# SEDIMENT TRANSPORT: AN APPRAISAL OF AVAILABLE METHODS

## VOLUME 2 – PERFORMANCE OF THEORETICAL METHODS WHEN APPLIED TO FLUME AND FIELD DATA

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Report No IT 119
November 1973
Second Impression
March 1978

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# NOTATION

| | |
|---|---|
| $A_*$ | Constant (Einstein) |
| $A_1$ | Constant of area (Bishop, Simons and Richardson) |
| $A_2$ | Constant of volume (Bishop, Simons and Richardson) |
| $A_3$ | Time scale proportionality factor (Einstein) |
| $A_3'$ | Constant of time scale (Bishop, Simons and Richardson) |
| $b$ | Stream breadth, surface width if not otherwise stated |
| $B_*$ | Constant (Einstein) |
| $C_1$ | Coefficient (Graf) |
| $C_2$ | Coefficient (Graf) |
| $C_D$ | Drag coefficient (Egiazaroff) |
| $d$ | Mean depth of flow |
| $d'$ | Mean depth related to the grain |
| $D$ | Sediment diameter |
| $D_{35}$ $D_{50}$ $D_{65}$ | Sediment diameters |
| $D_\alpha$ | Effective sediment diameter in a mixture |
| $D_i$ | Particular grain size |
| $D_m$ | Mean diameter (geometrical) for roughness determination |
| $D_{max}$ | Maximum sediment size |
| $D_o$ | Threshold diameter within a mixture |
| $D_{gr}$ | Dimensionless grain size |
| $e_b$ | Bed load transport efficiency (Bagnold) |
| $F$ | Dimensionless fall velocity |
| $F_{cg}$ | Sediment mobility, coarse grain (Ackers, White) |
| $F_{fg}$ | Sediment mobility, fine grain (Ackers, White) |
| $F_{gr}$ | Sediment mobility, general (Ackers, White) |

$g$ Acceleration due to gravity

$g_{st}$ Total transport, dry weight per unit width per unit time

$K$ Meander slope correction

$k_{se}$ Coefficient of total roughness, grain and form (Strickler)

$k_r$ Coefficient of particle friction, plane bed (Strickler)

$k$ Correction factor for models

$q_{st}$ Total load transport rate, submerged weight per unit width per unit time

$Q$ Water discharge

$Q_s$ That portion of Q whose energy is converted into eddying close to the bed

$R_*$ Particle Reynolds Number

$R_*'$ Particle Reynolds Number with respect to the grain

$s$ Specific gravity of sediment

$T$ Temperature

$t_1$ Exchange time of moving particles

$t$ Variable

$v_*$ Shear velocity

$v_*'$ Shear velocity related to the grain

$V$ Mean velocity

$V_{yd}$ Velocity at a level $y = 0.63D$

$w$ Fall velocity of sediment

$X$ Concentration by weight

$x$ Parameter for transition (smooth to rough)

$Y$ Dimensionless mobility number

$Y_C$ Critical mobility number

$(Y_C)_i$ Critical mobility number for particular shape, size and grading curve

$Y_C^{mix}$ Critical mobility number for a mixture

$Y'$ Mobility number ascribed to the grain

$y_*$ Distance above the bed

$y_D$ Distance above the bed (0.63D)

$Z$ Ratio of depth to particle diameter

$\alpha$ Percentage of sediment finer than a specific value

$\alpha$ Coefficient (Ackers, White)

$\gamma_s$ Specific weight of grain in fluid, $g(\rho_s-\rho)$

$\rho$ Density of fluid

$\rho_s$ Density of solids

$\nu$ Kinematic viscosity

$\lambda$ Dimensionless average jump length of particles

$\phi$ Intensity of transport

$\psi'$ Intensity of shear on representative particle

$\eta_o$ Constant (Einstein)

$\pi$ Circumference/diameter

$\xi$ Hiding factor

# SEDIMENT TRANSPORT:
# AN APPRAISAL OF AVAILABLE METHODS

## VOLUME 2 PERFORMANCE OF THEORETICAL METHODS WHEN APPLIED TO FLUME AND FIELD DATA

### CRITERIA FOR THE COMPARISON OF OBSERVED AND CALCULATED TRANSPORT RATES

The characteristics of the numerous sediment transport theories have been evaluated by several authors in a variety of ways. Comparisons between predicted and observed values have usually been achieved by plotting, for a given stream, both observed and calculated sediment transport rates against water discharge. On such a graph several theories can be compared directly with each other and with the observed data. However, a major drawback of this method is that the comparison is in the context of one very specific situation: it is related to a particular sediment in a particular flume or natural channel. Another way of evaluating a theory is to plot computed against observed sediment discharge but this method does not distinguish between any of the relevant parameters and where discrepancies arise the cause is not apparent. A further disadvantage is that only one theory can be depicted per graph. In both the above methods the order of magnitude of any errors is not immediately apparent.

The basic quantities which influence the process of sediment transport in two-dimensional, free surface flow are:-

$$\rho,\ \nu,\ \rho_s,\ D,\ d,\ v_*,\ g$$

where $\rho$ = density of fluid

$\rho_s$ = density of solids

$\nu$ = kinematic viscosity

$D$ = equivalent particle diameter

$d$ = water depth

$v_*$ = shear velocity $\sqrt{\tau_o/\rho}$

$g$ = acceleration due to gravity

Dimensional analysis yields the following grouping of these basic quantities

$$D_{gr} = \left\{\frac{\gamma_s D^3}{\rho\nu^2}\right\}^{1/3} = \left\{\frac{g(s-1)}{\nu^2}\right\}^{1/3} D \qquad \text{...Dimensionless grain size}$$

$$Y = \frac{\rho v_*^2}{\gamma_s D} = \frac{v_*^2}{(s-1)gD} \qquad \text{...Mobility number}$$

$$Z = \frac{d}{D} \qquad \text{...Relative grain size or Dimensionless flow depth}$$

$$s = \frac{\rho_s}{\rho} \qquad \text{...Relative density}$$

Hence, if we accept these four dimensionless parameters as a comprehensive set of significant variables, any mechanical property related to the movement of bed material in steady, uniform, two-dimensional flow is a function of these four dimensionless groups. In particular, the dimensionless sediment transport parameter will be a function of these groups, viz:-

$$\frac{q_t \, \rho^{\frac{1}{2}}}{\gamma_s^{3/2} \, D^{3/2}} = f\left\{ D_{gr}, \; Y, \; Z, \; s \right\} \qquad \ldots(1)$$

where the left hand side is the well-known Einstein transport function, $\phi$. (See Ref 7, Vol 1)

and $q_t$ = sediment transport rate as submerged weight per unit width per unit time.

That is

$$\phi = \frac{q_t \, \rho^{\frac{1}{2}}}{\gamma_s^{3/2} \, D^{3/2}} = \frac{q_t}{(s-1)^{3/2} \, g^{3/2} \, \rho \, D^{3/2}} \qquad \ldots(2)$$

The data available for evaluating sediment transport theories consists of measurements in a series of flumes and natural channels. In terms of the four groups on the right hand side of equation (1), each flume or natural channel exhibits constant or near constant values of $D_{gr}$ and s since these are primarily functions of the sediment and water characteristics. On the other hand, each set of data from a particular flume or river exhibits a range of values of Y and Z since these vary with flow.

In the present report $D_{gr}$ has been chosen as the principle variable. The available data covers the range $1 \leqslant D_{gr} \leqslant 1450$ which for sand in water at 15°C corresponds to a range of particle sizes $0.04 < D(mm) < 68$. Thus the influence of immersed weight and viscous forces are represented over a very wide range of particle sizes in the comparative plots.

Each data set, i.e. each series of measurements in a particular flume or river, covers a range of values of Y and Z (see Equation (1)). Thus errors due to the inability of a theory to cope with these two parameters correctly show up as scatter on the $D_{gr}$ plots. A detailed analysis of these two parameters would be virtually impossible since they both vary simultaneously and an attempt to do this has not been made in the present investigation. The effects of specific gravity, s, have been indicated by using different symbols for sand and "lightweight" materials.

Each theory has been applied to every measurement of sediment transport rate and the difference between observed and calculated values denoted by:-

$$\text{Discrepancy ratio} = \frac{X_{CALC}}{X_{ACT}} \qquad \ldots(3)$$

where X = transport rate of bed material expressed as a concentration by weight

CALC denotes calculated

ACT denotes actual.

Each data set is plotted according to its $D_{gr}$ value, see Figs 7-24 and 33, and the mean discrepancy ratio is indicated together with the minimum and maximum values. These limits give an indication of the spread of errors within a data set. These could be due to deficiencies in the theories in terms of Y, Z and s or simply errors in the observed values.

The theoretical formulae have been reduced to their simplest form (see Vol 1) such that they compute concentration, X, from the basic measured values of:

Depth (d)

Diameter (D)

Specific gravity (s)

Mean velocity (V)

Shear velocity ($v_*$)

Temperature (T)

Breadth (b) ... where applicable

Grading ... where applicable.

The sediment concentration, X, can be related to the sediment transport rate as dry or submerged weight per unit width per unit time as follows:-

$$g_t = X\ V\ d\ \rho\ g \qquad \ldots(4)$$

$$q_t = X\ V\ d\ \rho\ g\ \frac{(s-1)}{s} \qquad \ldots(5)$$

Some theoretical methods (Refs 7 and 18, Vol 1) include procedures for computing depth/discharge and hence depth/mean velocity relationships. However, in the present investigation measured depths and mean velocities have been used throughout. This eliminates any systematic errors in the above procedures and makes the comparison between observed and calculated transport rates more meaningful. In utilising the theories in practice, of course, the engineer must either use these procedures or measure the basic quantities before he can calculate transport rates.

Equation (3) gives the discrepancy ratio in terms of sediment concentrations. However, since the parameters which relate the concentration to the sediment transport rate and the dimensionless sediment transport parameters (equations (2), (4) and (5)) are all measured, equation (3) gives the same values as comparisons in terms of g, q, $\phi$ and $G_{gr}$ (see Refs 24 and 25, Vol 1).

Several equations examined in this investigation were presented originally as bed load equations but they have been compared with total load data. Our reasons for doing this were (i) that the definition of bed load is not universally acceptable and one man's bed load is another man's suspended load, (ii) some of the bed load equations have coefficients based on data which is not indisputably bed load data, and (iii) there are reports which suggest that, under certain circumstances, the bed load equations can be applied to total load data without introducing errors larger than those obtained from some of the so-called total load equations.

## ANALYSIS OF DATA CHARACTERISTICS

The general philosophy of this investigation has been to compare as many theories as possible against as much data as possible. This philosophy enables one to identify the characteristics of the theories and the characteristics of the individual data sets. One cannot assume that, where discrepancies arise, the data is right and the theory is wrong because of the obvious difficulties in measuring the relevant parameters.

At the extremes it is possible to eliminate bad theories and bad data sets. For example, if a theory consistently predicts 10 times the observed transport rate for 1000 measurements, it is reasonable to assume that the theory is inaccurate. Also, if 20 theories predict 100 times the observed transport rate for a particular data set, this casts severe doubts upon the data. In between these extremes, the interpretation is less straightforward, of course, but this does not reduce the argument for analysing a large number of theories and a large data "bank" covering a wide range of conditions. The author's definition of a good theory is one which can be applied with confidence to any channel in which non-cohesive solid particles are being transported by a fluid. It should not matter whether the channel is a miniature flume or a large river or whether the sediment is sand or wood grains.

### 1. Flume data

The data used in the present investigation was mainly acquired from literature and, for flume experiments, amounts to around 1000 measurements, (see Table 1). The experiments were carried out with uniform or near-uniform sediments with flow depths up to 0.4 m. For selecting the flume data an upper limit for the Froude Number of 0.8 was applied, thus avoiding the complexities associated with critical and supercritical flow conditions. No allowance has been made for side wall effects or bank friction.

## 2. Field data

The field data covers 11 sites with a total of 270 measurements as follows:-

Niobrara River, Cody, Nebraska, USA (Ref 1)

Middle Loup River, Dunning, Nebraska, USA (Ref 2)

Paraguay River, Km 385, PARAGUAY (Ref 18)

Atchafalaya River, Simmersport, Louisianna, USA (Ref 3)

Mississippi River, Tarbert Landing, Miss., USA (Ref 3)

Mississippi River, St Louis, Miss., USA (Ref 3)

Aare River, Brienzwiler, SWITZERLAND (Ref 4)

Elbow River, Bragg Creek, Alberta, CANADA (Ref 5)

Mountain Creek, Greenville, S. Carolina, USA (Ref 6)

Goose Creek, Oxford, Mississippi, USA (Ref 6)

Skive-Karup River, DENMARK (Ref 7)

The principal characteristics of the above data are summarised in Table 2. The transport measurements are based on different techniques which are described briefly in the footnote of the Table. More detailed information is given in the original papers.

## 3. Data classification

The data has been classified in terms of $D_{gr}$, Y, b/d, X and d/D in order to illustrate the coverage of the available information, to point out gaps where they exist and to compare the relative characteristics of flume and field data.

Dimensionless grain size:

The frequency distributions of the dimensionless grain size, $D_{gr}$, are shown for both flume and field data in Fig 1. There is clearly a deficiency of data for $D_{gr}$ less than about 3, between 15 to 20 and for $D_{gr}$ greater than 40. This latter range is very short of field data and also flume data for $D_{gr}$ greater than 70. There is a need for further measurements in this range.

Dimensionless mobility number:

The frequency distributions of the dimensionless mobility number are shown in Fig 2 and Table 3. Using the criterion of F Engelund (see later and Ref 8) approximately 43 per cent of the flume data and 25 per cent of the field data represent conditions where the material is transported close to the bed. This is a systematic tendency for flume data for $D_{gr}$ greater than about 15 and field data for $D_{gr}$ greater than about 20 as shown in Table 3.

The available flume and field data show a similar range of Y values, see Fig 2.

It is worth noting that a large percentage of the reported measurements for coarse materials show values of the dimensionless mobility number less than the critical value as given by Shields when computed with an effective diameter $D = D_{50}$. Flume data in the range $60 < D_{gr} < 110$ show this tendency and also the field data for the Elbow and Aare rivers ($D_{gr}$ = 687, 1013 and 1450). Many investigators have reported critical values well below the Shields function for coarse material and this has an important influence on several theories which are based on considerations of "excess shear".

Breadth/depth ratio:

The breadth/depth ratio is distinctly different for field and flume data. Half of the total flume measurements have a ratio below 5, see Fig 3 and Table 4. The frequency distributions of this ratio overlap only in the range between 5 and 20.

Although the breadth/depth ratio influences roughness problems when less than about 5 and the meandering of a stream when large, its influence on sediment transportation is thought to be minimal.

Sediment concentration:

As with the dimensionless mobility number the concentration shows a similar range of values in flume and field data. Both range from $10^{-1}$ to $10^{-6}$ as shown in Fig 4 and Table 5.

Relative grain size:

The frequency distributions of the relative grain size, d/D, are shown in Fig 5 and Table 6. The field data shows a wider range of values than the flume data. About 25 per cent of the field data has values between $10^4$ and $10^5$. These high values were recorded in data Refs 119, 118, 116, 153, 117, 150 and 151 (see Table 2 for details).

## 4. Transport rates

The interpretation of quoted transport rates is difficult, particularly in the case of field measurements where different sampling techniques are used and where there is often a wash load of very fine material not found in a bed sample. A few definitions are useful:-

(i) Bed load

That material which moves in close contact with the bed.

(ii) Suspended bed material load

That part of the suspended load consisting of particle sizes present in a bed sample. The description has been abbreviated to "suspended load" in the present report.

(iii) Total bed material load

That part of the total sediment discharge consisting of particle sizes present in a bed sample, i.e. the sum of (i) and (ii). The description has been abbreviated to "total load" in this report.

(iv) Wash load

That part of the total sediment discharge consisting of particle sizes smaller than those found in a bed sample or less than 0.06 mm.

There is little published data concerning the characteristics of the wash load although it can be of the same order as, and sometimes greater than, the total bed material load. However, as all the theories used in this study are either Bed Load Theories or Total Bed Material Load Theories, the data has been scrutinised with the view of eliminating the measured wash load.

Niobrara and Middle Loup Rivers (Refs 1 and 2):

Sediment concentrations for the Niobrara and Middle Loup rivers were measured in a contracted section and a flume respectively. For these situations the total bed material load was believed to be in suspension and suspended sediment samplers were used. Thus the quoted concentrations include the wash load. This was deducted using the grading curves of the measured material and the normal river bed material to eliminate sediment not present in the bed sample and also material less than 0.06 mm diameter. The wash load amounted to about 10 per cent of the total bed material load.

Paraguay River (Ref 18)

The Paraguay data consists of bed load data based on dune movement and suspended load data. The wash load was deducted as for the Niobrara and Middle Loup Rivers but the concentrations of fine sediment were much higher. The wash load was, on average, 300 per cent of the total bed material load.

Atchafalaya, Mississippi (Tarbert Landing), Mississippi (St Louis) (Ref 3):

Only the suspended load was measured in these rivers. The quoted values in Ref 3 include computed bed load data. In the present exercise the wash load has been eliminated as above and amounted to 600 per cent of the total bed material load.

Aare River and Elbow River (Refs 4 and 5):

References 4 and 5 report that all the material in suspension was finer than the bed material. The grain sizes of the suspended material were not to be found in a bed sample. Thus the reported bed load as measured with bed load samplers has been taken as the total bed material load. The wash load was not measured.

For gravel rivers the surface material is much coarser than the material found in depth. This effect was reported for both the Aare and the Elbow rivers.

It is not clear which grading curve should be used to compute sediment discharge or whether material in suspension falls into the wash load category or the suspended bed material load category. Based on a bed sample in depth, which will include smaller sizes, the material in suspension will be suspended bed material load but based on a surface sample the material in suspension will often be wash load. This latter interpretation has been adopted for the Aare river and Elbow river data. Thus the reported bed load and the grading curve of the surface layer are the basic data used in the computational procedures.

Mountain Creek, Goose Creek and Skive Karup River (Refs 6 and 7):

In these three rivers the sediment discharge was measured with bed load samplers. However, it must be accepted that, in certain cases, some of the bed material must have been travelling in suspension.

A common criterion for defining conditions in which the sediment transport takes place solely as bed load is $Y_C < Y < 0.4$. However, F Engelund (Ref 8) gives a more restrictive limit as follows:-

$$\frac{v_*}{w} < 0.85 \qquad \text{...(6)}$$

If the fall velocity, w, is expressed in terms of the Rubey equation (see Vol 1, Appendix 1) viz:-

$$w = \frac{Fv_*}{Y^{\frac{1}{2}}}$$

Then equation (6) reduces to the form

$$Y \leqslant 0.7225\ F^2 \qquad \text{...(7)}$$

F is a function of $D_{gr}$ (see Vol 1, equation (A6)) and takes a minimum value when $D_{gr} = 0$. This criterion for the initiation of suspended load conditions is plotted in Fig 6 together with the conditions for the initiation of bed load after Shields and Ackers, White.

Conditions for the three rivers can be summarised as follows:-

| | Mountain Creek | Goose Creek | Skive Karup |
|---|---|---|---|
| $D_{50}$ (mm) | 0.90 | 0.28 | 0.47 |
| $D_{gr}$ | 23 | 7 | 10 |
| $Y_{50}$ exceeded by only 1 per cent of tests | 0.18 | 0.64 | 0.36 |

where $Y_{50}$ is the Mobility Number related to the $D_{50}$ sediment size.

Based on these maximum $Y_{50}$ values, lines for the full bed material grading curves for the three rivers are shown in Fig 6. For the Goose Creek and Skive Karup rivers this diagram shows that a significant amount of suspended bed load transport is to be expected at the higher mobility numbers. In round figures up to 80 per cent and 50 per cent of the total transport respectively. This means that the reported transport rates are smaller than the actual transport rates. For the Mountain Creek data the suspended load only amounts to about 10 per cent of the total load and hence the error is not serious. Most theories overestimate transport rates for these data thus adding weight to the above argument.

5. Temperature effects

The dimensionless grain size is a function of particle diameter, particle specific gravity, fluid viscosity and the acceleration due to gravity. Viscosity in turn is a function of temperature. In most sets of observations for a particular flume or river these properties are constant thus providing a set of observations with constant $D_{gr}$. However, where the measurements have been extended over a period of time, variations in water temperature and sediment size have been noted. Where this has occurred the data sets have been broken down into sub-sets, each of which covers only a narrow range in terms of $D_{gr}$. The Middle Loup data is an example of this.

## DISCUSSION OF RESULTS

### 1. Performance of available methods

A SHIELDS (1936):

The Shields equation was originally proposed as a bed load equation. However, Fig 7 shows that the transport rates are overestimated over almost the whole range of $D_{gr}$ values for flume and field data. The proposed equation has an erratic performance for $D_{gr}$ less than 60 with calculated concentrations between 5 and 50 times the observed values. For coarse materials the discrepancy ratio is somewhat smaller.

For the data sets where the transport is mainly bed load ($D_{gr} > 15$) there is no significant improvement in the predicted values and for many of the coarse data sets (Data Ref 35, 36, 109, 113 and 114) the computed transport rate became zero. For these sets together with Data Refs 26 and 49 about 60 per cent of the individual measurements have values of the mobility number, Y, smaller than the critical value according to the familiar Shields expression. This casts doubt on the Shields threshold criterion for coarse sediments, both uniform and non-uniform, and is one of the major reasons why the Shields bed load equation cannot be used in this range. This argument applies to many other theories which are based on ambient shear values relative to the Shields critical values. The errors are, of course, most significant at low transport rates.

For most lightweight materials the Shields equation still over-estimates transport rates but to a lesser extent. Typical discrepancy ratios are between 1 and 10. However, other theories give better predictions for these materials.

The tendency of the Shields equation to over-estimate sediment discharges has also been reported in Ref 9.

A A KALINSKE (1947):

In 1947 Kalinske (Ref 2, Vol 1) proposed a theory which utilised basic physical principles of fluid dynamics. The resulting bed load formula was dimensionally homogeneous and was claimed by the author to fit laboratory and field data over a wide variety of conditions. Kalinske was one of the first people to look seriously at the problem of graded sediments.

The results shown in Fig 8 do not support the original claims. There is a tendency to overestimate transport rates for flume data and underestimate transport rates for field and lightweight sediment data. Superimposed on these systematic trends is a general scatter of the mean errors and scatter within each data set as indicated by the large difference between minimum and maximum discrepancy ratios.

The Kalinske equation is essentially the result of a simplified deterministic approach to the movement of bed material and there seems little scope for improvement in its present form. The limitations of the theory (see Vol 1) meant that 2 per cent of the data could not be analysed.

C INGLIS (1947):

The regime equation of C Inglis shows a similar general behaviour to that of A Shields. In this case, however, the overestimation of sediment transport rates reaches enormous proportions for flume data with predicted rates up to 200 times the observed rates, see Fig 9. A similar degree of scatter is shown when the theory is applied to lightweight sediments. This theory was based on observations of natural streams flowing under regime conditions. It is not too surprising, therefore, that it does not seem to apply to flume data at all.

When applied to field data there is little overall improvement although a systematic tendency seems to exist. For very fine material, $2 < D_{gr} < 6$, several data sets show close agreement between observed and calculated values but the comparison worsens with increasing $D_{gr}$. It seems therefore that this equation is best used for field conditions with fine sediments.

E MEYER-PETER AND R MULLER (1948):

The comparison of predicted and observed transport rates for the Meyer-Peter, Muller equation is given in Fig 10. The equation was presented as a bed load equation and should be judged in this light. However, from Fig 10, it appears that the equation gives better agreement for fine sediments than for coarse sediments even though the fine sediments are more likely to travel in suspension. This is a surprising result and the reasons appear to lie in a combination of three factors:-

(i) the coefficient 0.047, see equation (25), Vol 1,

(ii) the ratio $Q_s/Q$, see equation (26), Vol 1,

(iii) the ratio $k_{se}/k_r$, see equations (27-30), Vol 1.

The latter two ratios are always less than unity and transport rates diminish as they diminish, see equation (25) Vol 1. The ratio $Q_s/Q$ is a correction factor for wall friction and the ratio $k_{se}/k_r$ is a coefficient which arises in the separation of form and particle resistance. The coefficient 0.047 is, in fact, the critical mobility number which was taken by Meyer-Peter and Muller as constant.

The term in parentheses in equation (25), Vol 1, can be re-written as follows:-

$$\left\{\frac{Q_s}{Q}\left(\frac{k_{se}}{k_r}\right)^{3/2} - \frac{0.047}{Y}\right\}^{3/2}$$

The product $\frac{Q_s}{Q}\left(\frac{k_{se}}{k_r}\right)^{3/2}$ is always less than unity and the ratio $\frac{0.047}{Y}$ tends to unity as Y tends to 0.047. For Y values less than 0.047 the ratio $\frac{0.047}{Y}$ is clearly greater than unity and equation (25), Vol 1, becomes insoluble. Also when $\left|\frac{0.047}{Y}\right| > \left|\frac{Q_s}{Q}\left(\frac{k_{se}}{k_r}\right)^{3/2}\right|$ the equation is insoluble. These conditions occurred many times with the coarse data and it appears that the 0.047 value is too high, or the $\frac{Q_s}{Q}\left(\frac{k_{se}}{k_r}\right)^{3/2}$ composite correction factor is too low or both. This illustrates once more the difficulties surrounding the critical shear conditions for coarse materials. For the finer materials, values of Y tend to be well in excess of 0.047 and the difficulties with the excess shear ratio are far less pronounced. This could account, to some extent, for the improved performance of the theory in this range.

In his comparative study of the Meyer-Peter, Muller equation and the Einstein bed load function Chien (Ref 10) obtained good agreement for both equations with several sets of medium to coarse sediment. In particular the Gilbert and Murphy data ($D_{gr}$ = 18.5) agreed well with both sets of predictions. However, Chien assumed two-dimensional flow conditions and a plane bed taking $Q_s = Q$ and $k_{se} = k_r$. Making these substitutions in equation (25), Vol 1, leads to the expression

$$\phi = (4Y - 0.188)^{3/2}$$

which is the equation successfully used by Chien. In Fig 10 the Gilbert and Murphy data plots well below the discrepancy ratio of unity and the present results using the original Meyer-Peter Muller equation are less authentic than the Chien results. Thus the original $Q_s/Q$ and $k_{se}/k_r$ correction factors remain suspect.

H A EINSTEIN (Bed load, 1950):
H A EINSTEIN (Total load, 1950):

The H A Einstein bed and total load theories are two of the oldest amongst those theories based on probability concepts and which attempt to cope with graded sediments. A wide range of fluid flow conditions were considered.

The theory as formulated leads to complex and laborious computational procedures but has the advantage of being based on sound physical principles. Many other investigators have looked for support for their own formulae in the basic principles and parameters of the Einstein methods. Others have taken the basic Einstein concept and modified the method claiming improved accuracy in predictions of transport rates.

Before describing the results of present computations with the Einstein equations it is worth repeating (see Vol 1) that the measured total shear velocity has been utilised in the present study. The Einstein slope separation technique was utilised to determine the proportions of the grain and form shear velocities within the observed total. We consider that this is the best way of evaluating the theory because the comparison is a direct one between predicted and observed quantities.

Fig 11 shows the comparison between observed and computed transport rates for the bed load function. The function underestimates transport rates for $D_{gr}$ values less than about 8 (about 0.3 mm sand size), the extent of this underestimation increasing with decreasing $D_{gr}$. There is better agreement in the range $8 < D_{gr} < 40$ with a moderate overestimation of transport rates at higher values of $D_{gr}$. Thus, as would be expected with a bed load function, the theory underestimates transport rates for the finer materials which tend to travel mainly in suspension.

Fig 12 shows the comparison between observed and computed transport rates for the total load function. In the total load function the quantity of bed material travelling in suspension between the levels $y_* = 2D$ and $y_* = d$ is added to the bed transport. Despite this additional quantity of material there is little improvement in the accuracy of the method for the finer particle sizes. There is a good correlation between the discrepancy ratio and $D_{gr}$ which shows that the agreement between measured and computed transport rates deteriorates with decreasing $D_{gr}$.

Since the total load is, according to Einstein, proportional to the bed load (see Equation (53), Vol 1) the reason for the systematic variation with $D_{gr}$ may be found in the basic Einstein function (Equation (33), Vol 1) which determines the bed load.

At least two sets of investigators (see Refs 10 and 15, Vol 1) have made a critical analysis of the basic principles of the Einstein method and have found that one of the weak points in the theory is the assumption that the $A_*$ and $B_*$ quantities are constants. Bishop, Simons and Richardson proposed a modification in which $A_*$ and $B_*$ are expressed as a function of particle size, D. However, this is clearly adding a dimensional quantity to the transport and entrainment functions which is undesirable. Later in the present report we have converted these $A_*$, $B_*$ relationships in terms of $D_{gr}$ instead of D as part of a proposed modification of the Bishop, Simons and Richardson method. These functional relationships for $A_*$ and $B_*$ when introduced into the Einstein methods produce improvements in accuracy at the fine particle end of the size spectrum. (See later for details).

In deriving his relationships Einstein assumed that the average distance travelled in one movement of a bed particle was proportional to the particle diameter and equal to 100 grain diameters. It is difficult to estimate or measure the hop length of only those particles which move within the bed layer ($0 < y_* < 2D$) but evidence presented by B Krishnappen (see Ref 15) suggests that the hop length is not only a function of particle size but also of the ambient flow conditions. B A Christensen and T Y Chin (Ref 17) have suggested that the hop length is inversely proportional to the particle diameter.

Finally it is worth pointing out that in his original formulation of the bed load theory Einstein indicated that the sediment discharge was formed of those particles that in a given time travelled n times the length of the average jump where n denotes a statistically significant number of hops but he did not impose any restraint in terms of the height of jump. Thus, in fact, there is no reason to claim that the bed load function is confined to a layer from the bed to two diameters above the bed and it is not surprising that the total load theory should overestimate transport rates for the medium to coarse sediments since some of the predicted transport will consist of particles which have been "counted" twice.

The broad conclusion from the present analysis is that the Einstein methods should be used with care, particularly at low $D_{gr}$ values where there is a distinct tendency to underestimate transport rates. Confidence in the method rises above a dimensionless particle size of about 10 (0.4 mm sand size).

H A EINSTEIN AND C B BROWN (1950):

The comparison of the observed and computed sediment transport rates for the Einstein, Brown method is shown in Fig 13. There is a high degree of scatter in the plot and few systematic trends are apparent. There is a general but erratic tendency to overestimate transport rates, particularly for flume data with sands. Only at high $D_{gr}$ values do the flume data (sands) show reasonable agreement. The lightweight results, on the other hand, show better agreement.

