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MATHEMATICAL MODELLING OF WAVE CLIMATE NEAR OFFSHORE BREAKWATERS

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ABSTRACT

This report presents the results from mathematical models which have been used to calculate wave conditions in the vicinity of an island breakwater. The main purpose of the research described here was to provide engineers with guidance on the performance of offshore breakwaters in coast protection schemes. The report contains diffraction diagrams for both monochromatic and random incident waves for normal and obliquely incident waves. It also provides illustrations of the use of these diagrams in selecting breakwater layout, and in calculating other important parameters such as wave run-up and overtopping.

CONTENTS

1 INTRODUCTION

1.1 Background

During the past five years mathematical models have been developed (see Refs 1,2 and 3) to predict wave activity in the vicinity of an offshore breakwater. There are now several sites in the UK where offshore breakwaters are used in coastal defence schemes. However, there is very little information available to the engineer on the effect of breakwater layout on the performance of such schemes. The research reported here is intended to go some way towards providing such information, by examining and comparing wave height coefficients for a number of typical breakwater layouts. In addition, consideration is given to the application of these results in estimating the effect of introducing an offshore breakwater on wave run up and seawall overtopping. It is intended that the method and results contained in this report will provide useful guidance to the engineer involved in the preliminary design of the layout of an offshore breakwater scheme.

1.2 Terms of reference

Existing mathematical models were to be used to consider the following aspects of breakwater design layout.

(i) The length of coastline over which an offshore breakwater provides protection relative to its length and distance offshore. This was to be examined using both monochromatic and random waves for normal and oblique incidence.

(ii) The change in wave climate that can be anticipated along a coast due to the introduction of an offshore breakwater. This was to be investigated with particular reference to assessing the length of coast which experiences a significant reduction in wave height.

(iii) The effect of introduction of an offshore breakwater on sea wall overtopping and wave run-up.

In addition, some thought was to be given to the development of a software package which could be used by engineers for a preliminary assessment of offshore breakwater layout.

1.3 Outline of report

The main objective of the report in providing information on offshore breakwater layout is covered in Chapter 3 and 4. Chapter 3 discusses the effect of offshore breakwaters on nearshore wave conditions. In Chapter 4 details of the application of the model results in assessing, for example, wave run-up and seawall overtopping are given. It is anticipated that these two chapters will be of most direct interest to the engineer. The theoretical aspects of the models used to calculate diffraction effects are covered in Chapter 2. The conclusions and recommendations arising from this work are given in the final chapter.

2 DESCRIPTION OF MATHEMATICAL MODELS

The main purpose of the work described in this report is to demonstrate how mathematical models of wave climate near offshore breakwaters can be used to optimise the layout of a particular scheme. As it is the application of the models, rather than their development or validation, which is important here we will give only a brief description of each of the models used.

Two basic models were used to represent the effects of wave diffraction by island breakwaters. The validation of both models, and a detailed description is given in Reference 2. The first, referred to as the integral equation model, was originally developed by Brampton and Gilbert (Ref 4). It relies on finding a numerical solution to the governing integral equation, whose derivation is based on an assumption of constant water depth. This model has been shown to provide a reliable method for predicting wave conditions near an offshore breakwater in water of constant depth (Ref 1).

The second model, which was developed at Bristol University (Ref 5) uses a finite difference method to solve a parabolic approximation to the mild slope equation. It will therefore be referred to as the Parabolic Model. This model has some restrictions, which will be discussed in section 2.3, relating to the choice of incident wave angle, but has been shown for many cases (Ref 2) to provide a reliable method of estimating wave conditions. A significant advantage of this model compared with the integral equation model is that it includes variable depth terms; that is, refraction can be represented.

The methods used by both of these models are for unidirectional, monofrequency incident waves. As such they can be considered equivalent to using diffraction diagrams (see, for example, Ref 6) to calculate wave conditions near an offshore breakwater. Although, the models have the advantage of being applicable to any breakwater length/incident wave combination, whereas diagrams are only available for a few specific cases. To overcome the limitation of unidirectional monofrequency incident waves the integral equation model has been extended to use an incident wave spectrum with directional spreading as input. This brings the model results much closer to the situation in nature where breakwaters will be exposed to random incident waves. The technique used to do this is described fully in Reference 3. A similar approach has also been used by Goda (Ref 7).

In the remainder of this chapter both the integral equation model, section 2.2, and the parabolic model, section 2.3, are described more fully. An outline of the method used to represent 'random' incident waves is given in section 2.4.

2.2 Integral equation model

For a breakwater in water of constant depth the wave heights in its vicinity can be calculated using the integral equation technique described in Reference 4. This allows the wave height coefficient, ie the ratio of wave height at any point p to the incident wave height, to be estimated using the following method.

The wave height coefficient (H_p/H_i) can be expressed in terms of the velocity potential ϕ as,

$$\frac{H_p}{H_i} = \frac{|\phi_p|}{|\phi_i|} \qquad (1)$$

It can be shown that the velocity potential at any point p in the vicinity of an island breakwater is given by,

$$\phi_p = \phi_i + \phi_d, \qquad (2)$$

where ϕ_d is the velocity potential of the diffracted wave. This must satisfy the radiation condition,

$$\lim_{r \to \infty} r^{\frac{1}{2}} \left(\frac{\partial}{\partial r} - ik\right) \phi_d = 0$$

where $r = (x^2 + y^2)^{\frac{1}{2}}$. The origin of the Cartesian (x,y) co-ordinate system is taken at the centre of a breakwater of length 2a which is located along the x axis (see Fig 1). Waves approach the breakwater from the negative y direction. The incident potential is given by,

$$\phi_i(x,y) = \hat{a} \exp (ik(x\sin \beta + y\cos \beta)) \tag{3}$$

where $\hat{a}$ is the incident wave amplitude, k the wave number and β the incident angle. The diffracted potential is given by,

$$\phi_d(x,y) = \frac{i}{2}\int_{-a}^{a} g(x_o) \left[\frac{\partial H_o^{(1)}}{\partial y_o} (kR)\right]_{y_o=0} dx_o \text{ , } y \geqslant 0 \tag{4}$$

$$\phi_d(x,y) = -\phi_d(x,-y), \; y < 0,$$

where $R^2 = (x-x_o)^2 + (y-y_o)^2$,

$H_o^{(1)}$ is the Hankel function of the first kind, zeroth order and k is the wave number. The function $g(x_o), -a \leqslant x_o \leqslant a$ is the solution of the integral equation,

$$\int_{-a}^{a} g(x_o) \, H_o^{(1)} (k|x-x_o|)dx_o \tag{5}$$

$$= G(x) + Ae^{-ikx} + Be^{ikx}, \; -a \leqslant x \leqslant a,$$

where G(x) is a particular integral of the differential equation

$$\frac{\partial^2 G}{\partial x^2} + k^2 G = 2k\ \hat{a}\ \cos\beta\ \exp\ (ikx \sin\beta),$$

A and B are chosen to satisfy the boundary condition

$$g(x_o) = 0 \text{ at } x = \pm a.$$

Once (5) has been solved for $g(x_o)$, $-a \leqslant x_o \leqslant a$, the diffracted potential in the flow field can be calculated using (4) and the total potential recovered from (3) and (2). This will then allow the diffracted wave height to be calculated using (1).

2.3 Parabolic model

The parabolic equation used here (see Ref 3) is derived from the mild slope equation,

$$\underline{\nabla}.(c\ c_g\ \underline{\nabla}\phi) + \omega^2\ \phi\ c_g/c = 0, \tag{6}$$

with appropriate boundary conditions. Here $\phi(x,y)$ is the velocity potential, c is the phase, c_g the group velocity and ω the radian frequency. This equation was first derived by Berkhoff (Ref 8), it describes the propagation of periodic, small amplitude surface gravity waves over a seabed of mild slope. It will represent the combined effects of refraction, shoaling and diffraction. To derive the required parabolic approximation the reflected wave field is assumed to be small and is neglected, and forward travelling waves only are considered. This leads to the equation,

$$\frac{\partial\phi}{\partial x} = \frac{i}{2k}\frac{\partial^2\phi}{\partial y^2} + \{ik - \frac{1}{2k}\frac{\partial k}{\partial x}\}\phi, \tag{7}$$

where x is the main direction of wave propagation, y is the transverse direction. Deviations from the x direction are considered in the equation as oblique amplitude modifications.

Equation (7) is parabolic, whereas (6) is elliptic. The main advantage of a parabolic equation is that it permits a more rapid and straightforward method of solution than would be possible for an elliptic equation. This is because an elliptic equation defines a problem which is only properly posed, in general, when conditions are specified at all points around the boundary. It therefore requires that the equation is solved over the whole area of interest simultaneously. This necessitates a large amount of both computer storage and time. The form of the parabolic equation is such that, for a well posed problem, boundary conditions only need to be specified at the offshore boundary. A marching finite difference technique can be used to obtain the solution. This type of method only requires storage of one or two adjacent rows of solution points. As a consequence, is considerably less expensive in terms of cost and storage than the equivalent numerical solution to an elliptic equation.

There are of course drawbacks associated with these advantages. The primary one of these is that the parabolic approximation works best where the important effects occur in the direction of wave propagation, as transverse effects are only included in a weak sense. It has been found that this limitation is not significant provided that the grid over which the model is run is aligned close to (within ±30°) the main wave propagation direction. The lack of reflected waves in the model means that it does not give reliable estimates of wave conditions on the exposed side of the breakwater.

Despite these limitations it has been found (see Ref 2) that the parabolic equation (7) provides a useful and economic method for solving the type of problems of interest here.

2.4 Diffraction coefficients for random waves

The diffraction coefficients for a given breakwater configuration with fixed incident wave conditions is defined as the ratio of the wave height in the area affected by diffraction to the incident wave height. It is usually denoted by C_d where,

$$C_d = \frac{H_d}{H_i} , \tag{8}$$

and H_d and H_i are the wave heights in the area effected by diffraction, and the incident wave height respectively. In (8) it is assumed that H is the monochromatic wave height, and as a consequence of that C_d is a function of both the frequency and direction of the monofrequency incident wave.

In random waves the sea state is usually characterised in terms of the significant wave height (H_s) where,

$$H_s = 4[\int_0^\infty \int_0^{2\pi} S(f,\theta)\ d\theta\ df]^{½} , \tag{9}$$

and $S(f,\theta)$ is the spectral density which is a function of frequency (f) and direction (θ). Extending the definition given in (8) and (9) the diffracted wave height at a given location in random waves is given by,

$$(H_s)_d = 4\left[\int_0^\infty \int_0^{2\pi} C_d^2(f,\Theta)\ S(f,\Theta)\ d\Theta\ df\right]^{1/2}, \tag{10}$$

where $C_d(f,\Theta)$ is the diffraction coefficient at that location for a monofrequency wave with frequency f and direction Θ. Thus, for random waves the diffraction coefficient is given by

$$(C_d)_{ran} = \frac{(H_s)_d}{(H_s)_i} = \left[\int_0^\infty \int_0^{2\pi} C_d^2(f,\Theta)\ S(f,\Theta)\ d\Theta\ df\right]^{1/2} \Big/ \left[\int_0^\infty \int_0^{2\pi} S(f,\Theta)\ d\Theta\ df\right]^{1/2}. \tag{11}$$

In practise the values of the functions C_d and S will be known for certain discrete values of f and Θ and (11) will be approximated by

$$(C_d)_{ran} \simeq \frac{\left[\sum_{i=1}^{n} \sum_{j=1}^{m} C_d^2(f_i,\Theta_j)\ S(f_i,\Theta_j)\ \Delta\Theta_j\ \Delta f_i\right]^{1/2}}{\left[\sum_{i=1}^{n} \sum_{j=1}^{m} S(f_i,\Theta_j)\ \Delta\Theta_j\ \Delta f_i\right]^{1/2}}, \tag{12}$$

where n frequency and m direction components are being considered, and Δf_i and $\Delta\Theta_j$ are the width of the ith frequency and jth direction interval. The frequency and direction components used in (12) must be selected so as to fully cover the range of the incident wave spectrum.

Therefore to calculate the diffraction coefficient in random waves we need to specify the incident wave spectrum, and determine the diffraction coefficients for each of the discrete frequency and direction components in (12).

A typical spectrum is of the form,

$$S(f,\Theta) = S(f)\ G(\Theta),$$

where S(f) is the frequency distribution of wave energy and $G(\Theta)$ is the directional distribution of wave energy which is assumed to be independent of frequency. The frequency spectrum will normally be calculated using established formula for deep water waves such as Pierson-Moskowitz or JONSWAP. A summary of the formulae which are in common use is given in Chakrabarti (Ref 9). In the present work both Pierson-Moskowitz and JONSWAP spectra are used to provide examples of their effect on the predicted wave heights in lee of an island breakwater, (see section 3.3).

We also require a function describing the directional distribution of wave energy. Much research has been done on the choice of a function describing directional spreading of energy. In particular the work of Hasselman et al (Ref 10) which resulted in the formulation of the JONSWAP wave spectrum which uses a distribution based on $\cos^2(\Theta-\Theta_m)$, where Θ_m is the mean wave direction, to describe the directional spread of waves. There is some evidence to suggest that the use of a narrower directional spread, see Mitsuyasu (Ref 11), may be appropriate in shallow water. In the present work both $\cos^2$ and a $\cos^6$ spreading function are used.

