Sludge Disposal in Coastal Waters

Liverpool Bay Metal Transport Modelling Study

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ABSTRACT

The Water Directorate of the UK Department of the Environment commissioned Hydraulics Research Ltd to develop a mathematical model capable of simulating and predicting the physical dispersal of dissolved and adsorbed heavy metals associated with sewage sludge and dredged spoil disposal in Liverpool Bay. A matching set of two dimensional, two layer mathematical models were set up to simulate the tidal flows, suspended mud transport and heavy metal transport on a new AMT Distributed Array Processor. The model has three dynamically linked grids of size 2700m in the Eastern Irish Sea, 900m in Liverpool Bay and 300m in the Mersey Estuary. The Mersey Estuary was modelled in some detail because its mud is heavily contaminated with metals discharged from industry in the past. The exchange of water and suspended mud to and from the Mersey Estuary plays an important role in the dispersal of metals in Liverpool Bay. The model was divided into two layers to help resolve the vertical structure of the density and wind driven currents and the suspended solids profile. The model simulated the dynamic salinity - density field which drives the gravitational circulation in the Mersey Narrows and thereby helps contain suspended mud in the Mersey Estuary.

The tidal model was calibrated satisfactorily against observations of tidal levels, tidal velocities and salinity. The mud transport model included the processes of erosion, vertical turbulent exchange, suspension, settling, deposition and the effect of waves. The distribution of mud deposits on the bed was prescribed from observations. The rate of disposal of contaminated dredged mud in Liverpool Bay was estimated from records. The mud transport model was calibrated against observed concentrations of suspended mud. The much smaller quantity of slowly decaying organic sewage sludge was assumed to be wholly mixed and transported with the suspended mud.

The heavy metal transport model, which can handle one metal at a time, separates newly discharged metals from existing metals. It allows for metals to move with the water in solution in the dissolved state, and with the suspended mud in the adsorbed state. The rate of desorption and adsorption of metals to and from the mud is calculated on the divergence from equilibrium using a partition coefficient and a rate constant. The heavy metal model was calibrated by reference to observations in the Mersey Estuary.

The model is considered to be a practical tool for predicting the physical dispersal of heavy metals disposed in coastal waters and the methodology has already been applied to other UK coastal sites.

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- 1. Formulation of 2D two layer flow model
- 2. Formulation of 2D two layer mud transport model
- 3. Formulation of 2D two layer metal transport model

In 1982, Odd (Ref 1) suggested that it should be feasible to model the transport of heavy metals in Liverpool Bay using a mathematical model. By 1985 (Ref 2) a two dimensional two layer numerical model with three grids of different sizes (see Fig. 1) had been set up to model the Eastern Irish Sea, Liverpool Bay and the Mersey Estuary. The model simulated the tidal flow, influenced by a static imposed salinity field, the transport of mud and the transport of adsorbed new metal, which was input to the model at the locations of the main loads.

The model generated tidal currents in acceptable agreement with the observations, but the sense of the residual vertical circulation in Mersey Narrows was not as expected, being landward at the surface and seaward at the bed. The mud model gave suspended concentrations of the correct order in Liverpool Bay. The metal model results appear to be reasonable, but since only new metal was modelled it is not possible to compare the results with observations because the observations show much larger concentrations due to metal being discharged into Liverpool Bay for many years. The model showed that it is possible to model metal transport in Liverpool Bay but more work was needed to calibrate and validate the model for tidal flows, suspended solids and especially predicted metal concentrations.

In 1988, the research programme continued with an investigation into the physics of the process of desorption of metals from sludge (Ref 3). This work gave an estimate of the equilibrium partition coefficients and rate constants for use in the metal transport model. It was subsequently reviewed by Water Research Centre (Ref 8).

The present report describes the enhancements that have been made to the mathematical model since 1985 and presents the model results and comparisons with observations. The metal transport model in particular has been further developed, including the use of physical coefficients taken from the report on metal desorption. This model is now a practical tool to investigate metal transport in Liverpool Bay. The same methodology can also be applied to other estuarine and coastal areas and such a study has already been carried out for the Solent.

2. FLOW MODELLING

The original version of the two dimensional, two layer flow model of the Eastern Irish Sea, Liverpool Bay and the Mersey Estuary has been described in Reference 2. The layout of the three dynamically linked model grids, of grid sizes 300m, 900m, and 2700m is shown in Fig 1. The model is orientated at 15° to the west of north so as to align with Mersey Narrows. It has 5340 model cells in each of two layers and is run with a timestep of about 5 seconds. Variable width cells are used near the coast. The model datum is Ordinance Datum Newlyn (ODN).

The formulation of the flow model (TIDEFLOW2D-2L) is described in Appendix 1. The equations are solved using a finite difference method with explicit differences horizontally and implicit differences in the vertical. The flow model is written in FORTRAN Plus and run on an AMT DAP 605 parallel processing computer. This allows the computation at up to 4096 model cells to be carried out simultaneously.

2.1 Boundary conditions for the flow model

The flow model is driven by elevations on the two open boundaries - from Anglesey to the Isle of Man and from the Isle of Man to Whitehaven (see Fig. 1). Since the original work on the Liverpool Bay model was carried out a two dimensional model of the whole of the Irish Sea has been set up and run by HR using the M2 tidal constituent. To provide boundary conditions for the present model the Irish Sea model was also run for repeating M2 + S2 and M2 - S2 tides to give representative spring and mean tides. The levels in the Irish Sea model (which has a uniform 1500m grid) were extracted at the positions corresponding to the boundary cells of the Liverpool Bay model (which has a 2700m grid in the Eastern Irish Sea) in order to set up new boundary files. It was expected that these boundary conditions would give better representations of the flow particularly in the Eastern Irish Sea (because the Irish Sea model calibrated extremely well against observed M2 currents). At the top of the Mersey a constant river discharge of 50 m³/s is input.