The presence of the dimensionless fall velocity, F, in the $\phi$ versus $\psi$ relationship (see Equation (61), Vol 1) does not appear to give any improvement over the original Einstein function. In fact, the scatter and overall errors are much greater than in the basic Einstein method although the under-estimation of transport rates of the latter method is not apparent in the Einstein, Brown results. 2.7 per cent of the data could not be analysed by this method.

A A BISHOP, D B SIMONS AND E V RICHARDSON (1965):

Bishop, Simons and Richardson proposed a modification to the bed load equation of H A Einstein which represented a simplification of the computational procedures. The transport rates (total load) are no longer computed for individual grain sizes. Instead, the shape of the grading curve of the bed material is taken into account by using the $D_{35}$, $D_{50}$ and $D_{65}$ sizes as references. The method also includes functional relationships for the $A_*$, $B_*$ parameters (see Equation (67), Vol 1) in terms of particle size, D.

The results for this theory are shown in Fig 14. There is little scatter in the mean discrepancy ratios with a general tendency to underestimate transport rates by up to a factor of 4 for both flume and field data. However, there is scatter within each set as indicated by the difference between minimum and maximum discrepancy ratios. Two sets of field data (Data Refs 111 and 112) do not follow this pattern and calculated transport rates are well in excess of the observed values. However, as indicated earlier, the Goose Creek data does not include much of the suspended load in the observed "total" load. Data Ref 106 shows a similar but less pronounced effect. Most theories overestimate transport rates when applied to the Goose Creek data.

The results for coarse sediments are interesting because the Bishop, Simons and Richardson modifications of the basic Einstein method were largely concerned with the grading of the sediment and the shielding effects of larger particles. The coarse sediment data (Elbow River, $D_{gr} = 687$) and Aare River, $D_{gr} = 1013$ and 1450) shows a wide range of particle sizes at each site, see Fig 27, and hence is relevant in this context.

The Bishop, Simons and Richardson method overestimates, by a large margin, transport rates for these coarse sediments. The Einstein methods, on the other hand, show reasonable agreement for the Elbow River data and underestimate sediment transport for the Aare.

One reason for this difference could be found in the use of the hiding factor by Einstein, a correction not used by Bishop, Simons and Richardson. On the other hand, the $A_*$, $B_*$ versus D relationships have been extended well beyond the region investigated by Bishop, Simons and Richardson and this could be a source of error. The greatest diameter used by these authors was about 1 mm ($D_{gr} \simeq 25$) but the results up to $D_{gr} \simeq 100$ show that our assumed extrapolation is reasonable, see Fig 14.

A detailed analysis of individual test results shows a major underestimation of transport rates for many tests in which the $\psi'$ parameter exceeds about 20, especially for fine material. Major discrepancies also occur in terms of the lightweight data. This is not surprising, however, since the variation of $A_*$ and $B_*$ with D is not logical and D should be replaced by some dimensionless parameter containing D e.g. $D_{gr}$. This idea is expanded later.

The limitations of the method (see Vol 1) meant that 39.7 per cent of the data could not be analysed.

R A BAGNOLD (Bed load, 1956):
R A BAGNOLD (Total load, 1966):

R A Bagnold has proposed two sediment transport theories, one for determining the bed load (1956) and one for determining the total load of bed material (1966). The author was fairly emphatic about the roles of the two theories and suggested Y = 0.4 as a rough boundary between the two zones of applicability; see Equation (84), Vol 1 for the precise definition. In view of this the bed load equation has only been applied to those measurements in which the transport of material took place close to the bottom of the channel. The criterion of F Engelund (Ref 8) was used to define these conditions, the geometric mean being taken as the representative diameter where graded sediments were considered. See Fig 6 for details. This criterion meant that many tests were eliminated from each test series when applying the bed load equation and sometimes (fine materials) whole series were eliminated.

The total bed material load equation has been applied within the limits specified by Bagnold wherever these were specific. However, Bagnold's requirement for "adequate flow depth" such that "the thickness of the conceptual moving carpet can be neglected in comparison with the total depth of flow" has not been met. A more specific definition is needed.

In his total load theory Bagnold suggests, quite arbitrarily, that the effective fall velocity should be halved when substituting in the predictive equations.

However, initial comparisons with the present data suggested that this halving of the fall velocity introduced much larger errors and the full fall velocity was used thereafter. See Equation (77), Vol 1, for details.

The specified limitations of the total load theory mean that many tests and sets of tests are eliminated from the comparison. This is particularly noticeable for coarse and lightweight sediments where the mobility is generally low and conditions are within the zone close to threshold conditions. Some 68.5 per cent of the data was eliminated.

Fig 15 shows the results for the bed load theory. There is much scatter in this plot both in the mean values and in the differences between minimum and maximum discrepancy ratios. Computed transport rates vary from several times greater than the observed rates to several times less than the observed rates. 36.8 per cent of the data was outside the range of applicability of the bed load theory.

Fig 16 shows the results for the total load theory. There is good agreement in the range $1 < D_{gr} < 20$ with a marginal tendency to overestimate flume transport rates and underestimate field transport rates. The theory tends to overestimate transport rates for $D_{gr} > 20$ and there is an increasing number of results which are eliminated because of the minimum mobility criterion. The lightweights are eliminated because of mobility considerations, not because the theory does not apply to lightweights. It is worth noting, however, that the $e_b$ relationships are given for sand in water, see Vol 1.

The good results for flume data for the range $0 < D_{gr} < 20$ support the argument for using the full fall velocity as the effective quantity. If the effective fall velocity had been halved the predicted transport rates would have gone up by a factor approaching two, (see Equation (80), Vol 1). On the other hand there would have been a marginal improvement in the field data if the fall velocity had been halved.

The results for the total load theory show far less scatter than for the bed load theory both in terms of the means of the sets and the maximum and minimum errors for each set. It seems remarkable that with the total load theory the Willis flume data (Data Refs 56 to 58, $D_{gr} = 2.6$) fits into the general pattern for flume data. This is the only theory where this happens.

For $D_{gr}$ values less than about 20 (0.8 mm sand size) the total load theory can be used with confidence so long as the motion is well established, i.e. there is a significant suspended load. It is possible that the $D_{gr} \ngtr 20$ limitation represents the scope of experimental verification rather than a fundamental limitation. Experiments with coarse sediments usually cover only the early stages of sediment transport.

E M LAURSEN (1958):

Fig 17 compares predicted and observed sediment transport rates for the method proposed by E M Laursen. In spite of the scatter in some areas, particularly $D_{gr} < 6$, Fig 17 shows some correlation between the discrepancy ratio and the dimensionless particle size, $D_{gr}$. At low $D_{gr}$ values the theory underestimates transport rates, around $D_{gr} = 20$ there is reasonable agreement and for high $D_{gr}$ values the theory overestimates transport rates. The Goose Creek and Mountain Creek data do not fit this pattern but they are suspect for reasons stated earlier.

The E M Laursen equation (Equation (93), Vol 1) can be transformed into the form:-

$$\phi = 0.01 \left(\frac{V}{v_*}\right) \frac{Y^{\frac{1}{2}}}{sZ^{1/6}} \left\{ \frac{Y}{Y_C} \left(\frac{V}{v_*}\right)^2 \frac{1}{58Z^{1/3}} - 1 \right\} f\left(\frac{v_*}{w}\right) \qquad \ldots(8)$$

The dimensionless grain size parameter $D_{gr}$ occurs implicitly in the term $f\left(\frac{v_*}{w}\right)$ and probably in other terms as well. The fall velocity, w, can be expressed as follows:-

$$w = \frac{v_*}{Y^{\frac{1}{2}}} f_1(D_{gr}) \qquad \ldots(9)$$

(See Equations (A5) and (A6), Vol 1).

Hence, from (9)

$$f\left(\frac{v_*}{w}\right) = f\left(\frac{Y^{\frac{1}{2}}}{f_1(D_{gr})}\right) \qquad \ldots(10)$$

The above manipulations suggest that $f(v_*/w)$ is not unique as suggested by Laursen but is, in fact, a family of curves with the dimensionless particle size as one parameter. A schematic drawing showing one possible form for these curves is shown in Fig 25. The average curve presented by Laursen probably represents a $D_{gr}$ value of around 20; this being the point in Fig 17 where predicted and observed transport rates show reasonable agreement. The curves for higher values of $D_{gr}$ will be below this average curve, see Equation (10), and will yield a lower value of $f(v_*/w)$. Conversely curves for $D_{gr} < 20$ will plot above the average line.

The incipient stage of movement will correspond with the value:-

$$\left(\frac{v_*}{w}\right)_c = \frac{Y_c^{\frac{1}{2}}}{f_1(D_{gr})} \qquad \ldots(11)$$

where $$Y_c = f_2(D_{gr}) \qquad \ldots(12)$$

as shown in Fig 25. The straight lines "A" represent bed load transport and will tend to the horizontal as $D_{gr}$ falls to about 3 (see Fig 6).

It is interesting to note that J L Bogardi, using the Laursen method, suggested a family of curves defined by

$$f\left(\frac{v_*}{w}\right) = f_3(Y,D) \qquad \ldots(13)$$

although the form of the curves differed from the Laursen curve. See Ref 11 for details. In fact the quantity $f(v_*/w)$ cannot be a function of the dimensional grain size, D, but is probably a function of some dimensionless parameter involving D as suggested above.

The above discussion suggests how the $f(v_*/w)$ function might be modified to improve the overall accuracy of the method. However, the pronounced scatter in Fig 17 for $D_{gr} < 6$ and $D_{gr} > 40$ indicates some fundamental deficiency in the predictive equation when applied over a wide range of conditions. It is not possible to pinpoint this deficiency on the available evidence.

J ROTTNER (1959):

The results for J Rottner's bed load theory are shown in Fig 18 and the comparison between observed and predicted transport rates is good. Except for the Willis, Franco and Gilbert and Murphy flume data (Data Refs 55-58, 53 and 54, 18 and 19) a large percentage of the sets have mean discrepancy ratios close to unity with little internal scatter. One or two sets show high maximum discrepancy ratios but these are exceptions to the rule.

The lightweight sediments data fits in well with the general pattern and the field data show no systematic errors. Data Ref 111 (Mountain Creek) is again an anomaly but this is a characteristic of the data rather than the theory.

The theory was originally proposed as a bed load theory but the present evidence suggests that it can be used with confidence as a total load theory.

M S YALIN (1963):

Fig 19 indicates the performance of the bed load theory of M S Yalin. The results are poor with much scatter of the mean discrepancy ratios and also within each data set. The erratic behaviour is similar to that obtained with the Einstein-Brown method, see Fig 13.

The theory is based on a theoretical analysis of the motion of saltating particles. Assumptions include (i) plane bed conditions, (ii) fully developed turbulent flow and (iii) large depth/diameter ratios. Some of the data did not comply with these criteria and this may account for some of the scatter in the comparative plot.

The flume data used in this report in which the sediment transport is a bed process ($D_{gr} \gtrsim 15$), shows low values of the depth/diameter ratio.

For $D_{gr} > 30$ ; $Z < 200$.

For $10 < D_{gr} < 30$ ; $200 < Z < 1000$.

Similarly, the field data for bed load transport exhibits low depth/diameter ratios

| | |
|---|---|
| Goose Creek | ; $150 < Z < 1700$ |
| Skive-Karup | ; $Z \simeq 1700$ |
| Mountain Creek | ; $Z \simeq 100$ |
| Elbow River | ; $Z \simeq 30$ |
| Aare River | ; $Z \simeq 30$ |

Only the Mississippi River at St Louis shows high values of the depth/diameter ratio with a value $Z \simeq 30\,000$. However, only about 50 per cent of the individual measurements have values of Y less than 0.4 i.e. 50 per cent of the data constitutes bed load transport. Nevertheless the St Louis data ($D_{gr} = 16$ and 17) plots well on Fig 19.

In practice it is doubtful whether the two restrictions of plane bed and $Z \to \infty$ can occur simultaneously except in the transition range from dunes to antidunes. This observation was also made by C F Nordin and J P Beverage (Ref 12) who evaluated the Yalin method against field and flume data. Their evaluation was more favourable than the present findings.

Normally the value of Z is greater for field measurements than for flume measurements except at the coarse sediment end of the range. This could explain the slightly better agreement for field data up to a particle size of about 1 mm. However, it is not clear why the bed load theory should overestimate transport rates for a large proportion of both flume and field data.

T BLENCH (1964):

In proposing his regime equations Blench laid down many restrictions concerned with the range of application of the theory. In fact these conditions are so restrictive that practically all the flume data and a large proportion of the prototype data should be excluded. Furthermore, some of the conditions are not defined in a precise way.

Equation (109) (Vol 1) is valid only for fine sand and concentrations by weight less than $10^{-4}$. However, in the present analysis, a much less restrictive limit on concentrations has been employed. Only tests with X values greater than $5 \times 10^{-2}$ have been eliminated. This is the highest value for X given in Blench's graphical solution to equation (114) (Vol 1). The theory has been applied to the full range of sediment sizes.

The minimum breadth/depth ratio of 4, suggested by Blench, has been adhered to but the minimum flow depth of 0.4 m has been ignored. The "zero bed factor" has been assumed to be universally applicable whereas Blench suggests that it should be used with fine sands only, see equations (110) to (113) (Vol 1).

Although it seems unjust to ignore some of the restrictions suggested by Blench there seems no alternative. There would be very little data left for a comparative study. In the event only 31.2 per cent of the data was eliminated and the results indicate, see later, that some of the original restrictions are unnecessarily severe, particularly those which apply to flume data, and that Blench is being over-cautious in his views.

The evaluation of the meander slope correction term is a matter of judgement rather than a scientific procedure. The values adopted in the present study are as follows:-

| River | Data Ref | Meander slope correction, K |
|---|---|---|
| Atchafalaya, Simmersport | 118,119 | 2.00 |
| Mississippi, Tarbert Landing | 115,116,117 | 2.00 |
| Paraguay, km 385 | 153 | 2.00 |
| Niobrara, Cody | 107,108 | 1.25 |
| Goose Creek, Oxford | 112 | 1.00 |
| Middle Loup, Dunning | 101 to 105 | 1.25 |
| Skive;Karup | 106 | 2.00 |
| Mississippi, St Louis | 150,151 | 2.00 |
| Mountain Creek, Greenville | 111 | 1.25 |
| Elbow River, Bragg Creek | 109 | 1.25 |
| Aare River, Brienzwiler | 113,114 | 1.25 |
| Flume data | All | 1.00 |

The sediment transport equation presented by Blench is given in Vol 1, equation (114). It is worth noting that at the threshold conditions ($X \rightarrow 0$) the left hand side of the equation tends to unity. Thus the denominator and numerator on the right hand side of the equation must be numerically equal at the point of incipient motion.

The comparison of observed and predicted transport rates using the Blench equation is given in Fig 20. The flume data deemed inapplicable was eliminated as a result of the depth/breadth ruling.

Surprisingly, there is good agreement for many sets of flume data and in spite of the fact that the theory was developed from field observations the agreement with flume data is generally better than the agreement with field data. The theory underestimates transport rates for field data at the finer end of the size range and many sets indicated zero transport using the Blench approach. There are three similar results in the range $10 < D_{gr} < 20$.

Looking at the theory in terms of the prototype data the results appear to be good for some sets of data and very bad for others, see Fig 20. It is worth comparing these good and bad sets to determine their distinguishing features and hence to define more definitely the range of application of the theory. Good results were obtained when the theory was applied to data sets 108, 107, 112, 101, 102, 103, 104, 105, 111, 113 and 114. On the other hand data sets 119, 118, 116, 115, 117, 153, 106, 151, 150 gave poor agreement. The characteristics of these data sets are shown in Tables 2 to 6 and a detailed analysis suggests that the significant difference between the "good" and "bad" sets is the average value of the depth/diameter ratio as shown in Table 6. The "bad" results are related to those sets which have particularly high values of Z i.e. the theory seriously underestimates transport rates where the depth/diameter ratio is greater than about $10^4$. This seems to be an important limitation of the regime theory of T Blench.

Although according to Blench his equation includes both bed and suspended load the evidence of the fine prototype sediments does not support this view.

The lightweight data does not plot well on Fig 20 but this is to be expected since the theory is, strictly speaking, limited to quartz sediments.

F ENGELUND AND E HANSEN (1967):

Results using the Engelund and Hansen method, see Fig 21, are consistently good over the full range of sediment sizes and sediment specific gravities.

Comparisons with flume data are good with the exception of the three sets reported by Willis (Data Refs 56, 57 and 58), two sets report by Franco (Data Refs 53 and 54) and one of the sets reported by Gilbert and Murphy (Data Ref 19). Many theories predict low transport rates for these data and the discrepancies are probably errors in the data rather than the theory. There is a tendency, however, for the maximum discrepancy ratios in the plots to be much further from the mean than the minimum discrepancy ratios. Examination of the individual data sets where this occurs has shown that the theory tends to overestimate transport rates at low shear values. This appears to be a weakness in the Engelund and Hansen method. The predictive equation (Equation (117), Vol 1) was derived without considering the effects of viscous forces and nowhere does the kinematic viscosity of the fluid appear. Thus it is not surprising that the theory works best in the later stages of sediment transport where the influence of viscosity is less important. Table 3 shows the range of mobility numbers for the individual data sets.

Correlation with field data is also good, the only exceptions being Mountain Creek (Data Ref 111), Elbow River (Data Ref 109) and the second set of Aare River data (Data Ref 114). The peculiarities of these data sets have been mentioned previously.

The Engelund and Hansen method was established exclusively from experiments with sand (s = 2.65). However, the lightweight data shows up well on Fig 21 and the effects of specific gravity are clearly taken into account in the proposed method.

The great advantage of the Engelund and Hansen method is its simplicity. Equation (117), Vol 1, is probably the simplest of all predictive equations yet, in general, it is one of the most accurate. Its one shortcoming is the errors which are introduced at low transport rates.

W H GRAF (1965):

W H Graf proposed a relationship to describe the total sediment load in both open and closed conduits. The results for this theory are shown in Fig 22. There is a general tendency to overestimate transport rates and significant scatter between and within the data sets. There is no systematic variation of the performance of this theory in terms of the dimensionless particle size. The largest over-estimation of transport rates is, in general, associated with the flume data and the discrepancy is less for the prototype and lightweight sediments.

The results for the Graf equation bear a strong resemblance to the results for the Einstein, Brown approach (c.f. Figs 13 and 22). Both theories show a similar pattern of results with the same distinction between the results for the flume data (quartz materials) and the results for the prototype and lightweight materials. This similarity is, of course, due to the nature of the two predictive equations which resolve approximately to the forms:-

$$\phi = C_1 \, Y^3 \qquad \text{... Einstein, Brown.}$$

$$\phi = C_2 \, Y^{2.52} \qquad \text{... Graf.}$$

The coefficient $C_1$ in the Einstein, Brown expression is, however, related to the particle size (see Equation (64), Vol 1) and only becomes near constant at values of $D_{gr}$

greater than about 7. Also, the above form of the Einstein-Brown equation is only true for $Y > 0.1$ but this is not of practical significance. The coefficient $C_2$ in the Graf expression depends on the friction factor, $\lambda$. In spite of the different exponent there is little to choose between the performances of the two approaches.

F TOFFALETI (1968):

The comparison of predicted and observed transport rates for the Toffaleti method is shown in Fig 23. Below a $D_{gr}$ value of about 30 the method gives consistently good results with most of the mean discrepancy ratios between 1/4 and 4. In the range $D_{gr} > 30$ there is an increasing tendency to overestimate transport rates.

For two sets of flume data (Data Refs 26 and 19) and three sets of field data (Data Refs 109, 113 and 114) the measured parameters were outside the range to which the theory is applicable. The computer output indicated that the "FAC" coefficient exceeded a value of 2 in these sets, see Equation (157), Vol 1. Data Refs 109, 113 and 114 are gravel rivers and one would not expect this theory to be applicable in this range.

For flume data (quartz materials) the theory works well in the range $1 < D_{gr} < 25$ but there is an increasing systematic overestimation of transport rates at higher $D_{gr}$ values. It seems that the suggested correction factor $k = f(FAC)$ or the condition $Ak \geqslant 16$ (see Vol 1) are in some way introducing this systematic error but the computational procedures are so complex that it is difficult to be specific.

For the field data the theory works equally well in the range $1 \leqslant D_{gr} < 25$. Unfortunately there is no field data in the range $30 \leqslant D_{gr} < 100$ so it is not possible to say whether the systematic overestimation would develop in the case of field data as it did for flume data. The two sets of field data which have a mean discrepancy ratio between 6 and 8 are from Goose Creek and Mountain Creek (first set). Many theories over-predict for these data, see Figs 14, 17, 18 and 21, indicating systematic errors in the field measurements.

The theory underestimates transport rates for lightweight materials and the errors are large in many cases. However, the theory was derived using data for quartz materials and Toffaleti did not expect the theory to be applicable for materials with specific gravities other than 2.65.

The Toffaleti theory is one of the best theories for quartz materials in the range $1 < D_{gr} < 25$. However, the computational procedures are extremely cumbersome and the theory cannot be applied to lightweight materials. The limitations of usage suggested by Toffaleti eliminated 7.9 per cent of the data.

P ACKERS AND W R WHITE (1972):

The results for the general function proposed by Ackers and White are shown in Fig 24. These are consistently good and about 70 per cent of the mean discrepancy ratios are between ½ and 2.

With the exception of the Willis, Franco and some of the Gilbert and Murphy data all the mean discrepancy ratios for flume data are between 1/4 and 5. The only field data set falling outside this range is the Aare River (Data Ref 114) which shows a discrepancy ratio of zero i.e. $X_{CALC} = 0$. The theory overestimates transport rates for the Goose Creek and Mountain Creek (first set) by a factor of about 4 and joins the long list of theories which do likewise. There was undoubtedly some suspended load in these rivers which was not measured.

The theory works well for coarse materials even though the various coefficients were based originally on an analysis of flume data. The theory overestimated the Elbow river transport rates by a factor of three but within this set there are a few very low observed transport rates which distort the mean error. This is apparent from the high maximum discrepancy ratio for this set. The second set of data from the Aare River (Data Ref 114, $D_{gr} = 1450$, $D_{50} = 68$ mm) represents a very coarse gravel river for which almost all the theories underestimate the transport rates. The observed shears were, in general, well below the Shields threshold condition.

From Fig 24 it is possible to detect a very slight overestimation of transport rates for flume data (quartz and lightweights) and a modest underestimation for field data. It should be noted, however, that in their original papers (Refs 24 and 25, Vol 1) Ackers and White recommended the use of $D_{35}$ as a representative diameter for field conditions whereas the $D_{50}$ size has been used in the present comparison. This would explain the slight underestimation of field transport rates.

In deriving their general function (Equation (158), Vol 1), Ackers and White assumed that for coarse materials bed features would be small and that the mean velocity of flow would be given by the expression

$$\frac{V}{v_*'} = \sqrt{32}\,\log_{10}\left(\frac{\alpha d}{D}\right) \qquad \ldots(14)$$

where $v_*'$ is the shear velocity ascribed to the grains (equal to the total shear velocity if the bed is plane)

and $d$ is the mean depth.

The conventional mobility number is then given, for coarse material, by:-

$$F_{cg} = \left\{\frac{(v_*')^2}{gD(s-1)}\right\}^{\frac{1}{2}} = \frac{V}{\sqrt{32gD(s-1)}} \cdot \frac{1}{\log_{10}\left(\frac{\alpha d}{D}\right)} \qquad \ldots(15)$$

and for fine material where the grain shear becomes negligible relative to the total shear:-

$$F_{fg} = \left\{\frac{v_*^2}{gD(s-1)}\right\}^{\frac{1}{2}} \qquad \text{...(16)}$$

Equations (15) and (16) are then combined to express the mobility number in the transition range of sediment sizes as:-

$$F_{gr} = F_{fg}^{n} \cdot F_{cg}^{(1-n)} \qquad \text{...(17)}$$

where n is a transition exponent which depends on the dimensionless particle size, $D_{gr}$. Ackers and White then analysed a large quantity of flume data and concluded that fine materials (n=1) correspond to $D_{gr} = 1$ and coarse materials (n=0) are those in the range $D_{gr} > 60$ (i.e. 0.04 mm and > 2.5 mm sand sizes respectively). Since publication of their original reports (Refs 24 and 25, Vol 1) more data, both flume and field, have been obtained in the coarse range of sediment sizes and these have, for practical purposes, confirmed this upper limit. The predictions for coarse materials are, by sediment transport standards, accurate in the coarse range ($D_{gr} > 60$), see Fig 24. However, the coarse grain data includes references to minor bed features and irregularities and the theory could possibly be further refined by an asymptotic approach to the plane bed situation rather than the somewhat abrupt limit of $D_{gr} = 60$.

2. The influence of graded sediments

Of the theories analysed only Einstein, Laursen, Toffaleti and Bishop, Simons and Richardson seek to take the grading of a sediment into account. The other theories use an "equivalent" diameter which may be the $D_{35}$, $D_{50}$, $D_m$ or other size taken from an analysis of a bed sample. This is clearly a simplification since grading curves with a different shape will almost certainly have a different effective diameter. Furthermore, the effective sediment size will probably vary with the transport rate, particularly in gravel rivers where there is a wide spectrum of sizes and where the sorting of material is most pronounced.

It is useful to consider this problem in two separate sections; firstly the general problem of graded sediments in motion and secondly problems with armouring and the associated threshold conditions.

Graded sediments in motion:

The idea of an effective diameter related to the grading curve and the ambient flow conditions is illustrated diagrammatically in Fig 26. Assuming that the graded sediment can be defined by the range of sizes between $D_5$ and $D_{95}$, say, then sediment transport will occur within the zone defined by the limits $Y > Y_C$ and $D_{gr(5)} < D_{gr} < D_{gr(95)}$ if the armour plating effect is not significant.

Each size fraction has a value of $Y_C$ which is a function of its size. On the other hand the value of Y associated with each fraction will depend on the size of the fraction and the ambient flow conditions

$$Y_C = f_1(D_{gr}) \qquad \ldots(20)$$

$$Y = \frac{v_*^2}{gD(s-1)}$$

and substituting

$$D_{gr} = \left\{\frac{g(s-1)}{\nu^2}\right\}^{1/3} D$$

$$Y = \left(\frac{1}{g(s-1)\nu}\right)^{2/3} \cdot \frac{v_*^2}{D_{gr}}$$

i.e.

$$Y = f_2(v_*, D_{gr}) \qquad \ldots(21)$$

for a particular fluid and sediment density. Increasing the value of $v_*$ from zero the values of Y for all the size fractions will increase until $Y_5 = Y_{C5}$. At this point the 5 per cent fraction will start to move and the significant diameter, $D_\alpha$, will be $D_5$. At a somewhat higher value of $v_*$ the conditions could be as shown in the line AB in Fig 26. Several size fractions will be in motion and the significant diameter will be somewhere between $D_5$ and $D_{50}$ and increasing with $v_*$.

i.e. $$D_\alpha = f(Y) \qquad \ldots(22)$$

As $v_*$ increases the condition $Y_{95} = Y_{C95}$ will be reached and all the size fractions will then be in motion. This condition is illustrated by the line A'B' in Fig 26 with a significant diameter, $D_\alpha'$, somewhat larger than $D_\alpha$. At higher shear velocities, and certainly as $Y \to \infty$, there is likely to be little change in the magnitude of $D_\alpha$

i.e. $$D_\alpha = \text{constant} \quad (Y \to \infty) \qquad \ldots(23)$$

The above philosophy, which assumes that the individual size fractions have no influence on each other, thus leads to the broad conclusion that the significant or effective particle size in a graded material decreases with decreasing transport rates since the fractions moving are smaller than the bed material as a whole.

On the other hand the armouring effect where smaller particles hide behind, between or underneath the larger particles works in the opposite direction. It could be argued that if the bed is covered with material of the $D_{95}$ size (assuming a sample in depth) the significant particle size is initially the $D_{95}$ size not the $D_5$ size and the $D_\alpha$ falls from $D_{95}$ to its final intermediate value as Y increases.

Both views give the same answer when all the size fractions are in motion but there is plenty of scope for errors near the threshold conditions, see Fig 26. This presents enormous problems for the engineer particularly where the sediment is very coarse. Not only is there a wide range of sediment sizes in these rivers but, because of the large sediment sizes, the transport conditions are usually close to the threshold of movement. In fact, the larger the maximum sediment size the wider is the range of sizes. This is illustrated in Fig 27 where the cumulative percentage by weight is plotted against the grain size ratio $D_i/D_{50}$ for various rivers. Clearly, as $D_{50}$ rises so does the ratio $D_{95}/D_5$.

To sum up, therefore, those theories which assume an effective grain size which is a constant value related only to the grading curve are likely to introduce errors close to the threshold conditions. One improvement would be to relate the effective grain size to the mobility as well as the grading curve of the bed material. Also, the ratios $D_{95}/D_5$ or $D_{90}/D_{10}$ would be relevant parameters.

A few theories attempt to take size grading into account. Laursen, Toffaleti and Einstein base their computations on individual size fractions. Einstein also introduces a "hiding factor". Bishop, Simons and Richardson base their computations on three diameters $D_{35}$, $D_{50}$ and $D_{65}$.

Armouring and the associated threshold conditions:

Although the above mentioned theories attempt to take size grading into account they do so only in terms of established motion. Not one of the theories considers the initial movement of an armoured bed.

I V Egiazaroff made an interesting evaluation of this problem in deriving his own sediment transport theory. His theory has not been included in this review but his ideas on critical shear conditions are worth repeating here.

Egiazaroff derived the following expression for incipient motion of uniform sediments:-

$$Y_C = \frac{4}{3} \cdot \frac{1}{C_D} \cdot \left(\frac{v_*}{V_{yd}}\right)_c^2 \qquad \ldots(24)$$

where $C_D$ = drag coefficient

$V_{yd}$ = velocity at level $y = 0.63D$

and $c$ denotes critical conditions.

The ratio $v_*/V_{yd}$ was evaluated using the equation given by Einstein:-

$$\left(\frac{V_{yd}}{v_*}\right)_c = 5.75 \log\left(\frac{30.2}{k_s} \cdot 0.63Dx\right) \qquad \ldots(25)$$

where $x = f(k_s/\delta)$ ...(26)

See equations (48) and (49), Vol 1 for details.

For an arbitrary individual grain size, $D_i$, in a mixture under rough turbulent conditions and with $k_s$ equal to $D_m$, the geometrical average diameter for roughness based on the average figure for the material in motion and the total material (see Fig 29) is

$$D_m = (D_{m1} \text{ material in motion} + D_{m2} \text{ total material})/2 \qquad \ldots(27)$$

and equation (25) becomes:-

$$\left(\frac{V_{yd}}{v_*}\right)_c = 5.75 \log\left(\frac{19\ D_i}{D_m}\right) \qquad \ldots(28)$$

Substituting this value into (24) and taking $C_D = 0.4$ gives:-

$$(Y_C)_i = \frac{4}{3} \cdot \frac{1}{0.4} \cdot \frac{1}{\left[5.75 \log\left(\frac{19\ D_i}{D_m}\right)\right]^2}$$

$$= \frac{0.1}{\left[\log\left(\frac{19\ D_i}{D_m}\right)\right]^2} \qquad \ldots(29)$$

For the case of uniform material $D_i = D_m$ and equation (29) gives a value $Y_C = 0.06$. Fig 28 shows the curve represented by equation (29) together with the results from several experimental investigations.

