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EFFICIENT SOLUTION TO THE CURRENT-DEPTH DISPERSION EQUATION

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ABSTRACT

The current-depth dispersion equation is an analytically intractable equation which frequently occurs in computational models of wave-current interaction. Conventional techniques for the solution to this equation involve an iteration process which can be computationally time-consuming. An alternative quicker method involving interpolation in look-up tables is described in this report. The method is shown to predict the solution to an accuracy of 0.5% and is about three times faster in computational time than a method based on iteration.

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1 INTRODUCTION

Computational models of nearshore hydrodynamics often need to be run repeatedly for many input wave and current conditions, especially when long-term processes are investigated. In these circumstances, computational efficiency can become a major consideration.

A necessary calculation in such models is to determine the wavenumber, k, by solving the current-depth dispersion equation (Eq 1). The conventional technique involves an iteration method such as Newton-Raphson, but this is computationally time-consuming and can contribute significantly to the total run-time. A method of interpolation in a look-up table of values is a possible, and potentially quicker, alternative. Because there are two independent variables, a two-dimensional table is required. The number of values required for the table is therefore of manageable proportions, a few thousand at most. The aim of the work described in this report is to construct such a table enabling the current-depth dispersion equation to be solved by interpolation with acceptable accuracy for practical purposes, and with significantly less demand on computing time than a method based on iteration. An accuracy of 0.5% is regarded as sufficient for practical purposes, bearing in mind the uncertainties in the input quantities such as water depth. In order to avoid confusion between tables in this report and the look-up tables, the latter will be referred to as "L-tables".

A shortened version of this report has been published in Reference 1.

2 THEORY

The current-depth dispersion equation is

$$\omega_a - Uk \cos(\delta - \alpha) = (gk \tanh kh)^{1/2} \quad (1)$$

This equation is solved for the wavenumber, k, in terms of known values of the absolute wave angular frequency ω_a, the current speed U, the current direction δ, the wave orthogonal direction α, the acceleration due to gravity g, and the water depth h. Angle quantities are defined in Figure 1.

Equation 1 can be written in non-dimensional form as

$$A - Bx = (x \tanh x)^{1/2} \quad (2)$$

in which

$$x = kh \quad (3)$$

$$A = \frac{h\,\omega_a}{(gh)^{1/2}} \quad (4)$$

$$B = \frac{U \cos(\delta-\alpha)}{(gh)^{1/2}} \quad (5)$$

There are two independent variables A and B. The case B = 0 corresponds to the depth-only dispersion relation. There are analytical solutions to Equation 2 in deep water ($\tanh x = 1$),

$$x = \frac{1 + 2AB - (1 + 4AB)^{1/2}}{2B^2} \quad \text{for } B \neq 0$$

$$x = A^2 \quad \text{for } B = 0 \quad (6)$$

and in shallow water ($\tanh x = x$),

$$x = \frac{A}{1 + B} \tag{7}$$

Equation 2 can be regarded as

$$f(x) = g(\dot{x}) \tag{8}$$

in which

$$f(x) = A - Bx \tag{9}$$

$$g(\dot{x}) = (x \tanh x)^{1/2} \tag{10}$$

Graphs of these two functions are shown in Figure 2, and the solution to Equation 2 occurs at values of x where the graphs intersect. Four cases can be identified:

1. $B > 0$. One solution
2. $B = 0$. One solution (equivalent to the depth-only dispersion relation)
3. $B_c \leq B < 0$. Two solutions. B_c represents the critical slope where f(x) is a tangent to g(x). For reasons of continuity in B, the smaller of the two solutions is the correct one.
4. $B < B_c$. No solutions. This corresponds to the situation where the component of current velocity in the direction of k opposes, and is greater than, the wave phase velocity.

In the Newton-Raphson method for solving Equation 2, successive approximations are made to the correct solution for x by,

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \tag{11}$$

where $F(x) = A - Bx - (x \tanh x)^{1/2}$ (12)

$F'(x)$ is the derivative of $F(x)$ given by

$$F'(x) = -B - \frac{x(1 - \tanh^2 x) + \tanh x}{2(x \tanh x)^{1/2}} \qquad (13)$$

The subscript n denotes the nth approximation. The initial trial value of x must be chosen small enough to ensure convergence to the correct (ie smaller) solution in Case 3. This can be achieved by putting x initially to zero, but a closer approximation can be made by setting the initial x to the higher of the deep water and shallow water approximations (Eqs 6 and 7). For positive B, and small negative B, three or four iterations have typically been found necessary to obtain convergence to within 0.1%. For larger negative values of B near the critical value, more iterations are required.

3 CONSTRUCTION OF THE LOOK-UP TABLES

3.1 Range of A and B values

The ranges of A and B values for the look-up tables are chosen as

$$0 \leq A \leq 4$$

and $-4 \leq B \leq 4$

In practical applications to either prototype or laboratory situations it is very unlikely for A and B to lie outside these ranges. In the rare event of this occurring the full iteration method can be used.

The range of B at the negative end will be curtailed when the critical slope is exceeded. An expression

for the pairs of values of A and B at which this occurs can (in principle) be obtained by eliminating x between the equations $F(x) = 0$ and $F'(x) = 0$, but this is analytically intractable. However, two sufficient conditions which together cover most cases can be established.

The first condition is given by the gradient of $g(x) = (x \tanh x)^{\frac{1}{2}}$ at $x = 0$. The value of this gradient is one, corresponding to a value of $B = -1$. For higher x the gradient decreases monotonically (Fig 3) and therefore it can be concluded that if $B < -1$, there will be no solution, whatever the value of A. The second condition arises from considering the function $g(x) = x^{\frac{1}{2}}$ (ie putting $\tanh x = 1$ in the previously considered function). The value of $x^{\frac{1}{2}}$ at any x is larger than the value of $(x \tanh x)^{\frac{1}{2}}$ at the same x (Fig 3). This implies that those pairs of A and B which give a gradient which exceeds the critical gradient for the function $g(x) = x^{\frac{1}{2}}$ will also exceed the critical gradient for the function $g(x) = (x \tanh x)^{\frac{1}{2}}$, thereby resulting in no solution to the dispersion equation. Putting $\tanh x = 1$ into the equations $F(x) = 0$ and $F'(x) = 0$ results in two equations from which x can be eliminated analytically. Performing this calculation gives the condition $AB < -0.25$ for no solutions. Hence if either of the conditions

$$B < -1 \tag{14}$$

and $$AB < -0.25 \tag{15}$$

are satisfied, there will be no solution to the dispersion equation. There will, in addition, be a (relatively small) further range of pairs of A and B values where no solution exists. This is shown

graphically in Figure 3 where no solutions exist for the range of gradients, γ.

3.2 Intervals in A and B

The basic procedure for constructing the L-table of x values and then testing the accuracy of the interpolation procedure is as follows. Firstly, the full table is constructed using the Newton-Raphson method with what appears initially to be reasonable values of intervals in A and B so as to give a manageable total number of tabulated values (not exceeding five thousand). A second L-table of x values at the mid-points of the first table is then constructed by two methods, using Newton-Raphson iteration and by interpolation between the four adjacent values in the first table. The differences between the two methods indicates the errors involved in the interpolation method.

In constructing the initial L-table of x values, it was found that a more even spread of error differences was obtained if the table was split into three smaller tables. The first table consisted of positive and zero values of B, and all values of A. Intervals in both A and B were initially taken as 0.1, giving a total of 41 x 41 = 1681 entries. Error differences in this table showed a regular but small increase with increasing A. The variation with B was also regular, but much larger, giving errors at B = 0 of around 1-2%, while at large B errors were typically about 0.05%. It was apparent that a different function of B should be used in order to produce a more even spread of error differences. A new variable

$$C = \frac{1}{1 + B} \qquad (16)$$

was defined, ranging between 0.2 and 1.0 inclusive at intervals of 0.02 (corresponding to the B range of 4 to 0, and with the same number of divisions, 41). The form of this function of B is suggested by the shallow water approximation for x, Equation 7, and the effect is to produce finer intervals at small B and coarser intervals at large B. Using this new function a more even spread of error differences was obtained although still with a regular, but much smaller, increase in error difference towards lower B (higher C). Error differences were now within 0.5% everywhere.

L-tables 2 and 3 are for B less than zero. The need for two L-tables arises from the two different criteria for no solution, Equation 14 and Equation 15. A natural division occurs at A = 0.25. For A greater than 0.25 the no-solution condition given by Equation 15 will be reached first as B decreases from zero, while for A less than 0.25 the condition given by Equation 14 will be reached first. L-table 2 was constructed for values of A between 0 and 0.3, while L-table 3 covered the range of A from 0.3 to 4.0. The division was chosen at 0.3 rather than 0.25 because A was divided into intervals of 0.1; the exact choice of the division is not critical since around A = 0.25 there are a relatively large number of no-solutions not covered by criteria Equation 14 and Equation 15. From these two criteria, the logical choice of the B-variable in L-table 2 is simply B (ranging between 0 and -1) and in L-table 3, AB (ranging between 0 and -0.25). These ranges make maximum use of the available table space, avoiding as far as possible table entries corresponding to no solutions.

L-table 2 was initially constructed with four values of A between 0 and 0.3 inclusive at intervals of 0.1, and 50 values of B between 0 and -0.98 inclusive at intervals of 0.02. It was found that error

differences for values of B lower than about -0.6 increased rapidly until the no-solution region was reached. Accordingly values of B less than -0.6 were not tabulated, and the saving in table space was taken up by using a finer interval in A of 0.05, giving a total number of table entries of 31x7 = 217. The boundary of 0.5% errors followed an approximate straight line with equation

$$B = 0.4A - 0.61 \qquad (17)$$

Beyond this limiting B value the full iteration method was used to determine the correct solution or indicate if there is no solution.

L-table 3 consisted of 38 values of A between 0.3 and 4.0 inclusive at 0.1 intervals, and 25 values of AB between 0 and -0.24 inclusive at 0.01 intervals giving a total of 950 table entries. All error differences for A greater than 1.45 and for AB greater than -0.23 were found to be within 0.5%. For values of A below 1.45, the boundary of the 0.5% error difference roughly followed two straight lines with equations,

$$AB = -0.04A-0.172 \text{ for } 0.7<A\leq 1.45 \qquad (18)$$

$$\text{and } AB = -0.2714A-0.01 \text{ for } 0.3<A\leq 0.7 \qquad (19)$$

These equations, along with

$$AB = -0.23 \quad \text{for} \quad A \geq 1.45 \qquad (20)$$

were used to define the region within L-table 3 where interpolation could be used to an accuracy of 0.5%. Outside this region the full Newton-Raphson method would be employed.

The range and intervals of A and B are summarised in Table 1.

3.3 Programming details

The programming structure is shown as a flowchart in Figure 4 and the names and purposes of the subroutines are listed in the appendix. To increase the computational speed, the IF-THEN-ELSE structures used to choose the relevant table are nested in such a way as to minimise the number of lines of program actually executed for any given values of A and B. This can be seen from the fragment of pseudocode in Figure 5.

L-tables 1, 2 and 3 are shown in Tables 2, 3 and 4. Percentage differences of the mid-point values (between the interpolation and iteration methods) are shown in Tables 5, 6, 7 according to the formula:

$$\% \text{ difference} = (\text{Iter} - \text{Interp})/\text{Iter} \times 100\% \qquad (21)$$

Tests on computing speed were carried out separately for L-tables 1, 2 and 3. These tests involved generating the full set of mid-point values of each L-table using the iteration and interpolation methods. The iteration method was set up to give the same accuracy (0.5%) as the design accuracy of the interpolation method. The factors by which the interpolation method is faster than the iteration method are shown below.

L-Table No	Time taken to generate mid-points (s)		Speed Factor
	Iteration	Interpolation	
1	162.70	62.50	2.6
2	13.01	7.24	1.8
3	149.37	54.00	2.8

These times were obtained by running the L-table generation programs on a Compaq Deskpro 286 personal

computer running at 12 MHz. It can be seen that the time taken to generate L-table 1 is similar to that for L-table 3 using both methods, although the latter table is considerably smaller. In the iteration method, this is because the number of iterations required in the Newton-Raphson iterative process is greater in L-table 3. For the interpolation method, the reason is that there are some entries in L-table 3 that cannot be generated by interpolation and which therefore require iteration, whereas in L-table 1 there are no such entries. L-table 2 takes much less time to generate because of its smaller size.

As can be seen from the above table, the interpolation method is approximately three times faster.

4 CONCLUSIONS

In computational models of wave-current interaction solutions are required to the current-depth dispersion equation, Equation 1. The conventional method involves an iteration technique which can involve considerable computing effort. A different approach has been presented in this report which involves interpolation in look-up tables. A total of 2848 entries are needed in these tables, and the method is accurate to 0.5%, quite adequate for practical purposes. Time tests indicate the method is about three times quicker than one based on iteration.

5 REFERENCE

1. Southgate H N. "Aspects of computational efficiency in models of nearshore hydrodynamics", Int. Conf. on Computer Modelling in Ocean Eng. Venice, September 1988.

TABLES.

TABLE 1 Limits and intervals of the independent variables A and C in the L-tables.

		Lower Limit	Interval	Upper Limit	
L-Table 1: 1681 entries	A	0.0	(0.1)	4.0	
	C	0.2	(0.02)	1.0	[C = 1/(1 + B)]
L-Table 2: 217 entries	A	0.0	(0.05)	0.3	
	C	-1.0	(0.02)	0.0	[C = B]
L-Table 3: 950 entries	A	0.3	(0,1)	4.0	
	C	-0.24	(0.01)	0.0	[C = AB]

Total number of table entries = 2848

Table 2a L-Table 1

C \ A	.0000	.1000	.2000	.3000	.4000	.5000	.6000	.7000	.8000
.200	.0000	.0200	.0400	.0600	.0800	.1000	.1201	.1401	.1601
.220	.0000	.0220	.0440	.0660	.0880	.1100	.1321	.1541	.1762
.240	.0000	.0240	.0480	.0720	.0960	.1201	.1441	.1682	.1923
.260	.0000	.0260	.0520	.0780	.1040	.1301	.1562	.1823	.2084
.280	.0000	.0280	.0560	.0840	.1121	.1401	.1682	.1963	.2245
.300	.0000	.0300	.0600	.0900	.1201	.1502	.1803	.2105	.2407
.320	.0000	.0320	.0640	.0960	.1281	.1602	.1924	.2246	.2569
.340	.0000	.0340	.0680	.1021	.1361	.1703	.2045	.2388	.2731
.360	.0000	.0360	.0720	.1081	.1442	.1803	.2166	.2530	.2894
.380	.0000	.0380	.0760	.1141	.1522	.1904	.2287	.2672	.3058
.400	.0000	.0400	.0800	.1201	.1603	.2005	.2409	.2815	.3222
.420	.0000	.0420	.0840	.1261	.1683	.2106	.2531	.2958	.3386
.440	.0000	.0440	.0880	.1322	.1764	.2208	.2653	.3101	.3552
.460	.0000	.0460	.0921	.1382	.1845	.2309	.2776	.3245	.3718
.480	.0000	.0480	.0961	.1442	.1926	.2411	.2899	.3390	.3885
.500	.0000	.0500	.1001	.1503	.2007	.2513	.3022	.3535	.4053
.520	.0000	.0520	.1041	.1563	.2088	.2615	.3146	.3681	.4222
.540	.0000	.0540	.1081	.1624	.2169	.2718	.3270	.3828	.4392
.560	.0000	.0560	.1121	.1684	.2250	.2820	.3395	.3976	.4563
.580	.0000	.0580	.1162	.1745	.2332	.2924	.3521	.4124	.4736
.600	.0000	.0600	.1202	.1806	.2414	.3027	.3647	.4274	.4910
.620	.0000	.0620	.1242	.1867	.2496	.3131	.3773	.4424	.5086
.640	.0000	.0640	.1282	.1928	.2578	.3235	.3900	.4576	.5263
.660	.0000	.0660	.1323	.1989	.2660	.3340	.4028	.4729	.5442
.680	.0000	.0680	.1363	.2050	.2743	.3445	.4157	.4883	.5623
.700	.0000	.0700	.1403	.2111	.2826	.3550	.4287	.5038	.5807
.720	.0000	.0720	.1444	.2172	.2909	.3656	.4417	.5195	.5992
.740	.0000	.0740	.1484	.2234	.2992	.3763	.4549	.5354	.6180
.760	.0000	.0761	.1524	.2295	.3076	.3870	.4682	.5514	.6371
.780	.0000	.0781	.1565	.2357	.3160	.3978	.4815	.5676	.6564
.800	.0000	.0801	.1605	.2419	.3244	.4086	.4950	.5840	.6760
.820	.0000	.0821	.1646	.2480	.3329	.4196	.5086	.6006	.6960
.840	.0000	.0841	.1687	.2543	.3414	.4306	.5224	.6174	.7164
.860	.0000	.0861	.1727	.2605	.3499	.4416	.5363	.6345	.7371
.880	.0000	.0881	.1768	.2667	.3585	.4528	.5503	.6519	.7582
.900	.0000	.0901	.1809	.2730	.3671	.4640	.5645	.6695	.7798
.920	.0000	.0921	.1850	.2793	.3758	.4754	.5789	.6874	.8019
.940	.0000	.0941	.1890	.2856	.3845	.4868	.5935	.7056	.8245
.960	.0000	.0961	.1931	.2919	.3933	.4984	.6083	.7242	.8476
.980	.0000	.0982	.1972	.2982	.4021	.5100	.6233	.7432	.8715
1.000	.0000	.1002	.2013	.3046	.4110	.5218	.6385	.7626	.8960

Table 2b L-Table 1

C \ A	.9000	1.0000	1.1000	1.2000	1.3000	1.4000	1.5000	1.6000	1.7000
.200	.1802	.2003	.2204	.2405	.2606	.2807	.3009	.3211	.3413
.220	.1983	.2204	.2425	.2647	.2868	.3091	.3313	.3536	.3759
.240	.2164	.2405	.2647	.2889	.3132	.3375	.3618	.3862	.4106
.260	.2345	.2608	.2870	.3133	.3396	.3660	.3925	.4190	.4456
.280	.2527	.2810	.3093	.3377	.3662	.3947	.4234	.4521	.4808
.300	.2710	.3013	.3318	.3623	.3929	.4236	.4544	.4853	.5164
.320	.2893	.3217	.3543	.3870	.4197	.4526	.4857	.5189	.5522
.340	.3076	.3422	.3769	.4118	.4468	.4819	.5172	.5527	.5884
.360	.3260	.3627	.3996	.4367	.4740	.5114	.5491	.5869	.6251
.380	.3445	.3834	.4225	.4618	.5014	.5412	.5812	.6216	.6622
.400	.3631	.4042	.4455	.4872	.5291	.5713	.6138	.6566	.6998
.420	.3817	.4251	.4687	.5127	.5570	.6017	.6467	.6922	.7381
.440	.4005	.4461	.4921	.5385	.5853	.6325	.6802	.7283	.7769
.460	.4194	.4673	.5157	.5645	.6139	.6637	.7141	.7650	.8166
.480	.4384	.4887	.5395	.5909	.6428	.6954	.7486	.8024	.8570
.500	.4575	.5102	.5636	.6175	.6722	.7275	.7837	.8406	.8983
.520	.4768	.5320	.5879	.6445	.7020	.7603	.8194	.8795	.9406
.540	.4962	.5540	.6125	.6719	.7323	.7936	.8560	.9194	.9840
.560	.5158	.5762	.6374	.6997	.7631	.8276	.8933	.9603	1.0285
.580	.5356	.5986	.6627	.7280	.7945	.8624	.9316	1.0023	1.0744
.600	.5556	.6214	.6884	.7568	.8266	.8979	.9708	1.0454	1.1218
.620	.5759	.6445	.7145	.7861	.8593	.9343	1.0112	1.0900	1.1707
.640	.5964	.6679	.7411	.8160	.8929	.9717	1.0527	1.1359	1.2214
.660	.6171	.6917	.7681	.8466	.9272	1.0102	1.0956	1.1836	1.2741
.680	.6381	.7158	.7957	.8779	.9625	1.0499	1.1400	1.2330	1.3289
.700	.6594	.7404	.8239	.9099	.9989	1.0908	1.1860	1.2844	1.3862
.720	.6811	.7655	.8527	.9429	1.0363	1.1332	1.2337	1.3380	1.4461
.740	.7031	.7911	.8822	.9767	1.0750	1.1772	1.2835	1.3941	1.5091
.760	.7255	.8172	.9125	1.0116	1.1150	1.2229	1.3355	1.4530	1.5754
.780	.7484	.8440	.9436	1.0477	1.1565	1.2706	1.3900	1.5149	1.6455
.800	.7717	.8714	.9756	1.0850	1.1998	1.3204	1.4472	1.5804	1.7199
.820	.7955	.8995	1.0087	1.1237	1.2449	1.3727	1.5076	1.6497	1.7991
.840	.8198	.9284	1.0429	1.1639	1.2920	1.4278	1.5716	1.7236	1.8838
.860	.8447	.9582	1.0783	1.2059	1.3415	1.4860	1.6395	1.8025	1.9749
.880	.8702	.9889	1.1151	1.2497	1.3937	1.5476	1.7121	1.8873	2.0733
.900	.8965	1.0207	1.1534	1.2958	1.4488	1.6134	1.7900	1.9789	2.1801
.920	.9235	1.0536	1.1934	1.3443	1.5074	1.6837	1.8740	2.0784	2.2968
.940	.9514	1.0878	1.2353	1.3955	1.5698	1.7594	1.9651	2.1872	2.4253
.960	.9801	1.1234	1.2794	1.4499	1.6368	1.8414	2.0647	2.3069	2.5677
.980	1.0099	1.1607	1.3259	1.5080	1.7090	1.9308	2.1743	2.4399	2.7271
1.000	1.0408	1.1997	1.3752	1.5702	1.7874	2.0289	2.2961	2.5890	2.9073

Table 2c L-Table 1

C \ A	1.8000	1.9000	2.0000	2.1000	2.2000	2.3000	2.4000	2.5000	2.6000
.200	.3615	.3818	.4021	.4224	.4427	.4631	.4835	.5040	.5244
.220	.3982	.4206	.4430	.4655	.4880	.5105	.5331	.5557	.5784
.240	.4351	.4596	.4842	.5089	.5336	.5583	.5832	.6081	.6330
.260	.4723	.4990	.5258	.5527	.5796	.6067	.6338	.6610	.6883
.280	.5097	.5387	.5678	.5969	.6262	.6556	.6851	.7147	.7444
.300	.5475	.5788	.6102	.6417	.6734	.7052	.7371	.7692	.8014
.320	.5857	.6193	.6531	.6871	.7212	.7555	.7900	.8247	.8595
.340	.6243	.6604	.6966	.7331	.7698	.8067	.8439	.8812	.9188
.360	.6634	.7020	.7408	.7799	.8193	.8589	.8988	.9390	.9795
.380	.7031	.7443	.7858	.8276	.8697	.9122	.9550	.9981	1.0416
.400	.7434	.7873	.8315	.8762	.9213	.9667	1.0126	1.0588	1.1054
.420	.7843	.8311	.8783	.9259	.9740	1.0226	1.0716	1.1211	1.1711
.440	.8261	.8758	.9260	.9768	1.0281	1.0799	1.1324	1.1854	1.2389
.460	.8687	.9215	.9749	1.0289	1.0836	1.1390	1.1950	1.2516	1.3089
.480	.9123	.9683	1.0251	1.0826	1.1408	1.1999	1.2597	1.3202	1.3815
.500	.9569	1.0163	1.0766	1.1378	1.1999	1.2628	1.3266	1.3913	1.4568
.520	1.0026	1.0657	1.1297	1.1948	1.2609	1.3280	1.3961	1.4652	1.5353
.540	1.0497	1.1165	1.1846	1.2538	1.3242	1.3957	1.4684	1.5422	1.6171
.560	1.0981	1.1691	1.2413	1.3149	1.3899	1.4662	1.5438	1.6226	1.7026
.580	1.1481	1.2234	1.3002	1.3785	1.4584	1.5397	1.6225	1.7067	1.7923
.600	1.1998	1.2797	1.3613	1.4447	1.5298	1.6166	1.7050	1.7950	1.8865
.620	1.2535	1.3382	1.4250	1.5138	1.6046	1.6972	1.7917	1.8879	1.9858
.640	1.3092	1.3992	1.4916	1.5862	1.6831	1.7820	1.8830	1.9859	2.0907
.660	1.3672	1.4630	1.5613	1.6623	1.7656	1.8714	1.9794	2.0896	2.2017
.680	1.4278	1.5298	1.6346	1.7423	1.8528	1.9659	2.0815	2.1995	2.3197
.700	1.4914	1.5999	1.7118	1.8269	1.9450	2.0661	2.1900	2.3164	2.4453
.720	1.5581	1.6739	1.7934	1.9165	2.0430	2.1728	2.3056	2.4412	2.5795
.740	1.6284	1.7520	1.8799	2.0117	2.1473	2.2866	2.4291	2.5748	2.7233
.760	1.7028	1.8350	1.9719	2.1133	2.2589	2.4085	2.5617	2.7183	2.8781
.780	1.7817	1.9233	2.0702	2.2221	2.3787	2.5396	2.7045	2.8731	3.0452
.800	1.8657	2.0177	2.1756	2.3390	2.5077	2.6811	2.8589	3.0408	3.2264
.820	1.9556	2.1191	2.2891	2.4653	2.6473	2.8345	3.0266	3.2232	3.4239
.840	2.0522	2.2284	2.4119	2.6023	2.7990	3.0017	3.2097	3.4226	3.6402
.860	2.1565	2.3468	2.5454	2.7516	2.9650	3.1848	3.4106	3.6421	3.8787
.880	2.2696	2.4758	2.6913	2.9154	3.1474	3.3867	3.6327	3.8850	4.1433
.900	2.3931	2.6173	2.8519	3.0961	3.3493	3.6107	3.8799	4.1563	4.4395
.920	2.5287	2.7733	3.0297	3.2971	3.5746	3.8615	4.1574	4.4618	4.7742
.940	2.6788	2.9468	3.2283	3.5224	3.8282	4.1450	4.4724	4.8098	5.1570
.960	2.8463	3.1414	3.4522	3.7776	4.1168	4.4692	4.8343	5.2117	5.6010
.980	3.0349	3.3620	3.7075	4.0704	4.4499	4.8455	5.2568	5.6836	6.1256
1.000	3.2498	3.6152	4.0027	4.4113	4.8406	5.2903	5.7601	6.2500	6.7600

Table 2d L-Table 1

C \ A	2.7000	2.8000	2.9000	3.0000	3.1000	3.2000	3.3000	3.4000	3.5000
.200	.5449	.5655	.5861	.6067	.6273	.6480	.6687	.6895	.7103
.220	.6012	.6239	.6468	.6697	.6926	.7156	.7386	.7617	.7849
.240	.6580	.6831	.7083	.7335	.7588	.7842	.8096	.8351	.8607
.260	.7157	.7432	.7707	.7984	.8262	.8540	.8819	.9100	.9381
.280	.7742	.8042	.8343	.8645	.8948	.9252	.9558	.9864	1.0172
.300	.8338	.8664	.8990	.9319	.9649	.9980	1.0313	1.0647	1.0983
.320	.8946	.9298	.9652	1.0008	1.0366	1.0726	1.1087	1.1451	1.1816
.340	.9567	.9947	1.0330	1.0715	1.1102	1.1492	1.1884	1.2278	1.2674
.360	1.0202	1.0612	1.1025	1.1441	1.1859	1.2280	1.2704	1.3130	1.3559
.380	1.0854	1.1296	1.1740	1.2188	1.2639	1.3094	1.3551	1.4011	1.4475
.400	1.1525	1.1999	1.2477	1.2959	1.3445	1.3935	1.4428	1.4924	1.5424
.420	1.2216	1.2725	1.3239	1.3757	1.4279	1.4806	1.5337	1.5872	1.6410
.440	1.2930	1.3476	1.4027	1.4583	1.5145	1.5711	1.6282	1.6857	1.7437
.460	1.3668	1.4254	1.4845	1.5442	1.6045	1.6653	1.7266	1.7885	1.8508
.480	1.4435	1.5062	1.5695	1.6336	1.6982	1.7635	1.8294	1.8958	1.9628
.500	1.5232	1.5903	1.6582	1.7268	1.7962	1.8662	1.9369	2.0082	2.0801
.520	1.6062	1.6781	1.7508	1.8244	1.8987	1.9738	2.0496	2.1260	2.2032
.540	1.6930	1.7699	1.8478	1.9265	2.0062	2.0867	2.1679	2.2499	2.3326
.560	1.7838	1.8661	1.9495	2.0339	2.1192	2.2054	2.2925	2.3804	2.4690
.580	1.8792	1.9673	2.0565	2.1469	2.2383	2.3307	2.4239	2.5181	2.6130
.600	1.9795	2.0738	2.1693	2.2661	2.3640	2.4630	2.5629	2.6637	2.7654
.620	2.0853	2.1862	2.2886	2.3922	2.4971	2.6031	2.7101	2.8181	2.9271
.640	2.1972	2.3053	2.4149	2.5259	2.6382	2.7518	2.8665	2.9822	3.0990
.660	2.3158	2.4316	2.5491	2.6680	2.7884	2.9101	3.0330	3.1571	3.2822
.680	2.4419	2.5660	2.6919	2.8195	2.9486	3.0791	3.2109	3.3440	3.4782
.700	2.5764	2.7095	2.8446	2.9814	3.1199	3.2600	3.4014	3.5442	3.6883
.720	2.7202	2.8631	3.0081	3.1551	3.3038	3.4542	3.6062	3.7596	3.9145
.740	2.8745	3.0281	3.1840	3.3419	3.5018	3.6636	3.8271	3.9921	4.1588
.760	3.0407	3.2060	3.3737	3.5438	3.7160	3.8902	4.0663	4.2442	4.4239
.780	3.2204	3.3985	3.5793	3.7627	3.9485	4.1365	4.3266	4.5187	4.7128
.800	3.4155	3.6078	3.8032	4.0013	4.2022	4.4055	4.6113	4.8194	5.0296
.820	3.6285	3.8366	4.0481	4.2628	4.4806	4.7012	4.9245	5.1505	5.3791
.840	3.8621	4.0880	4.3178	4.5512	4.7880	5.0281	5.2715	5.5179	5.7672
.860	4.1201	4.3662	4.6167	4.8714	5.1300	5.3926	5.6588	5.9287	6.2020
.880	4.4072	4.6764	4.9507	5.2299	5.5138	5.8023	6.0952	6.3924	6.6937
.900	4.7293	5.0253	5.3274	5.6353	5.9489	6.2679	6.5922	6.9217	7.2563
.920	5.0944	5.4222	5.7571	6.0992	6.4481	6.8036	7.1657	7.5342	7.9089
.940	5.5136	5.8794	6.2541	6.6375	7.0295	7.4298	7.8382	8.2547	8.6790
.960	6.0021	6.4147	6.8385	7.2735	7.7193	8.1758	8.6429	9.1203	9.6079
.980	6.5826	7.0545	7.5411	8.0422	8.5577	9.0875	9.6314	10.1892	10.7609
1.000	7.2900	7.8400	8.4100	9.0000	9.6100	10.2400	10.8900	11.5600	12.2500

Table 2e L-Table 1

C \ A	3.6000	3.7000	3.8000	3.9000	4.0000
.200	.7312	.7521	.7730	.7939	.8149
.220	.8081	.8313	.8547	.8780	.9014
.240	.8864	.9121	.9379	.9638	.9898
.260	.9663	.9947	1.0231	1.0516	1.0802
.280	1.0481	1.0792	1.1103	1.1416	1.1730
.300	1.1320	1.1659	1.1999	1.2341	1.2684
.320	1.2183	1.2552	1.2922	1.3295	1.3668
.340	1.3072	1.3472	1.3875	1.4279	1.4686
.360	1.3990	1.4424	1.4860	1.5299	1.5740
.380	1.4941	1.5410	1.5882	1.6357	1.6834
.400	1.5927	1.6434	1.6944	1.7456	1.7972
.420	1.6953	1.7499	1.8049	1.8601	1.9158
.440	1.8021	1.8609	1.9201	1.9797	2.0396
.460	1.9136	1.9769	2.0406	2.1046	2.1691
.480	2.0303	2.0982	2.1667	2.2355	2.3048
.500	2.1525	2.2255	2.2990	2.3729	2.4473
.520	2.2809	2.3592	2.4380	2.5173	2.5971
.540	2.4159	2.4999	2.5844	2.6695	2.7550
.560	2.5583	2.6483	2.7389	2.8300	2.9217
.580	2.7087	2.8051	2.9022	2.9998	3.0981
.600	2.8679	2.9712	3.0752	3.1798	3.2851
.620	3.0369	3.1475	3.2589	3.3710	3.4838
.640	3.2167	3.3352	3.4546	3.5747	3.6956
.660	3.4084	3.5355	3.6635	3.7923	3.9219
.680	3.6135	3.7498	3.8871	4.0254	4.1645
.700	3.8336	3.9800	4.1275	4.2760	4.4255
.720	4.0706	4.2280	4.3866	4.5463	4.7072
.740	4.3269	4.4964	4.6672	4.8393	5.0127
.760	4.6051	4.7880	4.9724	5.1582	5.3454
.780	4.9088	5.1066	5.3060	5.5072	5.7099
.800	5.2420	5.4565	5.6729	5.8913	6.1115
.820	5.6101	5.8434	6.0791	6.3170	6.5570
.840	6.0194	6.2744	6.5321	6.7924	7.0552
.860	6.4787	6.7587	7.0419	7.3281	7.6174
.880	6.9991	7.3084	7.6215	7.9383	8.2588
.900	7.5957	7.9399	8.2888	8.6422	9.0000
.920	8.2896	8.6762	9.0686	9.4667	9.8703
.940	9.1110	9.5505	9.9974	10.4515	10.9127
.960	10.1056	10.6132	11.1304	11.6573	12.1936
.980	11.3464	11.9454	12.5579	13.1838	13.8228
1.000	12.9600	13.6900	14.4400	15.2100	16.0000

Table 3 L-Table 2 including out of range error denoted by 99.9999

C \ A	.0000	.0500	.1000	.1500	.2000	.2500	.3000
-.600	.0000	.1258	.2569	.4005	.5700	.8071	99.9999
-.580	.0000	.1197	.2437	.3776	.5307	.7253	1.0935
-.560	.0000	.1142	.2319	.3576	.4979	.6667	.9069
-.540	.0000	.1092	.2213	.3398	.4699	.6208	.8143
-.520	.0000	.1046	.2116	.3239	.4456	.5830	.7493
-.500	.0000	.1003	.2027	.3096	.4241	.5509	.6986
-.480	.0000	.0964	.1946	.2966	.4048	.5230	.6570
-.460	.0000	.0928	.1872	.2847	.3875	.4984	.6217
-.440	.0000	.0895	.1803	.2738	.3718	.4765	.5911
-.420	.0000	.0864	.1739	.2638	.3574	.4567	.5641
-.400	.0000	.0835	.1680	.2545	.3443	.4388	.5401
-.380	.0000	.0808	.1624	.2459	.3321	.4224	.5184
-.360	.0000	.0782	.1573	.2378	.3208	.4074	.4987
-.340	.0000	.0759	.1524	.2303	.3104	.3935	.4807
-.320	.0000	.0736	.1478	.2233	.3006	.3806	.4641
-.300	.0000	.0715	.1436	.2167	.2915	.3686	.4488
-.280	.0000	.0695	.1395	.2105	.2829	.3574	.4346
-.260	.0000	.0676	.1357	.2046	.2748	.3469	.4214
-.240	.0000	.0659	.1321	.1991	.2673	.3371	.4090
-.220	.0000	.0642	.1287	.1938	.2601	.3278	.3974
-.200	.0000	.0626	.1254	.1889	.2533	.3191	.3865
-.180	.0000	.0610	.1223	.1842	.2469	.3108	.3762
-.160	.0000	.0596	.1194	.1797	.2408	.3030	.3665
-.140	.0000	.0582	.1166	.1755	.2350	.2956	.3573
-.120	.0000	.0569	.1139	.1714	.2295	.2885	.3486
-.100	.0000	.0556	.1114	.1675	.2243	.2818	.3404
-.080	.0000	.0544	.1089	.1638	.2193	.2754	.3325
-.060	.0000	.0532	.1066	.1603	.2145	.2693	.3250
-.040	.0000	.0521	.1044	.1569	.2099	.2635	.3179
-.020	.0000	.0510	.1022	.1537	.2055	.2580	.3111
.000	.0000	.0500	.1002	.1506	.2013	.2526	.3046

Table 4a L-Table 3 including out of range errors denoted by 99.9999

C \ A	.3000	.4000	.5000	.6000	.7000	.8000	.9000	1.0000	1.1000
-.240	99.9999	99.9999	99.9999	99.9999	1.9521	2.1302	2.4459	2.8720	3.3993
-.230	99.9999	99.9999	99.9999	1.5894	1.6809	1.8871	2.1746	2.5434	2.9944
-.220	99.9999	99.9999	1.4300	1.4001	1.5334	1.7367	2.0022	2.3341	2.7364
-.210	99.9999	99.9999	1.2217	1.2843	1.4291	1.6265	1.8751	2.1804	2.5475
-.200	99.9999	1.1981	1.1069	1.1993	1.3479	1.5392	1.7745	2.0593	2.3992
-.190	99.9999	1.0116	1.0250	1.1317	1.2813	1.4670	1.6912	1.9596	2.2777
-.180	99.9999	.9078	.9609	1.0755	1.2248	1.4054	1.6204	1.8751	2.1752
-.170	.9500	.8338	.9082	1.0274	1.1757	1.3518	1.5588	1.8020	2.0868
-.160	.7905	.7759	.8633	.9854	1.1324	1.3044	1.5045	1.7377	2.0094
-.150	.6986	.7284	.8244	.9482	1.0937	1.2618	1.4558	1.6804	1.9406
-.140	.6329	.6882	.7900	.9147	1.0587	1.2233	1.4118	1.6288	1.8790
-.130	.5818	.6533	.7592	.8844	1.0268	1.1882	1.3718	1.5819	1.8231
-.120	.5401	.6227	.7314	.8566	.9975	1.1559	1.3350	1.5390	1.7722
-.110	.5051	.5954	.7061	.8311	.9704	1.1260	1.3011	1.4995	1.7255
-.100	.4750	.5708	.6829	.8075	.9453	1.0983	1.2696	1.4629	1.6823
-.090	.4488	.5486	.6615	.7856	.9219	1.0724	1.2402	1.4289	1.6423
-.080	.4257	.5282	.6417	.7651	.8999	1.0481	1.2127	1.3971	1.6049
-.070	.4050	.5096	.6232	.7460	.8793	1.0253	1.1869	1.3673	1.5701
-.060	.3865	.4923	.6060	.7280	.8599	1.0038	1.1626	1.3393	1.5373
-.050	.3697	.4764	.5898	.7110	.8415	.9835	1.1396	1.3128	1.5065
-.040	.3544	.4615	.5746	.6949	.8241	.9642	1.1178	1.2878	1.4774
-.030	.3404	.4476	.5603	.6797	.8076	.9459	1.0972	1.2641	1.4498
-.020	.3275	.4346	.5468	.6653	.7918	.9285	1.0775	1.2416	1.4237
-.010	.3156	.4225	.5340	.6515	.7769	.9118	1.0587	1.2201	1.3989
.000	.3046	.4110	.5218	.6385	.7626	.8960	1.0408	1.1997	1.3752

Table 4b L-Table 3 including out of range errors denoted by 99.9999

C \ A	1.2000	1.3000	1.4000	1.5000	1.6000	1.7000	1.8000	1.9000	2.0000
-.240	4.0131	4.6983	5.4455	6.2502	7.1111	8.0278	9.0000	10.0277	11.1111
-.230	3.5218	4.1155	4.7664	5.4695	6.2225	7.0245	7.8751	8.7745	9.7224
-.220	3.2077	3.7411	4.3291	4.9660	5.6490	6.3769	7.1491	7.9655	8.8260
-.210	2.9774	3.4659	4.0066	4.5942	5.2252	5.8982	6.6123	7.3673	8.1632
-.200	2.7967	3.2495	3.7525	4.3007	4.8904	5.5198	6.1880	6.8945	7.6393
-.190	2.6488	3.0722	3.5439	4.0594	4.6149	5.2083	5.8386	6.5052	7.2079
-.180	2.5243	2.9229	3.3680	3.8555	4.3818	4.9447	5.5428	6.1755	6.8426
-.170	2.4173	2.7945	3.2165	3.6797	4.1807	4.7171	5.2873	5.8907	6.5270
-.160	2.3237	2.6823	3.0840	3.5258	4.0044	4.5174	5.0632	5.6409	6.2501
-.150	2.2409	2.5831	2.9667	3.3894	3.8481	4.3402	4.8641	5.4189	6.0041
-.140	2.1667	2.4943	2.8619	3.2673	3.7079	4.1813	4.6856	5.2198	5.7833
-.130	2.0997	2.4143	2.7673	3.1571	3.5814	4.0376	4.5241	5.0396	5.5836
-.120	2.0388	2.3416	2.6814	3.0570	3.4662	3.9069	4.3770	4.8755	5.4017
-.110	1.9831	2.2751	2.6028	2.9654	3.3609	3.7871	4.2423	4.7251	5.2349
-.100	1.9317	2.2140	2.5307	2.8812	3.2639	3.6769	4.1182	4.5865	5.0812
-.090	1.8842	2.1576	2.4640	2.8034	3.1744	3.5749	4.0033	4.4583	4.9389
-.080	1.8400	2.1052	2.4022	2.7313	3.0912	3.4803	3.8967	4.3391	4.8067
-.070	1.7987	2.0564	2.3447	2.6642	3.0138	3.3921	3.7972	4.2280	4.6833
-.060	1.7601	2.0107	2.2910	2.6015	2.9415	3.3096	3.7042	4.1240	4.5679
-.050	1.7239	1.9679	2.2406	2.5427	2.8737	3.2323	3.6170	4.0264	4.4596
-.040	1.6897	1.9276	2.1933	2.4875	2.8100	3.1596	3.5350	3.9346	4.3576
-.030	1.6574	1.8897	2.1487	2.4356	2.7500	3.0912	3.4576	3.8480	4.2615
-.020	1.6268	1.8538	2.1066	2.3865	2.6934	3.0265	3.3845	3.7662	4.1705
-.010	1.5978	1.8197	2.0668	2.3401	2.6398	2.9653	3.3154	3.6887	4.0844
.000	1.5702	1.7874	2.0289	2.2961	2.5890	2.9073	3.2498	3.6152	4.0027

Table 4c L-Table 3 including out of range errors denoted by 99.9999

C \ A	2.1000	2.2000	2.3000	2.4000	2.5000	2.6000	2.7000	2.8000	2.9000
-.240	12.2499	13.4444	14.6943	15.9999	17.3609	18.7775	20.2497	21.7774	23.3606
-.230	10.7189	11.7641	12.8579	14.0002	15.1912	16.4308	17.7191	19.0559	20.4413
-.220	9.7307	10.6795	11.6724	12.7095	13.7907	14.9160	16.0854	17.2990	18.5567
-.210	8.9999	9.8776	10.7959	11.7551	12.7551	13.7959	14.8776	16.0000	17.1633
-.200	8.4223	9.2435	10.1029	11.0005	11.9363	12.9103	13.9225	14.9731	16.0617
-.190	7.9467	8.7215	9.5324	10.3793	11.2623	12.1813	13.1363	14.1273	15.1544
-.180	7.5439	8.2795	9.0493	9.8533	10.6915	11.5639	12.4706	13.4114	14.3865
-.170	7.1960	7.8976	8.6319	9.3988	10.1984	11.0306	11.8954	12.7928	13.7229
-.160	6.8906	7.5625	8.2656	9.0000	9.7656	10.5625	11.3906	12.2500	13.1406
-.150	6.6194	7.2648	7.9402	8.6457	9.3812	10.1467	10.9422	11.7677	12.6233
-.140	6.3760	6.9977	7.6482	8.3278	9.0362	9.7736	10.5398	11.3350	12.1591
-.130	6.1558	6.7559	7.3840	8.0401	8.7241	9.4359	10.1757	10.9435	11.7391
-.120	5.9551	6.5357	7.1433	7.7780	8.4396	9.1283	9.8440	10.5867	11.3563
-.110	5.7712	6.3338	6.9226	7.5376	8.1789	8.8462	9.5398	10.2596	11.0055
-.100	5.6016	6.1477	6.7192	7.3162	7.9385	8.5863	9.2595	9.9581	10.6821
-.090	5.4447	5.9754	6.5309	7.1111	7.7161	8.3457	9.0000	9.6790	10.3827
-.080	5.2988	5.8153	6.3558	6.9205	7.5092	8.1220	8.7588	9.4196	10.1044
-.070	5.1627	5.6659	6.1925	6.7427	7.3162	7.9132	8.5337	9.1775	9.8447
-.060	5.0354	5.5260	6.0397	6.5762	7.1356	7.7179	8.3230	8.9509	9.6017
-.050	4.9158	5.3947	5.8961	6.4199	6.9660	7.5344	8.1252	8.7382	9.3735
-.040	4.8033	5.2712	5.7610	6.2728	6.8064	7.3618	7.9390	8.5379	9.1587
-.030	4.6971	5.1546	5.6336	6.1340	6.6557	7.1988	7.7632	8.3489	8.9559
-.020	4.5967	5.0443	5.5130	6.0027	6.5133	7.0447	7.5971	8.1702	8.7642
-.010	4.5016	4.9398	5.3987	5.8783	6.3783	6.8987	7.4396	8.0008	8.5825
.000	4.4113	4.8406	5.2903	5.7601	6.2500	6.7600	7.2900	7.8400	8.4100

Table 4d L-Table 3 including out of range errors denoted by 99.9999

C \ A	3.0000	3.1000	3.2000	3.3000	3.4000	3.5000	3.6000	3.7000	3.8000
-.240	24.9994	26.6937	28.4436	30.2500	32.1111	34.0278	36.0000	38.0278	40.1111
-.230	21.8754	23.3580	24.8893	26.4692	28.0977	29.7748	31.5005	33.2748	35.0978
-.220	19.8586	21.2045	22.5946	24.0289	25.5072	27.0297	28.5963	30.2071	31.8619
-.210	18.3673	19.6122	20.8980	22.2245	23.5918	25.0000	26.4490	27.9388	29.4694
-.200	17.1885	18.3535	19.5567	20.7980	22.0776	23.3954	24.7514	26.1456	27.5779
-.190	16.2176	17.3168	18.4520	19.6232	20.8307	22.0741	23.3535	24.6689	26.0204
-.180	15.3957	16.4392	17.5169	18.6288	19.7749	20.9552	22.1698	23.4185	24.7014
-.170	14.6856	15.6810	16.7090	17.7696	18.8629	19.9887	21.1473	22.3384	23.5622
-.160	14.0625	15.0156	16.0000	17.0156	18.0625	19.1406	20.2500	21.3906	22.5624
-.150	13.5089	14.4245	15.3701	16.3458	17.3514	18.3871	19.4528	20.5485	21.6742
-.140	13.0121	13.8941	14.8049	15.7447	16.7133	17.7109	18.7375	19.7929	20.8772
-.130	12.5626	13.4141	14.2935	15.2008	16.1360	17.0991	18.0902	19.1092	20.1560
-.120	12.1530	12.9768	13.8275	14.7052	15.6099	16.5416	17.5004	18.4861	19.4989
-.110	11.7775	12.5758	13.4002	14.2508	15.1276	16.0306	16.9597	17.9150	18.8964
-.100	11.4315	12.2063	13.0065	13.8321	14.6831	15.5595	16.4614	17.3886	18.3412
-.090	11.1111	11.8642	12.6420	13.4444	14.2716	15.1235	16.0000	16.9012	17.8272
-.080	10.8133	11.5462	12.3031	13.0841	13.8890	14.7181	15.5711	16.4482	17.3493
-.070	10.5354	11.2494	11.9869	12.7478	13.5321	14.3398	15.1710	16.0255	16.9034
-.060	10.2753	10.9717	11.6910	12.4331	13.1980	13.9858	14.7964	15.6298	16.4861
-.050	10.0311	10.7109	11.4131	12.1376	12.8843	13.6534	14.4447	15.2584	16.0943
-.040	9.8012	10.4655	11.1516	11.8594	12.5891	13.3405	14.1137	14.9087	15.7254
-.030	9.5842	10.2338	10.9047	11.5969	12.3104	13.0452	13.8013	14.5787	15.3774
-.020	9.3791	10.0148	10.6713	11.3487	12.0469	12.7660	13.5059	14.2666	15.0482
-.010	9.1846	9.8071	10.4501	11.1134	11.7971	12.5013	13.2259	13.9708	14.7362
.000	9.0000	9.6100	10.2400	10.8900	11.5600	12.2500	12.9600	13.6900	14.4400

Table 4e L-Table 3 including out of range errors denoted by 99.9999

	A	
	3.9000	4.0000
-.240	42.2500	44.4444
-.230	36.9693	38.8895
-.220	33.5610	35.3041
-.210	31.0408	32.6531
-.200	29.0485	30.5573
-.190	27.4079	28.8315
-.180	26.0186	27.3703
-.170	24.8186	26.1077
-.160	23.7656	24.9999
-.150	22.8300	24.0158
-.140	21.9905	23.1326
-.130	21.2308	22.3336
-.120	20.5386	21.6054
-.110	19.9041	20.9379
-.100	19.3192	20.3227
-.090	18.7778	19.7531
-.080	18.2744	19.2236
-.070	17.8048	18.7296
-.060	17.3652	18.2671
-.050	16.9525	17.8330
-.040	16.5640	17.4243
-.030	16.1974	17.0386
-.020	15.8506	16.6739
-.010	15.5220	16.3282
.000	15.2100	16.0000

Table 5a % differences for L-Table 1.

A	.0500	.1500	.2500	.3500	.4500	.5500	.6500	.7500	.8500
.210	.0012	.0012	.0013	.0014	.0016	.0018	.0020	.0023	.0026
.230	.0015	.0016	.0017	.0018	.0020	.0022	.0025	.0028	.0031
.250	.0020	.0020	.0021	.0023	.0024	.0027	.0030	.0033	.0037
.270	.0025	.0025	.0026	.0028	.0030	.0033	.0036	.0039	.0043
.290	.0031	.0031	.0032	.0034	.0036	.0039	.0042	.0046	.0050
.310	.0037	.0038	.0039	.0041	.0043	.0046	.0049	.0053	.0058
.330	.0045	.0046	.0047	.0049	.0051	.0054	.0058	.0062	.0067
.350	.0054	.0055	.0056	.0058	.0060	.0063	.0067	.0071	.0076
.370	.0064	.0064	.0066	.0068	.0070	.0073	.0077	.0082	.0086
.390	.0074	.0075	.0077	.0079	.0081	.0085	.0089	.0093	.0098
.410	.0087	.0087	.0089	.0091	.0093	.0097	.0101	.0106	.0111
.430	.0100	.0101	.0102	.0104	.0107	.0110	.0114	.0119	.0125
.450	.0114	.0115	.0117	.0119	.0122	.0125	.0129	.0134	.0140
.470	.0130	.0131	.0132	.0135	.0138	.0141	.0145	.0151	.0156
.490	.0147	.0148	.0150	.0152	.0155	.0159	.0163	.0168	.0174
.510	.0166	.0167	.0169	.0171	.0174	.0178	.0182	.0187	.0193
.530	.0187	.0187	.0189	.0191	.0195	.0198	.0203	.0208	.0214
.550	.0208	.0209	.0211	.0213	.0216	.0220	.0225	.0231	.0237
.570	.0232	.0233	.0234	.0237	.0240	.0244	.0249	.0255	.0261
.590	.0257	.0258	.0260	.0263	.0266	.0270	.0275	.0281	.0288
.610	.0284	.0285	.0287	.0290	.0293	.0298	.0303	.0309	.0316
.630	.0313	.0314	.0316	.0319	.0323	.0328	.0333	.0340	.0347
.650	.0344	.0345	.0347	.0350	.0354	.0359	.0366	.0372	.0380
.670	.0376	.0378	.0380	.0383	.0388	.0393	.0400	.0408	.0416
.690	.0411	.0412	.0415	.0419	.0424	.0430	.0437	.0446	.0455
.710	.0448	.0449	.0452	.0457	.0462	.0469	.0477	.0487	.0496
.730	.0487	.0488	.0492	.0497	.0503	.0511	.0520	.0530	.0542
.750	.0528	.0530	.0534	.0539	.0547	.0556	.0566	.0578	.0591
.770	.0571	.0574	.0578	.0584	.0593	.0603	.0616	.0629	.0644
.790	.0617	.0620	.0625	.0632	.0642	.0654	.0668	.0684	.0701
.810	.0665	.0668	.0674	.0683	.0695	.0709	.0726	.0744	.0764
.830	.0715	.0719	.0726	.0737	.0750	.0767	.0787	.0809	.0832
.850	.0768	.0772	.0781	.0793	.0809	.0829	.0853	.0878	.0906
.870	.0824	.0829	.0838	.0853	.0872	.0896	.0923	.0954	.0987
.890	.0882	.0888	.0899	.0916	.0939	.0967	.1000	.1037	.1077
.910	.0943	.0950	.0963	.0983	.1010	.1043	.1082	.1127	.1174
.930	.1007	.1015	.1030	.1053	.1085	.1125	.1171	.1224	.1282
.950	.1073	.1082	.1100	.1128	.1165	.1212	.1268	.1332	.1402
.970	.1142	.1153	.1174	.1206	.1250	.1305	.1372	.1449	.1534
.990	.1215	.1227	.1252	.1289	.1341	.1406	.1485	.1578	.1684

Table 5b % differences for L-Table 1

	A +	.9500	1.0500	1.1500	1.2500	1.3500	1.4500	1.5500	1.6500	1.7500
.210	+	.0030	.0034	.0038	.0043	.0048	.0053	.0059	.0065	.0071
.230	+	.0035	.0039	.0044	.0049	.0055	.0060	.0067	.0073	.0079
C .250	+	.0041	.0046	.0051	.0056	.0062	.0068	.0074	.0081	.0088
.270	+	.0048	.0052	.0058	.0064	.0070	.0076	.0083	.0090	.0098
.290	+	.0055	.0060	.0065	.0072	.0078	.0085	.0092	.0100	.0107
.310	+	.0063	.0068	.0074	.0080	.0087	.0094	.0102	.0110	.0118
.330	+	.0072	.0077	.0084	.0090	.0097	.0105	.0113	.0121	.0129
.350	+	.0081	.0087	.0094	.0101	.0108	.0116	.0124	.0132	.0141
.370	+	.0092	.0098	.0105	.0112	.0120	.0127	.0136	.0144	.0153
.390	+	.0104	.0110	.0117	.0124	.0132	.0140	.0149	.0158	.0167
.410	+	.0117	.0123	.0130	.0138	.0146	.0154	.0163	.0172	.0181
.430	+	.0131	.0138	.0145	.0152	.0160	.0169	.0178	.0187	.0197
.450	+	.0146	.0153	.0160	.0168	.0176	.0185	.0194	.0203	.0213
.470	+	.0163	.0169	.0177	.0185	.0193	.0202	.0211	.0221	.0230
.490	+	.0181	.0187	.0195	.0203	.0212	.0221	.0230	.0239	.0249
.510	+	.0200	.0207	.0215	.0223	.0232	.0241	.0250	.0259	.0269
.530	+	.0221	.0228	.0236	.0244	.0253	.0262	.0271	.0281	.0290
.550	+	.0244	.0251	.0259	.0268	.0276	.0285	.0295	.0304	.0313
.570	+	.0268	.0276	.0284	.0292	.0301	.0310	.0320	.0329	.0338
.590	+	.0295	.0303	.0311	.0319	.0329	.0338	.0347	.0356	.0364
.610	+	.0324	.0332	.0340	.0349	.0358	.0367	.0376	.0385	.0393
.630	+	.0355	.0363	.0372	.0381	.0390	.0399	.0407	.0416	.0423
.650	+	.0388	.0397	.0406	.0415	.0424	.0433	.0441	.0449	.0456
.670	+	.0425	.0434	.0443	.0453	.0462	.0470	.0478	.0485	.0491
.690	+	.0464	.0474	.0484	.0494	.0503	.0511	.0519	.0525	.0532
.710	+	.0507	.0518	.0528	.0538	.0548	.0556	.0563	.0570	.0571
.730	+	.0553	.0565	.0576	.0587	.0597	.0605	.0612	.0613	.0617
.750	+	.0604	.0617	.0629	.0641	.0651	.0660	.0662	.0666	.0666
.770	+	.0659	.0673	.0687	.0700	.0711	.0716	.0721	.0721	.0719
.790	+	.0718	.0735	.0751	.0764	.0779	.0782	.0784	.0783	.0777
.810	+	.0784	.0803	.0821	.0838	.0846	.0853	.0854	.0850	.0841
.830	+	.0856	.0879	.0899	.0914	.0928	.0933	.0931	.0924	.0912
.850	+	.0934	.0962	.0988	.1004	.1018	.1022	.1018	.1006	.0990
.870	+	.1021	.1054	.1080	.1105	.1118	.1121	.1114	.1099	.1078
.890	+	.1117	.1159	.1190	.1217	.1232	.1233	.1223	.1202	.1177
.910	+	.1224	.1269	.1313	.1344	.1360	.1360	.1346	.1320	.1291
.930	+	.1344	.1400	.1452	.1488	.1507	.1505	.1486	.1456	.1423
.950	+	.1480	.1547	.1609	.1654	.1676	.1673	.1649	.1614	.1580
.970	+	.1623	.1714	.1790	.1846	.1872	.1868	.1839	.1802	.1771
.990	+	.1793	.1904	.2000	.2070	.2102	.2097	.2067	.2032	.2011

Table 5c % differences for L-Table 1

A	1.8500	1.9500	2.0500	2.1500	2.2500	2.3500	2.4500	2.5500	2.6500
.210	.0078	.0084	.0091	.0099	.0106	.0114	.0121	.0129	.0137
.230	.0086	.0094	.0101	.0109	.0116	.0125	.0133	.0141	.0150
.250	.0096	.0103	.0111	.0119	.0127	.0136	.0144	.0153	.0162
.270	.0105	.0113	.0121	.0130	.0139	.0147	.0156	.0165	.0174
.290	.0116	.0124	.0132	.0141	.0150	.0159	.0168	.0178	.0187
.310	.0126	.0135	.0144	.0153	.0162	.0171	.0181	.0190	.0199
.330	.0138	.0147	.0156	.0165	.0174	.0184	.0193	.0203	.0212
.350	.0150	.0159	.0168	.0178	.0187	.0197	.0207	.0216	.0226
.370	.0163	.0172	.0182	.0191	.0201	.0210	.0220	.0230	.0239
.390	.0176	.0186	.0195	.0205	.0215	.0225	.0234	.0244	.0253
.410	.0191	.0200	.0210	.0220	.0229	.0239	.0249	.0258	.0267
.430	.0206	.0216	.0226	.0236	.0245	.0255	.0264	.0273	.0282
.450	.0223	.0232	.0242	.0252	.0261	.0271	.0280	.0289	.0297
.470	.0240	.0250	.0259	.0269	.0278	.0287	.0296	.0305	.0313
.490	.0259	.0268	.0278	.0287	.0296	.0305	.0314	.0322	.0329
.510	.0278	.0288	.0297	.0306	.0315	.0324	.0331	.0339	.0346
.530	.0299	.0309	.0318	.0327	.0335	.0343	.0350	.0357	.0363
.550	.0322	.0331	.0340	.0348	.0356	.0363	.0370	.0376	.0381
.570	.0347	.0355	.0363	.0371	.0378	.0384	.0390	.0395	.0400
.590	.0372	.0380	.0387	.0394	.0401	.0406	.0412	.0418	.0418
.610	.0400	.0407	.0414	.0420	.0426	.0432	.0433	.0437	.0441
.630	.0430	.0436	.0442	.0448	.0450	.0454	.0457	.0460	.0462
.650	.0462	.0469	.0470	.0474	.0478	.0480	.0482	.0484	.0486
.670	.0498	.0500	.0504	.0506	.0507	.0508	.0509	.0510	.0511
.690	.0533	.0536	.0538	.0538	.0538	.0538	.0538	.0538	.0538
.710	.0574	.0574	.0574	.0573	.0571	.0570	.0569	.0568	.0568
.730	.0617	.0615	.0613	.0610	.0607	.0604	.0602	.0602	.0602
.750	.0663	.0660	.0655	.0650	.0646	.0642	.0640	.0640	.0641
.770	.0714	.0708	.0701	.0694	.0689	.0685	.0683	.0683	.0685
.790	.0770	.0760	.0751	.0743	.0737	.0733	.0731	.0732	.0736
.810	.0830	.0818	.0807	.0798	.0791	.0788	.0787	.0790	.0796
.830	.0897	.0883	.0870	.0860	.0853	.0852	.0854	.0860	.0869
.850	.0972	.0955	.0940	.0931	.0926	.0927	.0933	.0943	.0957
.870	.1056	.1037	.1023	.1015	.1013	.1019	.1030	.1045	.1065
.890	.1153	.1132	.1119	.1115	.1119	.1131	.1149	.1173	.1200
.910	.1264	.1245	.1236	.1238	.1251	.1272	.1300	.1334	.1373
.930	.1396	.1381	.1380	.1393	.1417	.1452	.1495	.1544	.1597
.950	.1557	.1551	.1563	.1591	.1634	.1689	.1752	.1823	.1899
.970	.1758	.1768	.1801	.1854	.1924	.2009	.2103	.2207	.2317
.990	.2017	.2054	.2121	.2216	.2331	.2462	.2601	.2758	.2926

Table 5d % differences for L-Table 1

C	A	2.7500	2.8500	2.9500	3.0500	3.1500	3.2500	3.3500	3.4500	3.5500
.210	+	.0146	.0154	.0162	.0171	.0179	.0188	.0197	.0205	.0214
.230	+	.0158	.0167	.0176	.0184	.0193	.0202	.0211	.0219	.0228
.250	+	.0171	.0180	.0189	.0198	.0207	.0215	.0224	.0233	.0242
.270	+	.0183	.0192	.0201	.0211	.0220	.0228	.0237	.0246	.0255
.290	+	.0196	.0205	.0214	.0223	.0233	.0241	.0250	.0259	.0267
.310	+	.0209	.0218	.0227	.0236	.0245	.0253	.0262	.0271	.0279
.330	+	.0222	.0231	.0240	.0249	.0258	.0266	.0275	.0283	.0291
.350	+	.0235	.0244	.0253	.0262	.0271	.0279	.0287	.0294	.0302
.370	+	.0248	.0257	.0266	.0275	.0283	.0291	.0298	.0306	.0313
.390	+	.0262	.0271	.0279	.0288	.0296	.0303	.0310	.0317	.0324
.410	+	.0276	.0285	.0293	.0301	.0308	.0315	.0322	.0329	.0335
.430	+	.0290	.0299	.0307	.0314	.0321	.0328	.0334	.0340	.0345
.450	+	.0305	.0313	.0321	.0328	.0334	.0340	.0346	.0351	.0356
.470	+	.0320	.0328	.0335	.0341	.0347	.0352	.0357	.0362	.0367
.490	+	.0336	.0343	.0349	.0355	.0360	.0365	.0370	.0374	.0378
.510	+	.0353	.0359	.0364	.0369	.0374	.0379	.0382	.0386	.0390
.530	+	.0369	.0375	.0380	.0384	.0388	.0392	.0396	.0400	.0404
.550	+	.0387	.0391	.0396	.0400	.0405	.0405	.0408	.0412	.0416
.570	+	.0406	.0407	.0411	.0415	.0419	.0422	.0425	.0428	.0430
.590	+	.0423	.0427	.0430	.0433	.0436	.0438	.0441	.0444	.0447
.610	+	.0443	.0446	.0449	.0451	.0454	.0456	.0459	.0462	.0465
.630	+	.0465	.0467	.0469	.0471	.0474	.0477	.0479	.0482	.0485
.650	+	.0487	.0489	.0491	.0493	.0496	.0499	.0502	.0505	.0509
.670	+	.0512	.0514	.0516	.0518	.0521	.0524	.0528	.0532	.0536
.690	+	.0539	.0541	.0543	.0546	.0549	.0553	.0558	.0562	.0567
.710	+	.0569	.0572	.0574	.0578	.0582	.0587	.0592	.0598	.0604
.730	+	.0604	.0606	.0610	.0614	.0620	.0626	.0632	.0639	.0647
.750	+	.0643	.0647	.0651	.0657	.0664	.0671	.0680	.0688	.0697
.770	+	.0688	.0693	.0700	.0708	.0716	.0725	.0735	.0746	.0756
.790	+	.0741	.0749	.0757	.0767	.0778	.0790	.0802	.0815	.0828
.810	+	.0804	.0814	.0826	.0839	.0853	.0867	.0882	.0898	.0913
.830	+	.0880	.0894	.0909	.0926	.0944	.0961	.0980	.0999	.1018
.850	+	.0973	.0991	.1011	.1033	.1055	.1077	.1100	.1124	.1147
.870	+	.1087	.1112	.1138	.1165	.1193	.1222	.1251	.1280	.1310
.890	+	.1231	.1263	.1297	.1333	.1369	.1406	.1443	.1481	.1518
.910	+	.1414	.1458	.1503	.1550	.1598	.1646	.1695	.1745	.1794
.930	+	.1654	.1714	.1776	.1839	.1904	.1969	.2036	.2103	.2170
.950	+	.1979	.2062	.2149	.2237	.2328	.2420	.2513	.2608	.2705
.970	+	.2435	.2558	.2685	.2815	.2943	.3080	.3219	.3362	.3507
.990	+	.3105	.3293	.3488	.3691	.3902	.4120	.4346	.4578	.4816

Table 5e % differences for L-Table 1

C	A: 3.6500	3.7500	3.8500	3.9500
.210	.0222	.0231	.0240	.0248
.230	.0237	.0245	.0254	.0262
.250	.0250	.0259	.0267	.0275
.270	.0263	.0271	.0279	.0287
.290	.0275	.0283	.0291	.0298
.310	.0287	.0295	.0302	.0309
.330	.0298	.0305	.0312	.0319
.350	.0309	.0316	.0322	.0328
.370	.0320	.0326	.0332	.0337
.390	.0330	.0335	.0341	.0346
.410	.0340	.0345	.0350	.0354
.430	.0350	.0355	.0359	.0363
.450	.0360	.0364	.0368	.0372
.470	.0371	.0374	.0378	.0381
.490	.0382	.0385	.0388	.0391
.510	.0393	.0396	.0399	.0402
.530	.0407	.0407	.0410	.0412
.550	.0419	.0421	.0424	.0426
.570	.0433	.0436	.0439	.0441
.590	.0449	.0452	.0455	.0458
.610	.0468	.0471	.0474	.0477
.630	.0489	.0492	.0496	.0500
.650	.0513	.0517	.0521	.0526
.670	.0541	.0545	.0550	.0555
.690	.0573	.0578	.0584	.0590
.710	.0610	.0617	.0624	.0631
.730	.0654	.0662	.0670	.0678
.750	.0706	.0715	.0725	.0734
.770	.0767	.0778	.0789	.0800
.790	.0841	.0853	.0867	.0880
.810	.0929	.0944	.0960	.0975
.830	.1037	.1056	.1075	.1093
.850	.1170	.1194	.1217	.1240
.870	.1339	.1368	.1397	.1426
.890	.1556	.1593	.1631	.1668
.910	.1844	.1893	.1942	.1992
.930	.2237	.2305	.2373	.2441
.950	.2803	.2901	.2999	.3097
.970	.3656	.3805	.3956	.4108
.990	.5061	.5312	.5569	.5832

Table 6 % differences for L-Table 2 including out of range error denoted by 99.9999. Solid line denotes boundary of 0.5% errors

C	A →	.0250	.0750	.1250	.1750	.2250	.2750
-.590	+	.5209	.5675	.6862	.9719	1.9419	99.9999
-.570	+	.4531	.4873	.5706	.7534	1.2360	4.3398
-.550	+	.3969	.4222	.4823	.6053	.8838	1.8032
-.530	+	.3498	.3689	.4131	.4989	.6734	1.1138
-.510	+	.3100	.3246	.3577	.4193	.5353	.7844
-.490	+	.2762	.2875	.3126	.3579	.4384	.5938
-.470	+	.2472	.2561	.2754	.3094	.3670	.4697
-.450	+	.2223	.2292	.2443	.2702	.3126	.3836
-.430	+	.2006	.2062	.2180	.2381	.2699	.3208
-.410	+	.1818	.1863	.1957	.2114	.2357	.2732
-.390	+	.1653	.1689	.1765	.1889	.2077	.2360
-.370	+	.1508	.1537	.1598	.1697	.1846	.2063
-.350	+	.1380	.1404	.1453	.1533	.1651	.1820
-.330	+	.1266	.1286	.1326	.1392	.1486	.1620
-.310	+	.1165	.1181	.1215	.1268	.1345	.1451
-.290	+	.1075	.1088	.1116	.1160	.1222	.1308
-.270	+	.0994	.1005	.1028	.1064	.1116	.1186
-.250	+	.0921	.0930	.0950	.0980	.1023	.1080
-.230	+	.0855	.0863	.0879	.0905	.0941	.0988
-.210	+	.0796	.0803	.0816	.0838	.0868	.0907
-.190	+	.0742	.0748	.0760	.0778	.0803	.0836
-.170	+	.0693	.0698	.0708	.0723	.0744	.0772
-.150	+	.0648	.0653	.0662	.0674	.0692	.0716
-.130	+	.0608	.0611	.0619	.0630	.0645	.0665
-.110	+	.0571	.0574	.0580	.0589	.0603	.0620
-.090	+	.0536	.0539	.0544	.0553	.0564	.0579
-.070	+	.0505	.0507	.0512	.0519	.0529	.0541
-.050	+	.0476	.0478	.0482	.0488	.0497	.0507
-.030	+	.0449	.0451	.0455	.0460	.0467	.0477
-.010	+	.0425	.0426	.0429	.0434	.0440	.0448

Table 7a % differences for L-Table 3 including out of range errors denoted by 99.9999. Solid line denotes boundary of 0.5% errors

C \ A	.3500	.4500	.5500	.6500	.7500	.8500	.9500	1.0500	1.1500
-.235	99.9999	99.9999	99.9999	99.9999	1.9524	1.4117	1.2995	1.2292	1.1530
-.225	99.9999	99.9999	99.9999	1.6137	.9450	.8210	.7878	.7473	.6843
-.215	99.9999	99.9999	2.6493	.9195	.6669	.6145	.6013	.5746	.5230
-.205	99.9999	99.9999	1.3109	.6676	.5340	.5069	.5025	.4848	.4432
-.195	99.9999	3.6849	.8916	.5365	.4535	.4401	.4397	.4285	.3950
-.185	99.9999	1.8968	.6813	.4543	.3994	.3926	.3965	.3885	.3620
-.175	99.9999	1.2688	.5537	.3974	.3597	.3576	.3625	.3583	.3374
-.165	5.1846	.9397	.4675	.3552	.3291	.3301	.3364	.3354	.3176
-.155	2.8844	.7363	.4051	.3226	.3047	.3078	.3150	.3153	.3029
-.145	1.9283	.5987	.3577	.2965	.2845	.2892	.2970	.2987	.2884
-.135	1.3914	.4987	.3206	.2750	.2677	.2735	.2816	.2844	.2760
-.125	1.0492	.4241	.2913	.2569	.2532	.2599	.2683	.2719	.2655
-.115	.8140	.3665	.2666	.2416	.2407	.2480	.2565	.2608	.2560
-.105	.6447	.3210	.2465	.2296	.2298	.2375	.2461	.2508	.2473
-.095	.5192	.2847	.2296	.2181	.2202	.2281	.2368	.2418	.2395
-.085	.4243	.2554	.2154	.2079	.2115	.2198	.2283	.2337	.2323
-.075	.3518	.2316	.2032	.1991	.2037	.2122	.2207	.2262	.2256
-.065	.2961	.2123	.1929	.1915	.1977	.2054	.2137	.2194	.2194
-.055	.2534	.1966	.1841	.1848	.1913	.1991	.2073	.2130	.2136
-.045	.2210	.1839	.1766	.1789	.1854	.1933	.2014	.2072	.2081
-.035	.1967	.1737	.1702	.1736	.1801	.1879	.1960	.2017	.2031
-.025	.1791	.1657	.1648	.1689	.1755	.1832	.1910	.1967	.1983
-.015	.1668	.1595	.1603	.1647	.1713	.1795	.1863	.1919	.1938
-.005	.1591	.1548	.1564	.1610	.1675	.1752	.1820	.1875	.1896

Table 7b % differences for L-Table 3 including out of range errors denoted by 99.9999. Solid line denotes boundary of 0.5% errors

C \ A	1.2500	1.3500	1.4500	1.5500	1.6500	1.7500	1.8500	1.9500	2.0500
-.235	1.0887	1.0459	1.0193	1.0017	.9887	.9783	.9697	.9625	.9562
-.225	.6212	.5748	.5428	.5231	.5097	.4994	.4907	.4834	.4772
-.215	.4636	.4152	.3823	.3609	.3467	.3363	.3277	.3189	.3125
-.205	.3895	.3415	.3068	.2837	.2681	.2566	.2476	.2402	.2339
-.195	.3471	.3006	.2649	.2404	.2237	.2118	.2026	.1951	.1888
-.185	.3197	.2754	.2393	.2136	.1960	.1835	.1741	.1665	.1601
-.175	.3003	.2586	.2226	.1960	.1775	.1645	.1548	.1470	.1406
-.165	.2855	.2466	.2111	.1839	.1647	.1510	.1410	.1331	.1267
-.155	.2735	.2374	.2029	.1753	.1554	.1412	.1308	.1228	.1162
-.145	.2632	.2302	.1967	.1691	.1486	.1339	.1231	.1149	.1082
-.135	.2555	.2241	.1920	.1644	.1434	.1282	.1172	.1087	.1020
-.125	.2471	.2186	.1881	.1609	.1396	.1239	.1125	.1038	.0970
-.115	.2397	.2149	.1850	.1581	.1366	.1205	.1087	.0999	.0929
-.105	.2332	.2102	.1820	.1559	.1342	.1178	.1057	.0967	.0896
-.095	.2272	.2059	.1801	.1538	.1324	.1157	.1033	.0940	.0868
-.085	.2216	.2022	.1775	.1521	.1310	.1140	.1013	.0918	.0844
-.075	.2164	.1986	.1754	.1515	.1296	.1127	.0997	.0900	.0825
-.065	.2114	.1952	.1734	.1500	.1291	.1114	.0985	.0885	.0808
-.055	.2067	.1920	.1715	.1490	.1281	.1104	.0974	.0872	.0794
-.045	.2022	.1889	.1696	.1479	.1274	.1103	.0964	.0861	.0782
-.035	.1980	.1858	.1678	.1470	.1268	.1095	.0960	.0851	.0772
-.025	.1940	.1829	.1660	.1460	.1263	.1091	.0953	.0848	.0761
-.015	.1902	.1801	.1642	.1451	.1258	.1087	.0948	.0839	.0758
-.005	.1865	.1773	.1625	.1442	.1254	.1084	.0944	.0834	.0749

Table 7c % differences for L-Table 3 including out of range errors denoted by 99.9999. Solid line denotes boundary of 0.5% errors

C \ A	2.1500	2.2500	2.3500	2.4500	2.5500	2.6500	2.7500	2.8500	2.9500
-.235	.9508	.9461	.9420	.9383	.9351	.9322	.9296	.9272	.9250
-.225	.4718	.4671	.4630	.4594	.4562	.4534	.4509	.4486	.4465
-.215	.3074	.3030	.2989	.2953	.2921	.2893	.2867	.2845	.2824
-.205	.2289	.2246	.2207	.2172	.2126	.2096	.2074	.2054	.2033
-.195	.1834	.1786	.1746	.1709	.1677	.1649	.1627	.1608	.1589
-.185	.1547	.1500	.1459	.1423	.1391	.1362	.1337	.1314	.1294
-.175	.1352	.1305	.1264	.1227	.1196	.1167	.1141	.1119	.1098
-.165	.1212	.1165	.1123	.1087	.1055	.1027	.1001	.0979	.0958
-.155	.1108	.1060	.1019	.0983	.0951	.0922	.0897	.0874	.0854
-.145	.1027	.0980	.0939	.0902	.0870	.0842	.0816	.0794	.0773
-.135	.0964	.0916	.0875	.0839	.0807	.0778	.0753	.0731	.0710
-.125	.0913	.0866	.0824	.0788	.0756	.0727	.0702	.0679	.0659
-.115	.0872	.0824	.0783	.0746	.0714	.0686	.0660	.0638	.0617
-.105	.0838	.0789	.0748	.0711	.0680	.0651	.0626	.0603	.0582
-.095	.0810	.0761	.0719	.0682	.0650	.0622	.0597	.0574	.0553
-.085	.0785	.0736	.0694	.0658	.0626	.0597	.0572	.0549	.0528
-.075	.0765	.0715	.0673	.0637	.0604	.0576	.0550	.0528	.0507
-.065	.0748	.0697	.0655	.0618	.0586	.0558	.0532	.0509	.0489
-.055	.0732	.0682	.0639	.0602	.0570	.0542	.0516	.0493	.0473
-.045	.0719	.0668	.0625	.0588	.0556	.0527	.0502	.0479	.0459
-.035	.0708	.0656	.0613	.0576	.0543	.0515	.0489	.0467	.0446
-.025	.0698	.0646	.0602	.0565	.0533	.0504	.0478	.0456	.0435
-.015	.0688	.0636	.0593	.0555	.0523	.0494	.0468	.0446	.0425
-.005	.0686	.0628	.0584	.0547	.0514	.0485	.0460	.0437	.0416

Table 7d % differences for L-Table 3 including out of range errors denoted by 99.9999. Solid line denotes boundary of 0.5% errors

C \ A	3.0500	3.1500	3.2500	3.3500	3.4500	3.5500	3.6500	3.7500	3.8500
-.235	.9231	.9213	.9206	.9200	.9188	.9177	.9168	.9159	.9151
-.225	.4447	.4430	.4415	.4401	.4388	.4377	.4366	.4356	.4347
-.215	.2806	.2789	.2774	.2760	.2747	.2736	.2725	.2715	.2706
-.205	.2015	.1998	.1983	.1969	.1956	.1945	.1934	.1924	.1915
-.195	.1572	.1541	.1525	.1514	.1504	.1492	.1482	.1472	.1463
-.185	.1275	.1258	.1243	.1232	.1223	.1213	.1190	.1180	.1170
-.175	.1080	.1063	.1047	.1034	.1021	.1009	.0999	.0988	.0979
-.165	.0940	.0923	.0907	.0894	.0881	.0869	.0859	.0848	.0839
-.155	.0835	.0818	.0803	.0789	.0776	.0765	.0754	.0744	.0735
-.145	.0755	.0738	.0722	.0709	.0696	.0684	.0674	.0664	.0655
-.135	.0691	.0675	.0659	.0646	.0633	.0621	.0610	.0600	.0591
-.125	.0640	.0624	.0608	.0594	.0582	.0570	.0559	.0549	.0540
-.115	.0599	.0582	.0566	.0553	.0540	.0528	.0518	.0508	.0498
-.105	.0564	.0547	.0532	.0518	.0505	.0493	.0483	.0473	.0464
-.095	.0535	.0518	.0503	.0489	.0476	.0465	.0454	.0444	.0435
-.085	.0510	.0493	.0478	.0464	.0451	.0440	.0429	.0419	.0410
-.075	.0489	.0472	.0457	.0443	.0430	.0418	.0408	.0398	.0389
-.065	.0470	.0454	.0438	.0424	.0412	.0400	.0389	.0379	.0370
-.055	.0454	.0437	.0422	.0408	.0396	.0384	.0373	.0363	.0354
-.045	.0440	.0423	.0408	.0394	.0381	.0370	.0359	.0349	.0340
-.035	.0428	.0411	.0396	.0382	.0369	.0357	.0347	.0336	.0328
-.025	.0417	.0400	.0385	.0371	.0358	.0346	.0336	.0325	.0317
-.015	.0407	.0390	.0375	.0361	.0348	.0336	.0326	.0316	.0307
-.005	.0398	.0381	.0366	.0352	.0339	.0327	.0317	.0307	.0298

Table 7e % differences for L-Table 3 including out of range errors denoted by 99.9999. Solid line denotes boundary of 0.5% errors

C \ A	3.9500
-.235	.9143
-.225	.4338
-.215	.2697
-.205	.1907
-.195	.1454
-.185	.1164
-.175	.0974
-.165	.0831
-.155	.0727
-.145	.0646
-.135	.0583
-.125	.0532
-.115	.0490
-.105	.0455
-.095	.0426
-.085	.0401
-.075	.0380
-.065	.0362
-.055	.0346
-.045	.0332
-.035	.0319
-.025	.0308
-.015	.0298
-.005	.0289

FIGURES.

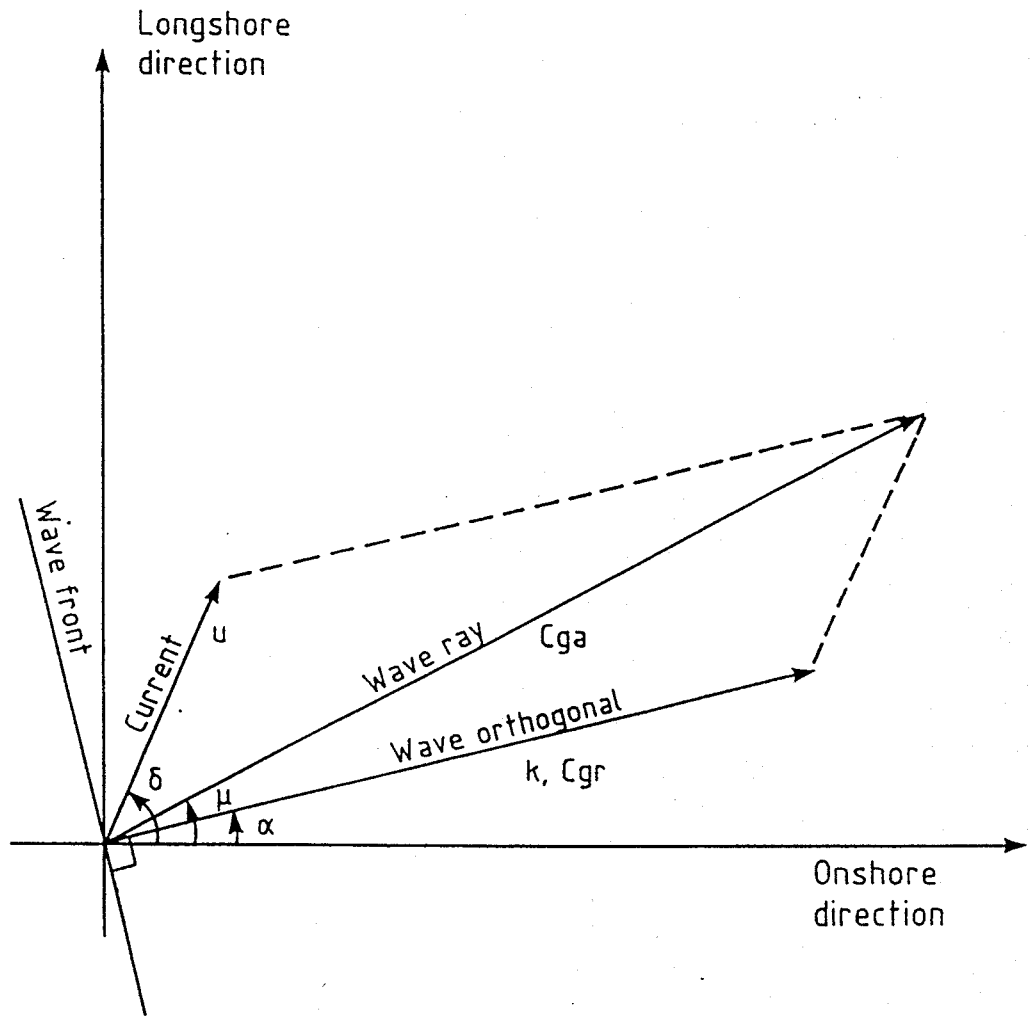

Fig 1 Geometry of currents, wave orthogonals and wave rays

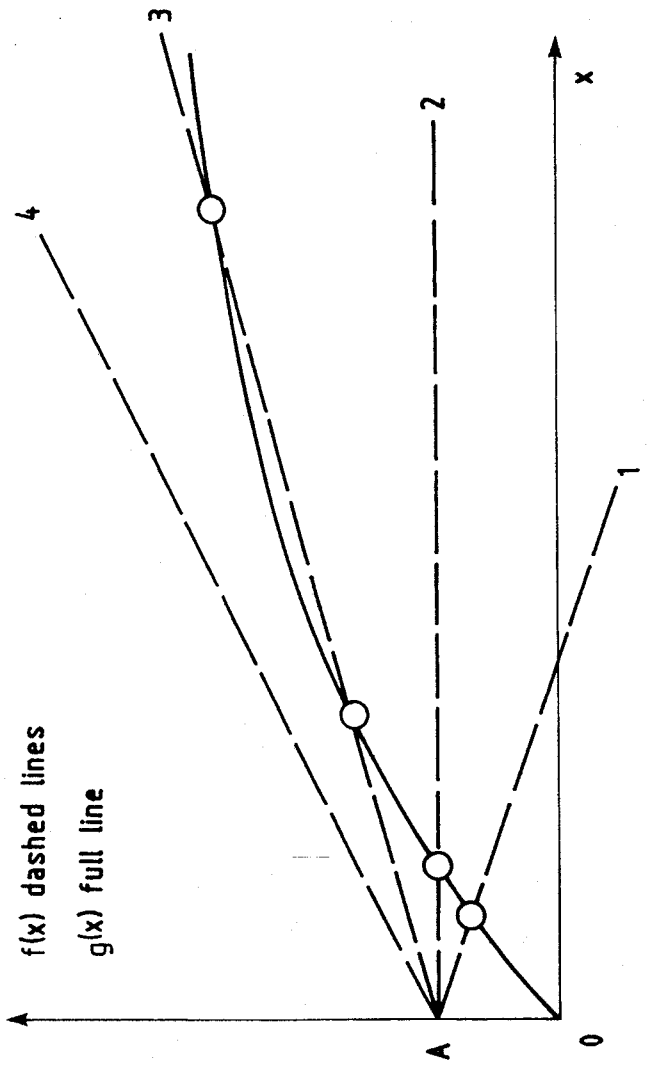

Fig 2 Four cases in the solution of the current-depth dispersion equation, Eq 2

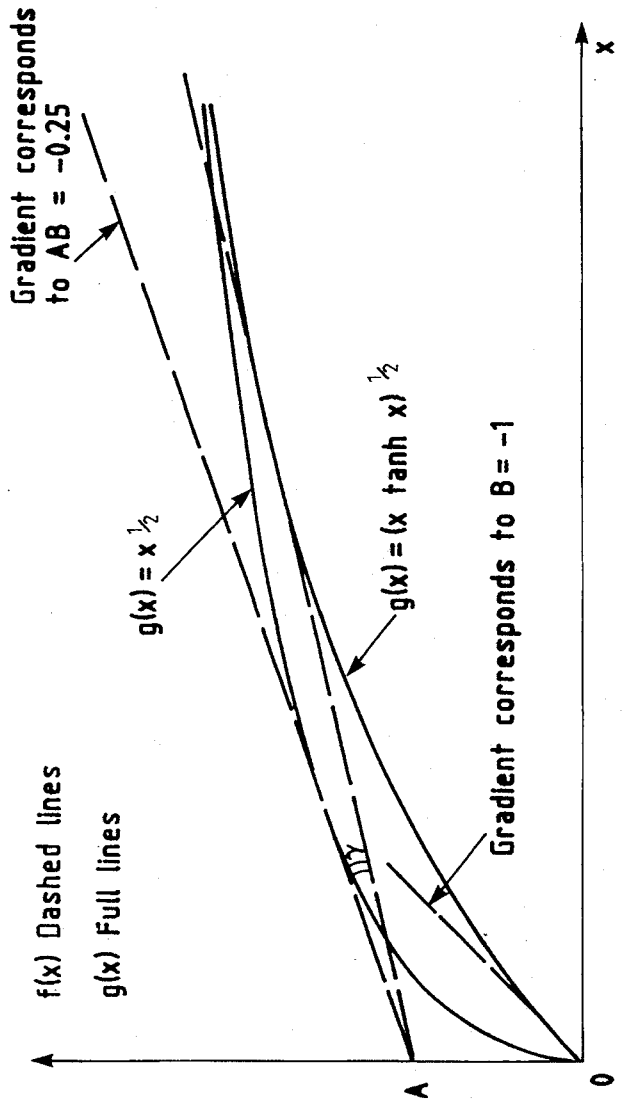

Fig 3 Showing the criteria for no solutions given by Eq 14 and Eq 15. γ represents a range of gradients of f(x) not covered by these criteria.

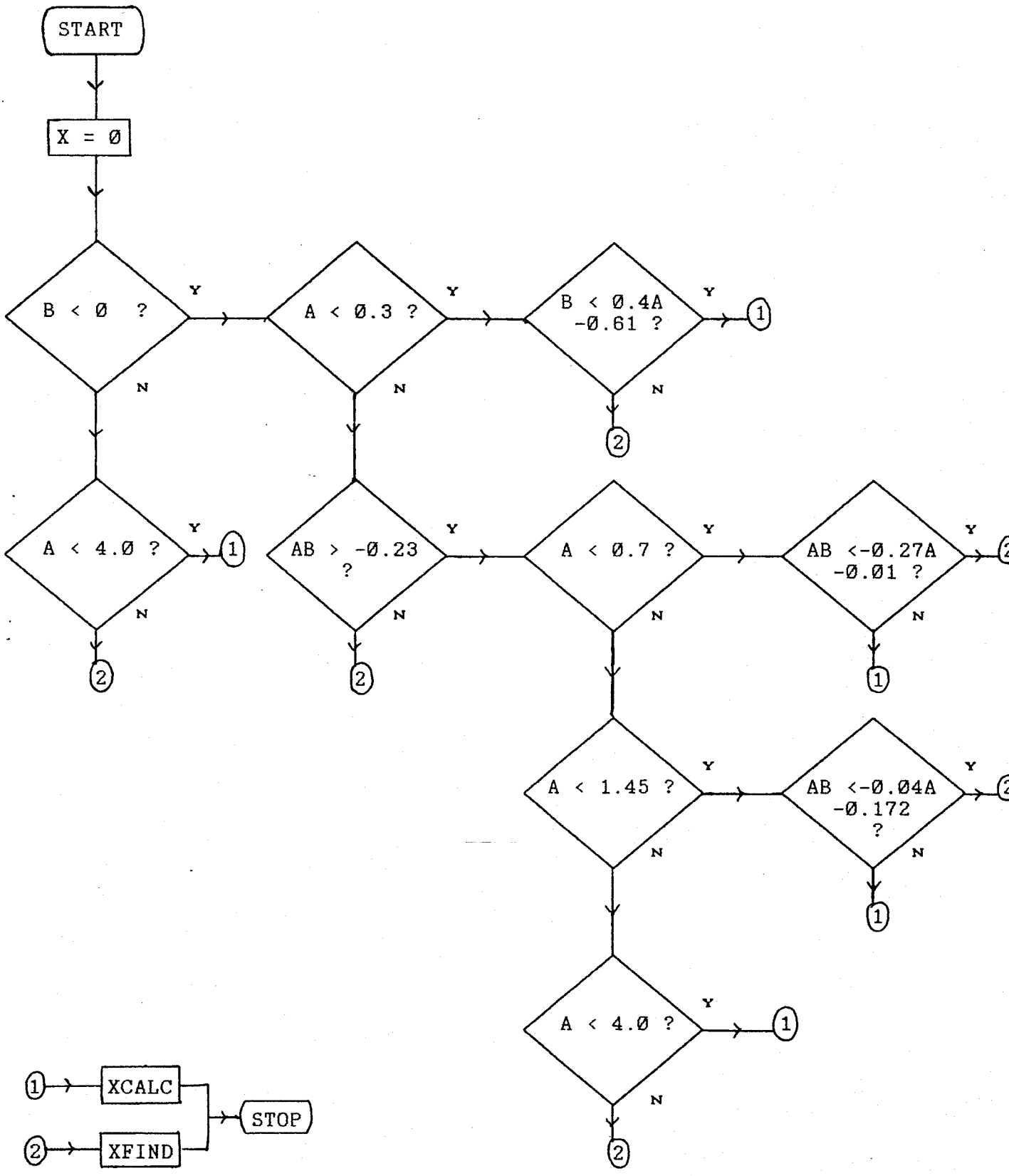

Fig 4 Flowchart of program structure

```
xcalc (A,B,x);

x = Ø;

if B < Ø then
   if A < Ø.3 then
      if B > Ø.4A-Ø.61 then
         evaluate x using Newton-Raphson iteration;
      else
         evaluate x using interpolation in L-table 2;
      endif;
   else
      if AB > -Ø.23 then
         if A < Ø.7 then
            if AB < -Ø.2714A-Ø.Ø1 then
               evaluate x using Newton-Raphson iteration;
            else
               evaluate x using interpolation in L-table 3;
            endif;
         else
            if A < 1.45 then
               if AB < -Ø.Ø4A-Ø.172 then
                  evaluate x using Newton-Raphson iteration;
               else
                  evaluate x using interpolation in L-table 3;
               endif
            else
               if A < 4.Ø then
                  evaluate x using interpolation in L-table 3;
               else
                  evaluate x using Newton-Raphson iteration;
               endif;
            endif;
         else
            evaluate x using Newton-Raphson iteration;
         endif;
      endif;
   endif;
else
   if A < 4.Ø then
      evaluate x using interpolation in L-table 1;
   else
      evaluate x using Newton-Raphson iteration;
   endif;
endif;

end;
```

Fig 5 Pseudocode showing program structure

APPENDIX.

APPENDIX

The main computational subroutine referred to in this text is called XCALC. Its purpose is to determine which look-up table to use for given values of A and B. The other subroutines and functions called are as follows:

TABFND - (SUBROUTINE) locates the lower limit of the intervals in A and C in which x lies.

FNTERP - (FUNCTION) performs a bivariate interpolation between the four values which surround the required value in the look-up table.

XFIND - (SUBROUTINE) performs Newton-Raphson iterations to find x for given A and B. Used for values of A and B which give errors of greater than 0.5% with XCALC.