2.2 Interface level

In the original work (Ref 2) the interface between the two model layers was set at 6.5m below ODN everywhere that the bed was at least 8m below ODN, elsewhere there was a single layer.

In the present work the interface has been set to 6m above the bed. This is useful throughout the two layer part of the model, as the near bed region usually has higher suspended solids concentrations than the rest of the water column. Nevertheless much

of the Mersey estuary can still only be modelled with one layer and this is a disadvantage.

2.3 Modelling salinity

In the original work an imposed static salinity distribution was assumed (see Fig. 2, this figure gives densities i.e 0.76 times the salinity). The purpose of this was to drive gravitational circulation in Mersey Narrows and elsewhere. Because of storage limitations in the DAP computer used for that work it was not possible to model the movement of the salinity field caused by the tidal currents so a static distribution was used.

It was found (Ref 2) that the residual circulation in the Mersey Narrows was not in the expected sense (it was landward in the upper layer and seaward in the lower). A check on the model formulation was carried out by running the model with no imposed tide. In this case the expected two layer flow did result.

Since 1985, HR has acquired a new DAP computer from Active Memory Technology Ltd which has 32 times as much memory as the old one. The model was therefore modified to include the transport of salt by the current as described in Appendix 1. The initial condition for salinity was the density field shown in The model also includes a freshwater Figure 2. discharge of 50 m³/s into the Mersey. Time histories of salinity from the model on a spring tide are shown in Figures 20-22. They agree reasonably with observations (taken from Ref 6). The observed tide ranges were 8.1m on 16th September 1982, 7.2m on 21st September 1983 and 9.7m on 8th March 1989 (Admiralty prediction). This compares to a model range at Liverpool of 7.7m. Contours of salinity are shown in Figures 23 and 24.

It was found, however, that despite the reasonable representation of the variation of salinity in both time and space the sign of the residual flow in the Narrows was still not as expected. In order to give a gravitational circulation in Mersey Narrows in agreement with expectation it was decided to use an ad-hoc multiplication factor to the density driving force. This compensates for the poor vertical resolution in the model (only two layers of which one is a thin bed layer) and for the fact that over most of the area of the Mersey Estuary the model includes only one layer (because it is so shallow) so the driving of the two layer residual circulation has to be confined to the Narrows rather than extending over the whole estuary and is consequently underestimated. The factor is applied only in the Mersey estuary and Narrows and is set to one for Liverpool Bay and the Eastern Irish Sea. By using a factor of four the expected sign of the tidal residual discharge is found as shown in Figures 15, 16 and 17. These model residuals (after 3 tides simulation) are for a spring tide, those for a mean or neap tide would be expected to be larger. Comparable residual discharges in Liverpool Bay for a spring tide with a steady wind of 10 m/s from the west are shown in Figures 18 and 19 after three tides of simulation.

2.4 Flow model calibration

In the model tests presented here the Manchester Ship Canal has not been included in the simulation because tidal flap gates have been constructed.

Tidal currents predicted by the two layer flow model in Liverpool Bay are compared with Admiralty Diamond observations for a mean spring tide in Figures 4 and 5. The upper model layer is that plotted in these

figures as it comprises most of the water column. A spring tide was chosen because the largest amount of sediment is in movement on a spring tide. The observation sites are shown in Figure 3. The results are shown for the third tide when transients are negligible. The agreement shown is generally very good. The model was run with a horizontal eddy viscosity coefficient of 20 m²/s and a bed roughness length of 0.2m corresponding to the generally sandy bed of Liverpool Bay. The model was run to simulate a condition without wind.

An example of the comparison of model spring tide currents with observed values at sites in the Mersey Estuary, including the Narrows is shown in Figures 7, 8, 9, 10, 13 and 14. These sites include Admiralty Diamond currents and also some results from the HR physical model of the Mersey. The location of the sites is shown in Figure 11. It can be seen that the 300m grid does not resolve the complicated estuary channels above about Eastham but the tidal currents in the Narrows are quite well represented. The difficulty of modelling the Mersey Estuary using a model with this grid size has been examined in detail in Reference 5.

Tidal elevations in the Mersey estuary on a spring tide (taken from Ref 4) are shown in Figure 12. It can be seen again that the 300m grid of this model cannot represent successfully the flow in the upper estuary.

The simulated flow field is considered satisfactory for predicting the movement of sediment and metal except in the upper part of the Mersey estuary where the 300m grid is too coarse to resolve the flow. This is unimportant in a model designed to study the lower Mersey estuary and Liverpool Bay.

3. SEDIMENT TRANSPORT MODELLING

Because much of the metal in Liverpool Bay is in an adsorbed state on the estuary mud, a model of mud transport was constructed (Ref 2). Details of the mud transport model are given in Appendix 2.

The mud transport model has been modified so as to accept the new interface position described above in section 2.2.

3.1 Effect of waves and treatment of

the bed

The model has been modified so as to represent the mud bed in two layers. The upper layer is the recently deposited low density fluffy mud and the lower layer is the stiffer mud which has been in place for much longer. At slack water mud settles from suspension into the upper bed layer and this mud deposit is supposed to consolidate into the lower layer only if it reaches a mass of more than 10 kg per square metre. Over most of the model area this does not happen as the slack water mud deposit is eroded either by the tidal current or by the effect of waves. The upper layer is assumed to erode at a bed stress of 0.4 N/m² and the lower bed layer is assumed for the purposes of these tests to be too stiff to erode. The effect of bioturbation and ripples is incorporated by defining the thickness of the lower mud layer (100 kg/m²) to equate to the depth of such mixing, which was set to be 120 mm.