Equation (29) gives the threshold conditions for an individual size fraction within a graded sediment. For the entire mixture, Egiazaroff suggested the use of $D_i = D_{50}$ and thus

$$Y_C^{mix} = \frac{0.1}{\log\left(\frac{19\ D_{50}}{D_m}\right)} \qquad \ldots(30)$$

As the geometric mean size, $D_m$, is usually larger than $D_{50}$

$$Y_C^{mix} > Y_C^{uniform}$$

According to Egiazaroff the $D_{50}$ value must be determined from the grading curve of the material in motion and equation (30) is only valid when there are motionless particles, i.e.

$$D_{max} \geqslant D_o = \frac{v_*^2}{0.06g(s-1)} \qquad \ldots(31)$$

When $D_{max} < D_o$ no accumulation of large bed material can take place and $D_i = D_o$ where $D_o$ is the equivalent threshold diameter for the mixture.

However, in a later work, C R Neill (Ref 14) raised two fundamental objections to the Egiazaroff philosophy.

(i) In his analytical derivation Egiazaroff applied the theoretical velocity profile at an elevation below the peaks of some of the larger particles. Neill argued that the physical picture in this area must be confused and the interaction between the flow and the smaller grain sizes must be dependent to a large extent on the disposition of the larger particles.

(ii) Egiazaroff's method of determining $D_m$ demands a knowledge of the sediment travelling in suspension which cannot be deduced from a grading curve of the original bed mixture.

The importance of obtaining a satisfactory method of determining critical mobility criteria for mixtures cannot be overemphasised. Many theories base their computations of sediment transport on the concept of "excess shear" i.e. $(Y^n - Y_c{}^n)$ and this term becomes crucial as $(Y^n - Y_c{}^n) \rightarrow 0$. A large proportion of the available data in the range $D_{gr} > 10$ constitutes bed load data and minor deviations in the value of $Y_c$ can cause very large errors in computed sediment transport rates. The theories of Shields, Meyer-Peter and Muller, Bagnold (1956), Laursen and Yalin are particularly susceptible to this effect.

It is interesting to note that the general function of Ackers and White incorporates a threshold condition which was derived from the analysis of established movement data rather than observations of the detachment of individual grains. The curve suggested by these authors is shown in Fig 6 together with the classical Shield's curve. For rough turbulent conditions ($D_{gr} > 60$), there is a considerable difference between the two curves. Ackers and White suggest $Y_c = 0.028$ while Shields suggests a value $Y_c = 0.060$. Both figures were, of course, derived from uniform sediment data but the lower value quoted by Ackers and White improves the accuracy of their predictions for the gravel river data from the Aare and Elbow Rivers.

It is clear that there is a need for further work to define, in a more precise way, the initial motion of a graded sediment and to investigate the established motion of a graded sediment at low transport rates.

3. A proposed modification of the Bishop, Simons and Richardson method

The proposed method of Bishop, Simons and Richardson is a modification of the basic Einstein approach in which the coefficients $A_*$ and $B_*$, assumed constant by Einstein, are related to the particle size, D. They also assumed that the same parameters determined the total transport rate regardless of whether the transport was in suspension or as bed load. The functional relationships between $A_*$, $B_*$ and D were derived from five sets of flume experiments.

According to Einstein (Ref 7, Vol 1), the value of $A_*$ is defined as

$$A_* = \frac{A_1 A_3}{A_2 \lambda} \qquad ...(32)$$

where $A_1$ = constant of area

$A_2$ = constant of volume

$\lambda$ = L/D, the dimensionless average jump length for a particle

$A_3$ = constant of time scale

If $A_3'$ represents the ratio of the Einstein "exchange time" to the time taken for a particle to fall a distance equal to its diameter, then

$$t_1 = A_3' \cdot \frac{D}{w} \qquad ...(33)$$

But

$$w = F\sqrt{gD(s-1)} \qquad ...(34)$$

where F = $f(D_{gr})$, see Equation (A5), Vol 1.

Hence from (33)

$$t_1 = \frac{A_3'}{f(D_{gr})} \sqrt{\frac{D}{g(s-1)}} \qquad ...(35)$$

This expression is similar to that used by Einstein (Ref 7, Vol 1, Equation (37)) but now

$$A_3 = \frac{A_3'}{f(D_{gr})} \qquad \ldots(36)$$

Therefore,

$$A_* = \frac{A_1 \; A_3'}{A_2 \lambda f(D_{gr})} \qquad \ldots(37)$$

with $A_3'$ a constant.

M S Yalin (Ref 15) made a similar analysis and also reported some measurements carried out by B Krischnappen which suggested that the dimensionless average jump length, $\lambda$, was related to the mobility, Y, and the dimensionless flow depth Z:-

$$\lambda = Y \; f_1 \left(\frac{Y}{Z}\right) \qquad \ldots(38)$$

Yalin also shows that the value of $B_*$ given by Bishop, Simons and Richardson in terms of the diameter, D, must, in fact, be related to the particle Reynolds number $R_*' = v_*'D/\nu$. Thus we can conclude that the Einstein approach could be improved (i) by relating $A_*$ to $D_{gr}$ (at least) and (ii) by relating $B_*$ to $R_*'$.

The expression used by Bishop, Simons and Richardson (Equation (69), Vol 1) is:-

$$\frac{A_*\phi}{1 + A_*\phi} = 1 - \frac{1}{\sqrt{\pi}} \int_{-B_*\psi' - 1/\eta_o}^{B_*\psi' - 1/\eta_o} e^{-t^2} \, dt \qquad \ldots(39)$$

However, the lower limit of the integral is not logical, as Yalin points out (Ref 15) since it predicts detachments of grains for high downward components of the pressure fluctuations. See Ref 19, p 131 for details. Thus in the present modification of the Bishop, Simons and Richardson method the following predictive equation is used:-

$$\frac{A_*\phi}{1 + A_*\phi} = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\xi B_*\psi' - 1/\eta_o} e^{-t^2}\, dt \qquad \ldots(40)$$

Note:-

(i) The intensity of transport, $\phi$, is related to the $D_{50}$ size as in the Bishop, Simons and Richardson method.

(ii) The intensity of shear on a representative particle, $\psi'$, is related to the $D_{35}$ size as in the Bishop, Simons and Richardson method.

(iii) $\eta_o$ is taken as 0.5 as suggested by Einstein and El Sammi, Ref 16.

(iv) The relationships between $A_*$ and $D_{gr}$, $B_*$ and $R_*'$ were derived from an analysis of the flume data and are shown in Fig 30.

(v) A hiding factor, based on the Elbow River data, is introduced to take into account some of the effects of mixtures, particularly the effects of large accumulations of coarser material on or near the surface of the bed at and just above threshold conditions. The hiding factor, $\xi$, is related to the ratio $D_{35}/D_{50}$ as shown in Fig 31. Analytically this curve is given by the following expressions:-

If $D_{max} \geqslant v_*^2/(0.1g(s-1))$ ; $\xi = 10^{3.27(\log(D_{35}/D_{50}))^2}$

If $D_{max} < v_*^2/(0.1g(s-1))$ ; $\xi = 1$

The curves shown in Fig 30 for $A_*$ and $B_*$ show strong similarities in shape with the plots provided by Bishop, Simons and Richardson and the comparison is an interesting one.

In using this new sediment transport equation certain limitations of the method became apparent.

(i) Computed transport rates were found to be excessively high when $\xi\psi'_{35}$ was less than 1. A lower limit $\xi\psi'_{35} = 1$ is suggested at the present time.

(ii) Transport rates are underestimated if $\xi\psi'_{35} \cdot B_*$ exceeds about 3. $\xi\psi'_{35} \cdot B_* < 3$ is the suggested working range. When $\xi\psi'_{35} \cdot B_*$ exceeds 3 conditions are very close to the incipient motion state and the difficulties are probably associated with threshold conditions for sediment mixtures.

Assuming $\eta_o$ to be genuinely constant, equation (40) represents a functional relationship between $A_*\phi_{50}$ and $\xi B_*\psi'_{35}$. A graphical solution is given in Fig 32.

Fig 33 compares observed and predicted transport rates for the new method. The results are encouraging and lend strong support to the arguments for relating $A_*$ and $B_*$ to $D_{gr}$ and $R_*'$ respectively, c.f. Fig 14. It is possible that further improvements could be made by relating $A_*$ to $D_{gr}$, Y and Z rather than simply to $D_{gr}$. Further analysis is desirable; particularly as the method, in its present form, was only applicable to 67.6 per cent of the total data available.

Details of the computational procedure for this new method are as follows:-

1. From the grading curve of a sample of the bed material note the $D_{35}$, $D_{50}$ and $D_{65}$ sizes.

2. Compute the value of $v_*'$ using Equations (45) to (48), Vol 1, or obtain the result from the graphs presented in Ref 23, Vol 1.

3. Calculate the shear intensity factor, $\psi'$, using the expression $\psi'_{35} = gD_{35}(s-1)/(v_*')^2$.

4. Evaluate $D_{gr}$ and $R_*'$ using:-

$$D_{gr} = D_{50} \left\{ \frac{g(s-1)}{\nu^2} \right\}^{1/3} \quad \text{and} \quad R_*' = \frac{V_*' D_{50}}{\nu}$$

5. Obtain values of $A_*$, $B_*$ and $\xi$ from Figs 30 and 31.

6. Obtain the value of $\phi_{50}$ either by using Fig 32 to find $\phi_{50} A_*$ and hence $\phi_{50}$ or by solving Equation (40) directly.

7. Convert $\phi_{50}$ to the sediment transport rate using equation (2) with $D = D_{50}$.

## 4. A brief comparison of the overall performance of the predictive methods

The theories have been evaluated by plotting the discrepancy ratio $X_{CALC}/X_{ACT}$ against the dimensionless grain size, $D_{gr}$. The mean, maximum and minimum discrepancy ratios for each set of data have been indicated on the plots. Hence the spread of errors within each data set could be due to deficiences in the theories in terms of Y, Z or s or simply errors in the observations.

As a guide to the overall performance of each theory it is useful to consider the amount of data falling within different ranges of errors. An analysis of individual measurements for each theory is shown in the form of histograms in Fig 34. The theories have been placed in order according to the proportion of the total amount of data for which the predictions are between ½ and 2 times the observed values. The dotted histograms indicated the performance of each theory when applied only to the data within its stated range of applicability. The better theories have up to 64 per cent of the data within this error band while several theories have less than 10 per cent and are not included in the diagram. The Ackers and White theory and the Engelund and Hansen method show up very well on this plot. The modified Bishop, Simons and Richardson approach is an improvement on the original theory and is very reliable within the stated range of application, see dotted histogram. However these limitations are such that 33 per cent of the total data pack was eliminated. Similarly, the Bagnold total load theory gives good results within its stated range of applicability but this range is very restrictive. Some 69 per cent of the data could not be analysed.

## CONCLUSIONS

1. Nineteen sediment transport theories have been examined with reference to flume and field data. The comparison has been based on almost 1000 flume experiments (quartz and light-weight materials) and 270 field measurements. Froude numbers in excess of 0.8 have been excluded and no allowance has been made for wall effects and bank friction.

2. The available data has been classified in terms of the dimensionless parameters X, $D_{gr}$, Y, Z and b/d. See Tables 3 to 6 and Figs 1 to 5.

3. The characteristics of the flume and field data vary considerably in terms of the breadth/depth ratio. See Table 4 and Fig 3.

4. Depth/diameter ratios for the flume data are generally much less than for the field data for similar dimensionless particle sizes. See Table 6 and Fig 5.

5. There is a significant lack of field data in the range $40 < D_{gr} < 500$. See Fig 1.

6. Using the criterion proposed by F Engelund most of the available data in the range $D_{gr} > 15$ consist of bed load measurements. This applies to both flume and field measurements.

7. Six sets of data produce computed concentrations well below the measured values when using many of the theoretical methods. This group of measurements must be regarded as suspect and comprises the data of Willis, Coleman and Ellis (Data Refs 56, 57 and 58), Franco (Data Refs 53 and 54) and Gilbert and Murphy (Data Ref 19).

8. The Goose Creek and Skive-Karup field data has been shown to lack a certain quantity of unmeasured suspended load and this view has been substantiated by several of the more reliable theories which predict transport rates in excess of the measured values.

9. Three individual measurements in the first set of field data from Mountain Creek (Data Ref 111) and two individual measurements in the Elbow River data (Data Ref 109) show very low measured concentrations which do not fit in with the general pattern. These have distorted the results for several predictive methods, principally in terms of the maximum discrepancy ratio.

10. A modification of the Bishop, Simons and Richardson method is proposed which expresses $A_*$ and $B_*$ in terms of $D_{gr}$ and $R_*'$ respectively. It also includes a hiding factor based on the $D_{35}/D_{50}$ ratio and a modification to the limits of the Einstein entrainment integral. The range of applicability of this new approach is, however, restricted to $\xi\psi'_{35} > 1$ and $\psi'_{35}B_*\xi < 3$. See Fig 32 for details. These restrictions are not too prohibitive in practice and the method can be applied to flume and field data so long as the transport rate is neither very low nor very high and so long as the $D_{gr}$ value is greater than about 2, i.e. > 0.08 mm sand sizes.

11. For the better theories the predicted transport rates were between ½ and 2 times the observed rates for about 60 per cent of the data and between ¼ and 4 times the observed rates for about 80 per cent of the data.

12. The theories can be divided into groups according to their general performance as follows:-

Group A:

Equations with the highest percentage of data with mean discrepancy ratios in the range ½ to 2, say about 50 per cent and with little scatter within the sets.

Group B:

Equations with 35 to 50 per cent of the mean discrepancy ratios in the range ½ to 2 with little scatter within the sets.

Group C:

Equations with a similar percentage of data in the range ½ to 2 as for Group B but with significant scatter within the data sets (indicating some deficiency in the form of the equation ).

Group D:

All other methods.

13. The theories fall into the above groupings as follows:-

Group A:

ACKERS and WHITE (1972)
ENGELUND and HANSEN (1967)
ROTTNER (1959)

Group B:

EINSTEIN (Total load, 1950)
MODIFIED BISHOP, SIMONS and RICHARDSON (1973)
TOFFALETI (1968)
EINSTEIN (Bed load, 1950)

Group C:

LAURSEN (1958)
GRAF (1968)

Group D:

ALL OTHER THEORIES INCLUDED IN THIS REVIEW

14. Two theories which have a limited range of application but which are reliable within these limits are:-

MODIFIED BISHOP, SIMONS and RICHARDSON (1973) $D_{gr} > 2$

$\xi\psi'_{35} > 1$

$\xi\psi'_{35}B_* < 3$

BAGNOLD (Total load, 1966) $Y > 0.4$

$2 < D_{gr} < 20$

Full fall velocity

The modified Bishop, Simons and Richardson approach eliminated 33 per cent of the data. The Bagnold method could not be applied to 69 per cent of the data.

15. The most reliable equation, applicable over a wide range of flow conditions and particle characteristics, is the general function of Ackers and White, see Fig 24. Some 64 per cent of the data was in the error band $\frac{1}{2} < X_{CALC}/X_{ACT} < 2$, see Fig 34 and some 46 per cent of the data was within the narrower limits $2/3 < X_{CALC}/X_{ACT} < 3/2$, see Fig 35. The Engelund and Hansen method, Fig 21, was also very reliable but errors were marginally higher. The Rottner bed load equation can be used with confidence as a total load theory, see Fig 18.

16. The behaviour of several of the theories which are based on excess shear velocities or excess mobility numbers together with numerous observations of sediment movement for coarse sediments apparently below the threshold conditions defined by Shields shows a clear need for further work in this area. Threshold conditions for coarse uniform and non-uniform sediments should be studied.

17. It would also be useful to examine the possibility of defining a significant diameter within a mixture in terms of the flow and sediment characteristics.

18. A summary of the theories which have been examined in this review and a list of those theories which were left out are presented in Vol 1.

## ACKNOWLEDGEMENTS

This is the third report on a continuing investigation into the movement of sediment by fluid flow. The work was carried out in Mr A J M Harrison's Fluvial Hydraulics Division by Dr W R White's section.

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**TABLES**

TABLE 1

Summary of flume experiments

| Investigator(s) | Date | Data Ref | Sediment | | | | No of tests in set |
|---|---|---|---|---|---|---|---|
| | | | Material | Size (mm) | Specific gravity | $D_{gr}$ | |
| **Quartz materials** | | | | | | | |
| Laursen | 1957 | 34 | Sand | 0.04 | 2.65 | 1.05 | 8 |
| Laursen | 1957 | 33 | Sand | 0.10 | 2.65 | 2.55 | 15 |
| Willis, Coleman, Ellis | 1972 | 56 | Sand | 0.10 | 2.65 | 2.57 | 28 |
| Willis, Coleman, Ellis | 1972 | 57 | Sand | 0.10 | 2.65 | 2.65 | 22 |
| Brooks | 1955 | 24 | Sand | 0.09 | 2.65 | 2.70 | 10 |
| Willis, Coleman, Ellis | 1972 | 58 | Sand | 0.10 | 2.65 | 2.76 | 28 |
| Kennedy and Brooks | 1965 | 20 | Sand | 0.14 | 2.65 | 3.70 | 8 |
| Vanoni and Brooks | 1957 | 21 | Sand | 0.14 | 2.65 | 3.74 | 14 |
| Nomicos | 1956 | 22 | Sand | 0.15 | 2.65 | 4.10 | 11 |
| Brooks | 1955 | 23 | Sand | 0.16 | 2.65 | 4.29 | 8 |
| Guy, Simons and Richardson | 1966 | 1 | Sand | 0.19 | 2.65 | 4.63 | 23 |
| Barton and Lin | 1955 | 25 | Sand | 0.18 | 2.65 | 4.70 | 29 |
| U.S.W.E.S. | 1935 | 27 | Sand | 0.21 | 2.65 | 5.10 | 14 |
| H.R.S. | 1972 | 55 | Sand | 0.25 | 2.65 | 6.30 | 7 |
| Guy, Simons and Richardson | 1966 | 2 | Sand | 0.27 | 2.65 | 6.50 | 13 |
| Guy, Simons and Richardson | 1966 | 3 | Sand | 0.28 | 2.65 | 6.50 | 24 |
| Gilbert and Murphy | 1914 | 14 | Sand | 0.30 | 2.65 | 6.95 | 6 |
| Guy, Simons and Richardson | 1966 | 6 | Sand | 0.32 | 2.65 | 8.30 | 16 |
| Guy, Simons and Richardson | 1966 | 7 | Sand | 0.33 | 2.65 | 8.30 | 6 |
| Gilbert and Murphy | 1914 | 15 | Sand | 0.37 | 2.65 | 8.60 | 28 |
| Guy, Simons and Richardson | 1966 | 8 | Sand | 0.33 | 2.65 | 8.80 | 10 |
| U.S.W.E.S. | 1935 | 28 | Sand | 0.31 | 2.65 | 8.80 | 11 |
| U.S.W.E.S. | 1935 | 29 | Sand | 0.35 | 2.65 | 8.95 | 26 |
| Guy, Simons and Richardson | 1966 | 4 | Sand | 0.45 | 2.65 | 10.10 | 19 |
| Stein | 1965 | 11 | Sand | 0.40 | 2.65 | 10.54 | 32 |
| U.S.W.E.S. | 1935 | 30 | Sand | 0.48 | 2.65 | 11.25 | 19 |
| Guy, Simons and Richardson | 1966 | 9 | Sand | 0.47 | 2.65 | 11.30 | 32 |
| Gilbert and Murphy | 1914 | 16 | Sand | 0.51 | 2.65 | 11.70 | 25 |
| U.S.W.E.S. | 1935 | 31 | Sand | 0.51 | 2.65 | 11.95 | 14 |
| Pratt | 1970 | 38 | Sand | 0.49 | 2.65 | 12.40 | 16 |
| U.S.W.E.S. | 1935 | 32 | Sand | 0.52 | 2.65 | 12.55 | 22 |
| Guy, Simons and Richardson | 1966 | 10 | Sand | 0.54 | 2.65 | 13.70 | 17 |
| Pratt | 1970 | 39 | Sand | 0.49 | 2.65 | 13.90 | 18 |
| Gilbert and Murphy | 1914 | 17 | Sand | 0.79 | 2.65 | 18.50 | 16 |
| Guy, Simons and Richardson | 1966 | 5 | Sand | 0.93 | 2.65 | 22.90 | 25 |
| Williams | 1970 | 121 | Sand | 1.35 | 2.65 | 31.80 | 25 |
| Williams | 1970 | 122 | Sand | 1.35 | 2.65 | 33.70 | 20 |
| Williams | 1970 | 123 | Sand | 1.35 | 2.65 | 35.40 | 22 |
| Williams | 1970 | 124 | Sand | 1.35 | 2.65 | 37.00 | 12 |
| Williams | 1970 | 125 | Sand | 1.35 | 2.65 | 38.10 | 19 |
| Liu | 1937 | 37 | Sand | 1.40 | 2.65 | 38.50 | 23 |
| Gilbert and Murphy | 1914 | 18 | Sand | 1.71 | 2.65 | 39.50 | 15 |
| Casey | 1935 | 49 | Sand | 2.45 | 2.70 | 58.33 | 28 |
| Liu | 1937 | 36 | Sand | 2.30 | 2.65 | 66.28 | 23 |
| Liu | 1937 | 35 | Sand | 3.41 | 2.65 | 98.52 | 15 |
| U.S.W.E.S. | 1935 | 26 | Sand | 4.08 | 2.65 | 100.35 | 10 |
| Gilbert and Murphy | 1914 | 19 | Sand | 4.94 | 2.65 | 113.27 | 5 |
| **Lightweight materials** | | | | | | | |
| U.S.W.E.S. | 1936 | 48 | Gilsonite | 0.90 | 1.07 | 6.60 | 15 |
| U.S.W.E.S. | 1936 | 45 | Coal | 1.10 | 1.35 | 14.40 | 17 |
| U.S.W.E.S. | 1936 | 43 | Haydite | 0.91 | 1.85 | 20.10 | 28 |
| U.S.W.E.S. | 1936 | 47 | Coal | 1.48 | 1.32 | 21.90 | 18 |
| U.S.W.E.S. | 1936 | 40 | Haydite | 1.07 | 1.85 | 23.50 | 25 |
| Franco | 1968 | 53 | Coal | 2.20 | 1.30 | 23.63 | 4 |
| U.S.W.E.S. | 1936 | 44 | Haydite | 1.33 | 1.74 | 26.10 | 27 |
| U.S.W.E.S. | 1936 | 42 | Gilsonite | 3.55 | 1.07 | 27.60 | 14 |
| Franco | 1968 | 54 | Coal | 2.20 | 1.30 | 31.50 | 4 |
| U.S.W.E.S. | 1936 | 46 | Coal | 3.20 | 1.32 | 48.00 | 15 |
| U.S.W.E.S. | 1936 | 41 | Coal | 3.30 | 1.35 | 53.80 | 14 |

TABLE 2

Summary of field data

| Location of measurements | Date | $D_{gr}$ | Concentration X (ppm) | Mobility No, Y | Relative grain size, Z | Breadth/Depth | Data Ref | Sediment Size (mm) | Sediment S.G. | No of tests in set | Temp ($^oC$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Atchafalaya River, Simmersport (a) | 1960-64 | 2.75 | $4<X<42$ | $0.43<Y<0.48$ | $10^4<Z<10^5$ | 45- 48 | 119 | 0.095 | 2.65 | 5 | 28.1 |
| | | 3.50 | $3<X<132$ | $0.15<Y<2.00$ | $10^4<Z<10^5$ | 36- 50 | 118 | 0.125 | 2.65 | 15 | 27.34 |
| 2 Mississippi River, Tarbert Landing (a) | 1964-65 | 3.91 | $28<X<238$ | $0.90<Y<1.60$ | $10^4<Z<10^5$ | 75-100 | 116 | 0.195 | 2.65 | 8 | 7.63 |
| | | 4.71 | $65<X<288$ | $1.27<Y<1.70$ | $10^4<Z<10^5$ | 75-100 | 115 | 0.195 | 2.65 | 6 | 17.8 |
| | | 5.68 | $53<X<122$ | $0.70<Y<1.29$ | $10^4<Z<10^5$ | 100-135 | 117 | 0.195 | 2.65 | 5 | 30.0 |
| 3 Paraguay River, km 385 (c) | 1972 | 6.10 | $9.5<X<29$ | $0.50<Y<1.00$ | $10^4<Z<10^5$ | 30- 70 | 153 | 0.240 | 2.65 | 4 | 20.0 |
| 4 Niobrara River, Cody, Nebraska Total load = Susp. load (b) | 1949-53 | 5.81 | $1299<X<1931$ | $1.40<Y<2.30$ | $10^3<Z<10^4$ | 40- 55 | 108 | 0.275 | 2.65 | 11 | 9.79 |
| | | 6.63 | $330<X<900$ | $1.07<Y<1.40$ | $10^3<Z<10^4$ | 40- 50 | 107 | 0.275 | 2.65 | 11 | 17.49 |
| 5 Goose Creek River, Oxford, Miss. Total load = Bed load (f) | 1942 | 7.01 | $355<X<2548$ | $0.26<Y<1.15$ | $10^2<Z<10^3$ | 20-100 | 112 | 0.280 | 2.65 | 19 | 20.0 |
| 6 Middle Loup River, Dunning, Nebraska Total load = Susp. load (d) | 1949-52 | 7.18 | $750<X<1980$ | $0.40<Y<0.70$ | $10^2<Z\leq10^4$ | 100-150 | 102 | 0.340 | 2.65 | 8 | 9.98 |
| | | 7.62 | $590<X<1690$ | $0.60<Y\leq0.70$ | $10^3$ | 100-150 | 105 | 0.340 | 2.65 | 3 | 13.06 |
| | | 8.00 | $960<X<2270$ | $0.40<Y<0.80$ | $10^3$ | 130-160 | 103 | 0.340 | 2.65 | 7 | 16.02 |
| | | 8.54 | $880<X<440$ | $0.70<Y<1.50$ | $10^3<Z\leq10^4$ | 40- 50 | 104 | 0.340 | 2.65 | 4 | 22.41 |
| | | 8.72 | $480<X<1380$ | $0.65<Y\leq0.75$ | $10^3$ | 120-150 | 101 | 0.340 | 2.65 | 5 | 20.94 |
| 7 Skive-Karup River, Total load = Bed load (f) | 1965 | 9.99 | 28.3 | 0.358 | 1723 | 16.67 | 106 | 0.470 | 2.68 | 1 | 10.0 |
| 8 Mississippi River, St Louis (a) | 1951-62 | 14.83 | $15<X<370$ | $0.40<Y\leq1.58$ | $10^4<Z<10^5$ | 40- 70 | 151 | 0.590 | 2.65 | 9 | 20.3 |
| | | 16.37 | $19<X<210$ | $0.24<Y\leq2.40$ | $10^4<Z<10^5$ | 30- 70 | 150 | 0.590 | 2.65 | 12 | 26.7 |
| 9 Mountain Creek, Greenville Total load = Bed load (e) | 1941 | 22.97 | $4<X<326$ | $0.059<Y<0.23$ | $10^2<Z<300$ | 30-150 | 111 | 0.900 | 2.65 | 21 | 19.21 |
| | | 24.25 | $259<X<665$ | $0.046<Y<0.20$ | $10<Z<200$ | 30- 90 | 111 | 0.900 | 2.65 | 21 | 24.47 |
| | | 24.60 | $154<X<248$ | $0.97<Y\leq0.18$ | $100<Z<200$ | 30- 50 | 111 | 0.900 | 2.65 | 20 | 25.50 |
| 10 Elbow River, Bragg Creek Total load = Bed load (g) | 1967-68 | 687.2 | $1.6<X<411$ | $0.04<Y<0.10$ | $20<Z<30$ | 50- 65 | 109 | 29.72 | 2.64 | 24 | 15.00 |
| 11 Aare River, Brienzwiler Total load = Bed load (g) | 1939 | 1013 | $21<X<102$ | $0.04<Y\leq0.06$ | $35<Z<50$ | 9- 13 | 113 | 47.50 | 2.70 | 23 | 10.00 |
| | | 1450 | $20<X<738$ | $0.029<Y<0.035$ | $19<Z<25$ | 10- 15 | 114 | 68.00 | 2.70 | 20 | 10.00 |

Footnotes:-

(a) Average of measured sand concentration applied to the discharge in the measured zone, plus the computed sand discharge for the unmeasured zone, Ref 21.
(b) Measured suspended sediment at contracted section based on depth integrated samples, Ref 1.
(c) Measured suspended sediment with point integrated sampler plus bed load computed from observed dune movement.
(d) Measured suspended sediment in a special flume based on depth integrated samples, Ref 2.
(e) Special measuring device for bed-load, Ref 7.
(f) Measured bed load with sampler developed at the Danish Health Society, Ref 8.
(g) Measured bed load with special sampler, Refs 4 and 5.