3 EFFECTS OF OFFSHORE BREAKWATERS ON NEARSHORE WAVE CLIMATE

3.1 Description of test programme

To meet the objectives (i) and (ii) of the terms of reference a number of different breakwater configurations and incident conditions needed to be examined. To simplify this process the breakwater lengths were defined in terms of the wavelength (L) of the incident wave. This effectively non-dimensionalises the cases examined with respect to period and water depth. (Wavelength is easily calculated from these two parameters). For tests with monofrequency waves the wavelength is calculated using the incident period. For the purpose of this report, wavelength for random waves refers to the peak period of the incident spectrum. The results from the tests are presented as wave height coefficients in the lee of the breakwaters, and as such are also non-dimensionalised with respect to incident wave height. It should be noted that both models used here assume that the breakwater is perfectly reflecting, ie has a reflection coefficient of unity.

Breakwaters of length 0.5, 1.0 and 2.0 wavelengths are examined in this study for both monofrequency, unidirectional waves and random incident waves. This range of breakwater lengths is representative of island breakwaters which have been constructed around the UK and other coasts worldwide. A range of incident directions from normal to 30° from normal are also considered. For random waves two directional spreading functions about the mean incident direction are also investigated.

For most cases a constant water depth was assumed. For most sites this assumption is sufficiently good for a first estimate of the effects of the breakwater on wave climate to be made. For subsequent more detailed, studies a site specific model with the physical bathymetry represented will need to be employed. This could be done using, for example, the parabolic model. To illustrate this point two idealised sloping bathymetries were also included in the calculations.

The complete model test programme is shown in Table 1. For all configurations wave height coefficients were calculated over an area of dimensions 6 wavelengths by 3 or 4 wavelengths in the immediate lee of the breakwater. The area covered is shown in Figure 1. The distance behind the breakwater in which the height coefficients were calculated is typical of the distances offshore at which many existing offshore breakwaters have been constructed.

The results from the model tests are presented as contour plots of wave height coefficients. The test results are discussed in section 3.2 for monofrequency waves in constant depth, section 3.3 for random waves in constant depth and section 3.4 for monofrequency waves in variable depth. In each section comparisons of the performance of the various configurations are given, together with a discussion of the relative accuracy of the models used.

3.2 Results for monofrequency, unidirectional incident waves (constant depth)

For the first set of tests (1 to 9) the integral equation model was used to predict wave height coefficients. The main purpose of these tests was to examine the area of coastline receiving significant sheltering by island breakwaters of various lengths. The results, shown in Figures 2,3 and 4, are presented in the same form as the conventional diffraction diagrams given in, for example, Reference 6. By assuming that the results shown in these figures along a line parallel to, and at a specified distance (say 0.5L, 1.0L and 2.0L) from the breakwater, are representative of the effects along a stretch of coast comments on the shelter afforded by various configurations can be made.

For the purpose of this report we consider areas where the wave height coefficient is below 0.5 as providing good shelter. In this case the energy reaching the lee of the breakwater will be less than 25% of the incident wave energy. Areas where the wave height coefficient is between 0.5 and 0.7, ie the energy penetrating the lee is between 25% and 50% of the incident, are regarded as providing adequate shelter.

If we first examine the breakwater of length 0.5L, see Figure 2. For all incident directions tested wave height coefficients in the lee of the breakwater are only rarely less than 0.7. At distances greater than 1.0L from the breakwater wave height coefficients are generally in excess of 0.8. To provide adequate shelter ($C_d \leqslant 0.7$) to a stretch of coastline of 2L the breakwaters would need to be 1L offshore. For a breakwater of this length there will be strong constructive interference between the diffracted waves emanating from the ends of the breakwater. This is leading directly to the relatively large values of wave height coefficient in the lee of the breakwater. The magnitude of this effect lessens as the length increases, so that breakwaters which are significantly longer can be expected to provide greater shelter in their immediate lee.

For a breakwater of length L contour plots of the wave height coefficients are shown in Figure 3. Cross-sectional plots of wave height coefficients along lines at distances 0.5L, 1.0L and 2.0L from the breakwater are shown in Figure 5. It can be seen that the breakwater provides adequate shelter ($0.5 \leqslant C_d \leqslant 0.7$) over lengths of 1.2L, 1.4L and 1.7L at distances 0.5L, 1.0L and 2.0L from the breakwater for normal incidence. In each case the length of shelter is reduced slightly and its location altered with change in incident angle. The quality of sheltered afforded is better with the breakwater at 1L offshore, with wave height coefficients over most of the protected area being less than 0.6.

Contour plots of wave height coefficient for a breakwater of length 2L are shown in Figure 4, with cross sectional plots in Figure 6. It can be seen that the breakwater provides adequate shelter ($C_d \leqslant .07$) over lengths of 2L, 2.2L and 2.3L at distances 0.5L, 1.0L and 2.0L from the breakwater at normal incidence. Good shelter ($C_d \leqslant 0.5$) is provided over lengths of 1.4L, 1.6L and 1.5L at distances 0.5L, 1.0L and 2.0L from the breakwater for normal incidence. Similar shelter is provided when the angle of incidence is changed, but the area protected is moved in line with the incident direction.

The wave height coefficients obtained for island breakwaters for monofrequency waves using any of the available methods should be viewed with caution when applying them to a real situation. This is because in nature the incident waves will combine many frequencies which will produce a different response to the monofrequency case. In many instances a monofrequency run at a representative frequency for the incident spectrum (say corresponding to the peak period) will give a reasonable approximation. Whilst the results may be sufficiently good for comparative purposes, they should only be regarded as a first estimate when considering the final design layout. However, results from monofrequency tests do provide a convenient, relatively fast method for an initial comparison. Provided they are viewed in this light they serve the stated purpose well. For a more comprehensive assessment of performance random incident waves should be used and these are described in the next section.

3.3 Results for random incident waves (constant depth)

Random incident waves were used as input to the model for breakwaters of length 1.0 and 2.0 wavelengths. Here the wavelength referred to is that which coincides with the peak period of the incident spectrum. Both Pierson-Moskowitz and JONSWAP spectra were used with $\cos^2$ and $\cos^6$ directional spreading functions, at various mean incident angles. For each run approximately 11 directional components and 8 frequency components were used. The programme for the random wave tests (10 to 19) is shown in Table 1. One of the aims in conducting a wide range of tests was to examine the response of wave height coefficients to changes in incident spectra and the shape of the spreading functions. This was in addition to examining the performance of the breakwaters over the area in their lee.

If we first examine the results of tests 10 to 12, see Figure 7. These will provide an indication of the shelter afforded by a breakwater one wavelength long, for different mean incident angles with a Pierson-Moskowitz input spectrum. By comparing these results with those in Figure 3 it is clear that using random wave input and directional spreading leads to generally larger wave height coefficients than for monofrequency waves. This can be seen more clearly in Figure 10, which shows values of wave height coefficients along lines parallel to the breakwater for random waves at distances 1L and 2L from it. Along these cross-sectional lines the wave height coefficient is less than 0.7 for a length 0.4L for the line at 1L for any of the mean incident directions. At 2L distance the coefficients do not fall below 0.7. This result indicates that the monochromatic unidirectional incident waves underestimate the wave height coefficients for random waves.

For a breakwater of length 2L with random incident waves, contours of wave height coefficient for different mean incident angles are shown in Figure 8. Cross sectional plots of wave height coefficient are shown in Figure 11. It is clear from this that the 2L breakwater offers significantly more shelter than the 1L breakwater. For the 2L breakwater stretches of coast of lengths 1.8L, 1.7L and 1.0L are sheltered with the breakwater at distances 0.5L, 1.0L and 2.0L from the coast. The length of coastline protected is similar for non-normally incident waves but location moves relative to normal incidence. As for the 1.0L breakwater case, the results for monochromatic incident waves (see Fig 4) underestimate the wave height coefficients obtained for the random wave case.

The remainder of the tests (16 and 17) were primarily intended to examine the effect on wave height coefficient for a breakwater 1L long of changes in incident wave spectra and directional spreading function. The results for test 16 (Pierson-Moskowitz spectrum, $\cos^6$ directional spreading) and test 17 (JONSWP spectrum, $\cos^2$ directional spreading) are given in Figure 9. By comparing these with the results of test 10 an assessment can be made of the effects of the choice of frequency and directional spectra on wave height coefficients. The differences between the various incident conditions can be seen most clearly on Figure 12.

The first point to note is that outside of the range $-L \leqslant x \leqslant L$ the results for all three cases are very similar. This indicates that for the configurations tested the choice of spectrum and spreading function is relatively unimportant outside of a length approximately twice that of the breakwater. This point should not be taken as a general statement without further, more extensive, testing. Inside the range $-L \leqslant x \leqslant L$ it can be seen from Figure 12 that these differences between the values of diffraction coefficients for different choices of spreading function. In general the $\cos^6$ function produces lower values of diffraction coefficient.

3.4 Results for monofrequency, unidrectioanl incident waves (variable depth)

Having considered the constant depth case we need to explore the effect of bed slope on diffraction coefficients near an island breakwater. Including a realistic bathymetry in this type of model is one further step towards a more accurate representative of the physical situation. For all of the model tests described in this section the parabolic model was used.

The model area was set up first for a breakwater of length 1L in constant depth and run for normally incident monofrequency waves (test 18). The layout was then modified to incorporate two different sloping beds, one of slope 1:100 and the other of slope 1:50. The model was then run again for normally incident monofrequency waves (tests 19 and 20). The results from these tests are shown in Figure 13.

Before running the tests for a sloping bed the parabolic model results for the flat bed layout incidence were compared to those of the same test using the Integral Equation Model (test 4). Profiles of predicted waveheight coefficients at 1L, 2L and 3L behind the breakwater, were compared for the two models and found to be in satisfactory agreement.

The effects of including a sloping bed can be seen clearly in Figure 13. Away from the centre line of the breakwater the wave height contours change shape for the sloping bed case. There is a tendancy for equivalent contours (ie those with the same value) to be pushed further from the centre line for the sloping bed cases. This effect is more marked for the 1:50 slope than for the 1:100 slope. It is the refraction process which is causing this, although its effects will be small as the slopes used are fairly gentle. However, the conclusion that the flat bed case will provide an overestimate of diffraction coefficients for the sloping bed case can be drawn from the results in Figure 13. Clearly, further tests are required to allow a more general statement to be made.

4 EXAMPLES OF APPLICATION

4.1 Introduction

The main purpose of this chapter is to illustrate how information on diffracted wave heights can be described, and used in subsequent calculations. In section 4.2 an example of the use of diffraction diagrams to calculate wave heights for optimising the position of a typical breakwater layout is given. In many situations the wave heights in the lee of the structure will be required for subsequent calculations of wave run-up or overtopping at a revetment or seawall. Examples of these applications are given in section 4.3.

4.2 Use of diffraction diagram

It is possible to use diffraction diagrams to do some preliminary optimisation of breakwater layout. To give an indication of how this is done we will consider a typical offshore breakwater intended to protect a specified stretch of coast. Clearly, this type of optimisation requires diffraction diagrams which cover the appropriate parameters (length, incident angle etc). The example given here has been selected so that the diagram shown in Figures 7 and 8 can be used.

The case we will consider is for a site where a length of coast of approximately 40m needs to receive protection. If we say that a breakwater of approximately 30m length in 5m depth is to be used, and that a typical wave condition is H_s = 1m and T_p = 5s with a direction approximately normal to the coast. The first stage in using diffraction diagrams will be to non-dimensionalise the problem with respect to wavelength. In a depth of 5m, a 5s period wave has a length (L) of 30m. This means that the area to be protected is approximately 1.3L and the proposed breakwater length is approximately 1L. As discussed earlier, more accurate results are achieved by using the diffraction diagrams for random wave input. If we assume that the incident wave conditions are in the form of a Pierson-Moskowitz spectrum then our optimisation can be based on the results shown in Figures 7 and 8. Clearly, if this is not the case then either the monochromatic approximations or a spectrum of the correct form will need to be used.

Examining first the results shown in Figure 7.1 for a breakwater 1 wavelength long with normally incident waves. It can be seen that positioning the breakwater one wavelength offshore protects a length of coast in its immediate lee of about 0.4L. Moving it further inshore to half a wavelength from the coast allows a stretch 0.6L long to be protected. In these regions the diffraction coefficient is predicted to be between 0.5 and 0.7. Therefore the incident waveheight (1.0m) will be reduced to between 0.5m and 0.7m in these areas. These values do not allow for wave breaking, or dissipation effects by the breakwaters and so are likely to overpredict the values occurring in nature.

In our original problem we stipulated that the length of coast to be protected was about 1.3L. It is evident from the results given above that wave heights between 0.5m and 0.7m cannot be achieved across this whole area with a breakwater of length 1L. Moving to a longer breakwater say 2L in length will allow this area to be protected. From Figure 8.1 it can be seen that positioning this breakwater up to 1L offshore will protect the requisite length of coast. It may also be possible to reduce the length to say 1.5L, and still provide effective protection over the necessary area.