Another important effect that has been introduced is that in areas of original mud bed (Fig. 25) the bed roughness is significantly lower than areas of rippled

sand so that the bed stresses are also low. To incorporate this in the model the bed stress is computed as

 $\tau_{\rm h} = \rho f u^2$

- where f is the friction factor (approximately 0.001 for smooth mud)
 - ρ is the density of water (kg/m³)
 - u is the current velocity (m/s).

In sandy areas a value of f = 0.004 was used as has been found appropriate elsewhere. The resulting peak bed stresses in Liverpool Bay for a spring tide are shown in Figure 26. Comparing this with the areas of original mud bed, Figure 25, it can be seen that much lower bed stresses prevail in the muddy areas.

It is also possible to see from this figure that on a spring tide, deposition, except briefly at slack water, is not possible anywhere in the area being modelled except on some of the mudbanks in the Mersey estuary.

3.2 Mud model calibration

The runs of the mud transport model presented here include the addition of half a million tonnes of mud per year on the bed at the dredged spoil disposal ground (Fig. 35). It was found that on a spring tide none of the new mud remained on the bed in the disposal area.

The mud model was run from an initial condition of 10 ppm suspended solids in the coarse grid (Eastern Irish Sea), 20 ppm in Liverpool Bay and 500 ppm in the Mersey Estuary.

The boundary condition in the Eastern Irish Sea was 10 ppm. The river Mersey was assumed to give a landward boundary value of 50 ppm. The model timestep was 45 seconds. The model parameters were, critical shear stress for deposition 0.1 N/m², critical shear stress for erosion 0.4 N/m² and settling velocity (m/s) 0.002 x mud concentration (kg/m³).

The suspended solids concentrations in the two model layers after running 10 repeating spring tides are compared with spring tide observations in Figures 27 and 28. These observations are for section 13 of the Mersey Narrows (Fig. 6). The observations are taken from Reference 6. It can be seen that the simulated mud concentrations are approximately repeating. Considering the variations between observed suspended concentrations on the two days and the variations across the width of the estuary the model gives acceptable agreement.

Plots of suspended sediment concentrations in surface water as predicted by the model are shown in Figures 33 and 34 and Plate 1. They show a continuous rise in concentration from below 50 ppm in Liverpool Bay up to more than 300 ppm in the Mersey Estuary.

Comparisons of model suspended solids concentrations with observations taken from the sewage sludge disposal ship on various spring tides in 1988 are shown in Figure 30. The locations are shown in Figure 29. The trend of rising concentration landward is similar in model and observations.

Further comparisons with NWWA and WRc observations in the Mersey Estuary are shown in Figures 31 and 32. The model predicts the increase in suspended solids concentrations to Eastham (the observations in Figure 31 were taken on a larger spring tide than that

modelled). Landward of Runcorn the coarse resolution of the model grid and choice of landward boundary condition give rise to poor representation of the suspended solids concentration. This, however, is not important in modelling Liverpool Bay where the results are generally satisfactory.

4. METAL TRANSPORT MODELLING

The metal transport model in particular has been much developed since the preliminary results reported in Reference 2. That work only looked at the dispersion of newly discharged metal and included no representation of the background metal distribution already present. Neither dissolved metal nor the desorption process was included.

4.1 Physical processes modelled

The model has been changed so that old and new metal are now computed separately. The distinction between old and new metal is one of convenience as the two forms are indistinguishable but it does allow us to predict the fate of the metal now being discharged into Liverpool Bay. If the total alone were computed, including the existing level of metal contamination, it would not be possible to distinguish where the new metal, whose discharge is computed over just a few tides, is going as the level of contamination is much lower than the existing level. By computing old and new metal, with present discharges all adding to the new metal, we can also add them together so as to be able to compare with observations.

The process of the desorption of metal from the adsorbed state on mud or sewage sludge into the

dissolved state (described in Ref 3) is also included in the model formulation. For a particular level of contamination of the suspended mud by metal there is an equilibrium level of dissolved metal. If the two are not in equilibrium then desorption or adsorption occurs at a rate proportional to the discrepancy to bring the adsorbed and dissolved metal towards equilibrium.

The transport processes modelled include advection by tidal and residual currents, settling of adsorbed metal on to the bed and its subsequent re-erosion and vertical turbulent mixing. The new and old metal undergo the same physical processes, the new metal just makes it possible to identify the metal in the model prediction that has been input during the time for which the model is run.

Bed mixing is represented by the choice of thickness of the original mud layer (100 kg/m²). Metal that gets into this lower bed layer is assumed to be mixed thoroughly.

The metal model has been set up so as to simulate the transport of one metal independent of the others. For the present work zinc has been chosen as the metal to be studied. Zinc was chosen because of its high concentration and because information was available on its rate of desorption. No details of metal complexes are modelled just the total amount of zinc.

4.2 Calibration of the

metal transport model

The sources of zinc modelled are shown in Figure 36. They were modelled as locations where adsorbed zinc was added to the bed material.

The loads were modelled as point loads (although the sewage sludge is discharged over a large area) and the discharges were modelled as continuous sources.

The initial condition on the old adsorbed zinc was set to 400/600/800 ppm of metal on mud both in suspension and on the bed in the three model grids. Initially there is no dissolved metal but desorption causes the dissolved level to rise and the adsorbed concentration to fall until an equilibrium is reached. The equilibrium between dissolved and particulate zinc and the rate of desorption were taken from Reference 3. The partition coefficient for zinc was taken as 10-4, ie 100 mg of adsorbed metal on 1 kg of mud is in equilibrium with 10 μ g/l of dissolved metal, and the desorption rate as 0.2/day. These values are appropriate for sewage sludge in sea water. The model boundary condition on all variables associated with metal concentrations was zero.