TABLE 3

FREQUENCY DISTRIBUTION OF MOBILITY NUMBER (Y)

| Y DGR | $Y_C$ | 0.1 | 0.4 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | MOBILITY NUMBER FREQUENCY DISTRIBUTION IN (%) | | | | | | | | | |
| Flume data, quartz material | | | | | | | | | | |
| 1.05 | 0 | 0 | 0 | 0 | 0 | 12 | 12 | 12 | 50 | 12 |
| 2.55 | 0 | 0 | 0 | 0 | 20 | 73 | 7 | 0 | 0 | 0 |
| 2.57 | 0 | 0 | 0 | 4 | 25 | 46 | 21 | 0 | 4 | 0 |
| 2.65 | 0 | 0 | 0 | 0 | 23 | 45 | 18 | 9 | 0 | 5 |
| 2.70 | 0 | 0 | 0 | 0 | 20 | 80 | 0 | 0 | 0 | 0 |
| 2.76 | 0 | 0 | 0 | 0 | 25 | 57 | 11 | 4 | 0 | 4 |
| 3.70 | 0 | 0 | 0 | 12 | 75 | 12 | 0 | 0 | 0 | 0 |
| 3.74 | 0 | 0 | 7 | 14 | 79 | 0 | 0 | 0 | 0 | 0 |
| 4.10 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 4.29 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 4.63 | 0 | 0 | 13 | 17 | 57 | 13 | 0 | 0 | 0 | 0 |
| 4.70 | 0 | 0 | 7 | 10 | 66 | 17 | 0 | 0 | 0 | 0 |
| 5.10 | 0 | 0 | 21 | 14 | 64 | 0 | 0 | 0 | 0 | 0 |
| 6.30 | 0 | 0 | 29 | 29 | 43 | 0 | 0 | 0 | 0 | 0 |
| 6.50 | 0 | 0 | 8 | 15 | 54 | 23 | 0 | 0 | 0 | 0 |
| 6.50 | 0 | 0 | 17 | 17 | 67 | 0 | 0 | 0 | 0 | 0 |
| 6.95 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.30 | 0 | 0 | 25 | 19 | 50 | 6 | 0 | 0 | 0 | 0 |
| 8.30 | 0 | 0 | 50 | 0 | 50 | 0 | 0 | 0 | 0 | 0 |
| 8.60 | 0 | 0 | 61 | 32 | 4 | 4 | 0 | 0 | 0 | 0 |
| 8.80 | 0 | 10 | 50 | 20 | 10 | 10 | 0 | 0 | 0 | 0 |
| 8.80 | 0 | 0 | 91 | 9 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.95 | 0 | 0 | 96 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10.10 | 0 | 5 | 58 | 16 | 21 | 0 | 0 | 0 | 0 | 0 |
| 10.54 | 0 | 0 | 3 | 0 | 37 | 56 | 3 | 0 | 0 | 0 |
| 11.25 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11.30 | 0 | 0 | 34 | 6 | 59 | 0 | 0 | 0 | 0 | 0 |
| 11.70 | 0 | 0 | 44 | 24 | 32 | 0 | 0 | 0 | 0 | 0 |
| 11.95 | 0 | 14 | 86 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12.40 | 0 | 0 | 87 | 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12.55 | 0 | 9 | 91 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13.70 | 0 | 6 | 0 | 12 | 53 | 29 | 0 | 0 | 0 | 0 |
| 13.90 | 0 | 11 | 61 | 6 | 22 | 0 | 0 | 0 | 0 | 0 |
| 18.50 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22.90 | 0 | 28 | 36 | 4 | 32 | 0 | 0 | 0 | 0 | 0 |
| 31.80 | 0 | 36 | 52 | 4 | 8 | 0 | 0 | 0 | 0 | 0 |
| 33.70 | 0 | 45 | 45 | 5 | 5 | 0 | 0 | 0 | 0 | 0 |
| 35.40 | 0 | 27 | 64 | 5 | 5 | 0 | 0 | 0 | 0 | 0 |
| 37.00 | 0 | 42 | 50 | 0 | 8 | 0 | 0 | 0 | 0 | 0 |
| 38.10 | 0 | 32 | 47 | 5 | 16 | 0 | 0 | 0 | 0 | 0 |
| 38.50 | 4 | 96 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 39.50 | 0 | 27 | 73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 58.33 | 32 | 68 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 66.28 | 61 | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 98.52 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 100.35 | 50 | 50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 113.27 | 0 | 40 | 60 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Flume data, lightweight material | | | | | | | | | | |
| 6.60 | 0 | 13 | 80 | 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14.40 | 0 | 35 | 65 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20.10 | 0 | 36 | 64 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21.90 | 0 | 50 | 50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23.50 | 0 | 40 | 60 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23.63 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26.10 | 0 | 67 | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27.57 | 14 | 64 | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31.50 | 0 | 75 | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48.02 | 0 | 67 | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 53.84 | 0 | 71 | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Field data, quartz material | | | | | | | | | | |
| 2.75 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3.50 | 0 | 0 | 53 | 13 | 7 | 7 | 20 | 0 | 0 | 0 |
| 3.91 | 0 | 0 | 0 | 0 | 12 | 75 | 12 | 0 | 0 | 0 |
| 4.71 | 0 | 0 | 0 | 0 | 0 | 33 | 67 | 0 | 0 | 0 |
| 6.10 | 0 | 0 | 0 | 25 | 75 | 0 | 0 | 0 | 0 | 0 |
| 5.68 | 0 | 0 | 0 | 0 | 40 | 60 | 0 | 0 | 0 | 0 |
| 5.81 | 0 | 0 | 0 | 0 | 0 | 45 | 45 | 9 | 0 | 0 |
| 6.63 | 0 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 7.01 | 0 | 0 | 63 | 11 | 21 | 5 | 0 | 0 | 0 | 0 |
| 7.18 | 0 | 0 | 0 | 37 | 63 | 0 | 0 | 0 | 0 | 0 |
| 7.62 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 8.00 | 0 | 0 | 0 | 14 | 86 | 0 | 0 | 0 | 0 | 0 |
| 8.54 | 0 | 0 | 0 | 0 | 25 | 75 | 0 | 0 | 0 | 0 |
| 8.72 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 9.99 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14.83 | 0 | 0 | 11 | 22 | 44 | 11 | 11 | 0 | 0 | 0 |
| 16.37 | 0 | 0 | 42 | 25 | 8 | 8 | 0 | 17 | 0 | 0 |
| 22.97 | 0 | 48 | 52 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24.25 | 0 | 38 | 62 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24.60 | 0 | 30 | 70 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 687.19 | 29 | 71 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1013.17 | 87 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1450.44 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE 4

FREQUENCY DISTRIBUTION RATIO BREADTH/DEPTH

| B/Y | 5.0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | |
|---|---|---|---|---|---|---|---|---|---|---|
| DGR | RATIO BREADTH/DEPTH FREQUENCY DISTRIBUTION IN (%) | | | | | | | | | |
| Flume data, quartz material | | | | | | | | | | |
| 1.05 | 25 | 75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.55 | 47 | 53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.57 | 54 | 46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.65 | 55 | 45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.70 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.76 | 75 | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3.70 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3.74 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.10 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.29 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.63 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.70 | 10 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5.10 | 36 | 64 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6.30 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6.50 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6.50 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6.95 | 17 | 83 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.30 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.30 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.60 | 64 | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.80 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.80 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.95 | 8 | 92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10.10 | 0 | 84 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10.54 | 72 | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11.25 | 21 | 79 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11.30 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11.70 | 64 | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11.95 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12.40 | 6 | 94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12.55 | 14 | 86 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13.70 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13.90 | 83 | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18.50 | 94 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22.90 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31.80 | 92 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 33.70 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 35.40 | 82 | 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 37.00 | 50 | 42 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 38.10 | 89 | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 38.50 | 0 | 91 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 39.50 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 58.33 | 79 | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 66.28 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 98.52 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 100.35 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 113.27 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Flume data, lightweight material | | | | | | | | | | |
| 6.60 | 60 | 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14.40 | 65 | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20.10 | 82 | 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21.90 | 61 | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23.50 | 88 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23.63 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26.10 | 74 | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27.57 | 79 | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31.50 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48.02 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 53.84 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Field data, quartz material | | | | | | | | | | |
| 2.75 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3.50 | 0 | 0 | 27 | 73 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3.91 | 0 | 0 | 0 | 0 | 12 | 87 | 0 | 0 | 0 | 0 |
| 4.71 | 0 | 0 | 0 | 0 | 33 | 67 | 0 | 0 | 0 | 0 |
| 6.10 | 0 | 0 | 25 | 25 | 50 | 0 | 0 | 0 | 0 | 0 |
| 5.68 | 0 | 0 | 0 | 0 | 0 | 0 | 60 | 40 | 0 | 0 |
| 5.81 | 0 | 0 | 9 | 91 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6.63 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7.01 | 0 | 0 | 11 | 21 | 37 | 26 | 5 | 0 | 0 | 0 |
| 7.18 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | 63 | 25 | 0 |
| 7.62 | 0 | 0 | 0 | 0 | 0 | 0 | 33 | 33 | 33 | 0 |
| 8.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 57 | 29 | 14 |
| 8.54 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8.72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 80 | 20 | 0 |
| 9.99 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14.83 | 0 | 0 | 11 | 78 | 11 | 0 | 0 | 0 | 0 | 0 |
| 16.37 | 0 | 0 | 25 | 50 | 25 | 0 | 0 | 0 | 0 | 0 |
| 22.97 | 0 | 0 | 48 | 48 | 0 | 0 | 0 | 0 | 5 | 0 |
| 24.25 | 0 | 0 | 29 | 67 | 0 | 5 | 0 | 0 | 0 | 0 |
| 24.60 | 0 | 0 | 65 | 35 | 0 | 0 | 0 | 0 | 0 | 0 |
| 687.19 | 0 | 0 | 0 | 71 | 29 | 0 | 0 | 0 | 0 | 0 |
| 1013.17 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1450.44 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## TABLE 5

### FREQUENCY DISTRIBUTION OF CONCENTRATION BY WEIGHT

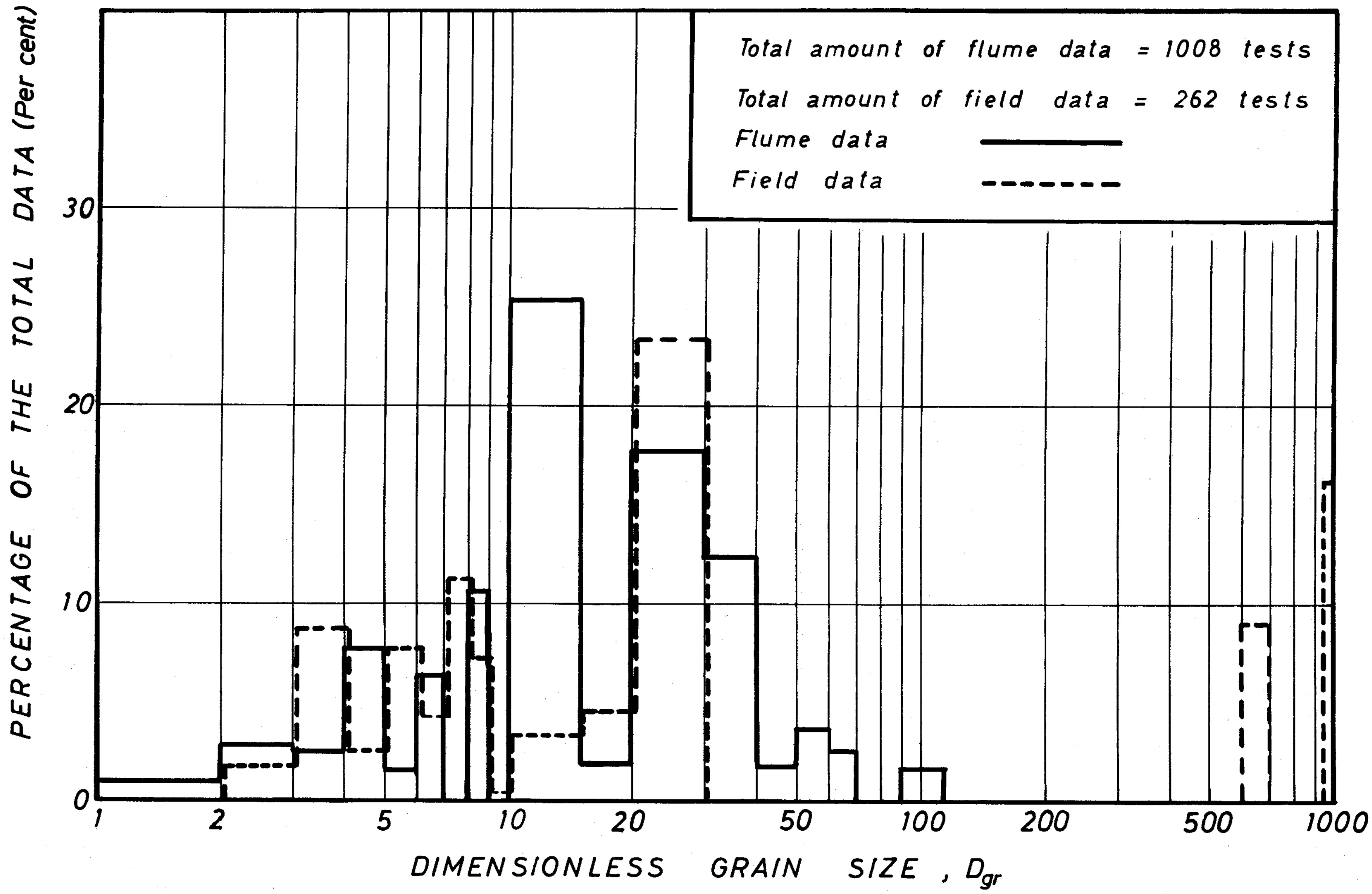
| X | | 10-1 | 10-2 | 10-3 | 10-4 | 10-5 | 10-6 |
|---|---|---|---|---|---|---|---|
| DGR | CONCENTRATION FREQUENCY DISTRIBUTION IN (%) | | | | | | |
| Flume data, quartz material | | | | | | | |
| 1.05 | 0 | 87 | 12 | 0 | 0 | 0 | 0 |
| 2.55 | 0 | 0 | 67 | 33 | 0 | 0 | 0 |
| 2.57 | 0 | 4 | 71 | 25 | 0 | 0 | 0 |
| 2.65 | 0 | 0 | 91 | 9 | 0 | 0 | 0 |
| 2.70 | 0 | 0 | 90 | 10 | 0 | 0 | 0 |
| 2.76 | 0 | 0 | 71 | 25 | 4 | 0 | 0 |
| 3.70 | 0 | 0 | 63 | 25 | 12 | 0 | 0 |
| 3.74 | 0 | 0 | 57 | 21 | 14 | 7 | 0 |
| 4.10 | 0 | 0 | 73 | 27 | 0 | 0 | 0 |
| 4.29 | 0 | 0 | 75 | 25 | 0 | 0 | 0 |
| 4.63 | 0 | 0 | 43 | 26 | 22 | 9 | 0 |
| 4.70 | 0 | 0 | 41 | 45 | 14 | 0 | 0 |
| 5.10 | 0 | 0 | 0 | 64 | 36 | 0 | 0 |
| 6.30 | 0 | 0 | 0 | 43 | 57 | 0 | 0 |
| 6.50 | 0 | 0 | 15 | 62 | 23 | 0 | 0 |
| 6.50 | 0 | 0 | 25 | 54 | 17 | 4 | 0 |
| 6.95 | 0 | 0 | 50 | 50 | 0 | 0 | 0 |
| 8.30 | 0 | 0 | 25 | 56 | 19 | 0 | 0 |
| 8.30 | 0 | 0 | 17 | 50 | 17 | 17 | 0 |
| 8.60 | 0 | 0 | 36 | 64 | 0 | 0 | 0 |
| 8.80 | 0 | 0 | 50 | 20 | 20 | 10 | 0 |
| 8.80 | 0 | 0 | 0 | 18 | 82 | 0 | 0 |
| 8.95 | 0 | 0 | 0 | 15 | 85 | 0 | 0 |
| 10.10 | 0 | 0 | 11 | 42 | 32 | 16 | 0 |
| 10.54 | 0 | 0 | 81 | 16 | 3 | 0 | 0 |
| 11.25 | 0 | 0 | 0 | 42 | 58 | 0 | 0 |
| 11.30 | 0 | 0 | 12 | 63 | 16 | 9 | 0 |
| 11.70 | 0 | 0 | 52 | 44 | 4 | 0 | 0 |
| 11.95 | 0 | 0 | 0 | 71 | 29 | 0 | 0 |
| 12.40 | 0 | 0 | 0 | 50 | 37 | 12 | 0 |
| 12.55 | 0 | 0 | 0 | 64 | 36 | 0 | 0 |
| 13.70 | 0 | 0 | 65 | 29 | 6 | 0 | 0 |
| 13.90 | 0 | 0 | 0 | 39 | 44 | 17 | 0 |
| 18.50 | 0 | 0 | 50 | 50 | 0 | 0 | 0 |
| 22.90 | 0 | 0 | 28 | 40 | 28 | 4 | 0 |
| 31.80 | 0 | 0 | 8 | 60 | 32 | 0 | 0 |
| 33.70 | 0 | 0 | 0 | 35 | 65 | 0 | 0 |
| 35.40 | 0 | 0 | 9 | 41 | 50 | 0 | 0 |
| 37.00 | 0 | 0 | 8 | 58 | 33 | 0 | 0 |
| 38.10 | 0 | 0 | 11 | 58 | 21 | 11 | 0 |
| 38.50 | 0 | 0 | 0 | 74 | 26 | 0 | 0 |
| 39.50 | 0 | 0 | 33 | 60 | 7 | 0 | 0 |
| 58.33 | 0 | 0 | 0 | 50 | 36 | 14 | 0 |
| 66.28 | 0 | 0 | 0 | 52 | 48 | 0 | 0 |
| 98.52 | 0 | 0 | 0 | 13 | 87 | 0 | 0 |
| 100.35 | 0 | 0 | 0 | 70 | 30 | 0 | 0 |
| 113.27 | 0 | 0 | 60 | 40 | 0 | 0 | 0 |
| Flume data, lightweight material | | | | | | | |
| 6.60 | 0 | 0 | 33 | 67 | 0 | 0 | 0 |
| 14.40 | 0 | 0 | 0 | 88 | 12 | 0 | 0 |
| 20.10 | 0 | 0 | 0 | 79 | 21 | 0 | 0 |
| 21.90 | 0 | 0 | 0 | 94 | 6 | 0 | 0 |
| 23.50 | 0 | 0 | 0 | 68 | 32 | 0 | 0 |
| 23.63 | 0 | 0 | 75 | 25 | 0 | 0 | 0 |
| 26.10 | 0 | 0 | 0 | 63 | 37 | 0 | 0 |
| 27.57 | 0 | 0 | 50 | 50 | 0 | 0 | 0 |
| 31.50 | 0 | 0 | 75 | 25 | 0 | 0 | 0 |
| 48.02 | 0 | 0 | 0 | 53 | 40 | 7 | 0 |
| 53.84 | 0 | 0 | 0 | 64 | 21 | 14 | 0 |
| Field data, quartz material | | | | | | | |
| 2.75 | 0 | 0 | 0 | 0 | 60 | 40 | 0 |
| 3.50 | 0 | 0 | 0 | 7 | 73 | 20 | 0 |
| 3.91 | 0 | 0 | 0 | 87 | 12 | 0 | 0 |
| 4.71 | 0 | 0 | 0 | 83 | 17 | 0 | 0 |
| 6.10 | 0 | 0 | 0 | 0 | 50 | 50 | 0 |
| 5.68 | 0 | 0 | 0 | 60 | 40 | 0 | 0 |
| 5.81 | 0 | 0 | 82 | 18 | 0 | 0 | 0 |
| 6.63 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 7.01 | 0 | 0 | 26 | 74 | 0 | 0 | 0 |
| 7.18 | 0 | 0 | 75 | 25 | 0 | 0 | 0 |
| 7.62 | 0 | 0 | 33 | 67 | 0 | 0 | 0 |
| 8.00 | 0 | 0 | 43 | 57 | 0 | 0 | 0 |
| 8.54 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 8.72 | 0 | 0 | 20 | 80 | 0 | 0 | 0 |
| 9.99 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 14.83 | 0 | 0 | 0 | 44 | 56 | 0 | 0 |
| 16.37 | 0 | 0 | 0 | 25 | 75 | 0 | 0 |
| 22.97 | 0 | 0 | 0 | 67 | 19 | 14 | 0 |
| 24.25 | 0 | 0 | 0 | 52 | 48 | 0 | 0 |
| 24.60 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 687.19 | 0 | 0 | 0 | 63 | 29 | 4 | 4 |
| 1013.17 | 0 | 0 | 0 | 17 | 83 | 0 | 0 |
| 1450.44 | 0 | 0 | 0 | 0 | 90 | 10 | 0 |

TABLE 6

FREQUENCY DISTRIBUTION OF Z

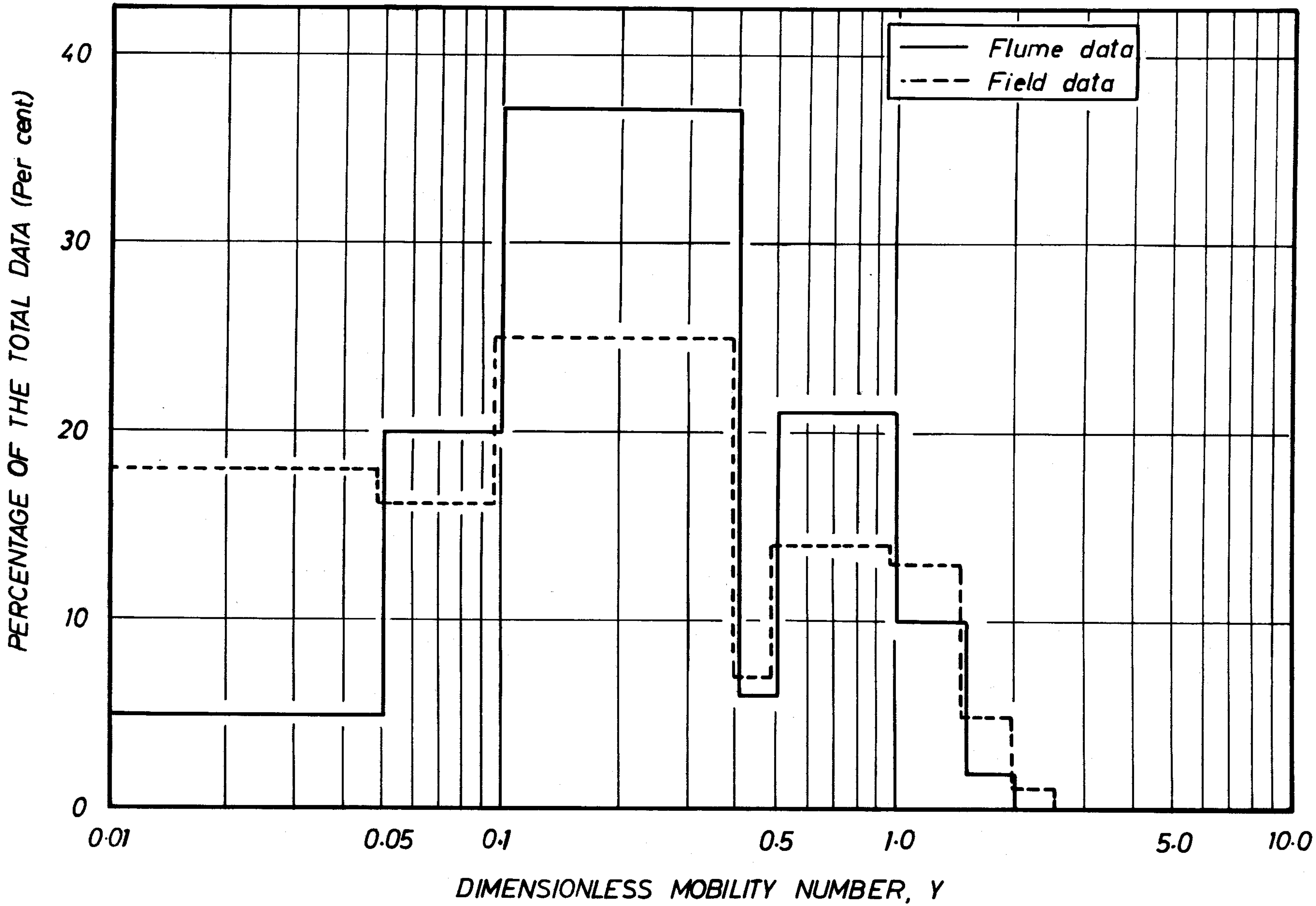
| Z | 10+1 | 10+2 | 10+3 | 10+4 | 10+5 | 10+6 | |
|---|---|---|---|---|---|---|---|
| DGR | RATIO DEPTH/GRAIN SIZE FREQUENCY DISTRIBUTION IN (%) | | | | | | |
| Flume data, quartz material | | | | | | | |
| 1.05 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 2.55 | 0 | 0 | 13 | 87 | 0 | 0 | 0 |
| 2.57 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 2.65 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 2.70 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 2.76 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 3.70 | 0 | 0 | 87 | 12 | 0 | 0 | 0 |
| 3.74 | 0 | 0 | 57 | 43 | 0 | 0 | 0 |
| 4.10 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 4.29 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 4.63 | 0 | 0 | 43 | 57 | 0 | 0 | 0 |
| 4.70 | 0 | 0 | 41 | 59 | 0 | 0 | 0 |
| 5.10 | 0 | 0 | 93 | 7 | 0 | 0 | 0 |
| 6.30 | 0 | 0 | 86 | 14 | 0 | 0 | 0 |
| 6.50 | 0 | 0 | 38 | 62 | 0 | 0 | 0 |
| 6.50 | 0 | 0 | 63 | 37 | 0 | 0 | 0 |
| 6.95 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 8.30 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 8.30 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 8.60 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 8.80 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 8.80 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 8.95 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 10.10 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 10.54 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 11.25 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 11.30 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 11.70 | 0 | 28 | 72 | 0 | 0 | 0 | 0 |
| 11.95 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 12.40 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 12.55 | 0 | 18 | 82 | 0 | 0 | 0 | 0 |
| 13.70 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 13.90 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 18.50 | 0 | 37 | 63 | 0 | 0 | 0 | 0 |
| 22.90 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 31.80 | 0 | 52 | 48 | 0 | 0 | 0 | 0 |
| 33.70 | 0 | 30 | 70 | 0 | 0 | 0 | 0 |
| 35.40 | 0 | 23 | 77 | 0 | 0 | 0 | 0 |
| 37.00 | 0 | 25 | 75 | 0 | 0 | 0 | 0 |
| 38.10 | 0 | 47 | 53 | 0 | 0 | 0 | 0 |
| 38.50 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 39.50 | 0 | 93 | 7 | 0 | 0 | 0 | 0 |
| 58.33 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 66.28 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 98.52 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 100.35 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 113.27 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| Flume data, lightweight material | | | | | | | |
| 6.60 | 0 | 67 | 33 | 0 | 0 | 0 | 0 |
| 14.40 | 0 | 82 | 18 | 0 | 0 | 0 | 0 |
| 20.10 | 0 | 43 | 57 | 0 | 0 | 0 | 0 |
| 21.90 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 23.50 | 0 | 52 | 48 | 0 | 0 | 0 | 0 |
| 23.63 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 26.10 | 0 | 78 | 22 | 0 | 0 | 0 | 0 |
| 27.57 | 0 | 93 | 0 | 0 | 0 | 0 | 0 |
| 31.50 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 48.02 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 53.84 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| Field data, quartz material | | | | | | | |
| 2.75 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 3.50 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 3.91 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 4.71 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 5.68 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 5.81 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 6.10 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 6.63 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 7.01 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 7.18 | 0 | 0 | 75 | 25 | 0 | 0 | 0 |
| 7.62 | 0 | 0 | 67 | 33 | 0 | 0 | 0 |
| 8.00 | 0 | 0 | 86 | 14 | 0 | 0 | 0 |
| 8.54 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 8.72 | 0 | 0 | 20 | 80 | 0 | 0 | 0 |
| 9.99 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 14.83 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 16.37 | 0 | 0 | 0 | 0 | 100 | 0 | 0 |
| 22.97 | 0 | 19 | 81 | 0 | 0 | 0 | 0 |
| 24.25 | 0 | 29 | 71 | 0 | 0 | 0 | 0 |
| 24.60 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 687.19 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 1013.17 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |
| 1450.44 | 0 | 100 | 0 | 0 | 0 | 0 | 0 |

**FIGURES**



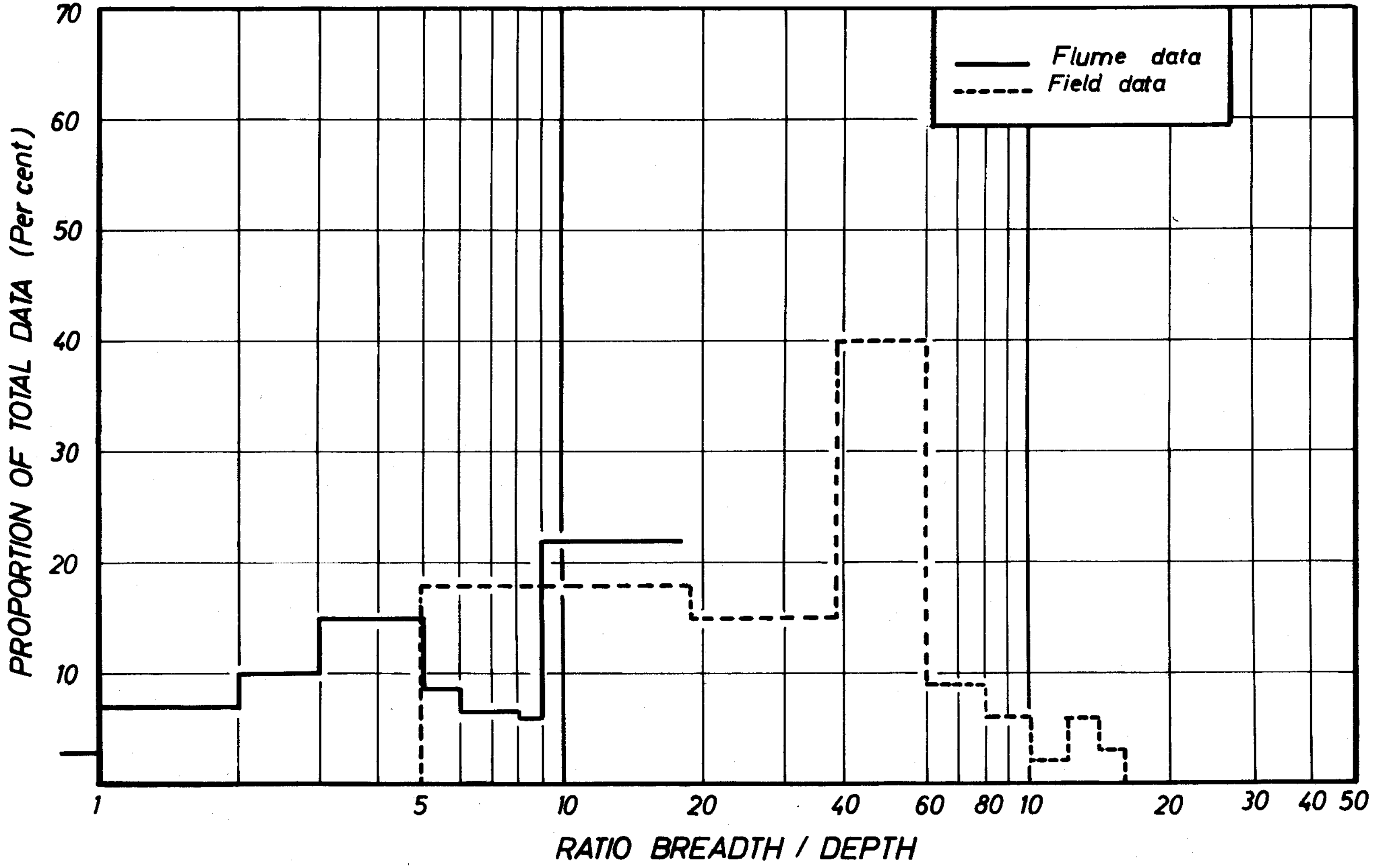

COMPARATIVE FREQUENCY DISTRIBUTIONS OF $D_{gr}$ FOR FLUME AND FIELD DATA

FIG 1



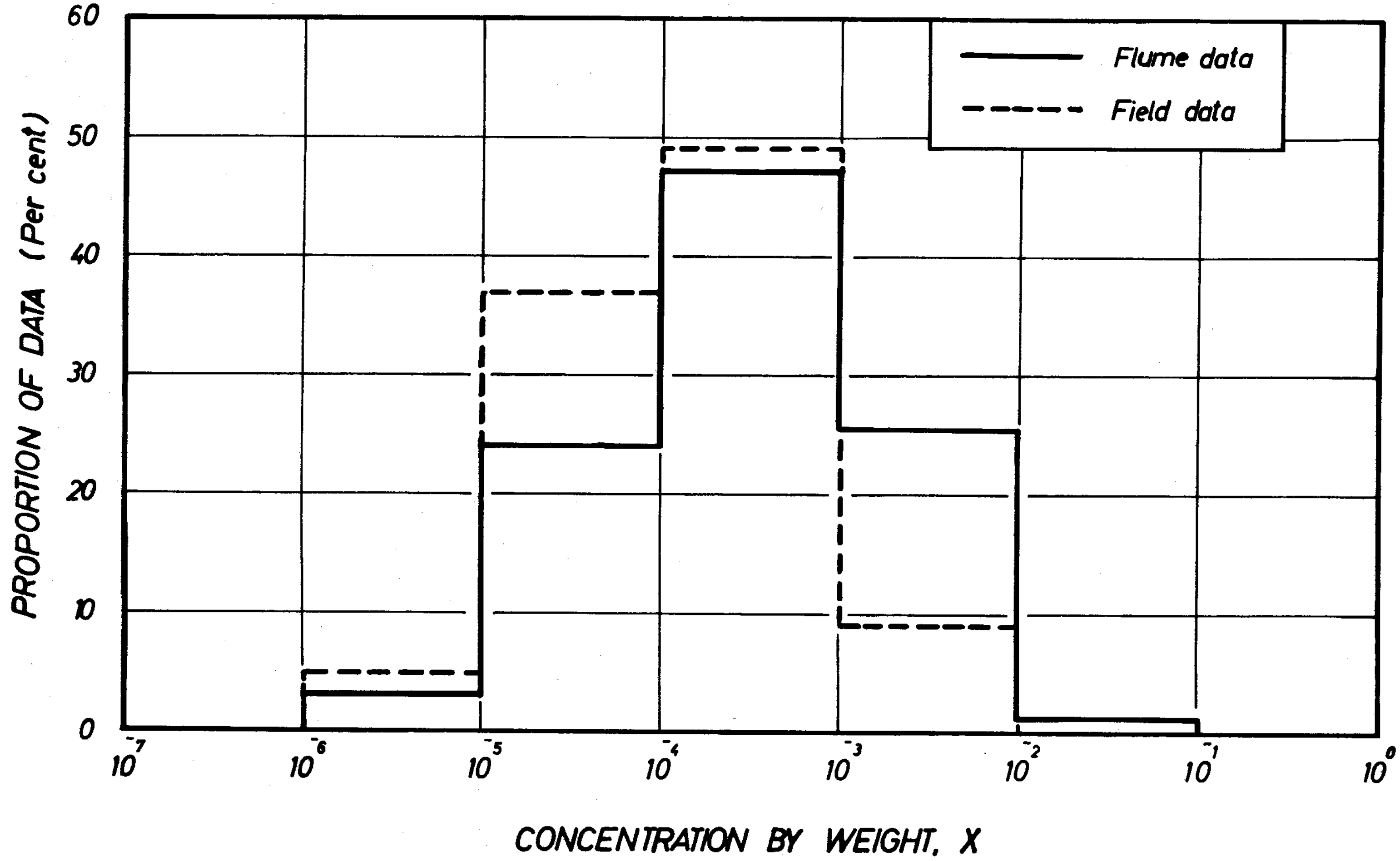

COMPARATIVE FREQUENCY DISTRIBUTIONS OF Y FOR FLUME AND FIELD DATA

FIG 2



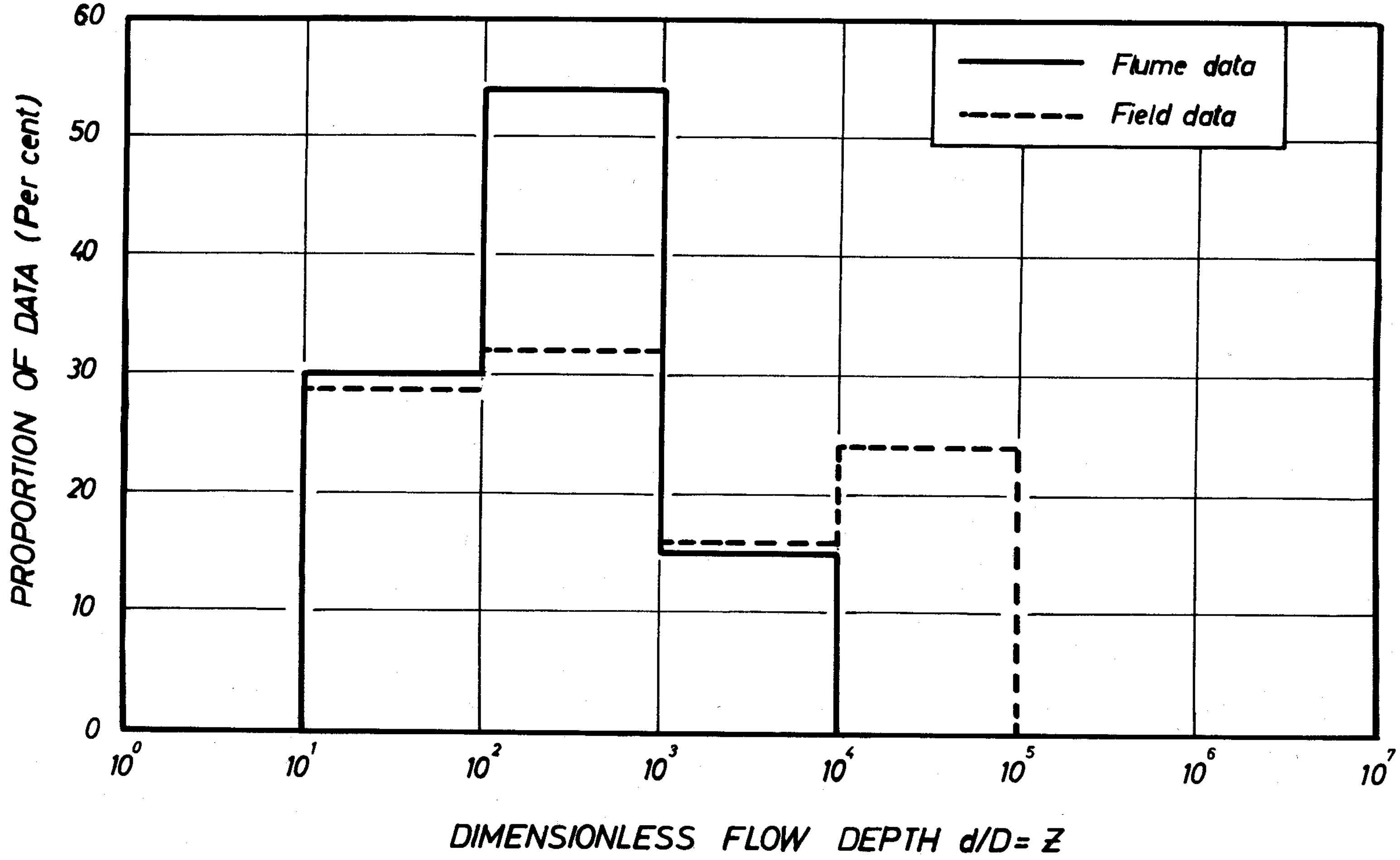

COMPARATIVE FREQUENCY DISTRIBUTIONS OF b/d FOR FLUME AND FIELD DATA

FIG 3



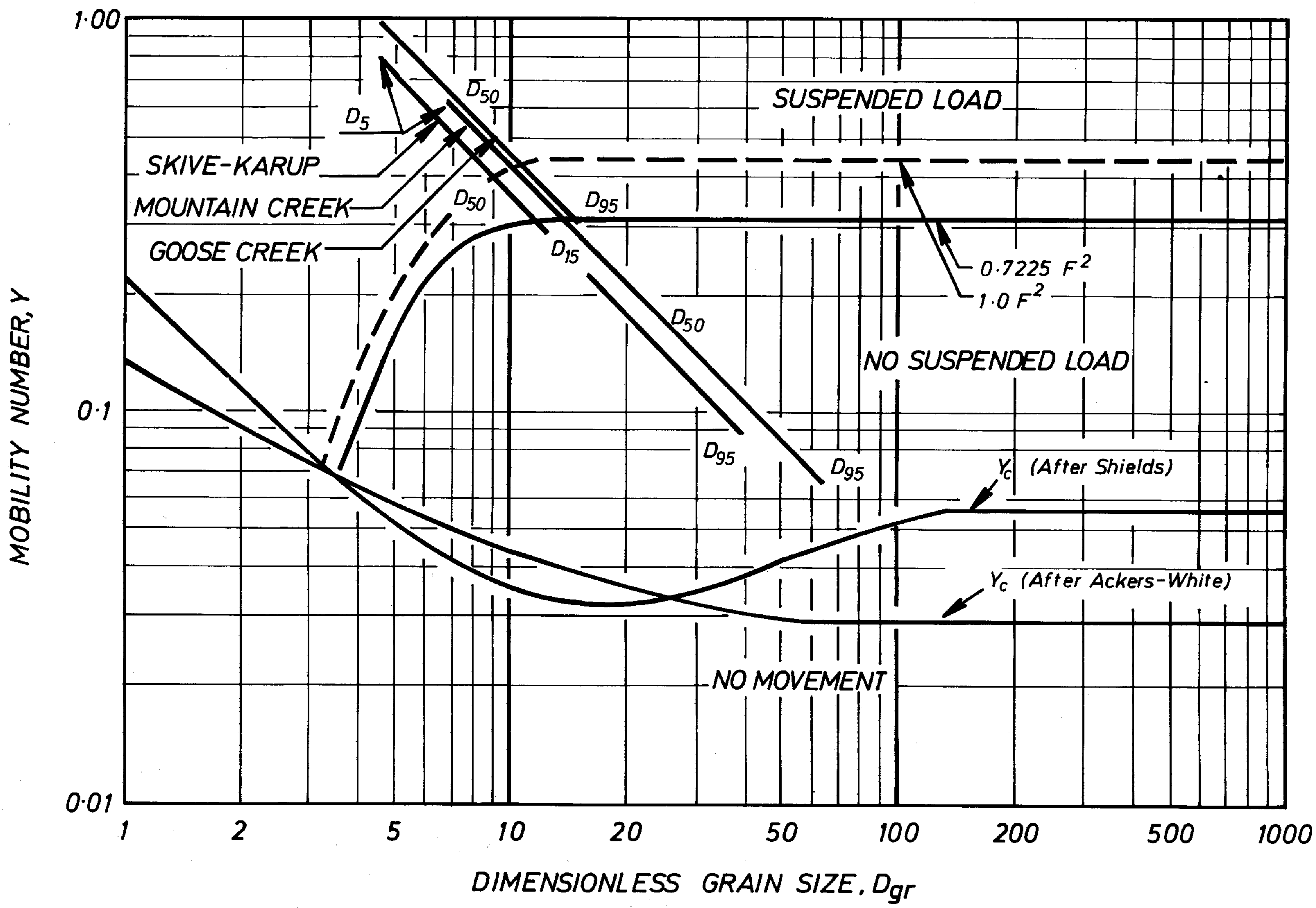

COMPARATIVE FREQUENCY DISTRIBUTIONS OF X FOR FLUME AND FIELD DATA

FIG 4



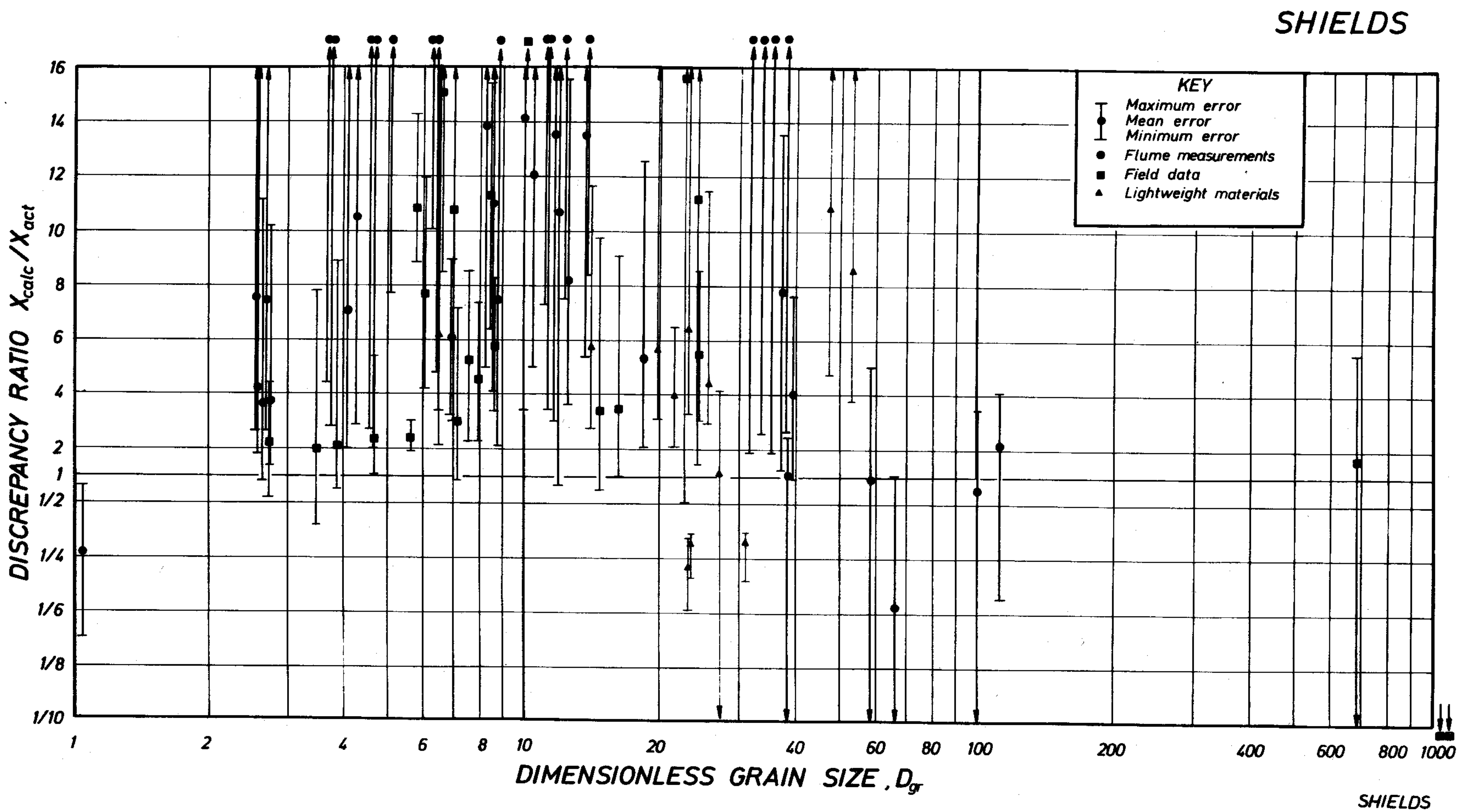

COMPARATIVE FREQUENCY DISTRIBUTIONS OF Z FOR FLUME AND FIELD DATA

FIG 5




CRITERIA FOR SUSPENSION OF THE SEDIMENT LOAD

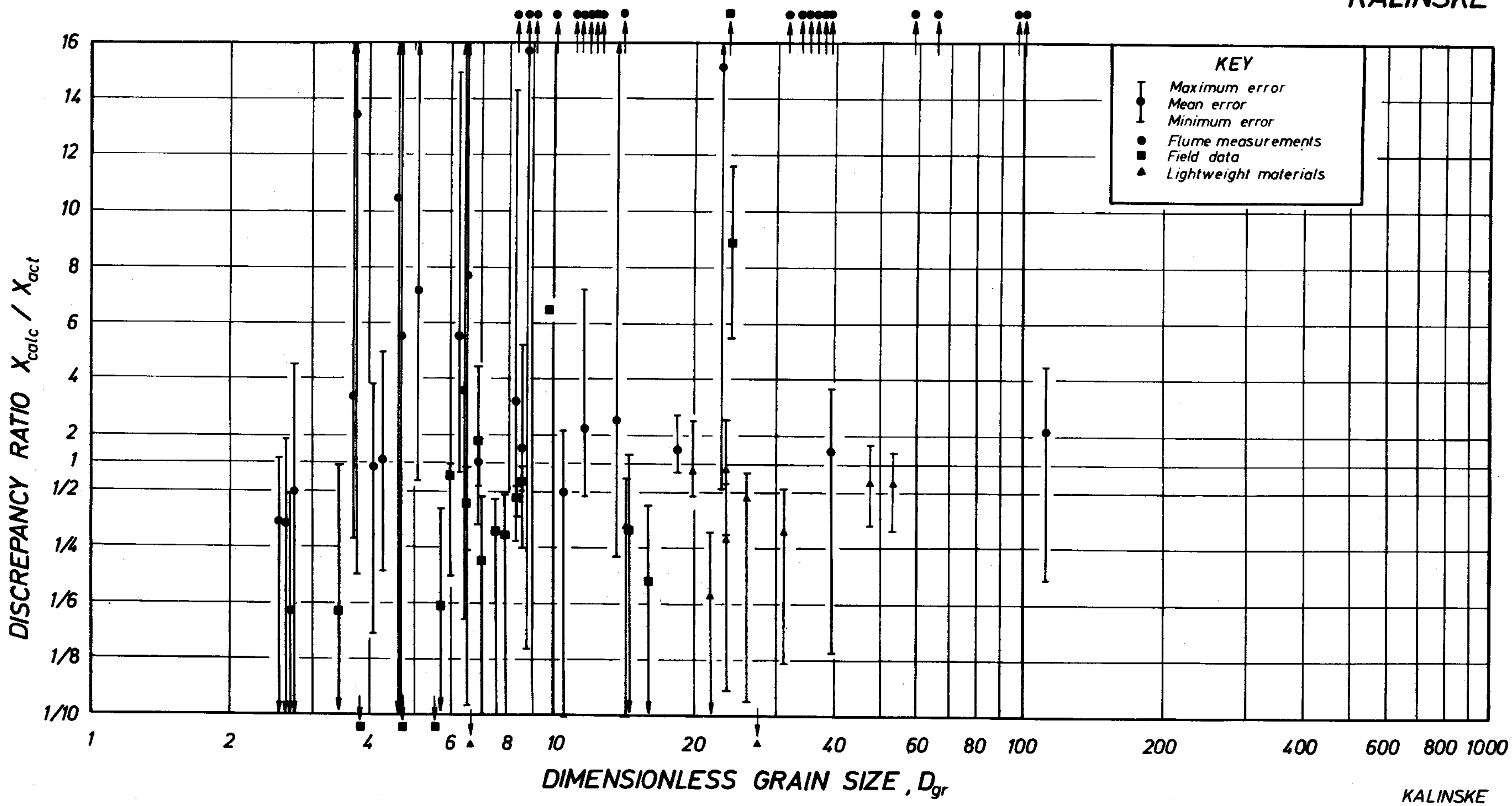
Fig 6




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 7

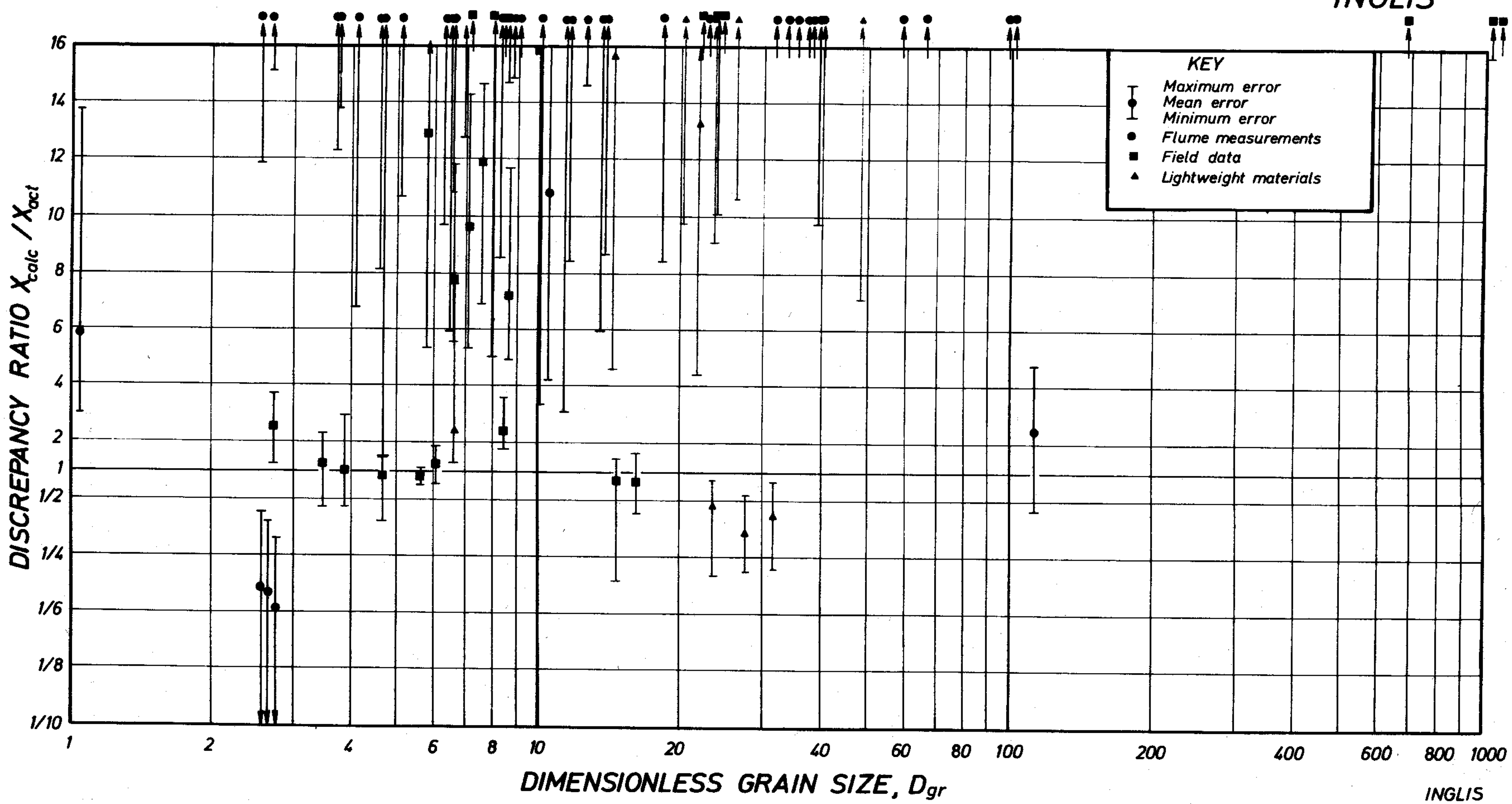
# KALINSKE




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 8

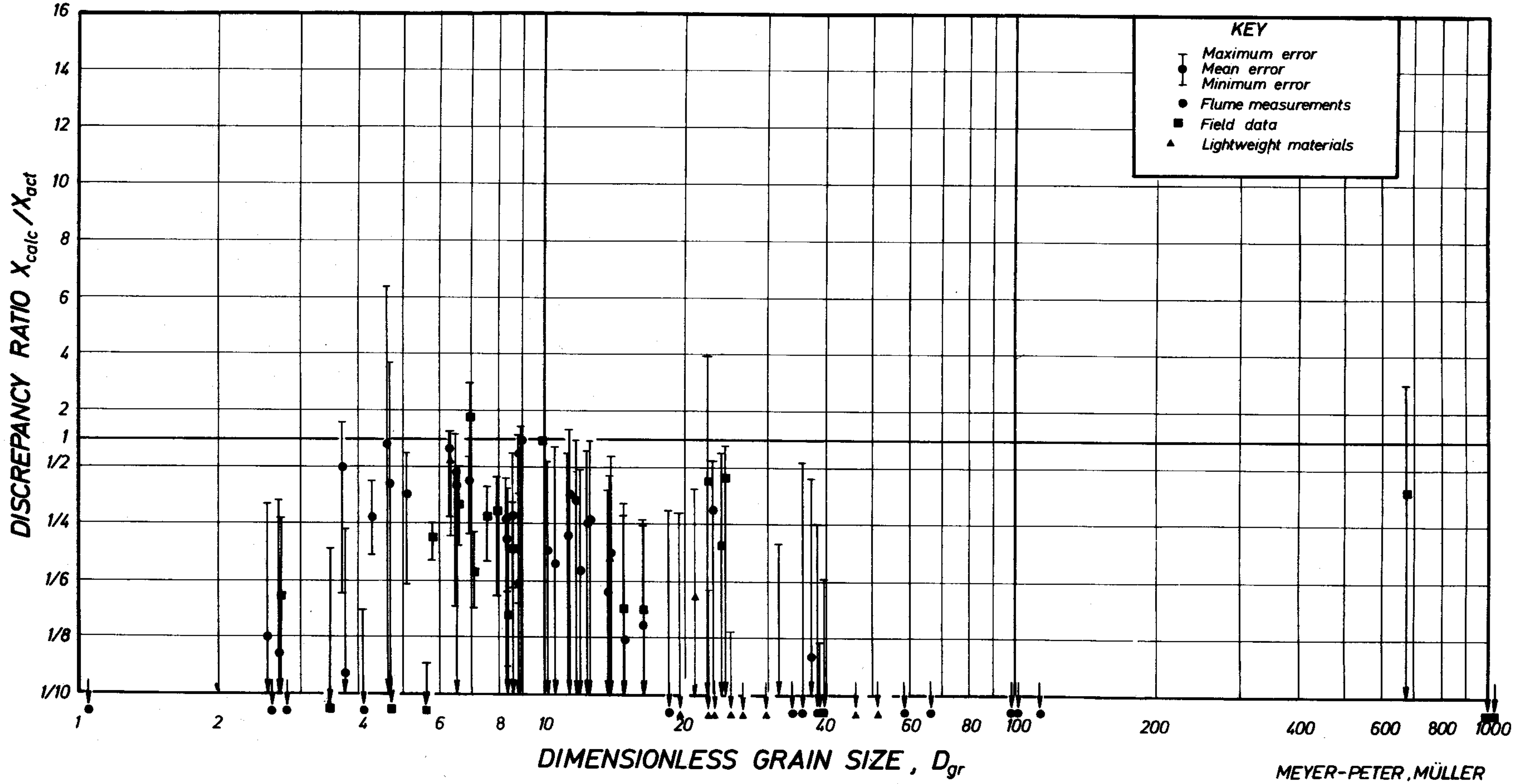
# INGLIS




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 9

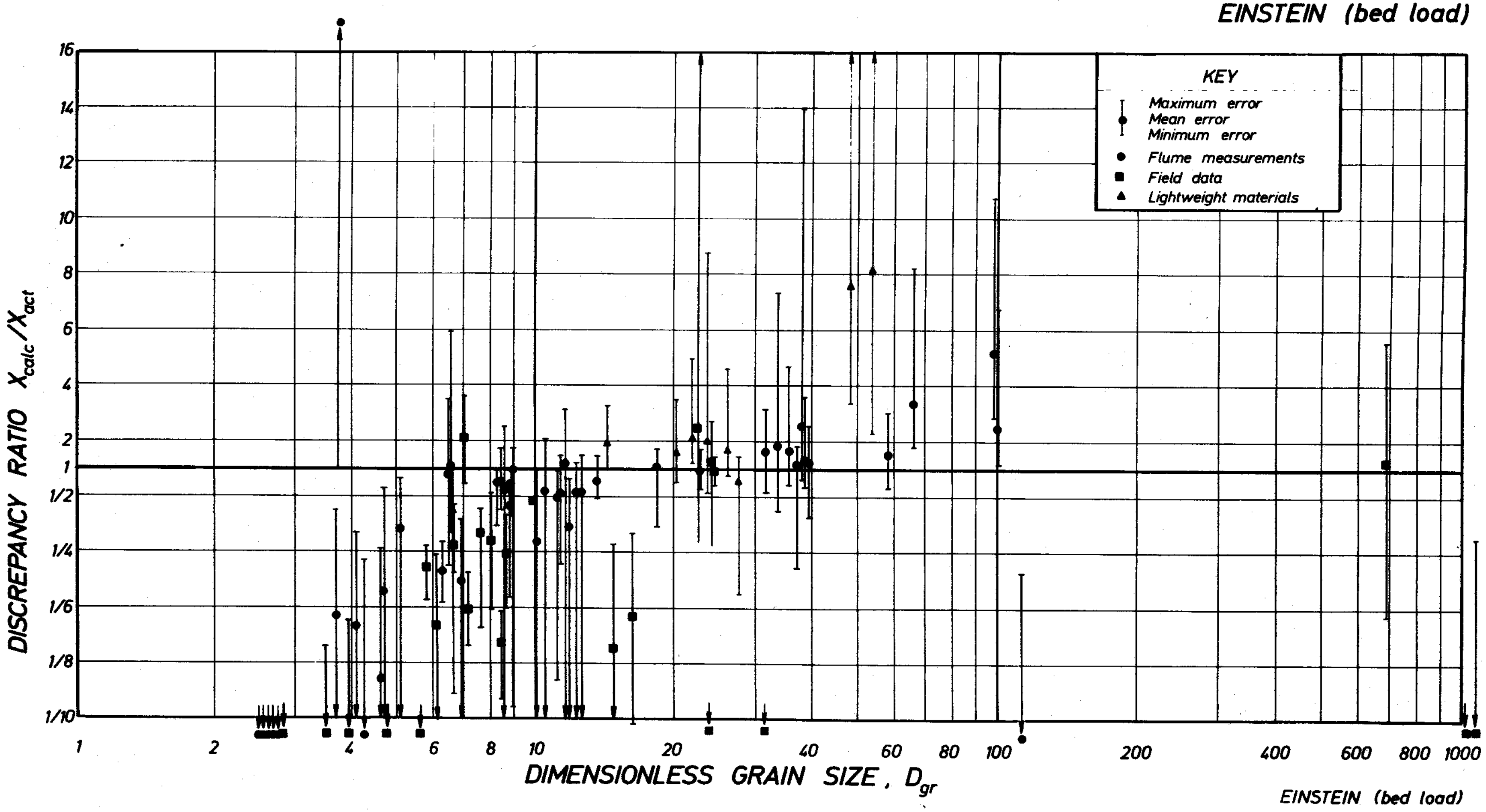
## MEYER – PETER , MÜLLER




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

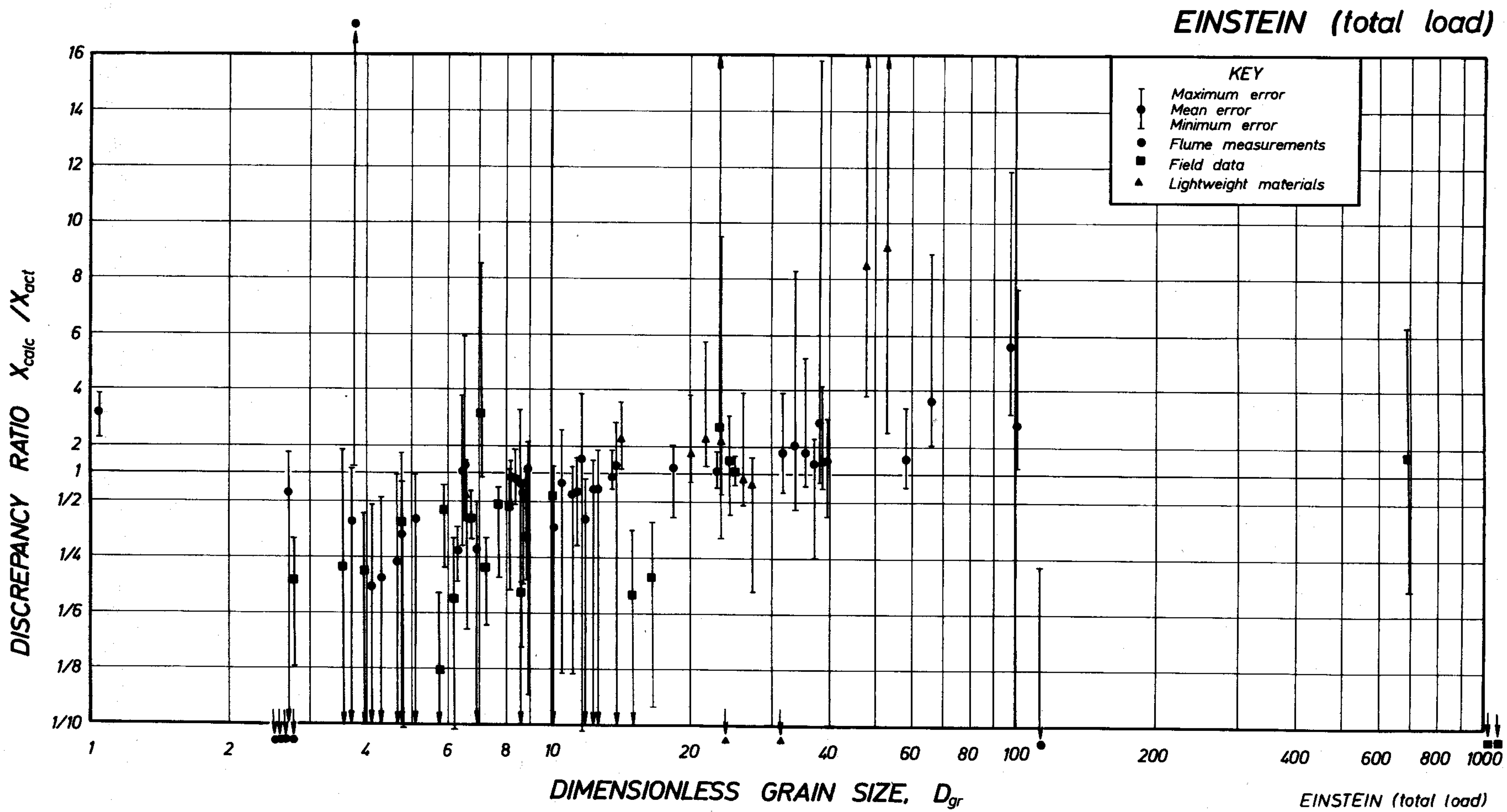
FIG 10

EINSTEIN (bed load)



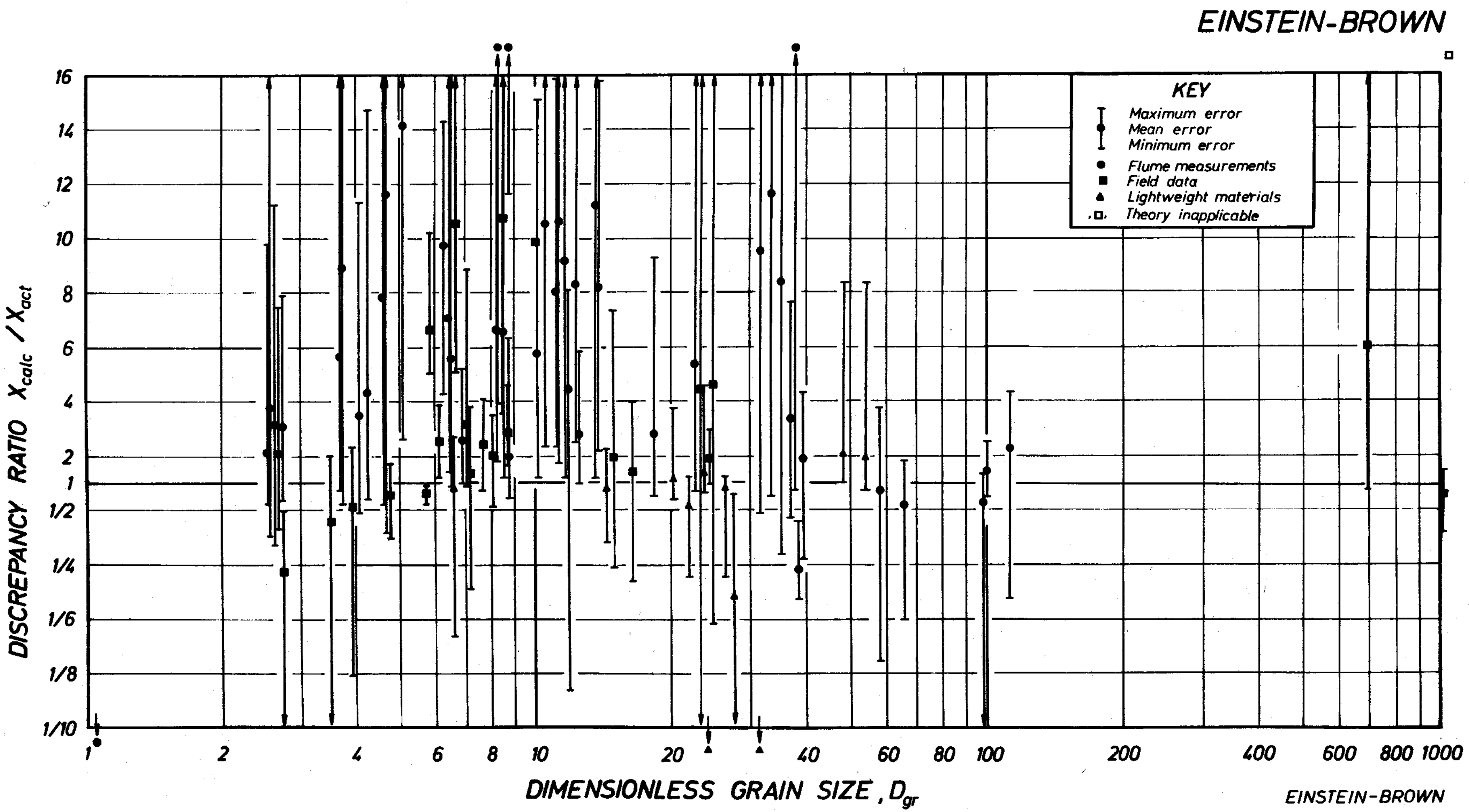

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 11




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

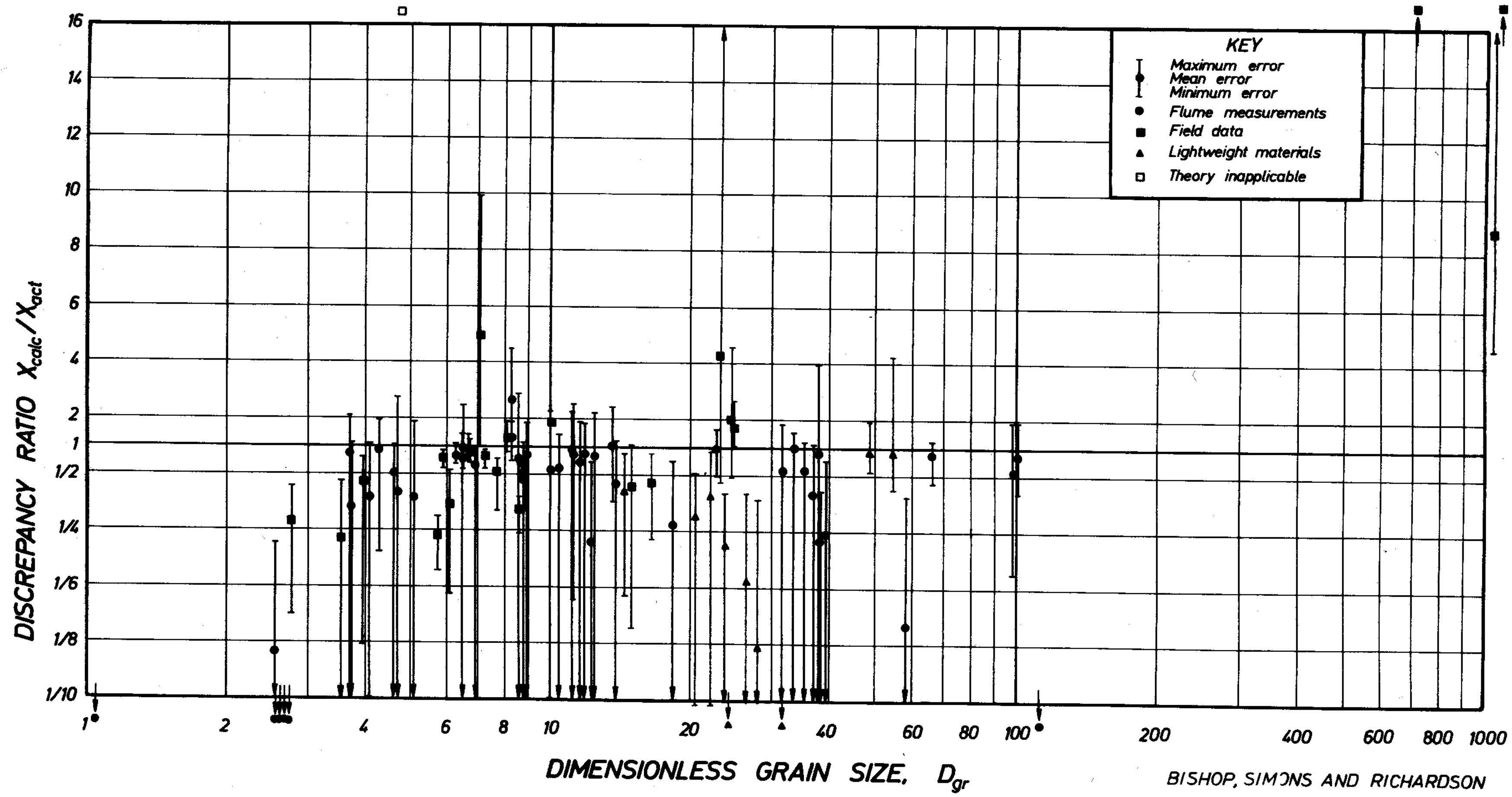
FIG 12




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

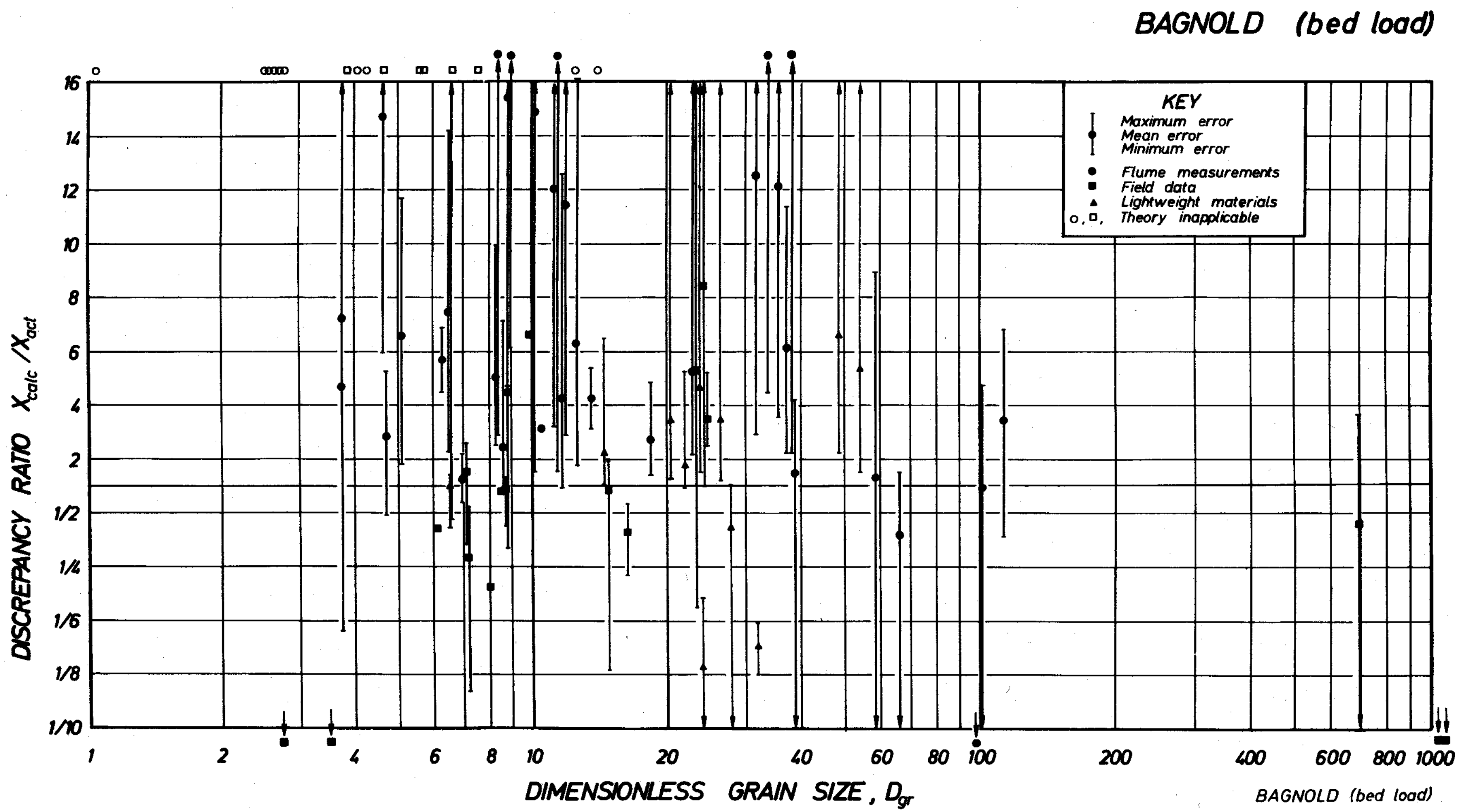
FIG 13

BISHOP, SIMONS AND RICHARDSON



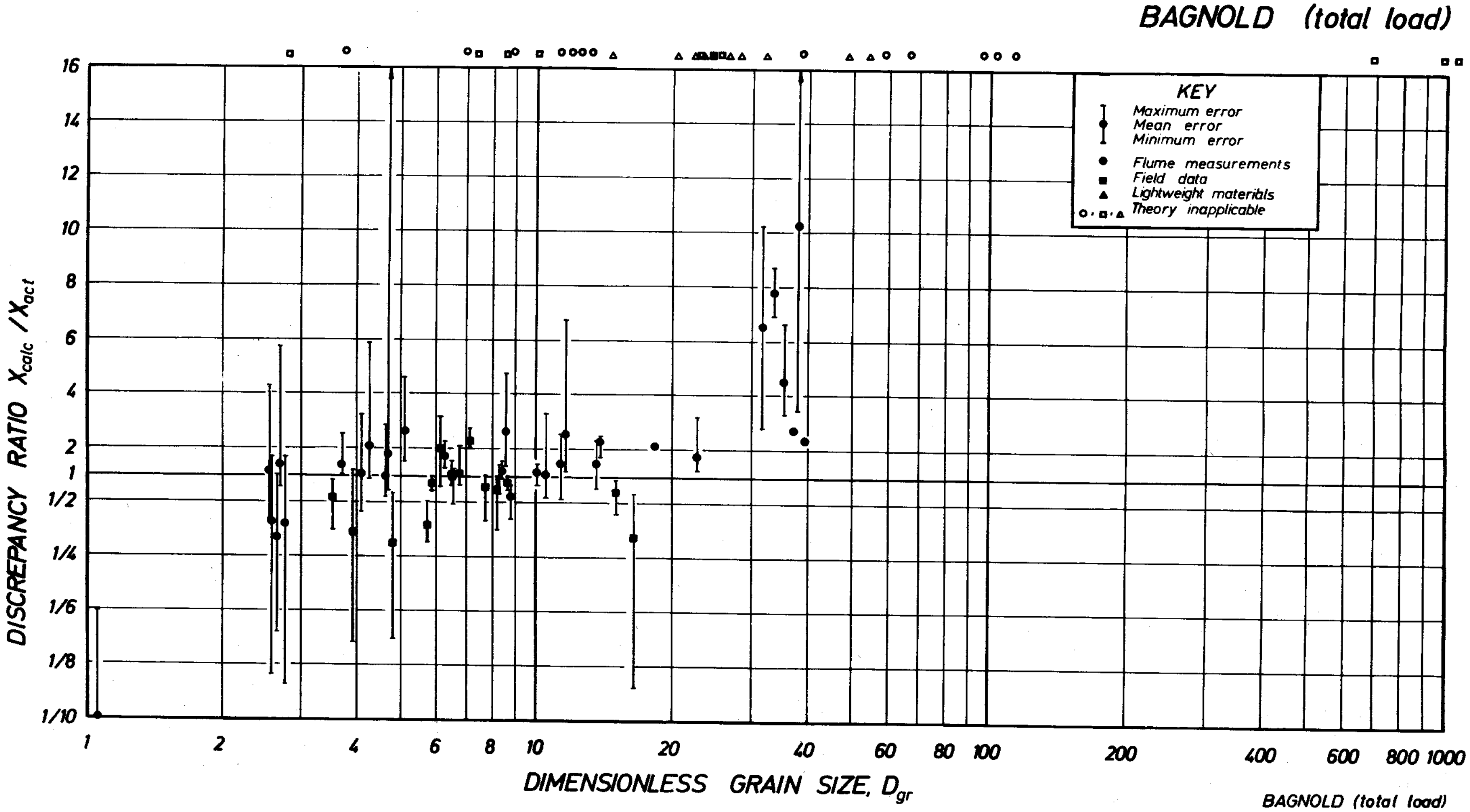

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 14




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

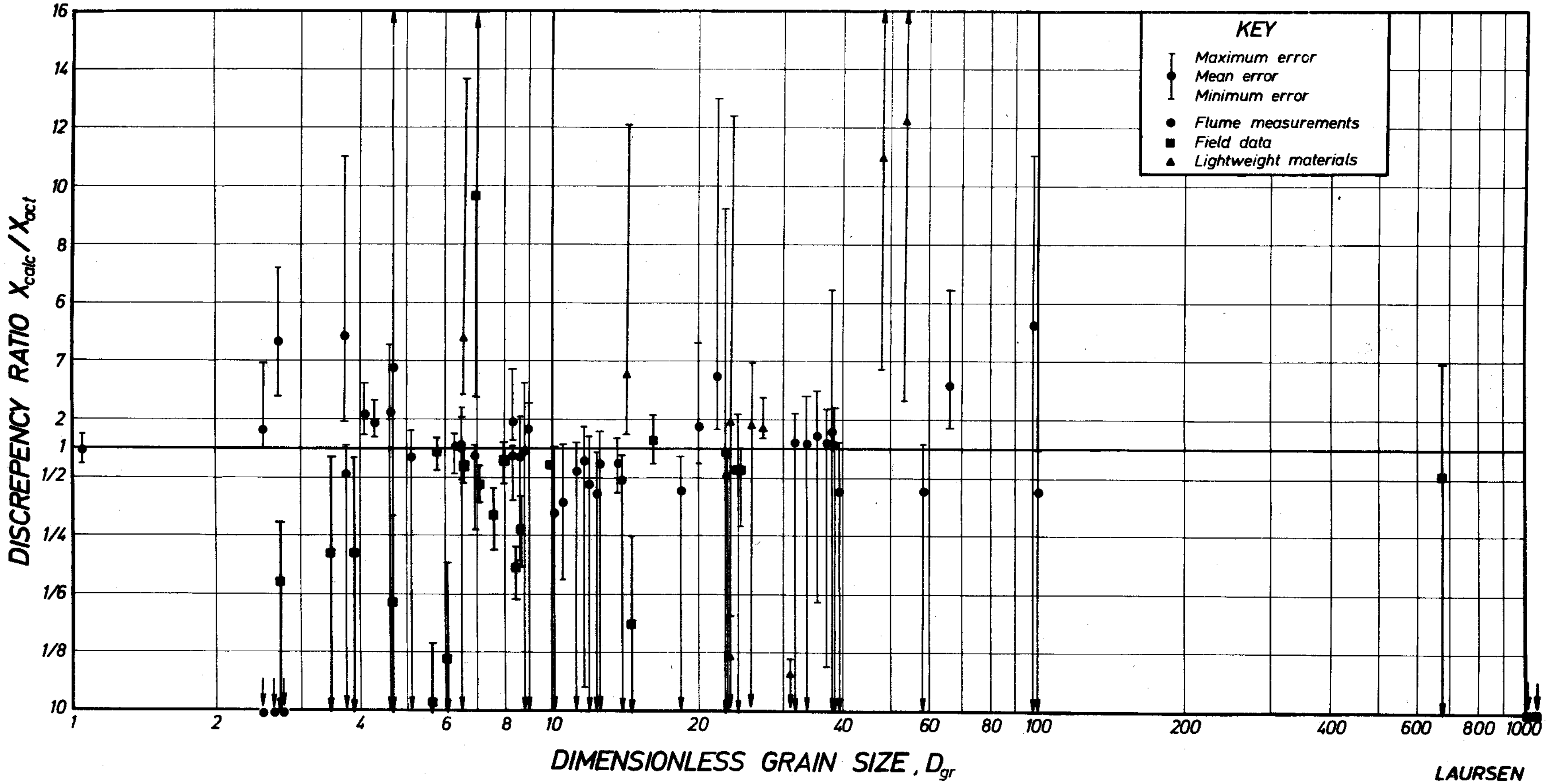
FIG 15




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 16

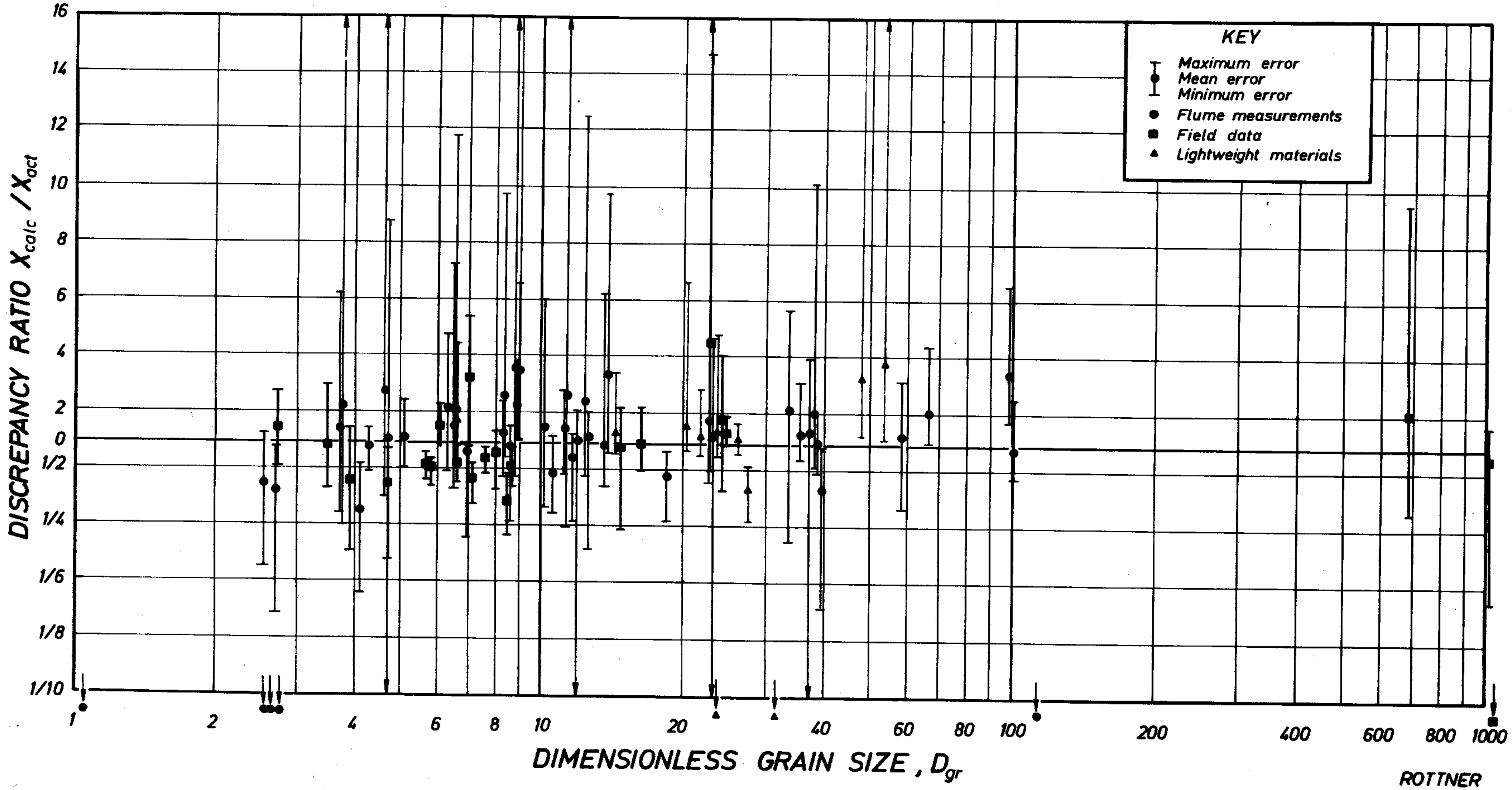
## LAURSEN




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

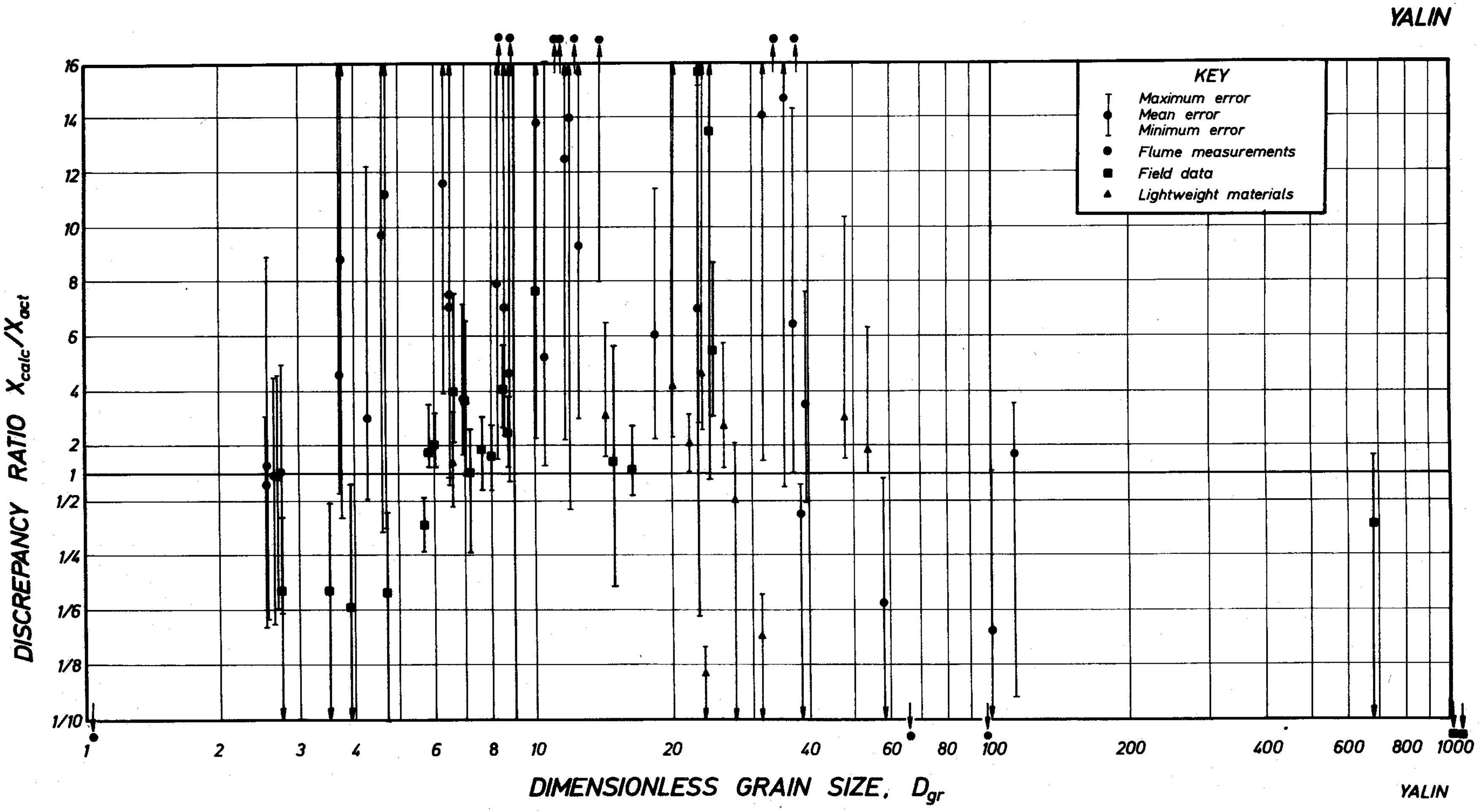
FIG 17

ROTTNER




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

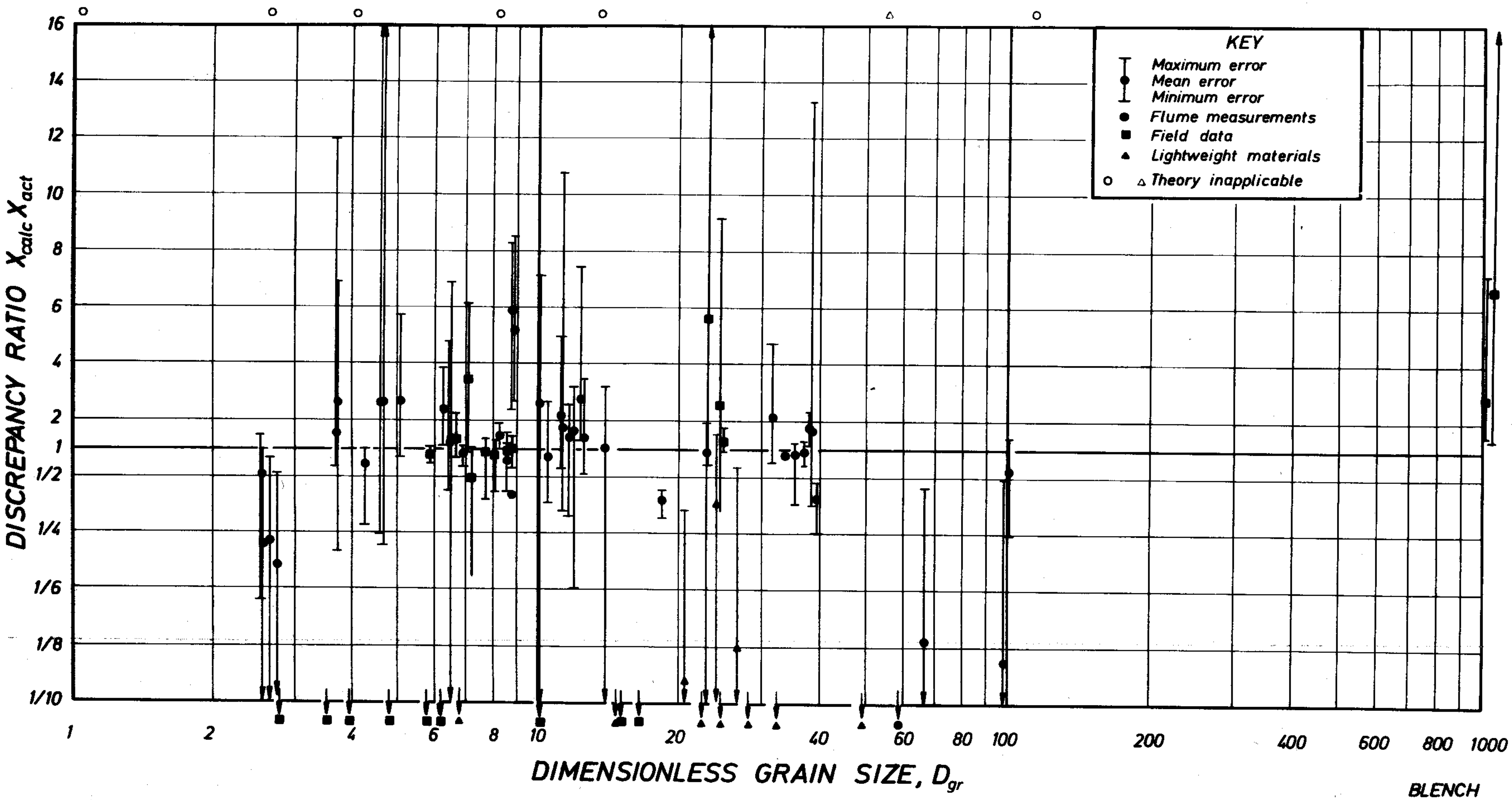
FIG 18




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 19

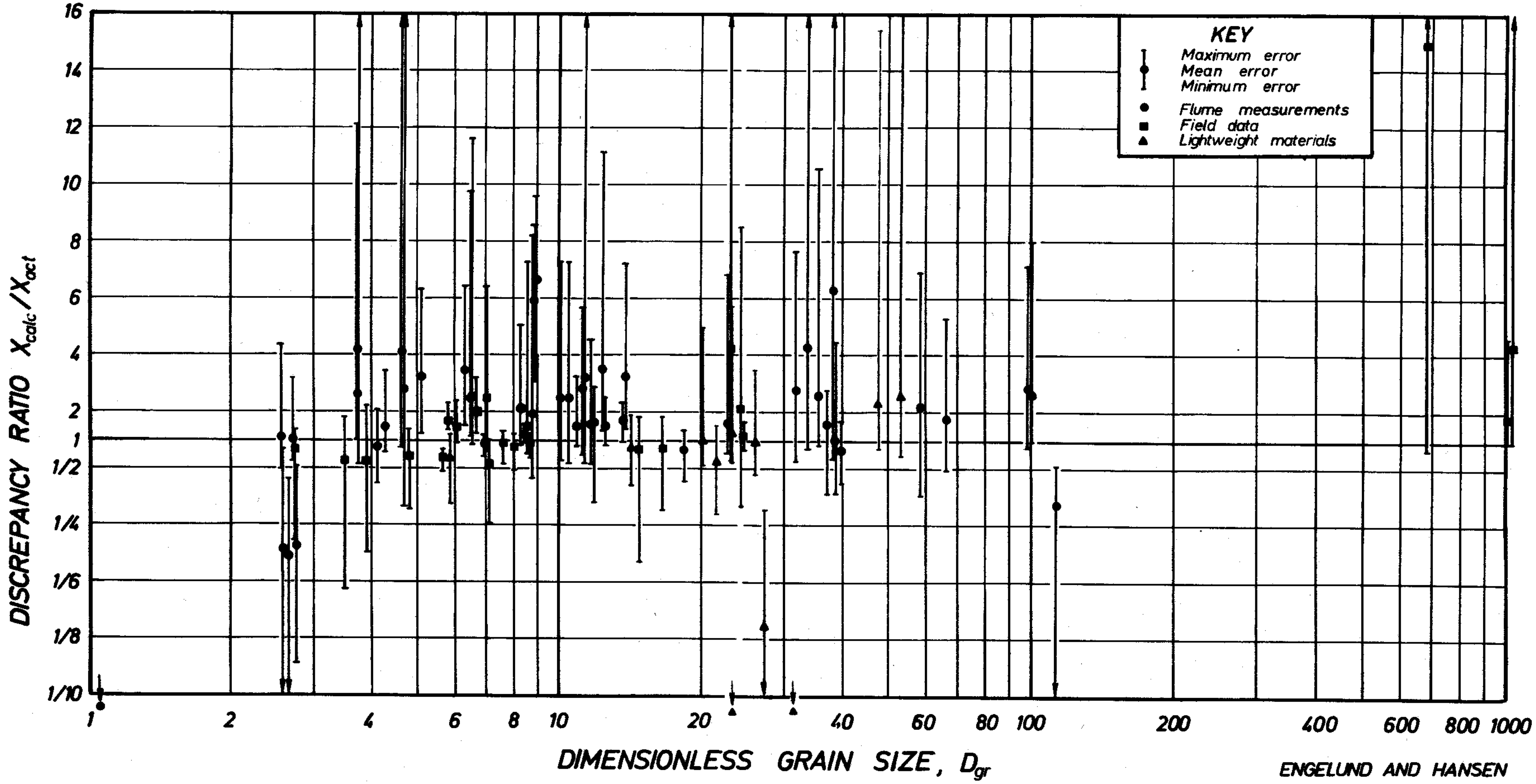
BLENCH




COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

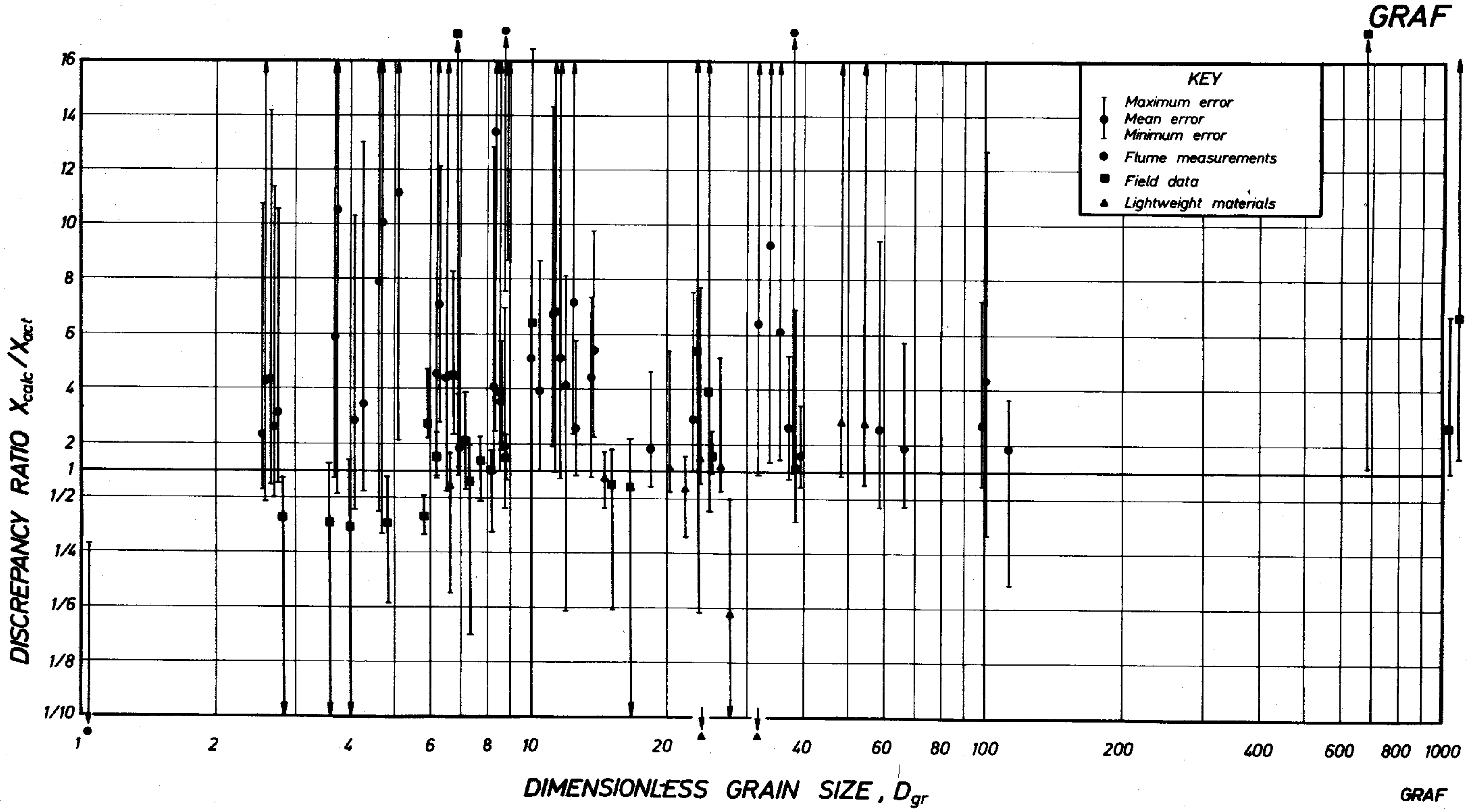
FIG 20

## ENGELUND AND HANSEN



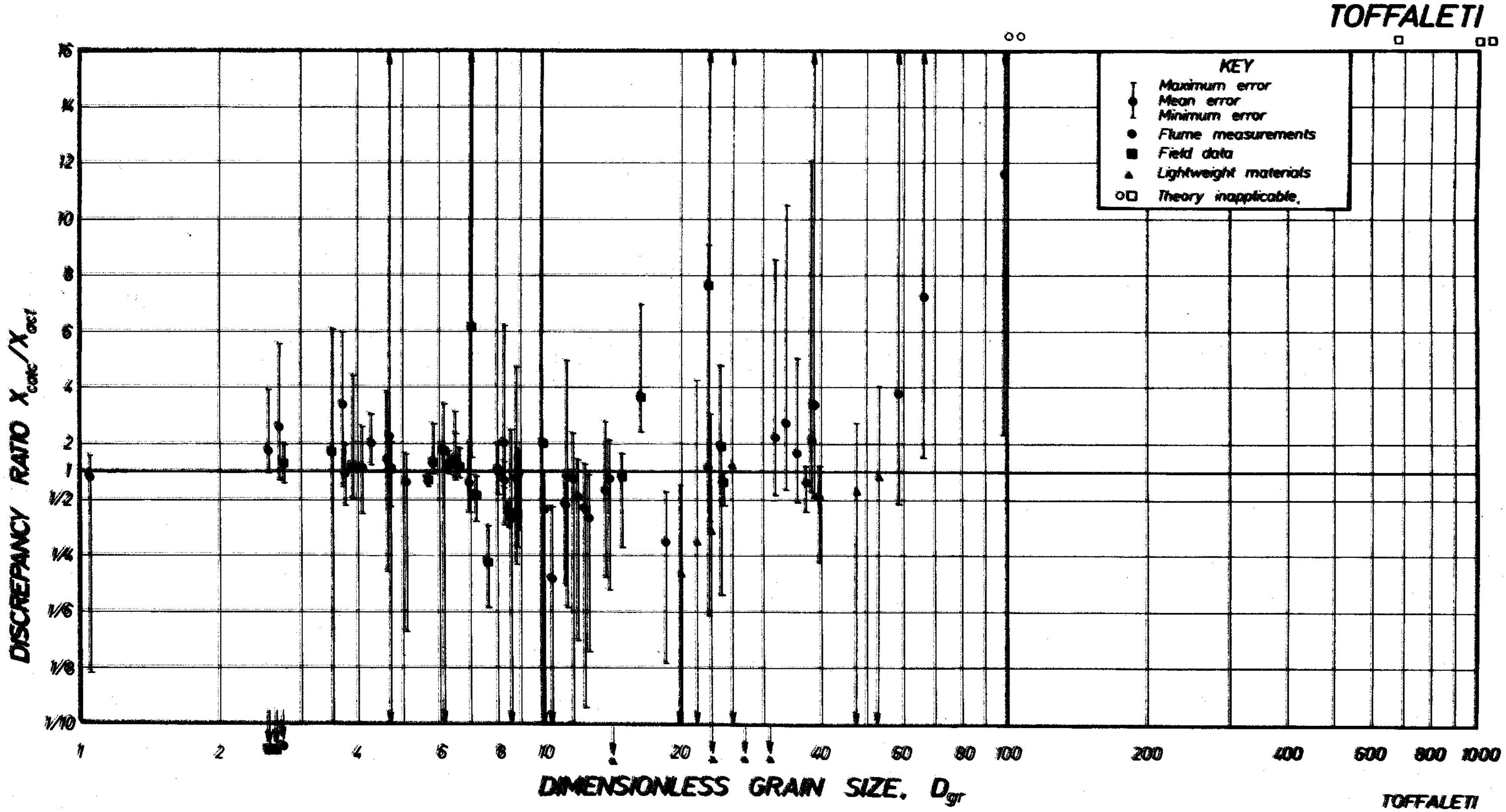

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 21



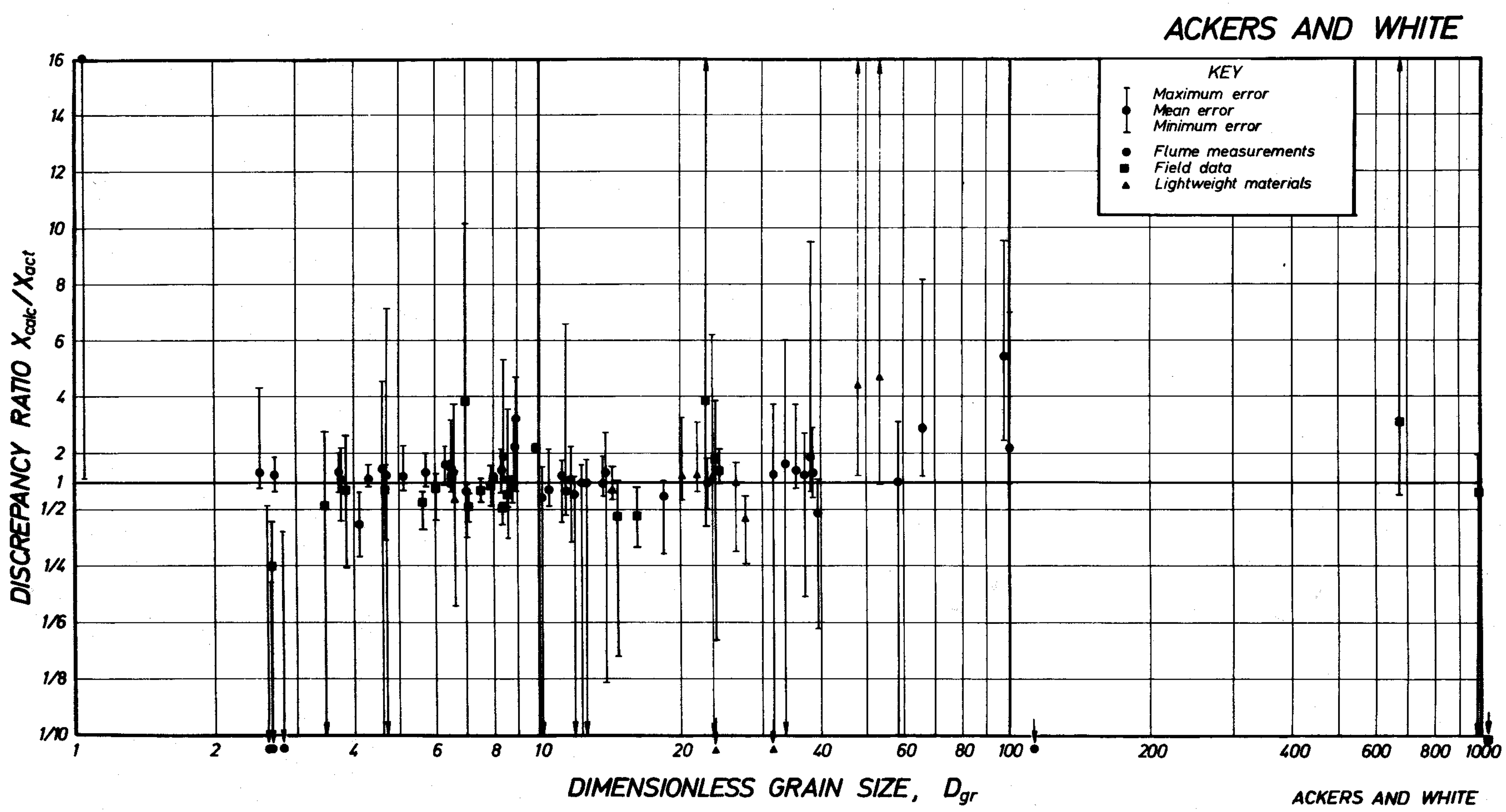

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 22



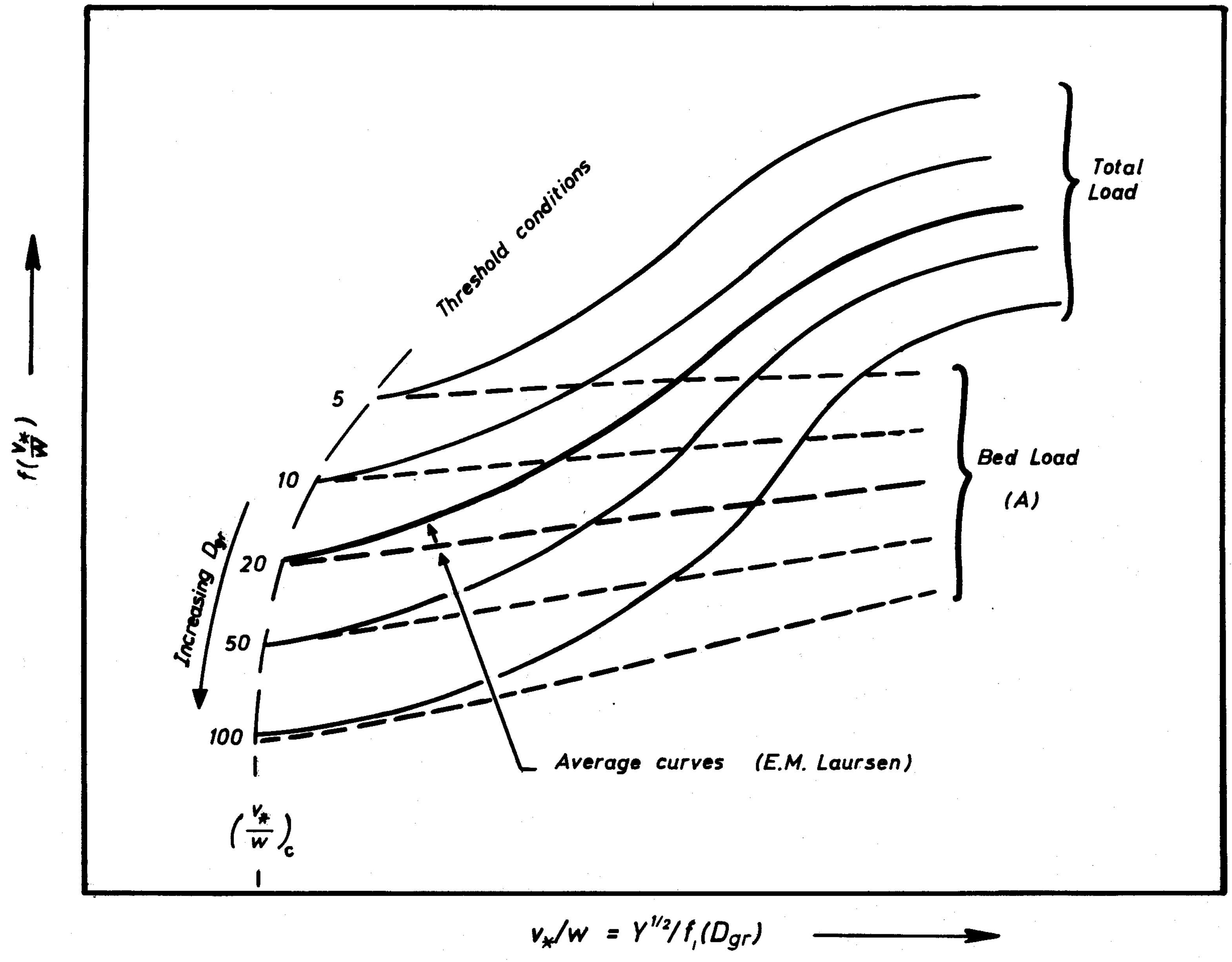

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 23



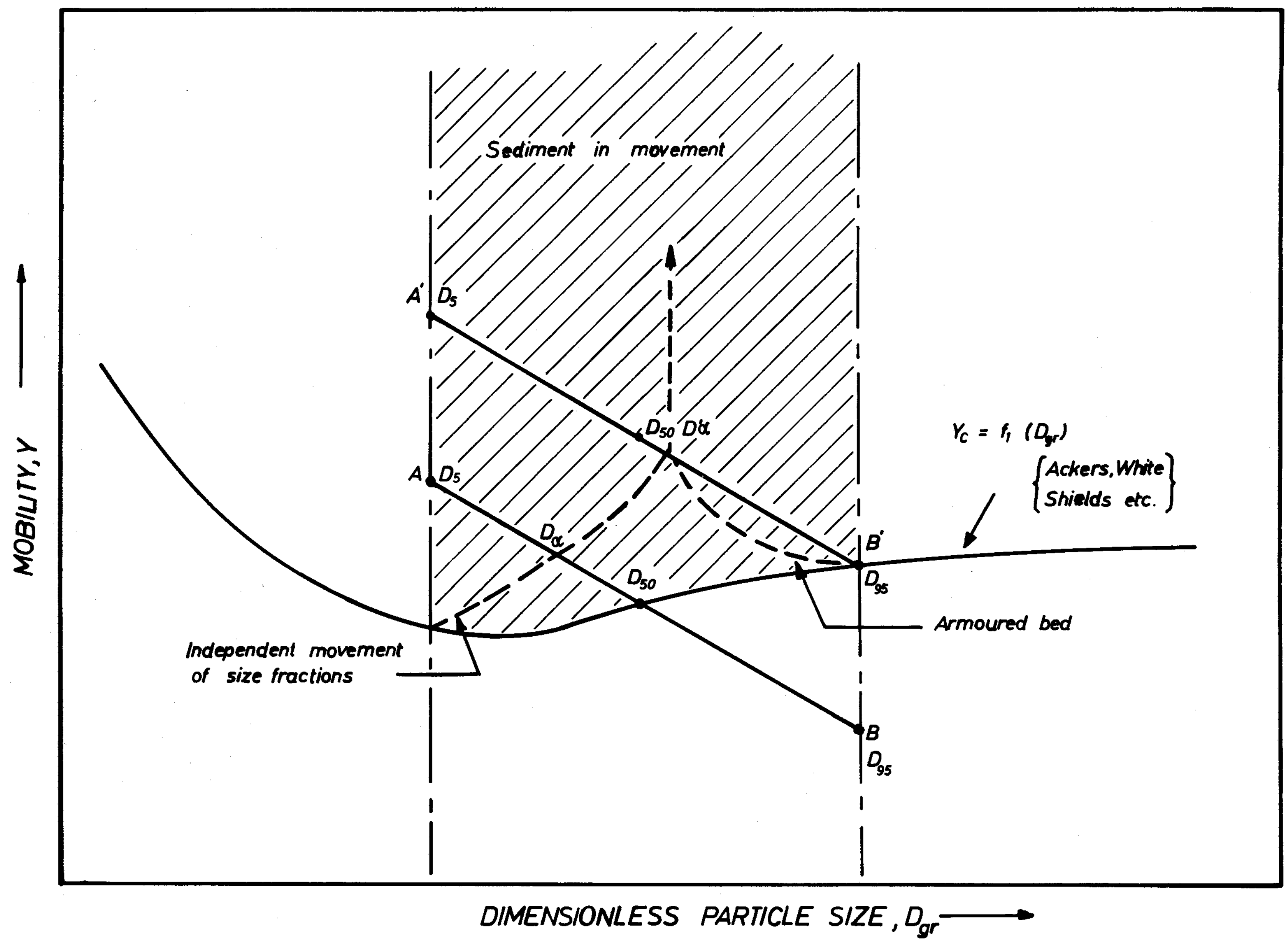

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 24



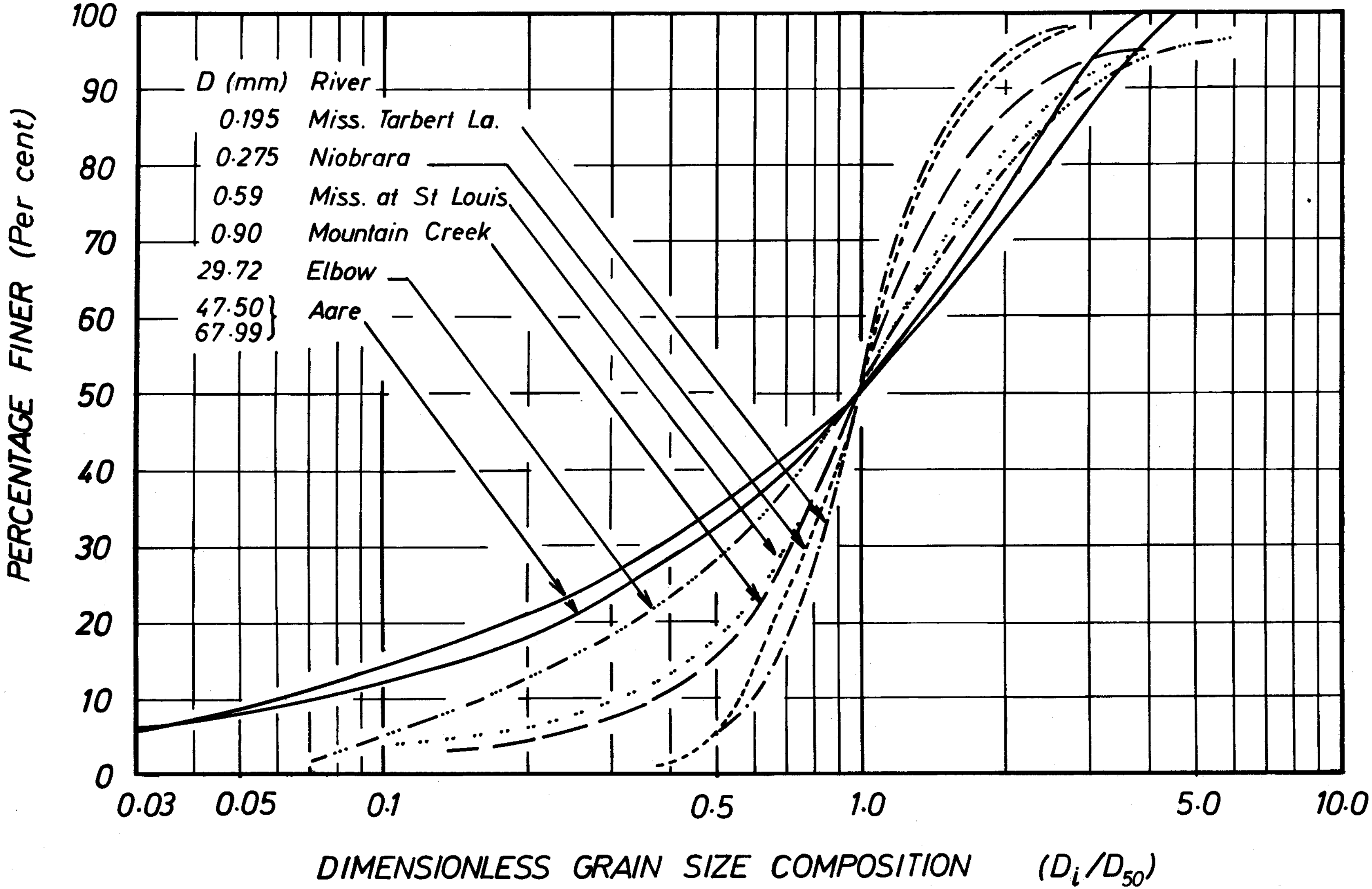

SCHEMATIC DIAGRAM DEFINING $f\left(\frac{v_*}{w}\right)$ AFTER E.M. LAURSEN

FIG. 25



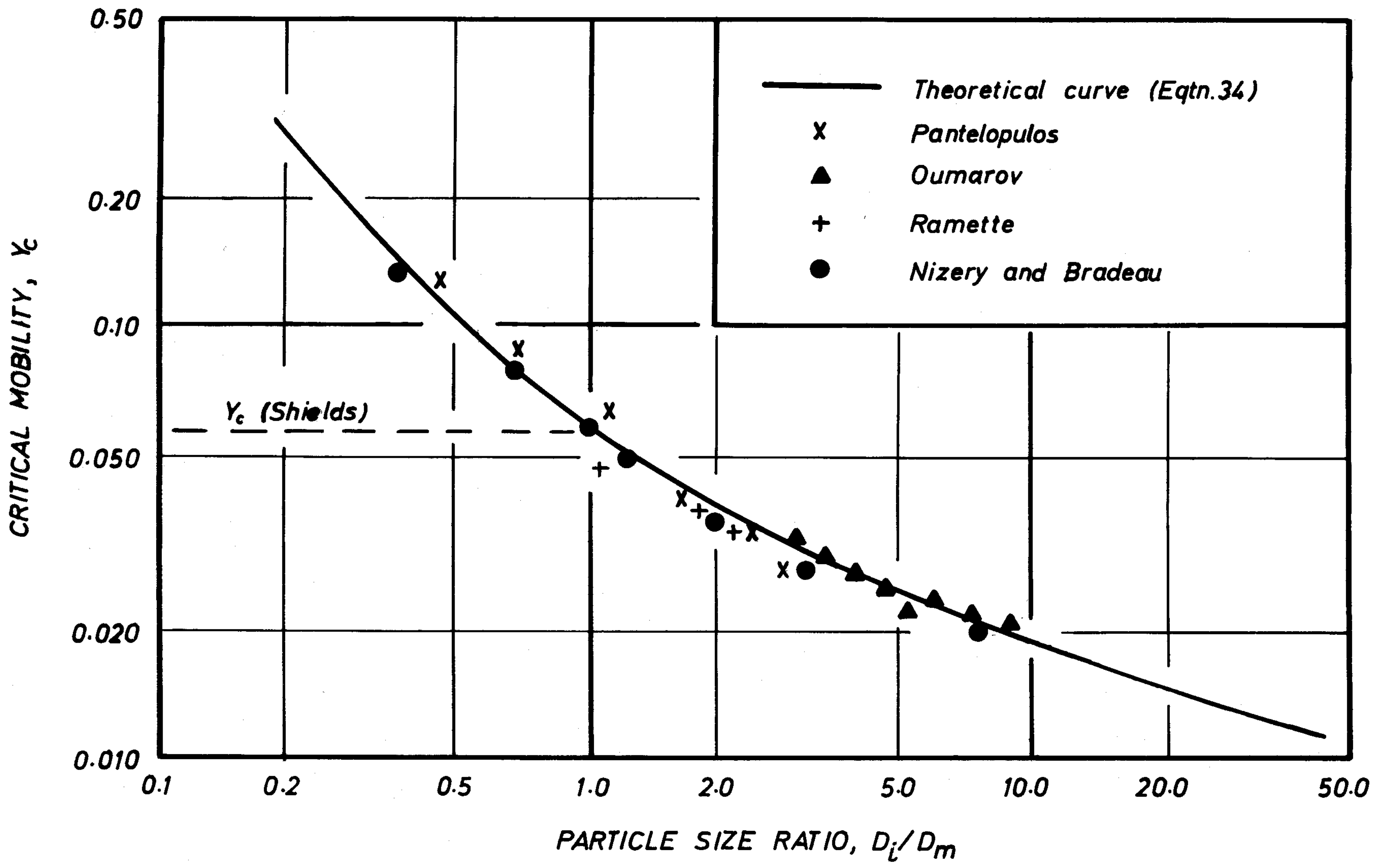

SCHEMATIC DIAGRAM SHOWING CHANGE OF EFFECTIVE DIAMETER WITH STAGE

FIG 26



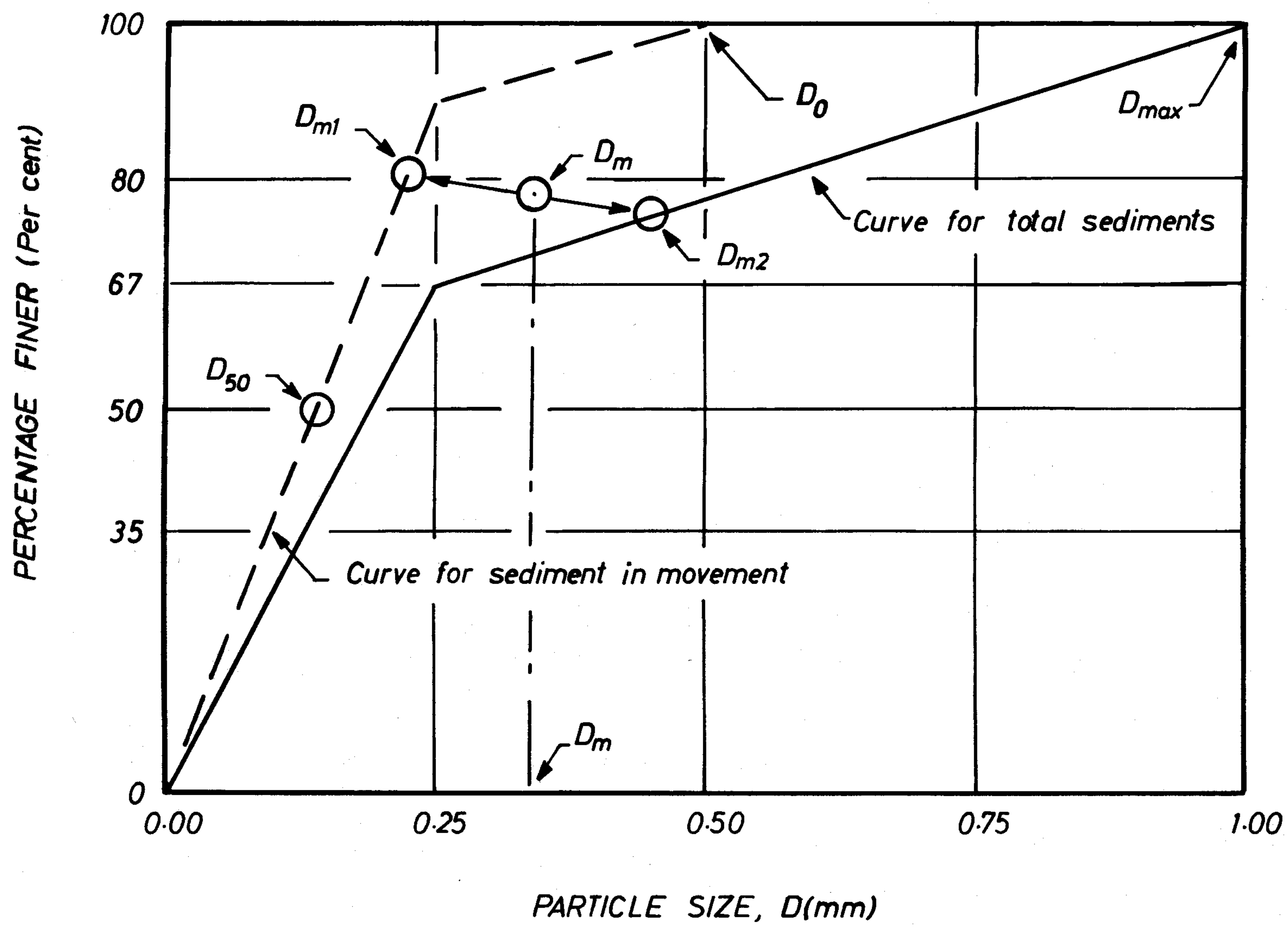

COMPARISON OF GRAIN SIZE DISTRIBUTION FOR FIELD DATA

FIG 27



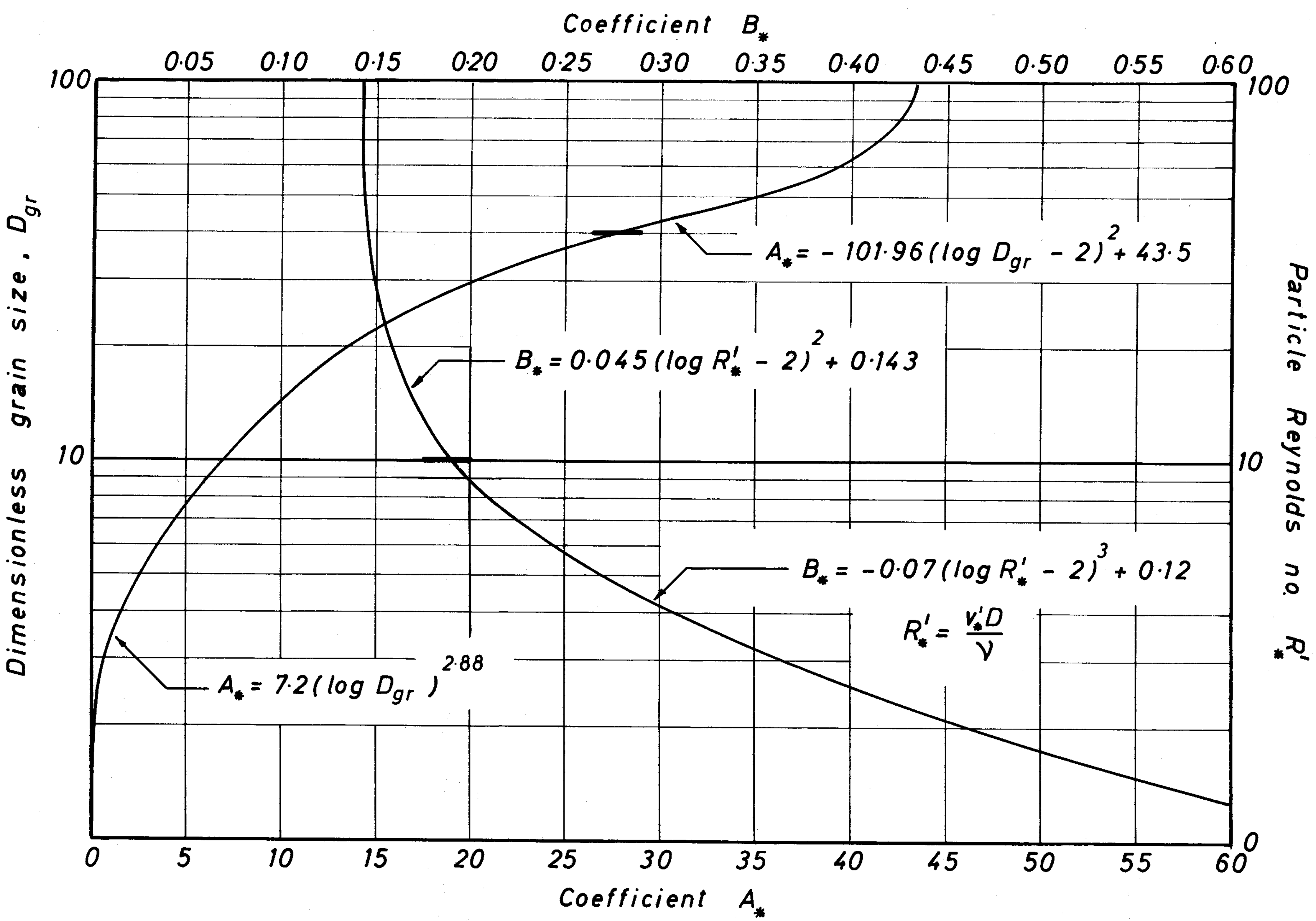

CRITICAL MOBILITY NUMBERS FOR GRADED SEDIMENTS, I.V. EGIAZAROFF

FIG 28



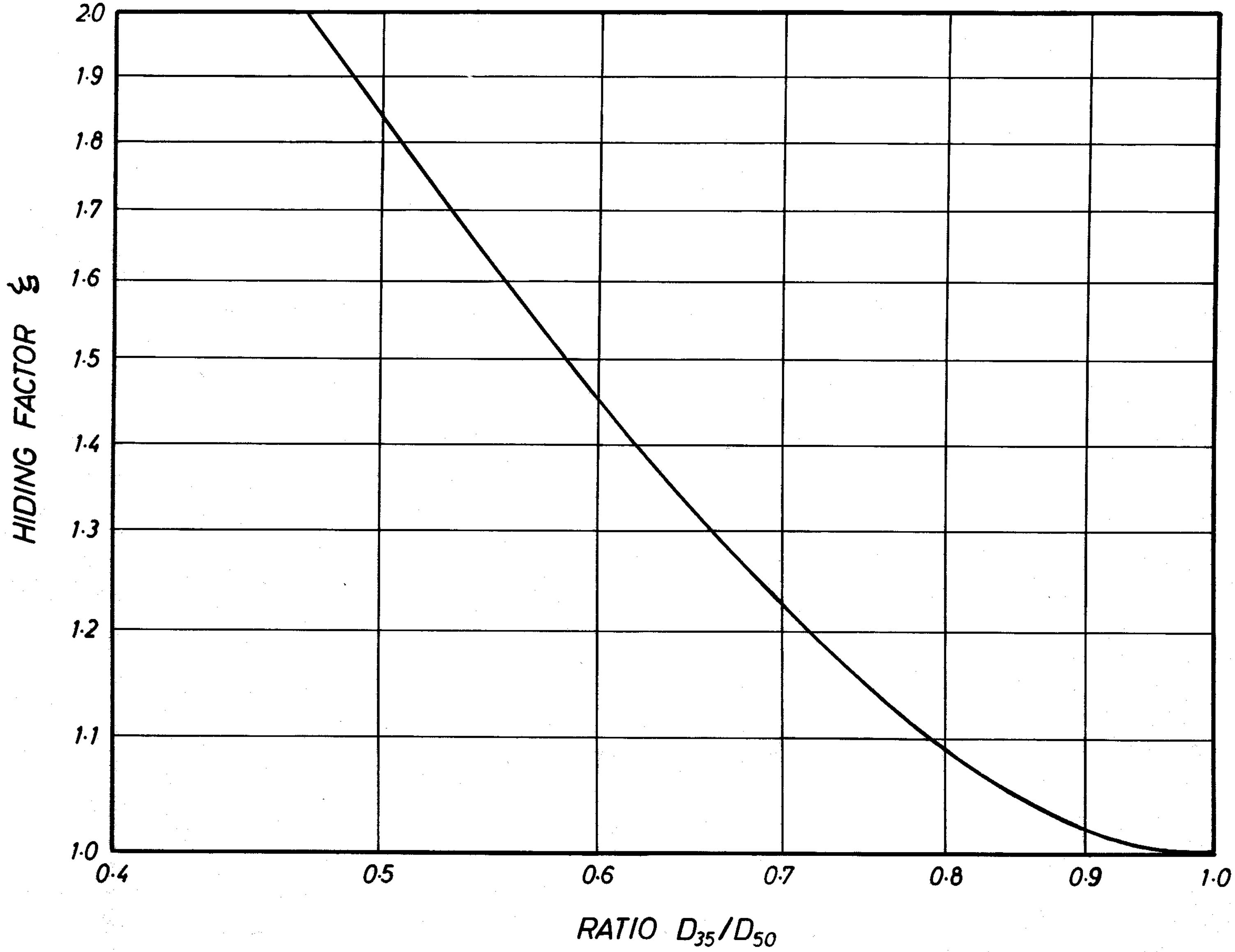

GRADING CURVES FOR MOVING AND TOTAL SEDIMENTS, I.V. EGIAZAROFF

FIG 29



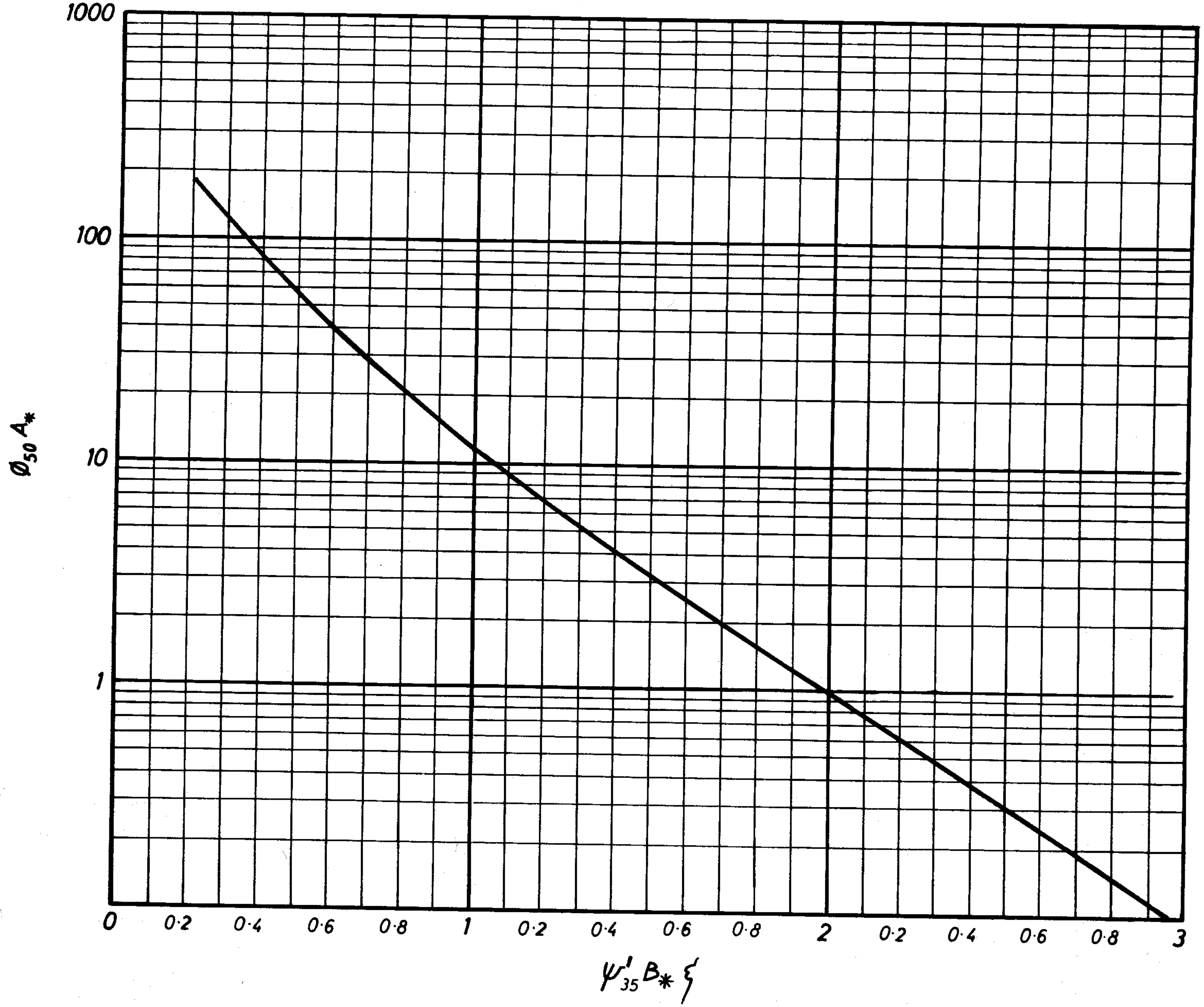

$A_*$ AND $B_*$ AS FUNCTIONS OF $D_{gr}$ AND $R'_*$ RESPECTIVELY

FIG 30




PROPOSED EMPIRICAL HIDING FACTOR

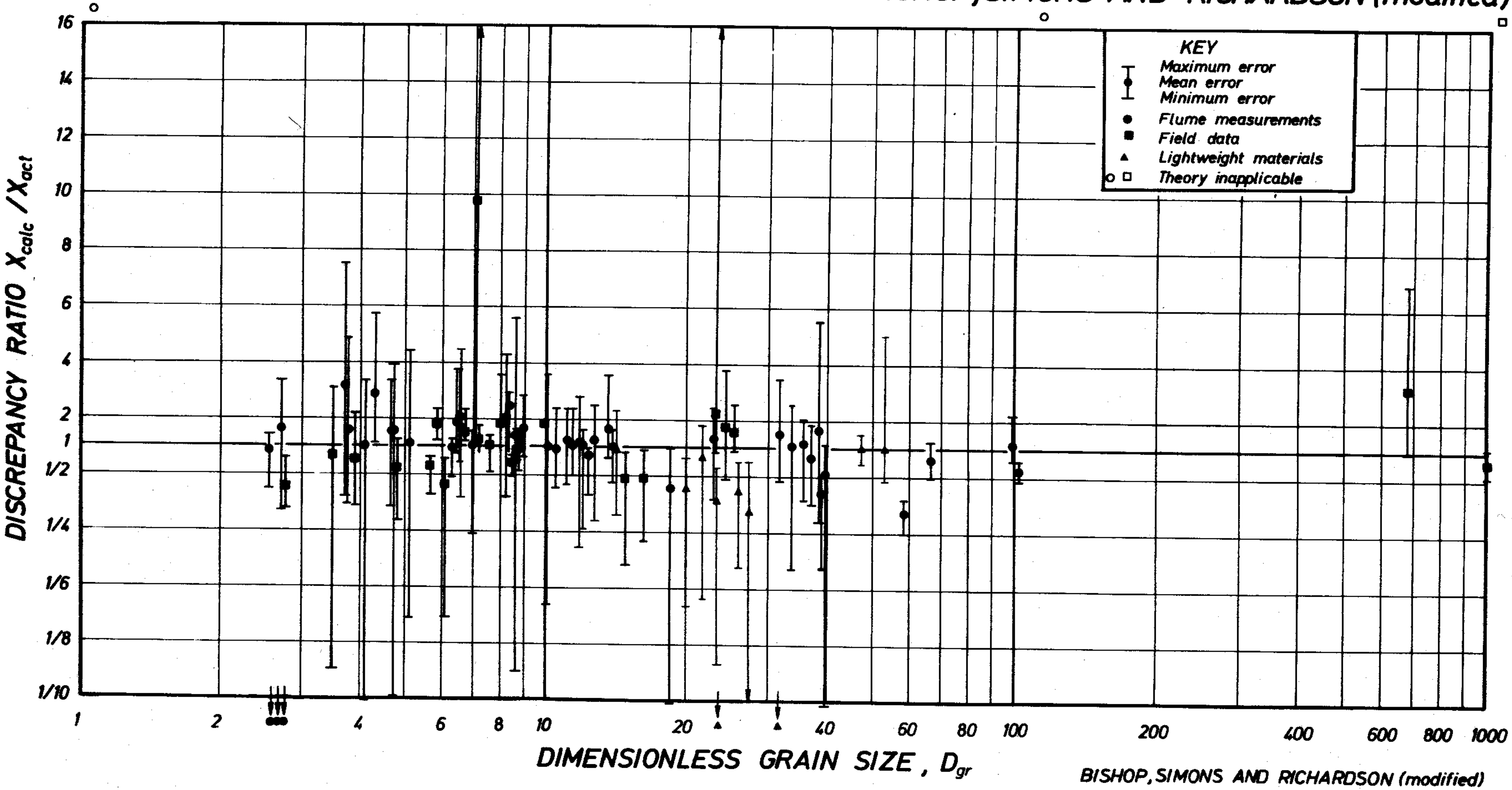
FIG 31




PROPOSED RELATIONSHIP $A_* \emptyset_{50} = f(\psi'_{35} B_* \xi')$

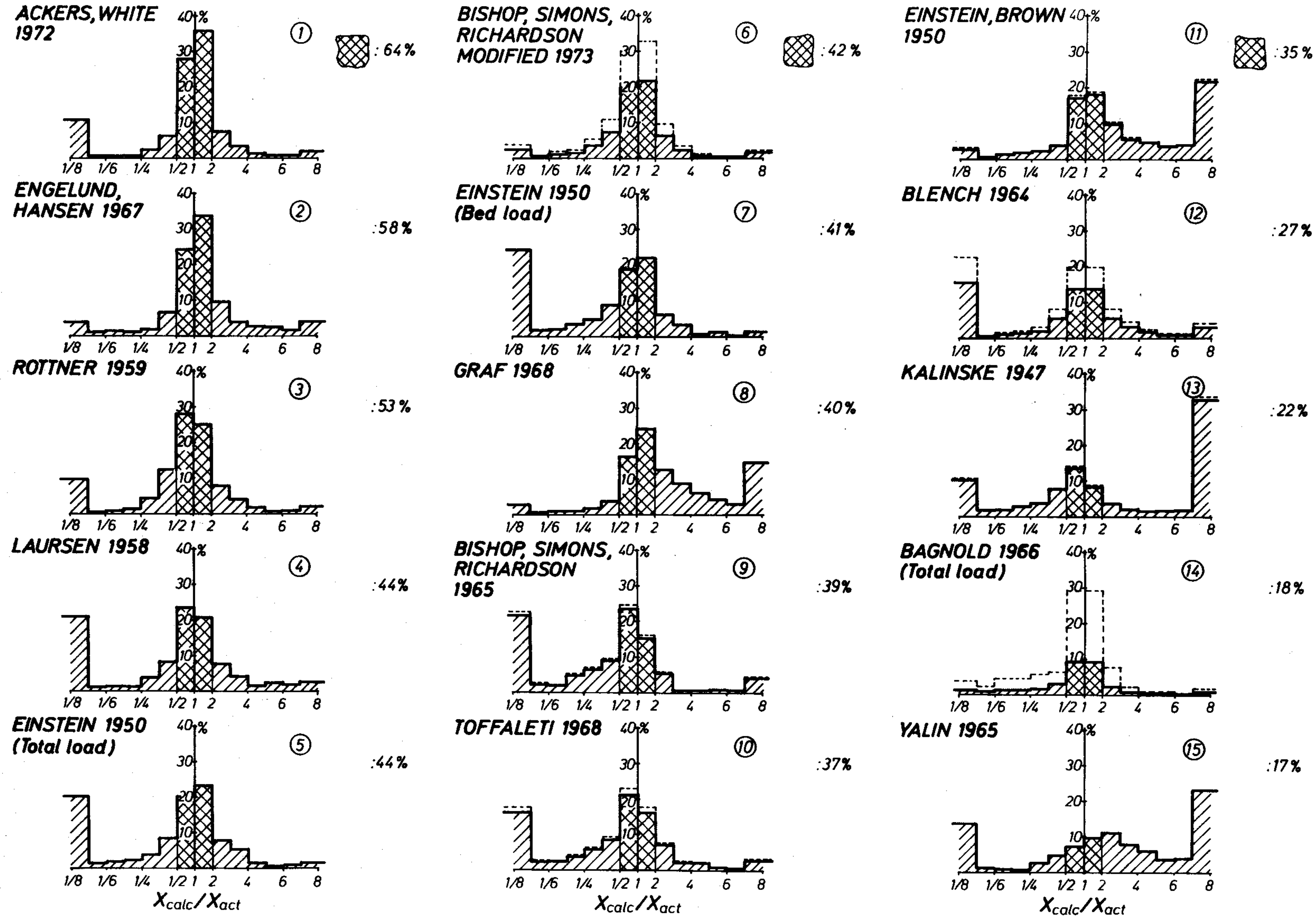
FIG 32

## BISHOP, SIMONS AND RICHARDSON (modified)



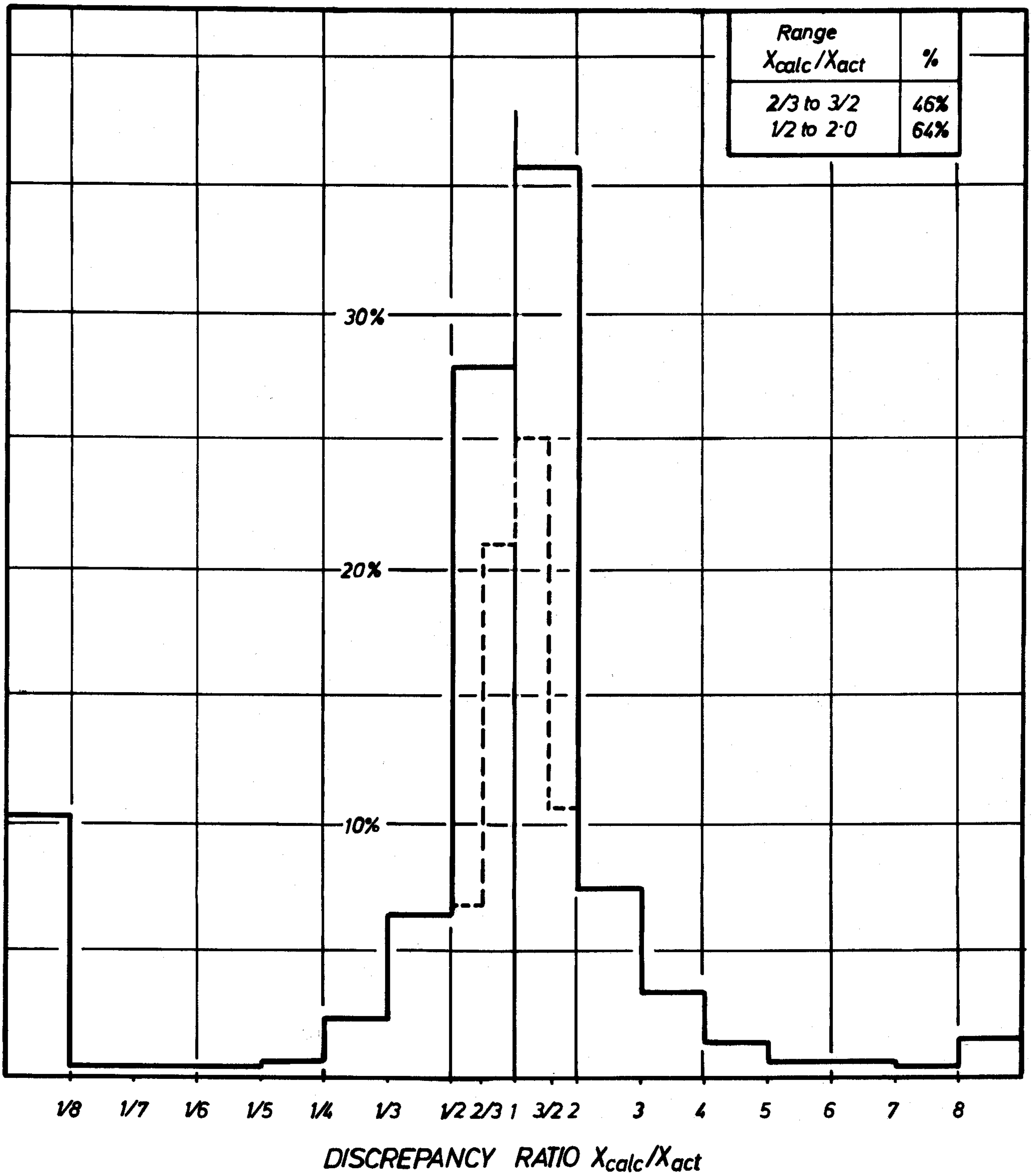

COMPARISON OF PREDICTED AND OBSERVED TRANSPORT RATES

FIG 33




A COMPARISON OF THE DISTRIBUTION OF ERRORS

FIG 34




DISTRIBUTION OF ERRORS OF PREDICTED TRANSPORT FOR INDIVIDUAL TESTS—ACKERS AND WHITE

FIG 35