Hydraulics Research
Wallingford

MATHEMATICAL MODELS OF WAVE REFRACTION, AND DIFFRACTION BY ISLAND BREAKWATERS

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Report No SR 171
February 1988

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This report describes work carried out under contract CSA 1035 "Shore protection islands" funded by the Ministry of Agriculture, Fisheries and Food. The study was carried out in the Maritime Engineering Department of Hydraulics Research.

The departmental nominated officer is Mr A J Allison. The Company's nominated project officer is Dr S W Huntington.

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ABSTRACT

This report describes two mathematical models which may be used for the prediction of wave effects by an island breakwater in varying depth. A brief description is given of the derivation of two models. Their results were compared for a test case of an island breakwater in uniform depth with those from a mathematical model which would normally be used in this situation.

Having established that the two models give a reasonable representation of wave diffraction by an island breakwater in uniform depth, a series of tests were then carried out for the varying depth. This allowed a qualitative assessment to be made of the performance of the models in the varying depth case. The report concludes by summarising the capabilities and limitations of each of the models, and by making suggestions for their future development.

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1 INTRODUCTION

1.1 Terms of Reference

One important aspect in the design of a system of offshore breakwaters is the prediction of their effects on the nearshore wave climate. An earlier report (Ref 1) described a mathematical model which would be used to give a first estimate of the changes in wave conditions due to wave diffraction and overtopping at offshore breakwaters. This model did not include the effects of refraction due to depth variation in the vicinity of the breakwater. In many situations where offshore breakwaters are being considered as part of a coast protection scheme the effects of refraction and shoaling will be significant. It is therefore important that, where necessary, such effects are included in mathematical models of wave propogation by offshore breakwaters. With this in mind we consider here two mathematical models which are capable of representing shoaling and refraction, and diffraction by breakwaters. These are both described briefly and their performance examined for uniform and varying water depths.

1.2 Outline of Report

In this report results from three mathematical models are presented. The first of these is the model described in Ref 1 which represents diffraction by breakwaters in water of uniform depth. The other two models also allow refraction and shoaling effects to be represented. The models are all described briefly, together with details of the approach used to examine their performance, in Chapter 2. A comparison of the results from all three models for the constant depth situation is given in Chapter 3, and results for varying depth are presented in Chapter 4. The conclusions and recommendations arising from this report are given in the final chapter.

2 MATHEMATICAL MODELS

2.1 Outline of approach

The mathematical model described in Ref 1 will represent the effects of wave diffraction by breakwaters in uniform water depth. Its results have been compared with theoretical solutions and found to be in good agreement. The mathematical technique used in this model relies on finding a numerical solution to the governing integral equation, whose derivation is based on an assumption of constant water depth. These equations do not admit any simple extensions which would allow depth variation to be included. Therefore in order to represent refraction and shoaling alternative models to that described in Ref 1 are required. In the present work two such models are investigated. In both cases their governing equations are derived from the mild slope equation (Ref 2) which represents both wave diffraction and refraction.

The first of these models was developed at Bristol University (Ref 3), and uses a finite difference method to solve a parabolic approximation to the mild slope equation. This is referred to throughout the report as the Parabolic model. The second model (Ref 4), which was developed at Liverpool University is also a finite difference model, but it solves a different approximation to the mild slope equation. This model is referred to as the Liverpool model. Both are described in the remainder of this chapter, together with brief details of the constant depth integral equation model for completeness.

One of the difficulties is assessing the performance of mathematical models of wave diffraction by breakwaters in varying depth, is the lack of analytical solutions or physical model test data to validate the numerical model results. For the constant depth case there are theoretical solutions available (see Refs 1 and 5). Therefore, the approach taken here was to examine the results from the mathematical models which could be used in varying depth for a test layout with constant water depth. This would enable a comparison to be made with results from the constant depth model discussed in Ref 1. The outcome of these comparisons are discussed in Chapter 3. The mathematical models were then applied to a similar breakwater layout but with varying water depth, and a quantitative assessment of their performance made. This is discussed in Chapter 4.

2.2 Integral Equation Model (Constant Depth)

For all of the mathematical models described here the main objective is to calculate the wave height in the vicinity of an island breakwater. For a breakwater in uniform water depth the required wave heights can be calculated using the integral equation technique described in Refs 1 and 5. This allows the wave height coefficient, ie the ratio of wave height at any point p to the incident wave height, to be estimated using the following method.

The wave height coefficient (H_p/H_i) can be expressed in terms of the velocity potential ϕ as,

$$\frac{H_p}{H_i} = \frac{|\phi_p|}{|\phi_i|} \qquad (1)$$

It can be shown that (see Ref 5) the velocity potential at any point p in the velocity of an island breakwater is given by,

$$\phi_p = \phi_i + \phi_d, \tag{2}$$

where ϕ_d is the velocity potential of the diffracted wave. This must satisfy the radiation condition,

$$\lim_{r \to \infty} r^{\frac{1}{2}} \left(\frac{\partial}{\partial r} - ik\right) \phi_d = 0$$

where $r = (x^2 + y^2)^{\frac{1}{2}}$. The origin of the Cartesian (x,y) co-ordinate system is taken at the centre of a breakwater of length 2a which is located along the x axis. Waves approach the breakwater from the negative y direction. The incident potential is given by,

$$\phi_i(x,y) = \hat{a} \exp (ik(x\sin \beta + y\cos \beta)) \tag{3}$$

where $\hat{a}$ is the incident wave amplitude and k the wave number. The diffracted potential is given by,

$$\phi_d(x,y) = \frac{i}{2} \int_{-a}^{a} g(x_o) \left[\frac{\partial H_o^{(1)}}{\partial y_o} (kR)\right]_{y_o=0} dx_o \ , \ y \geqslant 0 \tag{4}$$

$$\phi_d(x,y) = -\phi_d(x,-y), \ y < 0,$$

where $R^2 = (x-x_o)^2 + (y-y_o)^2$,

$H_o^{(1)}$ is the Hankel function of the first kind, zeroth order and k is the wave number. The function $g(x_o) -a \leqslant x_o \leqslant a$ is the solution of the integral equation,

$$\int_{-a}^{a} g(x_o)\, H_o^{(1)}\,(k|x-x_o|)\,dx_o \qquad (5)$$

$$= G(x) + Ae^{-ikx} + Be^{ikx}, \quad -a \leq x \leq a,$$

where $G(x)$ is a particular integral of the differential equation

$$\frac{\partial^2 G}{\partial x^2} + k^2 G = 2k\,\hat{a}\,\cos\beta\,\exp\,(ikx\,\sin\,\beta),$$

A and B are chosen to satisfy the boundary condition

$$g(x_o) = 0 \text{ at } x = \pm a$$

Once (5) has been solved for $g(x_o)$, $-a \leq x_o \leq a$, the diffracted potential in the flow field can be calculated using (4) and the total potential recovered from (3) and (2). This will then allow the diffracted wave height to be calculated using (1). A discussion of the results obtained using this method is given in Ref 1.

2.3 Parabolic model

The parabolic model whose results are given in this report was developed at Bristol University. A detailed account of the derivation of the governing equations and its application to a number of test problems is given in Dodd (Ref 3). A brief account of the derivation of the parabolic approximation and its limitations with respect to the present problem is given here.

The parabolic equation used here is derived from the mild slope equation,

$$\underline{\nabla}\cdot(c\,c_g\,\underline{\nabla}\phi) + \omega^2\,\phi\,c_g/c = 0, \qquad (6)$$

with appropriate boundary conditions. Here $\phi(x,y)$ is the velocity potential, c is the phase, c_g the group velocity and ω the radian frequency. This equation was first derived by Berkhoff (Ref 2), it describes the propagation of periodic, small amplitude surface gravity waves over a seabed of mild slope. It will represent the combined effects of refraction, shoaling and diffraction. To derive the required parabolic approximation the reflected wave field is assumed to be small and is neglected, and forward travelling waves only are considered. This leads to the equation,

$$\frac{\partial \phi}{\partial x} = \frac{i}{2k} \frac{\partial^2 \phi}{\partial y^2} + \left\{ik - \frac{1}{2k}\frac{\partial k}{\partial x}\right\}\phi, \qquad (7)$$

where x is the main direction of wave propagation, y is the transverse direction. Deviations from the x direction are considered in the equation as oblique amplitude modifications.

Equation (7) is parabolic, whereas (6) is elliptic. The main advantage of a parabolic equation is that it permits a more rapid and straightforward method of solution than would be possible for an elliptic equation. This is because an elliptic equation defines a problem which is only properly posed, in general, when conditions are specified at all points around the boundary. It therefore requires that the equation is solved over the whole area of interest simultaneously. This necessitates a large amount of both computer storage and time. The form of the parabolic equation is such that for a well posed problem boundary conditions only need to be specified at the offshore boundary. A marching finite difference technique can be used to obtain the solution; this type of method only requires storage of one or two adject rows of solution points and, as a consequence, is considerably less expensive in terms of cost and storage than the equivalent numerical solution to an elliptic equation.

There are of course drawbacks associated with these advantages. The primary one of these is that the parabolic approximation works best where the important effects occur in the direction of wave propagation, as transverse effects are only included in a weak sense. Also reflected waves are not represented in the model. However, with these limitations in mind, it will be demonstrated in Chapter 3 that the parabolic equation (7) provides a useful and economic method for solving the type of problems of interest here.

2.4 Liverpool model

The second model tested for representation of depth effects was developed at Liverpool University, and has been implemented at Hydraulics Research and used to obtain model results described in this report. A full account of the derivation of the governing equations and some verification tests may be found in Copeland (Ref 4). A brief description of the model is given here.

The governing equations used in this model are also derived from the mild slope equation (6). However, in this case a coupled pair of hyperbolic equations,

$$\nabla Q + \frac{c_g}{c} \frac{\partial \eta}{\partial t} = 0 \qquad (8)$$

$$\text{and} \quad \frac{\partial Q}{\partial t} + c\, c_g \nabla \eta = 0 \ , \qquad (9)$$

are obtained rather than a single parabolic equation. The assumptions made to arrive at (8) and (9) are that the solution to (6) is a harmonic steady state solution, and that the resulting expression can be split by introduction of the vertically integrated function of velocity Q.

Equations (8) and (9) will include the effects of reflection which equation (7) does not. They can be solved using a similar marching techniques to that used by the parabolic equation, although they will require more time and storage than the equivalent parabolic problem. They will, however, still require much less computational effort than obtaining a solution to the equivalent elliptical problem.

The wave heights from the model are obtained by averaging over several wave periods. There also appears to be a mathematical inconsistency in the introduction of the time variable in order to obtain the hyperbolic approximation to the mild slope equation which is essentially defined in terms of x and y. However, from the results presented in Ref 4 it is clear that the model can be used to provide solutions for a wide range of refraction/diffraction problems.

3 PERFORMANCE OF MATHEMATICAL MODELS - UNIFORM DEPTH

3.1 Description of test case

The layout which was used to compare the performance of the mathematical models consisted of an island breakwater of length 60m in a constant water depth of 5m. For incident waves with a period of 5s the length of the breakwater is approximately 2 wavelengths. This is a fairly typical length for offshore breakwaters used in shore protection applications around the UK coast.

The test programme consisted of running each of the mathematical models for the given layout with incident wave directions of normal and 45°. The results from the Parabolic and Liverpool model were then compared with those from the integral equation model. The comparison between the models are presented as both contours of wave height coefficient over the area represented, and as wave height coefficients along the profile lines A-B, A-C and A-D shown in Figure 1.

3.2 Discussion of results

Before discussing the results in detail it should be recalled that the main purpose of the tests for the uniform depth case was to check that the Parabolic and Liverpool models were representing the diffraction effects correctly. The first test which was carried out was for the Parabolic model with normally incident waves. A comparison between these results and those from the equivalent run of the integral equation model are given in Figure 2. It can be seen that the basic shape of the contours predicted by the Parabolic model is the same as those from the integral equation model. That is, both show the distinctive two "peaks" in their contours which can be thought of as corresponding to diffracted waves emanating from the breakwater ends. In general, the parabolic model appears to slightly under predict wave heights both in the immediate lee of the breakwater and outside the sheltered area.

A similar test was run using the Liverpool model, the results from this are shown in Figure 3. It can be seen that the contours of wave height coefficient from the Liverpool model are more confused than either of those from the integral equation or Parabolic models. The Liverpool model displays a tendancy to overpredict wave heights in the immediate lee of the breakwater, and to under predict these values elsewhere. The other notable feature Liverpool model is that the results which it produces for this case are not exactly symmetric about the line through the centre of the breakwater, as is expected for normally incident waves.

A comparison of the results of all three models for this case is given in Figure 4. This displays wave height coefficients along the profile lines A-B and A-C as shown in Figure 1. It can be seen that long line A-B, which goes through the centre line of the breakwater, the results from the Parabolic model are fairly close to those from the integral equation model. At most locations along this line the wave height coefficients calculated using the Parabolic model are within 10% of those from the integral equation model. The values calculated along the A-B line by the Liverpool model are between 35% and 60% larger than those from either the Parabolic or integral equation models.

For wave heights along the profiles line A-C an assessment of the performance of the models is less straightforward. Where the line A-C remains within the immediate shelter of the breakwater (distances along A-C less than about 1.4L) a similar trend to that for all values along the A-B line is preserved. That is, wave heights calculated using the Parabolic model are less than those from the integral equation model, whereas the Liverpool model values are larger. On leaving the immediate shelter of the breakwater (distances along A-C greater than about 1.4L) both the Parabolic and Liverpool models significantly underestimate the wave height coefficient calculated using the integral equation model.

For waves approaching the breakwater at 45° incidence the results of the model tests are given in Figures 5 to 7. It can be seen from Figure 5 that the wave height contours calculated using the Parabolic model follow a fairly similar shape to those from the integral equation model in the area behind the breakwater. In this area the wave heights from the parabolic model are slightly lower than those from the integral equation model. Outside of the immediate shelter of the breakwater the Parabolic model again appears to calculate slightly lower values than those from the integral equation model. Similar comments can be made for the results from the Liverpool model for this case. Figure 6 shows that the contours of wave height coefficient for the Liverpool model follow a similar pattern to those from the integral equation model. The values calculated by the Liverpool model appear to be very similar to those from the integral equation model both in the immediate shelter of the breakwater and outside it.

A comparison of the results for all three models along the profile lines A-B, A-C and A-D (see Figure 1) are shown in Figure 7. Along the line A-B the results from the Parabolic model follow a similar trend to those from the integral equation model, but the wave height coefficients which it predicts are 10% to 27% lower than those calculated using the integral equation model. Conversely, the results from the Liverpool model along this line display a different trend to those from the integral equation model, but the wave height coefficients calculated are generally within 15% of the values predicted by the integral equation model.

Along the profile line A-D, which is on the exposed side of the breakwater for this incident wave direction it can be seen that the results from both the Parabolic and Liverpool models follow similar trends. However, the wave height coefficients calculated by the Parabolic model are nearer to those from the integral equation model at distances greater than 0.7L from the breakwater.

From the tests carried out for the uniform depth case for normal and 45° incident waves it is possible to make some general comments on the performance of the Parabolic and Liverpool models. First, both models appear to give, in general, a reasonable representation of diffraction by an island breakwater. However, they do display some areas where the wave height coefficients calculated are outside of the range of accuracy which would be considered acceptable (say greater than 5-10% error in wave height coefficient). For normally incident waves the Parabolic model gives better results in the lee of the breakwater, whereas outside of this area neither model gives results to the required level of accuracy. For 45° incidence the Liverpool model gives acceptable results behind the breakwater, and both models appear to work reasonably well in the more exposed area.

For the parabolic model this behaviour can be explained by assumptions made in deriving the approximation. The consequence of which is that the model can be expected to work well where the important effects occur in the direction of wave propagation, rather than in the transverse direction. This was discussed in Chapter 2.3. However, for the Liverpool no such assumptions are made, and therefore the discrepancies between its results and those of he integral equation model are less easy to explain. These points will need to be recalled when reviewing the results from the model tests for the varying depth case.

4 PERFORMANCE OF MATHEMATICAL MODELS - VARYING DEPTH

4.1 Description of test case

For the varying depth case the same breakwater layout and incident wave conditions were used as those for the uniform depth tests. The bathymetry in the breakwater lee was modified to have a slope of 1:30, with a depth of 5m at the breakwater. The depth profile is shown in Figure 1. The Parabolic and Liverpool models were both tested using normally incident waves, and the results from these tests compared with those given in Figures 1 to 3. This will allow a qualitative assessment to be made of the ability of both models to represent the effects of refraction and shoaling, and diffraction by an island breakwater.

4.2 Discussion of results

One of the difficulties in assessing the performance of the mathematical models of wave diffraction by island breakwaters in varying depth is the lack of analytical solutions or physical model data against which to compare the results. Attempts were made at Hydraulics Research to carry out a series of physical model tests to measure wave heights in the lee of an offshore breakwater for a sinusoidal wave train. These experiments were set up to reproduce as closely as possible the assumptions made in formulating numerical models of wave diffraction (eg an idealised breakwater which was vertically sided and perfectly reflecting was used) in order to provide accurate verification data for the numerical models. However, the results from these tests proved to be disappointing as difficulties were experienced in generating regular waves of sufficiently consistent height at all points across the wave basin. As a means of overcoming this a second series of tests were then carried out using a random wave train, with diffraction coefficients being calculated for specific frequency components of the wave spectrum. However, this procedure was fraught with similar difficulties to those experienced when using sinusoidal waves. To investigate these problems it was decided to undertake numerical models tests prior to any further work with physical models. The numerical studies conducted to investigate the effect of diffraction of random waves by an island breakwater are reported in Ref 6.

One outcome of this is that when examining the results from the mathematical model tests for varying depth in the present study only a qualitative assessment of the performance of the models can be made. With this in mind it is worth making some general comments on the expected behaviour of the mathematical model before discussing their results. In addition to diffraction by breakwaters both shoaling and refraction also need to be considered. Broadly speaking shoaling can be expected to cause increases in wave height in the lee of the breakwater. Whereas refraction will lead to decreases in wave height, and changes in wave direction where the wave crests are travelling at an oblique angle to the bed contours. Shoaling will effect waves travelling at both normal and oblique incidence to the contours.

The results from the model tests are displayed as contours of wave height coefficient in Figures 8 and 10. In both cases the contours of wave height coefficient from the integral equation model for the constant depth case are also given for reference. On comparing the results of the Parabolic model for both cases it can be seen that for varying depth (Fig 8) a fixed height contour has moved further towards the model centre line when compared with the constant depth results (Fig 2), and that the general shape of the contours is more angular for varying depth.

This can be seen more clearly in Fig 9 which shows wave height coefficients along the profile lines A-B and A-C (see Fig 1) for both the varying and constant depth cases. Along the profile line A-B there is a slight increase in wave height for the varying depth case. Along this line wave crests will be travelling almost parallel to the bed contours and shoaling can be expected to dominate. It can be shown that if shoaling only is considered on increases about 4% and 11% can be expected in wave height in depths of 3m and 2m respectively. These depths are at distances of 2L and 3L along the line A-B. At these points the parabolic model predicts 3% and 11% increase in wave height. These are in good agreement with the expected behaviour of the solution. Along the A-C profile line a combination of refraction and shoaling can be expected to effect the wave heights calculated in varying depth. However, the values predicted by the parabolic model for varying depth do appear to be unexpectedly large compared with the constant depth results. However, it should be noted that the constant depth values of wave height coefficient from the Parabolic model along this line were significantly lower than those from the integral equation model. It is therefore possible that the Parabolic model is giving a reasonable estimate of wave height coefficients along this line for the varying depth case.

For the Liverpool model comparison of Figures 10 and 3 again shows that for varying depth the wave height contour with a given value is nearer the model centre line than for the constant depth case. A better impression of the behaviour of the Liverpool model for the varying and constant depth cases is given in Figure 11. Along the A-B line there is an increase in wave height for the varying depth case which is of the order of 10% at a distance 2L from the breakwater. This is larger than the value expected from shoaling behaviour alone. Along the A-C line wave heights remain the same until a distance 2L from the breakwater where the varying depth case predicts a larger values. At a distance of 2.5L along this line the wave height for the varying depth case is about 9%; this is slightly larger than would be expected by considering shoaling only, and would appear to indicate that refraction is having very little effect.

In summary it would appear that both the Parabolic and Liverpool model give a reasonable representation of wave effects in the vicinity of an island breakwater in varying water depth. The precise level of accuracy of the models is difficult to determine as there is a lack of analytical solutions or physical model data against which to compare the model results. One final point which should be made concerns the relative costs of running the various models. The integral equation model is the least expensive of the three to use, but the difference in cost between it and the parabolic model is marginal when compared with the large running time and cost of the Liverpool model. It is hoped that some optimisation of the computer code for the Liverpool model can be made in the near future to make it more economical to use than at present.

5 CONCLUSIONS AND RECOMMENDATIONS

1. The performance of two finite difference models which represent the effects of wave diffraction by an island breakwater in varying depth have

been evaluated. This was done by first comparing their results with those from an integral equation model in constant depth In order to determine how well the finite difference models represented diffraction by breakwaters. A qualitative assessment was then made of their performance for a similar breakwater layout in varying depth.

2. It was found that for constant depth the Parabolic model gave a better representation of diffraction effects in the shelter of the breakwater, outside of the shelter the Liverpool model was more accurate.

3. In varying depth the performance of the models was more difficult to assess. It appears that the Parabolic model again performs well in the shelter of the breakwater, but tends to overestimate wave heights outside this area. The Liverpool model seems to give reliable results in the less sheltered areas, but tends to slightly underestimate wave heights in the sheltered areas.

4. The Parabolic model was found to be considerably faster and less expensive to run than the Liverpool model.

5. One of the difficulties in making this assessment of performance was the lack of physical model data against which to compared the the finite difference model results. Using experience gained from a previous physical model tests, which were unsuccessful, it is recommended that another series of tests are carried out to investigate diffraction by an island breakwater

in varying depth. This will provide data for a full verification of the models described here.

6. Methods should be investigated for improving the running time, and thus reducing the cost, of the Liverpool model.

7. It is recommended that both the Parabolic and Liverpool models are developed further to include such effects as seabed friction and wave breaking.

6 ACKNOWLEDGEMENTS

The work described in this report was carried out in Dr S W Huntingtons Maritime Engineering Department. The author would like to thank Dr A H Brampton for his help and advice.

All of the models used in this study have been implemented at Hydraulics Research. The Parabolic model was developed by Mr N Dodd at Bristol University as part of his PhD research. He was funded by an SERC CASE award in conjunction with Hydraulics Research. The author is indebted to Mr Dodd for providing the Parabolic model results which are included in this report. The Liverpool model described here was developed in the Civil Engineering Department of Liverpool University. The author would like to thank Mr T S Hedges of this department for his assistance in transferring the software to Hydraulics Research, and for useful discussions on the models capabilities.

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FIGURES.

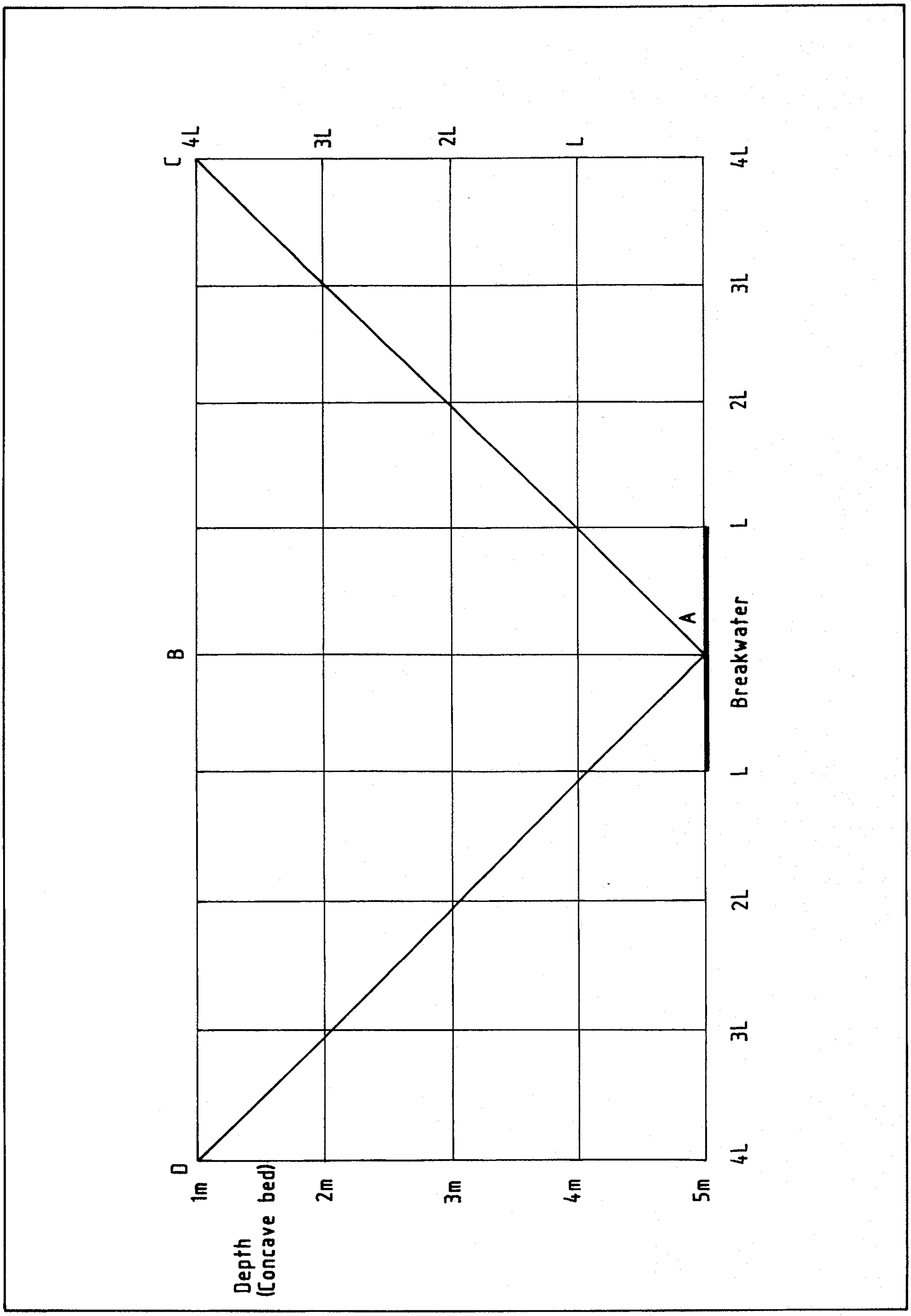

Fig 1   Layout for island breakwater tests

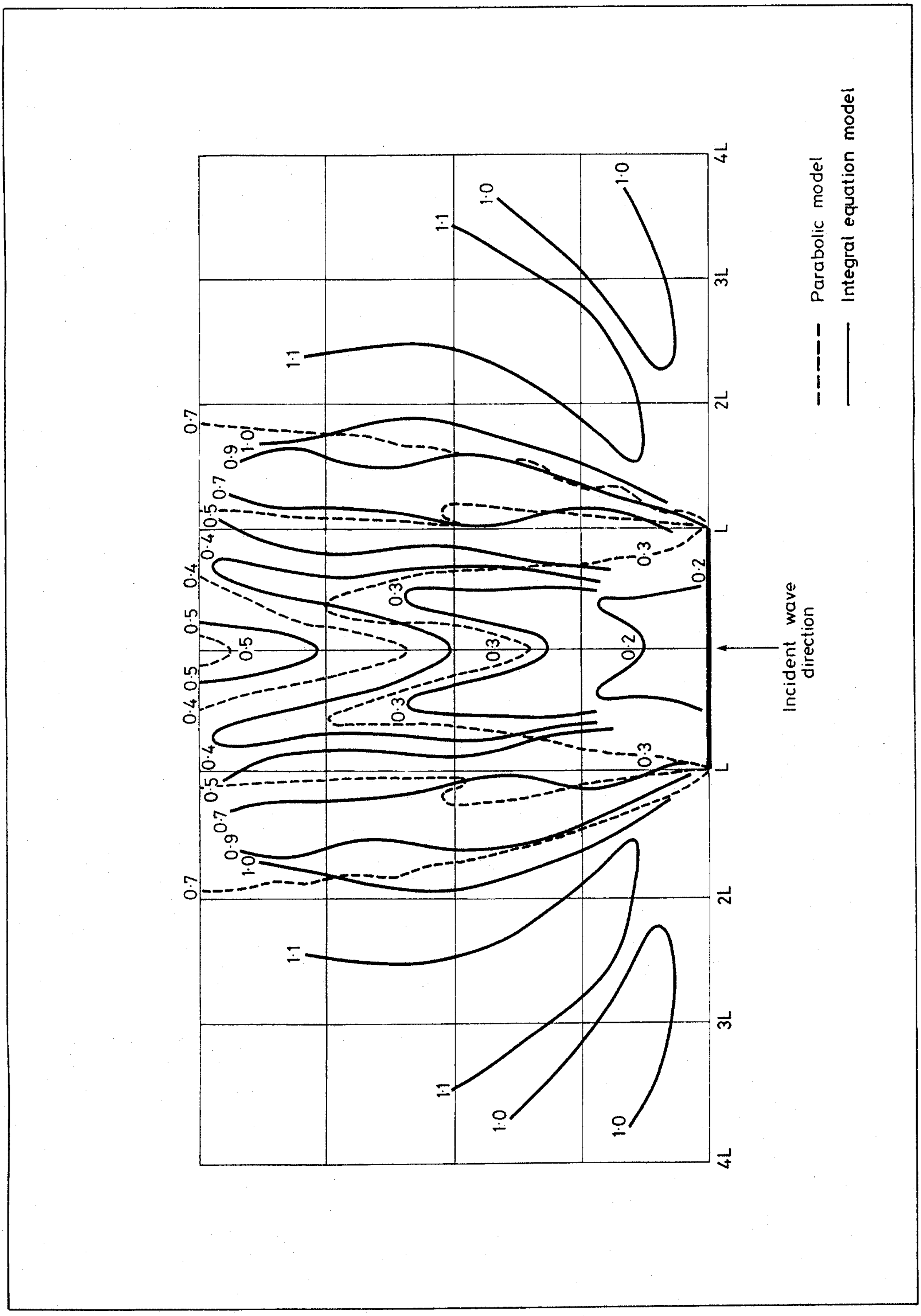

Fig 2 Parabolic model - wave height coefficients for an insular breakwater, length 2 wavelengths (L), constant depth profile, normal incidence

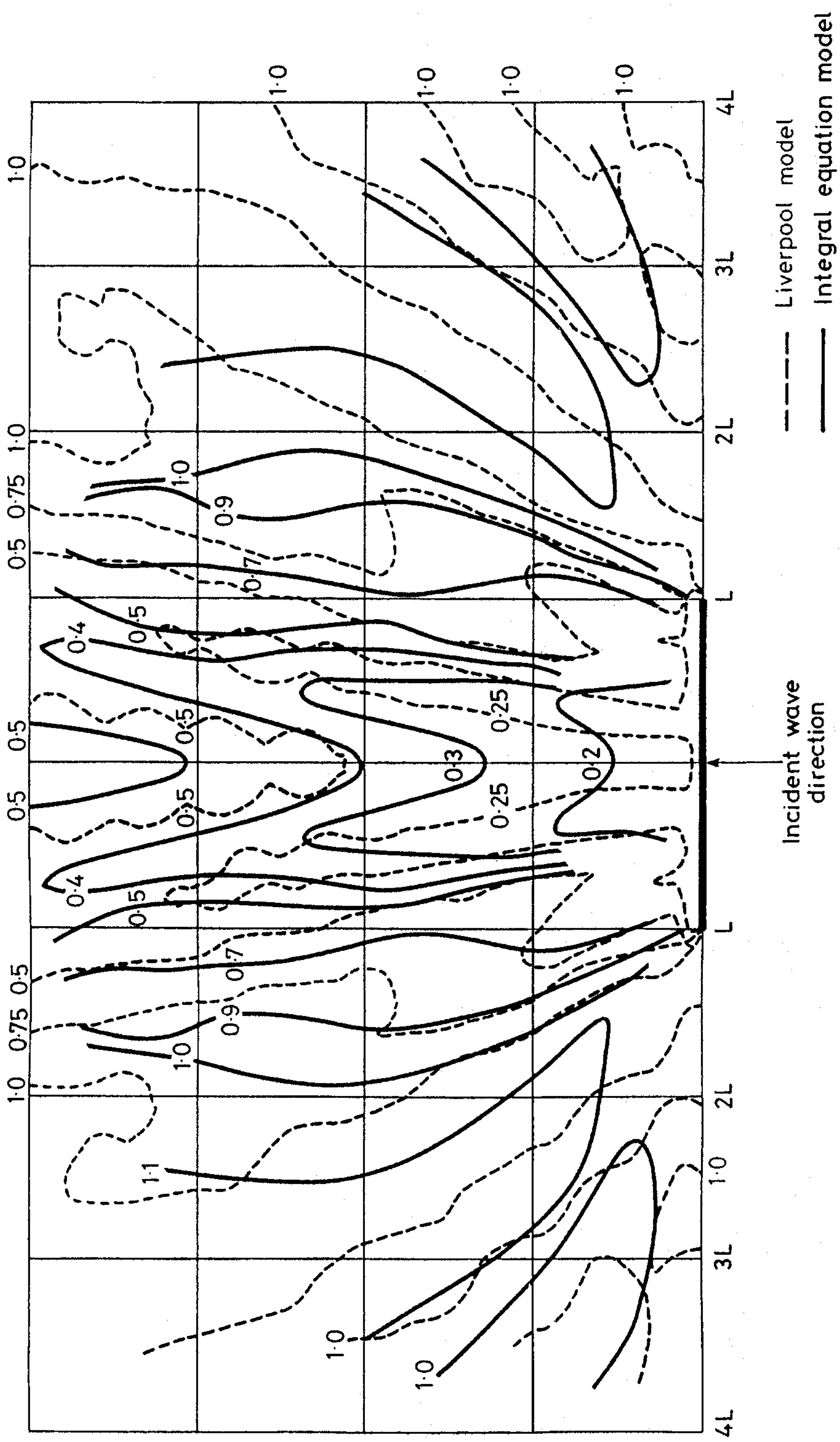

Fig 3 Liverpool model – wave height coefficients for an insular breakwater, length 2 wavelengths (L), constant depth profile, normal incidence

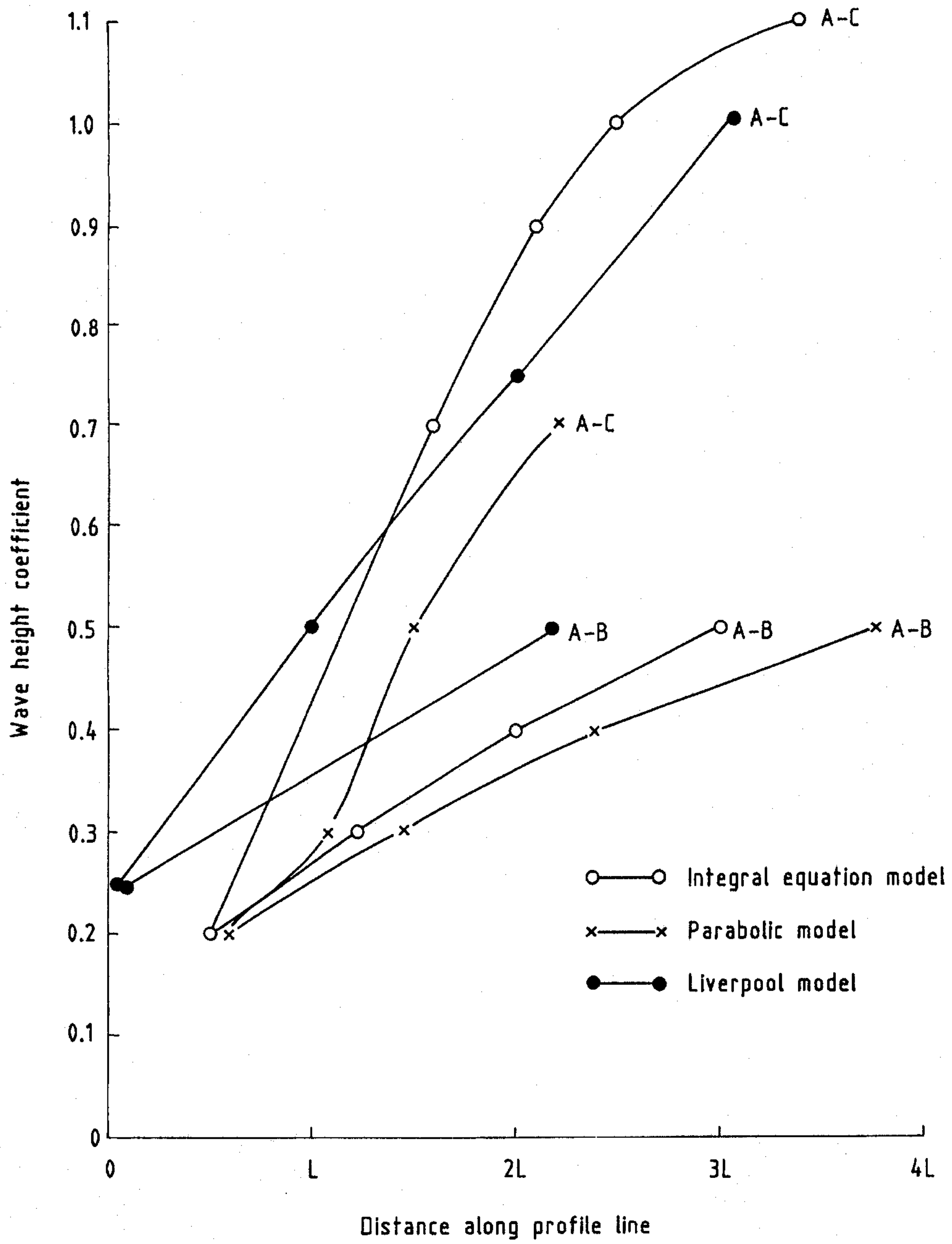

Fig 4 Comparison of model results – normal incidence, constant depth

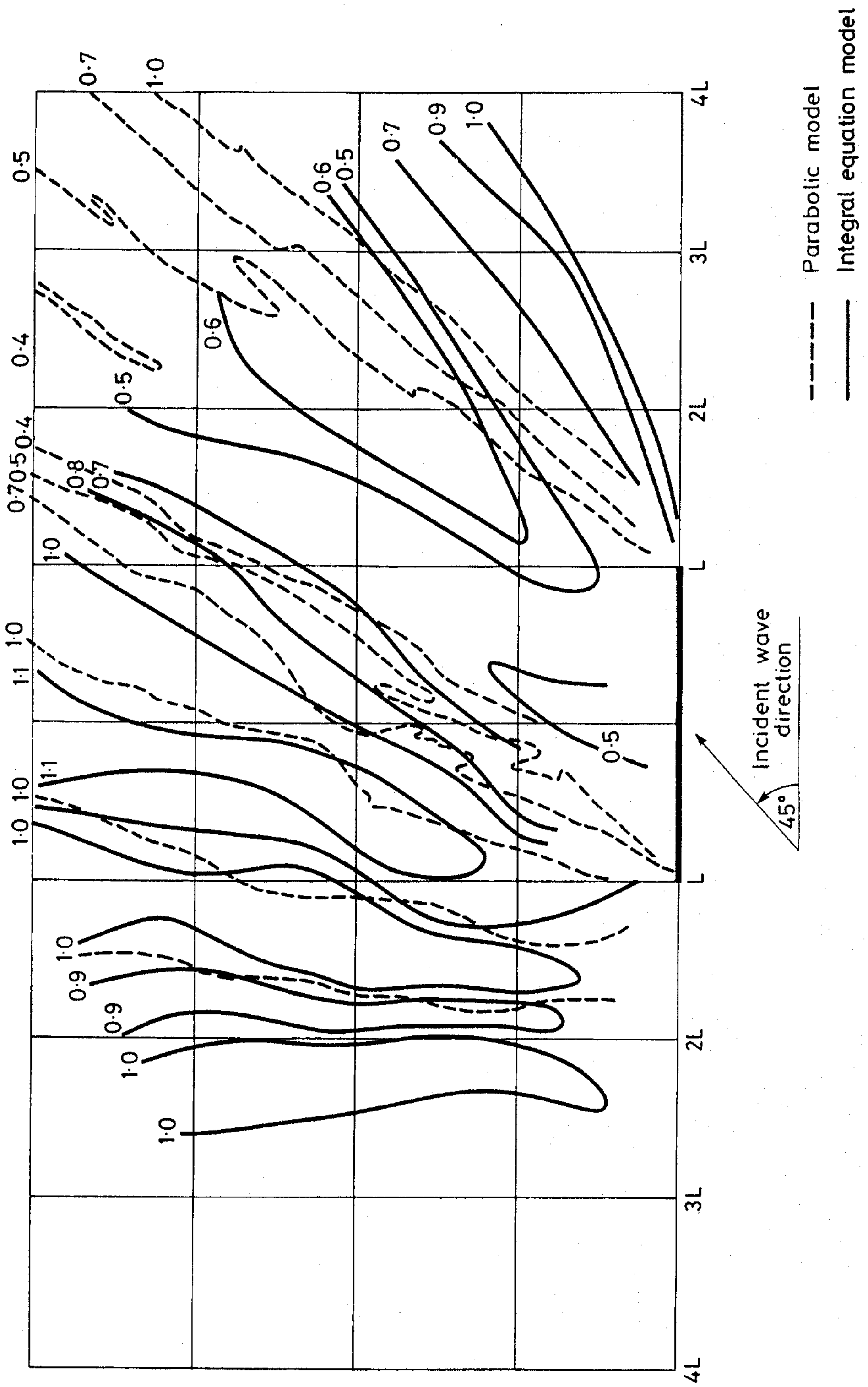

Fig 5 Parabolic model – wave height coefficients for an insular breakwater, length 2 wavelengths (L), constant depth profile, 45° incidence

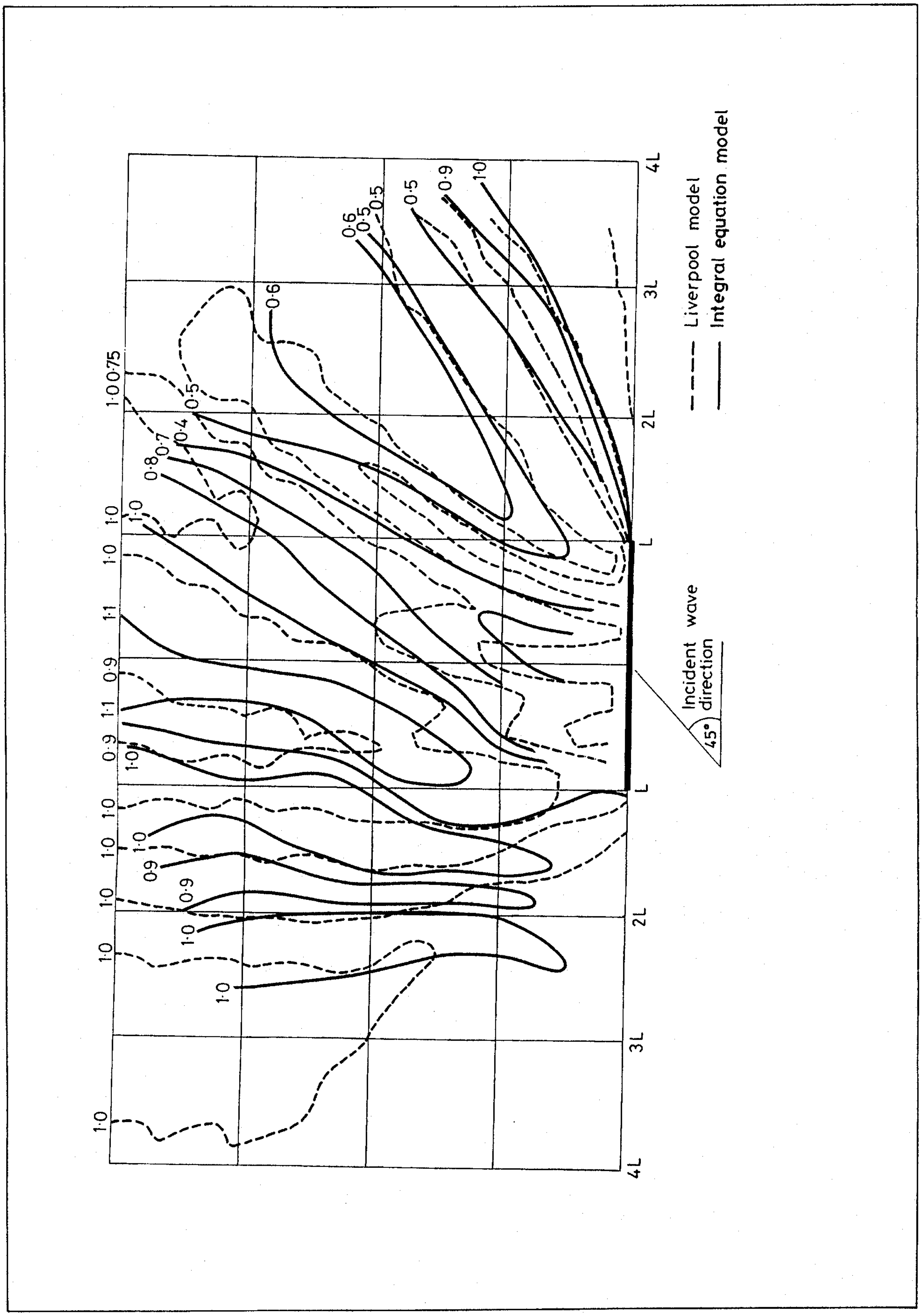

Fig 6 Liverpool model – wave height coefficients for an insular breakwater, length 2 wavelengths (L), constant depth profile, 45° incidence

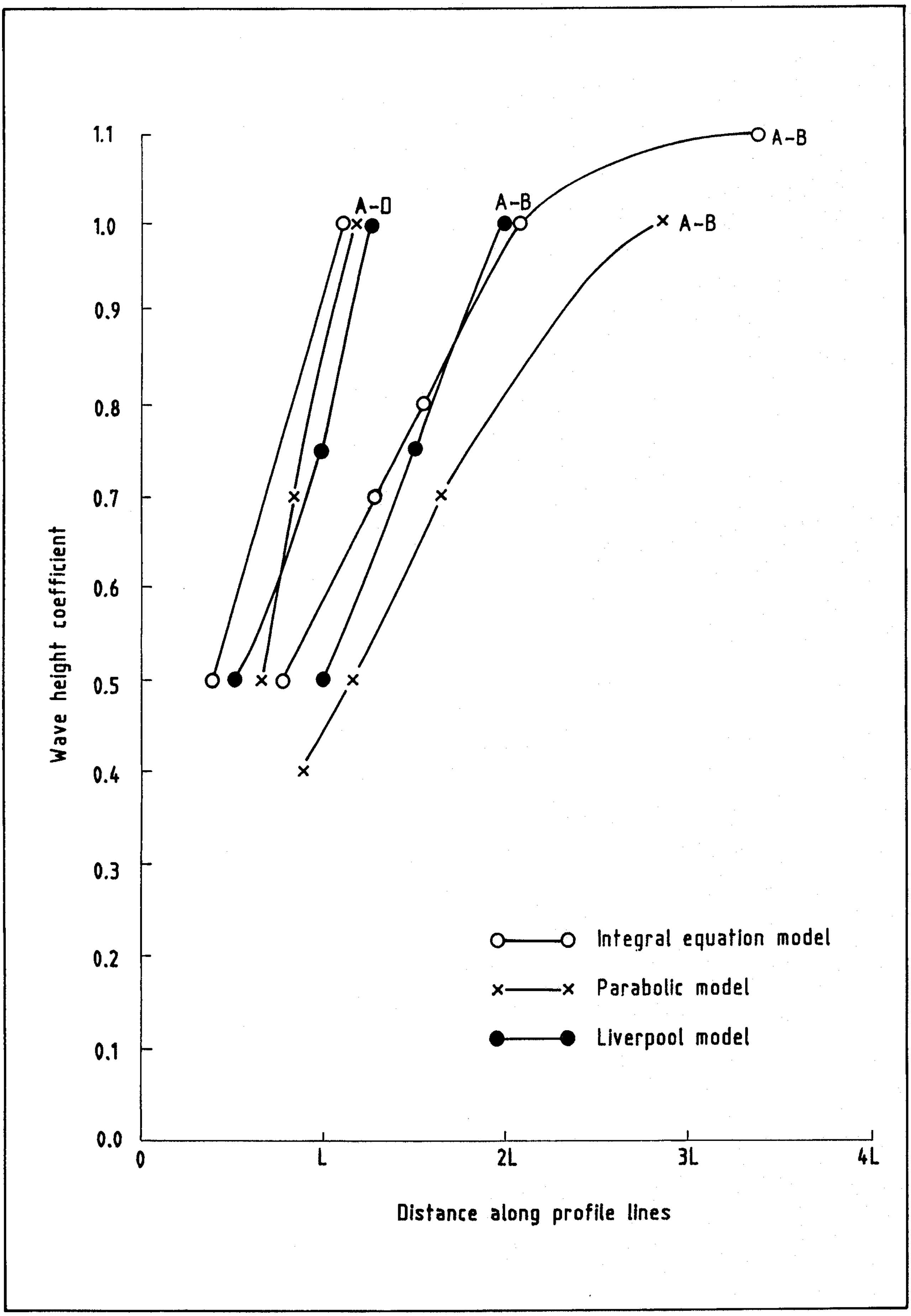

Fig 7 Comparison of model results - 45° incidence, constant depth.

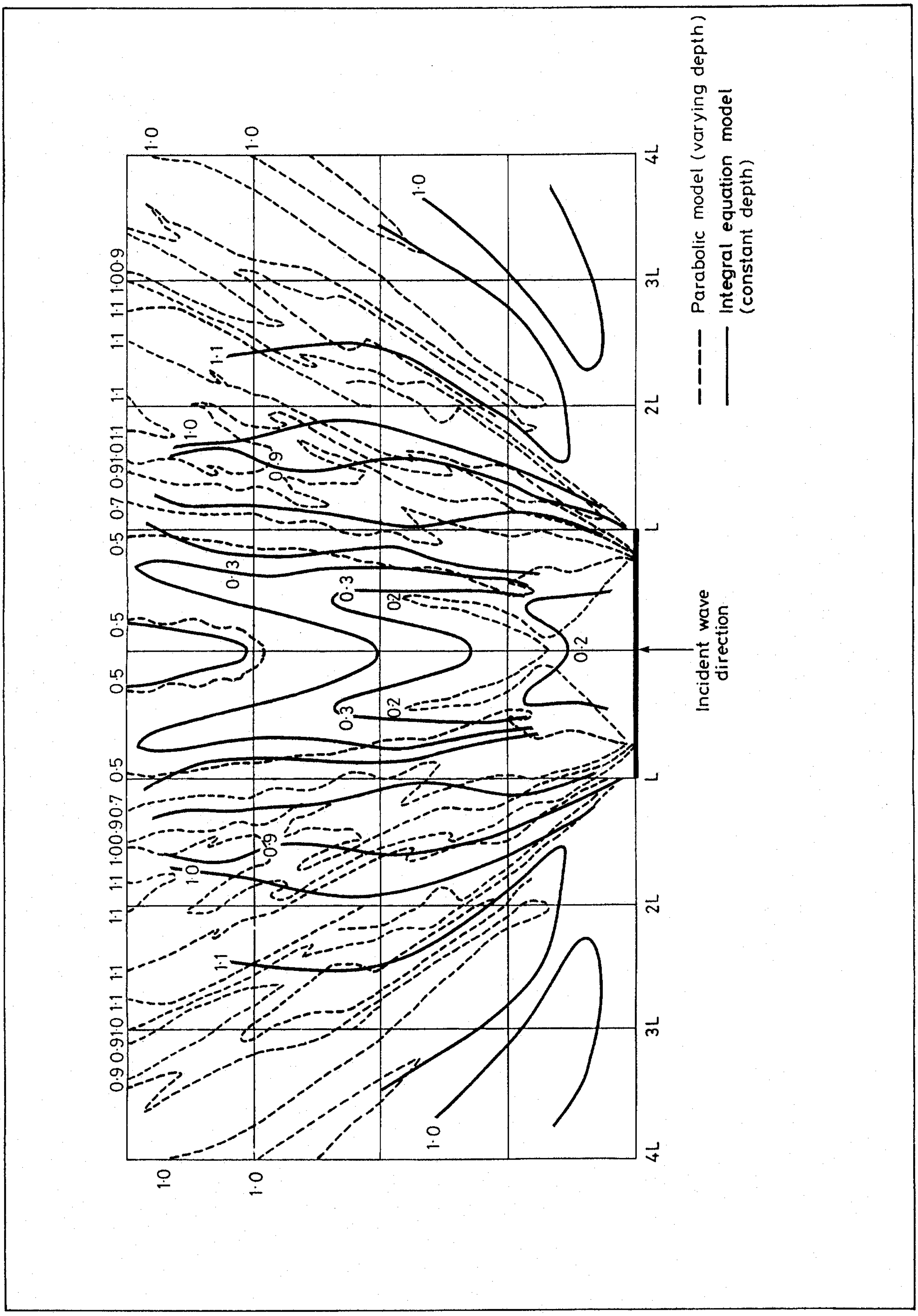

Fig 8 Parabolic model – wave height coefficients for an insular breakwater, length 2 wavelengths (L), varying depth profile