Total adsorbed zinc: in suspension in the surface water is shown in Figure 44 and Plate 2. The contour levels refer to the number of mg of zinc adsorbed on 1 kg of suspended mud. The results are shown after 10 repeating spring tides. It is clear that the concentration decreases away from the estuary. As only 10 tides are simulated the true value at the dumping ground will be underestimated.

The total adsorbed zinc on the bed at high water is shown in Plate 3. The contour levels are again mg of zinc per kg of mud. These values are temporary as mud forms a thin slack water deposit before being re-eroded on the ebb tide. The values reflect the adsorbed concentrations in suspension as shown in Plate 1. The areas with low values near the coast result from there being no mud on the bed in these

shallow areas where waves are assumed to prevent mud deposition from taking place.

If these values are compared with those observed on the bed of Liverpool Bay (Ref 7) the general order is correct but larger values are observed in certain parts of Liverpool Bay which the model underestimates because it was run for only 10 tides.

The total dissolved zinc is shown in Figure 45 and Plate 3. The values (given in parts per billion of water ie μ g per kg of water) are approximately in equilibrium with the adsorbed values as shown in Plate 1.

Comparisons of the model spring tide predictions for total particulate zinc and for total dissolved zinc with observations made by WRC in the Mersey Narrows on a tide of range 6.2m are shown in Figures 37 and 38. It can be seen that the metal concentrations are approximately repeating. These results can only be regarded as indicative because of the large difference between the ranges of the observed and modelled tides. It appears from Figure 38 that an equilibrium partition coefficient about 20% less than that obtained in Reference 3 would give excellent agreement with these data. Such a value of the partition coefficient would be within the suggested error bars given in Reference 3.

Model concentrations of adsorbed zinc on mud and dissolved zinc along an estuary centreline are plotted in Figures 39 to 41 as a function of salinity. Observational data from WRc and NWWA (Refs 9 and 10) are superposed. Again the dissolved concentrations tend to be rather larger than those observed. The comparison is generally good confirming the model calibration.

4.3 The fate of newly discharged metal

Model predictions of the fate of newly discharged adsorbed zinc in suspension and on the bed after 10 spring tides are shown in Figure 42 and Plates 5 and 6.

The model results show that in Liverpool Bay the areas of main contamination by new zinc after 10 tides are close to the sludge and spoil dumping grounds. At slack water an ephemeral layer of mud deposits forms on the bed of Liverpool Bay and this is contaminated by new zinc only near the dumping grounds (Plate 6). Subsequently at peak flood currents the slack water mud deposit is all eroded carrying the adsorbed metal back into suspension. For this reason Liverpool Bay is a dispersive area in which to carry out dumping.

After more tides, larger concentrations will be created in Liverpool Bay and the model could be used to study their influence.

5. CONCLUSIONS

A mathematical model has been developed which is capable of simulating and predicting the physical dispersal of dissolved and adsorbed heavy metals associated with sewage sludge and dredged spoil disposal in Liverpool Bay. A matching set of two dimensional, two layer mathematical models were set up to simulate the tidal flows, suspended mud transport and heavy metal transport on a new AMT Distributed Array Processor. The model has three dynamically linked grids of size 2700m in the Eastern Irish Sea, 900m in Liverpool Bay and 300m in the Mersey Estuary. The Mersey Estuary was modelled in some detail because its mud is heavily contaminated with heavy metals

discharged from industry in the past. The exchange of water and suspended mud to and from the Mersey Estuary plays an important role in the dispersal of metals in Liverpool Bay. The model was divided into two layers to help resolve the vertical structure of the density and wind driven currents and the suspended solids profile. The model simulated the dynamic salinity density field which drives the gravitational circulation in the Mersey Narrows and thereby helps contain suspended mud in the Mersey Estuary.

The tidal model was calibrated satisfactorily against observations of tidal levels, tidal velocities and salinity. The mud transport model included the processes of erosion, vertical turbulent exchange, suspension, settling, deposition and the effect of waves. The distribution of mud deposits on the bed was prescribed from observations. The rate of disposal of contaminated dredged mud in Liverpool Bay was estimated from records. The mud transport model was calibrated against observed concentrations of suspended mud. The much smaller quantity of slowly decaying organic sewage sludge was assumed to be wholly mixed and transported with the suspended mud.

The heavy metal transport model, which can handle one metal at a time, separates newly discharged metals from existing metals. It allows for metals to move with the water in solution in the dissolved state, and with the suspended mud in the adsorbed state. The rate of desorption and adsorption of metals to and from the mud is calculated on the divergence from equilibrium using a partition coefficient and a rate constant. The heavy metal model was calibrated by reference to observations in the Mersey Estuary.

The model is considered to be a practical tool for predicting the physical dispersal of heavy metals

disposed in coastal waters and the methodology has already been applied to other UK coastal sites.

