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EXAMINATION OF REFINED TURBULENCE MODELLING IN DEPTH AVERAGED MATHEMATICAL MODELS OF COASTAL FLOWS

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ABSTRACT

Mathematical models have been used for several years to simulate large scale tidal flows in the coastal environment. As computer resources improve with the introduction of relatively cheap but very fast modern computers, the representation of the physical processes in the models can be improved and the scope of the models can be increased. This becomes most meaningful as the spatial resolution of the models is also able to be improved by using the power of the modern new computers.

Refined turbulence models have been used successfully in the simulation of certain classes of flow in order to provide the mathematical model of the mean flow conditions with the required information on the relatively small scale and high frequency turbulent exchange of mass and momentum.

A large range of mathematical models of mean flows and turbulence models have been developed and combined to simulate a large number of particular types of flow. There is a need for reliable predictive models of recirculating flows in coastal waters and this report concentrates on, and presents an assessment of, the current and practicable representation of lateral turbulent exchange in two-dimensional depth averaged mathematical models of tidal flows.

This report is not intended as a review of current turbulence models; rather it presents an assessment of the usefulness of present day turbulence models in improving the accuracy and reliability of one type of mean flow model to simulate tidal flows in the coastal environment. The mean flow model of interest, widely used in the civil engineering industry, represents the coastal flows integrated over the water depth to provide a two-dimensional representation of the flow field.

As a result of the work done (which included the organisation and participation in seminars and workshops), it has been concluded that, within the restrictions imposed by a two-dimensional representation of flows where three-dimensional effects can be important in some areas of the model at some times in the tidal cycle, and within the restrictions on spatial resolution which can be achieved by even modern computing resources, the use of a refined turbulence model in a depth averaged mathematical model of tidal flows in the coastal environment is not warranted.

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1 INTRODUCTION

Mathematical models of tidal flows have been in use and at the service of the civil engineering industry for over 20 years. In that time, computer resources have improved dramatically both in speed of computation and in fast storage capacity. These improvements have allowed more detailed resolution of the simulated flows and the inclusion of the more complicated physical processes in the mathematical models.

There is an infinite range of flow problems where, while they are all definable in terms of the same set of fundamental differential equations describing conservation of momentum and conservation of mass, the dominant physical processes, or relative importance of various physical processes differ. As a result, a great many different types of model have been developed, incorporating different approximations or representations of some of the physical processes to take advantage of specific features of the type of flow being modelled while optimising the use of finite computing resources.

The research described in this report is concerned solely with two-dimensional mathematical models of coastal flows where the model simulates depth mean quantities and was undertaken in order to examine the representation of sub-grid scale details in this type of model using different turbulence models. This report is not intended to be a review of turbulence models and the widely varying flows in which different models have been found to be successful. Comprehensive reviews can be found in References 1 and 3. The work described in this report follows an earlier research programme sponsored by Hydraulics Research Ltd in the Department of Civil Engineering, University College of Swansea, University of Wales (Ref 2). At the start of this work, it was intended to use the particular turbulence model used in that research programme and examine its usefulness in simulating coastal flows compared with the more basic traditional approach which had been in use at Hydraulics Research and widely throughout the Civil Engineering industry for many years. As the research progressed, however, the apparent advantages to be gained from a more sophisticated representation of the sub-grid detail in depth averaged coastal flow models did not materialise.

Although the original intention of implementing and examining a refined turbulence model was not fulfilled, the research programme, which finally included a detailed examination of the numerical procedures employed in the current mathematical models and the attendance at and organisation of workshops, yielded useful conclusions.

As a result of the research described in this report, it is thought that it is now possible to apply a two-dimensional depth averaged model of coastal flows with greater confidence in the results and methods employed.

The equations describing the mean fluid motion and turbulent quantities and the details of their derivation can be found in many references, including References 1 and 2, and only those details required to illustrate the arguments in this report are reproduced.

2 DEPTH AVERAGED, TWO DIMENSIONAL FLOWS

2.1 The Fundamental Equations

The Navier Stokes momentum transport equations describe the conservation of momentum for a continuum fluid where the viscous stress is directly proportional to the rate of strain (Newtonian fluid). In a rectangular coordinate system rotating with the earth, as would be used in a mathematical model of tidal flows, these equations for an incompressible fluid with a free surface can be written in divergence form as:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_j u_i)}{\partial x_j} + \frac{1}{\rho}\frac{\partial P}{\partial x_i} + \Omega = \nu \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) \quad (j = 1,2,3) \tag{1}$$

where

u_i = velocity component in direction x_i $i = 1,2,3$
P = pressure
ν = coefficient of viscosity
Ω = $-2\ \omega u_2 \sin \phi$ $i = 1$
= $+2\ \omega u_1 \sin \phi$ $i = 2$
= 0 $i = 3$
ϕ = latitude (degrees)
ω = angular speed of rotation of earth = $2\pi/86400$

Equation (1) describes the instantaneous values of the velocity components and surface level. These instantaneous values in themselves are of little value in many cases and, considering the relatively high frequency of the turbulent fluctuations compared with tidal variations in the mean flow and the small length scales associated with these turbulent fluctuations, are impossible to resolve with present day computers in a model of large coastal area.

In order to reduce these equations to a useful form, the instantaneous velocity components are represented in terms of a turbulent mean component and a fluctuating component (Reynolds decomposition) as follows:

$$u_i = \overline{u}_i + u_i'$$

where the ' denotes the turbulent perturbation about the turbulent mean value denoted by $\overline{}$. The turbulent mean value is assumed to be obtained by averaging over a timescale which is long compared with the period of the turbulent fluctuations but short compared with variations in the mean flow. In tidal flows, the accelerations are small in general and such an averaging procedure is valid. Substituting this representation for u_i into equation (1), dropping the viscous terms which are small and are usually neglected, and averaging over time gives

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial(\overline{u}_j \overline{u}_i)}{\partial x_j} + \frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \overline{\Omega} =$$

$$\frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} \qquad (j = 1, 2, 3) \qquad (2)$$

where

$$\tau_{ij} = -\rho\,\overline{u_i' u_j'} \qquad (3)$$

In equations (2) and (3) '$\overline{}$' denotes time averaged values.

The non linear terms $\frac{\partial(u_i u_j)}{\partial x_j}$ give rise to the stress tensor τ_{ij}. The equations (2) and (3) describe the depth averaged turbulent mean flow and the problem in solving these equations lies in the representation of the Reynolds stresses ($\rho\,\overline{u_i' u_j'}$) in equation (3).

2.2 Depth Averaged Equations

In simulating coastal flows, where the length scales associated with the horizontal flow field are orders of magnitude greater than the water depth, equation (2) is integrated over the water depth. This has the effect of reducing the number of hydrodynamic equations by 1 making the resulting set of equations simpler to solve by taking advantage of the characteristic dimensions of the flow. In fact, in homogeneous conditions, in the absence of wind stress or rapidly varying bed levels, the horizontal velocity is usually a logarithmic function of distance above the bed. Depth averaging and neglecting the vertical variation in the horizontal velocity therefore departs from a true representation of the flow. In many coastal flows, however, this introduced discrepancy is small.

Integrating equation (2) over the depth and expanding in the two horizontal directions (x,y) for the depth averaged velocity components U, V gives:

$$\frac{\partial U}{\partial t} + \frac{\partial U^2}{\partial x} + \frac{\partial UV}{\partial y} + g\,\frac{\partial h}{\partial x} - 2\omega V \sin\phi$$

$$= \frac{1}{\rho d}\,\frac{\partial}{\partial x}\int_{-z}^{h}\{\tau_{xx} - (u-U)^2\}dz\ +$$

$$\frac{1}{\rho d}\,\frac{\partial}{\partial y}\int_{-z}^{h}\{\tau_{xy} - (u-U)(v-V)\}dz - \frac{1}{d}\tau_{xF} - \frac{1}{d}\tau_{xw} \qquad (4)$$

$$\frac{\partial V}{\partial t} + \frac{\partial UV}{\partial x} + \frac{\partial V^2}{\partial y} + g\,\frac{\partial h}{\partial y} + 2\omega U \sin\phi$$

$$= \frac{1}{\rho d}\,\frac{\partial}{\partial x}\int_{-z}^{h}\{\tau_{yx} - (u-U)(v-V)\}dz\ +$$

$$\frac{1}{\rho d}\,\frac{\partial}{\partial y}\int_{-z}^{h}\{\tau_{yy} - (v-V)^2\}dz - \frac{1}{d}\tau_{yF} - \frac{1}{d}\tau_{yw} \qquad (5)$$

where d = total depth = $h + z$

h = surface level relative to datum

z = bed level relative to datum

τ_{xF}, τ_{yF} = frictional stresses at the bed

τ_{xw}, τ_{yw} = wind stresses at the surface

The terms (u-U) and (v-V) arise as the result of the non-uniform velocity distribution over the depth. In most applications these terms are neglected or are assumed to combine with the Reynolds stress terms, although they are unrelated. Equations (4) and (5) now contain the integral over the depth of the Reynolds stresses and, in a depth averaged model, these stresses must be calculated, or at least represented in terms of the depth averaged velocity components.

3 MODELLING OF THE REYNOLDS STRESSES

3.1 Zero Equation Models

The oldest representation of the Reynolds stresses in terms of mean flow parameters is also perhaps the simplest, and it is still a basic feature of many more modern turbulence models. In this approach Boussinesq (1877) assumed by analogy with viscous effects in laminar flows that the Reynolds stresses could be expressed in terms of a coefficient of eddy viscosity and the gradient of the turbulent mean velocity:

$$\rho\ \overline{u'v'} = \nu_t\ \{\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\}$$

where ν_t is the coefficient of eddy viscosity. This coefficient, unlike the coefficient of viscosity, is not a property of the fluid but will vary according to the flow conditions. In any coastal region, the appropriate value for this coefficient will vary spatially depending on the local flows. In the simplest mean flow model, ν_t is assumed constant and uniform over the modelled area. This simple approach assumes that at any point, the intensity of turbulent exchange is in local equilibrium with the flow and does not allow the transport of turbulence from, for example, areas of high shear to areas of lower shear. In accelerating flows, it cannot reflect the historesis (Ref 4) which is observed where again, at any time, the rate of production and dissipation of turbulent kinetic energy is not in equilibrium with the mean flow.

Mixing length models define the eddy viscosity in terms of the local mean velocity gradient and a length scale as:

$$\nu_t = l^2 \left| \frac{\partial u}{\partial x} \right|$$

and the problem reduces to the specification of the mixing length. Depending on the flow being modelled, functional forms for the mixing length can be determined and this representation of the shear stress has advantages in certain flow models (Ref 12).

3.2 Higher Order Models

There are now a large number of models employing differential equations which describe the transport of turbulent quantities such as turbulent kinetic energy, turbulent diffusion and vorticity. Some of the equations used in these models are derived directly from the Navier Stokes equations while others are "modelled", being basically derived from the fundamental equations, but which have been simplified or modified on the basis of physical arguments.

For example, the $k - \epsilon$ model equations commonly used are not derived by depth integrating the full three-dimensional equations as in the derivation of the mean flow equations (Refs 1, 2 and 11). Rather, two-dimensional forms of the three dimensional $k - \epsilon$ equations are used to which, on the basis of physical arguments, are added terms representing the extra production and dissipation of turbulent kinetic energy expected to result from a non uniform flow field over the vertical. Further work (Ref 11) in modelling the Hydraulics Research flume experiment, which formed the basis for the model study described in Reference 2, showed that while good results were obtained, the problems encountered in simulating the distribution of turbulent kinetic energy in the earlier simulations (Ref 2) could be traced to these added terms which had a dominant effect on the solution to the $k - \epsilon$ equations.

(In modelling recirculating flows, for example, flow behind a breakwater, the flow is fully three-dimensional and attempting to model this situation with two-dimensional equations must result in some error in the solution which cannot be fully recovered by the use of a more sophisticated turbulence model. This is no criticism of the turbulence model, simply a restriction on the accuracy of the solution which can be derived).

Another class of models, the Reynolds Stress Models, employ differential transport equations for the individual Reynolds stresses $\overline{u_i' u_j'}$.

More recent model developments include Large Eddy Simulation (LES) and "Two-Fluid" models of turbulence. The Large Eddy Simulation technique involves applying a filter to the Navier Stokes equations. There is a great variation in scale between mean flow quantities and the turbulence quantities. The purpose of the filter was to separate the scales of motion which could be simulated directly from the smaller scale or sub-grid detail which has to be represented indirectly. This Large Eddy Simulation is computationally very expensive and is currently under development. The Two-Fluid model assumes that two fluids, a turbulent and non turbulent fluid share occupancy of space and the intermittent nature of turbulent flows can be modelled by consideration of the interchange of two fluids.

All of these higher order models require significant computer resources and some, such as the LES stress and Two-Fluid models, are still under development. There are a large number of different models and, within each model, e.g k - ϵ model, there may be important coefficients treated as constants but which are not truly universal.

There are many reported successes for each model in simulating different types of flows (jets in cross flows, flows in bends, wake flows) and some are given in Reference 1.

In recent years, higher order models have become more widely used in depth averaged models of relatively large scale tidal flows. It was the intention of this research to examine the existing 2-dimensional depth averaged mean flow models and include the most appropriate representation of the sub-grid detail.

3.3 Seminar on Refined Turbulence Modelling

At a relatively early stage in the examination of suitable turbulence models for civil engineering hydraulics, a one day seminar on Refined Turbulence Modelling was organised and held under this research programme at the suggestion of the IAHR UK Liaison Committee (Appendix I). It was felt that while refined turbulence modelling in the fields of mechanical engineering hydraulics and heat transfer had progressed rapidly, in civil engineering refined turbulence modelling was relatively infrequently used. The class of problem and spatial and temporal scales associated with each branch of engineering hydraulics differ greatly. It was hoped, however, that a one day meeting could encourage a useful exchange of ideas between the two groups.

The invited speakers were Dr A Hauguel, Department Laboratoire National d'Hydraulique, Electricité de France; Professor W Rodi, Institut for Hydromechanik, University of Karlsruhe, Professor D C Leslie, Queen

Mary College, Professor Launder and Professor Spalding. The subject areas covered by the speakers were Civil Engineering hydraulics, $k - \epsilon$ turbulence models, Large Eddy Simulation and two-fluid models of turbulence. The particular examples of flow problems studied covered a wide range of civil and mechanical engineering problems and the presentation and subsequent discussion proved valuable in determining the state of the art in turbulence modelling in different disciplines. The meeting, however, did not, and was not intended to, resolve the problem of the most appropriate turbulence model to use in the relatively narrow field of depth averaged mathematical modelling of coastal flows. It did, however, indicate that the more widely used $k - \epsilon$ model was at a stage of development where it could be used without incurring excessive computational overheads and it was decided to proceed with the implementation of this model as described in the earlier research programme supported by HR (Ref 2).

4 MEAN FLOW MODELLING NUMERICAL TECHNIQUES

There are a wide range of finite difference and finite element techniques available with which the governing differential equations can be solved. The solution procedure can, in fact, have an influence on the solution far more important than the representation of turbulence. For time dependent problems, the advantages of mesh refinement and, therefore, locally improved spatial resolution afforded by finite element techniques cannot always be exploited. In a model of tidal flows in a coastal region the area of high velocity or solute gradient, which may require a high degree of resolution, can move by many kilometers during a tidal cycle and a uniformly fine mesh is required over the whole area. Finite difference methods have been preferred because of the nature of the problem and their computational efficiency. The research programme at University College, Swansea (Ref 2) involved finite element solutions to the governing equations and, for the class of problem addressed by this research, it was concluded that there was no advantage to be gained from finite element techniques.

4.1 Finite Difference Solution Procedure

The Tideway system two-dimensional depth averaged models fall into two groups. There are the models which use the Distributed Array Processor (DAP), which employ relatively simple explicit numerical schemes, and there are those models which use conventional serial computers which employ both implicit and explicit numerical schemes. Descriptions of the finite difference schemes used can be found in References 5 and 6.

The implicit solution technique employs an Alternating Direction Implicit technique and this solution procedure, while computationally efficient, can lead to phase errors and an inaccurate solution (Ref 7) depending on the flow conditions, grid alignment and modelled area. The serial explicit model does not suffer from that deficiency but is computationally expensive to use. As a result, it was decided to examine the most recent DAP model. This model uses a relatively simple explicit numerical scheme which can exploit the extremely powerful array processor to give an accurate and economic solution to the governing differential equations.

5 MODEL APPLICATIONS

The DAP model has been used in many project studies and it is useful to examine a few results from these studies in order to define the class of problem of interest and identify the types of solution which have been obtained using a prescribed, uniform coefficient of eddy viscosity.

5.1 Large Scale Studies

5.1.1 Pentland Firth Model

A model of a large area of the coastal waters off the North Coast of Scotland (Fig 1) was set up to simulate tidal flows over the large area shown (Ref 8). The model used a 400m grid and had in excess of 20,000 active model cells. The 400m grid prevents the resolution of all but relatively large scale water movements and Figures 2-4 show some typical results compared with Admiralty tidal stream data. The model at least qualitatively reproduces the large eddy pattern indicated by the tidal stream data.

5.1.2 South East Dorset Model

Figure 5 shows the model area off the South East coast of Dorset modelled using a 400m grid in the DAP model (Ref 9). Figures 6 - 10 show some typical results of the recirculation off Portland Bill as resolved by the model compared with Admiralty tidal stream data.

5.1.3 Discussion

In both studies, a large area had to be modelled and a relatively large grid was used to resolve the main tidal flows. In both cases, the model reproduced these flows reasonably well. Varying the coefficient of eddy viscosity within the expected range (5 - 20m^2/s typically) had little effect on the flows and it is suggested that a refined turbulence model would not yield any improvement on this type of simulation.

In Reference 1, Rodi states: "In many situations, especially those of large scale, as for example, coastal waters, the terms involving the depth averaged turbulent stresses are negligible compared with the other terms and the only influence is through the bottom shear stress τ_b. In such cases, a model is needed only to relate τ_b to the depth averaged flow velocities."

Despite this, refined turbulence models are employed in coarse grid models. The problem reduces to determining when the problem and grid scale are sufficiently small to warrant refined turbulence modelling.

In fact, in examining the problem of modelling large scale recirculations, the finite difference techniques used can have an overriding influence on the solution. In the study of the Pentland Firth, it was found that the solution compared most favourably with the Admiralty data when, for the non linear terms in the governing equations (eqs 4 and 5), centred differences were used throughout, except in the neighbourhood of the coastline where upstream differences were employed. This type of modification to the model produced small but more noticeable changes to the flow field than varying the coefficient of eddy viscosity. (The model basically uses centred differences to minimise numerical diffusion effects. Upstream differences are of lower accuracy and more diffusive than centred differences but in the vicinity of coastlines, where centred differencing cannot be applied directly, either a more accurate higher order solution procedure could be implemented or a lower order diffusive representation of the non-linear terms could be used, such as that which gave the degree of qualitative comparison shown in figures 2 to 4 between the model and Admiralty tidal stream data).

The flow field in a coastal region is defined by the local tidal conditions and bathymetry and, using coarse grids of the order of several hundred meters, the unavoidable lack of resolution of bathymetric features could be more important than the relatively small correction which would be obtained by modelling turbulent exchange accurately. It is also important to note that at these grid sizes, where the water depth varies, the lack of resolution of the horizontal mean flow and the assumption that the flow is uniform over the depth are likely to introduce much larger fundamental short-comings in the solution than those resulting from a simple representation of turbulent exchange. Indeed, in these coarse grid models, the coefficient of eddy viscosity is not related to turbulent exchange but is a model parameter available to try to improve the models' simulations - by empirical adjustment within established ranges of values the Reynolds Stress terms simply permit convenient adjustment to the models' results. It should be noted, however, that in these large scale coastal flows, the Reynolds Stress terms in the governing equations are not of great importance.

5.2 Local Models of Coastal Flows

As the grid size reduces, the model should be able to resolve the variations in bathymetry and the variations in the mean flow more accurately. The relative importance of turbulent exchange in the solution should improve but the assumption that the flow is two-dimensional and can be depth averaged remains. It could be assumed that the vertical profile of velocity is logarithmic and the corresponding terms in the governing equations can be integrated to yield a result which can be included in the modelled equation. However, the velocity profile is not always logarithmic or unidirectional over the depth and the improvement, if any, to be gained from representing the effects of depth integration in this way is not easy to determine for the general case. In addition, examination of the relative magnitudes of the stress terms indicates that these terms are unimportant (Ref 1) for two-dimensional flows. Leendertse adds a factor α to the non linear terms, $\alpha u \frac{\partial u}{\partial x}$ where $\alpha > 1$, in order to allow for the effect of the non-uniform profile. It is found to have a similar effect on the bed friction term.

5.2.1 Swanage Bay Model

As part of one recent study, it was required to model tidal flows in Swanage Bay to examine the location of a sea outfall (Ref 10). Swanage Bay is contained in the South East Dorset model area (Figs 5, 11, 12) and results from this 400m grid model are shown in Figure 13. It can be seen that at this resolution, the coarse grid model is unable to resolve the local flows. A fine grid model data set was created using a 50m grid and the same mathematical model was run on the 50m grid using boundary conditions derived from the 400m grid model. A sequence of results is shown in Figures 14 to 18.

On the ebb tide, the flow expands into Swanage Bay. Around low water, the flow becomes confused and just floods round Peveril Point for between 1 and 2 hours before a well defined eddy develops with high shear along the line of shallower water as the flood tide flows develop. Eventually, the eddy fills the whole of the bay and, again, the flow near the shore of the bay is in an "ebb" direction on the flood tide.

Little field data was available with which to validate the fine grid model but local fishermen were able to describe the asymmetric flows in great detail. The model agreed well with these qualitative descriptions. Sensitivity tests were carried out on the importance of the constant coefficient of eddy viscosity. The model was run with different values and it was found, for example, that reducing the value from $10m^2/s$ to $5m^2/s$ produced little noticeable effect in the flow patterns but affected the magnitude of the velocities. For example, Figures 19 and 20 show time histories of velocity off the end of the pier. It can be seen that peak speeds reduce from 0.2m/s to 0.18m/s when the constant eddy viscosity is increased from $5m^2/s$ to $10m^2/s$. The value of the eddy viscosity is expected to lie in the range 1 - $10m^2/s$ and, without detailed, reliable field observations, it is not possible to optimise the value of this coefficient. However, within the range of expected values, its influence is small. Figure 21 shows the effect the coefficient has on the movement of the centre of the large eddy formed during the flood tide and the effect of varying the eddy viscosity can be seen.

5.3 Discussion

In the project studies described, qualitative agreement between model prediction and observed recirculating flows was achieved and the coefficient of eddy viscosity was not found to have a significant influence on the simulated flows, which would suggest that for these classes of flow problem, for the grid sizes used, a more refined turbulence model is unnecessary.

However, would this remain the case if even finer grids are used?

The representation of the very complex process of turbulent exchange using a prescribed eddy viscosity is, intellectually, unsatisfactory. However, bearing in mind the other approximations in applying numerical solution techniques, the assumptions that the flow is depth averaged and, more importantly, the type encountered in coastal waters, may in fact mean that the explicit inclusion of the Reynolds stress terms in the governing equations is unnecessary. Their retention does provide a means of calibrating a model but, for the coastal flows considered, the impact of the eddy viscosity is small. If this is the case, the need for refined turbulence modelling in depth averaged models of coastal flows vanishes.

6 MEETING ON REFINED MODELLING OF TURBULENT FLOWS IN COASTAL WATERS

In February 1988, a working group organised at the instigation of the IAHR sections on Maritime Hydraulics and Computational Hydraulics met to discuss "Refined Modelling of Turbulent Flows in Coastal Waters". It was intended that the participants should all present brief lectures on their particular research and the stated objective of the meeting was: "this working meeting, on invitations and without proceedings, is rather informal and is aimed as an exchange platform for discussion between specialists in the main field of 2D modelization of coastal flows; in order to enable large and efficient discussions, the communications should be rather short and be clearly focused on problems relating to numerical modelization of turbulence in 2D coastal flows."

The meeting was organised by and held at Electricité de France Laboratoire National d'Hydraulique, Chatau, Paris and the list of participants and a report on the meeting is given in Appendix II.

In fact, the 3-day meeting included presentations on stratified flows and simulations of the vertical structure of the flow and 3-dimensional simulations but, on the whole, was a useful conclusion to this research programme.

Appendix II contains a report on the meeting and summarises the conclusions of the discussions. It is useful, however, to examine the conclusions of several of the papers presented which concentrated on relatively large scale and small scale two-dimensional depth averaged models of coastal flows.

6.1 Large Scale Models

G K Verboom stated that his objective had been to "find a formulation for ν_t to compute (large scale) recirculation flow in practical situations for steady and unsteady flow". He illustrated his presentation by describing a model study of Anna Friso Polder. Essentially, he was modelling the flows at the entrance to a small harbour where the depth increased rapidly from 10 to 40m over a distance of approximately 150m. He carried out several experiments using constant values of eddy viscosity of ν_t = 10, 1, 0.1, 0m²/s with model grid sizes of 45m and 22.5m and different model timesteps in his implicit finite difference model. His conclusions were that reducing ν_t from 10m²/s to 1m²/s had a strong influence but further reduction from $1m^2/s$ to $0m^2/s$ had no influence - except if $\nu_t = 0m^2/s$ there was no steady state solution for a grid size of 22.5m. He concluded that the effects of depth variations can be as important as the eddy viscosity and that a constant eddy viscosity can be used to simulate the flow. It was thought, however, that the importance of the eddy viscosity would increase if the modelled area had a flat bottom.

M B Abbott described a model study of Yell Sound in the Shetland Islands (Ref 13). The flows in Yell Sound are extremely complex with eddies forming behind islands and headlands. He concluded that in problems where the depth is considerably less than the model grid size, the eddy viscosity is unimportant and he suggested that the numerical scheme, provided it was isotropic and the model used a square finite difference grid, should generate the required momentum exchange (and the solution should be independent of the accuracy of the numerical scheme).

He went further to state that a differential equation containing an eddy viscosity is a "madness" because the equation is a statement at a point and cannot contain diffusion.

P Pechon described a simulation of recirculating flows in a harbour using constant coefficients of eddy viscosity and found that, in fact, the treatment of the boundaries using either "slip" or "no-slip" boundary conditions was of greater importance in influencing the solution than the coefficient of eddy viscosity.

6.2 Small Scale Models

Some of the presentations at the meeting were concerned with the simulation of classical flow problems observed in laboratory experiments such as flows past a breakwater or past a step. In these cases, the use of a constant eddy viscosity - or zero eddy viscosity, M B Abbott, - produced good agreement with observation provided the model grid was sufficiently fine. '$k - \epsilon$' models also produced good results but it was concluded (Appendix II) that further work would be required to improve the model for coastal applications.

7 CONCLUSIONS

In the course of this research project, attention has concentrated on two-dimensional depth averaged mathematical modelling of coastal flows and the most appropriate representation of the lateral exchange of momentum.

In coarse grid models, (grid size 100m or more) the turbulent exchange is unimportant compared with the other terms (Ref 1). In finer grid models with grid sizes of the order of 50m or less, where the bathymetry varies within the model area, the bathymetric effects are still much more important than the lateral exchange of momentum.

The coefficient of eddy viscosity has been found to have a minor influence on the results of model simulations and, while some researchers suggest it should be dropped from the governing differential equations, its retention does permit minor tuning of the model results if data on the flow being simulated is available. In these circumstances, the eddy viscosity should be considered just that, a fine tuning parameter, and its use should not be confused with a realistic representation of lateral turbulent exchange of momentum.

In very fine grid models such as have been used to simulate laboratory experiments, equally good results have been reported using, for example, $k - \epsilon$ models or constant coefficients of eddy viscosity. The most appropriate value for the constant eddy viscosity may have to be determined by empirical adjustment. The use of constant eddy viscosity in, for example, a model of flows behind a breakwater (Ref 11), where flow conditions vary greatly in relatively short distances, may be successful as a result of the formulation of the Reynolds stress term in terms of the product of the velocity gradient and the coefficient of eddy viscosity. This term can only be important where the velocity gradient is large and, provided the chosen coefficient is relevant to the area of high velocity gradient, its value is unimportant in the remainder of the flow field where velocity gradients are small. The problem remains in selecting the appropriate value for the coefficient of eddy viscosity.

The meeting on refined turbulence modelling (Appendix II) suggested that, provided the model grid is sufficiently fine, the simulation of these small scale flows should not be heavily dependent on the chosen coefficient.

As a result of the work done so far, it is intended to re-examine the finite difference schemes employed in the two-dimensional depth averaged model to minimise the dependence of the solution on the numerical techniques used in the vicinity of boundaries, but to retain the relatively simple use of a constant coefficient of eddy viscosity. The flow fields which will be simulated by the model, whether large scale or local coastal flows where the bathymetry can vary, have been found to be insensitive to this parameter. It is retained purely as a device with which to optimise the model results, if necessary, when relevant observations are available. The use of a more refined turbulence model is not considered relevant in two-dimensional depth averaged models of coastal flows.

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FIGURES

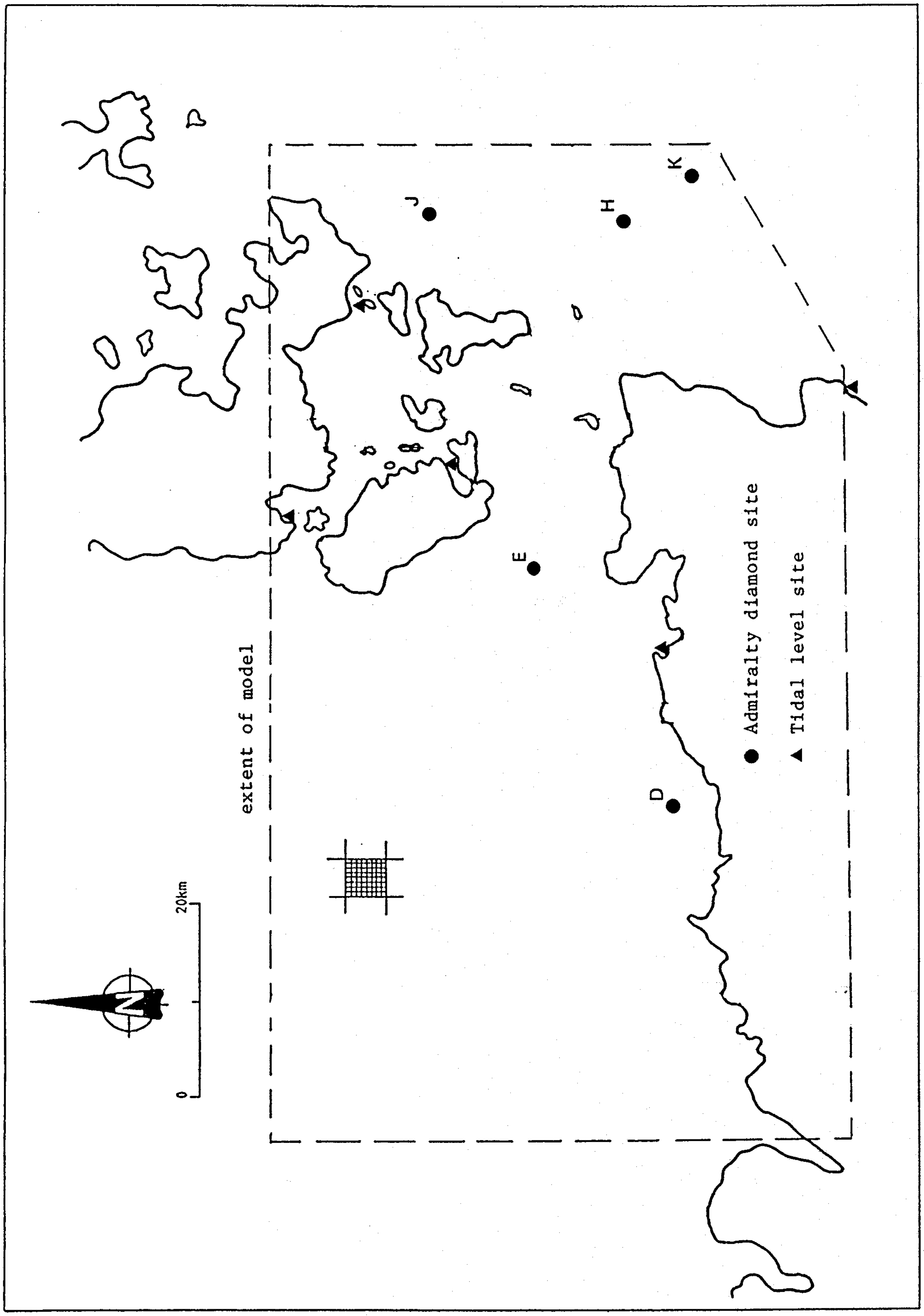
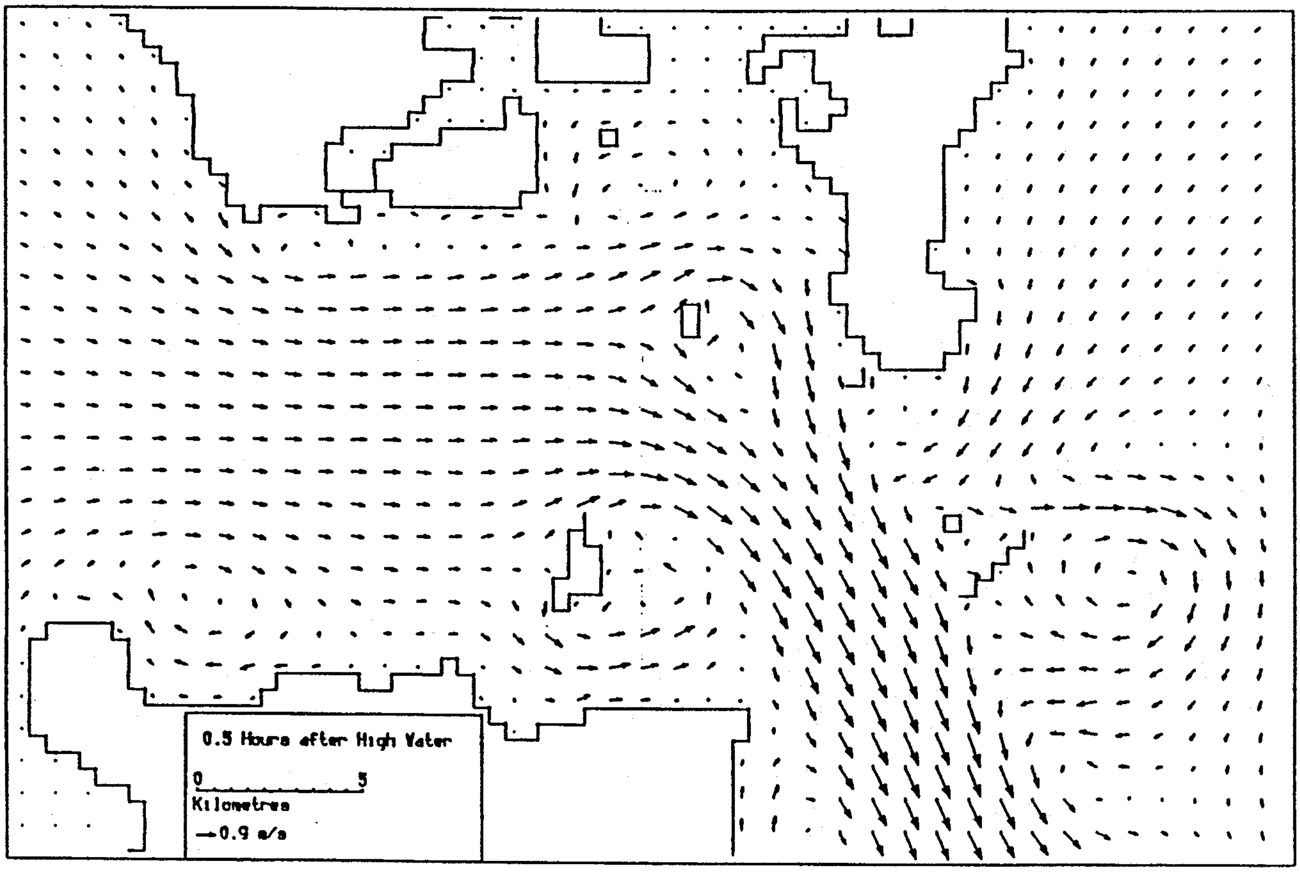


