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THE MATHEMATICAL FORMULATION OF
NON-LINEAR WAVE FORCES ON SHIPS

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The mathematical formulation of non-linear wave forces on ships

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ABSTRACT

The importance of non-linear wave forces in causing ranging of vessels on their moorings is explained. Various mathematical representations of these forces have been investigated with the help of published data. Due to their complexity the amount of computational effort needed to calculate the forces exactly is very great and approximate methods have been sought.

It has been found that for one type of force which can be expressed as a product of first order wave pressures and vessel movements a well known approximation, suggested by Newman and used in offshore applications, will lead to large underestimates in the force. This is due to spatial gradient effects, ignored in the Newman approximation, becoming more important for the coastal applications of interest here where resonant periods of oscillation of moored vessels are generally shorter than the resonant periods of structures moored offshore. Therefore, more exact expressions for this type of force have been formulated.

A second type of non-linear wave force requires solution of the diffraction of random waves, by the vessel, to second order in the wave amplitude. This problem also requires a large computational effort to obtain an exact solution and three different approximations have been investigated. By applying the approximate treatments to a relatively simple situation studied experimentally with regular wave groups, where second order diffraction was shown to be a controlling factor in the resultant non-linear wave force, it has been possible to identify the best of the three approximations.

As a result of this work, mathematical equations have been obtained which can be expected to provide a good description of non-linear wave forces and moments on ships. However, these expressions will require more programming than originally planned due to the need to represent exactly non-linear forces of the first type. The further work needed to incorporate non-linear forces and moments into the computer models UNDERKEEL and SHIPMOOR has been described. This work is needed to provide a more comprehensive check on the suggested approximate treatment of the second order diffraction problem and on time domain representations of non-linear forces and moments. Such work will lead ultimately to a final validation of the computer model of a moored ship against full scale data.

CONTENTS

1 INTRODUCTION

Engineering feasibility studies for the development of new ports and the extension of existing ones to accommodate the large vessels of today are increasingly taking on a common form. Initially, computer models are required to investigate a wide range of parameters. Having established favoured schemes, a random wave physical model is then used for final optimisation.

A suite of computer models is presently under development at Hydraulics Research (HR) to satisfy the requirement for a realistic first estimate of harbour and ship response to wave action. The work described in this report covers the formulation of a suitable set of equations for describing non-linear wave forces on ships. Once programmed these equations will extend the capabilities of UNDERKEEL (Ref 1) a computer model of the linear response of a vessel to waves, to enable important non-linear wave forces to be represented. The implementation of these equations on a computer will be described in a subsequent report. In this report various methods of representing non-linear wave forces are evaluated with the aid of published data in the literature and through comparisons between theoretical and experimental results. This leads to a recommended set of equations for describing non-linear wave forces on ships.

Although the forces under discussion are smaller than the (linear) forces at the wave period they usually cause larger movements of moored vessels. This process is illustrated schematically in Figure 1. A typical wave spectrum, in this case with a peak in spectral density S(f) at a frequency (f) of 0.1 Hertz or a period of 10 seconds, will produce a force with a spectrum $S_F(f)$ like that shown in Figure 1. There is a non-linear wave force which occurs at wave group periods, ie at low frequencies, in addition to the main linear force at ordinary wave periods. Once moored, vessels of 5,000 tonnes displacement and above have resonant periods of oscillation on their moorings of 20 seconds to several minutes depending on the size of the vessel and the compliance of the moorings. This is illustrated in Figure 1 by the sharply peaked frequency response function at a (resonant) period longer than the wave period ie at a low frequency. The reason for this shape of the resonant response is that being long period, the hydrodynamic damping is low and this tends to give a very narrow response function centered on the resonant frequency. Multiplying the frequency response function by the force spectrum will give rise to a response spectrum like that in Figure 1. It can be seen that although the low frequency force is smaller than the wave frequency force, it produces a larger response and the loads in the moorings can be many times the non-linear wave force. This means moored vessels will tend to range (move horizontally) on their moorings at wave group periods (low frequencies) with the response at wave periods occurring mainly in the vertical motions of heave, pitch and roll. It can be seen, therefore, that the accurate representation of non-linear wave forces is vital if realistic estimates of moored ship movements are to be obtained and any subsequent berth "downtime" defined.

The approach described above can be developed further to enable a time history of the non-linear wave forces to be defined and this is the ultimate aim of the work. Then non-linear wave forces can be represented in the computer model SHIPMOOR (Refs 2,3) along with the linear forces at wave periods already represented. Linear forces are calculated using the present version of UNDERKEEL to define the relevant hydrodynamic coefficients for a given ship. These coefficients allow the time histories of the linear forces to be constructed and then used in SHIPMOOR which integrates the ship's equations of motion in the time domain. It is necessary to solve these equations in the time domain to allow for non-linearities present in conventional mooring systems. The most significant non-linearity is caused by fenders being stiffer than mooring lines which in turn leads to subharmonic sway motions of a vessel on and off the fenders at a period which is a multiple of the wave period (Refs 2,3) an effect first described by Lean (Ref 4). Such subharmonic motions could never be represented using just a frequency domain computer model like UNDERKEEL. Once the combined UNDERKEEL/SHIPMOOR model is fully developed, with the capability of representing both linear and non-linear wave forces, it will provide a realistic description of the response of moored ships to waves with mainly wave period vertical motions and long period horizontal motions on the moorings due to a mixture of subharmonic response (caused by non-linearities in the moorings) and wave grouping response (caused by non-linear wave forces). Such complex moored ship behaviour has been observed in many random wave physical models of harbours carried out at HR over the years and it is only by including the basic physics within the mathematical formulation that realistic estimates of berth tenability can be obtained from computer models.

It is of interest to compare a "dynamic" description of a moored vessel in waves, like that outlined above, with the approach often used in the offshore industry for the design of moorings. A "deterministic" approach has been much used which involves defining a design wave (typically a maximum wave occurring during an extreme storm event) to estimate the maximum mooring load just at the wave period and adding this to the mean loads produced by steady wind, current and wave forces. In taking this approach all long period dynamic effects due to subharmonic responses and wave group responses are ignored and the probabilistic nature of real responses is side-stepped. Hence the description "deterministic" or, as the approach is sometimes called, "quasi-static". A comparison was made, for a barge shaped vessel moored in deep water, of the mooring loads calculated using a deterministic, or quasi-static method, and loads calculated using a fully dynamic model (Ref 5). In the case of the dynamic model a source method (Ref 6) was used to calculate the response at wave periods and the Newman approximation (Ref 7) was used to represent the non-linear wave forces. It was found that due to long period responses the maximum mooring loads resulting from the dynamic model were 25 to 35% higher than those predicted by the quasi-static method. It is also interesting to note that the parameters chosen in Reference 5 for this comparison, a relatively short period sea of 8s zero crossing period and very long resonant periods of some 250 seconds, are the most favourable for the Newman approximation. It will be demonstrated in the next Section of this report that for seas with longer periods, and for shorter resonant periods of moored vessels (which are more appropriate for coastal applications) the Newman approximation can result in large underestimates of the slowly varying force. This indicates that even for long resonant periods the quasi-static approach will lead to much larger underestimates than 25% in maximum mooring loads for extreme seas, where wave periods are longer. The fact that mooring failures have not been widespread in the offshore industry may well be due to the large safety factors applied in mooring designs. With the drop in the price of oil, however, there is now pressure to trim most costs including that of the moorings and so there is a clear requirement for more realistic models which take full account of the long period dynamic behaviour. In addition to realistic physical modelling more accurate methods of calculating non-linear wave forces are available (Refs 8,9) and these will be considered further in subsequent sections of this report.

Perhaps it is worth making the point that the necessary funding to enable large safety factors to be applied in mooring design for port applications has never been available. This means accurate modelling techniques are also needed in harbour design both to describe wave action in harbours and the resulting movements and mooring loads for the vessels using harbours. The "dynamic" modelling technique outlined above, in which both non-linear moorings and non-linear wave forces are taken into account, will provide realistic estimates of berth tenability provided the total spectrum of wave energy, including the disturbance at wave group periods, is defined at berths within the harbour. A computer model capable of providing a description of both the ordinary waves and wave group disturbances is under development in a separate research contract entitled "Further development of the Boussinesq model of waves in harbours". Used together these realistic computer models of vessel and harbour response will form a powerful design tool in feasibility studies.

2 APPROXIMATE METHODS

To understand approximate methods of calculating non-linear wave forces it is necessary to appreciate the mechanisms causing these forces. The four main components for a vessel in regular waves (single period waves of uniform height) were first identified by Pinkster (Ref 8). They are illustrated schematically in Figure 2. Once these effects are understood it will be easier to generalise to the case of irregular or random waves. As drag forces on ships are negligible in most circumstances relative to inertial forces, they are ignored in what follows.

The surface elevation term in Figure 2 is the integral of the wave pressure over the area between the surface elevation and the displaced equilibrium water line on the vessel. Because the integral is taken over an area that is of first order in the wave amplitude, it leads to a non-lienar term that is related to the square of wave amplitude.

The second term is the integral, over the mean submerged area, of the quadratic velocity term in the equation for the water pressure (a Bernoulli effect). Being quadratic this term is proportional to the square of wave amplitude.

To understand the third term it is necessary to appreciate that the pressure acting on the submerged surface of the vessel, after movement by the waves, gives rise to a second order force. This effect is described by (Taylor) expanding pressure on the displaced body surface about the equilibrium position of the vessel keeping terms to second order and then integrating over the mean submerged area of the vessel.

The fourth term is caused by rotation of the vessel ie roll, pitch and yaw. Wave forces on the vessel act normal to the body surface and as the body rotates in the waves, resolutes of the first order wave force will develop in horizontal and vertical directions. These resolved forces will be second order in the sense that they are given by the product of first order wave forces, including hydrostatic restoring forces, with first order angular rotations of the vessel.

In regular waves all of the above effects lead to a steady force, sometimes called a steady drift force, only if the vessel scatters the waves. This occurs because there is a steady flux of momentum in a regular wave which is proportional to the square of wave amplitude. Scattering of the waves by the vessel will induce a steady second order force due to the change produced in the momentum flux. If scattering does not occur, ie the vessel moves with the waves, then the momentum flux carried by the waves remains intact and no steady drift force develops (Ref 10).

We can now generalise these results to the case of more realistic wave motion. The simplest representation of irregular waves consists of the superposition of two waves with frequencies f_1 and f_2, say. These waves produce a beating effect as they pass into and out of phase with one another to produce regular wave groups. The resultant wave amplitude or envelope fluctuates with a period given by the inverse of the difference frequency $|f_2-f_1|$ (Fig 3). It can be seen that the momentum flux carried by such waves, which is proportional to the square of wave amplitude, will also fluctuate at the difference frequency and scattering of the waves by the vessel will then produce both a mean and a slowly varying force at the difference frequency via the mechanisms outlined above. This is illustrated schematically in Figure 3 where the resultant force is assumed to lag the wave envelope with a phase difference of α_{12}.

In the limit of no wave scattering we have seen that the mean drift force tends to zero but this does not happen to the slowly varying component of the drift force. There will be a spatial gradient in the second order pressure effects acting along the surface of the vessel, in the direction of the waves, whether or not the waves themselves are scattered by the vessel. For example, such a gradient exists in the incident waves because the wave envelope has a wavelength given by $2\pi/|k_2-k_1|$ where the individual wavelengths are $2\pi/k_1$ and $2\pi/k_2$. This means all of the main mechanisms leading to the steady drift force in regular waves, as illustrated in Figure 2, produce a slowly varying force in irregular waves for two reasons. One is associated with scattering of the waves by the vessel and the second is because of the existence of spatial gradients in the slowly varying drift force.

The spatial gradient effect is the (second order) counterpart of the (first order) Froude-Kryloff wave force which is defined by integrating the pressure in the incident wave field over the submerged surface of the vessel. For a vessel in a head sea, wave scattering effects are small and the Froude-Kryloff force forms a good approximation to the total wave force (Ref 1). In similar fashion, the spatial gradient effects in the slowly varying drift force will form a good approximation to the total drift force when there is little wave scattering. This approximation was used by Bowers (Ref 11) to explain long period ranging of container ships in a physical model of a proposed seaport (Ref 12). When moored in a proposed berth near the seaport entrance a large container vessel, some 280m in length at full scale, was found to surge on its moorings with a periodicity of a minute or so, and with movements approaching the wave height, in random wave head seas with periods of only some 5 seconds. In explaining this behaviour it was demonstrated in Reference 11 that there is an additional mechanism leading to a slowly varying drift force. This is a long period disturbance which travels with groups of waves and acts like a long wave in producing a force on a vessel. This disturbance is called set-down beneath wave groups because Bernoulli pressures in groups of large waves cause a reduction in water pressure which leads to a depression in the mean water level beneath the large waves (Ref 13). There is a compensating rise in mean level between groups of large waves. This surface effect induces a long period wave-like flow beneath the surface which acts on the submerged part of a ship to produce a slowly varying drift force. Set-down itself is produced by spatial gradients in the incident waves and so the force due to set-down is allied to the gradient effects already described in the second order drift force.

The four mechanisms in Figure 2 for producing both steady and slowly varying drift forces, together with the slowly varying force due to set-down, help to explain the non-linear wave forces that excite the long period resonances of moored ships.

2.1 Newman's approximation

In the light of the discussion of the mechanisms causing long period non-linear wave forces it can be seen that pairs of wave components with frequencies f_m, f_n will cause a number of somewhat complex forces at the difference frequency $|f_m - f_n|$. In an irregular or random sea there will be a large number of wave components and an even larger number of possible pairs of components: for N wave components there will be N^2 possible pairs. This shows that even with interpolation of a more limited number of non-linear components, representation of all the non-linear wave effects is a large task.

The approximation suggested by Newman (Ref 7) is to use simply the steady part of the non-linear force, as given by m = n, to represent the total force. This involves calculating effectively N non-linear wave force components instead of N^2 components and, in addition, using a far field calculation for obtaining the horizontal components of the steady non-linear wave force. This far field calculation is considerably easier to carry out than the integrations of second order effects, over the vessel's surface, that are required to obtain the complete non-linear wave force. The basis of the Newman approximation is that the resonant periods (T_R) of interest for horizontal motions of ships on their moorings are long enough for the coefficients of the wave forces at the relevant difference frequencies to be approximated by the steady force coefficients,

ie $\frac{1}{T_R} = \left| f_m - f_n \right| \cong 0.$

This in turn means that only those non-linear forces arising from wave scattering by the vessel are represented. In particular the long period spatial gradient effects present in the absence of scattering, which include the force due to set-down, are ignored.

The Newman approximation has been much used in offshore applications as it enables a "dynamic" model to be developed for long period resonances of structures on their moorings without having to carry out complex calculations. The resonant periods of interest are sometimes very long. For example, 'fishtailing' of a tanker on a single point mooring typically occurs at periods of 10 minutes or more although tethered buoyant platforms can have resonances in surge, sway and yaw at one to two minutes in water depths of 200m to 400m. Nevertheless, it is clear that in the limit of a long enough resonant period, the spatial gradient effects present in the absence of scattering will become small because the relevant difference frequencies are small and spatial gradients associated with those difference frequencies will also tend to be small. Set-down is also small in deep water which tends to reduce its contribution to the long period non-linear wave force. For these reasons it has been claimed by a number of authors that the Newman approximation can be expected to lead to a reasonable representation of non-linear wave forces for offshore applications. This point has been made in Reference 14 where comparisons are presented between Newman's approximation and other more exact calculations of the long period non-linear wave force on cylinders of various cross section in beam seas in deep water. But, yet again the example given is for a short period sea of zero crossing period 5.5 seconds. This will favour the Newman approximation because wave scattering is strong for short period seas. Extreme seas typically have zero crossing periods longer than 10 seconds and even though scattering of waves is much reduced the resulting forces are higher because the waves themselves are larger. There appears to be less information in the literature on the accuracy of the Newman approximation for extreme sea states although in one application to a semisubmersible platform it was found that the approximation seriously underestimated the responses even with a resonant period of about 3 minutes (Ref 15).

For the coastal applications of interest here the reduced water depth has two main consequences. One is that resonant periods of interest are generally shorter because the length of mooring lines is that much less and this results in stiffer characteristics. The second consequence of shallower water is that set-down is amplified. Indeed, where the primary waves themselves are deemed shallow water waves, the non-linear wave force due to set-down becomes the largest part of the spatial gradient force present in the absence of scattering. Both these consequences of a reduced water depth will tend to reduce the accuracy of the Newman approximation. It is necessary, therefore, to study the accuracy of the approximation in the light of the requirements for coastal applications.

The force due to set-down can dominate the spatial gradient effects in coastal applications but the Newman approximation, taken on its own, ignores all such effects. It will be necessary, therefore, to allow for a separate set-down force and use the Newman approximation to represent all the other forces if more lengthy calculations are to be avoided. To judge the accuracy of this process we use data in the literature in Section 2.2 to check the ability of the Newman approximation to represent all the non-linear wave forces apart from that due to set-down. The accuracy of approximate methods of representing the set-down force, or as it is sometimes called the force associated with the second order potential, will be considered subsequently (see Section 4).

2.2 Check on accuracy

Here we use data presented by Standing (Ref 16) for the case of a vessel moored in quartering seas (Fig 4). In this work long period non-linear wave forces were calculated by two methods. One involved using the Newman approximation and the other was a more exact calculation of the required forces. Responses of the moored vessel were then estimated for comparison with experimental data. The model used for the experimental work had the following dimensions.

Length	4.7m
Beam	0.76m
Draught	0.29
Water depth	7.62m

This model represented a drill-ship just under 100m long at a scale of about 1 to 20. However, the calculations of non-linear wave forces were only presented for wave periods of 12 seconds or less at full scale which is not representative of the periods of extreme sea states. In order to use the available data to judge the accuracy of the Newman approximation over a greater range of wave periods we can assume a scale of 1 to 50 instead of 1 to 20. Then, the longest wave period considered becomes 19 seconds at full scale and the vessel dimensions become:

Length	235m
Beam	38m
Draught	14.5m

These dimensions are representative of a 85,000 tonne vessel. For smaller ships there will be less scattering of the waves, making the Newman approximation less accurate. The opposite will hold for larger vessels.

In a random sea the incident wave elevation can be defined as the sum of N wave components:-

$$\eta = \sum_{m=1}^{N} a_m \cos(\omega_m t + \epsilon_m), \tag{1}$$

where the amplitudes of the wave components a_m are given by the wave spectrum S(f)

$$a_m^2 = 2\ S(f_m)df,$$

the radian wave frequency is defined by

$$\omega_m = 2\pi f_m,$$

and ϵ_m is a random phase. The general expression for the long period non-linear wave force is then given by:-

$$F(t) = \sum_{m=1}^{N} \sum_{n=1}^{N} a_m a_n \{P(\omega_m,\omega_n)\cos[(\omega_m-\omega_n)t + \epsilon_m-\epsilon_n]$$

$$+ Q(\omega_m,\omega_n)\sin[(\omega_m-\omega_n)t + \epsilon_m-\epsilon_n]\} \qquad (2)$$

The approximation proposed by Newman is the following:-

$$P(\omega_m,\omega_n) \cong P(\frac{\omega_m+\omega_n}{2}, \frac{\omega_m+\omega_n}{2}),$$

$$Q(\omega_m,\omega_n) \cong 0$$

The steady component of F(t) is defined by the sum of terms in (2) of the form $a_m^2\, P(\omega_m,\omega_m)$ and so we see the Newman approximation involves using the steady force coefficient $P(\frac{\omega_m+\omega_n}{2}, \frac{\omega_m+\omega_n}{2})$ as the coefficient of the long period force at the difference frequency $(\omega_m-\omega_n)$.

To calculate non-linear wave forces it is necessary to solve first for the response of the vessel at ordinary wave periods. This is achieved by Standing through the use of oscillating sources placed on surface elements that cover the hull. The source strengths are chosen to satisfy the boundary condition on flow normal to hull surface (Ref 17).

Having obtained these source strengths it is then possible to calculate directly the mean components of the surge and sway non-linear wave forces, as well as the mean component of the non-linear yaw moment, through the use of expressions derived by Faltinsen and Michelsen (Ref 6). These expressions are based on the change in wave momentum caused by scattering of the waves by the vessel and they make use of relationships obtained originally by Newman (Ref 18). The expressions are relatively easy to evaluate because they can use a "far field" assumption. This means the mean components of the horizontal non-linear wave force can be obtained without too much calculation. Through this approach Standing was able to calculate the coefficients of the mean forces used to represent the long period non-linear forces in the approximation suggested by Newman (Ref 7). This was done for the vessel moored in quartering seas.

In addition to estimating the long period non-linear wave forces via the Newman approximation, Standing also carried out more exact "near field" calculations of the forces. This involved integrating second order expressions, for the effects described in Figure 2, over the hull of the vessel after first using the source method to define the vessel's responses at ordinary wave periods. After these lengthy calculations it was possible to define a matrix of coefficients for the long period components of both the surge and sway forces. These matrix elements are given by:-

$$F_{mn} = \sqrt{P^2_{mn} + Q^2_{mn}}$$

where P_{mn} and Q_{mn} are the coefficients defined in Equation (2), ie

$$P_{mn} = P(\omega_m, \omega_n),$$
$$Q_{mn} = Q(\omega_m, \omega_n).$$

The matrix F_{mn} is symmetric and calculations were carried out for the vessel moored in quartering seas using 10 basic wave frequencies. This meant that coefficients for 55 pairs of frequencies were required including the 10 values on the diagonal defined by $m = n$. The results appear in Tables 1 and 2 for the long period surge and sway forces, respectively. It was possible to compare the more exact "near field" calculations of the diagonal coefficients, which apply to the steady non-linear wave forces, with the simpler "far field" calculations of those same coefficients. Good agreement was found and this formed a check on both sets of calculations.

Having obtained exact expressions for the long period forces on the moored vessel we are in a position to check the accuracy of the more approximate method proposed by Newman. This can be done for a range of wave conditions and a range of resonant periods (for the vessel on its mooring) by the following method. We choose the period of the peak of the spectrum for the wave condition to equal $\frac{1}{f_n}$ and choose the resonant period (T_R) to equal $\frac{1}{f_m - f_n}$. By comparing the exact coefficient F_{mn} with the coefficient $|\bar{F}|$ on the diagonal at frequencies $\frac{f_m + f_n}{2}$, $\frac{f_m + f_n}{2}$ we obtain the following from Tables 1 and 2.

Periods (s)		Surge force (KN/m^2)			Sway force (KN/m^2)		
$T_p(f_n)$	$T_R(f_m)$	F_{mn}	$\|\bar{F}\|$	% error	F_{mn}	$\|\bar{F}\|$	% error
19.2(.052)	23(.096)	106	12.4	-88	160	22.8	-86
	56(.070)	35	2.0	-94	40	5.0	-86
	111(.061)	12.5	1.8	-86	15	4.1	-73
14.3(.070)	20(.120)	69.5	63.3	- 9	175	109	-38
	56(.088)	51.5	18	-65	75	20	-73
	111(.079)	18.5	13.6	-27	25	22	-12
8.9(.112)	59(.129)	80.5	128.1	+59	580	725	+25
	125(.120)	137.5	144.3	+ 5	635	650	+ 2

It is clear from the above results that, overall, the Newman approximation leads to large errors in the estimation of that part of the long period non-linear wave force that excludes the force due to set-down. The best result is obtained for a wave condition with a spectral peak at 8.9s and with a resonant period of the vessel on its mooring of 125s. This is consistent with the fact that the Newman approximation requires both strong scattering of the primary waves (short T_p) and small gradient effects in the absence of scattering (long T_R). For the particular ship considered here the requirements appear to be $T_p < 9$s and $T_R > 2$ minutes. Such limitations are unacceptable for coastal applications where spectral peak periods often exceed 9s and resonant periods of moored vessels typically range from about 20s for a ferry up to some 2 minutes for a large tanker.

The approximation is seen to lead to particularly large underestimates of the long period non-linear wave force for a long peak period of about 19s. In this case the amount of wave scattering is small making the (diagonal) steady force coefficients small while the (off-diagonal) spatial gradient effects occurring in the absence of scattering are very much larger over the whole range of resonant period of interest ie 20s to 2 minutes.

On this basis it is clear that a more exact if more lengthy method of calculating long period non-linear wave forces has to be considered. The method employed by Standing is based on the use of oscillating sources on the submerged surface of the vessel. However, this technique is not satisfactory for coastal applications where the underkeel clearance can be small and, for this a more direct method has been developed and programmed in the form of a computer model called UNDERKEEL for vessel responses at primary wave periods (Ref 1). UNDERKEEL can be used as the basic model upon which one can build a description of long period non-linear wave forces. To achieve this it is necessary to derive general expressions for these forces in terms of the response of a vessel at ordinary wave periods. Such general expressions have been derived by Standing (Ref 9) but they differ in some respects from expressions derived earlier by Pinkster (Ref 8). These aspects are considered further in Sections 3 and 4 of this report.

3 MORE EXACT EXPRESSIONS

The notation used to describe vessel motion is consistent with that already given in References 1,2 and 3. Referring to Figure 5 for a vessel displaced from its mean position, it is assumed a fixed right handed coordinate system GXYZ, with axis GZ vertical, lies with its origin at the equilibrium position of the ship's centre of gravity. After undergoing a surge S_1 along GX, a sway S_2 parallel to GY and a heave S_3 parallel to GZ the centre of gravity moves to G' and a new system of coordinates G'X'Y'Z' can be defined with its (moving) origin at G' but with axes parallel to the fixed GXYZ system. Forces and moments acting on the vessel will be evaluated relative to the G'X'Y'Z' system of axes.

Angular rotations of the vessel are then assumed to occur about the following axes. A yaw S_6 about axis G'Z' to give G'x'y'Z'. A pitch S_5 about axis G'y' to give G'xy'z'. And finally a roll S_4 about G'x to give G'xyz. The axes G'xyz can be considered to be fixed in the moving vessel.

For a vector $\underline{x}$ with coordinates (x,y,z) in the system of axes G'xyz moving with the vessel we find the coordinates (X,Y,Z) of the same point relative to the fixed axes GXYZ are defined by,

$$\underline{X} = \underline{X}_G + \underline{R}.\underline{x} \tag{3}$$

where $\underline{X} = (X,Y,Z)$,

$\underline{X}_G = (S_1, S_2, S_3)$,

$$\underline{R} = \begin{bmatrix} \cos S_5 \cos S_6 & \sin S_4 \sin S_5 \cos S_6 - \cos S_4 \sin S_6 & \cos S_4 \sin S_5 \cos S_6 - \sin S_4 \sin S_6 \\ \cos S_5 \sin S_6 & \sin S_4 \sin S_5 \sin S_6 + \cos S_4 \cos S_6 & \cos S_4 \sin S_5 \sin S_6 - \sin S_4 \cos S_6 \\ -\sin S_5 & \sin S_4 \cos S_5 & \cos S_4 \cos S_5 \end{bmatrix}$$

The matrix $\underline{R}$ describes rotations of the vessel and it will be used in what follows to describe perturbations in the direction of the outward pointing normal $\underline{n}$ to the surface of the hull caused by vessel movement. Thus, up to the second order in the wave amplitude:

$$\underline{n} = \underline{n}_o + \underline{n}^{(1)} + \underline{n}^{(2)} + \ldots..$$

where,

$$\underline{n}^{(1)} = \underline{R}^{(1)}.\underline{n}_o\ , \tag{4}$$

$$\underline{n}^{(2)} = \underline{R}^{(2)}.\underline{n}_o\ , \tag{5}$$

$$\text{and, } \underline{R}^{(1)} = \begin{bmatrix} 0 & -S_6^{(1)} & S_5^{(1)} \\ S_6^{(1)} & 0 & -S_4^{(1)} \\ -S_5^{(1)} & S_4^{(1)} & 0 \end{bmatrix} \quad (6)$$

$$\underline{R}^{(2)} = \begin{bmatrix} -\tfrac{1}{2}(S_5^{(1)2}+S_6^{(1)2}) & S_4^{(1)}S_5^{(1)}-S_6^{(2)} & S_5^{(2)}+S_4^{(1)}S_6^{(1)} \\ S_6^{(2)} & -\tfrac{1}{2}(S_4^{(1)2}+S_6^{(1)2}) & S_5^{(1)}S_6^{(1)}-S_4^{(2)} \\ -S_5^{(2)} & S_4^{(2)} & -\tfrac{1}{2}(S_4^{(1)2}+S_5^{(1)2}) \end{bmatrix} \quad (7)$$

Forces and moments acting on the vessel up to second order in the wave amplitude will be considered in the following sub-sections.

3.1 Non-linear wave forces

We assume in what follows that the force $\underline{F}$, pressure P and the surface elevation η can all be expanded in powers of the wave amplitude, just as in a conventional Stokes expansion of the basic wave equations. A suffix o will be used to denote a quantity of zero order and superfixes (1) and(2) will denote first and second order quantities as shown above in the expansion of the normal $\underline{n}$.

The second order non-linear wave force acting on the vessel can be expressed in the form,

$$\underline{F}^{(2)} = -\int_{S_o} P^{(2)}\underline{n}_o \, dS - \int_{S_o} P^{(1)}\underline{n}^{(1)} dS - \int_{S_o} P^{(0)}\underline{n}^{(2)} dS - \int_{S_1} P^{(1)}\underline{n}_o dS \quad (8)$$

In this equation the integrals extend over the submerged area of the vessel and, in particular, either the area S_o defined by the vessel in its equilibrium position or the additional area s_1 near the surface defined by movement of the vessel's (equilibrium) waterline relative to the wave elevation.

Expressions for the pressure are obtained by using Bernoulli's equation together with a Taylor expansion to relate pressure on the surface of the displaced vessel to pressure on the surface of the vessel in its equilibrium position. Thus,

$$P^{(0)} = \rho g(d-(Z + c)), \tag{9}$$

$$P^{(1)} = -\rho g Z^{(1)} + \rho\phi_t^{(1)}, \tag{10}$$

$$P^{(2)} = -\rho g Z^{(2)} + \rho\phi_t^{(2)} + \rho(\underline{X}^{(1)}.\nabla)\phi_t^{(1)} - \tfrac{1}{2}\rho(\nabla\phi^{(1)})^2 \tag{11}$$

In the above expressions the centre of gravity of the vessel in equilibrium is assumed to lie a distance c above the seabed and the water depth is denoted by d. First and second order motions of the vessel $\underline{X}^{(1)}$ and $\underline{X}^{(2)}$ are defined by equation (3). The velocity potential ϕ includes all wave motions due to incident and scattered waves and waves created by motions of the vessel. The partial derivative of ϕ with respect to time is denoted with a suffix t.

UNDERKEEL can be used to define first order vessel motions and the first order velocity potential as these all occur at the wave period. However, to describe the second order velocity potential $\phi^{(2)}$ exactly, and ultimately second order vessel motions, it is necessary to solve the diffraction of waves by the vessel to second order in the wave amplitude. Such an approach would require a distribution of sources over the surface of the water surrounding the vessel as well as sources on the vessel itself. A very large number of calculations would then be needed to solve for the source strengths needed to define second order pressure forces on the vessel. In view of these difficulties some approximations have been sought.

It has been proposed by Standing (Ref 9) that, leaving aside the second order vessel movement potential, the second order potential associated with just the incident waves ($\phi_I^{(2)}$) which describes set-down beneath wave groups be used in the relevant component of the second order force in equation (8) ie

$$-\rho \int_{S_o} \phi_{It}^{(2)} \underline{n}_o \, dS.$$

This approach was developed originally by Bowers (Ref 11) when describing the force due to set-down on a large vessel moored in short period head seas. While this approximation proved to be satisfactory for that particular situation it is not clear that it is adequate for the general case.

A different approach has been suggested by Pinkster (Ref 8). He suggests that set-down be treated like a long wave with a modified wave number and that diffraction of this modified long wave by the vessel be taken into account in calculating the second order wave force. In more detail, the complete second order potential can be expressed the form,

$$\phi^{(2)} = \phi_o^{(2)} + \phi_d^{(2)} + \phi_b^{(2)} , \tag{12}$$

where $\phi_b^{(2)}$ denotes the potential associated with flows set-up in the water by second order vessel movements, $\phi_d^{(2)}$ denotes the potential of the (second order) diffracted wave and $\phi_o^{(2)}$ denotes the potential of the (second order) disturbance created in the vicinity of the vessel both by incident and scattered waves and by waves created by first order motions of the vessel. Thus, $\phi_o^{(2)}$ contains as one of its components the set-down potential $\phi_I^{(2)}$ for the second order disturbance created by just the incident waves. The boundary condition that flow normal to the vessel's surface must match the normal velocity can be expressed in the form,

$$\nabla\phi_b^{(2)}.\underline{n}_o = \underline{v}^{(2)}.\underline{n}_o \ , \tag{13}$$

$$[\nabla(\phi_o^{(2)}+\phi_d^{(2)})+(\underline{X}^{(1)}.\nabla)\nabla\phi^{(1)}].\underline{n}_o + \nabla\phi^{(1)}.\underline{n}^{(1)} = \underline{v}^{(1)}.\underline{n}^{(1)} \tag{14}$$

Equation (13) describes the boundary condition on the potential for second order vessel motion with velocity $\underline{v}^{(2)}$. This enables added mass and damping coefficients to be determined for the (second order) long period vessel movement and there is no particular difficulty in solving for $\phi_b^{(2)}$. The difficulty arises in solving for $\phi_o^{(2)}$ exactly and here Pinkster suggests retaining just the set-down potential $\phi_I^{(2)}$ associated with the incident waves and then solving for the diffraction potential $\phi_d^{(2)}$ using Equation (14) with terms involving first order quantities put to zero.

Both Standing's and Pinkster's approximate treatments have been applied to the case of a horizontal cylinder moored beam onto regular wave groups in a situation where the non-linear wave force due to the second order velocity potential is known to be an important part of the total non-linear wave force. The results are described in Section 4 where a comparison has been made with experimental data. It is found that neither approximation results in good agreement with the experimental data. It is also shown, however, that an allied approach in which the set-down potential $\phi_I^{(2)}$ is assumed dominant and then diffracted by the cylinder retaining first order body motions, results in a better agreement with the experimental data. This is just one particular situation, though, and further experimental work with a ship model in random waves is needed to determine the true accuracy of these approximations.

Returning to the rest of the terms in the second order wave force we find the first of the four integrals on the right-hand side of Equation (8) gives rise to the following expression, after substituting for the second order pressure from Equation (11).

$$-\int_{S_o} P^{(2)}\underline{n}_o dS = -\rho\int_{S_o} (\phi_t^{(2)}+(\underline{X}^{(1)}.\nabla)\phi_t^{(1)}-\tfrac{1}{2}(\nabla\phi^{(1)})^2)\underline{n}_o dS$$

$$-(C_{33}\; S_3^{(2)}+C_{35}\; S_5^{(2)}+C_{33}(c-d)(\frac{S_4^{(1)2}+S_5^{(1)2}}{2}))(0,0,1)$$

$$+\rho g V(-S_5^{(2)},\; S_4^{(2)},\; -\tfrac{1}{2}(S_4^{(1)2}+S_5^{(1)2})) \qquad (15)$$

where,

$$C_{33} = \rho g \int_L Bdx,$$

$$C_{35} = -\rho g \int_L Bxdx,$$

$$V = \int_L BDdx,$$

In these latter expressions the vessel's beam at position x is denoted by B, and its draught by D. The integrals for the coefficients are taken over the length L of the ship and the quantity V is seen to be the volume of water displaced by the vessel.

The second of the integrals in Equation (8) can be expressed in the form,

$$- \int_{S_o} P^{(1)} \underline{n}^{(1)} dS = - \underline{R}^{(1)} . \int_{S_o} P^{(1)} \underline{n}_o dS \qquad (16)$$

The total first order wave force on the vessel is given by,

$$\underline{F}^{(1)} = - \int_{S_o} P^{(1)} \underline{n}_o dS - \int_{S_o} P^{(0)} \underline{n}^{(1)} dS,$$

$$\text{ie } - \int_{S_o} P^{(1)} \underline{n}_o dS = \underline{F}^{(1)} + \underline{R}^{(1)} . \int P^{(0)} \underline{n}_o dS \qquad (17)$$

Hence, substituting (17) in Equation (16) we obtain,

$$- \int_{S_o} P^{(1)} \underline{n}^{(1)} dS = \underline{R}^{(1)} . \underline{F}^{(1)} - [\underline{R}^{(1)}]^2 . \underline{F}^{(0)} \qquad (18)$$

where to zero order we have just the buoyancy force:-

$$\underline{F}^{(0)} = - \int P^{(0)} \underline{n}_o dS = (0, 0, \rho g V)$$

The third integral in Equation (8) takes the form

$$- \int_{S_o} P^{(0)} \underline{n}^{(2)} dS = -\underline{R}^{(2)} . \int_{S_o} P^{(0)} \underline{n}_o dS$$

$$= \underline{R}^{(2)} . \underline{F}^{(0)} \tag{19}$$

Finally, the fourth integral in (8) can be expressed in the form,

$$- \int_{s_1} P^{(1)} \underline{n}_o dS = - \int_L dx \int_o^{\eta_r^{(1)}} \rho g (\eta_r^{(1)} - \epsilon) \underline{n}_o \, d\epsilon$$

$$= -\rho g/2 \int_L (\eta_r^{(1)})^2 \, \underline{n}_o dx \tag{20}$$

In evaluating this integral the following hydrostatic approximation for pressure near the surface has been used,

$$P^{(1)} \equiv \rho g \, [\eta^{(1)} - Z^{(1)} - \epsilon],$$

$$= \rho g \, [\eta_r^{(1)} - \epsilon].$$

Here $\eta_r^{(1)}$ is the wave elevation relative to the displaced (by $Z^{(1)}$) equilibrium waterline of the vessel and ϵ is the (small) variable of integration for the s_1 area, ie

$$Z = d - c + \epsilon.$$

Having assembled all four integrals on the right hand side of Equation (8) we can express the non-linear wave force in the form,

$$\underline{F}^{(2)} = -\tfrac{1}{2}\rho g \int_L (\eta_r^{(1)})^2 \, \underline{n}_o dx$$

$$+ \tfrac{1}{2}\rho \int_{S_o} (\nabla\phi^{(1)})^2 \, \underline{n}_o dS$$

$$- \rho \int_{S_o} (\underline{X}^{(1)}.\nabla) \, \phi_t^{(1)} \, \underline{n}_o dS$$

$$+ \underline{R}^{(1)}.\underline{F}^{(1)}$$

$$- \rho \int_{S_o} \phi_t^{(2)} \, \underline{n}_o dS$$

$$- \tfrac{1}{2} C_{33}(c-d)((S_4^{(1)})^2 + (S_5^{(1)})^2) \; (0,\, 0,\, 1)$$

$$- (C_{33} \, S_3^{(2)} + C_{35} \, S_5^{(2)})(0,\, 0,\, 1) \qquad (21)$$

In adding the four contributions together some cancellation occurs between terms on the right hand sides of Equations (15), (18) and (19).

The first four terms in Equation (21) correspond to the four mechanisms causing non-linear wave forces that are shown schematically in Figure 2 (see discussion at the beginning of Section 2). Both Pinkster and Standing agree on the form of these terms.

The fifth term is the force due to the second order velocity potential and here some approximation is necessary due to the complexity of solving the second order diffraction problem. These approximations have already been discussed and their accuracy is considered further in Section 4 of this report.

The sixth term is a second order buoyancy effect found by Standing but ignored by Pinkster. The derivation given in this report confirms Standing's result.

The final term describes the usual restoring forces due to buoyancy but to second order in heave and pitch.

Using results from UNDERKEEL for first order quantities, together with a suitable approximation for the second order velocity potential force, we can use Equation (21) to calculate the non-linear wave force on moored vessels. This approach can be expected to yield a more accurate description than that possible using the Newman approximation.

In the next sub-section we address the problem of obtaining the second order wave moments.

3.2 Non-linear wave moments

A general form for the moment about the moving centre of gravity G', of the pressure acting on a surface element dS at position $\underline{X}'$ in the G'X'Y'Z' system of axes, is the following:-

$$d\underline{M} = - P\,\underline{X}' \times \underline{n}\,dS.$$

This can be expressed in terms of the vector $\underline{x}$ for the same position with coordinates (x,y,z) in the system of body axes G'xyz moving with the vessel. Hence,

$$\underline{M} = - \int_S P(\underline{R}.\underline{x}) \times \underline{n}\,dS$$

Allowing for expansions in all three quantities P, $\underline{R}$ and $\underline{n}$ we can express the second order moment in the form,

$$\underline{M}^{(2)} = -\int_{S_o} P^{(2)}\underline{x} \times \underline{n}_o dS - \int_{S_o} P^{(1)}[(\underline{R}^{(1)}.\underline{x}) \times \underline{n}_o + \underline{x} \times \underline{n}^{(1)}]dS$$

$$-\int_{S_o} P^{(0)}[(\underline{R}^{(2)}.\underline{x}) \times \underline{n}_o + (\underline{R}^{(1)}.\underline{x}) \times \underline{n}^{(1)} + \underline{x} \times \underline{n}^{(2)}]dS$$

$$-\int_{S_1} P^{(1)} \underline{x} \times \underline{n}_o \, dS \tag{22}$$

The various quantities in this equation have already been defined. The first of the four integrals on the right hand side leads to the following expression after substituting for $P^{(2)}$ from (11),

$$-\int_{S_o} P^{(2)}\underline{x} \times \underline{n}_o \, dS = -\rho \int_{S_o} (\phi_t^{(2)} + (\underline{X}^{(1)}.\nabla)\phi_t^{(1)} - \tfrac{1}{2}(\nabla\phi^{(1)})^2)\underline{x} \times \underline{n}_o dS$$

$$- (C_{44}\, S_4^{(2)},\; C_{35}\, S_3^{(2)} + C_{55}\, S_5^{(2)},\; 0)$$

$$- C_{35}\,(c-d)\,\tfrac{1}{2}\,(S_4^{(1)2} + S_5^{(1)2})(0, 1, 0) \tag{23}$$

where,

$C_{44} = \rho Vg.GM,$

$C_{55} = \rho g \int_L Bx^2 \, dx,$

with,

GM = metacentric height above the centre of gravity.

The second of the integrals in Equation (22) can be expressed in the following form for a freely floating vessel.

$$- \int_{S_o} P^{(1)} \left[(\underline{R}^{(1)}.\underline{x}) \times \underline{n}_o + \underline{x} \times (\underline{R}^{(1)}.\underline{n}_o) \right] dS$$

$$= \underline{R}^{(1)}.\underline{M}^{(1)}, \tag{24}$$

where the first order moment is given by,

$$\underline{M}^{(1)} = - \int_{S_o} (P^{(1)} \underline{x} \times \underline{n}_o + P^{(0)} (\underline{R}^{(1)}.\underline{x}) \times \underline{n}_o + P^{(0)} \underline{x} \times \underline{n}^{(1)}) dS$$

The third integral in (22) can be shown to vanish for a freely floating vessel.

The fourth integral in (22) takes the form

$$- \int_{S_o} P^{(1)} \underline{x} \times \underline{n}_o \, dS = - \rho g/2 \int_L (\eta_r^{(1)})^2 \, \underline{x} \times \underline{n}_o \, dx \tag{25}$$

Summing up the four integrals on the right hand side of Equation (22) we can express the second order moment in the form,

$$\begin{aligned}
\underline{M}^{(2)} = & - \tfrac{1}{2} \rho g \int_L (\eta_r^{(1)})^2 \, \underline{x} \times \underline{n}_o dx \\
& + \tfrac{1}{2} \rho \int_{S_o} (\nabla\phi^{(1)})^2 \, \underline{x} \times \underline{n}_o dS \\
& - \rho \int_{S_o} (\underline{X}^{(1)}.\nabla)\phi_t^{(1)} \, \underline{x} \times \underline{n}_o dS \\
& + \underline{R}^{(1)}.\underline{M}^{(1)} \\
& - \rho \int_{S_o} \phi_t^{(2)} \, \underline{x} \times \underline{n}_o dS \\
& - \tfrac{1}{2} C_{35} \, (c-d)(S_4^{(1)2} + S_5^{(1)2})(0, 1, 0) \\
& - (C_{44} \, S_4^{(2)}, \, C_{35} \, S_3^{(2)} + C_{55} \, S_5^{(2)}, \, 0)
\end{aligned} \tag{26}$$

The first four terms in Equation (26) correspond to the moment effects of the mechanisms shown schematically in Figure 2. Again, Pinkster and Standing agree on the form of these terms.

The fifth term involving second order potentials has to be treated approximately. These approximations have been described already.

The sixth term was obtained by Standing but ignored by Pinkster.

The final term describes the usual restoring couples due to buoyancy for second order roll and pitch.

Equation (26) is taken to be the non-linear wave moment which can be evaluated using UNDERKEEL to describe the first order response together with a suitable approximation for the moment associated with the second order velocity potential.

4 SECOND ORDER POTENTIALS

The difficulties attached to solving exactly for the second order diffraction potential have been discussed in Section 3.1. Three different approximations were also considered, one due to Pinkster (Ref 8) one due to Standing (Ref 9) and a third allied approach. Here we apply all three approximations to a situation where second order potential effects were identified in experiments as being an important part of the total non-linear wave force.

Experimental work on second order wave forces on wave power devices has been carried out at HR (Ref 19). Tests were performed in a wave flume with a model moored across the flume and subjected to regular groups. The model was able to surge freely ie move horizontally along the flume on its mooring at the wave group period. Tests were carried out with and without a control signal to the wave-maker to compensate for set-down beneath wave groups. This was done to see how important set-down compensation was in tank testing of moored wave power devices. As set-down is an important part of the second order potential effect, these tests also demonstrated the conditions for which second order potential effects from an important component of the non-linear wave force.

In what follows the experiments are described and results produced to show when second order potential effects become important. By comparing theoretical results, obtained using various approximations to the second order diffraction problem, with the relevant experimental results it is possible to show the accuracy of the various approximations.

4.1 Experimental results

A wave flume equipped with a wedge type wave generator was used in the experimental investigation. The wave-maker was position controlled by an electro-hydraulic system. In tests with regular wave groups the electrical signal to the wave-maker consists of a sum of two frequencies so that the wave group period equals the inverse of the difference between the two wave frequencies.

The model length scale was 1 to 100, making the time scale 1 to 10 with Froude scaling. All quantities given here and in subsequent sections are expressed in full scale terms unless stated otherwise.

The mooring arrangement consisted of four mooring lines made of rubber. Figure 6 shows the layout in plan. The ends of the lines attached to the device were mounted just below the water level and the other ends were held in rigid supports so that all four lines were horizontal and parallel with the sides of the flume. In all cases the device extended the full width of the flume (1.2m). To allow free movement of the device horizontally along the flume (surge) and vertically (pitch and heave) a small gap was left at each end of the device between it and the flume walls. The mooring stiffness chosen made the resonant period in surge approximately 62s. This ensured a significant surge response in regular wave group tests where the group period was consistently about 52s. A gently sloping shingle beach was built behind the device. This beach was a good absorber of the primary waves with reflection coefficients of less than 10%. To minimise the effect of long wave reflections on the surge response of the device, it was placed at an anti-nodal point for a reflection system produced by any long waves at the group period travelling towards the wave-maker and undergoing perfect reflection. Since wave slope is zero at an anti-nodal point such a reflection system should then not produce a horizontal force.

Wave height in the model was measured using twin wire wave probes. Horizontal movement of the device was measured with a Selspot system. This consisted of an infra-red light source mounted on the device and a camera mounted outside the flume that registered movement of the light source as the device moved. The Selspot system allowed large surge movements to take place. Signals from the measuring instruments were fed to mini-computer capable of performing a spectral analysis.

The two wave periods (T_1 and T_2) used to produce regular wave groups covered a range of conditions from relatively short waves (T_1 = 10.0s, T_2 = 8.4s) to long period waves (T_1 = 20s, T_2 = 14.5s). Wave periods were chosen so that the wave group period remained the same for each pair, ie 52s approximately. As the Selspot system allowed large surge movements to occur, tests were carried out with wave heights of up to 22m represented for the longer period waves (T_1 = 20s, T_2 = 14.5s).

The water depth in the flume represented 60m at full scale.

The device consisted of a circular cylinder of diameter 10m that was ballasted such that it floated with a draught of 8.7m. The four horizontal mooring lines were attached to the sides of the cylinder at a depth of 2.4m below the water line. The stiffness of each line was 7.2 tonnes/m and this produced a resonant period for surge of approximately 62s.

Each experiment with regular wave groups was carried out twice, once in the presence of any spurious free long wave at the wave group period due to the wave-maker and then again with the free long wave minimised by an additional movement of the wave-maker at the wave group period to compensate for set-down. The data were analysed with a fast Fourier transform computer program to give spectra of wave height and device movement. The three peaks of interest in the spectra occur at the wave group period and at the primary wave periods. The amplitude (half the wave height or half the total movement) of each component was obtained from the area under each peak by using

$$\tfrac{1}{2}\,(\text{amplitude})^2 = \text{area}$$

Experimental values of the disturbance at the wave group period at various positions down the flume are compared with theoretical values in Table 3. By carrying out measurements at nodal points (distances of 1¾, 2¼L etc from the wave-maker) for a reflection system produced by any free long waves travelling towards the wave-maker and undergoing perfect reflection, it is possible to compare the measured long wave amplitudes with theoretical predictions for a purely progressive wave system leaving the wave-maker (Ref 20). In column (a) of Table 3 the amplitudes are compared for the case with free waves present due to the wave-maker. In column (b) the comparison is made with the free long wave minimised by movement of the wave-maker at the group period. To compensate for set-down. When theoretical values under the two columns are compared it will be seen that unequal values (column (a)) at various nodal points should become equal (column (b)) if the free long wave from the wave-maker is eliminated. This is explained by the interference pattern produced by set-down and the free wave being removed when the free wave is eliminated. This leaves just set-down beneath waves with the amplitude value shown under theory in column (b). Results are given for a range of primary wave amplitudes for each pair of primary wave periods. In general there is qualitative agreement between experiment and theory in that unequal experimental values under column (a) tend to become more equal and closer to the theoretical value in column (b) when an appropriate secondary movement of the wave-maker is used. In making this comparison it should be borne in mind that the twin wire wave probes used for these measurements are accurate to within 0.2mm (0.02m full scale).

The resulting non-linear surge of the cylinder at the wave group period is plotted in Figures 7,8 and 9 for each pair of frequencies. The solid line denotes results with set-down compensation and the dashed line results without compensation. As the non-linear surge is a second order effect its amplitude should be proportional to the product of the primary wave amplitudes. While this appears true for the large surge movements recorded in Figure 7 some variation from this behaviour is apparent in Figures 8 and 9 where straight lines cannot be drawn through the experimental results. It is thought that this is largely due to inaccuracies in the measurement of the smaller surge movements by the Selspot system.

The main experimental result is, however, the obvious indication that second order potential effects only become important in this particular situation when the primary wave periods are long ie T_1 = 20s, T_2 = 14.5s. For shorter wave periods there is little difference between results obtained with and without set-down compensation. As the spurious free long waves at the wave group period, present in the absence of set-down compensation, are of the same order of magnitude as set-down itself, an experimental result which shows their presence to be unimportant also indicates that set-down is unimportant and vice versa. These results are consistent with the discussion in Section 2.2 of this report where it was found that the contribution of spatial gradient effects to the total non-linear wave force becomes dominant in the absence of scattering of the primary waves ie as the primary wave period becomes long. Set-down is yet another spatial gradient effect and so it too can be expected to become important for long primary wave periods.

4.2 Comparison of theory with experiment

The experimental work described above has identified a situation where second order potential effects make an important contribution to the non-linear wave force ie the set of results shown in Figure 7. We can now apply the three approximate methods of calculating second order potential effects to this situation and compare the results with the experimental data.

Denoting the cylinder surge by s we can write the equation of motion in the form,

$$\ddot{s} + \beta \dot{s} + w_o^2 s = \frac{F}{M} \tag{27}$$

where, β is the coefficient of damping at the wave group period,

$$w_o^2 = \frac{S_t}{M}$$

S_t is effective (linear) stiffness of model mooring ie 29gm/cm

M is total mass of model cylinder including added mass ie 28.15kg

F is the non-linear wave force at the wave group period.

The above figure for the total mass of the cylinder, including the added mass, was obtained by perturbing the moored device in still water and noting its resonant period (about 6.25s in the model). The coefficient of damping was obtained by noting the decay rate of the resulting surge oscillations. We can then solve Equation (27) for surge s once we know the wave force F ie.

$$s = \frac{|F|}{M[(w^2 - w_o^2)^2 + \beta^2 w^2]^{\frac{1}{2}}} \tag{28}$$

where w is the (radian) wave group frequency.

Given the relatively simple cylindrical shape of the device it is possible to form a good estimate of the non-linear surge force defined in Equation (21) by analytical calculation. This was done initially using Standing's approximate treatment of the second order potential wave force in order to compare with results obtained by Brendling (Ref 21) using Standing's model. The two theoretical results for the amplitude of surge motion are shown by the (short) dashed lines in Figure 10. It can be seen that good agreement was obtained between data from the analytical model and data given in Reference 21 which arose from using a source method and computation with the model developed by Standing (Ref 9). However, both sets of theoretical results show large underpredictions of the surge measured in the experiments (see ▫ symbols in Fig 10). It is important to note that the experimental data obtained with set-down compensation are plotted because the effects of spurious free waves from the wave-maker have not been considered in any of the theoretical results shown in Figure 10.

The very small values of surge predicted using Standing's approximate treatment of the second order potential force arise through a significant amount of cancellation between that approximate second order potential force and the other non-linear wave forces present in Equation (21).

When Pinkster's approximate treatment of the second order potential force is used a significant overprediction of surge occurs (see long dashed line in Fig 10). In this case set-down is diffracted as if it were a free long wave with a modified wave number to match the wave number of set-down. The approximation increases the effective set-down force making it 133% of the resultant force. Cancellation of some of this large set-down force by the other non-linear wave forces leads to the resultant. A tendency for Pinkster's method to overestimate the second order potential force has been noted by Pinkster himself (Ref 8).

Finally, we apply the allied approximation for the second order potential force described in Section 3.1 of this report. This leads to the solid line in Figure 10 and much better agreement with the experimental data. Using this approach we can then also allow for forces due to spurious free long waves present in the experiments performed without set-down compensation at the wave-maker. This results in an increased surge as shown by the dashed line in Figure 11 and again good agreement is obtained with experimental data. This gives some confidence in the third method of approximating the second order potential force and it is suggested that this approximation be used when evaluating the non-linear forces and moments defined by Equations (21) and (26).

It should be borne in mind, though, that the three approximations have only been tested here against a relatively simple experimental situation using regular wave groups. Further validation is needed using more comprehensive random wave model experiments to check the suggested approximation. The example described here also demonstrates the need for set-down compensation at the wave-maker in experimental work to ensure that non-linear wave forces and moments are well represented.

5 CONCLUSIONS

A description has been given of non-linear wave forces acting on moored ships. It has been shown how these forces cause significant ranging of vessels on their moorings at wave group periods.

Due to the large computational effort needed to represent these forces exactly some approximations have been sought. The non-linear forces and moments can be divided into two basic types.

One type, involving (first order) movements of the ship at the wave period, can be expressed in terms of products of first order quantities only. The first four terms on the right hand side of Equations (21) and (26) together with the sixth term are of this type. They can be evaluated once the first order problem of wave diffraction around the vessel is solved but in their exact form, ie Equations (21) and (26), they will require considerable computation for a random sea. The Newman approximation (Ref 7) has been suggested for evaluating these terms in order to reduce the amount of computation. Although this approximation is much used in offshore applications it has limitations in the coastal applications of interest here. In particular it has been found that non-linear forces due to spatial gradients in all the second order pressure terms, effects not represented in Newman's approximation, tend to be more important in coastal applications because the resonant periods of moored ships are generally shorter than resonant periods of structures moored offshore. As a result the Newman approximation has been shown to lead to large underestimates of non-linear wave forces of the first type. In view of this result there appears to be little alternative to use of the exact expressions as shown in Equations (21) and (26) for non-linear wave forces of the first type.

The second type of non-linear wave force involves solution of the diffraction problem to second order in the wave amplitude. This would require even more computation than that needed for the non-linear forces of the first type, if second order potentials were to be calculated exactly. This type of effect is represented by the fifth term on the right hand side of Equations (21) and (26). It is also clear that second order potential effects will be more important in coastal applications than in offshore ones due to the increase in the magnitude of set-down beneath wave groups as waves approach the coastline: set-down being one of the main components in the second order potential. Three approximate methods of calculating forces due to second order potentials have been applied to experiments with a moored cylinder carried out at HR as part of the Wave Power research programme (Ref 19). One approximation proposed by Standing (Ref 9) ignores diffraction of set-down and it was found to lead to significant underestimates in the resultant effect for experiments where second order potential effects were known to be important (see short dashed line in Fig 10). A different approximation proposed by Pinkster (Ref 8) was found to lead to significant overestimates (see long dashed line in Fig 10). A third, allied, approximation has been suggested and found to be in better agreement with the experiments (see solid line in Fig 10). It is felt, however, that further validation of the suggested approximation for second order potential effects is needed using more comprehensive experiments with ship models moored in random waves. In this regard it is important that set-down compensation at the wave-maker is used to ensure the correct representation of second order potential effects in model experiments.

Taken overall, the work described in this report indicates that non-linear wave forces can be expected to be well described provided forces and moments are evaluated using Equations (21) and (26) with the suggested approximation for representing second order potential effects.

6 RECOMMENDATIONS

It is recommended that Equations (21) and (26) be employed to represent non-linear wave forces on moored ships. By using UNDERKEEL (Ref 1) to calculate (first order) responses of the vessel at the wave period it will be possible to evaluate all the terms on the right hand side of equations (21) and (26) provided the suggested approximation for second order potential effects is used. However, programming these expressions for non-linear forces and moments will involve more work than that needed had it been possible to make use of the Newman approximation in evaluating these forces. In drafting the programme of research to be carried out under the present contract it was hoped that the Newman approximation would prove adequate and, as a result, the amount of work needed to programme non-linear wave forces has been underestimated.

In parallel with the work described in this report, an extension of UNDERKEEL to allow for a vessel moored against a quay face, has been under investigation. It was anticipated that a relatively minor amount of work would be involved but this has not proved to be the case. An important simplification, that proved possible in developing UNDERKEEL for a free ship, can still be used for a vessel moored against an open (piled) jetty but the flows created around a vessel moored against a quay face have been found to require a different set of assumptions. This work will be described in a subsequent report. Once completed, however, the extended UNDERKEEL can be used to derive the first order responses of a vessel moored against the quay face and these responses can, in turn, be used in Equations (21) and (26) to calculate non-linear wave forces and moments for that case.

It can be appreciated that the extra work needed to complete the two aspects described above means that final validation of the computer model cannot be carried out under the present contract as originally hoped. However, collection of the necessary field data on moored ship movements should be possible under an extension of the present contract. And it should be possible to carry out some validation of the computer model in frequency space (for a ship on linear moorings) using published data. It is intended that the latter aspect be described in a separate report on the programming of non-linear wave forces which will be produced under the existing contract.

The work described in this report has highlighted a need for more experimental data to check the approximate treatment of second order potential effects. Due to the complexity of non-linear wave forces it is likely to prove impossible to check this approximation using full scale data. This is due to the large number of competing effects in a realistic situation. The advantage of model experiments is that more control can be exercised over the particular factors of interest and experiments can be designed to allow those particular factors to be investigated in detail. The experiments described in this report were able to do this for the simple situation of a cylinder moored in beam waves with regular wave groups and this has helped to judge the accuracy of various approximate methods for calculating second order potential effects. However, the model under development is for ships moored in random waves and more comprehensive experiments are needed for that situation. It is essential in those experiments that compensation for set-down beneath wave groups is used to ensure the correct representation of second order potentials.

There are also a number of aspects related to time domain representations of non-linear wave forces, that require study. This is the step necessary before final validation of the computer model can be carried out against full scale data. It involves using the non-linear forces given by Equations (21) and (26) to derive coefficients that can be used to construct a time series that, in turn, can be used in SHIPMOOR (Refs 2,3). It is necessary to move into the time domain to allow for the fact that conventional ship mooring systems are non-linear making a frequency space model like UNDERKEEL unsuitable for realistically moored ships when used alone. This is why SHIPMOOR has been developed in parallel with UNDERKEEL so that together the two models can describe realistic situations. The complication with time domain representations is that non-linear wave forces have non-Gaussion statistics and it will be necessary to ensure that the mathematical representations reproduce the correct statistics. This requires checking both via experiments using random waves and by comparing results of simulations with analytical predictions like those derived recently for second order wave quantities (Ref 22).

Taking all of the above points into account it is clear that following completion of the present contract, further work will be necessary to enable a satisfactory validation of the computer model of a moored ship to be made. This work must encompass; a more comprehensive experimental study to check the suggested approximate treatment of second order potential effects, and experimental and analytical work to check the statistics of time domain representations of non-linear wave forces. Only then can a final validation of the computer model take place against full scale data.

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TABLES.

TABLE 1 Long period surge force coefficients (F_{mn}) for 85,000 tonne vessel moored in a quartering sea (ignoring forces due to set-down)

f_n \ f_m	0.129	0.120	0.112	0.104	0.096	0.088	0.079	0.070	0.061	0.052
0.129	-85.0									
0.120	97.0	-133.5								
0.112	80.5	137.5	-155							
0.104	54.5	114.0	141.0	-127.5						
0.096	33.0	83.0	113.5	101.5	- 68.5					
0.088	21.5	58.5	95.5	92.5	57.5	-27.0				
0.079	45.0	39.0	84.0	99.0	77.0	43.0	-18.0			
0.070	52.0	69.5	103.5	113.0	86.5	51.5	18.5	- 8.0		
0.061	34.0	53.5	94.5	116.5	101.5	72.5	40.5	23.5	- 2.0	
0.052	30.5	43.5	85.0	113.0	106.0	82.0	53.5	35.0	12.5	-1.5

Units : f_m, f_n in H_z
F_{mn} in KN/m²

TABLE 2 Long period sway force coefficients (F_{mn}) for 85,000 tonne vessel moored in a quartering sea (ignoring forces due to set-down)

f_n \ f_m	0.129	0.120	0.112	0.104	0.096	0.088	0.079	0.070	0.061	0.052
0.129	765									
0.120	710	720								
0.112	580	635	580							
0.104	415	490	460	345						
0.096	250	345	355	250	120					
0.088	140	255	310	255	120	30				
0.079	105	190	300	315	220	110	20			
0.070	95	175	230	210	145	75	25	25		
0.061	25	105	190	205	155	100	50	30	5	
0.052	25	70	155	190	160	115	70	40	15	3

Units : f_m, f_n in H_z
F_{mn} in KN/m²

TABLE 3 Regular wave group conditions for the flume experiments

Primary wave periods T(s) and amplitudes a(m)		Distance of wave probe from wave-maker in terms of wavelength (L) of free long wave	Amplitude (m) of disturbance at the wave group period: (a) Without set-down compensation		(b) With set-down compensation	
			Theory	Experiment	Theory	Experiment
T_1 = 20	a_1 = 0.69	1¾L	0.01	0.01	0.02	0.02
T_2 = 14.5	a_2 = 0.75	2¼L	0.02	0.02	0.02	0.02
"	a_1 = 1.39	1¾L	0.04	0.04	0.06	0.05
	a_2 = 1.42	2¼L	0.06	0.06	0.06	0.05
"	a_1 = 3.02	1¾L	0.17	0.14	0.27	0.20
	a_2 = 2.89	2¼L	0.26	0.24	0.27	0.22
"	a_1 = 5.61	1¾L	0.62	0.24	0.98	0.98
	a_2 = 5.67	2¼L	0.95	1.09	0.98	0.76
T_1 = 12.4	a_1 = 0.74	1¾L	0.02	0.02	0,01	0.02
T_2 = 10.0	a_2 = 1.05	2¼L	0.02	0.01	0.01	0.01
"	a_1 = 1.61	1¾L	0.06	0.06	0.04	0.04
	a_2 = 1.86	2¼L	0.07	0.06	0.04	0.04
"	a_1 = 2.33	1¾L	0.12	0.11	0.09	0.08
	a_2 = 2.67	2¼L	0.13	0.08	0.09	0.06
T_1 = 10.0	a_1 = 1.07	1¾L	0.005	0.01	0.014	0.02
T_2 = 8.4	a_2 = 1.01	2¼L	0.007	0.01	0.014	0.02
"	a_1 = 2.37	1¾L	0.013	0.02	0.03	0.03
	a_2 = 0.99	2¼L	0.016	0.03	0.03	0.04
"	a_1 = 1.81	1¾L	0.012	0.02	0.04	0.02
	a_2 = 1.55	2¼L	0.018	0.03	0.04	0.05
"	a_1 = 4.21	1¾L	0.02	0.04	0.06	0.06
	a_2 = 1.06	2¼L	0.03	0.08	0.06	0.09

FIGURES.

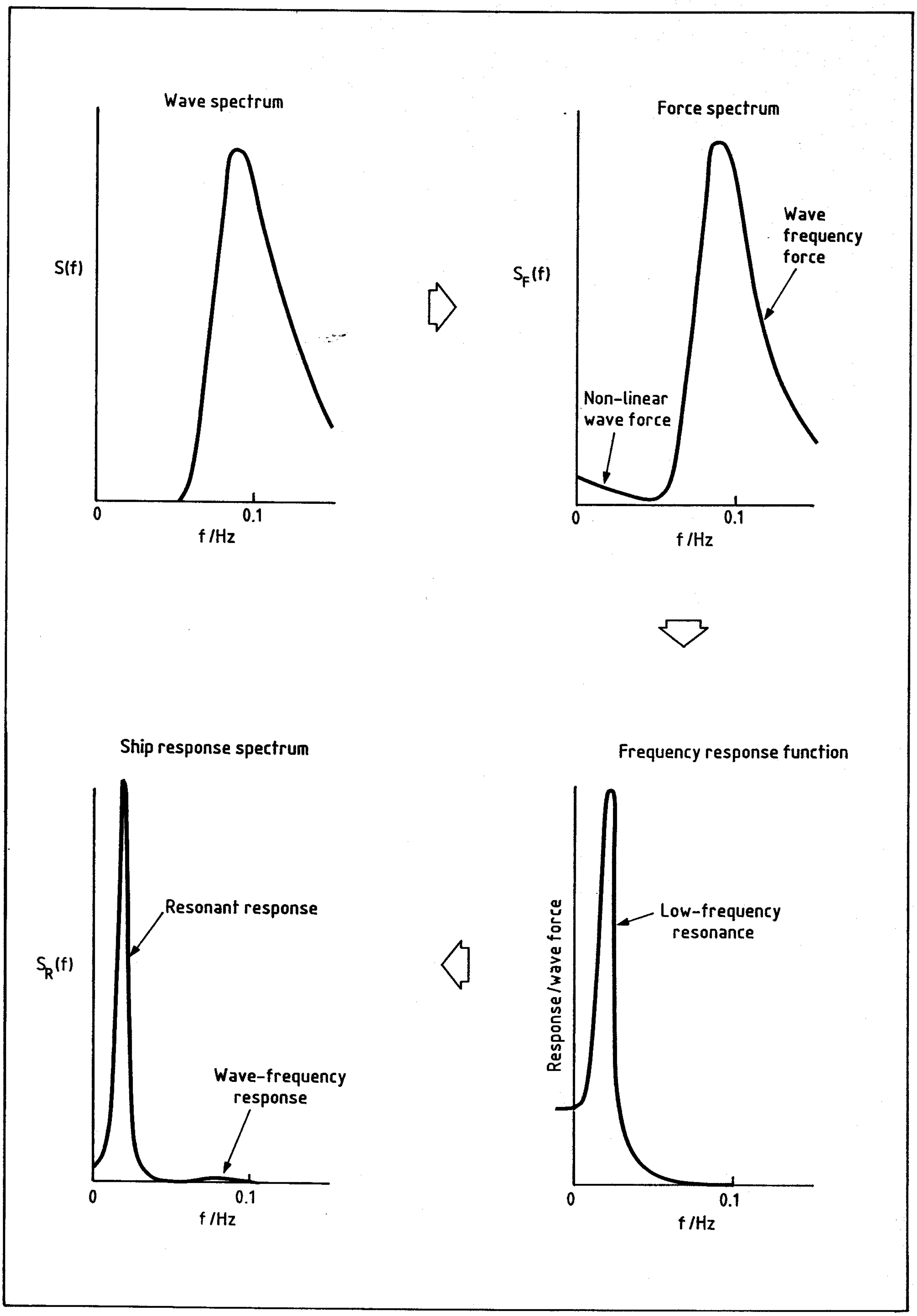

Fig 1 Schematic diagram of resonant excitation of a ship on its moorings

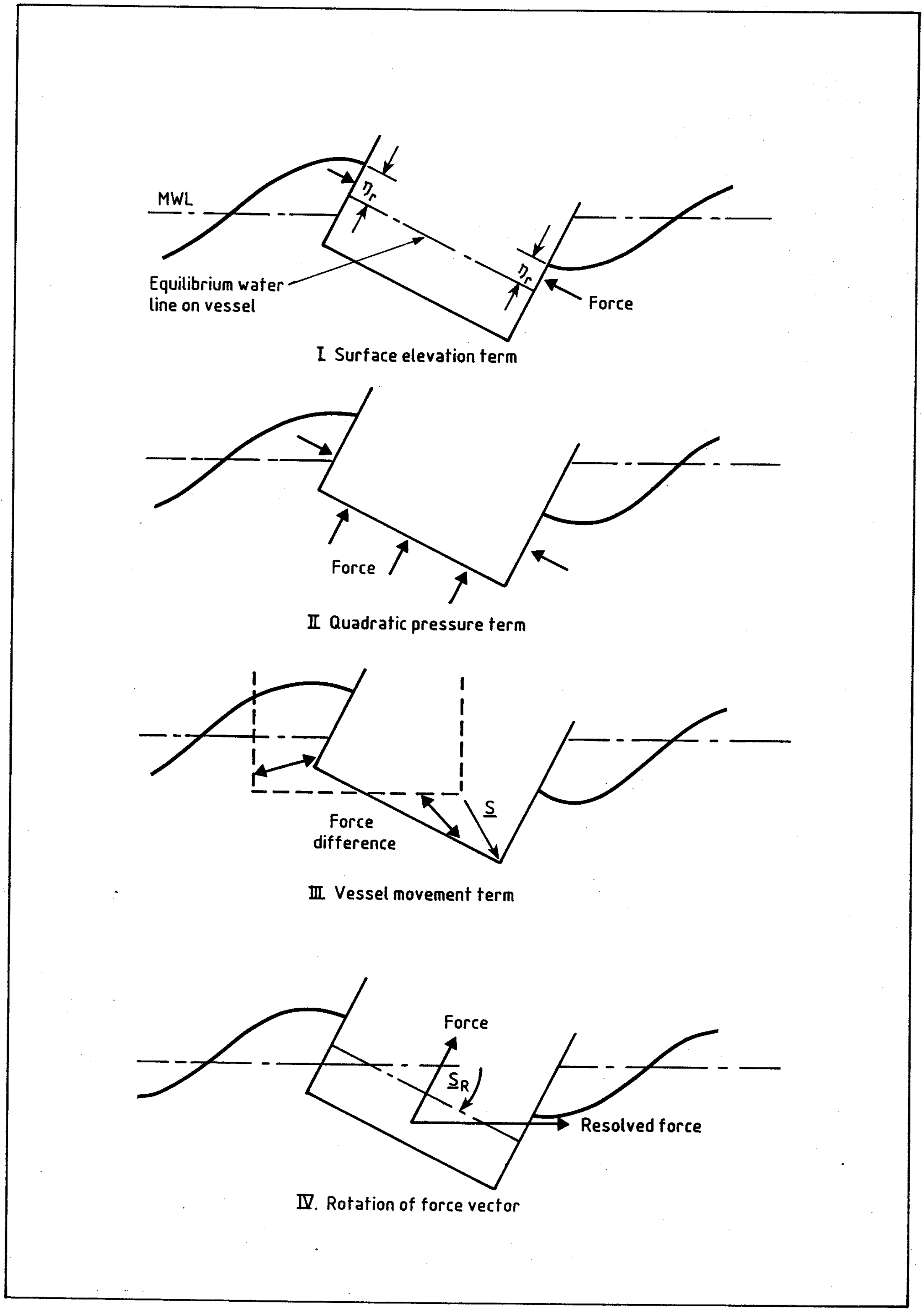

Fig 2 Mechanisms causing non-linear wave forces

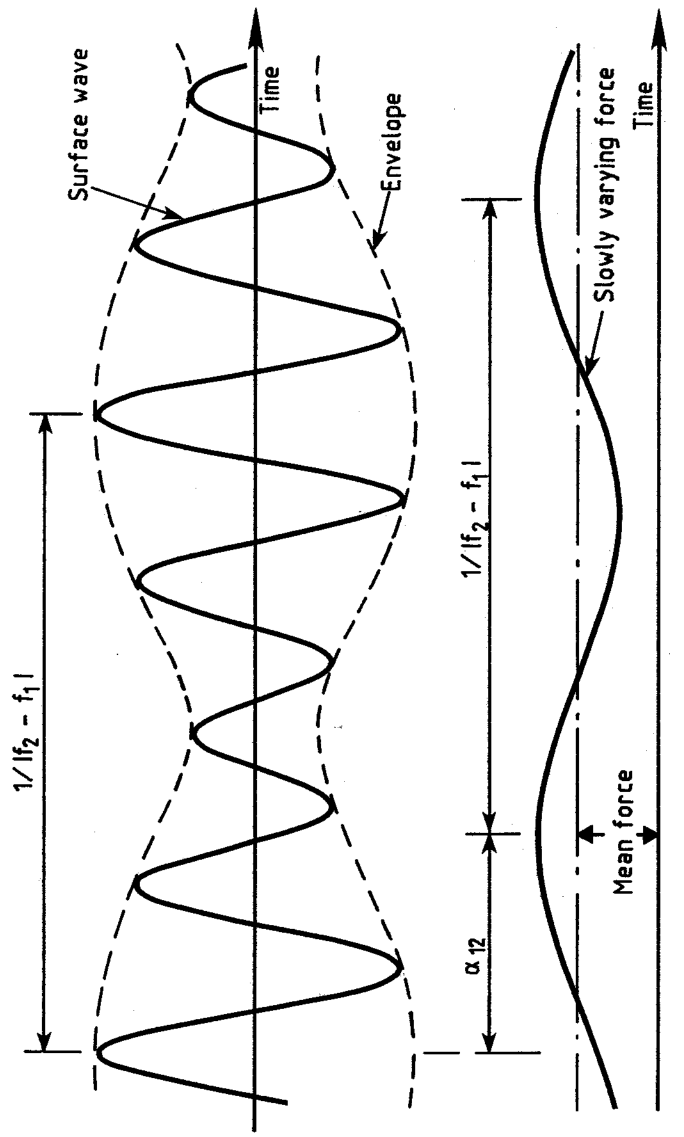

Fig 3 Regular wave groups and resulting non-linear wave force

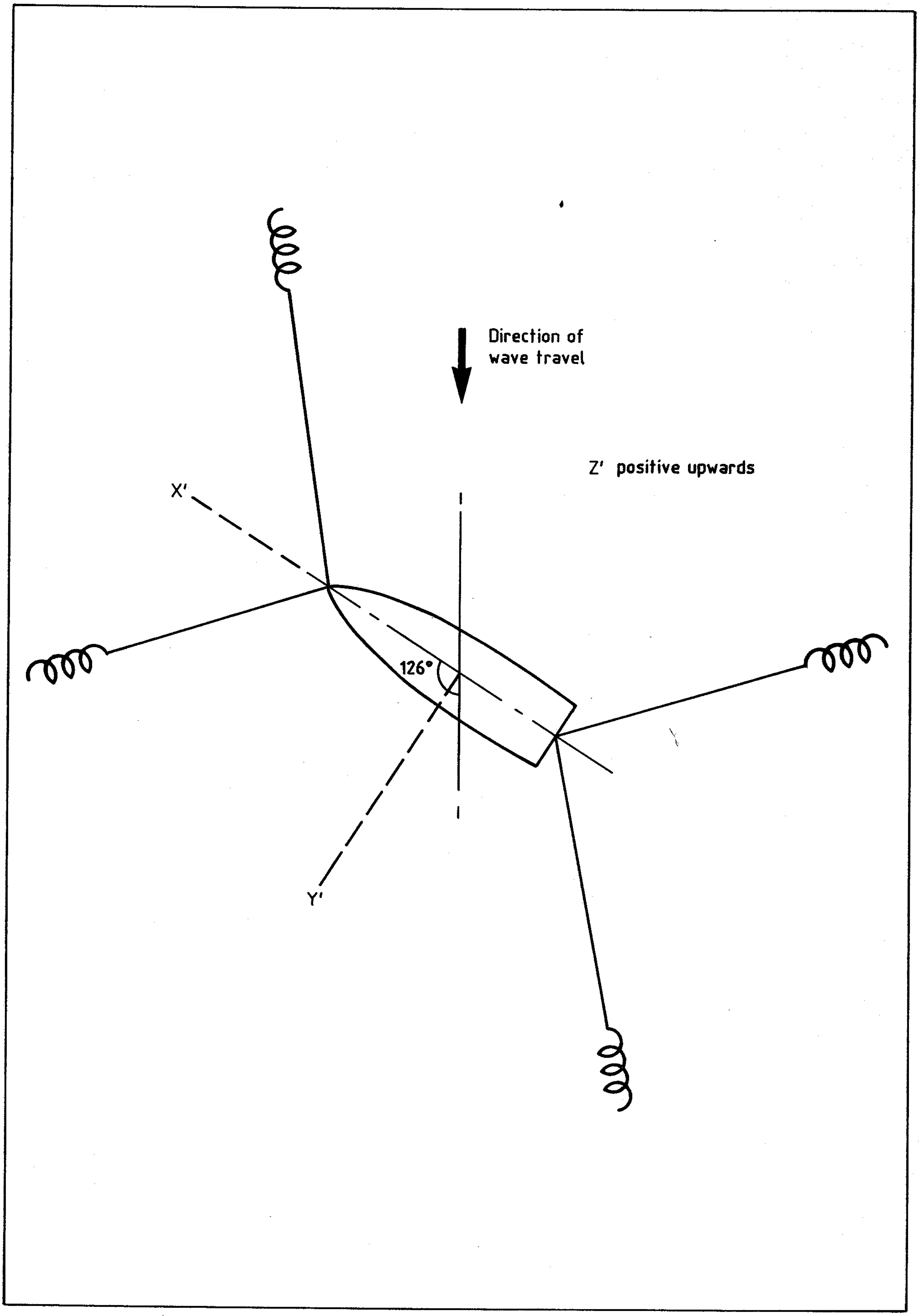

Fig 4 Mooring layout of a vessel in quartering seas

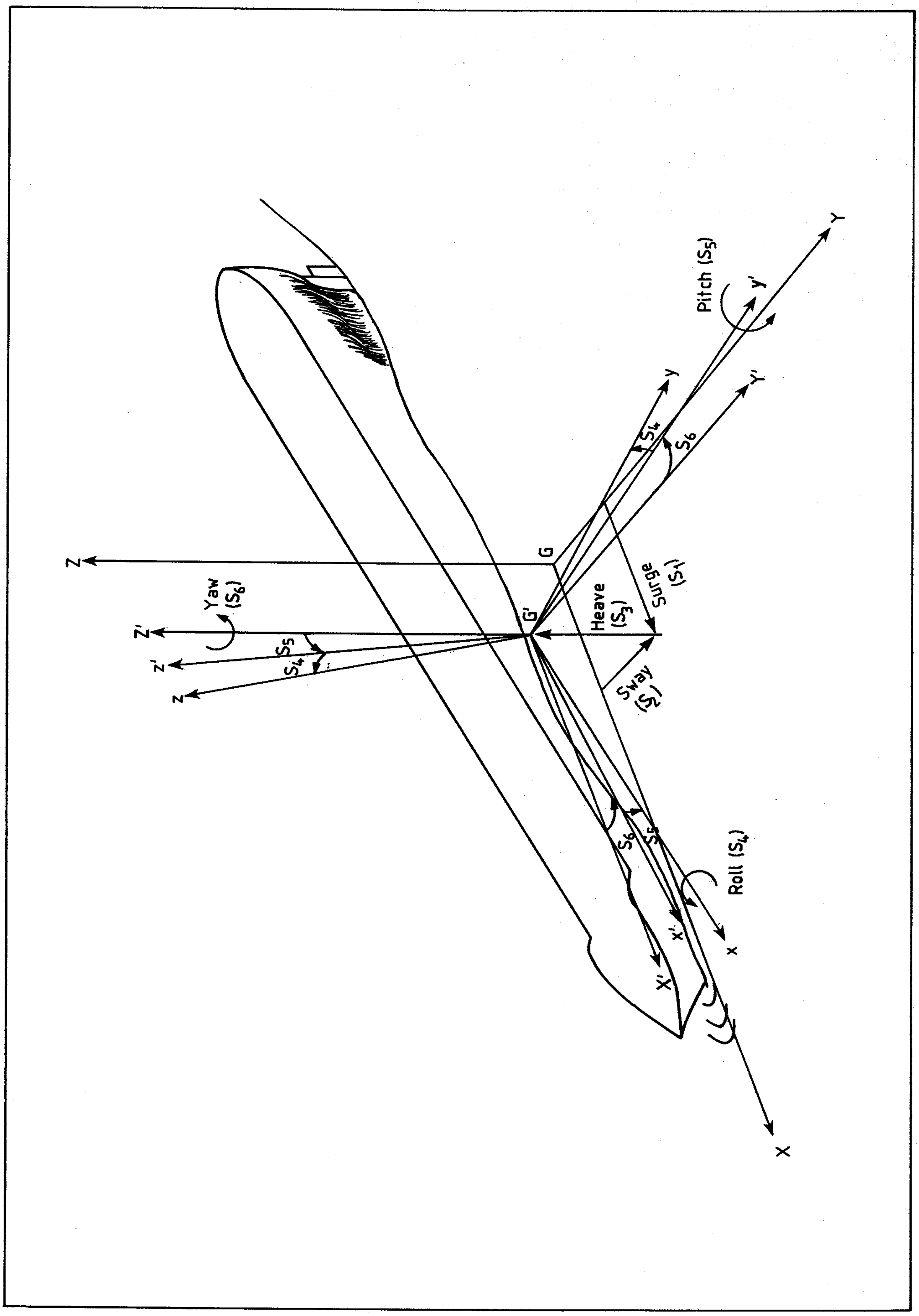

Fig 5 Definition sketch of vessel movements and coordinate systems

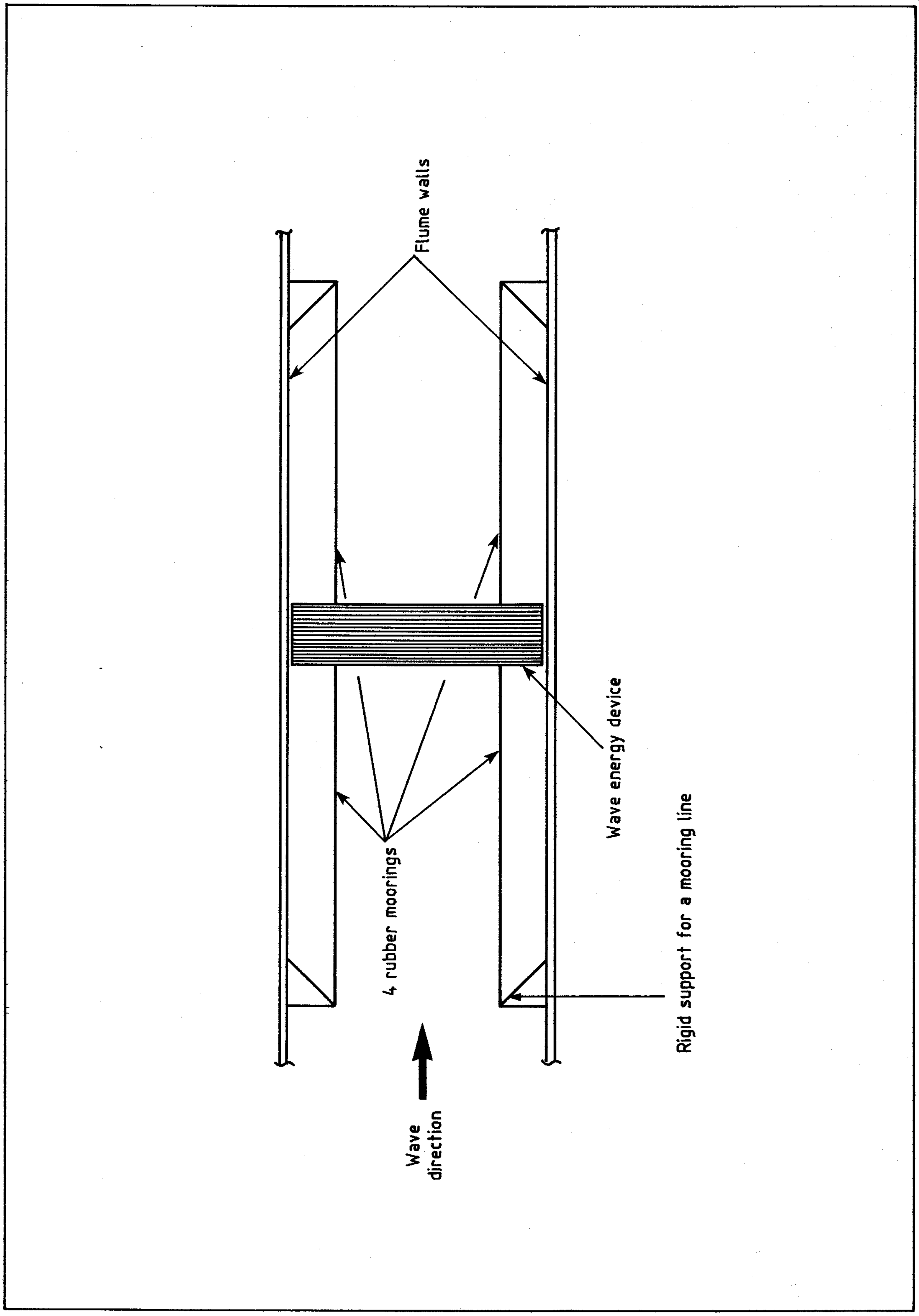

Fig 6 Layout of wave flume experiments

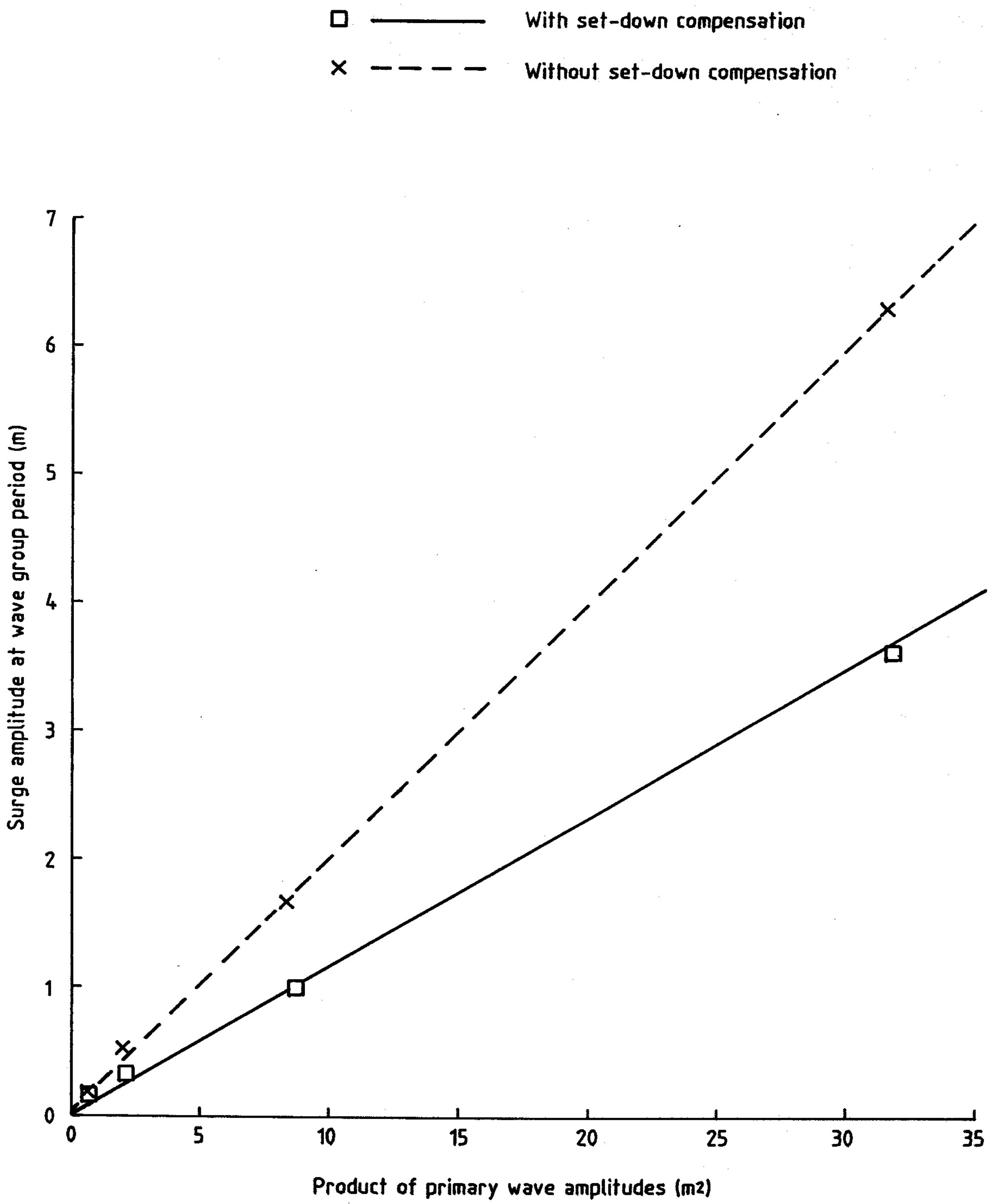

Fig 7 Experimental non-linear surge (T_1 = 20s, T_2 = 14.5s)

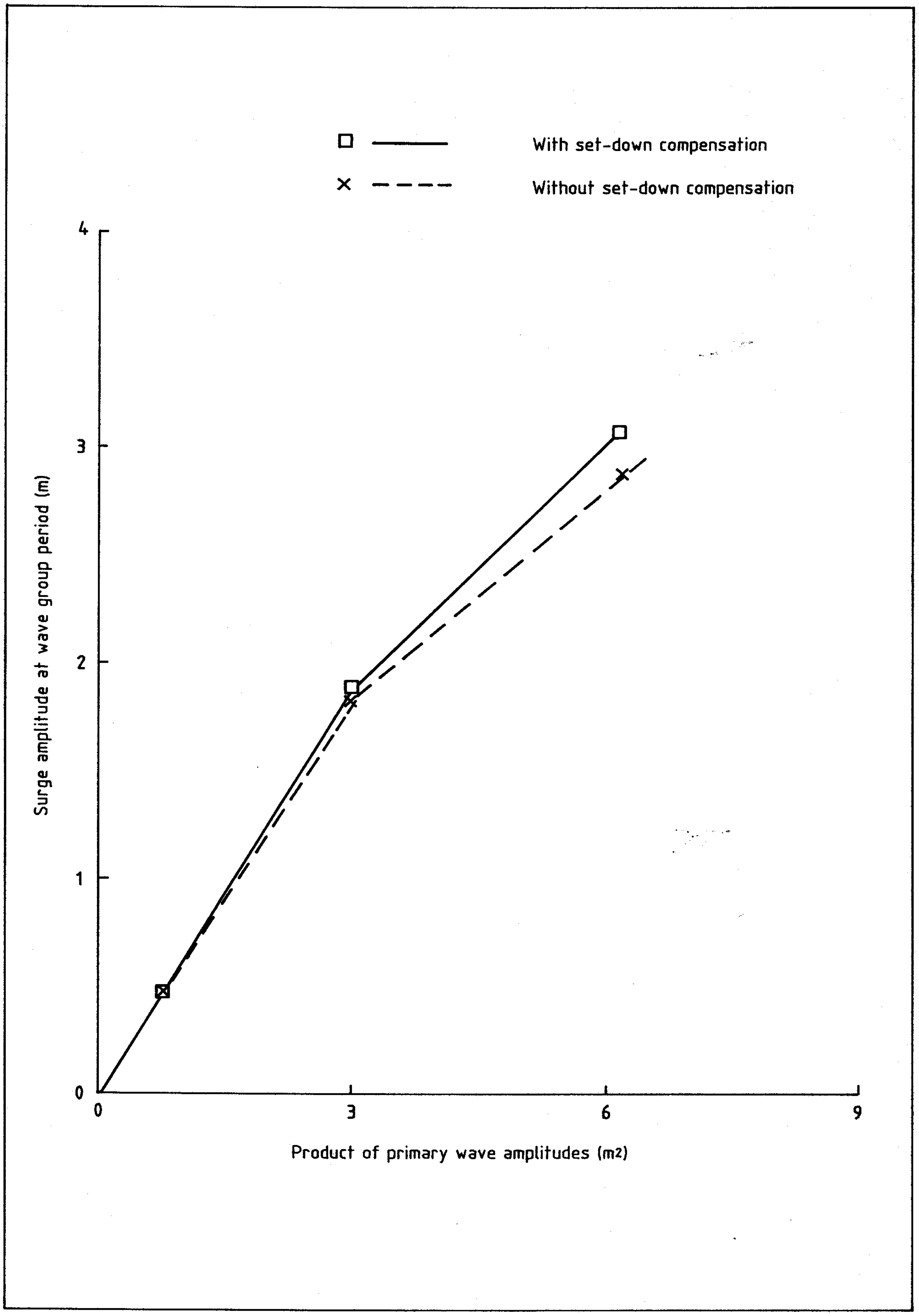

Fig 8 Experimental non-linear surge (T_1 = 12.4s, T_2 = 10.0s)

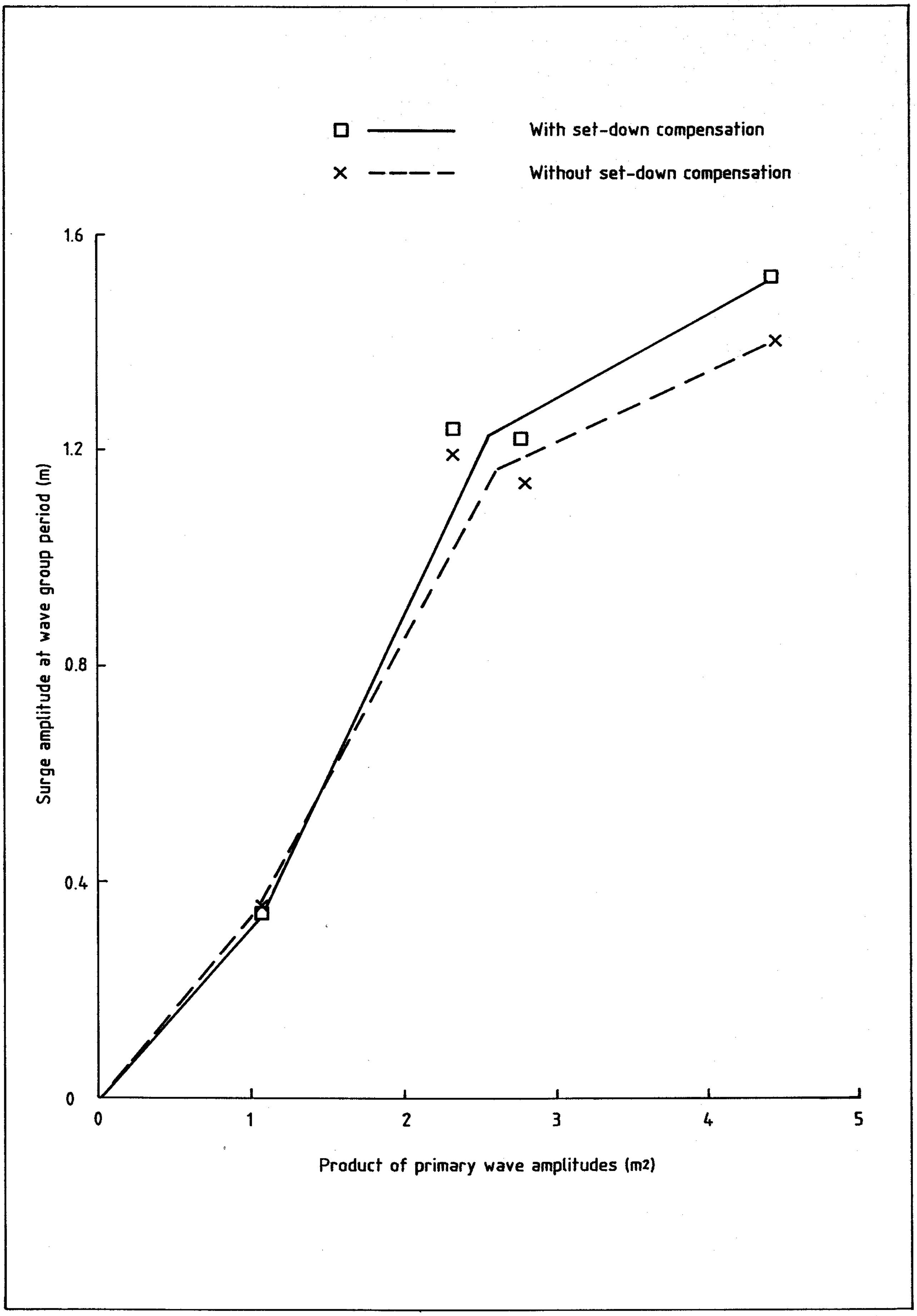

Fig 9 Experimental non-linear surge (T_1 = 10.0s, T_2 = 8.4s)

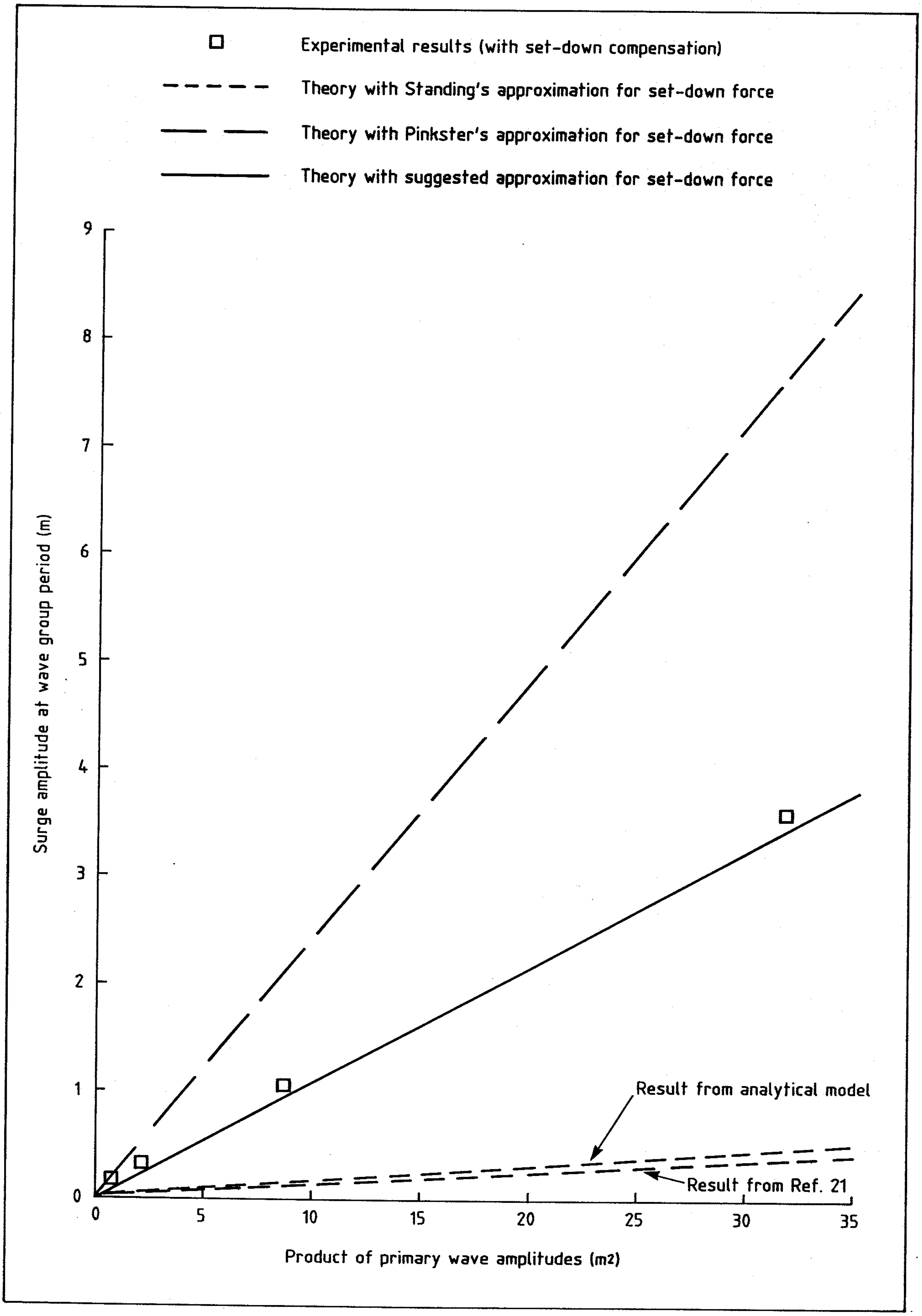

Fig 10 Experimental and theoretical non-linear surge (T_1 = 20s, T_2 = 14.5s)

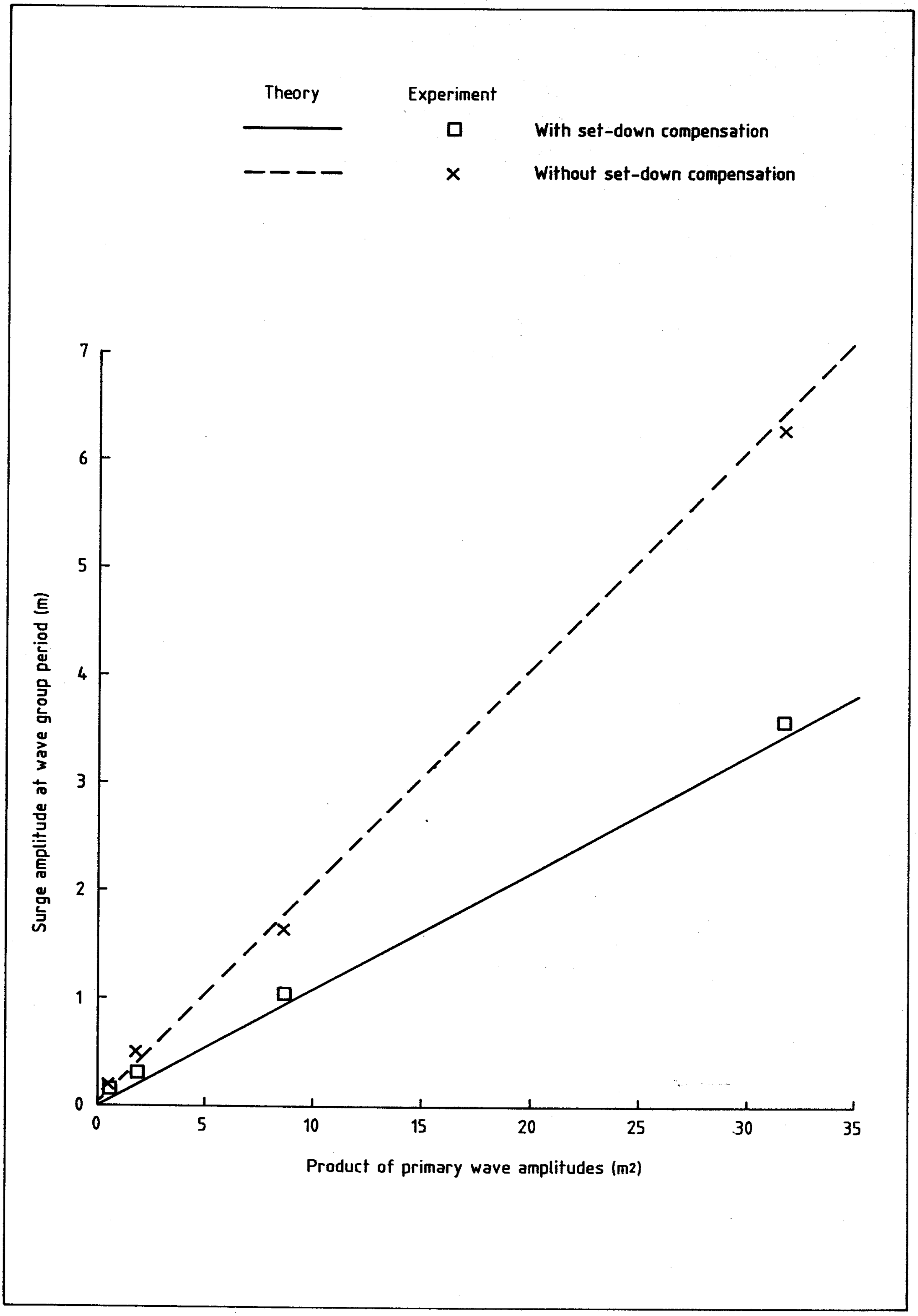

Fig 11 Comparison of experimental results with theory using suggested approximation for set-down force (T_1 = 20s, T_2 = 14.5s)