6. ACKNOWLEDGEMENTS

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FIGURES





Fig 2 Prescribed density field: Liverpool Bay



Fig 3 Location of Admiralty diamonds used for current calibration Charts 1826 and 1978



Fig 4 Velocities calibration Spring tide - surface layer



Fig 5 Velocities calibration Spring tide - surface layer



Fig 6 Data positions in Mersey Narrows



observations made by HRL 16 Sept 1982

Fig 7 Velocities in the Mersey Narrows Spring tide - surface layer



Fig 8 Velocities in the Mersey Narrows Spring tide - bed layer





Fig 9

Velocities in the Mersey Narrows Spring tide - surface layer





Fig 10 Velocities in the Mersey Narrows Spring tide - bed layer


Fig 11 Data positions in Mersey Estuary



Fig 12 Tidal level calibration - Spring tide



Fig 13 Spring tide currents in the Mersey Estuary - surface layer



Fig 14 Spring tide currents in the Mersey Estuary - surface layer



Fig 15 Residual discharges in Liverpool Bay - upper layer



Fig 16 Residual discharges in Liverpool Bay - lower layer



Fig 17 Residual discharges in Mersey Estuary - lower layer



Fig 18 Residual discharges in Liverpool Bay with 10m/s wind from West-upper layer



Fig 19 Residual discharges in Liverpool Bay with 10m/s wind from West-lower layer



Fig 20 Salinity in Mersey Narrows spring tide









observations by NWW 8 Mar 1989

Fig 22 Salinity in Mersey Estuary spring tide



Fig 23 Salinity contours at high water



Fig 24 Salinity contours at low water



Fig 25 Areas of mud initially on the bed



Fig 26 Peak bed stress during a spring tide





Fig 27 Suspended sediment in Mersey Narrows Observations of 21/9/83



Observations of 16/9/82



Fig 29 Positions of observations by sewage sludge disposal ship during 1988



Fig 30 Suspended sediment in Liverpool Bay Observations during 1988





Fig 31 Suspended sediment in Mersey Estuary Observations of 8/3/89



Fig 32 Suspended sediment in Mersey Narrows Observations of 9/8/84



Fig 33 Suspended sediment contours at high water, surface waters



Fig 34 Suspended sediment contours at low water, surface waters



Fig 35 Location of sewage sludge and dredged spoil disposal grounds



Fig 36 Prescribed sources of zinc







Fig 38 Dissolved zinc in the Mersey Narrows observations of 4/8/84









Fig 42 New contamination of mud on bed at peak ebb



Fig 43 Wave action zones



Fig 44 Total adsorbed zinc in suspension in surface waters at low water



Fig 45 Total dissolved zinc in surface waters at low water
PLATES



Hydraulics Research Tideway System

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Hydraulids Research Tideway System





Hydrophics Research Tideway System



APPENDICES

APPENDIX 1: Formulation of 2D two layer flow model

The model, TIDEFLOW-2D2L, is based upon the differential equations describing conservation of mass, momentum and salt averaged over the depth of each layer or over the total depth in areas too shallow for a two-layer representation. In the shallow areas, the lower layer vanishes and the surface layer only is used, as in a depth-averaged model.

In order to simulate the complex interaction between the variation in water density (created by differences in salinity) and the tidal flows, the simulation of salt movement was included in the overall model using a version of SALTFLOW-2D2L. The simulation of tidal flows and salt movement is then carried out interactively in order to simulate the dynamic effects of the changing salinity and, therefore, density distribution over the tidal cycles.

The first part of this appendix describes the hydrodynamic part of the model and the second part describes the salt model.

<u>1 The Hydrodynamic Equations</u>

The hydrodynamic equations solved in the model, apart from the interaction between the layers, are similar to, and are derived in a similar way to, the depth averaged equations. The layer averaged equations are as follows:

Conservation of water mass

31

^{∂uz} B +	$\partial vz_B + w = 0$	
Эх	ду	(1)

011	ouu .	ovu		
2+ +	211 +	$\frac{1}{2\pi} - w = 0$	1 g	
σι	OX.	dy		

(2)

Conservation of momentum

<u>Bed layer</u>

$$\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{g\partial}{\partial x} + \frac{p}{x} + \frac{fu(u^{2}+v^{2})^{\frac{N}{2}}}{z_{B}} - D\left(\frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial y^{2}}\right) - \Omega v =$$

$$\frac{\frac{1}{m^{2}}}{z_{B}} \frac{\partial(u^{2}+v^{2})^{\frac{N}{2}}}{\partial z}, \quad \frac{\partial u}{\partial z}$$

$$(3)$$

$$\frac{\partial}{\partial t} + \frac{u\partial v}{\partial x} + \frac{v\partial v}{\partial y} + \frac{g\partial h}{\partial y} + \frac{p}{y} + \frac{fv(u^{2}+v^{2})^{\frac{N}{2}}}{z_{B}} - D\left(\frac{\partial^{2}v}{\partial x^{2}} + \frac{\partial^{2}v}{\partial y^{2}}\right) + \Omega u =$$

$$\frac{\frac{1}{m^{2}}}{z_{B}} \frac{\partial(u^{2}+v^{2})^{\frac{N}{2}}}{\partial z}, \quad \frac{\partial v}{\partial z}$$

$$(4)$$

$$where:$$

$$P_{x} = \frac{gd}{\rho} \cdot \frac{\partial\rho}{\partial x} + \frac{gz_{B}}{2\rho} \frac{\partial\rho}{\partial x}$$

$$(3a)$$

$$P_{y} = \frac{gd}{\rho} \cdot \frac{\partial\rho}{\partial y} + \frac{gz_{B}}{2\rho} \frac{\partial\rho}{\partial y}$$

$$(4a)$$

$$\frac{Surface layer}{\partial t} + \frac{u\partial u}{\partial z} + \frac{v\partial u}{\partial z} + \frac{g\partial h}{\partial x} + \frac{gd}{2\rho} \cdot \frac{\partial\rho}{\partial x} - D\left(\frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial y^{2}}\right)$$

$$-\Omega v = -\frac{1}{m^{2}} \frac{\partial(u^{2}+v^{2})^{\frac{N}{2}}}{\partial z}, \quad \frac{\partial u}{\partial z} + \tau_{XW}$$