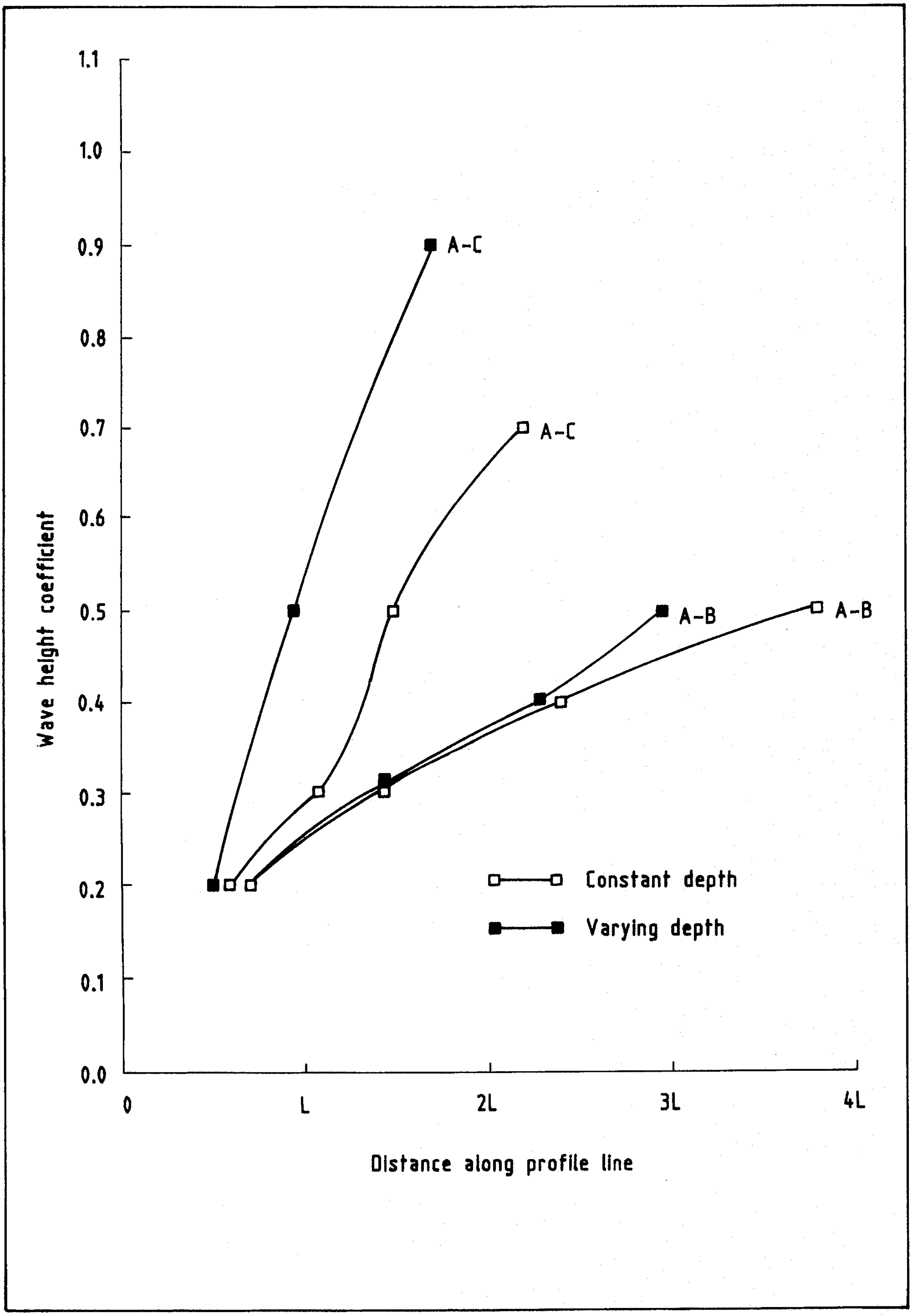

Fig 9 Comparison of Parabolic model results for constant and varying depth cases

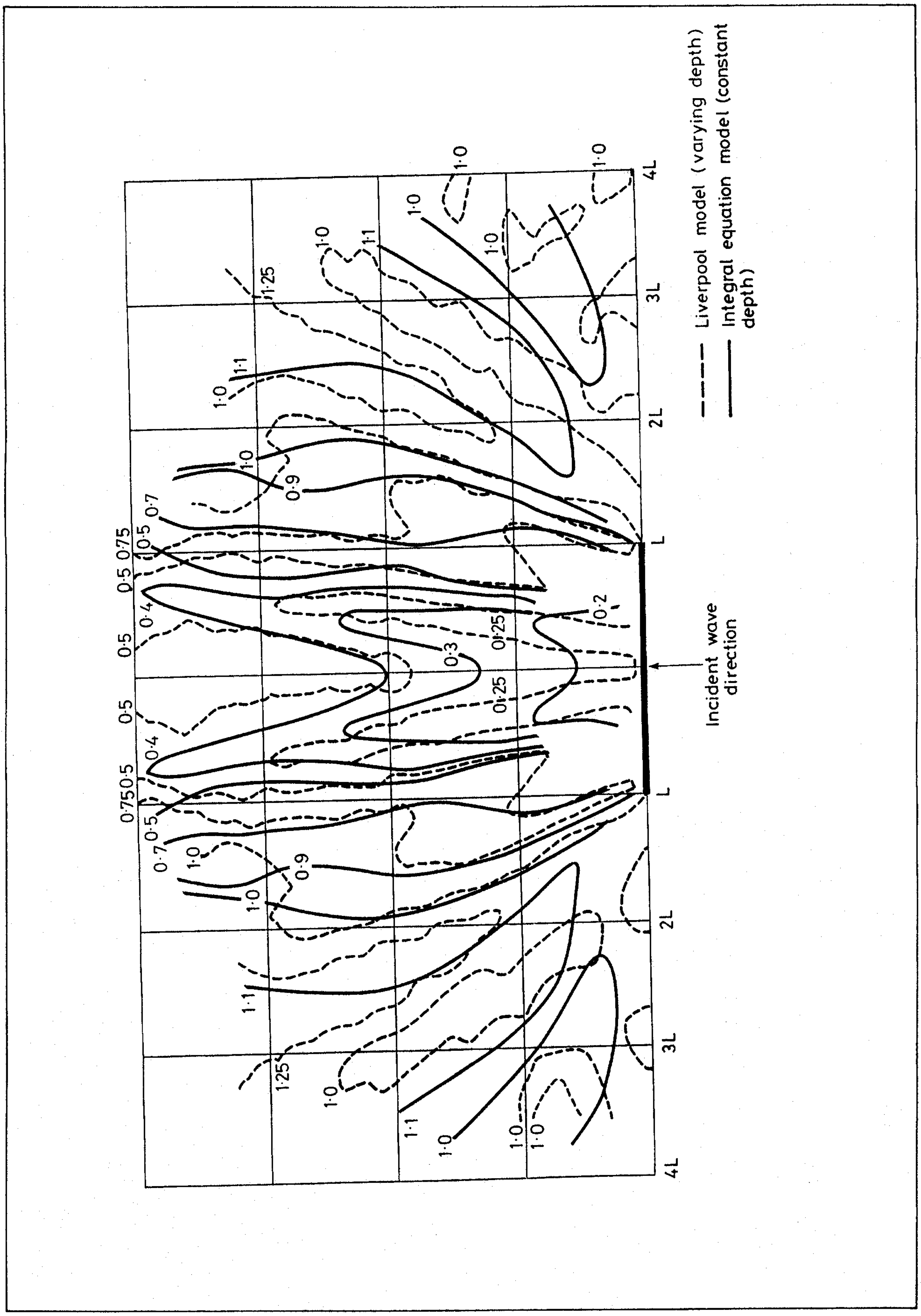

Fig 10 Liverpool model – wave height coefficients for an insular breakwater, length 2 wavelengths (L), varying depth profile, normal incidence

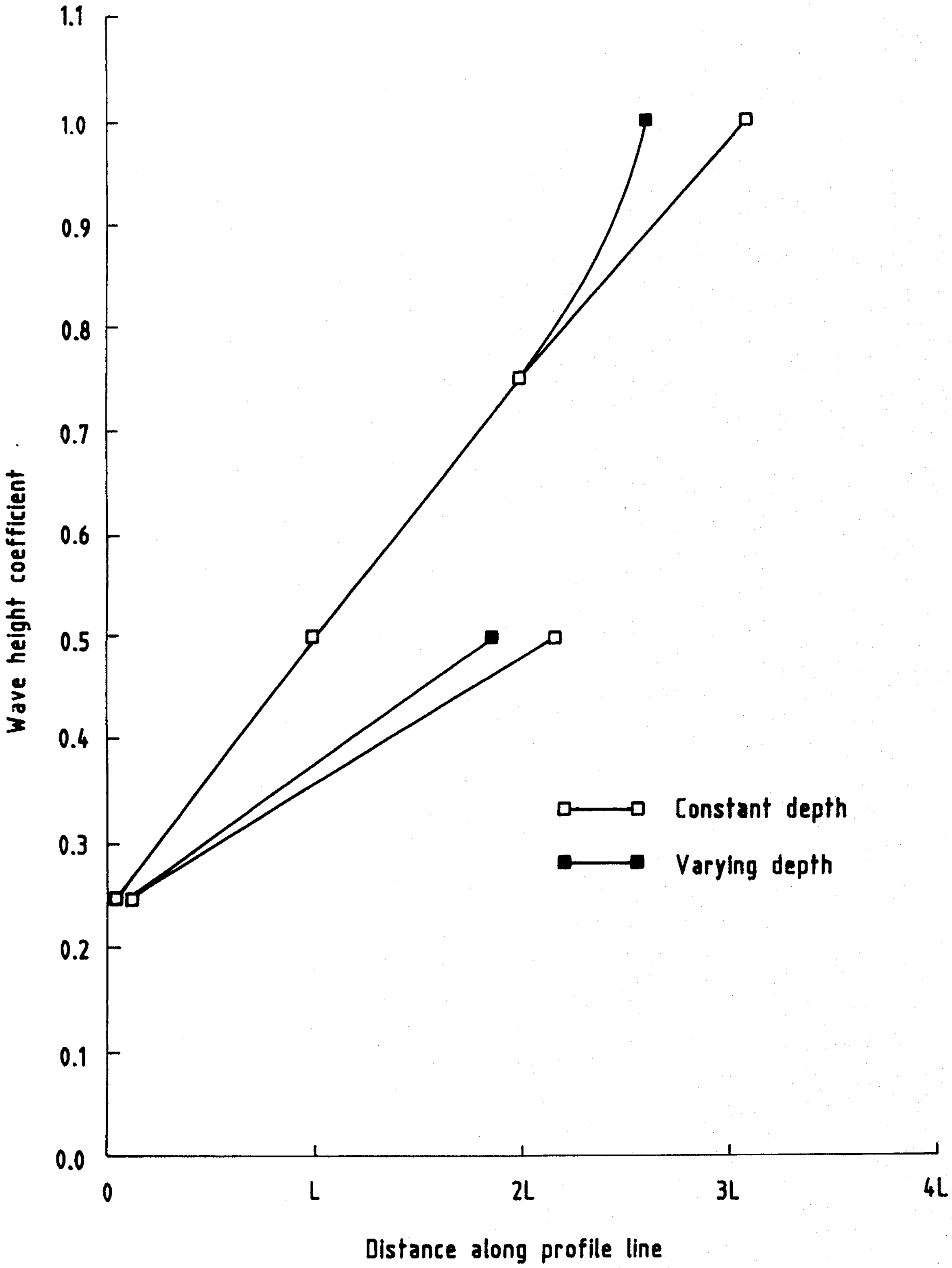

Fig 11 Comparison of Liverpool model results for constant and varying depth cases