Fig.1 Pentland Firth Model Boundaries



Model simulation

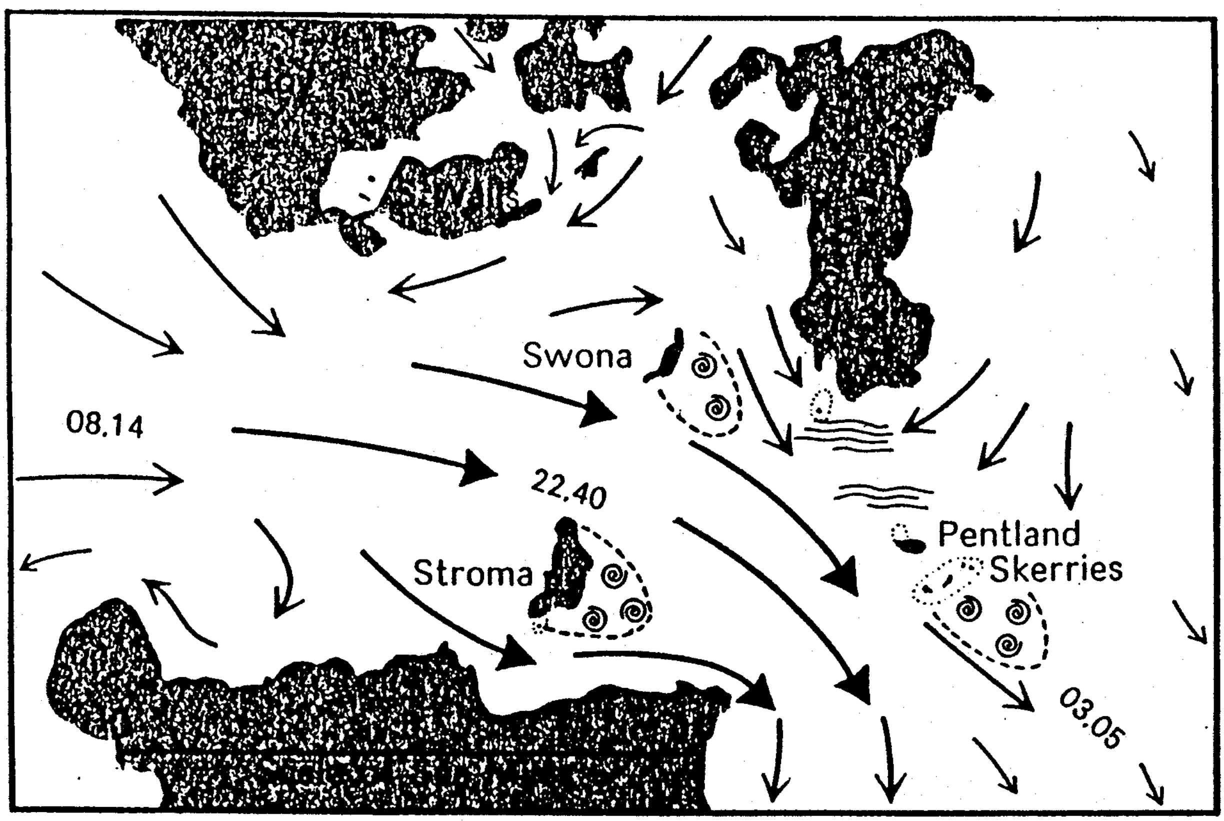
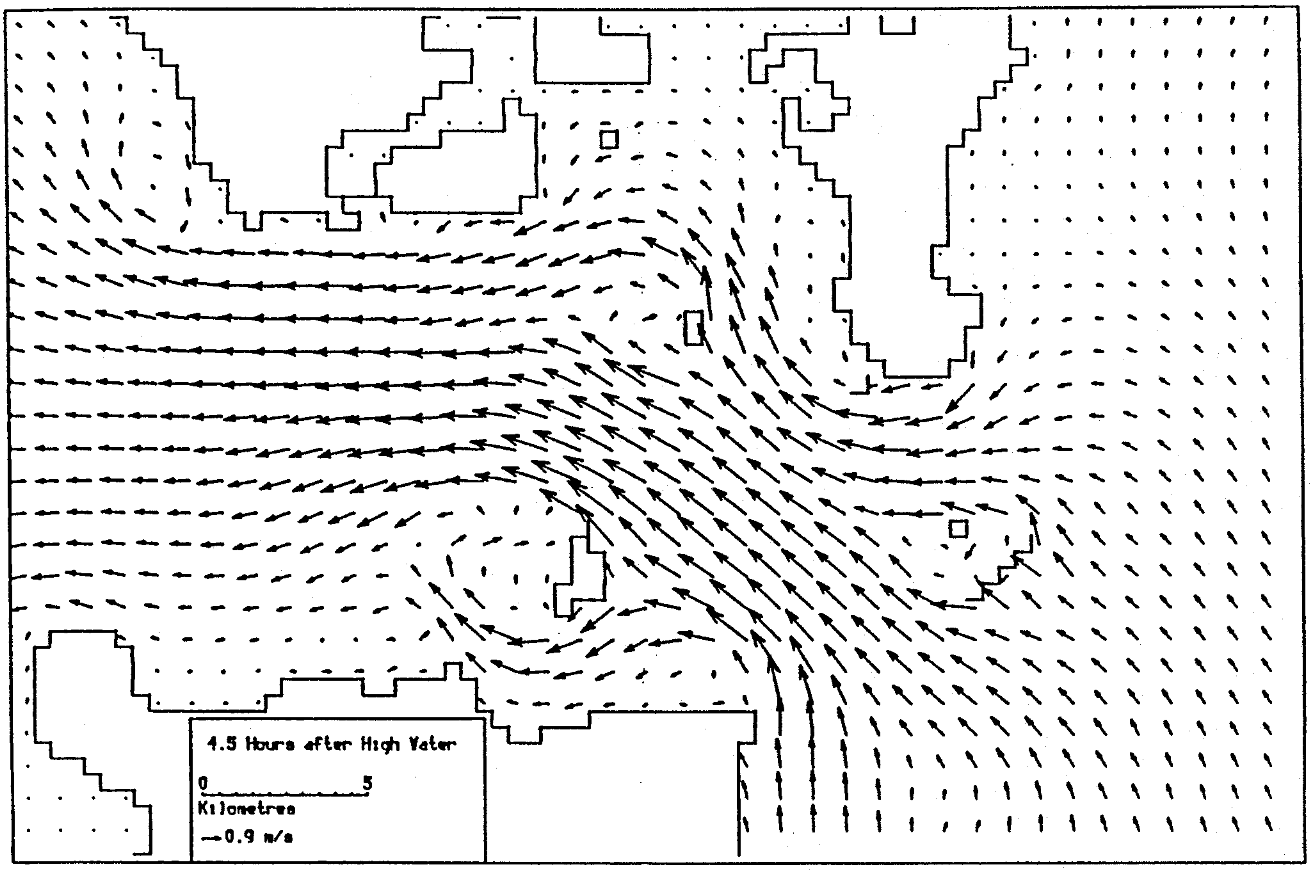


Fig.2 Comparison of Admiralty tidal streams and model simulation, Pentland Firth



Model simulation

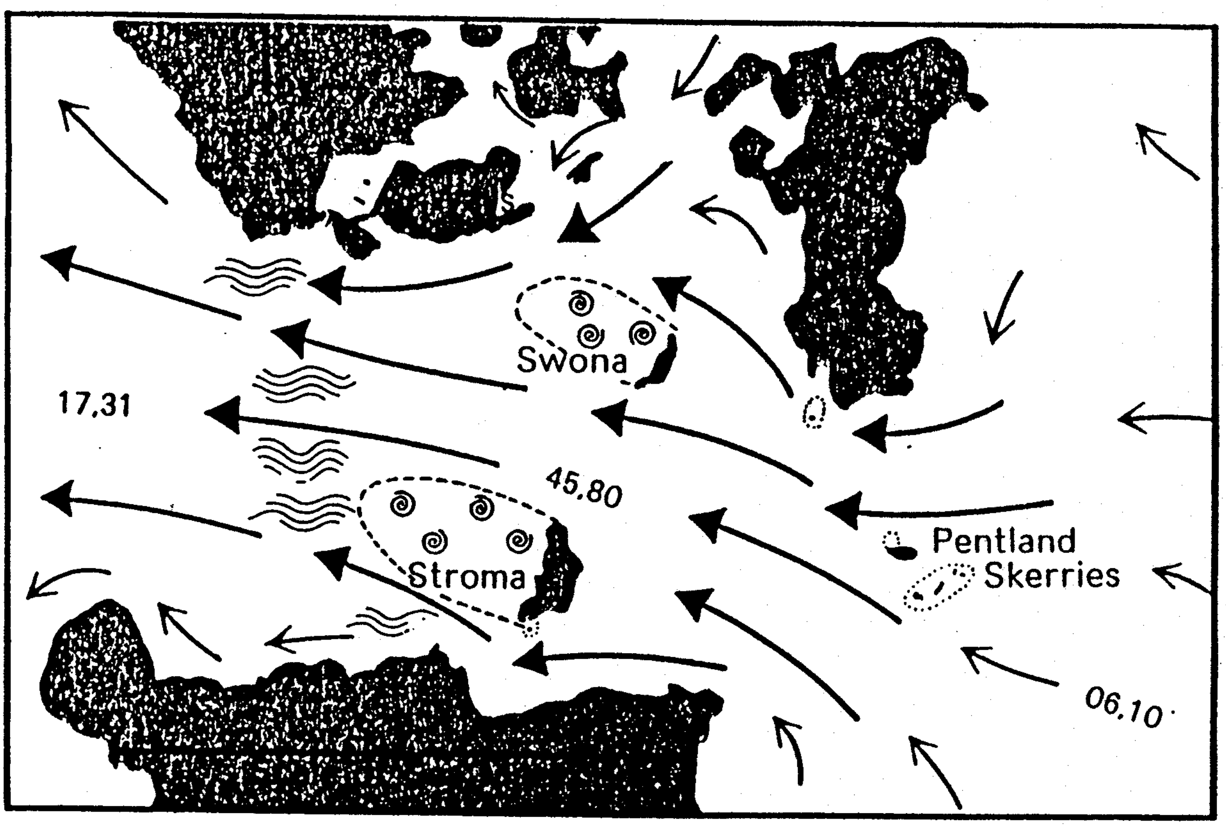
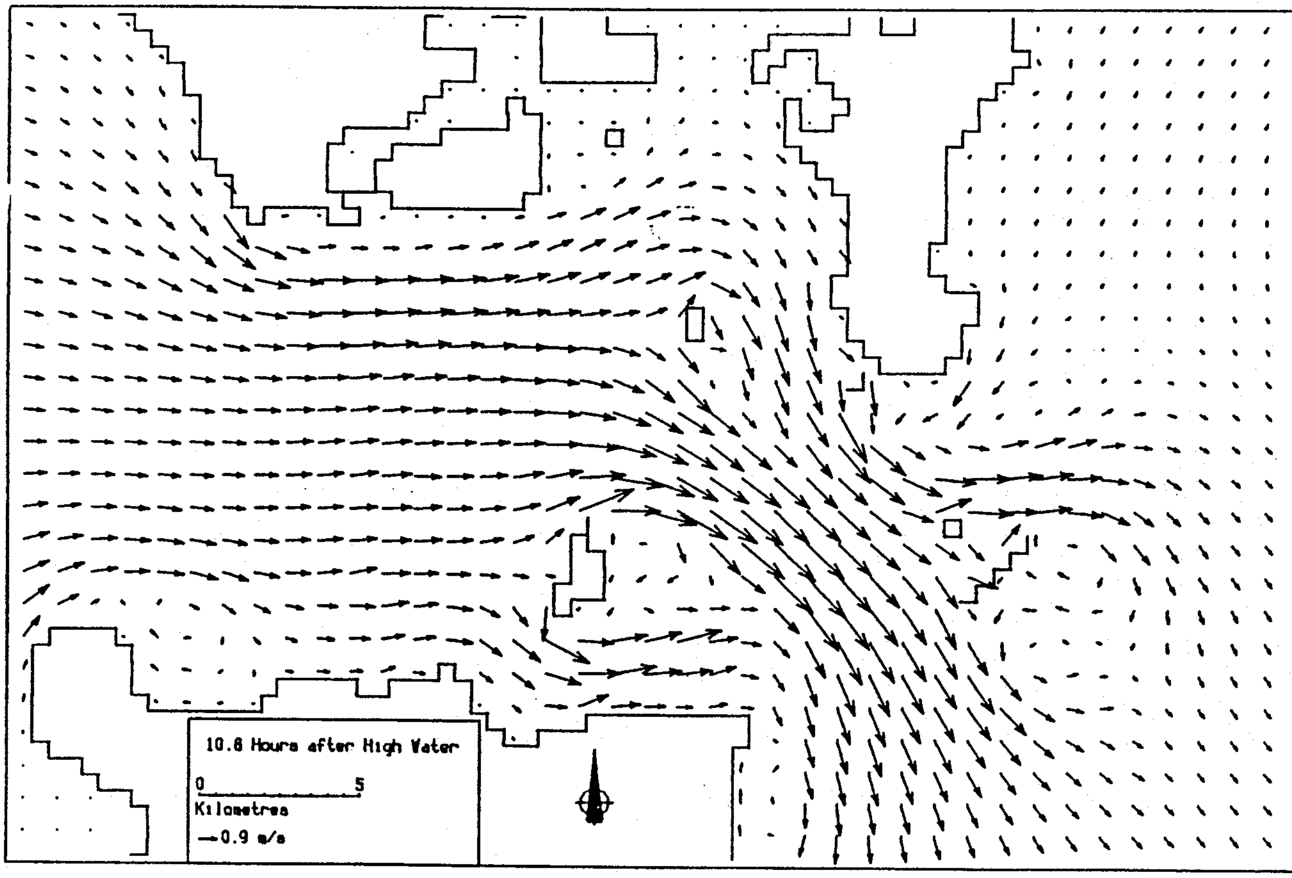


Fig.3 Comparison of Admiralty tidal streams and model simulation, Pentland Firth



Model simulation

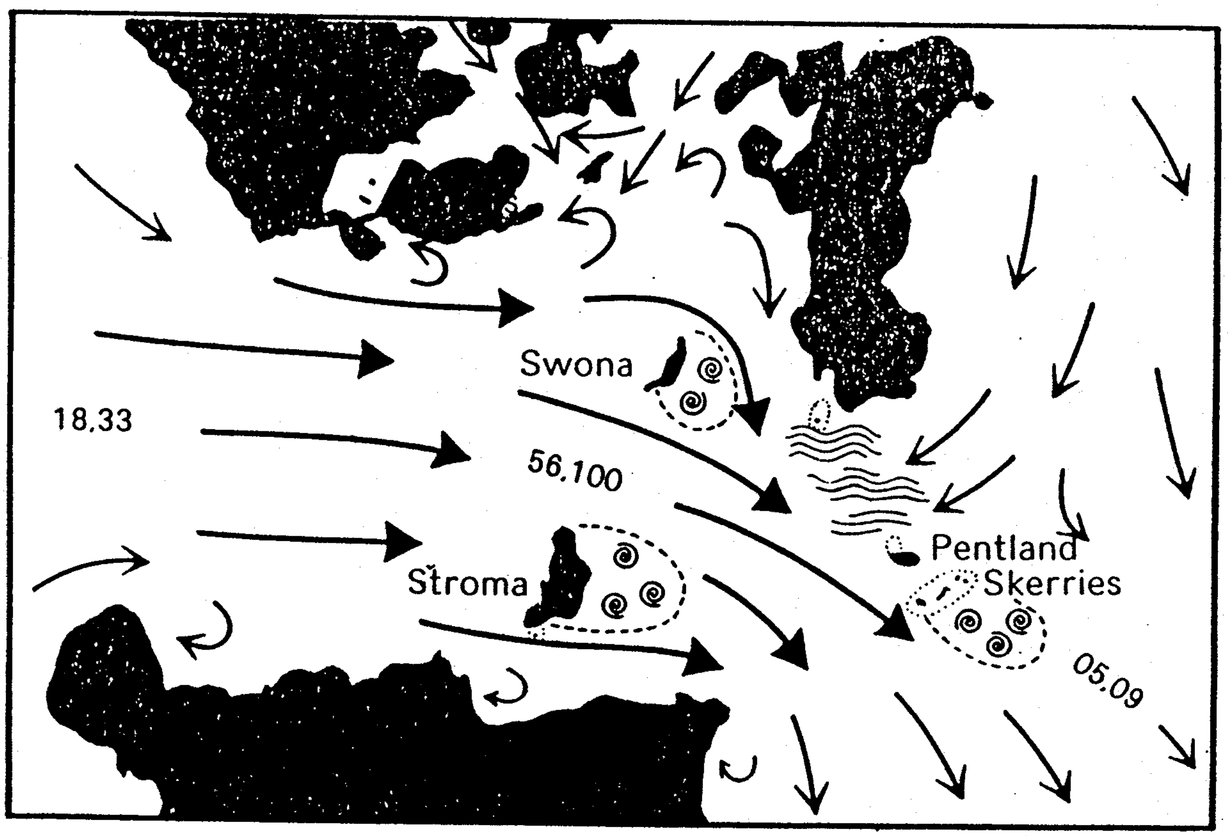


Fig.4 Comparison of Admiralty tidal streams and model simulation, Pentland Firth

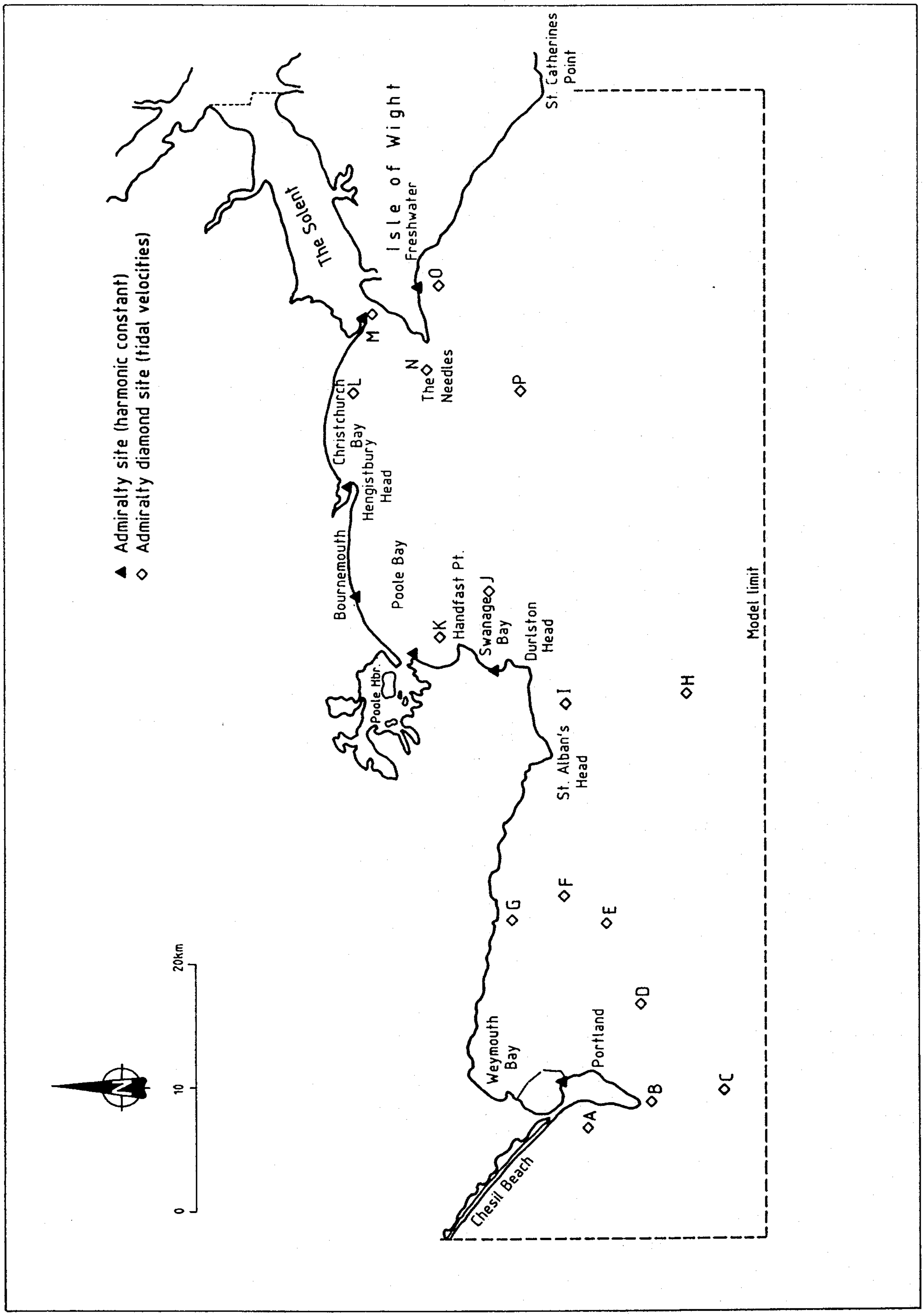
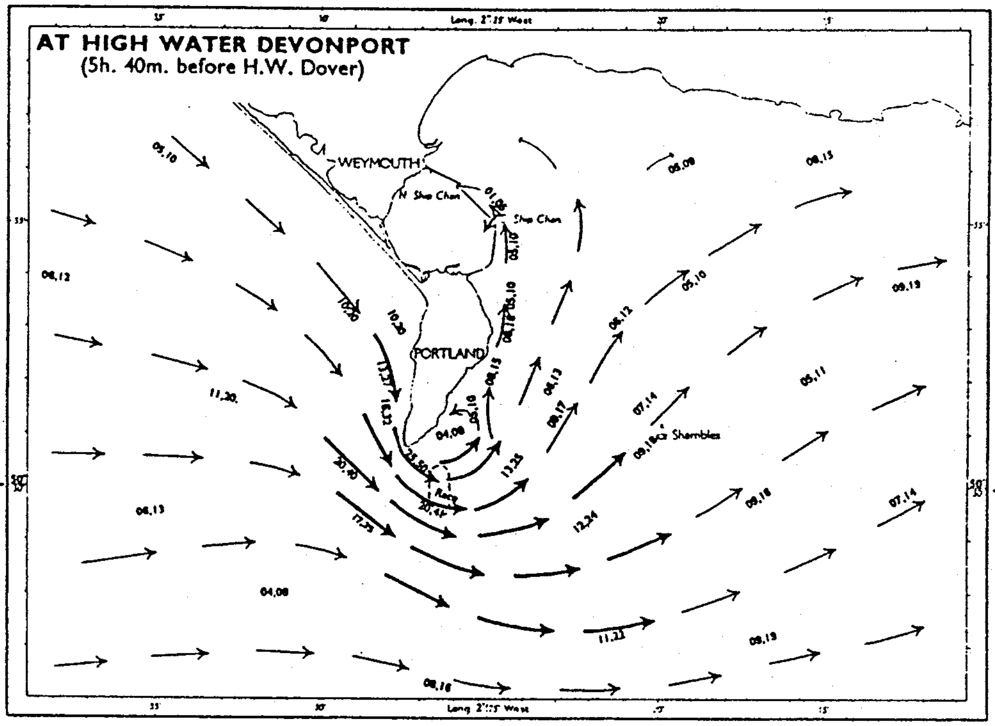
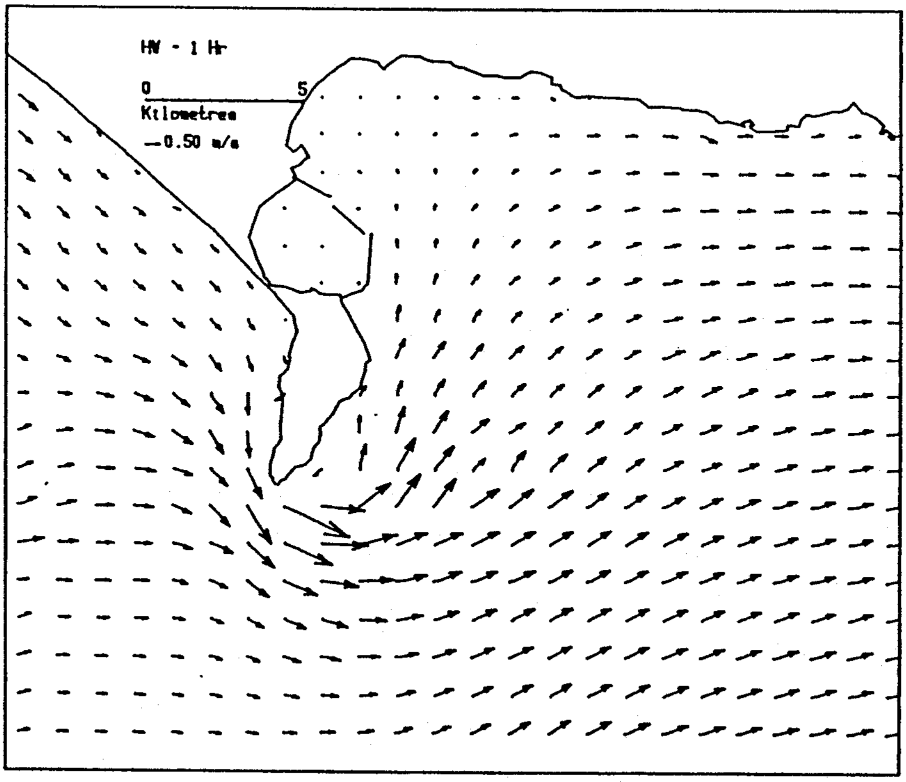


Fig.5 South East Dorset Model Boundaries





Model simulation

Fig.6 Simulated tidal velocities, spring tide, comparison with Admiralty tidal streams

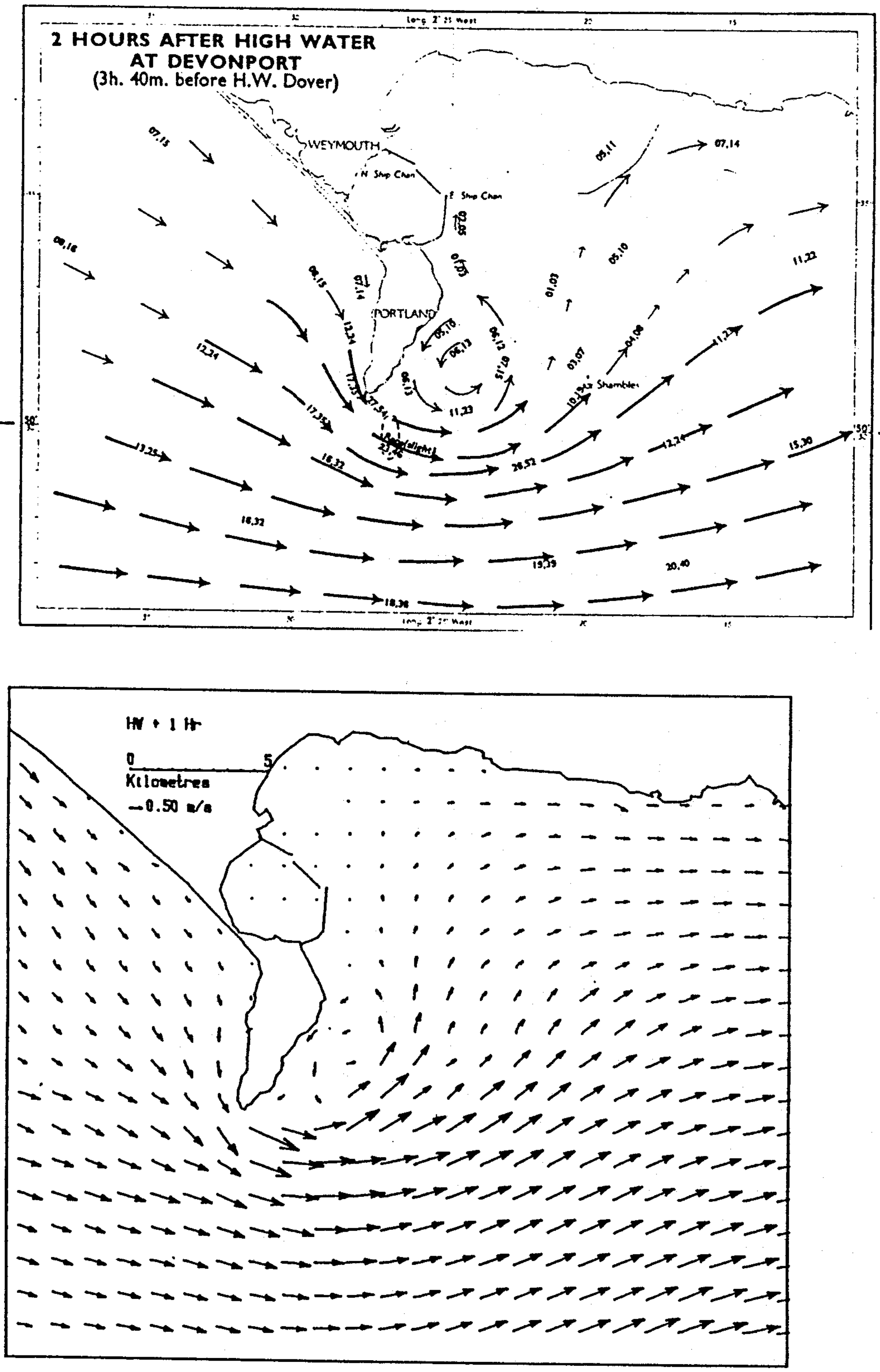
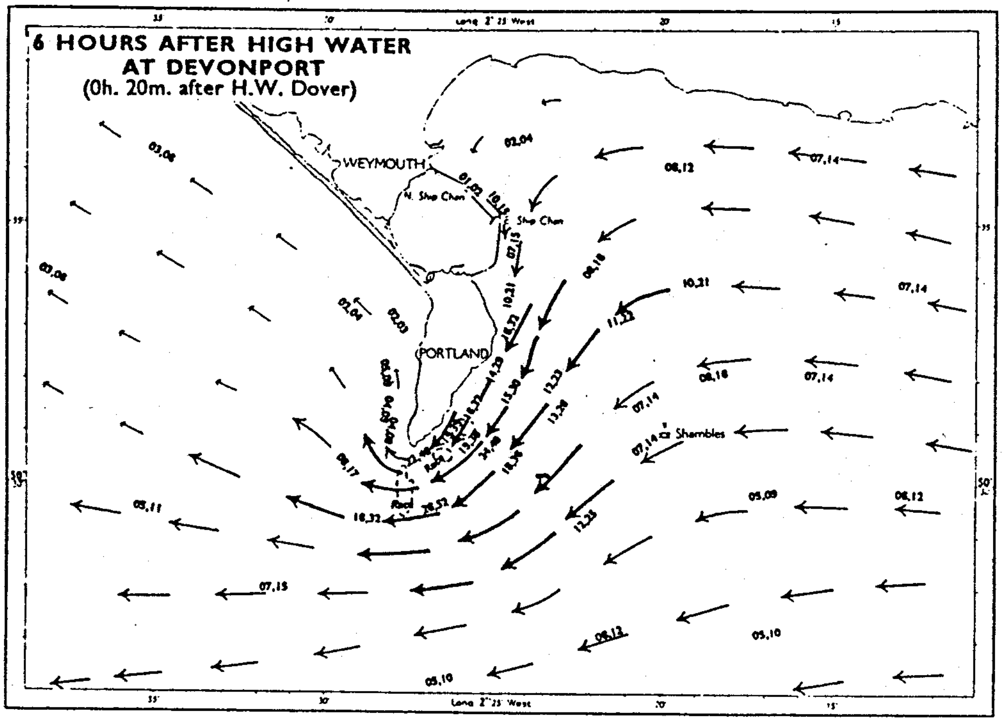


Fig.7 Simulated tidal velocities, spring tide, comparison with Admiralty tidal streams.



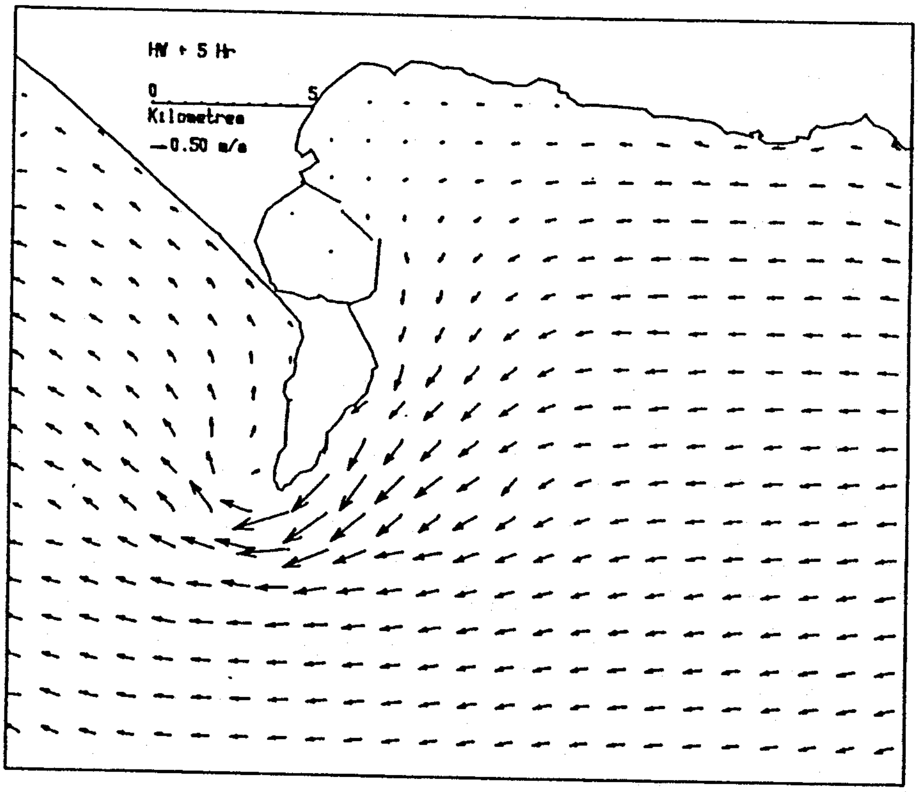
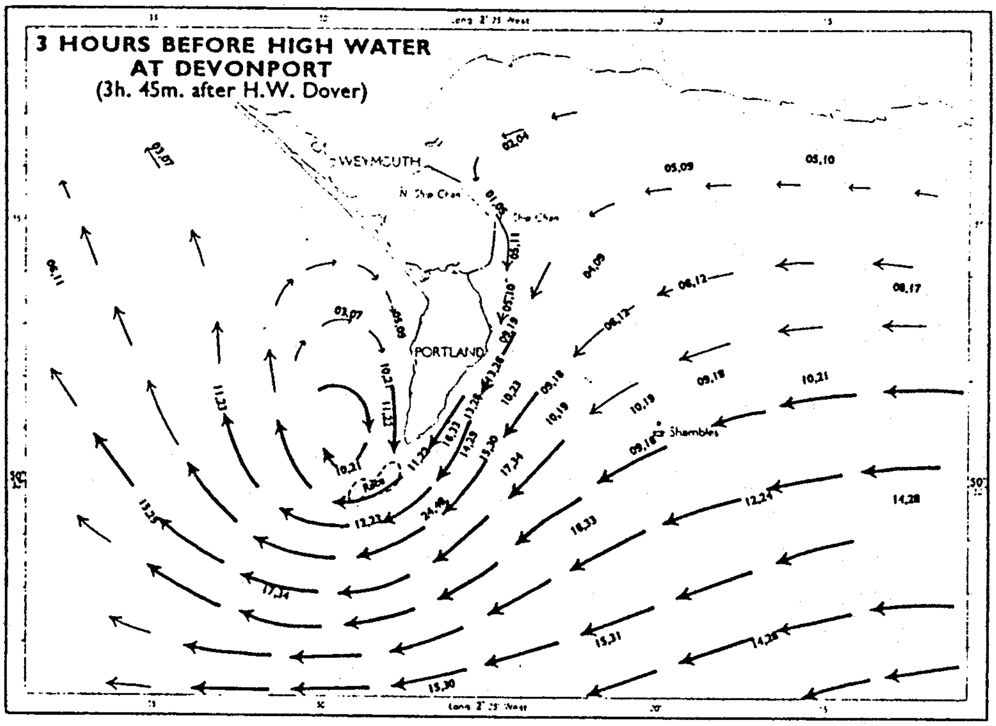


Fig.8 Simulated tidal velocities, spring tide, comparison with Admiralty tidal streams



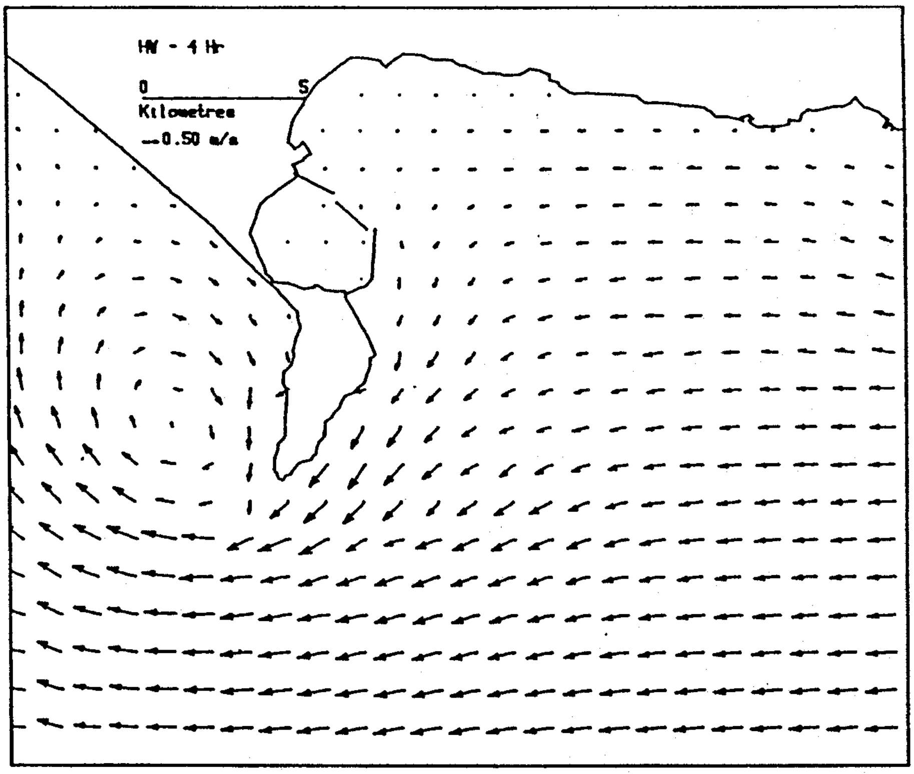
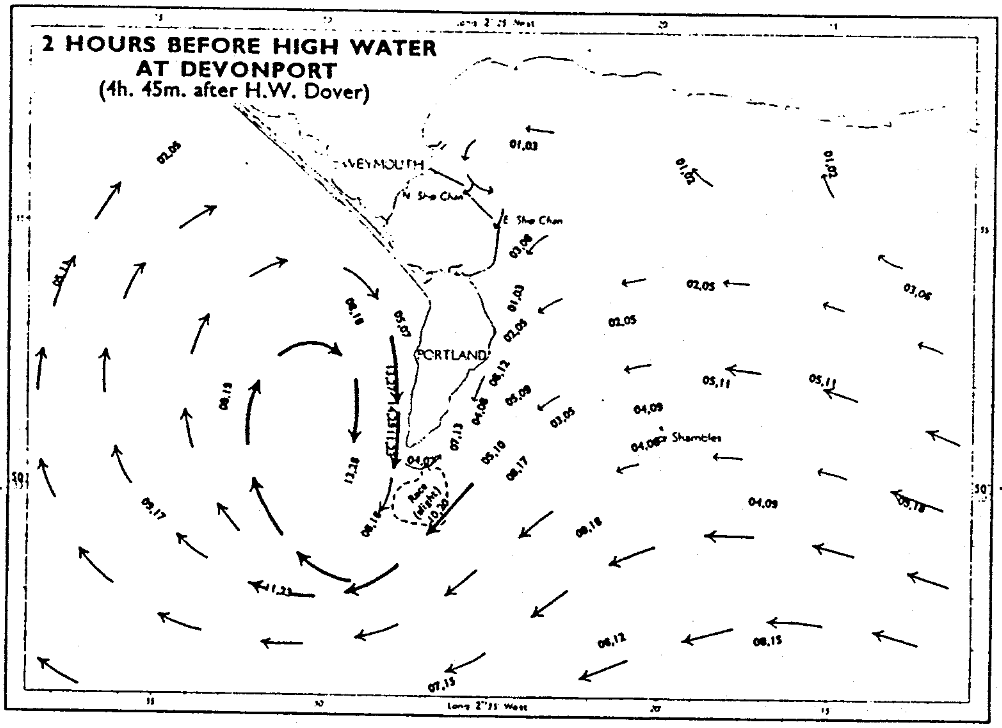


Fig.9 Simulated tidal velocities, spring tide, comparison with Admiralty tidal streams.



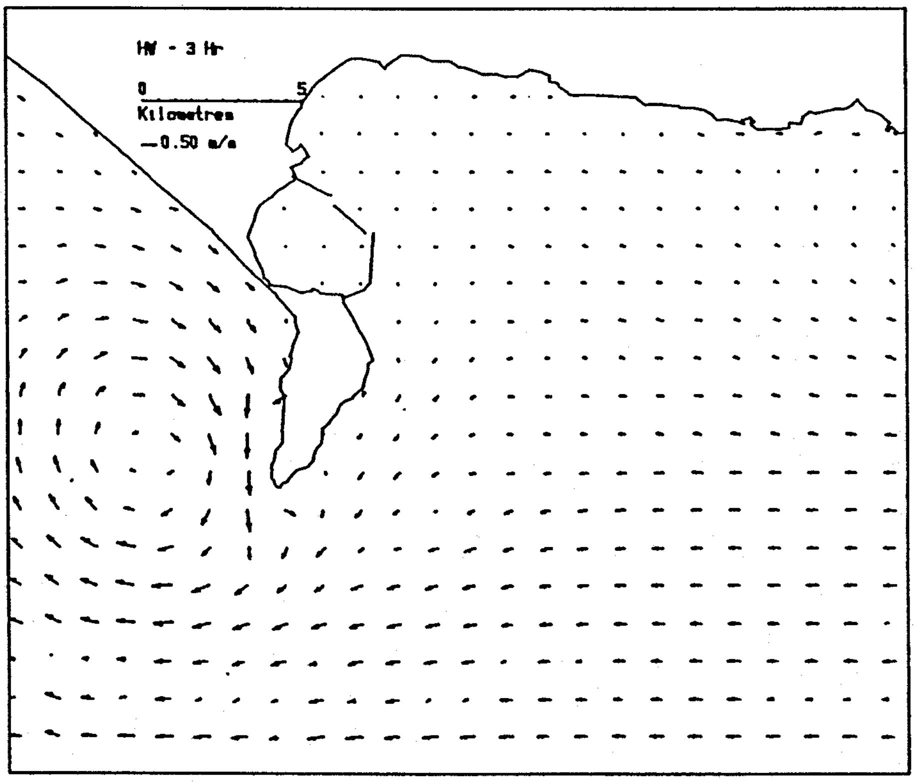


Fig.10 Simulated tidal velocities, spring tide, comparison with Admiralty tidal streams

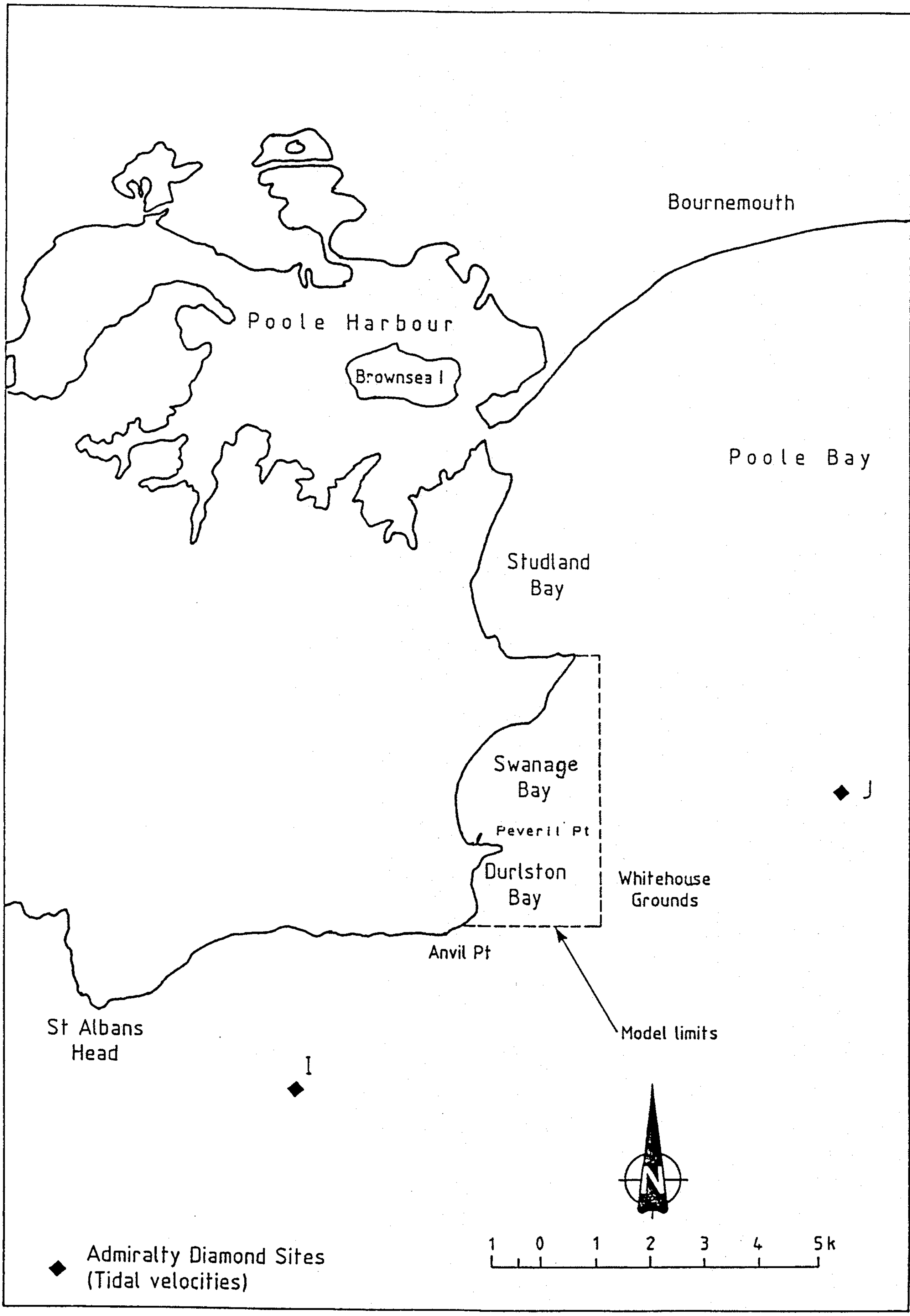


Fig.11 Location plan - Swanage Bay model area

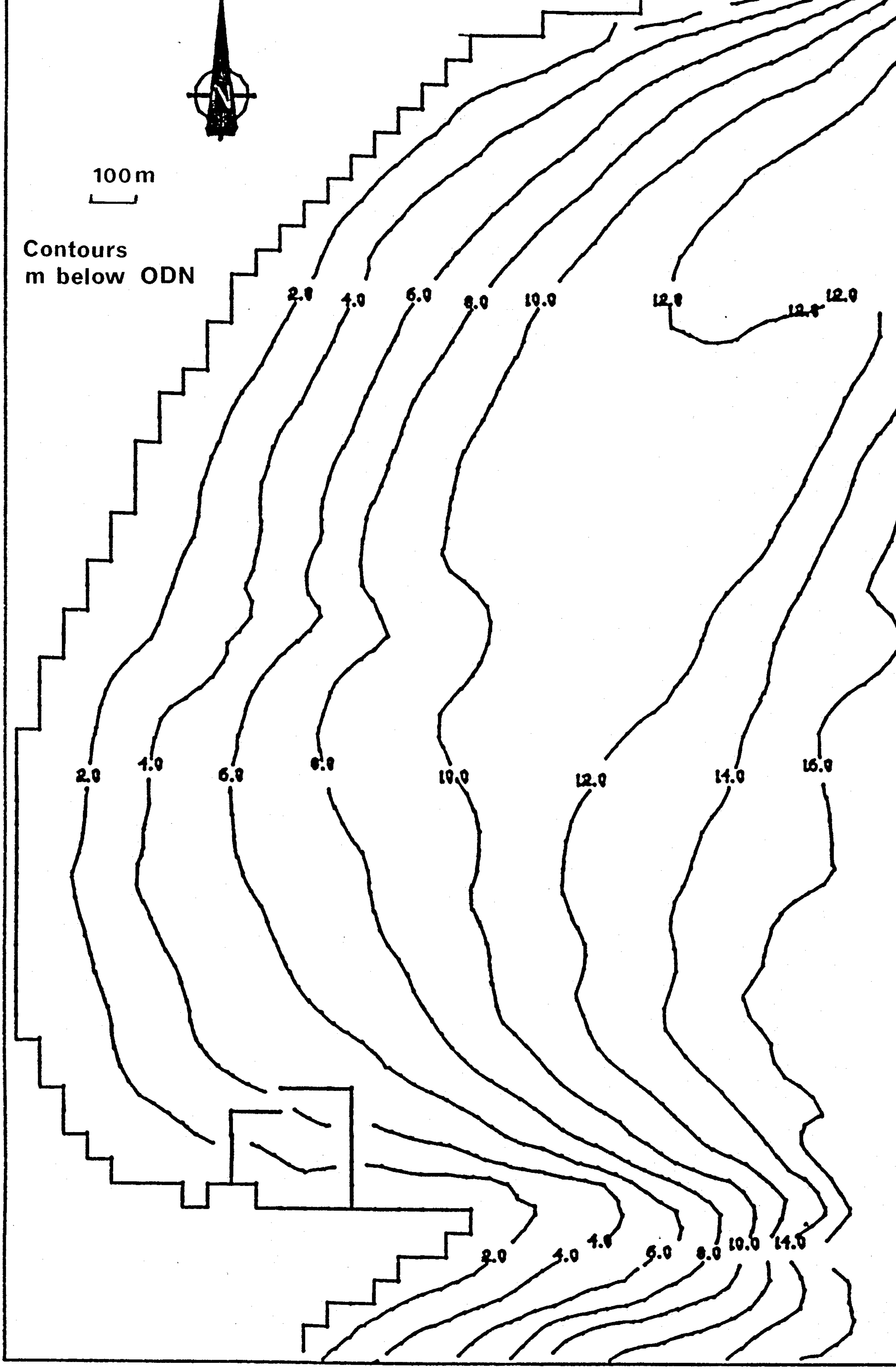


Fig.12 Swanage Bay Bathymetry

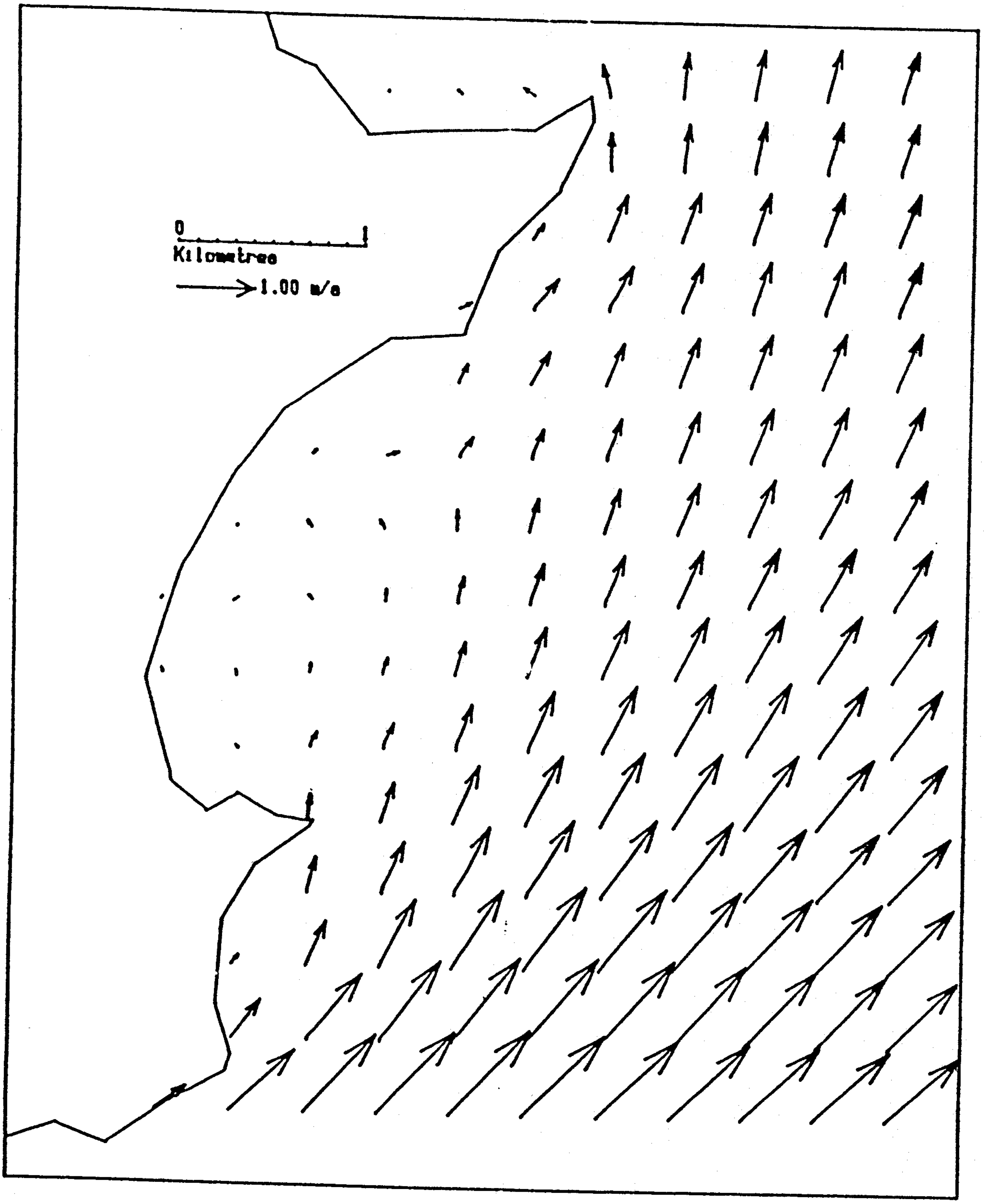


Fig.13 Swanage Bay tidal velocities from SE Dorset model

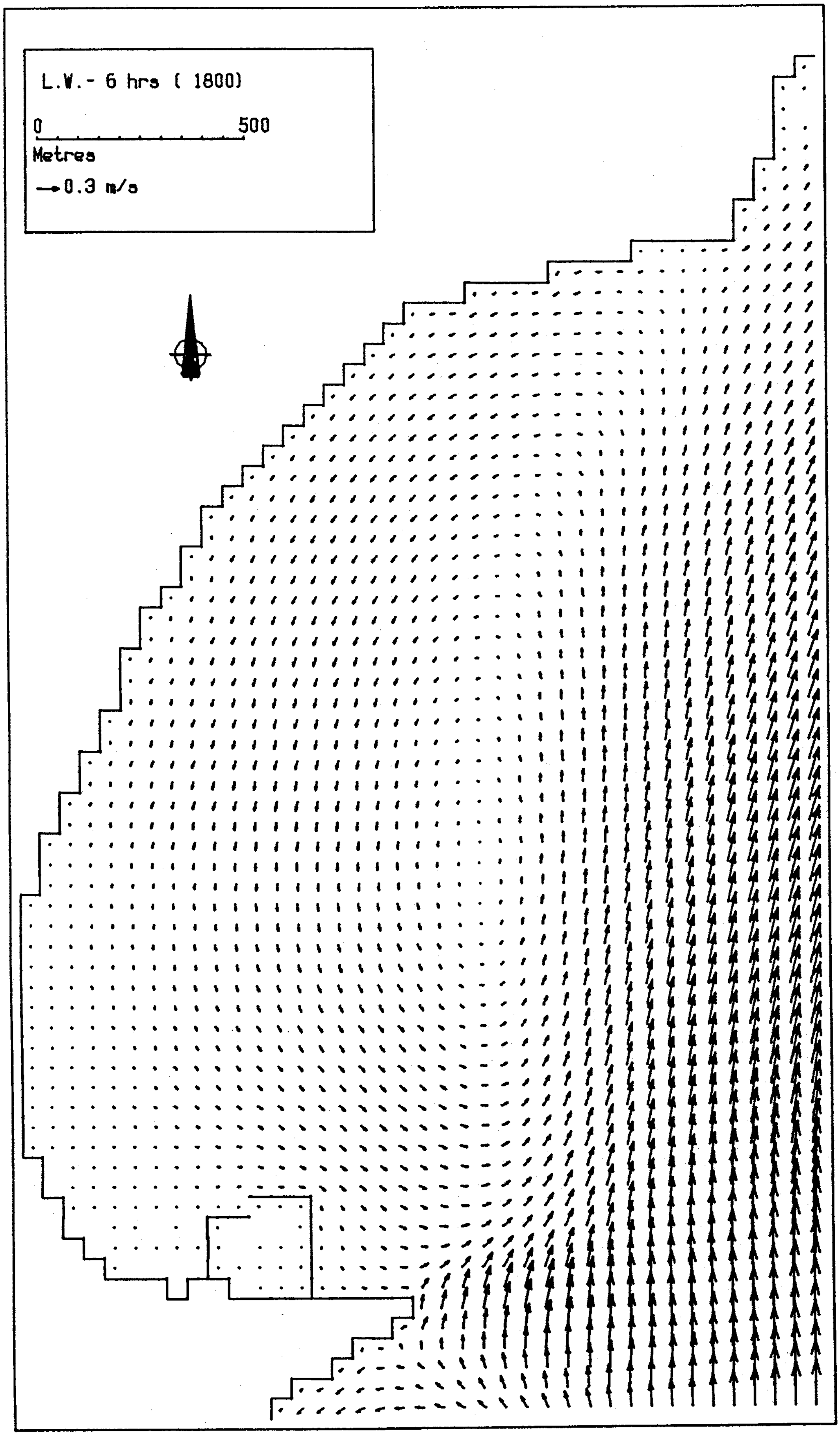


Fig.14 Swanage Bay model - spring tide velocities

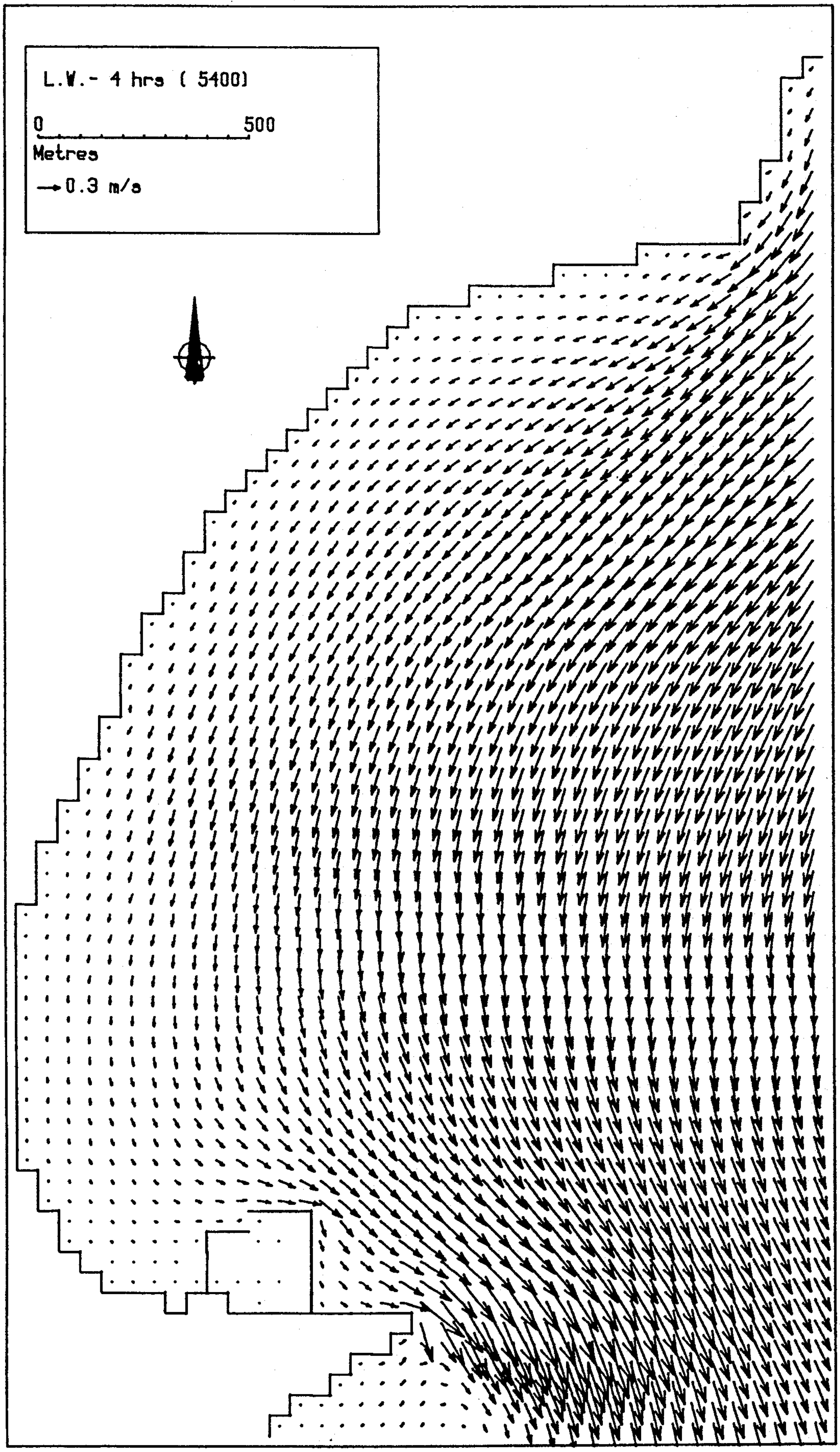


Fig.15 Swanage Bay model - spring tide velocities.

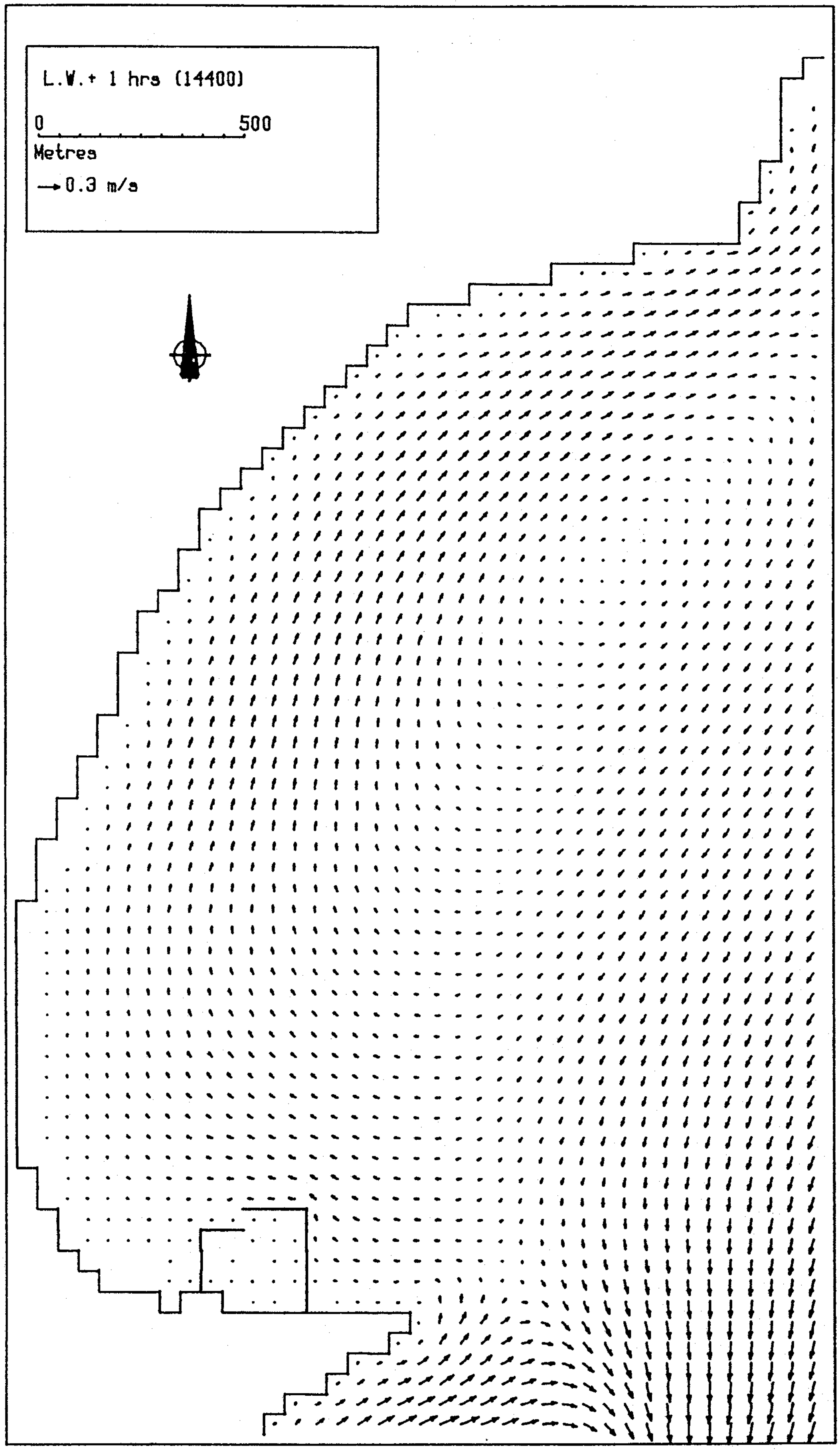


Fig.16 Swanage Bay model - spring tide velocities

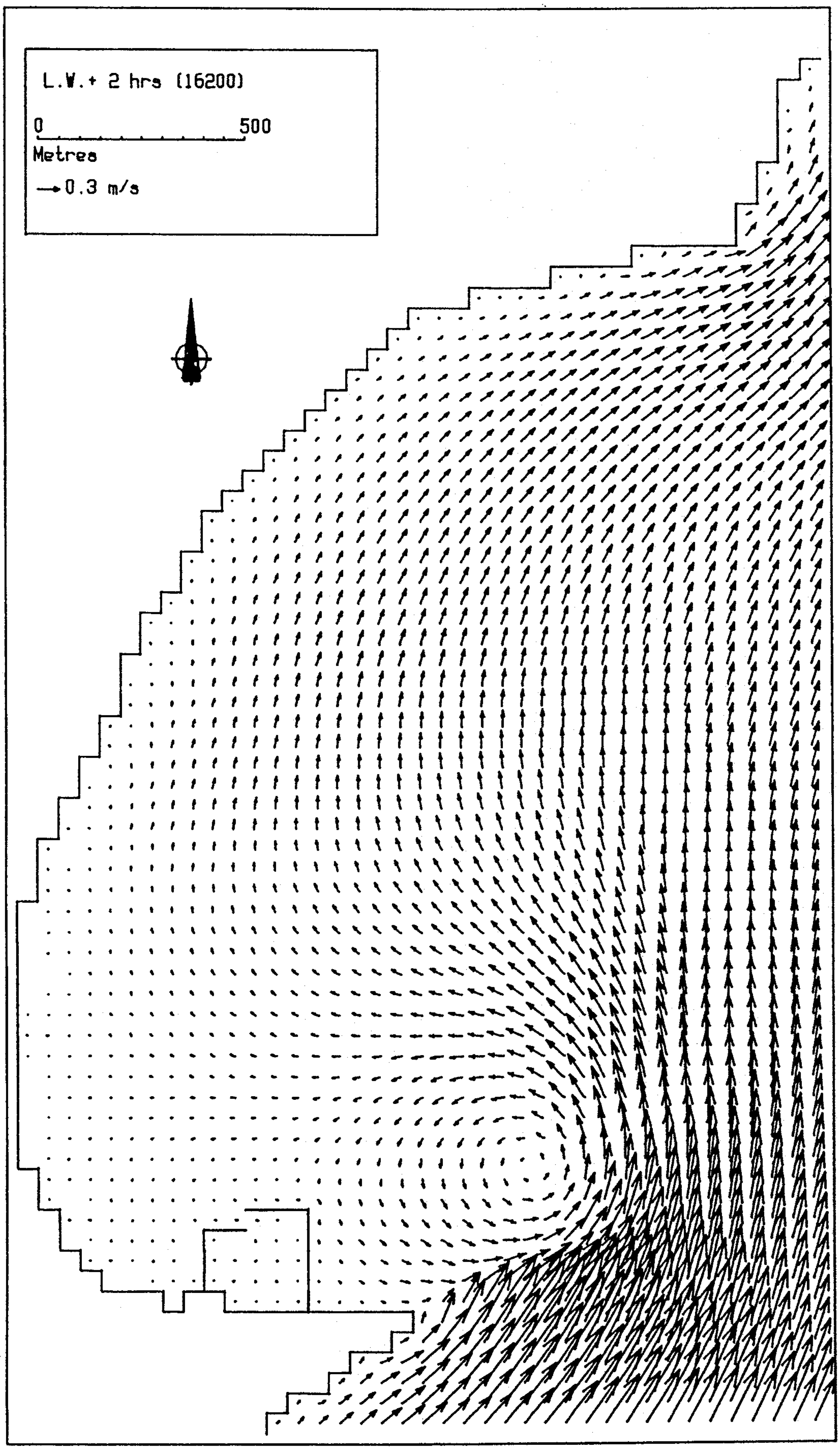


Fig.17 Swanage Bay model - spring tide velocities

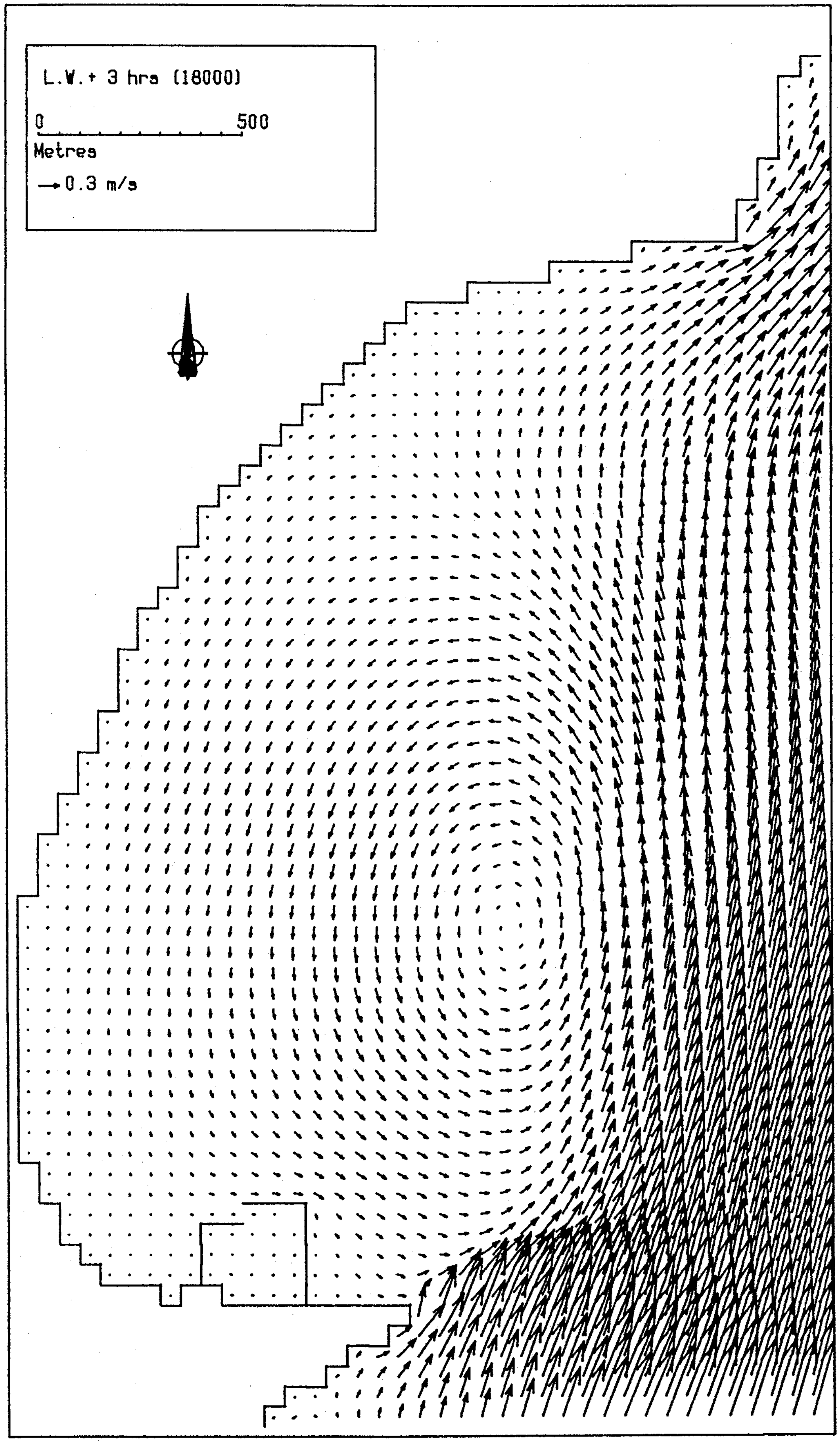


Fig.18 Swanage Bay model - spring tide velocities

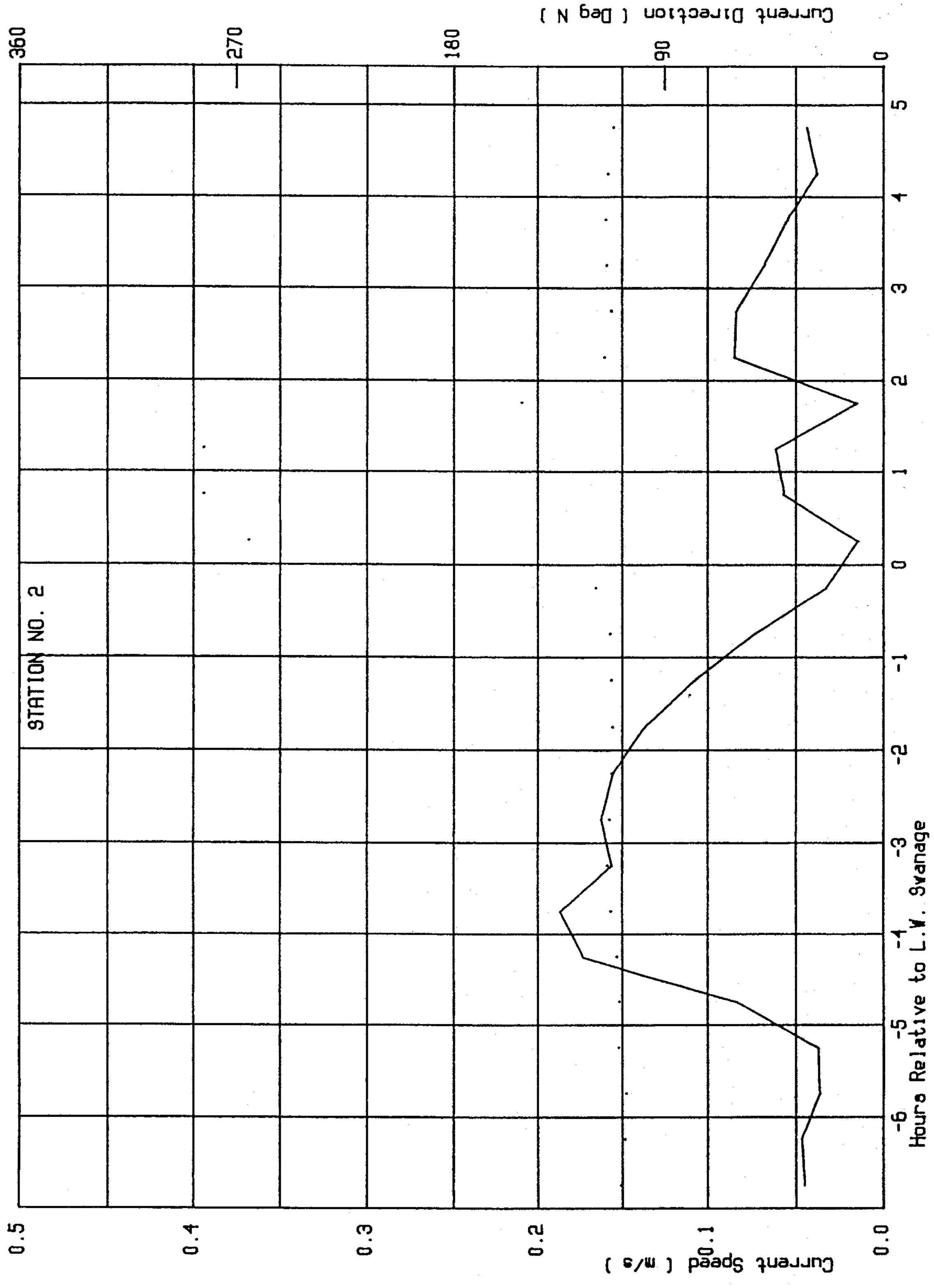


Fig.19 Swanage Bay model - spring tide currents off the pier - Diffusion $10m^2/s$

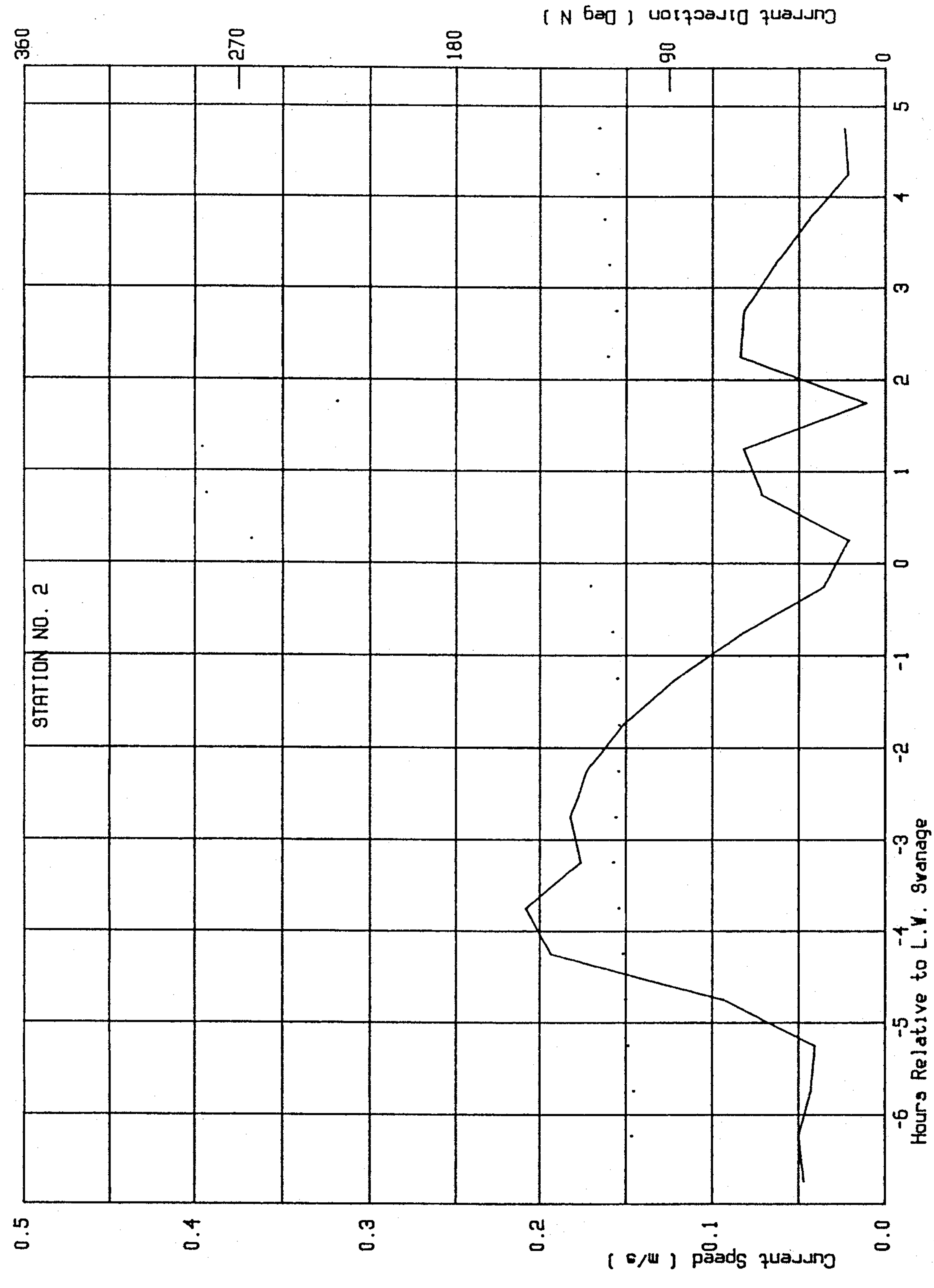


Fig.20 Swanage Bay model - spring tide currents off the pier - Diffusion $5m^2/s$

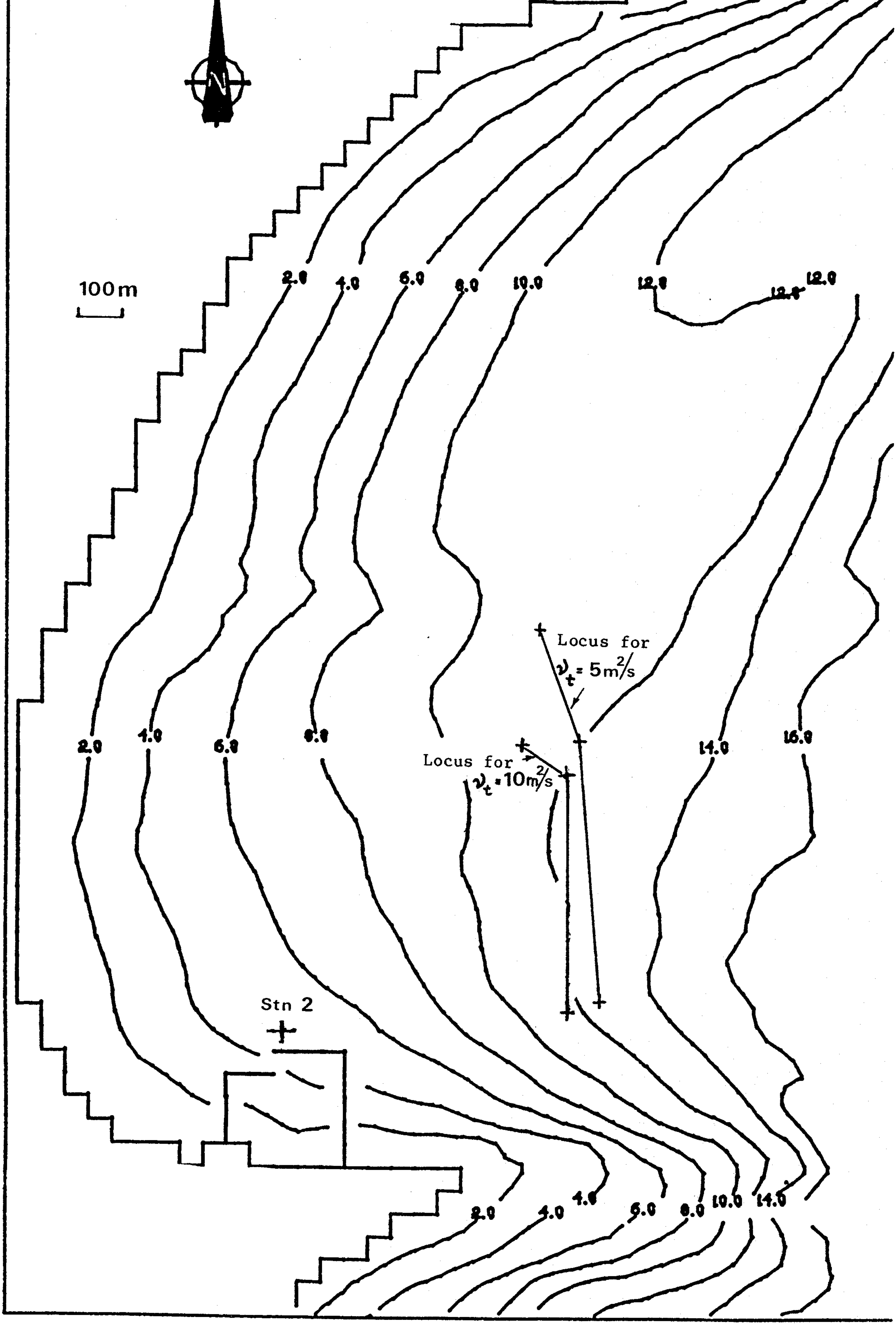


Fig.21 Locus of centre of Recirculation on Flood Tide.

APPENDICES.

APPENDIX I

Refined Turbulence Modelling One Day Meeting

Hydraulics Research
Wallingford

Refined Turbulence Modelling One Day Meeting

Thursday 28 November 1985

Research groups have been applying numerical models to hydraulic problems for the civil engineering industry for many years. Typically, these problems involve modelling the flow, solute and suspended solids transport in large coastal regions, tidal lagoons or estuaries. In the past available computing power restricted the spatial resolution which could be achieved in a solution and necessitated the use of relatively simple turbulence models. With recent increases in computer power together with the development of more sophisticated turbulence models, it should now be possible not only to improve the solution to existing problems but also to increase the scope of civil engineering problems which can be studied.

In the field of mechanical engineering hydraulics and heat transfer problems, refined turbulence models have been used and developed over many years.

At the suggestion of the UK Liaison Committee of the IAHR a one day meeting will be held on Thursday 28th November 1985 in the Fountain Building, Hydraulics Research Limited, Wallingford. The purpose of this meeting is to define the state of the art of turbulence modelling in both engineering fields and, it is hoped, that this will encourage a useful exchange of ideas between the two groups.

In order to promote informal discussion, the number of participants will be restricted. The preliminary programme is given overleaf and, in addition to the three invited speakers, time has been left for discussion and brief descriptions of practical problems currently of interest to any participant.

Programme

09.30	Assembly and Coffee	
10.00	Introduction by Dr T J Weare	Managing Director Hydraulics Research Limited
10.15	Dr A Hauguel	Department Laboratoire National d'Hydraulique Electricite de France
11.15	Prof W Rodi	Institut fur Hydromechanik University of Karlsruhe
12.15	Discussion	
13.00	Lunch	
14.00	Prof D C Leslie	Director of the Turbulence Unit Queen Mary College
15.00	Discussion/Brief presentations from participants	
16.30	Tea and optional tour of laboratory	

APPENDIX II

IAHR Meeting on Refined Modelling of Turbulent Flows in Coastal Waters

Report on the Meeting on Refined Modelling of Turbulent Flows in Coastal Waters

Held on February 25/26 1988
at LNH Chatou - France

OBJECTIVES AND ORGANIZATION

The meeting was co-sponsored by the IAHR Sections on "Maritime Hydraulics" and "Computational Hydraulics", and organized by the Laboratoire National d'Hydraulique of Electricité de France - Chatou.

Twenty-five Representatives of major Universities, Research Laboratories and Consulting firms from seven countries attended the meeting (Denmark, France, Federal Republic of Germany, Italy, The Netherlands, United Kingdom, USA) ; the list of participants is attached.

The aim of the meeting was to initiate discussions between specialists of turbulence modelling in maritime coastal processes, through specific papers presenting the state of the art on the subject, and the problems encountered and to be solved in this field. The meeting focused on vertically integrated models of marine currents, without limitation on the modelled area (small recirculation behind a jetty, large eddies in coastal zones...).

The three following themes were introduced by a general lecture on the subject, followed by shorter communications :

Theme a : "Basic Characteristics of Turbulence in Coastal Areas"

This theme focused on basic physical processes, available measurements, and importance of turbulence for application to numerical modelling of flows in coastal areas.

- Chairman : K.P. HOLZ (Hannover University)

- Introduction by M.B. ABBOTT (IHE Delft)

- Communications :

  . K.P. HOLZ : A Database for Recirculating Flow (from experiment).
  . R. UITTENBOGAARD : Turbulence measurements in a steady stratified mixing layer.
  . C. FLOKSTRA : Reproduction of mixing layer by a depth-averaged model.
  . P. PECHON : The need of turbulence models in practical applications.

Theme b : "Conventional Modelling of Turbulence

This theme focused on the characteristics and limitations of the models currently in use, with on the other hand, encouraging results of refined turbulence models applied to 3D smaller scale problems.

- Chairmain : D. LAURENCE (LNH Chatou)

- Introduction by W. RODI (Karlsruhe University)

- Communications :

  . J.M. USSEGLIO-POLATERA : Ability of conventional 2-D codes in modelling small scale turbulence.
  . M.M. GIBSON : A Reynolds-stress closure study of the boundary layer beneath surface waves.

. D.J. CARRUTHERS : The structure of turbulence and waves near density interfaces.

. J.G. RODGER : Some recent simulations of coastal flows.

. R.A. FALCONER : Modelling Tidal Eddies in the lee of Rattray Island (Australia) and in Rectangular Harbours.

. J.M. HERVOUET : Comparison of different turbulence models.

. R. UITTENBOGAARD : Simulation of steady stratified mixing layer with k-epsilon model(s).

<u>Theme c : "Large Eddy Simulation for Coastal Processes"</u>

This theme discussed the possibilities of application of LES to the modelling of flows in coastal areas, through the experience of the participants in modelling the larger unsteady features of coastal flows with models containing a large number of nodes.

- Chairman : M.B. ABBOTT (IHE Delft)

- Introduction by D. LAURENCE (LNH Chatou)

- Communications :

. M.B. ABBOTT : The Paradox at LES in Two-Dimensional Flows.

. K.P. HOLZ : Automatic Grid Refinement for Reproducing Recirculating Flows.

. K. VERBOOM : Simulation of Recirculations in areas with steep bottom.

. I.R. WARREN : Model Resolution Requirements for LES.

. R.F. HARLEMAN : Filtering techniques applied to shallow water flow.

. B. AUPOIX : Subgrid scale modelling in homogeneous turbulence.

. J.J. LEENDERTSE : Advection term approximation dependency of residual circulation in an estuary.

. W.G. GRAY : Experience with finite element wave equation models.

. J.J. LEENDERTSE : A coupled weather/hydro model for surface water movements.

SYNTHESIS OF THE MEETING

The discussion which ended the meeting was animated by A. JAMI. The chairman of each theme briefly summarized his session.

The main points of the conclusion were the followings :

I - Measurements

Some of the participants exhibited interesting data collected in experimental facilities or in prototype cases ; these data are available and could be provided for collaboration and test comparisons between those laboratories which could be interested.

Some of these data were collected in a flume where circulation flows due to groins were measured. Other investigations with stratified flow were presented. They suggested that a significant part of the energetic fluctuations were due to internal waves rather than turbulence. Circulation flows in a harbour or behind an island were also presented.

II - Numerical simulations

The various presentations can be divided in four different types of modelling : 2D depth-averaged "regional" and "local" flows, 2D vertical simulations and 3D simulations.

A - 2D depth averaged "regional" models

The considered domains are of the order of tens of kilometers. The main feature of the flow is interaction of momentum and pressure with bottom topography.

Some applications pointed out the effect of the bathymetry and of the size of the mesh grid, which can be as important as the turbulence one.

In this case diffusive transport of momentum through vertical planes is negligible. All that is required for a numerical code, is correct conservation of momentum and vorticity together with an adequate representation of bottom friction. As pointed out by Pr. Abbott, the problem can be directly adressed in a discret sense, instead of enforcing discretisation of model expressed in the space of continuous fonctions. As with the cellular automata approach, isotropic properties of the grid are required (e.g. LEENDERTSE type). Indeed, many simulations where able to reproduce observed gyres, whatever the value of the constant viscosity chosen is.

The 2D simulations of currents in large domains compared generally well with measurements. So it appeared that in many cases where the only knowledge of the currents was required, a sophisticated turbulence model was not necessary and a constant diffusion coefficient could be used. On the other hand, this is generally no more the case when problems related to pollution and sedimentology are involved, and where a good reproduction of the current drifts is necessary.

B - 2D depth averaged "local" models

We are now considering the problem at the scale of, say a harbour. The size of the mesh step is of the order of the depth or less. Now diffusion fluxes are to be considered. Conventionnal modelling of turbulence as introduced by Pr. Rodi is required.

In this case results are sensitive to the chosen turbulent viscosity. In some cases results are unchanged if a constant viscosity is used instead of a transport equation or mixing length model. The problem is how to prescribe the value of this constant as far as prediction is concerned.

It was also pointed out that refined turbulence modelling is required if water quality is studied (passive scalar transport).

Tests in confined areas with depth-averaged k-epsilon model showed reasonable agreement with measurements. However some further investigations have to be done to confirm or possibly to improve this model for coastal applications.

C - 2D vertical simulations

Some presentations where related to modelling phenomena in the vertical direction, such as buoyancy effects interaction of turbulence and gravity waves, and the bottom boundary layer. These close up views of the problem can help modelling in the previous depth-averaged context.

D - 3D simulations

In the 3D context turbulence modelling has very much improved, ranging from simple Prandtl mixing length theory to second-order-closure schemes. Conventionnal 3D Large Eddy simulation can be used to test or improve the former model as non-measurable data is exhibited. Performance of second-order closures are still good when buoyancy is involved. For this case a general model can only be found at this level of closure.

As for LES, modelling of the sub-grid stresses is also in progress.

It was pointed out that for 2D L.E.S. one should not directly use 3D sub-grid stress models such a Smagorinsky's since the underlying physics of 2D turbulence are completly different from that of the 3D case.

III - Conclusion

Finally it was proposed to separate the problems according to the different scales of the problem (2D or 3D turbulence) and to plan workshops, each of them being limited to modelling on a specific scale. This proposal will be further discussed inside the concerned IAHR sections.

MEETING ON REFINED MODELLING OF TURBULENT FLOWS IN COASTAL WATERS

List of participants

M.B.	ABBOTT	- Int. Inst. Hyd. & Env. Eng. - Delft - The Netherlands
B.	AUPOIX	- ONERA/CERT - Toulouse - France
E.P.	CARATELLI	- Inst. Univ. Navale - Napoli - Italy
D.J.	CARRUTHERS	- University of Cambridge - UK
Y.	COEFFE	- Laboratoire National d'Hydraulique - Chatou - France
R.A.	FALCONER	- University of Bradford - Bradford - UK
C.	FLOKSTRA	- Delft Hydraulics - Delft - The Netherlands
M.M.	GIBSON	- Imperial College - London - UK
W.G.	GRAY	- University of Notre Dame - Notre Dame - USA
D.R.F.	HARLEMAN	- Ralph M.Parsons Laboratory - Cambridge - USA
J.M.	HERVOUET	- Laboratoire National d'Hydraulique - Chatou - France
K.P.	HOLZ	- Universitat Hannover - Germany
A.	JAMI	- Laboratoire National d'Hydraulique - Chatou - France
B.	LATTEUX	- Laboratoire National d'Hydraulique - Chatou - France
D.	LAURENCE	- Laboratoire National d'Hydraulique - Chatou - France
J.J.	LEENDERTSEE	- The RAND Corp. - Santa Monica - USA
B.	MANOHA	- Laboratoire National d'Hydraulique - Chatou - France
P.	PECHON	- Laboratoire National d'Hydraulique - Chatou - France
J.D.	RODGER	- Hydraulics Research - Wallingford - UK
W.	RODI	- University of Karslruhe - Karlsruhe - RFA
J.C.	SALOMON	- IFREMER - Brest - France
U.	SVENSSON	- SMHI - Suède
R.	UITTENBOGAARD	- Delft Hydraulics - Delft - The Netherlands
J.M.	USSEGLIO	- CEFRHYG - Echirolles - France
G.K.	VERBOOM	- Delft Hydraulics - Delft - The Netherlands