$$(5)$$

$$\frac{\partial v}{\partial t} + \frac{u\partial v}{\partial x} + \frac{v\partial v}{\partial y} + \frac{g\partial h}{\partial y} + \frac{gd}{2\rho} \cdot \frac{\partial\rho}{\partial y} - D\left(\frac{\partial^{2}v}{\partial x^{2}} + \frac{\partial^{2}u}{\partial y^{2}}\right)$$

$$+ \Omega u = -\frac{1}{m^{2}} \frac{\partial(u^{2}+v^{2})^{\frac{N}{2}}}{\partial z}, \quad \frac{\partial v}{\partial z} + \tau_{YW}$$

$$(5)$$

where

(u,v) = depth averaged horizontal velocity (either layer) (m/s) w = vertical velocity component between layers (m/s) h = surface level relative to datum (m) z_B = bed layer depth (m) d = surface layer depth (m) d_T = total depth (m) P_x, P_y = density component of pressure gradient (m/s²) f = friction parameter D = coefficient of horizontal eddy viscosity (m²/s) Ω = Coriolis parameter (1/s) ^Txw, ^Tyw = surface wind stress components (N/m²) l_m = momentum mixing length (m) ρ = density (kg/m³)

The equations incorporate the assumptions that the flow is incompressible and well mixed, that vertical accelerations are negligible (the hydrostatic pressure assumption) and that a quadratic friction law is valid.

The water density is prescribed as a function of the salinity which is given by:

$$\rho = 1000 + 0.76s$$

where s = salt concentration (kg/m³)

Interfacial Mixing

The turbulent exchange of momentum between the two layers was represented in terms of a momentum mixing length (1_m) in equations (3) to (6). The mixing length at the interface is a function of the total depth and the bed layer thickness and the degree of stratification between the layers represented in terms of a bulk Richardson number defined as:

Ri =
$$\frac{z_B^g \Delta \rho}{\rho[(u_1 - u_2)^2 + (v_1 - v_2)^2]}$$

(7)

Subscripts 1 and 2 refer to the upper and lower layers respectively.

$$l_m = 0.4z_B(1\frac{z_B}{d_T})^{\frac{1}{2}}$$

Frictional Resistance

The friction factor f is defined in terms of a roughness length (k_s) where k_s is related to the size of the protuberances on the bed, either directly in the form of particle sizes (especially in the case of shingle and stones etc) or indirectly in the form of ripple lengths (in the case of fine particles, ripple lengths are about 1000 times median grain size) (see, for example, Ref A2).

Eddy Viscosity

The formula for the eddy viscosity coefficient, D, is not well determined: Fischer (Ref A3) discusses various formulae. As a first approximation D= O(Ud). Fortunately the solutions to the equations are not in general critically dependent on D. However, the size of D does have an effect on the size of tidal eddies and so by comparing model eddy sizes with observations, the value of D used could be roughly confirmed as being reasonable.

2 Salt Conservation Equations

The two-dimensional two-layer model of salt movement solves the set of equations describing the conservation of salt. These equations are similar to those employed in the two-layer mud transport model (ref A4) and can be written as: Surface layer

 $\frac{\partial ds}{\partial t} + \frac{\partial uds}{\partial x} + \frac{\partial vds}{\partial y} - ws - \frac{\partial}{\partial x} (Dd \ \frac{\partial s}{\partial x})$ $-\partial \left(\frac{Dd \ \frac{\partial s}{\partial y}}{\partial y} + 1 \right)_{s} \frac{\partial \left(u^{2} + v^{2}\right)^{\frac{1}{2}}}{\partial z} \quad \left| \frac{\partial s}{\partial z} \right| = 0$ (8)

Bed layer

 $\frac{\partial z_{B}s}{\partial t} + \frac{\partial u z_{B}s}{\partial x} + \frac{\partial v z_{B}s}{\partial y} + ws - \frac{\partial}{\partial x}(Dz_{B}\frac{\partial s}{\partial x})$ $- \frac{\partial}{\partial y}(Dz_{B}\frac{\partial s}{\partial y} - 1_{s}1_{m} \left| \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right| \frac{\partial s}{\partial z} = 0$ (9)

where

s = salt concentration (kg/m³) (in upper or lower layer)
l_m = momentum mixing length (m)
l_s = solute mixing length (m)
D = coefficient of dispersion (m²/s)

In use, the model solves the flow equations each timestep to calculate water surface levels and water velocity components in each model grid cell. The model then solves the equations describing conservation of salt using these calculated values, so updating the salt concentration in each grid cell and therefore the water density. This new density distribution is then used in the flow equations at the next timestep.

The results from the model, consisting of water surface levels, salinity and the two components of the water velocity in each model cell are stored at frequent intervals. The results are then available for analysis or use by, for example, the two-layer mud transport model or other water quality models which form part of the overall suite of models.

References

- Al Hydraulics Research Ltd. Saline Intrusion and Sediment Transport in the Rotterdam Waterways. Two-layer numerical model simulation studies. HR Report EX 883, 1979.
- A2 Yalin, M.S. Mechanics of sediment transport. Pergamon Press, Oxford. 1972.
- A3 Fischer, M.B. Mixing and dispersion in estuaries. Ann Rev Fluid Mech, 8, pp 107-133. 1976.
- A4 Hydraulics Research Ltd. Hydraulic and Water Quality Studies in Victoria Harbour. Two-layer Mathematical Model Simulation of Mud and Particulate Effluent Transport. User Manual. HR Report No EX1689, May 1988.

