees of vessel movement.



Fig 2 Definition sketch



Fig. 3a Surge response of stationary ship at 0.04 Hz.

.



Fig. 3b Surge response of stationary ship at 0.06 Hz.



Fig. 3c Surge response of stationary ship at 0.08Hz.



Fig. 3d Surge response of stationary ship at 0.10 Hz.







Fig. 4b Sway response of stationary ship at 0.06Hz.



Fig. 4c Sway response of stationary ship at 0.08Hz.

.



Fig. 4d Sway response of stationary ship at 0.10 Hz.







Fig. 5b Heave response of stationary ship at 0.06 Hz.



Fig. Sc Heave response of stationary ship at 0.08 Hz.



Fig. 5d Heave response of stationary ship at 0.10 Hz.



Fig. 6a Roll response of stationary ship at 0.04 Hz.



Fig. 6b Roll response of stationary ship at 0.06Hz.

--**F**-...



Fig. 6c Roll response of stationary ship at 0.08Hz.



Fig. 6d Roll response of stationary ship at 0.10 Hz.


Fig. 7a Pitch response of stationary ship at 0.04 Hz.

.



Fig. 7b Pitch response of stationary ship at 0.06 Hz.



Fig. 7c Pitch response of stationary ship at 0.08 Hz.





Fig. 8a Yaw response of stationary ship at 0.04 Hz.



Fig. 8b Yaw response of stationary ship at 0.06Hz.

....



Fig. 8c Yaw response of stationary ship at 0.08 Hz.



Fig. 8d Yaw response of stationary ship at 0.10 Hz.



Fig 9 Line drawing of tanker



Fig 10 Swell with 19s spectral peak

3...



Fig 11 Swell with 14.5s spectral peak



Fig 12 Storm wave with 13s spectral peak

.



Fig 13 Storm wave with 11s spectral peak

.





Fig 15a Heave at 12 kts, 0°





Fig 15c Heave at 12 kts, 60°



Fig 15d Heave at 12 kts, 75°

. .





Fig 15f Heave at 12 kts, 105°





Fig 15h Heave at 12 kts, 150°



Fig 15i Heave at 12 kts, 180°



Fig 16a Roll at 12 kts, 0°



Fig 16b Roll at 12 kts, 30°



Fig 16c Roll at 12 kts, 60°



Fig 16d Roll at 12kts, 75°



Fig 16e Roll at 12 kts, 90°





Fig 16g Roll at 12 kts, 120°



Fig 16h Roll at 12 kts, 150°



Fig 16i Roll at 12 kts, 180°



Fig 17a Pitch at 12 kts, 0°



Fig 17b Pitch at 12 kts, 30°



Fig 17c Pitch at 12 kts, 60°



Fig 17d Pitch at 12 kts, 75°




Fig 17f Pitch at 12 kts, 105°



Fig 17g Pitch at 12 kts, 120°



Fig 17h Pitch at 12 kts, 150°



PLATE.





Plate 1 Overall view of wave basin layout

APPENDIX.



APPENDIX 1

Effect of vessel speed on transfer coefficients

Salvesen et al give the following equations relating the transfer function $T_{kj}(U)$ for a vessel underway and T_{kj} for a stationary vessel. These relationships were obtained for a vessel in deep water but here they have been assumed to apply in shallow water. (We have used a sign of i, the square root of -1, which is opposite to that used by Salvesen et al.)

$T_{11}^{(U)} = T_{11}$,	$T_{13}^{(U)} = T_{13}$,	$T_{15} \stackrel{(U)}{=} T_{15} - \gamma T_{13}$
$T_{22}(U) = T_{22}$,	$T_{24} \frac{(U)}{(U)} = T_{24}$,	$T_{26} (U) = T_{26} + \gamma T_{22}$
$T_{31}^{(0)} = T_{13}$,	$T_{33}^{(0)} = T_{33}$,	$T_{35} \begin{pmatrix} 0 \\ -\gamma \\ T_{35} \end{pmatrix} = T_{35} - \gamma T_{33}$
$T_{42}(0) = T_{24}$,	$T_{44} \begin{pmatrix} 0 \\ - \end{array} = T_{44} ,$	$T_{46} = T_{46} + \gamma T_{24}$
$T_{51}^{(U)} = T_{15} + \gamma T_{13},$	$T_{53}^{(U)} = T_{35} + \gamma T_{33},$	$T_{55}^{(U)} = T_{55} - \gamma^2 T_{33}$
$T_{62}^{(U)} = T_{26} - \gamma T_{22},$	$T_{64}^{(U)} = T_{46} - \gamma T_{24},$	$T_{66}^{(U)} = T_{66} - \gamma^2 T_{22}$
$T_{17}^{(U)} = T_{17},$	$T_{27}^{(U)} = T_{27},$	$T_{37}^{(U)} = T_{37}$
$T_{47}^{(U)} = T_{47},$	$T_{57}^{(U)} = T_{57} + \gamma T_{37},$	$T_{67}^{(U)} = T_{67} - Y T_{27}$

In the above expressions $\Upsilon = U/i\omega_{\rho}$.