Obviously at this stage other factors like cost, amenity value and the practicalities of construction will need to be taken into account, but this can now be done in the light of information being available on the appropriate position of the breakwater.

There are two main drawbacks. First, the diffraction diagrams used for the example do not include refraction or wave breaking effects which may be significant in some instances. This problem can be overcome by using the parabolic model described earlier with an empirical criterion for breaking. However, it is suggested that this is done once initial optimisation has been made using a constant depth model. The second drawback is availability of diffraction diagrams for the particular range of incident conditions and breakwater configurations of interest. This will allow the engineer involved in the design of offshore breakwaters to make a more effective optimisation of their layout, in terms of the area protected.

4.3 Application of diffraction diagrams in predicting the effect of offshore breakwaters on wave run-up and seawall overtopping

One of the possible uses of an offshore breakwater is to protect a seawall or coastal revetment from the worst effects of wave action. For example, the offshore breakwater at Rhos on Sea (see Ref 12) was construction to reduce wave activity at an existing seawall, which in times so severe weather had been overtopped. The overtopping had lead to flooding of the residential area behind the wall. The alternative of raising the wall was considered, but rejected as being visually instrusive from an amenity point of view. So far, the breakwater at Rhos has proved to be successful in achieving its stated aim.

To illustrate the effect of introducing an offshore breakwater on run-up and overtopping, typical techniques have been considered. In each case empirical formulae have then been applied to assess the effect of introducing an offshore breakwater on the important parameters. This approach is primarily intended to illustrate the methods which can be used to assess the effect of an offshore breakwater on wave conditions at an existing seawall, or coastal revetment.

As an example we will consider the effect of introducing an offshore breakwater on wave run-up at a smooth faced sloping coastal revetment. An expression for the significant run-up (R_s) at such a structure has been derived by Allsop et al (Ref 13) as:

$$R_s = H_s\ (2.11 - 0.09\ Ir') \qquad (13)$$

Here the modified Irribarren number is defined as:

$$Ir' = \tan\alpha\ /\ ((H_s/L_p)^{1/2})$$

where $L_p = gT_p^2/(2\pi)$ is the deepwater wavelength at the peak period. The expression (13) was based on the results of flume tests using a Pierson-Moskowitz spectrum for smooth slopes at 1:2, 1:1.5 and 1:1.33; it is valid for the range $2.8 \leqslant Ir' \leqslant 6.1$.

From (13) the run-up at the revetment following the introduction of an offshore breakwater, R_{sd} can be written in terms of the diffraction coefficient (C_d) and the significant wave height (H_s) and run-up (R_s), before introduction of the breakwater as:

$$R_{sd} = C_d^{1/2} R_s - 2.11 H_s C_d^{1/2} (1 - C_d^{1/2}) \qquad (14)$$

It is clear from (14) that provided the offshore breakwater diffraction coefficients are less than unity at the revetment then the significant run-up will be reduced by its introduction.

To illustrate this point we will examine the case of a coastal revetment with a smooth face at a slope of 1:2. If we assumed the incident waves have a Pierson-Moskowitz spectral shape then expressions (13) and (14) can be used. We will take the incident wave height to be 1m and the peak period to be 5s. Before any offshore breakwater is introduced the significant run-up will be, from (13), 1.83m.

If we then assume that an offshore breakwater one wavelength long, situated half a wavelength from the revetment toe is proposed. From Figure 7.1 it can be seen that in the immediate lee of the structure the diffraction coefficient will be between 0.5 and 0.7 at the revetment toe. From (14) this will reduce the significant run-up to between 0.85m and 1.24m. Therefore introduction of the offshore breakwater will substantially reduce run-up at the revetment. Similar calculations to this could be made for other breakwater configurations and types of revetments using the diffraction diagrams given here, and the empirical formulae for run-up in Reference 13.

A second example of the application of diffraction diagrams to physical situations is provided by considering the effect of an offshore breakwater on seawall overtopping. Rather than providing a specific example, as seawall types and approach bathymetrics will vary widely, we will simply describe how the calculation can be made. First, in appropriate diffraction diagram will be required to predict wave conditions along the line of the seawall. These will clearly vary with the locations along the seawall relative to the sheltering effect of the breakwater. The wave heights predicted along the seawall can then be used, together with parameters describing the seawall profile, with established formulae for estimating overtopping discharge (eg Ref 14) to calculate overtopping at various locations along the wall. The relation between overtopping discharge and wave height is not linear, so this will need to be done at frequent intervals along the wall to ensure a good resolution in the calculations. It can be expected that for normally incident waves the greatest reduction in overtopping discharge will occur at a point on the seawall corresponding to the centre line of the offshore breakwater. The overtopping discharge will increase gradually with distance from this point until it reaches the level expected without the breakwater. For example with an offshore breakwater one wavelength long located two wavelengths offshore there will be a substantial reduction in overtopping discharge over a two wavelength stretch of wall. Overtopping discharge levels will return to near their original values at around two wavelengths either side of the breakwater centre line.

Clearly, it is possible to consider a very wide range of cases using the methods described above. We have deliberately here only considered the techniques in order to illustrate how the engineering aspects of the effects of offshore breakwater construction may be considered. One important feature which has not been included here is the effect of an offshore breakwater on the beaches in its vicinity. The physical processes occurring in this situation are complex and still not well understood. At present it is advisable that the effects of an offshore breakwater scheme on the beaches in its immediate vicinity, are investigated using a mobile bed physical model. For long term effects over a large coastal area, research will be required to examine the possibility of linking existing beach plan shape models (eg Ref 15) with an offshore breakwater diffraction model. If this could be achieved it would be a very useful tool for the engineer involved in the design of an offshore breakwater scheme.

5 CONCLUSIONS AND RECOMMENDATIONS

1. Existing mathematical models have been used to predict wave height coefficients in the lee of an offshore breakwater. Diffraction diagrams for various breakwater lengths and incident angles were presented for both monofrequency and random incident wave conditions for the constant depth case.

2. A comparison of the results for random and monofrequency waves, at the peak period and mean direction of the spectrum, lead to the conclusion that using monofrequency results may lead to an underestimate of wave heights in the lee of the breakwater. The results from monofrequency tests will be adequate for the purpose of comparison of schemes, but for a more detailed examination random incident waves should be used.

3. To make the random incident wave calculations does require the use of a computer. One

possibility is that the necessary diffraction calculations could be made by engineers using micro-computer software. This has the advantage of allowing many layouts and incident wave conditions to be considered.

4. A parabolic model was used to include depth varying terms in the calculations. This was found to be successful, although further work is required to define the range of conditions which can be investigated using this model. In principle, it could also be modified to allow random waves to be represented, but this was beyond the scope of the present study.

5. Examples have been provided of the application of the diffraction calculations in selecting breakwater layout, and predicting consequent run up and overtopping. It is hoped that these examples together with the diffraction diagrams will provide the engineering with general guidance in the layout of offshore breakwaters.

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TABLE

TABLE 1 Test programme

Test No	Breakwater length (L = 1 wavelength)	Incident[+] angle/ spectrum	Bathymetry[x]	Model[*] used	Fig No
Monochromatic, unidirectional waves					
1	0.5L	0°	U.D	I.E	2.1
2	0.5L	15°	U.D	I.E	2.2
3	0.5L	30°	U.D	I.E	2.3
4	1.0L	0°	U.D	I.E	3.1
5	1.0L	15°	U.D	I.E	3.2
6	1.0L	30°	U.D	I.E	3.3
7	2.0L	0°	U.D	I.E	4.1
8	2.0L	15°	U.D	I.E	4.2
9	2.0L	30°	U.D	I.E	4.3
Random waves					
10	1.0L	$0°/P.M/\cos^2$	U.D	I.E	7.1
11	1.0L	$15°/P.M/\cos^2$	U.D	I.E	7.2
12	1.0L	$30°/P.M/\cos^2$	U.D	I.E	7.3
13	2.0L	$0°/P.M/\cos^2$	U.D	I.E	8.1
14	2.0L	$15°/P.M/\cos^2$	U.D	I.E	8.2
15	2.0L	$30°/P.M/\cos^2$	U.D	I.E	8.3
16	1.0L	$0°/P.M/\cos^6$	U.D	I.E	9.1
17	1.0L	$0°/J/\cos^2$	U.D	I.E	9.2
Monochromatic, unidirectional waves					
18	1.0L	0°	U.D	P	13.1
19	1.0L	0°	1:100 slope	P	13.2
20	1.0L	0°	1:50 slope	P	13.3

Note

+ Incident angle for random wave input refers to mean direction. See Fig 1 for definition of angle convention

x U.D - Uniform depth

* I.E - Integral equation model, P - Parabolic model.

FIGURES.

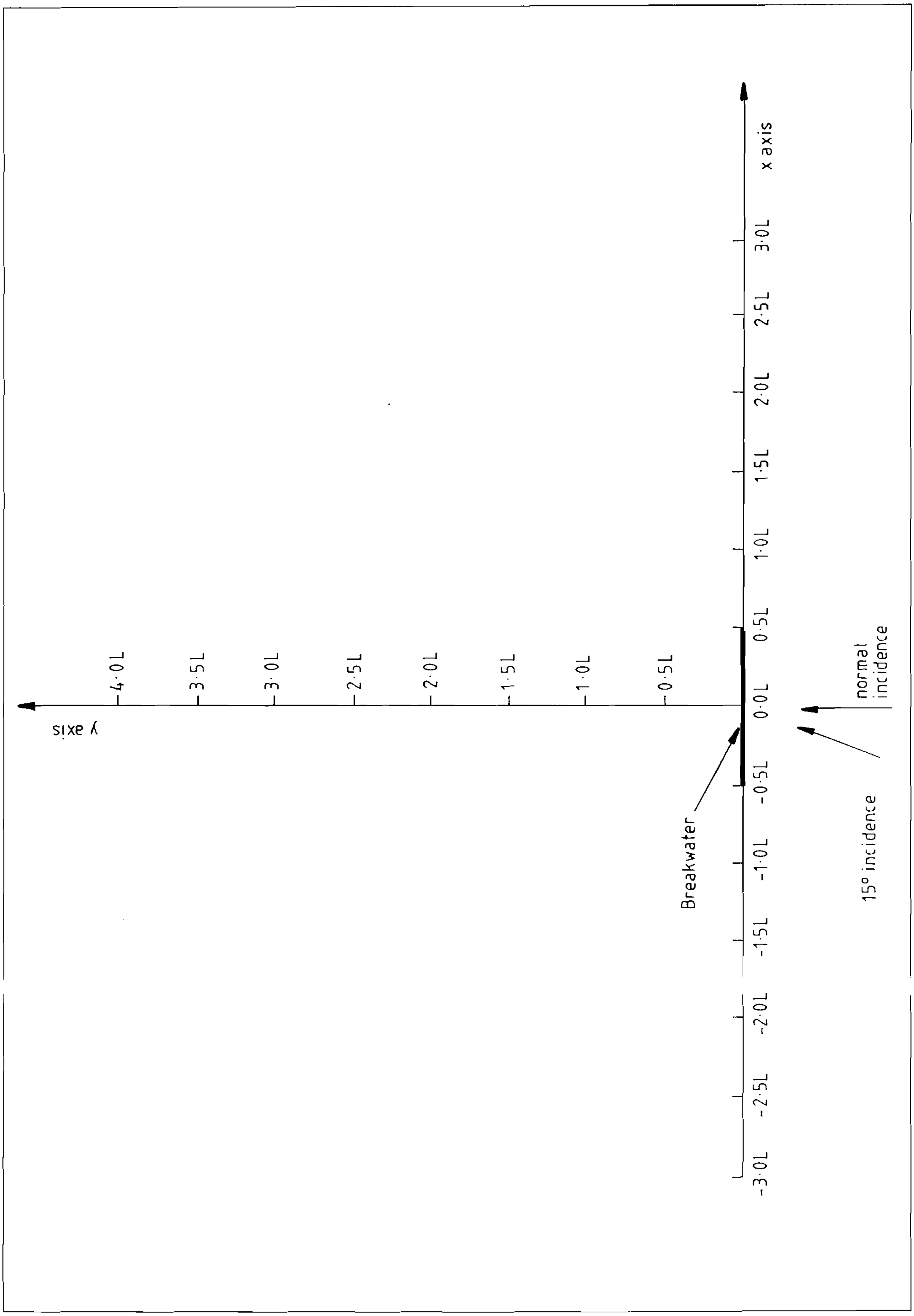

Fig 1 Breakwater layout (L = 1 wavelength)

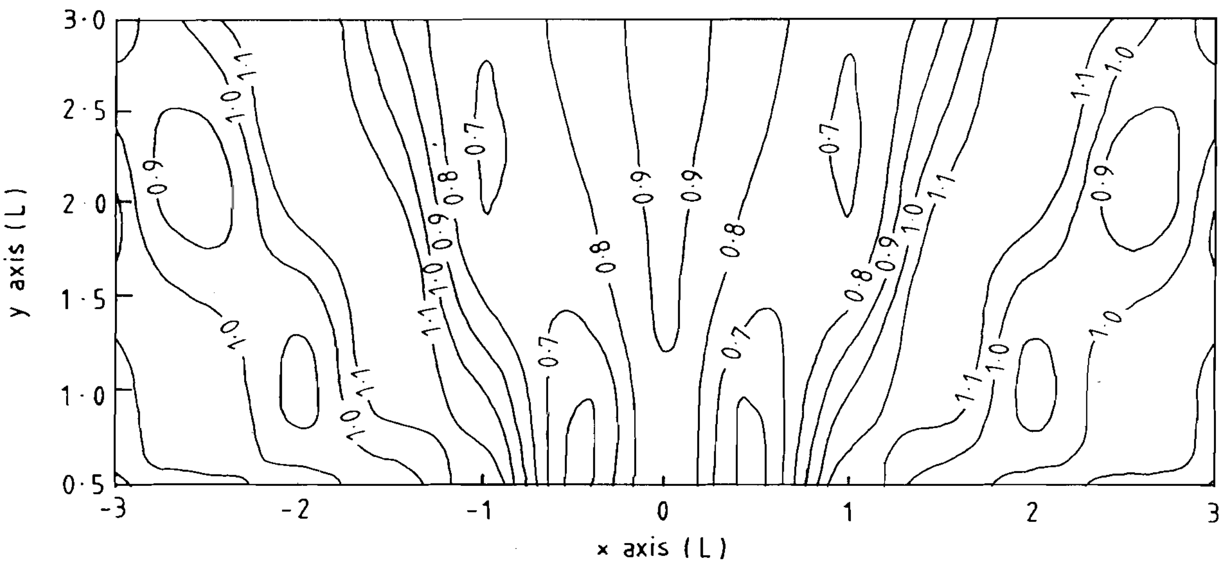

Fig 2·1 Incident direction 0°.

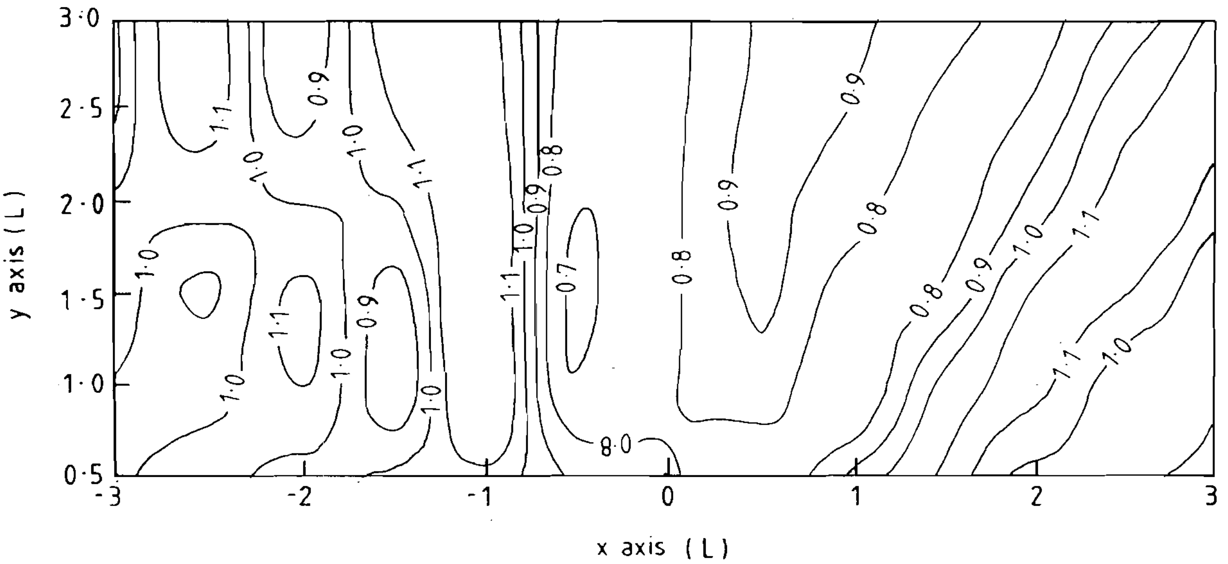

Fig 2·2 Incident direction 15°

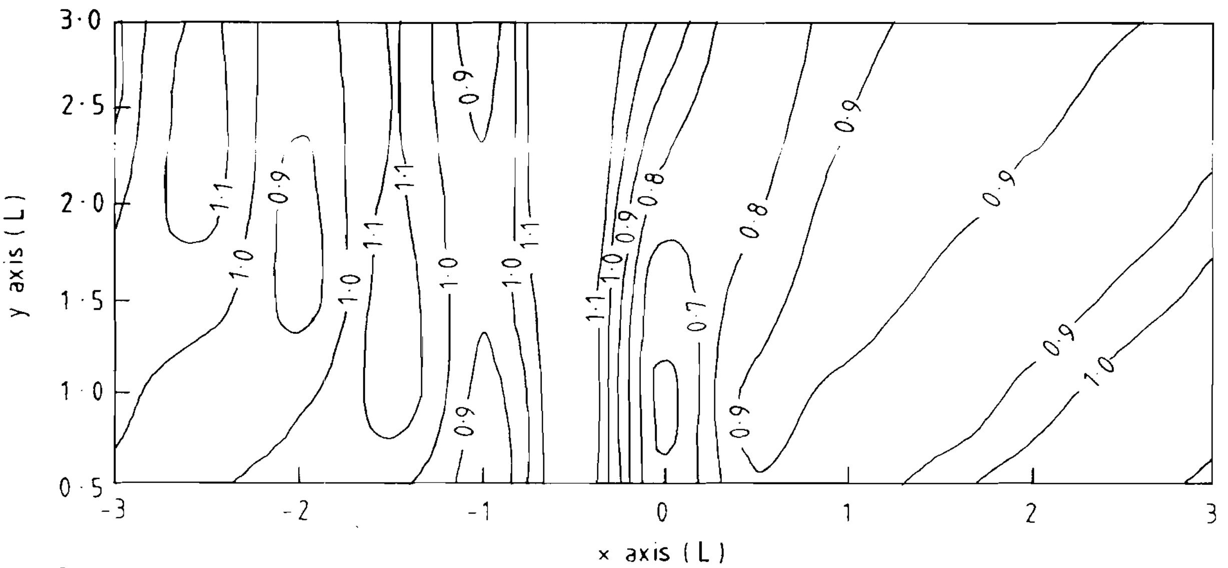

Fig 2·3 Incident direction 30°

Fig 2 Wave height coefficients for monofrequency waves, breakwater length 0.5L

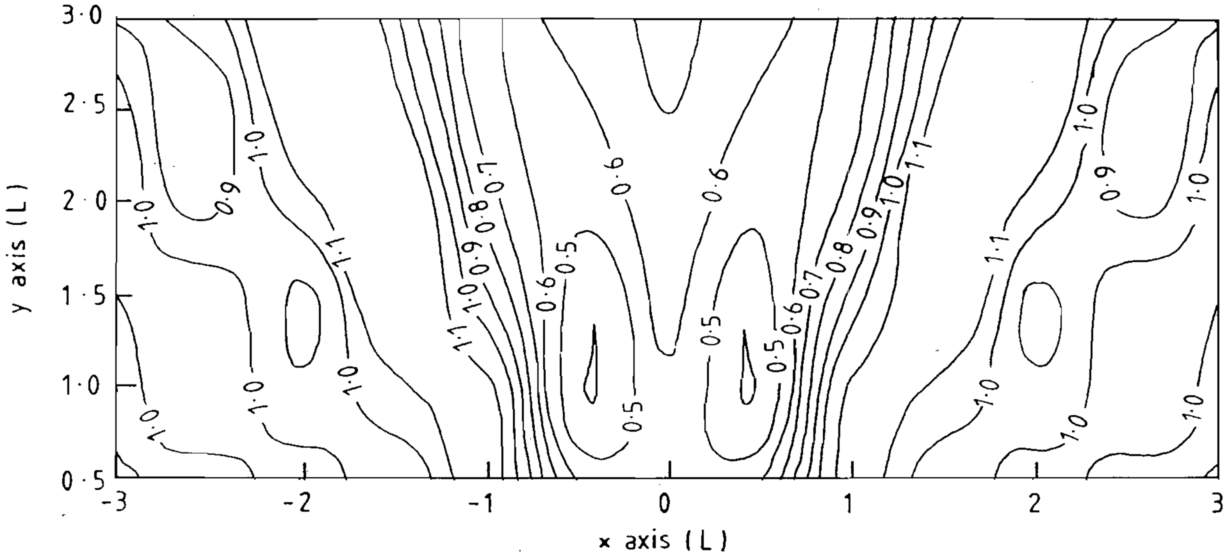

Fig 3·1 Incident direction 0°

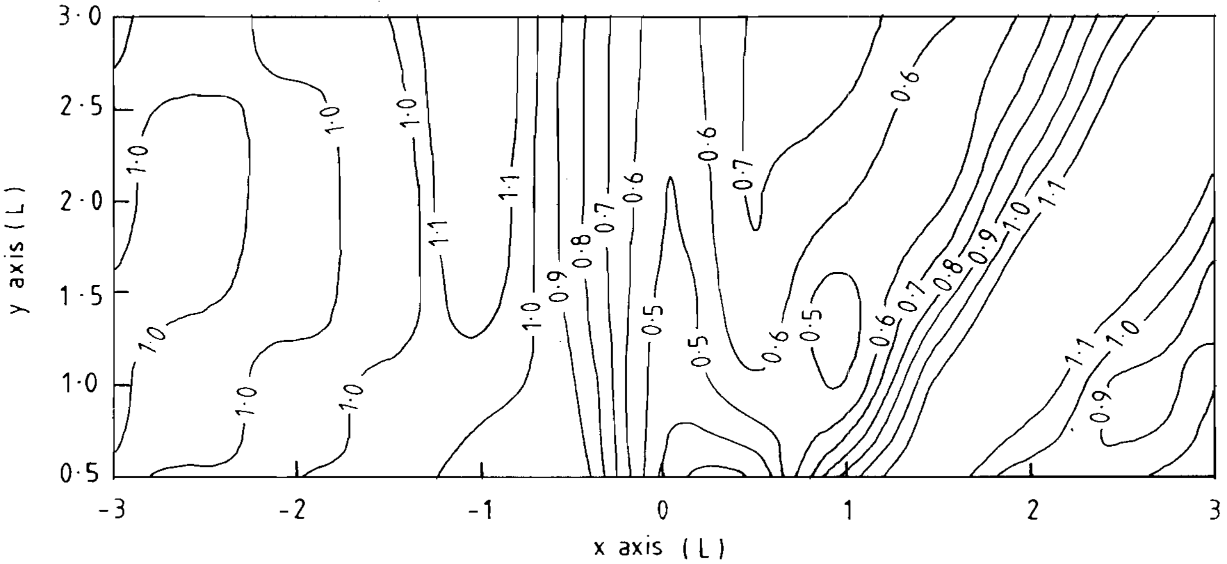

Fig 3·2 Incident direction 15°

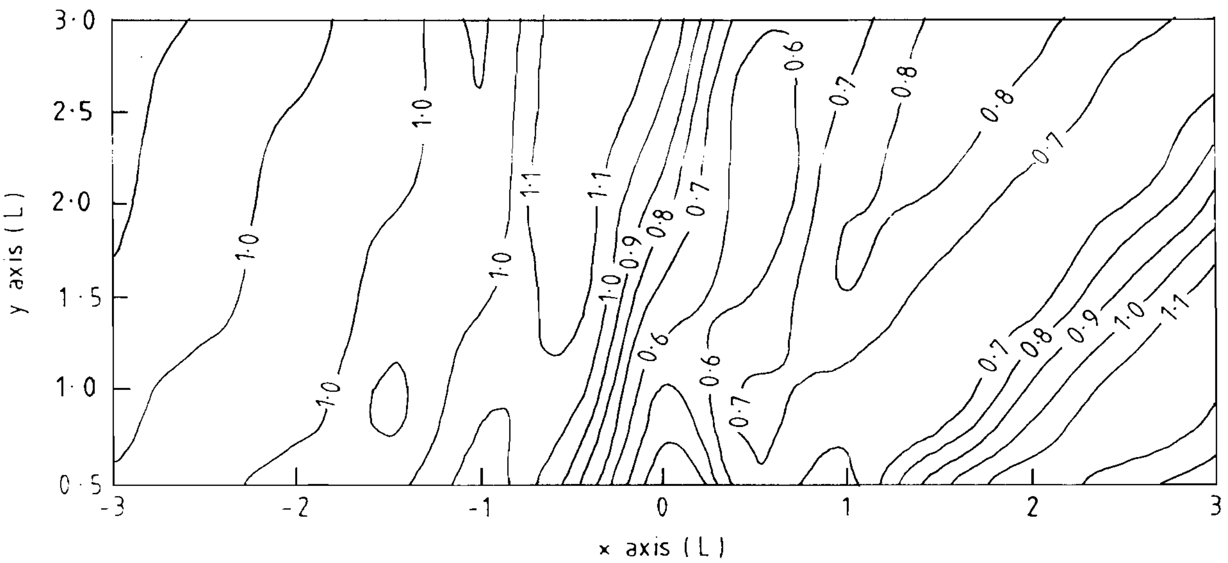

Fig 3·3 Incident direction 30°

Fig 3 Wave height coefficients for monofrequency waves, breakwater length 1.0L

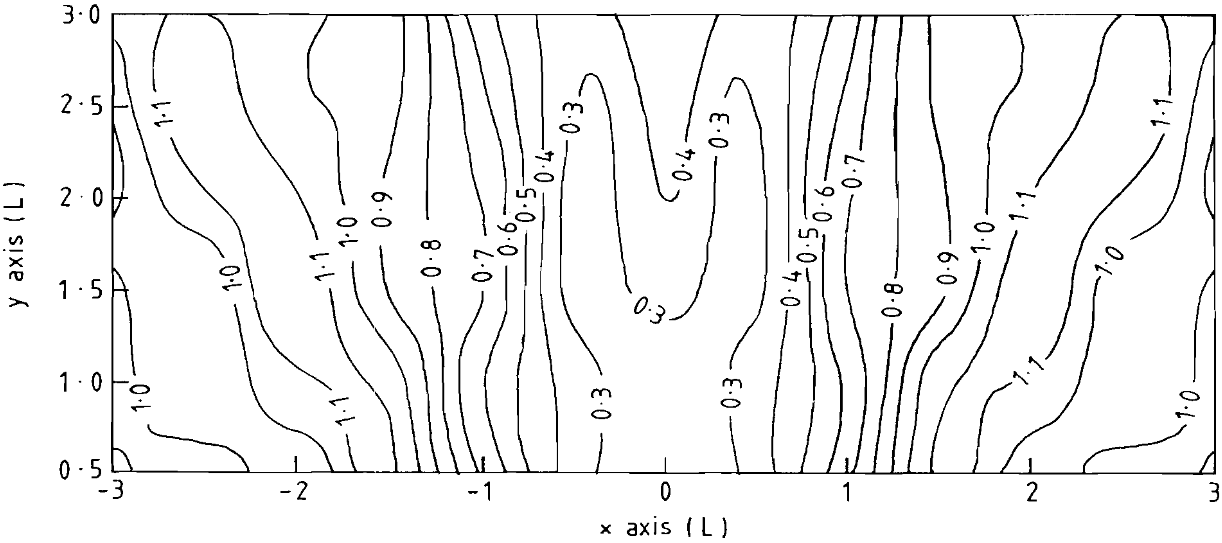

Fig 4·1 Incident direction 0°

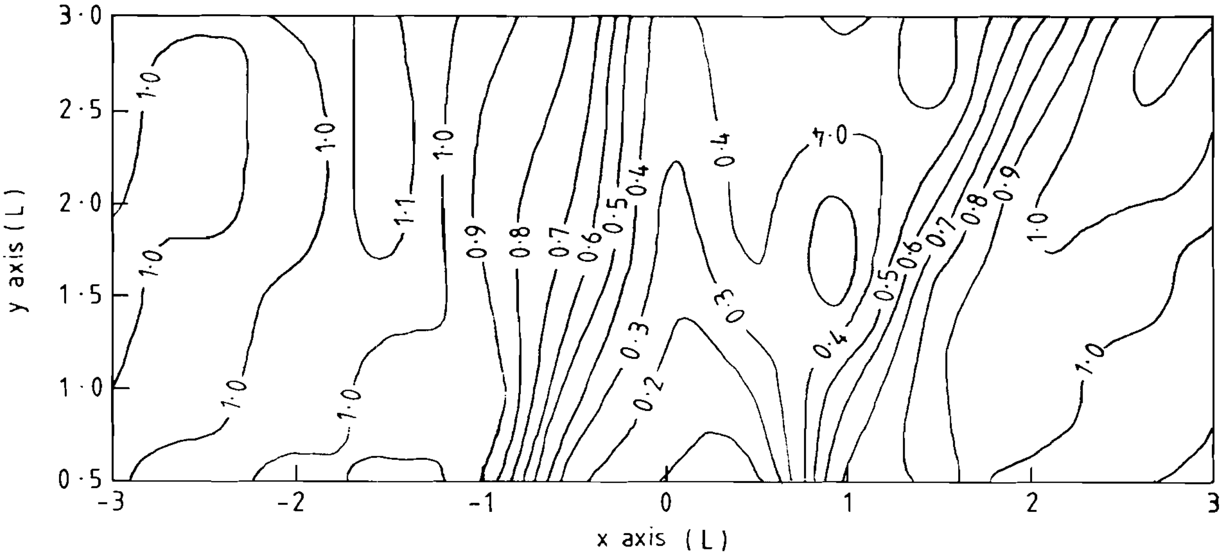

Fig 4·2 Incident direction 15°

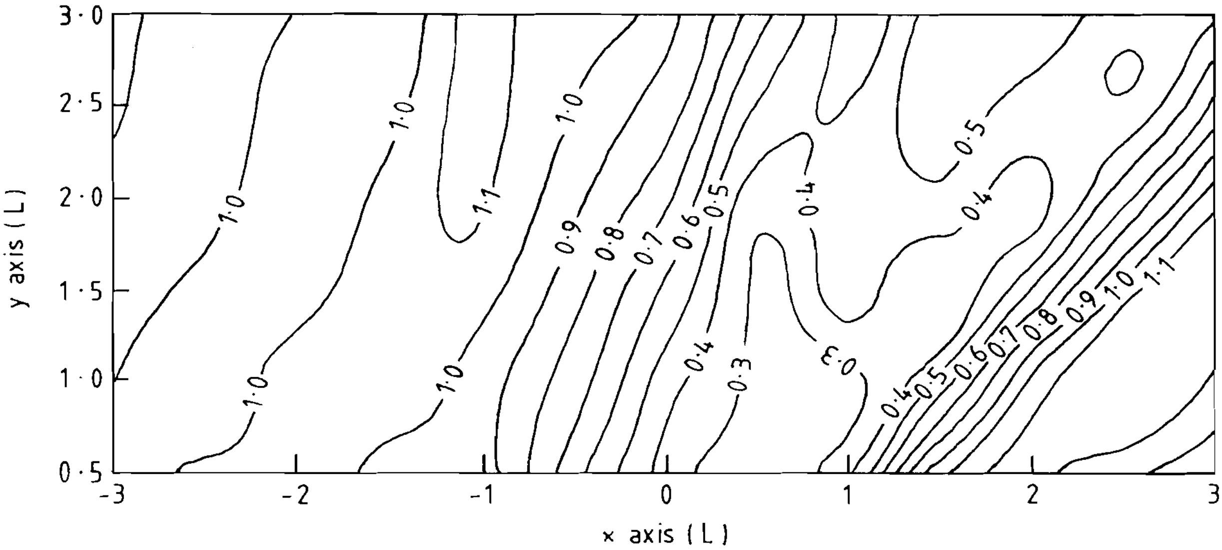

Fig 4·3 Incident direction 30°

Fig 4 Wave height coefficients for monofrequency waves, breakwater length 2·0L

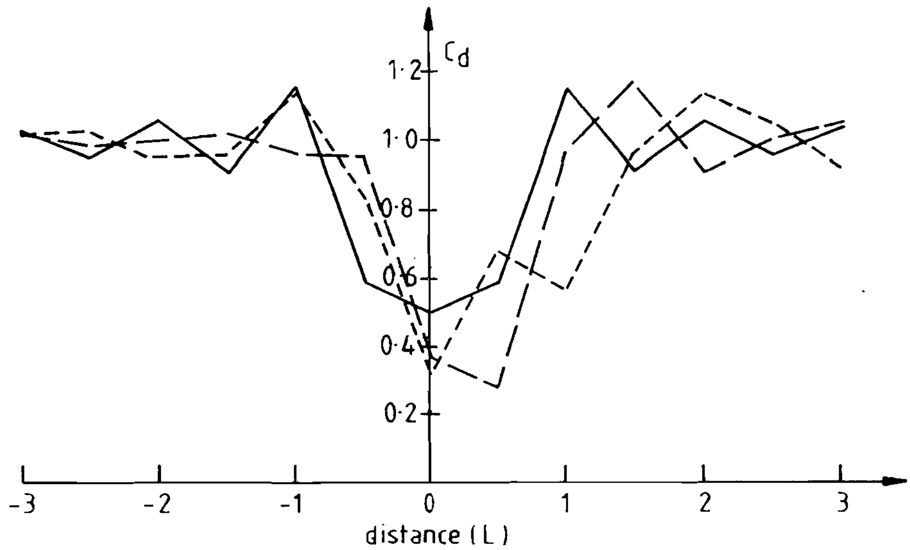

Fig 5·1 Cross section 0·5L from breakwater

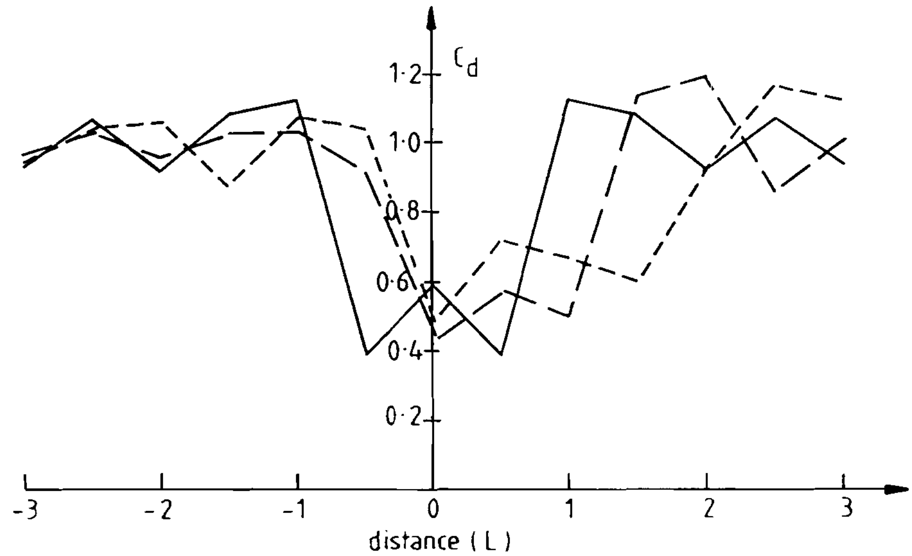

Fig 5·2 Cross section 1L from breakwater

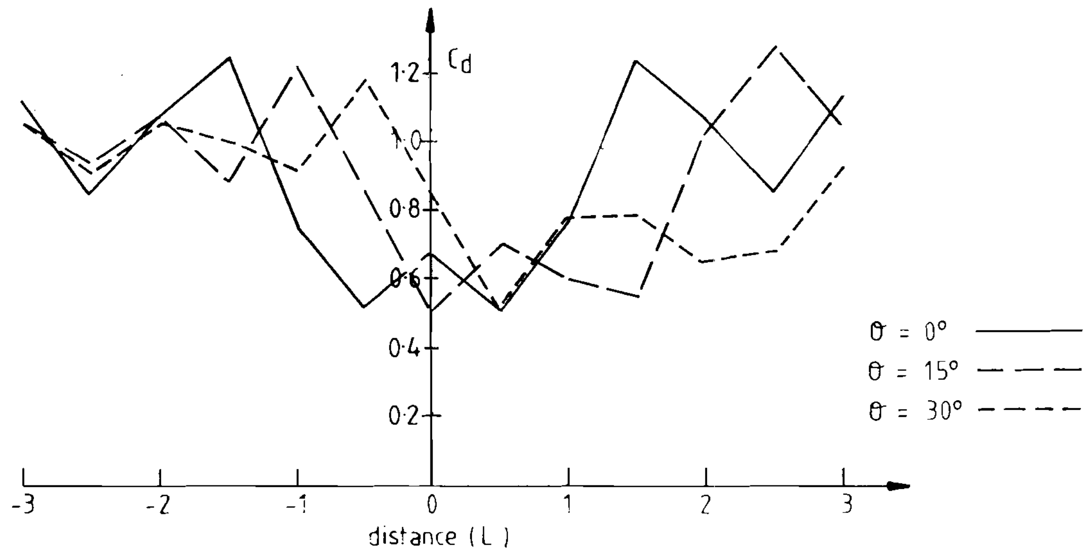

Fig 5·3 Cross section 2L from breakwater

Fig 5 Comparison of wave height coefficients (Cd) for breakwater of length 1L

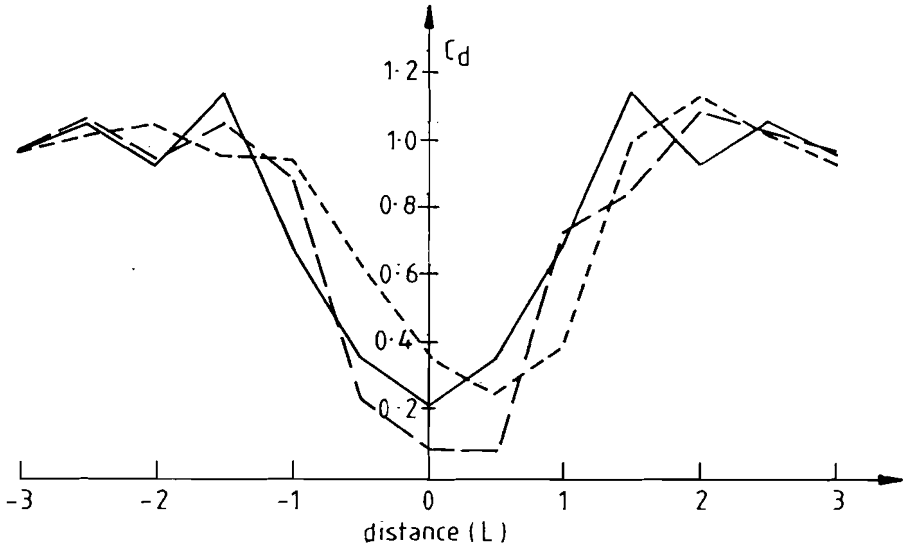

Fig 6·1 Cross section 0·5L from breakwater

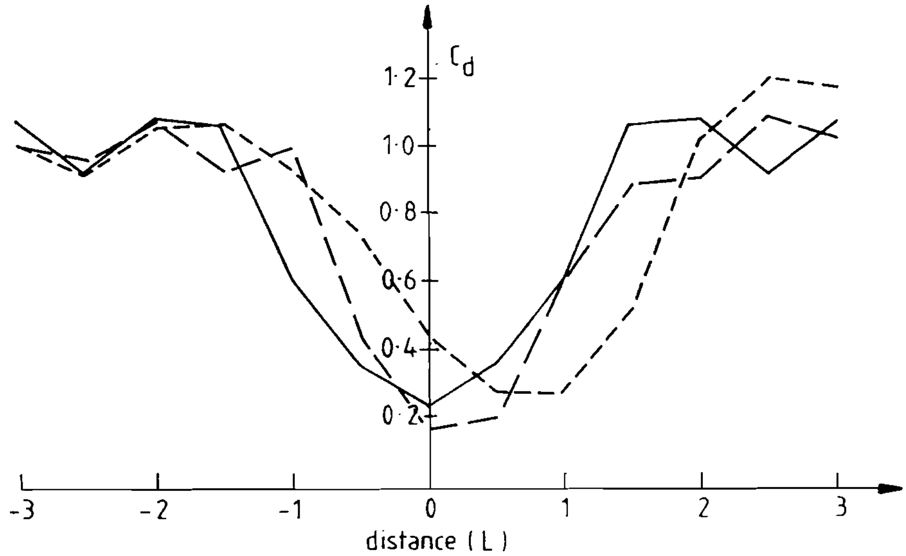

Fig 6·2 Cross section 1L from breakwater

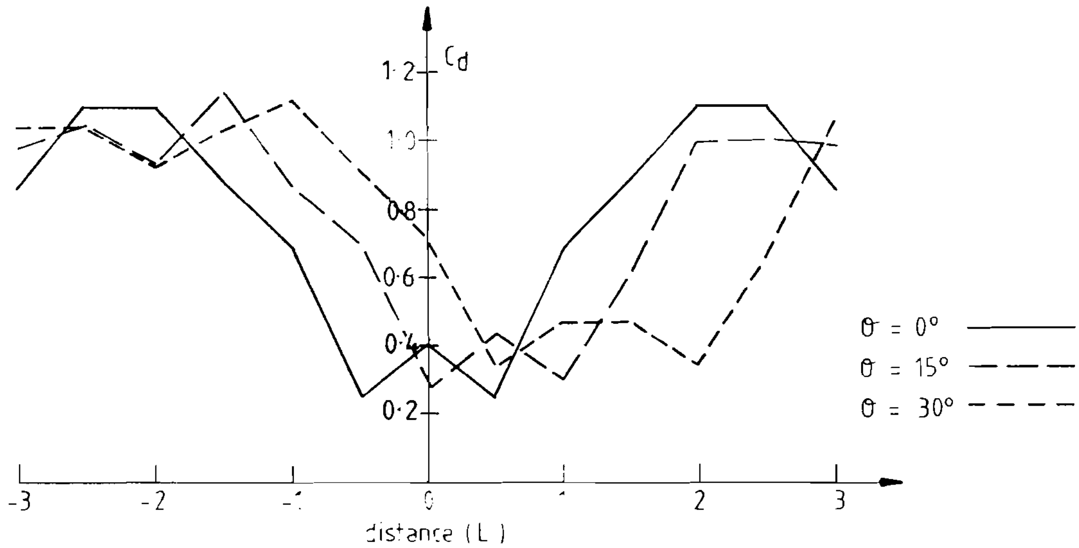

Fig 6·3 Cross section 2L from breakwater

Fig 6 Comparison of wave height coefficients (Cd) for breakwater of length 2L

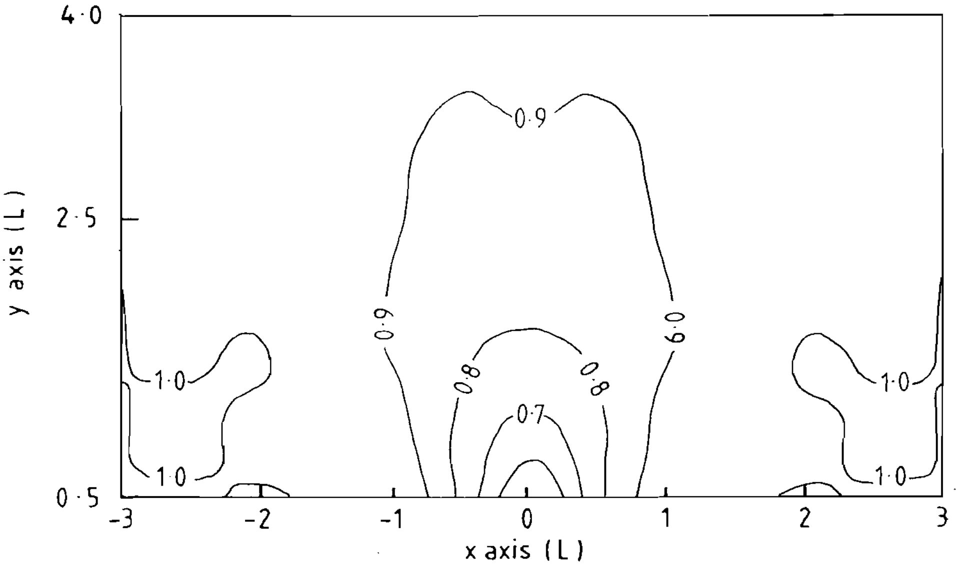

Fig 7·1 Mean direction 0°

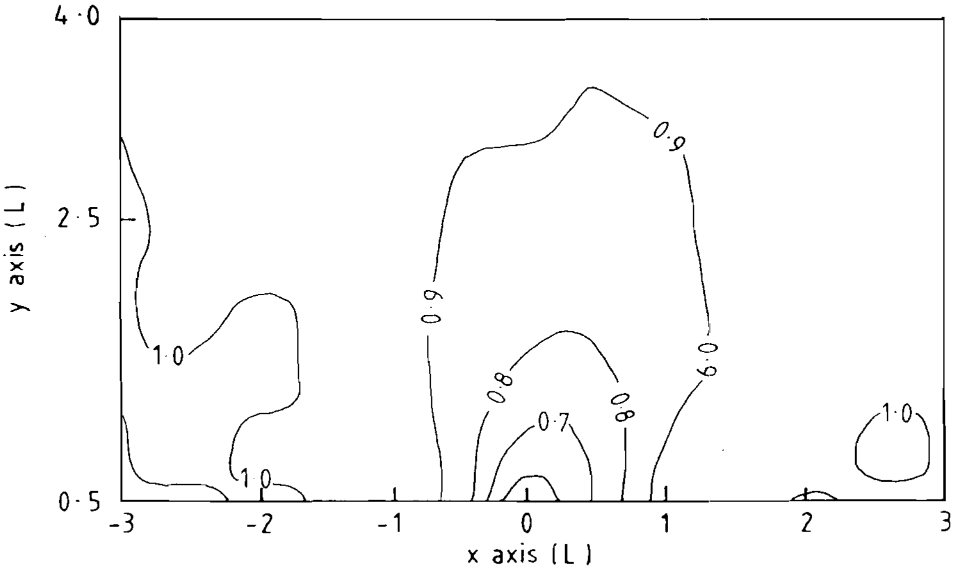

Fig 7·2 Mean direction 15°

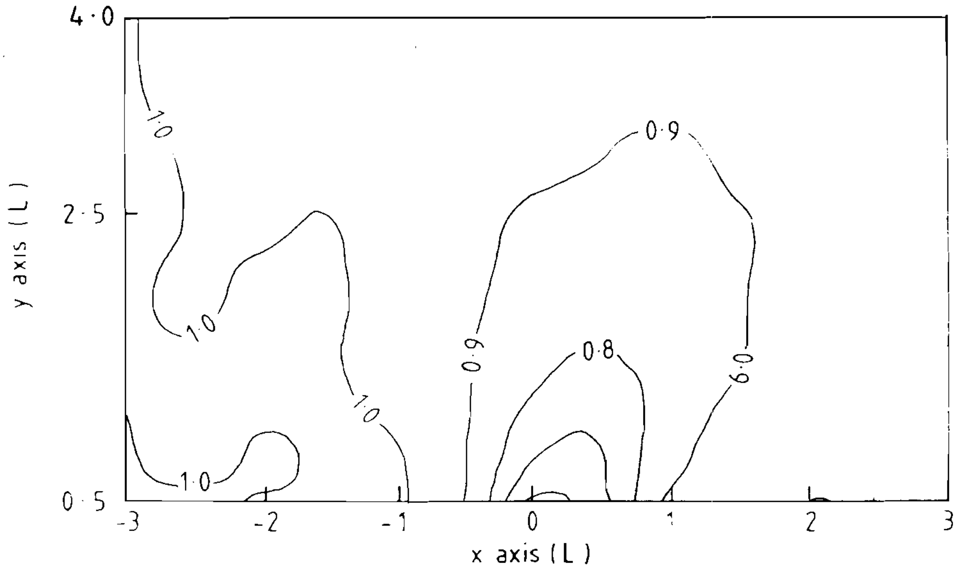

Fig 7·3 Mean direction 30°

Fig 7 Wave height coefficients for random waves, breakwater length 1.0L, P-M spectrum, $\cos^2$ spreading

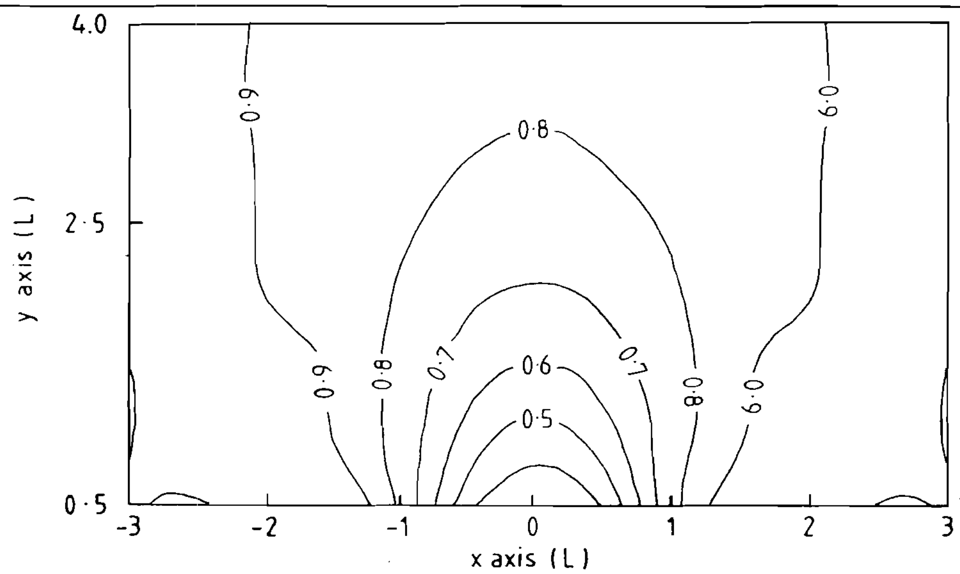

Fig 8·1 Mean direction 0°

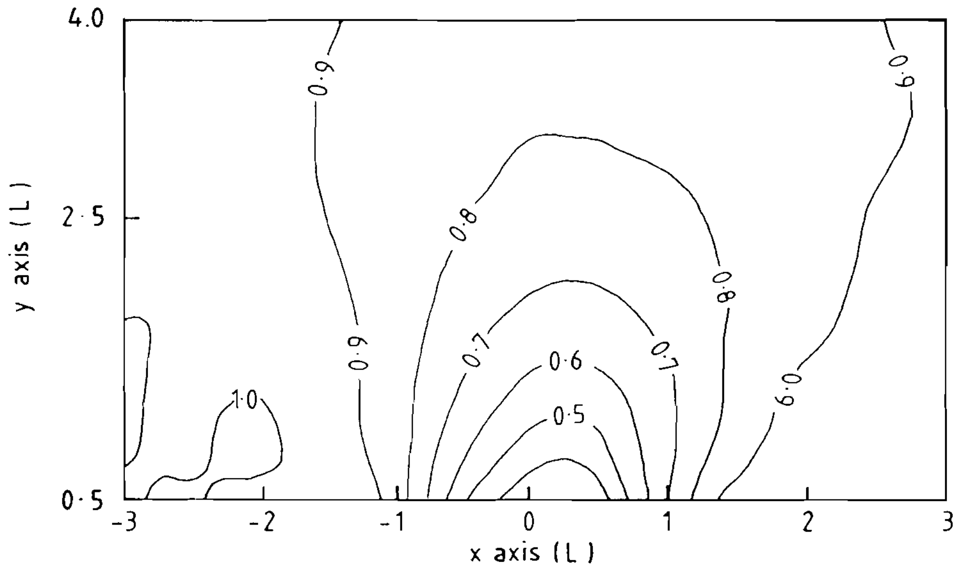

Fig 8·2 Mean direction 15°

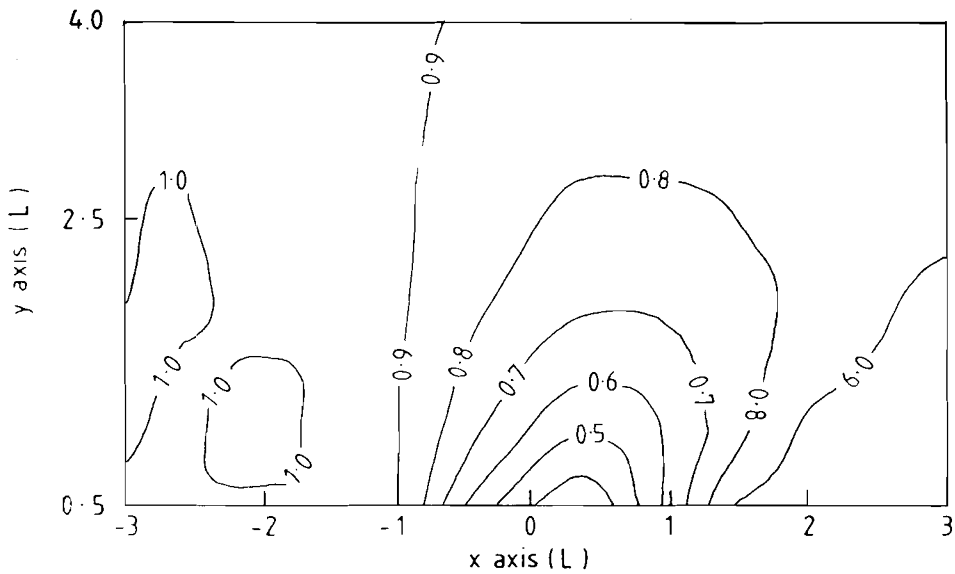

Fig 8·3 Mean direction 30°

Fig 8 Wave height coefficients for random waves, breakwater length 2.0L, P-M spectrum, $\cos^2$ spreading

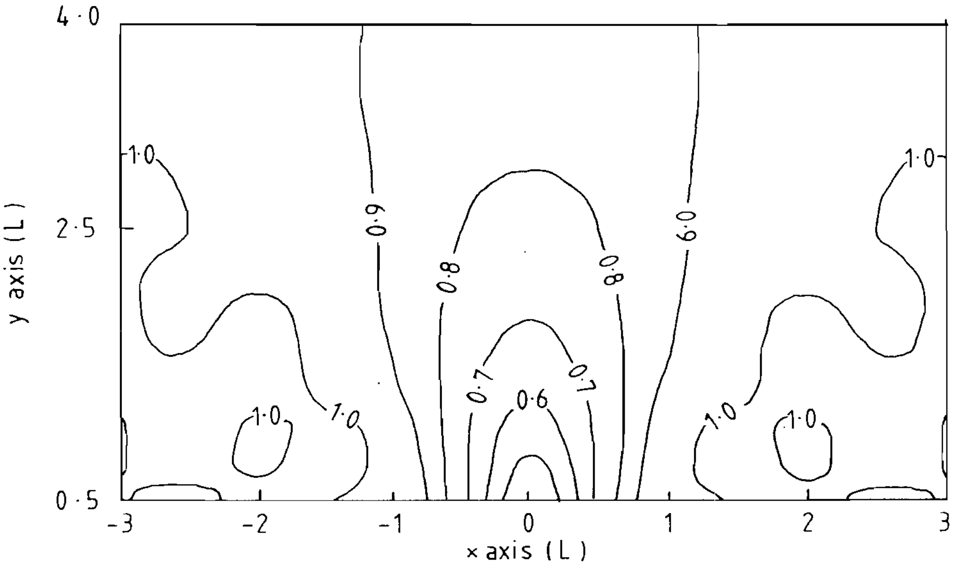

Fig 9·1 P-M spectrum, $\cos^6$ spreading

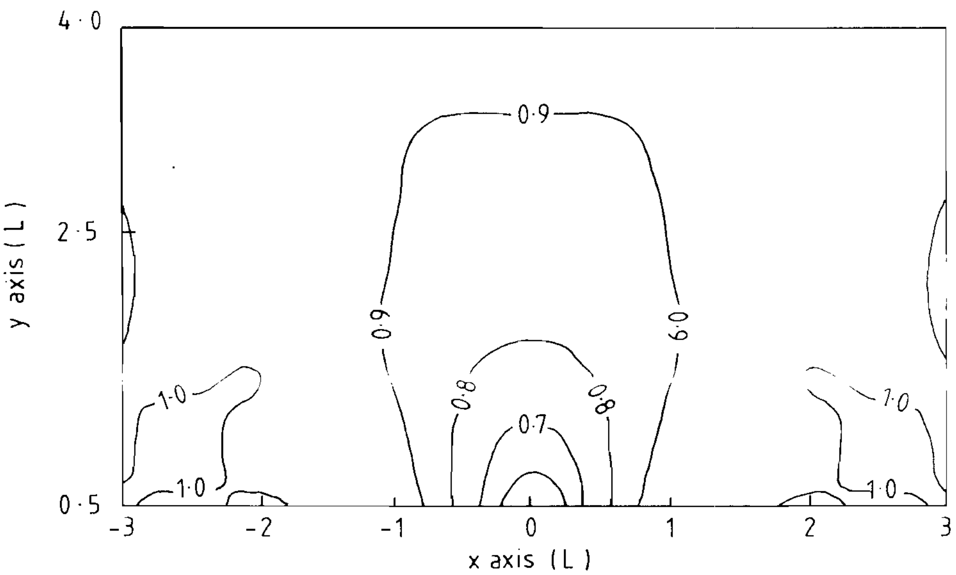

Fig 9·2 JONSWAP spectrum, $\cos^2$ spreading

Fig 9 Wave height coefficients for random waves, breakwater length 1·0L, normal incidence

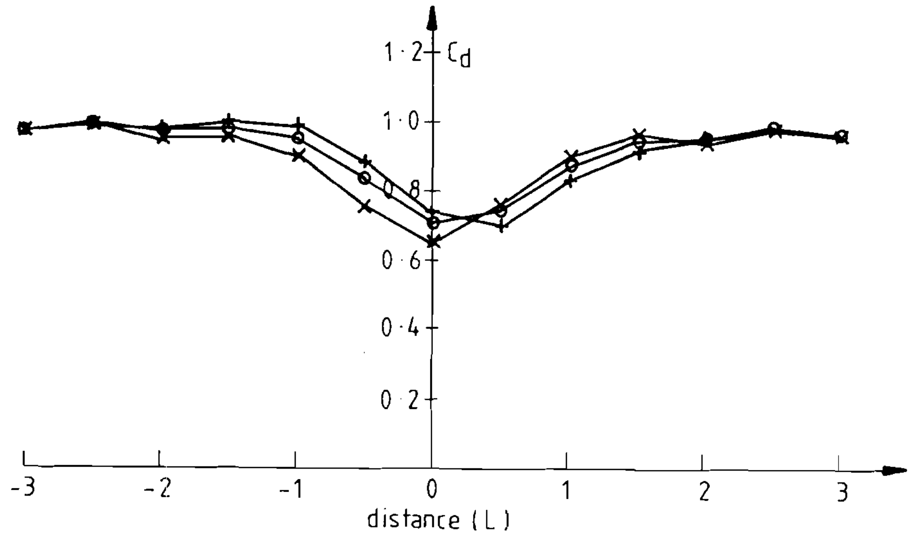

Fig 10·1 Cross section 1L from breakwater

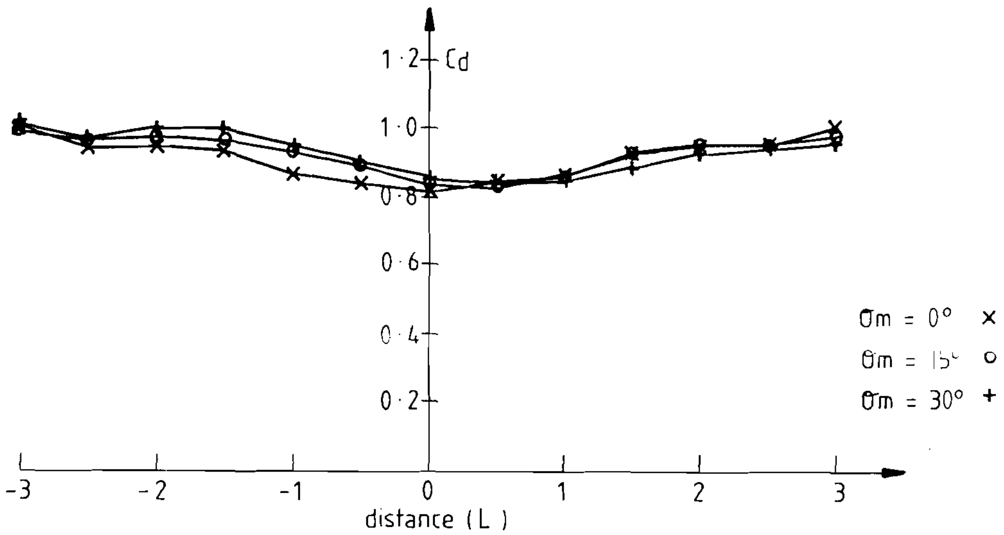

Fig 10·2 Cross section 2L from breakwater

Fig 10 Comparison of wave height coefficients (Cd) for breakwater of length L (P-M spectrum)

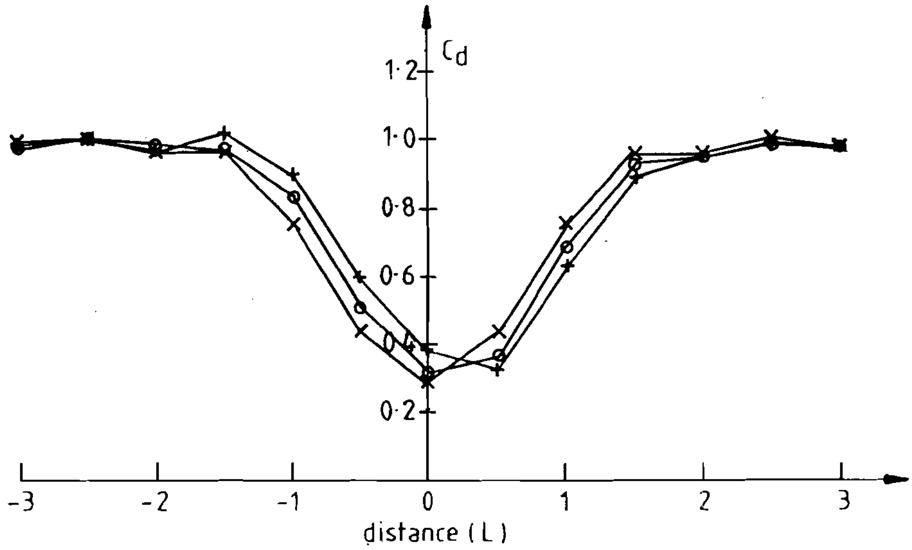

Fig 11·1 Cross section 0·5L from breakwater

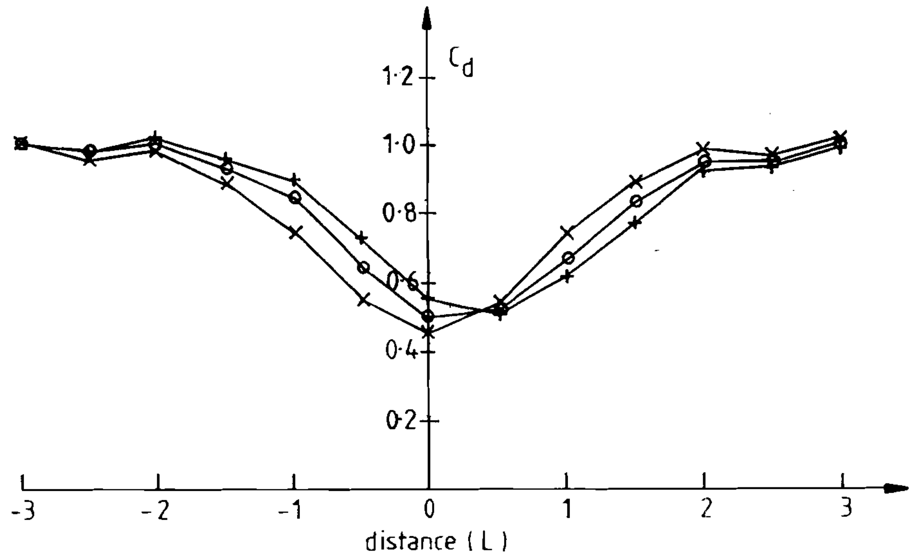

Fig 11·2 Cross section 1L from breakwater

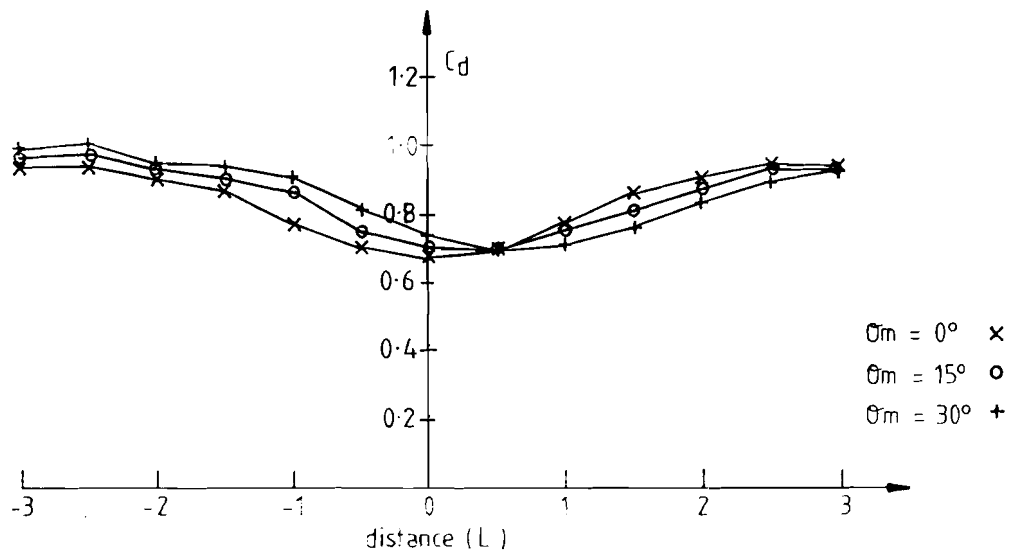

Fig 11·3 Cross section 2L from breakwater

Fig 11 Comparison of wave height coefficients (Cd) for breakwater of length 2L (P-M spectrum)

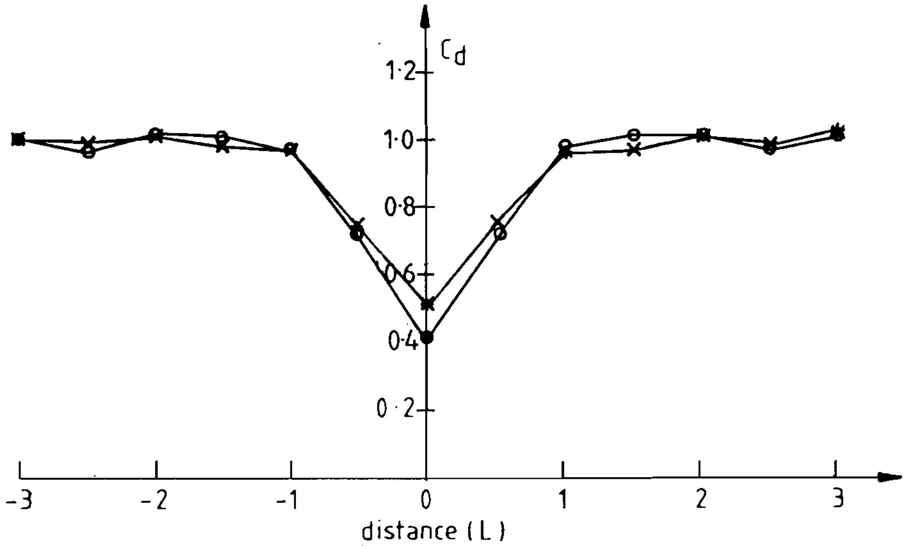

Fig 12·1 Cross section 0·5L from breakwater

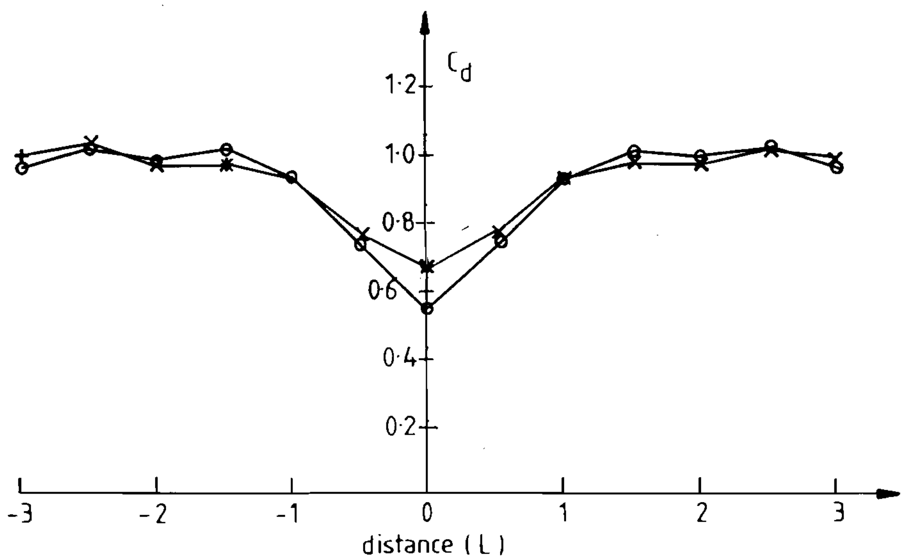

Fig 12·2 Cross section 1L from breakwater

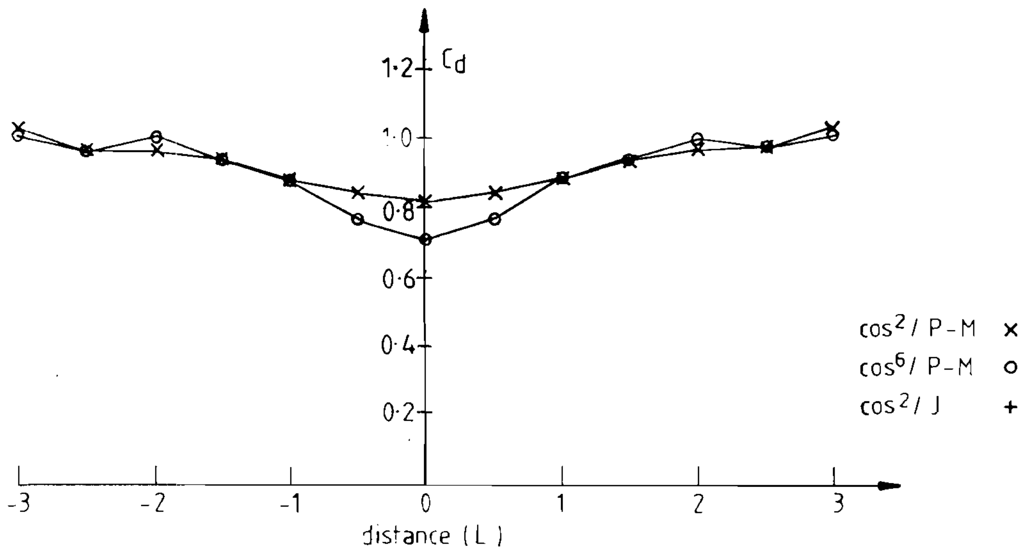

Fig 12·3 Cross section 2L from breakwater

Fig 12 Effect of different incident spectra, breakwater length 1L

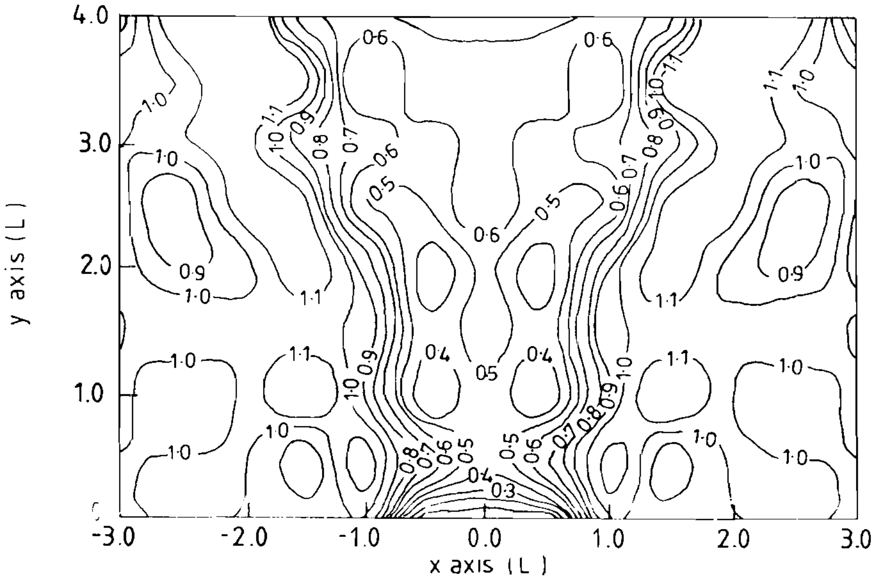

Fig 13·1 Uniform depth

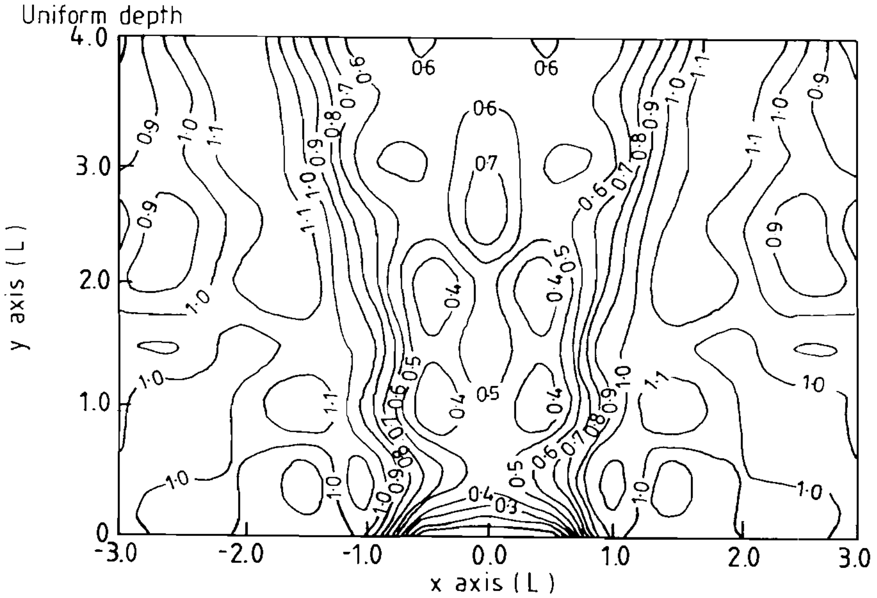

Fig 13·2 1: 100 slope

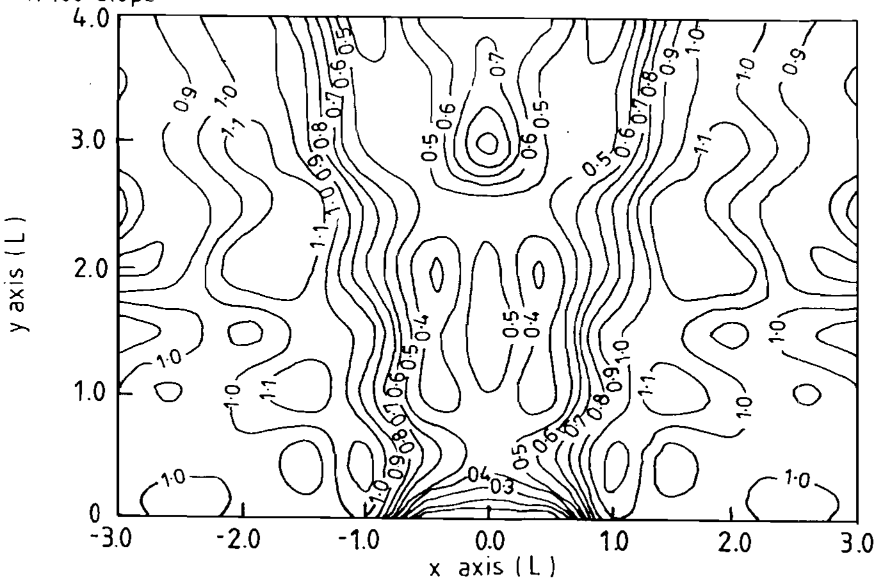

Fig 13·3 1: 50 slope

Fig 13 Wave height coefficients for variable depth cases, breakwater length 1·0L, normal incidence