The sediment movement model uses the stored results from the two-layer flow model and is based on the equations describing conservation of mass. These equations are similar to those used in the HR MUDFLOW two-dimensional depth averaged models and can be written as follows:

Surface layer

$$\frac{\partial dc}{\partial t} + \frac{\partial udc}{\partial x} + \frac{\partial vdc}{\partial y} - (w - w_s) - \frac{\partial}{\partial s} (D_s d_s) + \frac{\partial c}{\partial s} = 0$$

$$- \frac{\partial}{\partial n} \left(\frac{D}{n} d_s \frac{\partial c}{\partial n} + \frac{1}{c} \frac{1}{m} \right) \left(\frac{\partial (u^2 + v^2)}{\partial z} \right) \left(\frac{\partial c}{\partial z} = 0$$

Bed layer

$$\frac{\partial z_{B}c}{\partial t} + \frac{\partial uz_{B}c}{\partial x} + \frac{\partial vz_{B}c}{\partial y} + (w - w_{S})c - \frac{\partial}{\partial s}(D_{S}z_{B}\frac{\partial c}{\partial s})$$
$$- \frac{\partial}{\partial n}(D_{n}z_{B}\frac{\partial c}{\partial n} - 1_{c}t_{m} \left| \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right| \frac{\partial c}{\partial z} = \frac{dm}{dt} + L_{mud}$$

where

d = surface layer depth (m)

 z_{p} = bed layer depth (m)

- u,v = horizontal velocity components referred to the cartesian coordinates x and y (in upper or lower layer) (m/s)
- w = the vertical velocity component at the layer interface referred to the cartesian coordinate z (m/s)

t = time (sec)

s,n = intrinsic coordinates, parallel with and normal
 to the local flow direction respectively (m)

D_c = longitudinal (shear flow) dispersion coefficient (m^2/s) Dn = lateral (turbulent) diffusion coefficient (m^2/s) = suspended mud concentration (kg/m³) (in upper С or lower layer) 1____ = momentum mixing length (m) l_c = solute mixing length (m) w_s = settling velocity (m/s) dm/dt = bed exchange (kg/m²/s) (erosion or deposition) L_{mud} = loading (kg/m²/s) (can be included in either upper or lower layer) c = suspended solids concentration in upper layer if $(w-w_s) \le 0$ (kg/m^3) ē = suspended solids concentration in lower layer if $(w-w_s) \ge 0$ (kg/m^3)

Similar equations define the movement of biodegradable suspended solids discharged from sea outfalls, except for the addition of extra terms (- γ dc and - $\gamma z_{\rm B}$ c in the upper and lower layers respectively, where γ is a specified decay rate (l/s)) representing the oxidation (decay) of the biodegradable matter.

The turbulent exchange of suspended solids between the two layers is represented by a mixing length technique, where the solute mixing length (1_{c}) and the momentum mixing length (1_{m}) are given by:

 $l_{m} = \frac{l_{o}}{1 + 16Ri}$ $l_{c} = \frac{l_{o}}{1 + 33Ri} \quad \text{if } Ri \le 0.7$

or

$$l_{c} = \frac{0.035 l_{o}}{\sqrt{Ri}} \qquad \text{if } Ri \ge 0.7$$

where the momentum mixing length for neutral conditions (l_0) and the Richardson number (Ri) are defined by:

$$l_{o} = \frac{0.4 (z_{B} + d) (1 - z_{B}/(z_{B} + d))^{3/2}}{\log_{e} ((z_{B} + d)/z_{B})}$$

Ri = $\frac{(z_{B} + d) g \Delta \rho}{\rho [(\Delta u)^{2} + (\Delta v)^{2}]}$

where:

g = acceleration due to gravity (m²/s) ρ = density of the surface layer (kg/m³) $\Delta \rho$ = density difference between the layers (kg/m³) Δu , Δv = velocity (u,v) difference between the layers (m/s)

As can be seen from the above equations, turbulent mixing between the layers is determined by the relative depth of the bed layer, the horizontal velocity differences between the layers and the salinity difference between the layers, which determines the density difference.

The erosion or deposition of mud at the bed is prescribed by relationships which can be summarised as follows:

Deposition
$$\frac{dm}{dt} = w_s c (1 - \frac{\tau_b}{\tau_d}) \text{ when } \tau_b \le \tau_d$$

Erosion $\frac{dm}{dt} = M (\tau_b - \tau_e) \text{ when } \tau_b \ge \tau_e$

where

 $\tau_b = bed stress (N/m^2)$ $\tau_d = critical stress for deposition (N/m^2)$ $\tau_e = critical stress for erosion (N/m^2)$ $w_s = settling velocity (m/s)$ M = erosion constant (kg/s/N) The settling velocity is assumed to depend on the suspended mud concentration through the relationship:

$$w_{\rm s} = \beta c$$

where β is an empirical constant. $\tau_e^{},$ the critical stress for erosion is related to the dry density of the exposed mud bed ($\rho_d^{})$ through the equation:

$\tau_{e} = 0.0013 \rho_{d}$

APPENDIX 3: Formulation of 2D two layer metal transport model

The metal movement model like the mud model uses the stored results from the two-layer flow model and is based on the equations describing conservation of mass. These equations can be written as follows:

1 Dissolved metal

Surface layer

$$\frac{\partial dc_{d}}{\partial t} + \frac{\partial udc_{d}}{\partial x} + \frac{\partial vdc_{d}}{\partial y} - w \ \overline{c_{d}} - \frac{\partial}{\partial s} (D_{s} d \ \frac{\partial c_{d}}{\partial s})$$
$$- \frac{\partial}{\partial n} \left(\frac{D}{n} d \ \frac{\partial c_{d}}{\partial n} + 1_{c} \frac{1}{m} \right) \left(\frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right) \left(\frac{\partial c_{d}}{\partial z} = 0 \right)$$

