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TEST ON A DIRECTIONAL WAVE RECORDER: THE SEA DATA 621

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Report No SR 158
February 1988

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This report describes work funded by the Department of the Environment under Research Contract PECD 7/6/054 (PIF No HRL 3/85) for which the DoE nominated officer was Dr R P Thorogood. It is published on behalf of the Department of the Environment but any opinions expressed in this report are not necessarily those of the funding Department. The work was carried out in the Maritime Engineering Department of Hydraulics Research, Wallingford, under the management of Dr S W Huntington.

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ABSTRACT

Directional wave effects are of importance in coastal engineering although directional wave properties are seldom measured. This report describes HR's testing of an instrument for measuring directional waves, the Sea Data 621. Tests took place first in a wave basin, then in the sea off Aberdeen. Due to instrument malfunctions, data from the field trial was of poor quality and useful results were not obtained, although the instrument does have the capability of measuring the required directional parameters.

CONTENTS

1 INTRODUCTION

The ability to measure waves is essential in coastal engineering. Field measurements have been made for many years. Special instruments, such as wave-rider buoys and pressure transducers, have been developed and used to record waves in coastal waters and standard analysis methods exist to process the data.

Less attention has been paid by researchers to recording the directions of waves at sea. Collecting data for directional analysis requires more complicated instruments which are expensive, and at present, less reliable than those used to measure just wave height. Directional analysis needs greater amounts of data and more computer processing than wave height analysis. As a result it is usually the case that wave directional characteristics are inferred from wave predictions made using wind conditions at or near any given site: wind directions being easier to measure or otherwise obtain than wave directions. For all these reasons, wave direction measurements have been taken infrequently in the past.

But there are many circumstances in which directional characteristics of waves are important. Ship responses, for example, can be much greater for waves from one direction than for waves from another (Ref 1). For another example, wave refraction depends on direction, so to use a refraction model to estimate inshore wave heights from offshore data, one needs to know offshore wave directions as well as heights (Ref 2).

Not only the mean direction of propagation of a wave train is important, the extent to which some components may deviate from the mean can be significant too. It is therefore often useful to be able to get a measure of what is termed directional wave spread.

Even at inshore sites, where it is often assumed that refraction will make waves effectively unidirectional, wave spread can be important. Long period waves (set-down) exist in association with all short period random waves. They contribute to ranging of moored ships (Ref 3). Some wave-measuring devices, eg waverider buoys, which rely on measuring acceleration, cannot measure long period (greater than about 30 seconds) waves, and so cannot be used to measure set-down. Alternatively, there are circumstances in which it is necessary to distinguish between set-down and long period waves due to other causes. Whether in the latter case or using a waverider, set-down can only be estimated by mathematical calculation. This can be done using recorded short period wave data. But set-down is sensitive to even small amounts of wave spread as well as to wave height (Ref 4). Estimating set-down at any given site accurately therefore requires knowledge of the directional spread. Knowing spread, in turn, requires making directional wave field recordings.

Field data on directional sea waves can therefore be seen to be much needed and such data has been collected only occasionally in the past (eg Refs 5,6).

Now, with improvements in micro-electronics making greater computing power available to process the masses of data involved, more sophisticated and reliable instrumentation is available 'off the shelf' and it is becoming practicable to collect directional wave data routinely rather than just occasionally.

This report describes Hydraulics Research's (HR's) trial test of a commercially produced instrument for measuring and recording directional waves - the Sea Data 621.

The Sea Data 621 consists of a pressure transducer and an electro-magnetic current meter together with a compass and electronic control and recording equipment housed in a pressure capsule (Ref 7). It is deployed immersed, usually at or near the sea bed in shallow water. The current meter measures two horizontal perpendicular components of water flow past the instrument, and an on-board processor resolves them into northerly and easterly components using the compass reading. North and east flow velocities, and pressure are recorded. This is sufficient data to deduce directional information about waves on the sea surface (Section 2).

2 DIRECTIONAL ANALYSIS THEORY

This analysis method has been expounded in a previous HR report (Ref 8). We shall only repeat the more important results here. SI units shall be assumed throughout: pressures are in Newtons per square metre, velocities are metres per second. Frequencies, ω, shall be in radians per second. The analysis uses spectral methods to estimate wave height at the surface, mean direction of propagation, and spread.

The starting point for the analysis is calculating the frequency spectra of pressure, $S_{pp}(\omega)$, and two horizontal flow velocity components, $S_{uu}(\omega)$ and $S_{vv}(\omega)$, We also compute the cross-spectra $S_{pu}(\omega)$, $S_{pv}(\omega)$ and $S_{uv}(\omega)$.

Two complex functions of frequency, α_1, and α_2, are defined:

$$\alpha_1(\omega) = \frac{\rho\omega}{k} \frac{\cosh kh_p}{\cosh kh_v} \frac{(S_{pu} - i\, S_{pv})}{S_{pp}} \quad (1)$$

$$\alpha_2(\omega) - \left(\frac{\rho\omega}{k} \frac{\cosh kh_p}{\cosh kh_v}\right)^2 \left(\frac{S_{uu} - S_{vv} - 2iS_{uv}}{S_{pp}}\right) \quad (2)$$

where: ω = (radian) frequency

ρ = water density

h_p = height of pressure transducer above sea bed

h_v = height of current meter above sea bed

k = wave number

Mathematically, α_1 and α_2 are the first two Fourier components of the normalised directional distribution of wave energy at frequency ω. But this fact need not concern us here; the important point is that they enable us to calculate spread and mean direction at that frequency.

Mean wave direction, $\bar{\Theta}$, is calculated from α_1:

$$\bar{\Theta}(\omega) = - \arg(-\alpha_1) = \arctan\left(\frac{-S_{pv}(\omega)}{-S_{pu}(\omega)}\right) \quad (3)$$

The Sea Data 621 resolves flow into northerly and easterly components. It is convenient for us to take u as the northerly velocity and v as the easterly; $\bar{\Theta}$ is then the mean wave direction relative to north. Note that our $\bar{\Theta}$ definition differs by 180° from that given in Reference 8 where $\bar{\Theta}$ is a direction of propagation because we have chosen to use the more usual convention under which a wave direction is the direction from which a wave is coming.

The Sea Data 621 does not provide sufficient data to calculate wave directional distributions completely. As a result, although our mean wave direction, $\bar{\Theta}$, is well defined, there are ambiguities in estimating spread. We can define various measures of spread using the Sea Data data, but it cannot be guaranteed that they will agree.

One possible measure is the waves' directional variance, σ^2:

$$\sigma^2 = 1 - \alpha_1 \alpha_1^* = 1 - |\alpha_1|^2 \tag{4}$$

The value of variance can vary between 0 and 1. It is 0 if (and only if) all the wave energy is concentrated in a single direction; a variance of 1 may indicate, for example, either wave energy uniformly distributed in all directions, or energy equally distributed between two opposites. The disadvantage of variance as a measure of spread is that, except in the special case of zero variance, there are no simple interpretations of what directional distributions values physically represent. There are generally infinitely many different distributions showing the same variance. Each will give a different interpretation of what the variance 'means'.

But some also yield measures which provide insight into waves' spreading behaviour.

For example, consider a directional distribution in which the wave energy is uniformly distributed between the angles $\bar{\Theta} - \Theta_s$ and $\bar{\Theta} + \Theta_s$. The spreading angle, Θ_s, is a measure of that distribution's spread. If we can find a uniform distribution which is a good fit to our recorded one, then we can use its Θ_s value as a measure of our recorded spread. A good fit is often defined by having the same α_1 values (ie the same variance). Thus a spreading angle for our recorded data can be calculated using the equation:

$$\frac{\sin \Theta_s}{\Theta_s} = |\alpha_1| \tag{5}$$

Similar methods of finding a good fit can be used with other distributions too. A popular directional distribute for theoretical work is the 'cos 2s' model:

$$\text{wave energy density} \ \alpha \ \cos^{2s} \left(\frac{\Theta - \bar{\Theta}}{2}\right) \tag{6}$$

Here, the parameter 's' is a measure of spread; large values correspond to narrow spreads, and s = 0 gives a uniform spread over all directions.

We have a choice of methods for calculating s, depending on whether we base calculations on α_1 or α_2. Unless the real wave energy distribution really does fit the cos 2s model (and it generally does not), there is no reason for them to agree; they are best considered as independent measures of spread. That based on α_1 is likely to be more reliable.

$$S_1 = \frac{|\alpha_1|}{1 - |\alpha_1|} \tag{7}$$

$$S_2 = \frac{1 + 3|\alpha_2| + (1 + 14 \ |\alpha_2| + |\alpha_2|^2)^{\frac{1}{2}}}{2 \ (1 - |\alpha_2|)} \tag{8}$$

<u>Wave height analysis and effective wave number</u>

The Sea Data 621 contains a pressure transducer, and the usual method of calculating the wave elevation spectrum at the surface, $S_{\eta\eta}(\omega)$, is from the recorded pressure spectrum and linear wave theory:

$$S_{\eta\eta}(\omega) = \left[\frac{\cosh kh}{\rho g \ \cosh kh_p}\right]^2 S_{pp}(\omega) \tag{9}$$

(Where: h is water depth, g is gravitational acceleration.)

But the elevation spectrum can alternatively be calculated from the velocity spectra which the instrument also records:

$$S_{\eta\eta}(\omega) = \left[\frac{\omega \cosh kh}{kg \cosh kh_v}\right]^2 (S_{uu}(\omega) + S_{vv}(\omega)) \qquad (10)$$

We prefer to use only expression (9) in our wave height analysis, and expression (10) is not used.

Expression (10) does however give us a basis for checking the accuracy of our instruments and, incidentally, for calculating an alternative set of wave spread parameters. By eliminating $S_{\eta\eta}$ in (9) and (10), we get:

$$k \frac{\cosh kh_v}{\cosh kh_p} = \rho\omega \left(\frac{S_{uu} + S_{vv}}{S_{pp}}\right)^{\frac{1}{2}}$$

In practice, using recorded spectra, the expression above cannot be exact. It is safer to define an effective wave number, k_e, by solving:

$$k_e \frac{\cosh k_e h_v}{\cosh k_e h_p} = \rho\omega \left(\frac{S_{uu} + S_{vv}}{S_{pp}}\right)^{\frac{1}{2}} \qquad (11)$$

Comparing the effective value, k_e, with the theoretical wave number, k, gives a measure of pressure and velocity readings' accuracy. Large discrepancies may indicate that either the pressure transducer or the current meter is faulty.

And we can use k_e to define alternative estimates for α_1 and α_2; substituting k_e for k in (1) and (2), we get:

$$\alpha_{1e} = \frac{S_{pu} - i\, S_{pv}}{\sqrt{[S_{pp}\,(S_{uu} + S_{vv})]}} \qquad (12)$$

$$\alpha_{2e} = \frac{S_{uu} - S_{vv} - 2i\, S_{uv}}{S_{uu} + S_{vv}} \qquad (13)$$

These can be used in the same way as α_1, α_2 were earlier, using the same expressions to derive alternative values for the spreading parameters: σ^2, Θ_s, s_1, s_2. Mean heading, $\bar{\Theta}$, remains the same whichever formula for α_1 is used.

Checking the theory

A computer program has been written at HR to do the analysis described above. Before applying it to recorded field data, it was tested.

We used a synthesiser to generate a simulated wave recording. The synthesiser was set up to output pressure and velocity time series with correct phase relationships (see Appendix). A Pierson-Moskowitz wave spectrum was simulated. Pressure and velocity depth attenuations were included; we simulated recordings taken with apparatus 1m above the seabed in a total depth of 10m. The analysis program correctly reproduced the synthesized wave height.

Three tests were done, each modelling a uniform directional wave energy distribution but with different spreading angles and mean directions. Spreading angle was independent of frequency in each test as was mean direction. Values are listed below:

	$\bar{\Theta}$	Θ_s
Test 1	22½°	22½°
Test 2	45°	45°
Test 3	66°	66°

As spectral analysis was carried out, we estimated values for $\bar{\Theta}$ and Θ_s over narrow frequency bands. Fifteen bands covered the range from 0.05Hz to 0.16Hz - roughly the range of frequencies at which there was significant wave energy.

We list below the mean, largest and smallest estimates for $\bar{\Theta}$ from our fifteen frequency bands for each of the three tests:

	True	Estimated		
	Θ	Θ_{mean}	Θ_{max}	Θ_{min}
Test 1	22½°	25½°	37°	16°
Test 2	45°	47°	61°	33°
Test 3	66°	65°	78°	49°

The results show that the anlaysis estimates mean wave direction well. Mean estimated values are very close. No maximum or minimum estimate differs from the true value by more than 17°. It appears that in practice we should be fairly confident of estimating mean wave directions to within ±15° using this analysis.

Spreading angle values were less well estimated. There were two sets of results: using linear theory, and using effective wave numbers. They were similar, so for brevity we list only the linear theory set here:

	True	Estimated		
	Θ_s	$\Theta_{s\ mean}$	$\Theta_{s\ max}$	$\Theta_{s\ min}$
Test 1	22½°	55°	97°	0°
Test 2	45°	50°	65°	36°
Test 3	66°	65°	82°	43°

Test 1 is in error but, to date, we have been unable to establish the cause. Results for tests 2 and 3 are satisfactory although the variance of results is greater than that found for mean wave direction.

Analysis using other spread measures, σ and s, gave results of similar quality.

It is worth noting that Θ_s is not an unbiased estimator for spread and there will be a trend for the estimated Θ_s values to be too large, particularly for narrow spreads. The effect does not, however, explain our getting larger estimated Θ_s values for test 1 than we got for test 2.

In conclusion, our tests demonstrated that the analysis method works well in estimating mean wave direction. Wave spread could usually be estimated satisfactorily though with less accuracy than mean direction, and doubts remain about the analysis's usefulness at estimating narrow spreads. The Sea Data 621 is, we anticipate, most likely to be used sitting near the sea-bed in shallow water. Spreads in shallow water are usually narrow due to refraction effects. There is, therefore, a need to improve the analysis of spread for use with this type of instrument in shallow coastal waters.

3 TESTING THE SEADATA 621

The Sea Data 621 was tested first under controlled conditions in a wave basin and then by deployment in the sea.

3.1 Wave basin test

We had done preliminary tests on the instrument in a towing tank before carrying out the test described here. That test had demonstrated the current meter's accuracy in steady flows but cast doubt on compass accuracy (Ref 9).

The next test was to put the instrument under waves in laboratory conditions and see how well it performed.

The test took place in a wave basin at HR. The Sea Data 621 was mounted on the bottom and the basin filled to a depth of 1.35m. Regular and irregular waves were generated and run over the instrument.

With the recorder set to record bursts of just eight pressure and velocity values at a time, spectral analysis of the recordings was not sensible. Hand analysis was done. It showed recorded pressures and velocities to be compatible with input wave heights.

Compass readings were unsteady; the compass was swinging through many degrees - more than the one or two degrees the SEADAT manual (Ref 7) said was normal in service. We thought this excessive fluctuation might have been caused by stray electric currents causing magnetic fields in and around the wave basin. If so, the fluctuation was an experimental artefact and we need not worry about it in the field.

Of more concern was a fault in the pressure record. Values came in bursts of eight; the second and sixth pressure values of each burst were always wrong by four instrument units (about 0.07psi). No explanation of this fault was forthcoming from Sea Data or their UK agents when we enquired about it and, as yet, the fault has not been corrected.

HR's investigation of the pressure and compass problems was severely hampered by our not being able to inspect output from the instrument. We had to record data on tape, even for laboratory bench tests, and send it to IOS Wormley (see also Section 3.2.2) for translation before we could inspect it. IOS's translation equipment was not infrequently on one of their ships on a cruise. Thus it sometimes took months for us to get test results we could have obtained in minutes if we had had Sea Data's reader in our laboratory.

The four unit error did not seem significant compared to pressures we anticipated measuring in the field and we could always remove erroneous values and replace them by interpolation. Winter, the prime wave recording season, was ending by this time. We decided to press on with field deployment as quickly as possible.

3.2 Field test

3.2.1 Deployment

The SEADATA 621 was deployed on the seabed on about the 7 metre contour off Aberdeen Harbour (Fig 2) on 11 February 1987. It was mounted in a frame to put the pressure transducer and current meters about 1m above the bed. It shared the site with a Waverider Buoy HR had deployed in connection with another project. Waverider readings were to be used to corroberate Sea Data findings. And the buoy was also a marker to make finding the Sea Data instrument on the seabed easy. Unfortunately, the buoy broke free of its moorings in a storm; the SEADATA 621 was lost for a while, and was only finally recovered in May instead of being picked up after four weeks as we had intended. On retrieval, the data tape was observed to be full.

3.2.2 Data format and quality

The instrument had been set to record bursts of 2048 pressure and velocity readings in DW format (Ref 7 for explanation of format) at a scanning rate of 2Hz. Bursts were recorded at six hourly intervals. At this recording rate, we anticipated that the magnetic tape cassette in the logger would have sufficient capacity for twenty eight days' recording.

The tape was sent to IOS at Wormley for translation to a format readable on HR's ICL 2972 computer. After a delay caused by reading the data cassette backwards on the first attempt at translation (which gives no error messages and appears to work, but gives nonsense results), translation was done successfully. But all this took many months to complete.

Preliminary inspection showed the correctly translated tape to contain 108 recordings, all of 2048 scans, with very few parity errors. At four recordings per day, this represented 27 days' recording - almost exactly what we had expected.

Compass readings fluctuated a bit, less than in the wave tank test, but more than the manual (Ref 7) suggested is normal. There was also a detectable long period drift: mean readings at the start of the recording period differed from those just before the tape ran out.

The pressure recording problem identified in the wave tank test also recurred though in a slightly different form. In the recording format we were using, values were recorded in blocks of eight scans. We found that, in most recordings, either one or two pressure values were corrupt in every block. The corruption occurred at the same positions within blocks fairly consistently throughout each recording, but at different positions in different recordings. For example, in one recording the fifth pressure value in every block might be faulty; in the next recording, it might be that the seventh value was wrong. To make anything of the recordings, it was necessary to detect and remove corrupt values from the record and replace them by data interpolated from neighbouring readings. Even then, there were noticeable effects on the pressure spectrum (see below) and when the problem was severe it made some recordings unusable.

3.2.3 Wave height and pressure analysis results

Calculating the mean pressure over each recording gave us an estimate for the sea depth which varied with the Tide (Table 3). This analysis indicated the site was about 9m below CD (lowest astronomical tide level) ie rather deeper than had been planned.

After subtracting mean values and removing long period tidal variations from our recordings, we calculated pressure and velocity spectra.

Pressure spectra often showed a fault which was presumably caused by our having to interpolate to remove corrupt data (see above). Faults tended to occur regularly once every eight scans, which at a scanning rate of 2Hz implies a fault frequency of 0.25Hz. This appeared subsequently as a spike in the pressure spectrum at 0.25Hz in most recordings. The spike imposed an effective upper frequency limit on our analysis. However most wave energy was at lower frequencies in a storm so the spike was not in itself a serious problem. A greater concern was that the fault might also affect the pressure spectrum seriously at wave frequencies; and this we cannot be sure about. Effective wave numbers (11) disagreed with theory, but not badly.

The surface wave spectrum was calculated from the pressure spectrum using equation (9). From this, we could calculate significant wave heights at the surface.

Three storms were identified; on 26/27 February, 1/2 March (during which the Waverider stopped transmitting) and 7 March. For the first of them, we can compare the Sea Data 621's wave heights with those from the Waverider (Table 1).

Results do not agree well. The Sea Data 621 has a definite tendency to over-estimate wave heights (by more than 50% in some cases). The reason for this error is not known. We derived Sea Data wave heights from the pressure record, so the fault must lie with the pressure data. It is possibly connected with the fault mentioned above ie the one that gave us the 0.25Hz spike. Alternatively, our analysis indicated greater water depths than we expected at the site, so perhaps our depth results are wrong because of our faulty pressure transducer, and we consequently over-compensated for depth attenuation (Equation 9). Either problem could account for our too large heights.

Yet a third interpretation is that the Waverider broke free of its mooring earlier than the 1st of March but went on transmitting data. Up to 18.00 on the 27/2/87 the Sea Data Unit gives slightly larger wave heights than the Waverider. After that, if the waverider broke free and drifted towards the shore, it would register lower heights. In this case the Sea Data Unit could be giving a realistic measure of the wave height.

3.2.4 Directional analysis results

An example of our directional analysis results for a recording during the storm of 2 March 1987 is given in Table 2 and a summary of the more important results is given in Table 3. We have no independent field data against which we can check the accuracy of our estimated wave spread parameters; all we can say is that they look plausible for the site and known wind conditions.

The mean wave directions however are obviously absurd. Comparison of Table 3 with Figure 1 shows that our results suggest 3 and 4m waves coming from a coastline about one mile away; generating such large waves from such a small fetch is impossible.

Obviously, something is wrong either with our analysis or our data. The analysis has been tested (Section 2) and checked. Thus the data is at fault in some way. We have checked that the compass was installed in the right alignment in the Sea Data 621. But further checks on the instrument have been restricted because we cannot translate the data ourselves. It seems likely, however, that the compass is faulty.

4 CONCLUSIONS

Our Sea Data 621 is a troublesome instrument. There is known to be a fault which is sometimes serious, in its pressure circuitry. We suspect something is wrong with the compass as well.

As it stands, it cannot be used to measure wave directions at sea as we intended.

A major problem is that we cannot inspect output from the instrument at HR so we have no way of checking that it is functioning properly in the laboratory.

We do not know why we have got wrong wave directions in our analysis; if we could read the Sea Data output ourselves for some very simple tests, we could at least eliminate some possible reasons for our problem and we could probably solve it. The method of data analysis has been checked via independent tests and so we know the errors are not caused by the specially written software.

Having to send all data tapes to Wormley for translation causes excessive delays whenever IOS is using their translation equipment on a cruise.

Sooner or later HR will have to do directional wave recording. The Sea Data 621 has the required capability but the example we have bought does not work properly.

5 ACKNOWLEDGEMENTS

This work was done at Hydraulics Research Ltd under the supervision of Dr E C Bowers. Mr D Perrott conducted the laboratory tests of the instrument. Mr C J Teal supervised the field deployment off Aberdeen. Mr J M A Spencer wrote and ran the analysis programs (and wrote this report). We are grateful to Dr C H Clayson at the Institute of Oceanographic Sciences Wormley for translating our data tapes for us.

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TABLES.

TABLE 1 **Comparison of Sea Data and Waverider results**

Date	Time	Waverider H_s(m)	Sea Data H_s(m)
26/2	18.00	2.67	2.60
	21.00	2.66	
27/2	00.00	2.85	3.32
	03.00	2.52	
	06.00	2.60	2.68
	09.00	2.27	
	12.00	1.99	2.28
	15.00	2.42	
	18.00	1.64	1.80
	21.00	1.57	
28/2	00.00	1.79	2.72
	03.00	1.45	
	06.00	1.15	1.80
	09.00	1.34	
	12.00	1.61	2.64
	15.00	1.30	
	18.00	1.16	1.68
	21.00	1.42	
1/3	00.00	1.61	2.28
	03.00	1.39	
	06.00	1.15	1.64

TABLE 2 Spectral analysis results for recording 12.00 2 March

Freq Hz	$S_{\eta\eta}(f)$ m^2/Hz	$\bar{\theta}$	θ_s Lin theory	θ_{se}	σ	s_1
0.044	0.4	197°	42°	63°	0.41	11.
0.052	0.5	204°	23°	48°	0.23	38.
0.060	1.1	202°	43°	46°	0.42	9.9
0.068	3.9	195°	47°	35°	0.45	8.4
0.076	32.2	195°	54°	28°	0.51	6.2
0.084	50.2	190°	50°	30°	0.48	7.0
0.091	44.8	189°	54°	31°	0.51	6.1
0.099	11.9	193°	64°	39°	0.59	4.2
0.107	6.1	198°	56°	37°	0.53	5.5
0.115	4.9	192°	69°	53°	0.63	3.4
0.123	3.2	189°	68°	50°	0.62	3.6
0.130	4.5	203°	74°	46°	0.66	3.0
0.138	3.9	203°	75°	56°	0.68	2.8
0.146	3.7	190°	67°	49°	0.62	3.6
0.154	6.2	201°	82°	63°	0.72	2.3
0.162	7.5	201°	69°	56°	0.63	3.4
0.169	5.4	208°	72°	66°	0.65	3.1
0.177	5.0	208°	77°	59°	0.69	2.6
0.185	2.7	205°	98°	84°	0.81	1.4
0.193	2.3	196°	101°	80°	0.83	1.3
0.201	2.4	213°	89°	73°	0.77	1.8

TABLE 3 Summary of Sea Data results for three storms

Date	Time	Sea depth m	F peak Hz	Hs m	$\bar{\Theta}$	Θ_s
26/2	18.00	9.62	0.14	2.60	179°	68°
27/2	00.00	13.22	0.11	3.32	175°	67°
	06.00	10.16	0.11	2.68	183°	61°
	12.00	13.54	0.12	2.28	187°	67°
	18.00	9.87	0.11	1.80	198°	56°
28/2	00.00	13.24	0.10	2.72	200°	62°
1/3	18.00	10.52	0.099	2.68	200°	53°
2/3	00.00	11.80	0.091	4.64	189°	63°
	06.00	11.31	0.084	4.28	190°	43°
	12.00	12.68	0.084	5.20	190°	50°
	18.00	11.72	0.084	3.54	185°	50°
3/3	00.00	11.49	0.091	3.36	187°	51°
	06.00	11.96	0.099	2.40	180°	60°
6/3	18.00	12.97	0.11	3.12	163°	53°
7/3	00.00	10.94	0.11	2.92	173°	55°
	06.00	12.55	0.11	3.68	167°	52°
	12.00	10.78	0.099	4.36	168°	50°
	18.00	12.57	0.099	4.16	164°	39°
8/3	00.00	11.33	0.099	3.44	218°	21°
	06.00	12.54	0.11	3.44	225°	13°
	12.00	11.45	0.11	3.24	230°	59°

FIGURE.

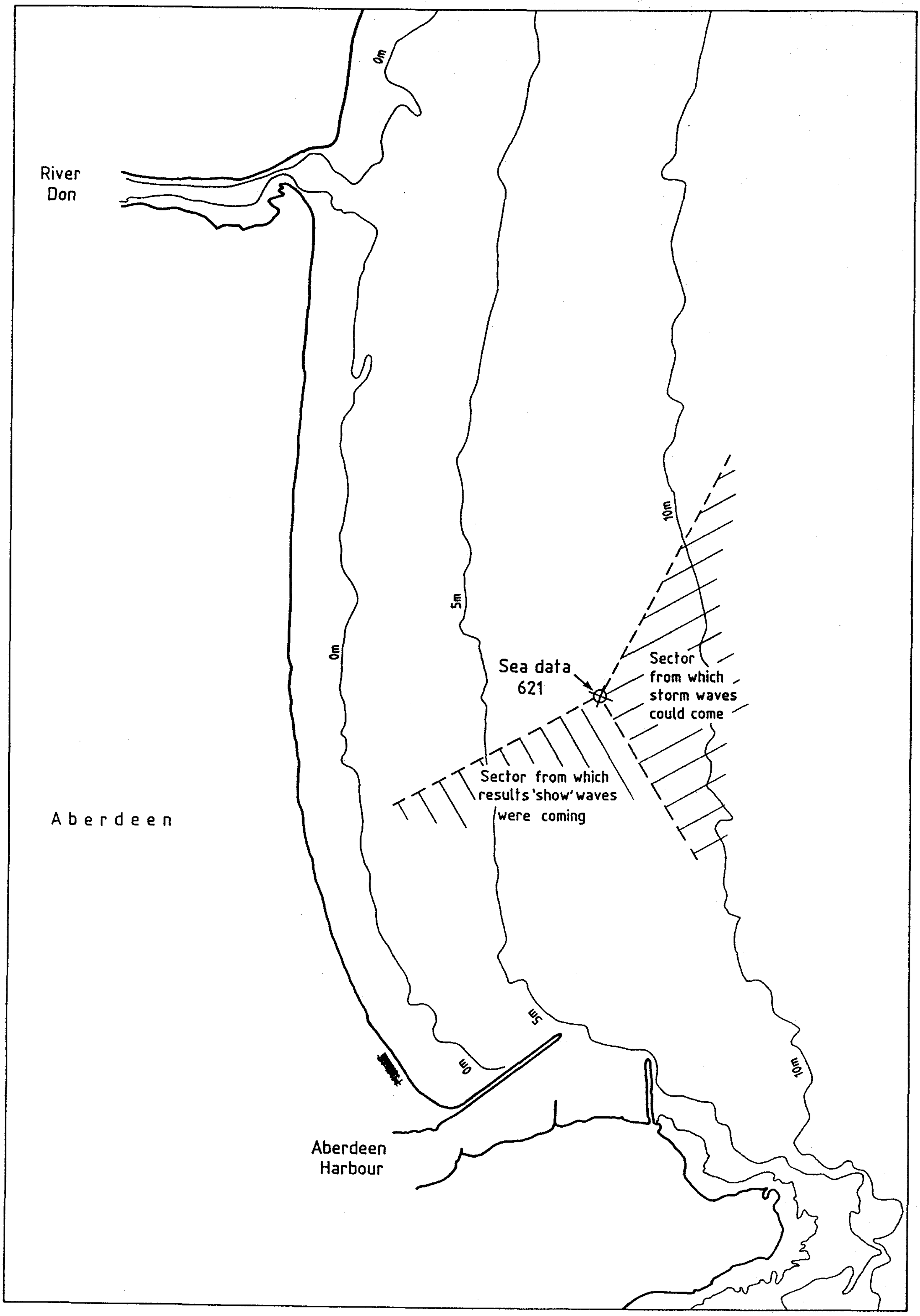

Fig 1 Aberdeen field test site

APPENDIX.

APPENDIX I

Synthesizing two or more related random signals

This appendix describes a method that we have used to synthesize two or more simultaneous, random but related signals. In this case the signals were pressure and current velocities beneath a simulated random water wave which we wanted for testing (Section 2), but the method is quite general and can be used in other cases involving more complicated relationships between signals, for example to synthesize all six forces and moments acting on a ship hull in random waves.

The normal synthesizer method works by filtering a white noise signal, N(t) (which is often obtained from a quasi-random shift register). If we want to get a signal with a certain spectrum, S(ω), we can first define a Fourier Transform:

$$A(\omega) = \left(\frac{S(\omega)}{d\omega}\right)^{\frac{1}{2}} \qquad (1)$$

taking its Inverse Fourier Transform, we can define a filter function:

$$a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)e^{-i\omega t}d\omega \qquad (2)$$

Then a random signal, f(t), with the required spectrum is got by applying the integral:

$$f(t) = \int_{-\infty}^{\infty} a(\tau)\ N(t-\tau)\ d\tau \qquad (3)$$

In practice, a(t) is always small for large values of t, so the integral in (2) needs only to be taken over a finite range.

Synthesizing two or more related signals requires only slight modifications of the above procedure. We assume we know how the signals' Fourier Transforms are related and that the relationships can be expressed by functions of frequency: suppose, for example, we have three signals: $A_1(\omega)$ defined from a spectrum as in Equation (1) and:

$$A_2(\omega) = B_2(\omega) . A_1(\omega) \qquad (4a)$$

$$A_3(\omega) = B_3(\omega)\ A_1(\omega) \qquad (4b)$$

B_2 and B_3 will generally be complex with $B(\omega)$ and $B(-\omega)$ conjugate. We can calculate three different filter functions (a_1, a_2, a_3) from (2). Each filter function applied to (3) will give a different signal, and if they are applied using the same white noise signal, N(t), the phase relationships expressed by the transfer functions (B_2, B_3) will be conserved. We shall have three related random signals.

As an example, consider the pressure and current velocities beneath a wave. The pressure spectrum is obtained from surface elevation:

$$S(\omega) = \left(\rho g \frac{\cosh\ kh}{\cosh\ kh} p\right)^2\ S_{\eta\eta}(\omega)$$

(NB: Same notation as used in section 2.)

From $S(\omega)$, we get $A_1(\omega)$. Linear theory gives relationships between pressure and horizontal current velocities for regular waves from which we derive B_2

and B_3. If Θ is the angle between the wave's direction of travel and our x-axis, we know:

$$u = \frac{k}{\rho\omega} \frac{\cosh kh_v}{\cosh kh_p} \cos\Theta . P$$

$$v = \frac{k}{\rho\omega} \frac{\cosh kh_v}{\cosh kh_p} \sin\Theta \; P$$

Where P represents pressure, and (u,v) is horizontal velocity.

Hence:

$$B_2 = \frac{k}{\rho\omega} \frac{\cosh kh_v}{\cosh kh_p} \cos\Theta$$

$$B_3 = \frac{k}{\rho\omega} \frac{\cosh kh_v}{\cosh kh_p} \sin\Theta$$

Knowing B_2 and B_3 we can calculate A_2 and A_3 (4); from $A_1(\omega)$, $A_2(\omega)$, $A_3(\omega)$ we can calculate the corresponding fitter functions $a_1(t)$, $a_2(t)$, $a_3(t)$; and applying them in (3) we obtain synthesized pressure and velocity signals which are random but correctly related to one another.