Bed layer

$$\frac{\partial z_{B}c_{d}}{\partial t} + \frac{\partial uz_{B}c_{d}}{\partial x} + \frac{\partial vz_{B}c_{d}}{\partial y} + wc_{d} - \frac{\partial}{\partial s} (D_{s}z_{B}\frac{\partial c_{d}}{\partial s})$$
$$- \frac{\partial}{\partial n} (D_{n}z_{B}\frac{\partial c_{d}}{\partial n} - 1_{c}l_{m} \left| \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right| \frac{\partial c_{d}}{\partial z} = L_{metal}$$

where

- d = surface layer depth (m)

 $z_{R} = bed layer depth (m)$

- w = the vertical velocity component at the layer interface referred to the cartesian coordinate z (m/s)

t = ti	me (sec))
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- s,n = intrinsic coordinates, parallel with and normal
 to the local flow direction respectively (m)
- $D_s = longitudinal (shear flow) dispersion$ coefficient (m²/s)
- $D_n = lateral (turbulent) diffusion coefficient (m²/s)$
- l_m = momentum mixing length (m)
- l_{c} = solute mixing length (m)
- L = loading (kg/m²/s) (can be included in either upper or lower layer)
- \bar{c}_d = dissolved metal concentration in upper layer if w $\leq 0 \ (kg/m^3)$
- \bar{c}_{d} = dissolved metal concentration in lower layer if w > 0 (kg/m³)
- 2 Adsorbed metal

Surface layer

$$\frac{\partial dc_{m}}{\partial t} + \frac{\partial u dc_{m}}{\partial x} + \frac{\partial v dc_{m}}{\partial y} - (w - w_{s}) c_{m} - \frac{\partial}{\partial s} (D_{s} d \frac{\partial c_{m}}{\partial s})$$
$$- \frac{\partial}{\partial n} \left(D d \frac{\partial c_{m}}{\partial n} + 1 c_{m}^{1} \right) \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \left| \frac{\partial c_{m}}{\partial z} = 0 \right|$$

Bed layer

$$\frac{\partial z_{B}c_{m}}{\partial t} + \frac{\partial uz_{B}c_{m}}{\partial x} + \frac{\partial vz_{B}c_{m}}{\partial y} + (w - w_{s}) \bar{c}_{m} - \frac{\partial}{\partial s} (D_{s}z_{B} \frac{\partial c_{m}}{\partial s})$$
$$- \frac{\partial}{\partial n} (D_{n}z_{B} \frac{\partial c_{m}}{\partial n} - 1_{c}l_{m} \left| \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right| \frac{\partial c_{m}}{\partial z} = \frac{dm}{dt} + L_{metal}$$

where

$$\begin{split} & w_{g} = \text{settling velocity of mud (m/s)} \\ & dm/dt = \text{bed exchange (kg/m^{2}/s) (erosion or deposition)} \\ & L_{meta\overline{1}} \quad \text{loading (kg/m^{2}/s) (can be included in either upper or lower layer)} \\ & \overline{c}_{m} = \text{adsorbed metal concentration in upper layer if} \\ & (w-w_{s}) \leq 0 (kg/m^{3}) \\ & \overline{c}_{m} = \text{adsorbed metal concentration in lower layer if} \\ & (w-w_{s}) \geq 0 (kg/m^{3}) \end{split}$$

The turbulent exchange of dissolved or adsorbed metal between the two layers is represented by a mixing length technique, where the solute mixing length (1_c) and the momentum mixing length (1_m) are given by:

$$l_{m} = \frac{l_{o}}{1 + 16Ri}$$

$$l_{c} = \frac{l_{o}}{1 + 33Ri} \quad \text{if } Ri \le 0.7$$
or

$$l_{c} = \frac{0.035 \ l_{o}}{\sqrt{Ri}} \qquad \text{if } Ri \ge 0.7$$

where the momentum mixing length for neutral conditions (1_0) and the Richardson number (Ri) are defined by:

$$l_{o} = \frac{0.4 (z_{B} + d) (1 - z_{B}/(z_{B} + d))^{3/2}}{\log_{e} ((z_{B} + d)/z_{B})}$$

Ri = $\frac{(z_{B} + d) g \Delta \rho}{\rho [(\Delta u)^{2} + (\Delta v)^{2}]}$

where:

g = acceleration due to gravity (m^2/s)

 ρ = density of the surface layer (kg/m³) $\Delta \rho$ = density difference between the layers (kg/m³) Δu , Δv = velocity (u,v) difference between the layers (m/s)

As can be seen from the above equations, turbulent mixing between the layers is determined by the relative depth of the bed layer, the horizontal velocity differences between the layers and the salinity difference between the layers, which determines the density difference.

The erosion or deposition of metal adsorbed on mud at the bed is prescribed by relationships which can be summarised as follows:

Deposition
$$\frac{dm}{dt} = w_s c \left(1 - \frac{\tau_b}{\tau_d}\right)$$
 when $\tau_b \le \tau_d$
Erosion $\frac{dm}{dt} = M \left(\tau_b - \tau_e\right)$ when $\tau_b \ge \tau_e$

where

$$\tau_b$$
 = bed stress (N/m²)
 τ_d = critical stress for deposition (N/m²)
 τ_e = critical stress for erosion (N/m²)
 w_s = settling velocity (m/s)
M = erosion constant (kg/s/N)

The settling velocity is assumed to depend on the suspended mud concentration through the relationship:

$$w_{s} = \beta c$$

;

where β is an empirical constant. $\tau_e,$ the critical stress for erosion is related to the dry density of the exposed mud bed (ρ_d) through the equation: