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DRIFTKEEL: A mathematical model of long period, second order forces on, and motion of, ships in shallow waters

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ABSTRACT

Long period ship movements can be a problem to port operators. Due to resonance and small hydrodynamic damping, horizontal excursions can be large with a consequent risk of mooring lines breaking, or of cargo handling being hindered. Accurate modelling of long period forces and motions is essential for economically moderating the problem in designs for port developments. Both mathematical and physical models can be used. But little mathematical modelling has been done because suitable, accurate models did not exist.

This report describes a mathematical model of long period and steady drift forces on ships in shallow water developed by Hydraulics Research and called DRIFTKEEL.

Forces driving long period motions are predominantly due to second order effects with magnitudes proportional to wave height squared. Second order forces are described in Section 2 of this report.

Results are presented in Sections 3 and 4 showing long period and steady drift surge, sway, heave and pitch forces and movements and long period surge motions. For the long crested wave conditions investigated, forces associated with set-down were dominant among slowly varying forces - an effect ignored in the Newman approximation. Steady drift forces computed using DRIFTKEEL were similar to published experimental results. Further verification was obtained by comparison with a simpler two dimensional model of long period sway forcing, and by finding some correlation between measurements of surge motion of a model ship in a random sea and estimates made using DRIFTKEEL.

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1. INTRODUCTION

Modelling has been established as a useful procedure in the design of harbour developments for many years. A good model will highlight the salient aspects of a design and show up any shortcomings so that they can be eliminated long before the developer is committed to the great costs of construction.

Both physical and mathematical models are used to test hydraulic characteristics of harbours. Each has its advantages. Physical models automatically include all the physics of wave behaviour and are thus comprehensive and well suited to refining final designs. Mathematical models represent only those physical effects that have been chosen to be included by the modeller, and it is never feasible to describe even those effects exactly. A mathematical model is therefore only approximate, but it is generally far cheaper to set up and run than a physical model and provided it is realistic it is well suited to feasibility studies where comparisons of a large number of different possible designs are required.

Wave disturbance models of harbours have been frequently used in the past, models of ship response in those waves less so. But it must be remembered that the ultimate purpose of a harbour is to provide a sheltered place for loading and unloading ships. The important factor deciding harbour design is not wave height within the harbour in itself but the effect of those waves on ships sheltering there and on the ease of handling those ships. Wave period and direction are just as important as wave height in this respect. Considering wave heights in a harbour in isolation without also considering ship response will therefore lead to grossly inaccurate estimates of berth tenability.

It is fair to say that in feasibility studies consideration of ship response has generally been made in the past by rule of thumb methods rather than scientific modelling. Such methods though are inexact; there must be a tendency for the practitioner to choose to err on the side of caution, so it seems probable that many harbours will have been excessively conservatively designed because the wave protection chosen was greater than it needed to be.

In the past, rule of thumb methods had to be used because general purpose berths had to cater for a wide variety of types and sizes of vessel each of which had its own response characteristics in waves - anticipating motion and refining berth design to control it were not realistic options. Being conservative was the prudent way to proceed in those circumstances.

But now berths are commonly designed for use by just one type of ship, or even specifically for one vessel. This opens up the possibility of far greater refinement of berth design; not just the design of quayside facilities such as cargo handling equipment, but also controls on ship motion such as mooring arrangements and wave protection at minimum cost. Testing these measures to ensure that vessel movement at a berth is small requires accurate modelling of ship response to waves. Once these models have been developed, engineers will be able to design harbours confident in the knowledge that the proposed port will provide adequate wave protection. The end result will be a more economic design: less wave protection will be needed per berth, less money will be wasted building superfluously long harbour breakwaters for protection that is not needed, and (since longer breakwaters cause narrower harbour entrances) harbours will be produced that make it easier for ships to enter and leave.

At the same time, advances in computer technology are making even more sophisticated numerical models ever more practicable. It can be said that mathematical modelling of ship response in waves is an idea whose time has come.

At Hydraulics Research we have already developed UNDERKEEL, a numerical model of ship motion in waves applicable to a vessel either moving or stationary in shallow water but not moored (Ref 1). UNDERKEEL has been successfully employed in repayment work to provide initial estimates of underkeel clearances needed for ships to sail navigation channels with minimal risk of bed contacts in bad weather (Refs 2, 3 and 4). We have also developed a model incorporating non-linear mooring forces, called SHIPMOOR, for simulating moored ship movements once the forces exerted by waves on the ship have been computed (Refs 5 and 6). And QUAYSHIP (Ref 7) has been developed to calculate linear wave forces for a ship close to a solid quay in waves.

The types of waves on the sea or a lake ordinarily visible to the naked eye are adequately described by linear theory for our purposes and so can be considered to be examples of linear waves. The essence of linear wave theory is that quantities such as water pressure and hence resultant force on a ship are linearly proportional to wave height. Generally speaking, linear wave effects are the dominant cause of short period ship motions, which in this context means periods less than about twenty five seconds - periods corresponding to those of typical sea waves. Ship vertical motion is mostly short period.

However, the predominant horizontal movements of large ships when moored are at much longer periods - typically one or two minutes. This is because mooring lines act as relatively soft springs attempting to restrain the hugely massive ship, the ship is pulled back into position only slowly, so the natural period of ship motion is long. We also find damping effects at these long periods are not large. Large resonant motions can result if any long period excitation acts on the ship.

Such large motions can be damaging in at least two possible ways: There is firstly the obvious danger that they will cause mooring lines to break, which is most likely to happen in a storm. But large long period motions in less severe weather conditions can be highly inconvenient too by making cargo handling difficult or impossible; this is particularly a problem at container ports, where the cranes used to load and unload containers need to be manoeuvered with precision to place them without damage (Ref 8), and it is a problem at oil and gas ports also, where, for safety, loading must stop once vessel movement exceeds some threshold value (Ref 9). Long period motions are therefore both a safety hazard and a commercial problem, and they need to be controlled. As was explained earlier, economical and effective control can most easily and effectively be obtained with the help of accurate models of ship motions.

Hydraulics Research has carried out studies of long period ship response by field measurement and physical modelling (eg Ref 10). A special aspect of it, subharmonic response (Ref 11) has been studied by mathematical modelling using SHIPMOOR (Refs 5 and 6). Subharmonic response is an effect caused by the relative stiffness of fenders compared to mooring lines whereby short period, linear wave forcing can in some circumstances excite a long period resonance.

But direct long period forcing, which is another important cause of long period response, has not been mathematically modelled at HR. This is because the mechanisms driving long period forcing are different from those operating at shorter periods, so UNDERKEEL and QUAYSHIP are not applicable. Linear wave theory describes short period forcing: it turns out that long period forcing is predominantly due to second order effects, which cause forces proportional to wave height squared. General equations describing these forces have been developed in preparation for computer modelling (Ref 12).

This report describes a mathematical model, called DRIFTKEEL, developed at HR for computing long period second order forces on free ships in shallow water. It can also be used for modelling long period responses of ships with linear moorings (ie where all mooring forces are proportional to movement) at open quays; an exercise of this sort is described in Section 4. In reality however, typical ship moorings are non-linear in very many respects, for example in that fenders are much less compliant than mooring lines, so DRIFTKEEL can only model this typical case by linearised approximation. We need to uprate the model in future to enable us to deal with non-linear moorings using DRIFTKEEL in conjunction with SHIPMOOR; but this is dependant on research funding being made available. Funding is also needed for another planned development, DRIFTSHIP, a model for computing long period forcing on a ship close to a quay wall. This is an important, common mooring condition found in most ports, but the presence of a reflecting quay adds considerably to the already complex physics and mathematics of second order forces.

2. THEORY

Prior to DRIFTKEEL being developed, research into the theory of second order wave forces was undertaken, the results of which are given in Reference 12. The theory used in the DRIFTKEEL model, which shall be described below follows closely the formulation given in that report.

But first we must explain why second order effects are important and what they are. The introduction described how the natural periods of large ships moving at their moorings (particularly in surge, sway and yaw: see Fig 1) are long and how lightly damped long period motions generally are. In these circumstances any forcing at the requisite natural period is liable to set up a large resonant response.

Ordinary surface waves with such long periods (greater than about thirty seconds) simply do not occur on the earth - they would require huge (unphysical) wind speeds for long periods with vast uninterrupted fetches. Consequently, linear waves cannot excite long period resonances of large ships.

But second order effects will generate long period forcing even when ordinary linear waves of long period are not present. This forcing will be very weak compared to the first order forcing that acts at shorter periods, but resonance can make its effects large. Second order forcing is therefore significant.

A working definition is that a second order effect is one whose magnitude is proportional to first order wave height squared. We shall refer to linear waves, and the ship responses they produce, as first order phenomena; first order magnitudes are all linearly proportional to wave height. Forces proportional to wave height squared or involving products of ship motions with wave height are therefore by definition second order.

Bernoulli Force

Perhaps the simplest second order force to understand is that described by the quadratic term in Bernoulli's equation. With potential theory, Bernoulli's equation gives the fluctuating pressure as:

$$p = \rho\left[\frac{\partial\phi}{\partial t} - \tfrac{1}{2}(\nabla\phi)^2\right] \qquad (1)$$

If ϕ is the potential associated with a linear wave, its magnitude will be proportional to wave height and the "$-\tfrac{1}{2}(\nabla\phi)^2$" term describes a second order pressure which integrated over the ship hull will exert a resultant second order force and moment. For convenience, this force will be referred to as the 'Bernoulli force'.

Difference Frequencies

The Bernoulli pressure can be used as an example to demonstrate how second order effects cause long period forcing. Suppose there are two sinusoidal, regular first order wave components present with frequencies ω_1 and ω_2; the associated potentials can be expressed (and here, *, denotes complex conjugate) in complex Fourier component form:

$$\phi_1 = \tfrac{1}{2}\left[\Phi_1(\underset{\sim}{x}) \exp(-i\omega_1 t) + \Phi_1^*(\underset{\sim}{x}) \exp(i\omega_1 t)\right]$$
$$\phi_2 = \tfrac{1}{2}\left[\Phi_2(\underset{\sim}{x}) \exp(-i\omega_2 t) + \Phi_2^*(\underset{\sim}{x}) \exp(i\omega_2 t)\right]$$

Multiplying to calculate Bernoulli pressure, an expression is obtained that contains the following terms:

$$P = -\frac{1}{8}\rho\ [\nabla\Phi_1.\nabla\Phi_2^* \exp(-i(\omega_1-\omega_2)t) + \nabla\Phi_2.\nabla\Phi_1^* \exp(i(\omega_1-\omega_2)t)]$$

The expression can be seen to represent a pressure wave with a frequency equal to the difference between the two first order wave frequencies. In fact second order effects always produce signals at difference frequencies. Since in a natural sea first order wave energy is continuously distributed across a spectrum of frequencies, difference frequencies may be tiny. Small frequencies equate to long periods, so second order effects generate long period forcing.

In the simple example (used above) with just two wave frequencies, if the difference frequency is small then the two first order waves will interfere, producing beats or alternate groups of large and small waves. The beat (or group) frequency will equal the difference frequency. An association between wave grouping and second order forces can thus be demonstrated; slowly varying drift forces occur with wave groups. It is true in random seas as well as in the simple case.

Steady Drift Force

Second order forces occur at all difference frequencies down to zero difference frequency, where they produce steady drift forces. Steady drift forces' effects are easily seen: the tendancy of all floating objects of all types and sizes to drift in the same direction any waves are going is due to a second order steady drift force.

The usual tendency of the Bernoulli force if it acted alone would be to pull floating objects up into the waves. This does not happen because a more powerful force acts in the opposite direction. That force is surface stress.

Surface Stress

Surface stress is a force that is treated as acting at and along the ship's waterline. Like many second order forces (not including Bernoulli) it can be considered as a relatively small correction to first order force computations. In first order calculations, force is calculated by integrating pressure over the wetted area of the ship hull only below the equilibrium waterline; in reality waves have a definite height and water pressure acts on the hull all the way up to the wave surface. Surface stress is a correction to account for the discrepancy. It causes a force per unit length acting perpendicular to the hull surface equal to:

$$F/d\ell = \frac{1}{2} \rho g \, \eta^2 \qquad (2)$$

Where η is wave elevation, which we take relative to the equilibrium waterline of the vessel displaced by the vessel's vertical movement (Ref 12).

<u>Pressure Gradient Effect</u>

Another second order force that can be described as a correction to first order values is the pressure gradient force. This is the correction for first order force calculations being made taking the vessel at a fixed, mean position, whereas in reality it moves.

First order pressure is given by the linear term in Bernoulli's equation:

$$p = \rho \frac{\partial \phi}{\partial t}$$

Now consider a point $\underline{x}$ on the vessel hull, suppose the ship moves so that point goes to $\underline{x} + \underline{X}$, then we can approximate pressure at the new position using the first terms of a Taylor expansion:

$$p(\underline{x} + \underline{X}) = \rho(\frac{\partial \phi}{\partial t}(x) + X. \nabla \frac{\partial \phi}{\partial t} (\underline{x})) \qquad (3)$$

The second term on the right hand side represents a second order pressure (or rather a second order correction to pressure) which when integrated over the submerged hull area gives rise to a second order force. Second order pressure being proportional to first order pressure gradient, we call this the pressure gradient effect.

First order flow boundary conditions can be used to simplify some pressure gradient calculations. As an example, we simplify calculations for sway motion. Many modern ship hulls have vertical sides over much of their length. On these sides, we can use the boundary condition relating potential gradient to hull velocity ($\underline{V}$):

$$-\nabla \phi . \underline{n} = \underline{V} . \underline{n}$$

If $\underline{X}$ in (3) is purely a sway translation, $\underline{X}$ and $\underline{n}$ are co-directional, $\underline{V} = \frac{dX}{dt}$; the pressure gradient term can be evaluated as follows:

$$\underline{X}.\nabla \frac{\partial\phi}{\partial t} = \underline{X}. \left(\frac{\partial}{\partial t} \nabla\phi\right)$$

$$= \underline{X}. \frac{\partial}{\partial t} (-\underline{V})$$

$$= - \underline{X}. \frac{d^2}{dt^2} \underline{X}$$

The above result holds good at any point on the hull for the component of motion $\underline{X}$ normal to the surface. Potential gradient forces from other components of motion still need to be evaluated by other means.

Force Rotation

Force rotation is another second order correction term to first order forces. It is the correction for the assumption used at first order that forces may be calculated for the vessel in its equilibrium orientation. In fact it rotates in roll, pitch and yaw; as it rotates, the orientation of the hull changes with time, all normals to the hull are rotated, and hence the force on any small area of the hull (given by pressure multiplied by area acting in the direction of the normal) is also rotated. The net effect is that resultant first order hydrodynamic force and moment acting on the vessel both rotate as the ship rotates. The difference between the rotated and the unrotated force and moment is a second order force - the force rotation force.

Using $\underline{F}^{(1)}$ to denote total first order force, $\underline{F}^{(2)}$ to denote (second order) force rotation, $\underline{X}^{(1)}$ to denote (first order) movement, and subscripts 1, 2, 3,...,6 for surge, sway, heave, roll, pitch, yaw respectively (see Fig 1), so for example X_4 is roll movement, $F_6^{(1)}$ is first order yaw moment and $F_3^{(1)}$ is first order heave force, the following six expressions for the components of force rotation can be obtained:

$$F_1^{(2)} = X_5^{(1)} F_3^{(1)} - X_6^{(1)} F_2^{(1)} \qquad (4a)$$
$$F_2^{(2)} = X_6^{(1)} F_1^{(1)} - X_4^{(1)} F_3^{(1)} \qquad (4b)$$
$$F_3^{(2)} = X_4^{(1)} F_2^{(1)} - X_5^{(1)} F_1^{(1)} \qquad (4c)$$
$$F_4^{(2)} = X_5^{(1)} F_6^{(1)} - X_6^{(1)} F_5^{(1)} \qquad (4d)$$
$$F_5^{(2)} = X_6^{(1)} F_4^{(1)} - X_4^{(1)} F_6^{(1)} \qquad (4e)$$
$$F_6^{(2)} = X_4^{(1)} F_5^{(1)} - X_5^{(1)} F_4^{(1)} \qquad (4f)$$

Buoyancy Effect

There is a second order buoyancy effect that causes a heave force and a pitch moment. It comes about because we take rotations to be about the ship's centre of gravity. If the centre of gravity is beneath water level, rotations in roll or pitch will increase the vessel's immersed volume and so increase buoyancy:

$$F_3^{(2)} = \tfrac{1}{2}\, C_{33}C\, (X_4^{(1)2} + X_5^{(1)2}) \qquad (5a)$$
$$F_5^{(2)} = \tfrac{1}{2}\, C_{35}C\, (X_4^{(1)2} + X_5^{(1)2}) \qquad (5b)$$

where:

- C = Distance below waterline of centre of gravity
- C_{33} = ρ.g. waterplane area
- C_{35} = ρ.g. 1st moment of waterplane area.

Second order wave potential forces

All the forces and effects described above are products of first order quantities - either first order waves, or flows associated with those waves, or first-order ship responses. We now move on to consider another family of effects - forces generated by waves that are themselves of second order magnitude. Calculating these effects exactly would require excessive computation, therefore an approximation described in Reference 12 is used here.

The first and best known second order wave is set-down. Set-down has been described many times and in many researchers works, see for example Reference 13. A simple description is that it is a long period wave and pressure disturbance intimately associated with wave grouping; it propagates with wave groups, producing a depression in mean surface level in groups of large waves, and always occurs whenever and wherever wave groups do. Set down amplitude is proportional to first order wave heights squared.

There is a potential flow associated with set down, but the expression for potential, though well established, is long and complicated and will not be repeated here; refer to Reference 13 instead. Suffice it to say that the normal Bernoulli's Equation laws apply for calculating pressures, and that the linearised form of the equation is sufficient for calculations to second order:

$$p^{(2)} = p\, \frac{\partial \phi^{(2)}}{\partial t}$$

In our theory we include only set-down associated with waves incident on the ship. Waves radiating from the ship (ie diffracted waves and waves generated by ship motions) also cause set-down, but the effect is usually negligibly small. Omitting that small set-down component is a reasonable approximation.

Diffraction of incident set-down around the vessel was shown to be significant in Reference 12. Set-down diffraction causes waves radiating from the ship like ordinary first order diffraction; these waves are not tied to wave groups, they obey the same dispersion relationships as first order waves, flow boundary conditions are the same at the sea bed and the free water surface while at the ship hull there is a similar boundary condition:

$$- \nabla(\phi_d^{(2)} + \phi_s^{(2)}) . \underline{n} = 0 \qquad (6)$$

where $\phi_s^{(2)}$ is set-down potential
$\phi_d^{(2)}$ is set-down diffraction potential

Set-down diffraction may therefore be treated in the same way as a low frequency first order, linear wave. We use the same methods for computing set-down diffraction forces in DRIFTKEEL as we used in UNDERKEEL (Ref 1) for linear diffraction.

Another second order diffraction wave arises because of first order ship movement. The no flow hull boundary condition states:

$$-\nabla\phi.\underline{n} = \dot{\underline{X}}.\underline{n}$$

$\dot{\underline{X}}$ is the local hull motion velocity. First order wave potentials satisfy this equation exactly for the hull placed in its equilibrium position. But higher order potentials need to be added to satisfy boundary conditions once the hull is realistically assumed to change its position.

Taylor series expansion of the potential gradient $\nabla\phi$ gives (cf. equation (3)):

$$\nabla\phi(\underline{x} + \underline{X}) = \nabla\phi(\underline{x}) + (\underline{X}.\nabla)\nabla\phi(x) + \ldots$$

Also, extra terms need to be added to normal vectors to reflect hull rotations:

$$\underline{n} = (n_1, n_2, n_3) + (X_5 n_3 - X_6 n_2, X_6 n_1 - X_4 n_3, X_4 n_2 - X_5 n_1)$$

Letting:

$$\underline{n}^{(0)} = (n_1,n_2,n_3)$$
$$\underline{n}^{(1)} = (X_5n_3-X_6n_2,\ X_6n_1-X_4n_3,\ X_4n_2-X_5n_1)$$

The expanded equation for the boundary conditions is:

$$-\nabla(\phi^{(1)} + \phi^{(2)}).(\underline{n}^{(0)} + \underline{n}^{(2)}) - (X^{(1)}.\nabla)\ \nabla\phi^{(1)}.\underline{n}^{(0)} = \dot{\underline{X}}^{(1)}.\ (\underline{n}^{(0)}+ \underline{n}^{(1)})$$

First order terms equate. Retaining only second order terms, we are left with boundary conditions for the second order motion diffraction potential:

$$-\nabla\phi^{(2)}.\underline{n}^{(0)} = (\dot{\underline{X}}^{(1)} + \nabla\phi^{(1)}).\underline{n}^{(1)} + (\underline{X}^{(1)}.\nabla)\nabla\phi^{(0)}.\underline{n}^{(0)} \quad (7)$$

Like set-down diffraction, this second order potential ($\phi^{(2)}$) represents a free wave of a similar nature to a first order linear wave; we solve it in DRIFTKEEL by similar methods as were used in UNDERKEEL. Terms on the right hand side of (7) may all be considered known quantities. There is therefore no fundamental theoretical difficulty in finding a solution although evaluating the boundary condition can be complicated, particularly at sharp edges (eg bilge keels) on the hull.

Implementation

All the second order forces described above are evaluated in the DRIFTKEEL model. Not all components of all of them are computed however. The importance to us of second order forces arises from their effects at low frequencies. Second order forces do also have short period components, but these are comparatively inconsequential for ship movement because resonant periods are too long to be excited by them, so they are omitted from DRIFTKEEL. Roll movement predominantly happens at the natural period of roll in response to first order waves. Since second order forces are unimportant in driving rolling motions, DRIFTKEEL does not include second order roll moment calculation. First order roll motions and moments are important and included. We model long period, second order forces and moments in surge, sway, heave, pitch and yaw; results are discussed in following sections of this report.

Symmetry arguments for a free ship or a ship at an open jetty justify some simplification of second order force calculations. For example, sway forces can only result from pressure differences from side to side of the hull, so effects that produce the same pressure on both sides of the hull generate no sway force. In general, waves around a ship may be divided into those components symmetrical about the ships longitudinal axis and those anti-symmetrical; side to side ship movements (sway, roll, yaw) are associated with anti-symmetrical waves and longitudinal and vertical motions (surge, heave, pitch) go with symmetrical ones. Symmetrical and anti-symmetrical waves have symmetrical and anti-symmetrical potentials respectively. An anti-symmetric pressure is needed to generate a sway force. For illustration, consider Bernoulli pressure:

$$p = -\tfrac{1}{2}\rho\ (\nabla\phi)^2$$

Divide potential into symmetric (ϕ_s) and anti-symmetric (ϕ_a) parts, Bernoulli pressure expands to:

$$p = -\tfrac{1}{2}\rho[(\nabla\phi_s)^2 + 2\nabla\phi_a\nabla\phi_s + (\nabla\phi_a)^2]$$

We find that: $[(\nabla\phi_s)^2 + (\nabla\phi_a)^2]$ is a symmetric quantity and so incapable of giving a sway force (although it will force surge, heave and pitch); $\nabla\phi_a\nabla\phi_s$ gives an anti-symmetric pressure that will drive sway.

In general, anti-symmetric second order pressures arise only from combinations of a first order symmetric quantity (eg wave height or potential) with an anti-symmetric - as in the Bernoulli pressure example shown above. Only half the possible number of first order combinations therefore needs to be considered as possible contributors to sway force, and conversely only the other half can contribute to surge, heave and pitch forcing.

DRIFTKEEL is a frequency domain model; that is it calculates forces on ships exerted by regular waves of given frequencies. Given these forces, responses may then be found. In second order theory pairs of regular primary wave frequencies are taken; first order forces and responses are calculated using UNDERKEEL methods (built into DRIFTKEEL); and DRIFTKEEL then calculates second order forces. Second order forces will generally vary in time sinusoidally with a frequency equal to the difference of the two original primary wave frequencies. A special case occurs when the two primary frequencies are identical: this is a second order force arising from a wave interacting with itself, such forces always occur in any wave pattern and, difference frequencies being zero, the result is a steady drift force.

Any vessel may naturally move in response to second order forcing at difference frequencies. This motion will generate waves in the surrounding water exactly as first order motions do, and (again like first order motions) there will be hydrodynamic added inertias and damping forces acting on the ship. When associated with second order motions these are technically second order forces; computation of them and flow boundary conditions, however, are identical to first order procedures and boundary conditions as implemented in UNDERKEEL.

Responses may be calculated by solving the equation of motion:

$$(\underline{M}+\underline{A})\ \ddot{\underline{X}}^{(2)} + \underline{B}.\dot{\underline{X}}^{(2)} + \underline{C}.\underline{X}^{(2)} = \underline{F}^{(2)} \qquad (8)$$

Where: $\underline{M}$ is the ship's inertia matrix
$\underline{A}$ is the added inertia matrix
$\underline{B}$ is the added damping matrix
$\underline{C}$ is a matrix combining buoyancy and mooring restoring forces and moments
$\underline{X}^{(2)}$ is second order response
$\underline{F}^{(2)}$ is total second order forcing

The form of equation (8) allows for linearised mooring forces. The frequency domain approach adopted in DRIFTKEEL is applicable to linear moorings, but breaks down with non-linear mooring forces, so the model cannot be used for the common mooring arrangement of a ship against fenders. To model these cases, an adaptation to second order of the impulse response function (Ref 6) needs to be added to SHIPMOOR. Linear mooring forces can be included in DRIFTKEEL calculations - both in calculating the first order responses needed in second order force calculations and in the ultimate calculation of long period responses.

Second order forcing and response spectra

Output from DRIFTKEEL is in the form of dimensionless second order forces and responses for pairs of unit amplitude regular first order wave trains each of specified frequency and direction. Forces and responses are given in complex Fourier component form retaining phase information so force and motion time series may be constructed if wished with correct relationships to primary waves. But often we need only second order spectral information; a second-order response spectrum (for example) may be constructed as follows:

Let: $S^{(1)}(f,\Theta)$ be a (first order) wave elevation directional spectrum giving energy density of waves propagating at frequency f and direction Θ.

$X^{(2)}(f,f,\Theta_1,\Theta_2)$ be the complex second order response function giving a ship response at the difference frequency, f^-, to two regular, unit amplitude wave trains: one with frequency f and direction Θ_1, the other with frequency $f + f^-$ and direction Θ_2.

Second order response spectrum is given by:

$$S_X^{(2)}(f^-) = 2 \int_{f=o}^{\infty} \int_{\Theta_1=o}^{2\pi} \int_{\Theta_2=o}^{2\pi} \left| X^{(2)}(f^-,f,\Theta_1,\Theta_2) \right|^2 S^{(1)}(f,\Theta_1)\, S^{(1)}(f+f^-,\Theta_2)\, df\, d\Theta_1\, d\Theta_2 \quad (9)$$

3. SWAY FORCE AND YAW MOVEMENT

DRIFTKEEL has been developed to calculate second order sway forces and yaw movements - the two may be taken together as the calculations involved are almost identical. No yaw moment results are presented in this report.

3.1 Comparison of DRIFTKEEL with a two dimensional model

DRIFTKEEL is a three dimensional model in which the ship being modelled has length, breadth and depth, and flow variations in all three directions around the hull are represented realistically, But along a long ship in a beam sea lengthwise flows can be expected to be slow and small; in these circumstances, a two dimensional model with just transverse and vertical flows may be used to calculate sway forces per unit length fairly accurately. A two dimensional model is relatively simple to implement.

For verification of DRIFTKEEL, a two dimensional second order sway force model was written using similar theory to DRIFTKEEL except for the omission of first order roll for the sake of simplicity. Surge, pitch and yaw motions cannot be represented in these two hull-cross-sectional dimensions (and will be small anyway), leaving just heave and sway present in the 2-D model.

Two dimensional model results are presented in Figs 2 and 3; comparable results using DRIFTKEEL with first order roll constrained to be zero are in Figs 4 and 5. Figures 2 and 4 show steady drift forces; 3 and 5 slowly varying forces with fifty second period. In all cases dimensionless long period sway force magnitudes per unit length for pairs of unit amplitude wave trains are plotted against the two waves' average frequency. Difference frequencies are kept constant. The different types of second order force discussed in section 2 (Bernoulli, Surface Stress, etc) are plotted separately in addition to total force. Generally, the sum of the different type component force magnitudes is larger than total force magnitude; this is not a mistake, it arises because plotted curves represent absolute values of complex numbers.

The ship hull used in these calculations was prismatic in shape with a rectangular cross section and a flat bottom. In DRIFTKEEL, the bow and stern ends of the hull were closed by vertical, transverse surfaces: no attempt was made to taper the hull shape in any way. The two dimensional model cannot, by its nature, represent flows at the ends of the hull; it effectively works assuming an infinitely long ship. The vessel's cross section was identical to the midships section of the ship we also used for surge force tests (section 4 of this report):

Hull length	310m	(DRIFTKEEL)
	∞	(2D model)
beam	47.17	(semi-beam, Y = 23.59m)
draught	18.90m	
Water depth	22.7m	

The underkeel clearance was therefore 20% of hull draught or 16.67% of water depth.

Forces generated by long crested waves from only one wave direction are shown; all waves are propagating perpendicular to the vessel.

It will be seen that forces computed using the two different models are mostly similar. They thus provide some mutual corroboration.

The (fairly small) differences in both second order diffraction forces between Figs 3 and 5 are not unexpected. The forces are being generated by a fifty second period (0.02 Hz difference frequency) diffraction wave scattered from the vessel. Such a wave has a wavelength of about 700m - rather longer than the ship length, so end effects and the finite hull length will profoundly affect wave generation and diffraction in the DRIFTKEEL model. The same cannot happen in two dimensions.

Both Figure 3 and Figure 5 show set-down diffraction as being the dominant effect causing slowly varying sway drift forces. Over the whole range of primary wave frequencies it is a dominant enough effect that it alone would give a good approximation to total force magnitude. But these results have been obtained using long crested primary waves. It is known (Ref 13) that set-down is at its largest in that wave condition. In a natural short crested sea, we expect set-down and set-down diffraction to be a relatively less dominant cause of sway forcing on any vessel than these results would suggest.

Set-down diffraction does not contribute to steady drift forces - set-down diffraction pressures being proportioned to rates of change of potentials (Bernoulli's equation) can have no steady component. Newman's approximation (Ref 14) has been proposed as an approximate method for estimating slowly varying drift forces; it is derived from consideration of steady drift effects, and therefore omits set-down and set-down diffraction forces. Our slowly varying sway force calculations confirm (see eg Ref 12 for an earlier statement) the approximation's inadequacy in many circumstances in shallow water.

3.2 Steady drift force

Steady drift forces, the zero difference frequency component of second order force, have been studied both for their own importance and because the Newman approximation was seen as a method for computing slowly varying forces. There is therefore some literature describing studies into the effect; examples are Refs 15 and 16. These studies are concerned with effects on ships in deep water, in which the Newman approximation is more often valid; we are not aware of any work specifically concerned with second order forces on vessels with small underkeel clearances such as we are considering here. The previous work is therefore not always directly comparable with our model. But important grounds for comparison do exist. In particular, there are many points connected with steady drift forces which are not sensitive to water depth.

Our sway steady drift force results are presented in Figs 2 (2D model), 4 (DRIFTKEEL without roll) and 6 (DRIFTKEEL with roll). Consideration of Fig 6 shall be deferred to the next sub-section, in this section we shall concentrate on Figs 2 and 4. Figures 2 and 4 show a similar pattern, and one which resembles closely transverse steady drift forces in Ref 15: Very little force is exerted by very low frequency primary waves. There is a peak force at heave resonant frequency (about 0.06 Hz), and then the force tends to a high frequency limit value fairly rapidly as primary frequency increases. Surface stress is the largest force throughout. Bernoulli force is generally significant and has a high frequency limit half the size of surface stress. Pressure gradient force is significant only near heave resonance. Other types of forces make no contribution to steady drift force if there is no roll, pitch or yaw present.

Surface stress pushes a vessel in the direction of wave travel; Bernoulli and pressure gradient pull it in the other direction. The resultant is that total force is smaller than surface stress but pushing in the same direction.

Looking at Figures 3 and 5 showing slowly varying (not steady) drift forces for a moment; we see that the variation with frequency of those force components that contribute to steady drift is very similar when there is non-zero difference frequency. Newman's approximation is only invalid because extra forces come into play with slowly varying drift forces; it will be valid in circumstances where those extra forces are small. These extra forces will be smaller in deep water and at very high primary wave frequencies; Newman's approximation might then be good. It happens that our interest is in shallow water ship behaviour, not always in particularly short period waves, so the approximation is not sufficient as explained in Reference 12.

Drift force is very small for very low frequency first order waves. This is because at these frequencies the ship moves very closely with the water flow associated with incident waves and very little wave energy is scattered from the vessel. Without wave scattering there is no mean pressure difference across the width of the ship and hence no effective force. An alternative interpretation is that drift forces can be shown to be associated with transfers of momentum between waves and the vessel. Newton's third law states that for every action there must be an equal and opposite reaction; in this case, drift force is a reaction to momentum flux bound up in scattered wave groups. Absence of scattered waves implies no drift force.

At heave resonance on the other hand, the large motions will generate large scattered waves and so a lot of drift force. And large ship motions relative to the orbital motions of water particles below the incident waves produce large pressure gradient forces in particular, which can be seen in Figures 2 and 4.

At high frequencies the ship moves very little in response to incident primary waves. There is consequently very little wave generated by ship motion. But there is nearly perfect reflection of incident waves off the 'upwave' side of the ship and near total wave shelter downwave, so wave diffraction is very significant. This diffraction produces drift force.

The high frequency situation can be examined via the simple two dimensional model. Suppose the upwave side of the ship is at $y = Y$, the downwave side at $y = -Y$. The incident wave elevation is:

$$\eta(y,t) = \cos\ (ky + \omega t)$$

The reflected wave in $y \geqslant Y$ is (assuming perfect reflection):

$$\eta(y,t) = \cos\ (k(2Y-y) + \omega t)$$

Total wave in $y \geqslant Y$ is:

$$\eta(y,t) = \cos\ (ky + \omega t) + \cos\ (k(2Y-y) + \omega t)$$

$$= 2\cos k(Y-y)\ .\ \cos\ (kY + \omega t)$$

On the downwave side, $y \leqslant -Y$, perfect reflection upwave implies perfect shelter on the other side and there is no wave.

Resultant steady surface stress per unit ship length is (eg (2)):

$$F = \tfrac{1}{2}\rho g\ |\eta|^2$$

$$= \tfrac{1}{2}\rho g\ \tfrac{1}{2}\ 2^2$$

$$= \rho g \qquad (10)$$

The other significant force acting is Bernoulli. Orbital water flows at the ship's side will be purely vertical. If the surface is at $z = 0$ and water is deep:

$w = -2\omega \sin (kY + \omega t) \exp kz \qquad \text{at } y = Y$

Bernoulli force per unit length is

$$F = -\tfrac{1}{2}\rho \int_{-\infty}^{0} |w|^2 \, dz$$

$$= -\tfrac{1}{2}\rho \, \tfrac{1}{2}(2\omega)^2 \int^{0} \exp 2kz \, dz$$

$$= -\tfrac{1}{2}\rho \frac{\omega^2}{k}$$

But the dispersion relationship for linear waves in deep water gives $\omega^2 = gk$, so:

$$F = -\tfrac{1}{2} \rho g \qquad (11)$$

Comparing (10) and (11) we can see Bernoulli force is half surface stress and acting in the opposite direction - a result Figures 2 and 4 illustrate at their high frequency ends.

It is notable that high frequency, short wavelength primary waves, which cannot ever cause significant motions of large ships by first order forces, can nevertheless cause second order long period forces through surface stress and Bernoulli effects, They can thus indirectly cause significant long period movements but not short period ones. Short period waves, of the order of six or seven seconds are common in moderate sea states in European and North Atlantic waters. They do not cause problems to shipping in well protected ports, but they might be expected regularly to cause large ship movements at exposed berths with inferior mooring arrangements.

Set-down and set-down diffraction are also likely to generate significant long period forcing from high frequency waves. Figures 3 and 5 show set-down diffraction as being the dominant effect even at the highest primary frequencies displayed although its effects are also relatively smaller at smaller difference frequencies and for short crested seas; we have shown here forces for a difference frequency of 0.02 Hz(50 second period), set-down forces would be roughly only half as big at a 0.01 Hz (100 second period) difference frequency while Bernoulli and surface stress forces would be relatively unchanged.

In general, set-down diffraction, Bernoulli force and surface stress may all exert significant long period second order sway forcing from short period waves; the forces that contribute to steady drift force, ie Bernoulli and surface stress, tend to be relatively more significant at small difference frequencies and shorter primary wave periods.

3.3 Roll Effects

All the rotational modes of ship motion (roll, pitch, yaw) are either absent or very small in all the results discussed so far in this report. The ommission was useful as a simplification during the development of DRIFTKEEL. But in reality, rotational modes (and particularly roll) may have significant effects on second order sway forces; it is therefore necessary that they are present in the final, developed version of DRIFTKEEL. Results obtained including roll, pitch and yaw in the model are shown in Figures 6 and 7.

First order roll motions can influence second order sway forces via a number of mechanisms. Each of the types of force described in section 2 (except set-down and set-down diffraction) is affected in a different way. Bernoulli force is affected by the hull's rolling motion pushing water flows around the vessel; the currents have velocity and so cause Bernoulli pressures. There is a pressure gradient effect due to roll motion moving every point on the hull surface to a new position with a different pressure. A similar mechanism operates to generate a motion diffraction wave and so a second order pressure. Roll motion translates components of heave force into sway by the force rotation effect. Surface stress is strongly influenced by roll because we measure wave height from a waterline fixed in the hull; as the ship rolls, the waterline rolls with it so our surface evaluation is changed and a surface stress produced.

The results presented in Figures 6 and 7 were obtained using the definitive version of DRIFTKEEL (including roll). They show dimensionless sway steady drift force per unit length with a period of fifty seconds (Fig 7). Each force is produced by the interaction of effects from two regular unit amplitude first order incident waves; the graphs show variation of force magnitudes with incident wave frequency. All incident waves are long crested, propagating perpendicular to the ship.

The ship is the same as that modelled for the earlier results shown in Figs 4 and 5; water depth is also unchanged, so Figs 4 and 6 are directly comparable, as are Figs 5 and 7; the only significant difference is the inclusion of roll in the model for Figs 6 and 7. The vessel's roll natural frequency is about 0.06 Hz (sixteen second period).

Comparing Figs 4 and 6, we see that sway drift force is similar whether or not roll is in the model at high and low ends of the primary frequency range examined. But roll has dramatic effects around roll resonance where roll response is large.

Finding roll's effect to be big when big roll motions occur is not surprising. It is notable however that roll's effect on each of the types of force (Bernoulli, surface stress, etc) taken in isolation is greater than its effect on total force - there is considerable cancellation of effects taking place.

Similar cancellation can be seen when we examine the slowly varying forces in Figs 5 and 7. Roll causes only a fairly small kink in the total force curve at a mean frequency of 0.07 Hz (due to interactions between large, near resonant roll at 0.06 Hz and other motions and waves at 0.08 Hz). Big roll effects can be seen on surface stress, Bernoulli, pressure gradient, force rotation and motion diffraction forces; but effects cancel and total force closely follows set-down.

diffraction (a force type unaffected by roll) in both graphs. Roll, it seems, affects steady drift forces significantly, but makes less difference to slowly varying sway forces.

4. SURGE, HEAVE AND PITCH FORCES AND RESPONSES

Surge, heave and pitch modes are similar (and distinct from sway, roll and yaw) for reasons of symmetry. Sway, roll and yaw are anti-symmetric; they are forced by differences in pressure from side to side of the vessel, that is they are forced by pressure distributions anti-symmetric about the hull axis, and they also generate anti-symmetric wave patterns when the ship moves. On the other hand, a surge, heave or pitch ship movement will generate a symmetric wave pattern about the ship, and the modes are forced by symmetric pressures, so we call these modes symmetric. As was touched on in Section 2 this symmetric/anti-symmetric distinction makes considerable differences to the details of second-order force calculations. Within DRIFTKEEL, surge, heave and pitch force calculations are similar in many ways and different from sway and yaw. There is therefore some logic in grouping them together in this report.

4.1 Surge Force and Motions

Second order surge forces calculated using DRIFTKEEL (including first order roll pitch and and yaw) are shown in Figs 8 and 9. Figure 8 shows steady drift force; Fig 9 shows slowly varying force at fifty second period. In both cases, the force plotted is a dimensionless force per unit length. Force per unit length was chosen rather than force per unit beam (which would be the natural choice for surge) to give comparability with sway force magnitudes shown in Figures 6 and 7.

As in the sway study unit amplitude, regular, first order, primary, incident waves were taken to cause second order forcing with first order response motions and scattered waves. Incident waves for surge force calculations (Figs 8 and 9) were long crested bow seas (direction 180° in Fig 1).

The ship used here was similar to but not identical to that used earlier for sway forces: overall dimensions were identical, but instead of being a rectangular prism, this hull had a genuine ship hull-form. It was in fact a reproduction of a ship model first used by Van Oortmerssen (Ref 17).

Ship Length	310m
Beam	47.17m
Draught	18.90m
Water Depth	22.70m

Comparison of surge figures with sway show second order surge forces to be much smaller.

The general pattern of surge steady drift forces variation with primary frequency is similar to sway steady drifts without roll (Fig 4). Low frequency force is negligible, high frequency force is fairly constant and there is a peak in between. The reasons for this pattern for surge are the same as those given earlier (Section 3.1) for sway steady drift. Peak force occurs at a slightly lower frequency in surge than sway because it happens with pitch rather than heave resonance, and the large pitch response came at a somewhat longer wavelength. Roll resonance will affect surge second order force in general, but it was small in this case forced by a bow sea, and in a bow sea there is less tendancy for the vessel to move with water flow at low wave frequencies (because it is relatively long compared to wavelengths) than there is in a beam sea, so we see a bit more force at low primary frequencies in surge than we did in sway (Fig 4).

Slowly varying surge drift force (Fig 9) is dominated by set-down force except at the highest frequencies where surface stress and Bernoulli are significant. The very small set-down force there however is only due to wave group length - and hence set down 'wavelength' - equalling the length of the ship at those frequencies: set-down force can be significant at even higher frequencies than those shown. Set-down diffraction force is negligible at all frequencies. This contrasts with second order sway forcing, in which diffraction is dominant. But diffraction is known not to be significant in forcing first order surge either, and the finding is not surprising.

Long period responses and comparison with experiment

Once slowly varying forcing is calculated, and added masses and damping found by UNDERKEEL methods, linear mooring forces and buoyancy forces may be added; then the ship's equation of motion (8) can be solved to find second order motions. Surge, heave and pitch are coupled and treated as being so in all DRIFTKEEL calculations although the connection between surge and the other two is weak.

Figure 10 shows a graphical form the outcome of one such set of computations. It shows surge response functions for the Van Oortmerssen ship, response amplitudes for pairs of unit amplitude waves, varying with primary frequency. A total mooring compliance in surge of 60 tonnes/m was assumed.

This is the linearised stiffness in surge of the mooring system Oortmerssen used in experiments described in Reference 17. His experiments involved both physical and mathematical modelling. The ship was the same one we modelled for long period surge force (dimensions above). His thesis (Ref 17) describes measured responses in several wave conditions, but we are here concerned with the responses he measured in a long crested, bow, random sea. Very considerable long period ranging of the ship was observed (Table 1).

For corroboration of DRIFTKEEL, we set out to replicate the results. Van Oortmerssen used a mooring arrangement with head and breasting mooring lines, an open berth and low friction fenders. We assumed friction to be zero, calculated vessel surge response functions (similar to Fig 10) in surge, and reconstructed Van Oortmerssen's wave spectrum from his published data; the calculations described in Section 2 (equation 9) were performed to complete the modelled vessel's response spectrum.

Results of the calculations are given in Table 1 together with measurements from Reference 17 for comparison. Van Oortmerssen actually only prints figures in his results tables for physical model rms total motion (not high and low frequency ranges separately); however his graphs of response spectra show short period surge was very small in his experiments, and we have taken the published value for rms surge in his mathematical model without long period forcing as representative of short period response.

Comparing our results with his. We find excellent agreement on short period rms values. And (although we have not reproduced his response spectra here), our long period surge spectrum is similar in shape to that shown in Reference 17. Surge energy is concentrated at the, 120 second, natural period. But we over-estimate long period movement by a factor of three.

There are possible explanations for the discrepancy. One is that surge response is sensitive to mooring line stiffness - in a trial we reduced rms surge to 0.72m by changing mooring compliance to 54 tonnes/m. Reference 17 is not absolutely clear about mooring stiffnesses, so we may not be representing the actual value used in the experiment.

Friction is another possibility. We neglected fender friction in our mathematical modelling, but it probably was present in Van Oortmerssen's tests to some extent. Experience with physical models at HR indicate fender friction can reduce motion of a model ship on fenders to about a half the motion in a similar, but fenderless, mooring line arrangement. Friction may therefore easily account for much of the discrepancy in this case.

There also seem to us to be shortcomings in Van Oortmerssen's experimental procedure that could have affected his results. He was representing a random sea wave spectrum by generating only a fairly small number (seventeen in this case) of primary wave frequency components. This was standard procedure at the time the work was done. But it has major drawbacks. One is that the wave pattern will cyclically repeat after a relatively short period of time. In Van Oortmerssen's tests we think this cycle time was 125.7 seconds, which is very close to the natural period of ship movement in surge. Experimenters now know it is very difficult to get results for phenomena happening at the cycle period which are truly representative of what happens in nature. It is therefore possible that the surge respnses Van Oortmerssen measured are not typical of long term average responses that would be observed in the truly random wave conditions our mathematical model represented.

Given these results, we can be fairly confident DRIFTKEEL models long period surge forces well. Using the same theories to model forces in other modes as we do, we can also be fairly confident of DRIFTKEEL's realistic use in sway, yaw, heave and pitch forces. However, this is only a limited validation of the model and comparison with more general physical model results in surge, sway and yaw on a linear mooring is needed.

4.2 Heave Force and Pitch Moment

Heave and pitch are crucial modes of ship motion in navigation channel design. Obviously, any channel has to be designed deep enough to ensure that the risk is remote of vessels heaving and pitching in waves so much that they strike bottom and damage themselves. HR has done studies for several clients testing navigation channels using UNDERKEEL (Refs 2, 3 and 4).
But to responses at wave periods, we need to add long period ship motions. These are expected to be of the order of tens of centimetres in size even in severe sea states with significant wave heights of 3 or 4 metres. But every extra centimetre required to be dredged adds considerably to the costs of a harbour. Up to now we have always assumed the dominant long period vessel motion would in most cases be due to set-down. Set-down causes a depression of the mean water surface, and so a downwards vessel movement, in groups of large waves. Exactly when ship movement can be expected to be at its most violent and the risk of bed contact greatest, set-down always shifts the vessel down and adds to the risk. It seems likely that even small long period vertical movements may be significant.

In studies for clients, we have always conservatively added the whole set-down wave height to estimates of required safe underkeel allowances (Refs 2,3). There are two purposes in modelling heave and pitch forces using DRIFTKEEL: to confirm whether set-down is indeed the dominant vertical force, and to investigate whether a less conservative approach could be taken.

A different set of effects is involved in forcing the vertical motions heave and pitch from that forcing sway, yaw and pitch: Surface stress has no vertical effect on a wall sided ship, but second order buoyancy effects (which are ineffective horizontally) do affect vertical motions. In our graphs of calculated forces and moments, however, second order buoyancy effects have been subsumed into force rotation, so do not appear.

Second order heave forces and pitch movements calculated by DRIFTKEEL are shown in Figures 11-14. The ship modelled is the same as that described in Section 4.1 and used in surge force and motion modelling. As before, we show steady and slowly varying, fifty second period forces and movements plotted against primary frequency. Unit amplitude incident waves are assumed. Heave forces are dimensionless force magnitudes per unit length; plotted pitch values are also magnitudes of complex Fourier components non-dimensionalised with division by ship length squared. Waves are long crested with a direction 45° to the vessel.

Both steady forces are dominated by the rotation buoyancy effect (Equation (5)) at roll resonance. The effect as shown here would have dramatic effects: raising the vessel a steady half metre in regular, metre high (crest to trough) waves with sixteen second periods, for example. It it were true, this would drastically reduce required water depths for some ships in some circumstances. But the effect has undoubtedly been over-estimated here: it is proportional in magnitude to roll squared, and resonant roll is sensitive to the damping applied. Roll damping is known in nature to be largely due to effects such as drag and eddy shedding, for which we know of no entirely satisfactory theory practical for our purposes, and which are therefore not present in our model. Roll damping is consequently under-represented, so we get too much roll at resonant frequencies, and the rotation buoyancy effect is too big. The only way to solve the problem would be to develop and improve our representation of first order roll response.

Rotation buoyancy has no significant effect on slowly varying heave force (Fig 12) or pitch moment (Fig 14) because the frequency bandwidth of resonant roll response is narrow. Only at very small difference frequencies, less than that bandwidth, can rotation buoyancy effects be large.

Slowly varying forces are largely caused by set-down as we had anticipated. Other effects do add to the force however. All of them (Bernoulli, force rotation, potential gradient, motion diffraction) will be at their largest whenever the ship is moving most extremely in response to first order waves. Figure 12 in particular shows total force being slightly greater than set-down alone. They could therefore add significantly to risks of bed contact.

Table 2 shows long period vertical motion compared to set-down wave height. Wave conditions are the same as we used earlier in modelling surge motions, ie a long crested, random bow sea with a significant wave height of 2.58m and the spectrum given in Reference 12. The bow and the stern are the points on the vessel that move vertically the most. Table 2 shows that maximal vessel vertical motions are very similar in size to set-down, as we had anticipated. Bow motion is, in this case, slightly greater, and stern movement slightly less than set-down magnitude. The finding confirms the correctness of using set-down in our earlier work for clients to estimate long period ship vertical movement, and it gives some additional corroboration of DRIFTKEEL's accuracy.

5. CONCLUSIONS

Long period motions of moored ships can be large and have the potential to damage moorings and otherwise disrupt cargo handling. Measures therefore must be taken to restrict them. We believe sometimes an over conservative approach has been taken to harbour design in this respect, and more wave protection has been provided than is needed; providing superfluous protection wastes money on construction and may result in a harbour in which ship manoeuvering is harder than it need be.

Port design can be improved by testing alternatives using accurate models. Either physical or mathematical models or both are feasible. Physical models are guaranteed to include all physical processes so are more reliable, but are expensive to set up; they are, however, needed in refinement of final designs. Mathematical models are generally much cheaper to use and are suitable for testing large numbers of possible options at an earlier design stage.

To date, however there has been no accurate mathematical model developed of long period forcing an moored ships in shallow water. This report describes one called DRIFTKEEL, produced by Hydraulics Research to answer this need. Predictions of forces and ship motions made using DRIFTKEEL agree well qualitatively and quantitatively with the few experimental results that have been published for the case of small underkeel clearances.

The model as it is, is applicable only to free ships or those at linearisable moorings either at open jetty or far from a quay wall. This does cover some important cases and is a useful development in itself, but mooring systems are usually significantly non-linear, and very many ships are berthed against fenders on solid quays. The presence of a quay well transforms the hydraulics of water-wave flows around the vessel (Ref 7). Further work and funding is needed to model vessel long period motions in these common cases.

Long period forces on ships are mostly generated by second order effects, proportional to wave heights squared. They are thus fundamentally different from forces at wave periods, which are predominantly first order and linearly proportional to wave heights. Different calculations are therefore needed to model long period focus on ships from those at shorter periods. First order models are not adequate for long period motions - hence the need to create DRIFTKEEL. Effects causing second order forces are described in Section 2 of this report.

Results presented in this report obtained using the model show that set-down effects are dominant in generating most of the long period force observed. Incident set-down waves directly generated most surge and heave force and pitch moment (Figs 9, 12, 14); diffraction of set-down generated more sway force (Fig 7). This finding is similar to observations made regarding first order Froude-Krylov and diffracted first order wave forcing.

Observed significant slowly varying force effects due to phenomena other than set-down were restricted to relatively narrow ranges of wave frequencies. Only then did the effects causing steady drift forces (see below) become important. Because of this and other inaccuracies (Ref 12) the Newman approximation cannot be assumed valid for use modelling long period forces in shallow water.

Results presented in this report will however tend to overstate set-down effects in that they have been obtained assuming long crested waves. It can be shown (Ref 13) that set-down is smaller in short crested and crossing seas. Since natural seas are always short crested, effects like surface stress and Bernoulli force are relatively more significant in real situations than these results show.

Long period vertical motions modelled using DRIFTKEEL indicate that using set-down alone (as has been done in the past) results in an accurate estimate of long period vertical ship movement in shallow navigation channels.

Surface stress is the largest effect contributing to steady drift forces in surge and sway. Bernoulli forces are significant, asymptotically tending to half surface stress at high wave frequency. Large steady forces are liable to be found around heave, pitch and roll resonant frequencies - generated as a consequence of large, resonant movements.

Resonant roll motions can be seen (comparing Figs 4 and 6) to generate particularly dramatic effects: introducing roll into the model causes huge changes in each of the types of force modelled when each is taken alone. But the changes induced cancel each other out to a considerable extent. The difference in total force is, though still significant, not so dramatic.

There is a paucity of published results against which DRIFTKEEL can be checked. Work has been done concerning second order forces, but much of it has been in connection with deep water applications. Results from few controlled experiments on ships in shallow water are available. There is, therefore, a clear need for experiments to be carried out of a linearly moored vessel in random waves.

Verification of DRIFTKEEL has been obtained by: comparison with a simpler two dimensional model of sway forces (Section 3.1), assessment of steady drift force calculations (Section 3.2), comparison of model estimates of surge motion in a sea with measurements made by experiment (section 4.1), and comparison of vertical movement with set-down wave amplitudes. In each case agreement was at least satisfactory. We may therefore have confidence that DRIFTKEEL is an accurate model of long period, second order wave forces acting in a free ship in shallow water. It can be used to describe the behaviour of moored ships in waves provided the moorings are linear but much further work is needed to model a ship moored against fenders.

6. ACKNOWLEDGEMENT

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7. REFERENCES

1. Lean G H, Bowers E C and Spencer J M A. UNDERKEEL: a computer for vertical ship movement in waves when the underkeel clearance is small, HR Report SR 160, 1988.

2. Hydraulics Research. "Underkeel allowance for deep draughted vessels in the Dover Strait. Phase 1, preliminary study to establish critical parameters. Phase, 2, final estimates for a safe allowance." Report Nos EX 1309, 1985 and EX 1432, 1986.

3. Hydraulics Research. "Use of the approach channel to Port Muhammad bin Qasim, Pakistan, by Panamax container vessels." Report No EX 1841, 1988.

4. Hydraulics Research. "Mathematical modelling of a vessel's vertical motion in waves in shallow water." Report NO EX 1897, 1989.

5. Spencer J M A and Bowers E C. Mathematical Simulations of Moored Ships in the Time Domain. HR Report SR 103, August 1986.

6. Spencer J M A. Mathematical simulations of a Ship Moored in Waves and of the Effects of a passing vessel on a Moored Ship. HR Report SR 145, January 1988.

7. Lean G H, Bowers E C and Spencer J M A. "QUAYSHIP: A computer model of a ship against a quay in the presence of waves". HR Report SR 232, February 1990.

8. Slinn P J B. 'Effect of ship movement on container handling rates', Dock and Harbour Authority, Vol 60, No 705, p117-120, August 1979.

9. Hydraulics Research. "Effect of passing vessels on a moored ship", HR Report EX 566, July 1971.

10. Hydraulics Research. 'Port of Acajutla, random wave model study'. HR Report EX 739, July 1976.

11. Lean G H: 'Subharmonic motions of moored ships subjected to wave action'. Trans. Roy. Inst. Nav. Archit. Vol 113, p387, 1971.

12. Bowers E C. 'The Mathematical Formulation of Non-Linear Wave Forces on Ships'. HR Report SR 193, February 1989.

13. Bowers E C. 'Infragravity waves and harbour design', HR Report SR 226, January 1990.

14. Newman J N. Second order, slowly varying forces on vessels in irregular waves, Proc. Int. symp. on the Dynamics of Marine Vehicles and Structures in Waves, London, 1974.

15. Pinkster J A. Mean and low frequency wave drifting forces on floating structures. Ocean Engng. Vol 6, p593, 1979.

16. Standing R G and Dacunha N M C. Slowly-varying and mean second-order wave forces on ships and offshore structures, 14th ONR Symposium on Naval Hydrodynamics, Ann Arbor, 1982.

17. Van Oortmerssen G: 'The motions of a moored ship in waves', NSMB Report No 510, 1976.

TABLES.

Table 1 : Comparison of surge motion as modelled by DRIFTKEEL with experimental results.

Wave conditions : 2.58 m significant wave height 180° wave direction (long crested) random sea.

	Freq'y (Hz)	DRIFTKEEL	Van Oortmerssen expt.	Van Oortmerssen math'l model
Surge spectrum (m^2/Hz)				
	0.008	163.6		
	0.016	0.607		
	0.024	0.638×10^{-1}		
	0.032	0.522×10^{-2}		
	0.040	0.243×10^{-2}		
	0.048	0.111×10^{-2}		
Long period rms surge (m)		1.14	0.38	0.36
	0.068	0.525×10^{-1}		
	0.076	0.890×10^{-1}		
	0.084	0.986×10^{-1}		
	0.092	0.565×10^{-1}		
	0.099	0.395×10^{-1}		
	0.107	0.530×10^{-1}		
	0.115	0.339×10^{-1}		
	0.123	0.134×10^{-1}		
	0.131	0.133×10^{-1}		
	0.139	0.258×10^{-2}		
	0.147	0.155×10^{-2}		
	0.155	0.142×10^{-2}		
	0.163	0.159×10^{-3}		
	0.171	0.300×10^{-3}		
	0.179	0.553×10^{-4}		
Short period rms surge (m)		0.06	Small	0.07
Total rms surge (m)		1.14	0.38	0.36

Table 2 : Comparison of long period vessel motion spectra modelled by DRIFTKEEL with set down

Wave conditions : 2.58 m significant wave height 180° wave direction (long crested) random sea.

Frequency	Heave	Pitch	Bow Motion	Stern Motion	Set-down amplitude
Hz	m^2/Hz	Deg^2/Hz	m^2/Hz	m^2/Hz	m^2/Hz
0.008	$0.151x10^{-1}$	$0.170x10^{-3}$	$0.214x^{10-1}$	$0.114x10^{-1}$	$0.242x10^{-1}$
0.016	$0.736x10^{-2}$	$0.213x10^{-2}$	$0.234x10^{-1}$	$0.224x10^{-1}$	$0.188x10^{-1}$
0.024	$0.197x10^{-2}$	$0.288x10^{-2}$	$0.235x10^{-1}$	$0.226x10^{-1}$	$0.155x10^{-1}$
0.032	$0.243x10^{-3}$	$0.233x10^{-2}$	$0.196x10^{-1}$	$0.151x10^{-2}$	$0.138x10^{-1}$
0.040	$0.259x10^{-3}$	$0.621x10^{-3}$	$0.570x10^{-2}$	$0.393x10^{-2}$	$0.114x10^{-1}$
0.048	$0.383x10^{-3}$	$0.580x10^{-4}$	$0.925x10^{-3}$	$0.692x10^{-3}$	$0.793x10^{-2}$
RMS values	0.0142	0.0081	0.0274	0.0246	0.0270

FIGURES.

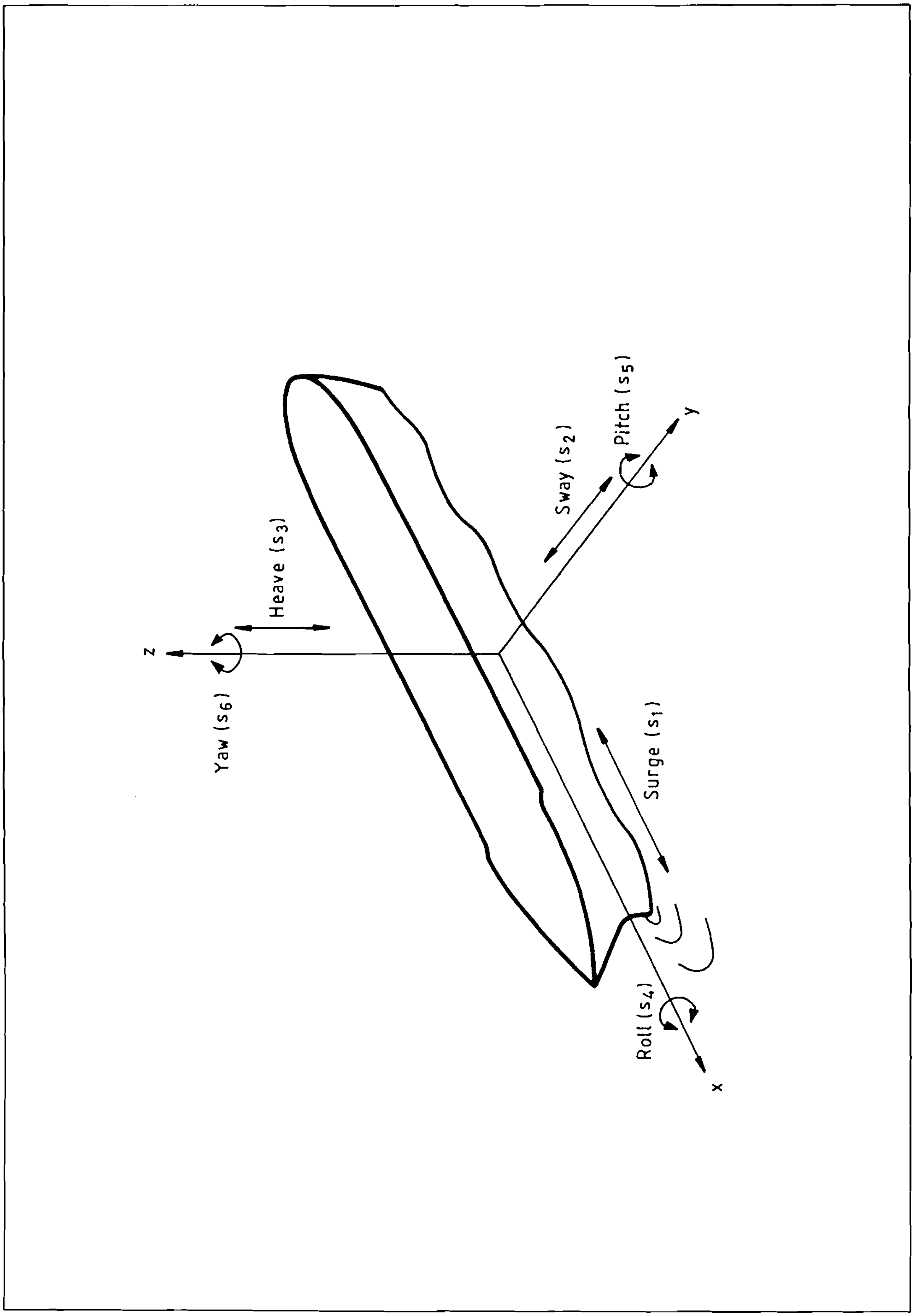

Fig. 1 The six degrees of vessel movement.

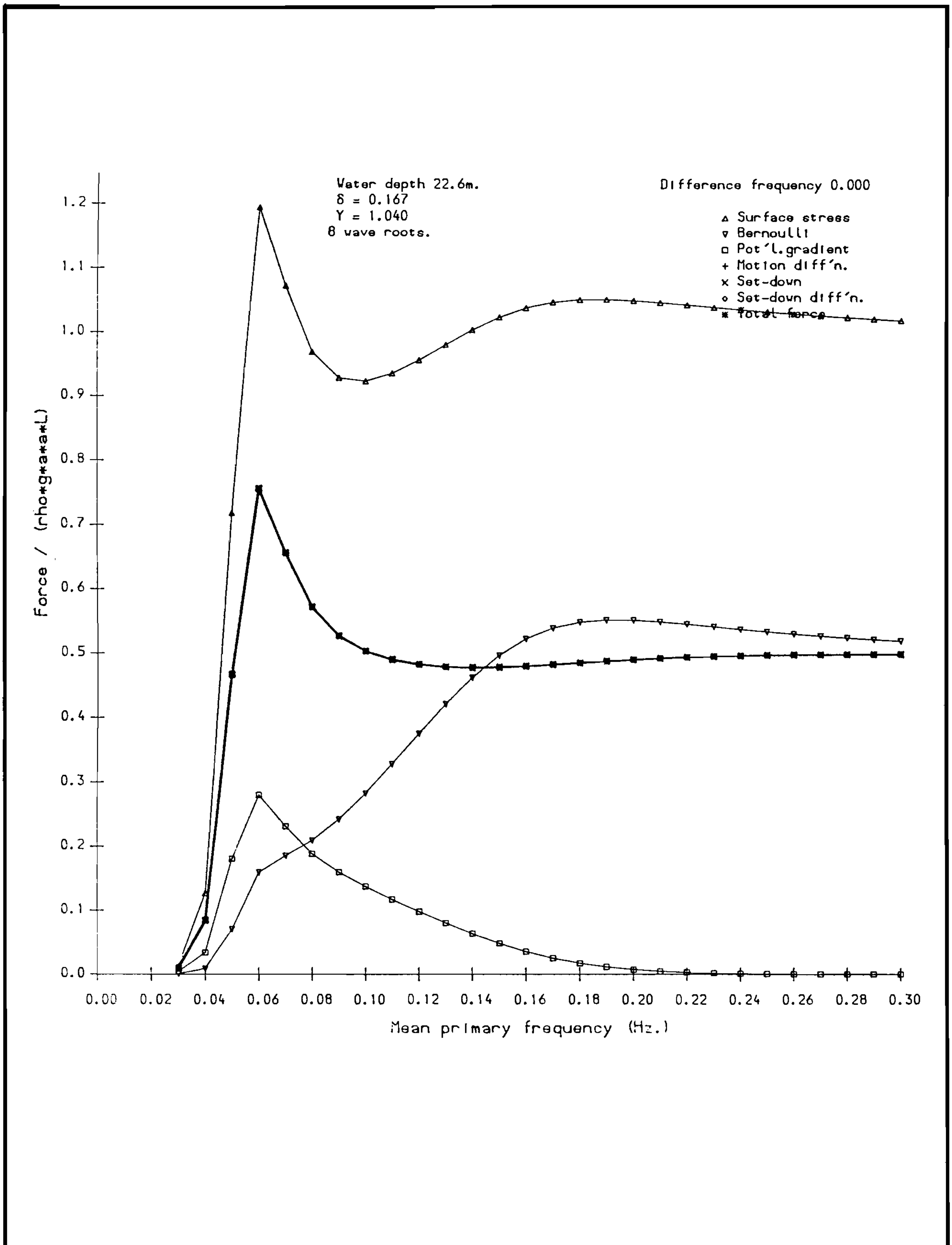

Fig 2 Sway steady drift forces - 2D model , beam sea.

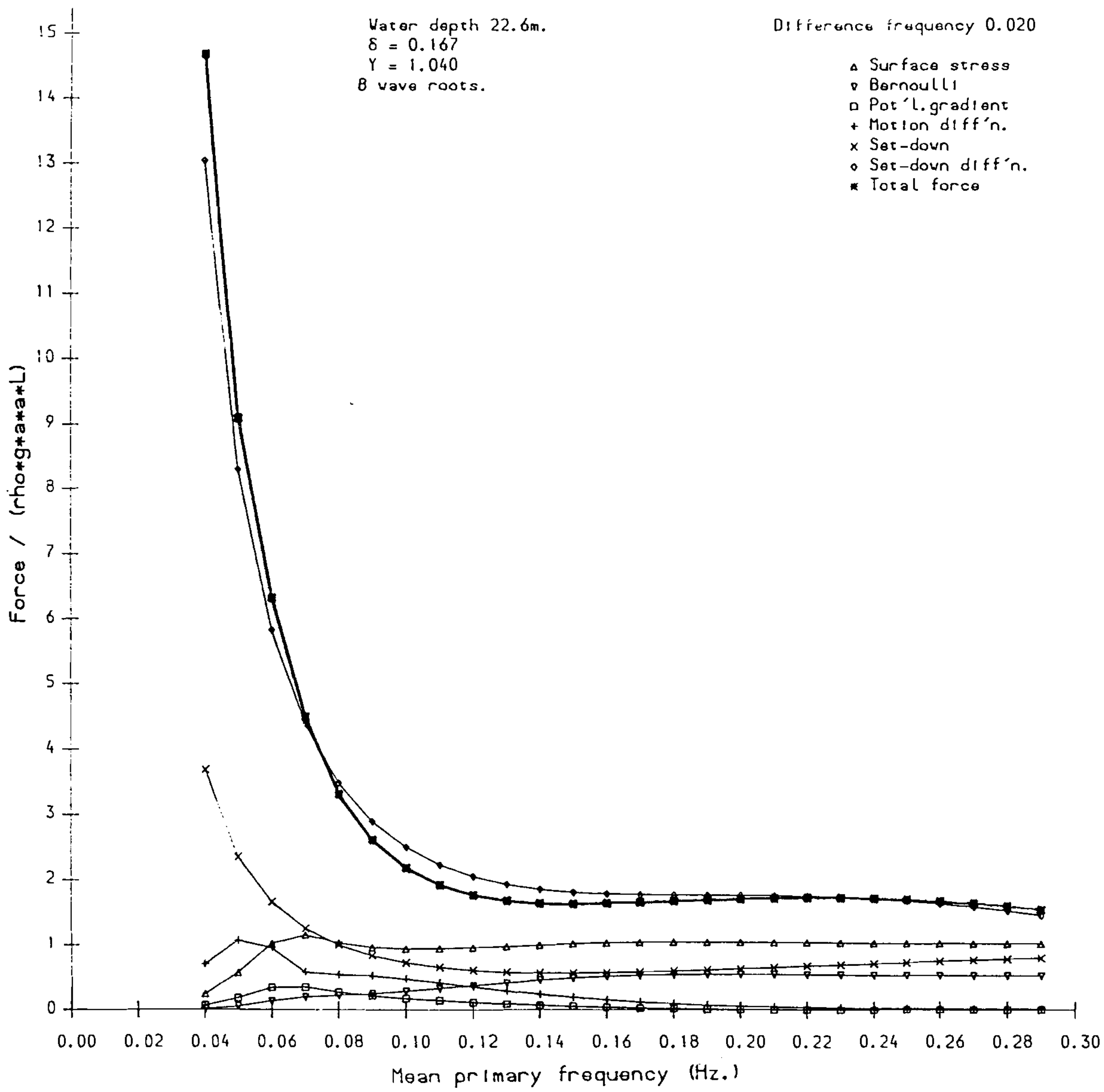

Fig 3 Sway force , 50 second period - 2D model , beam sea.

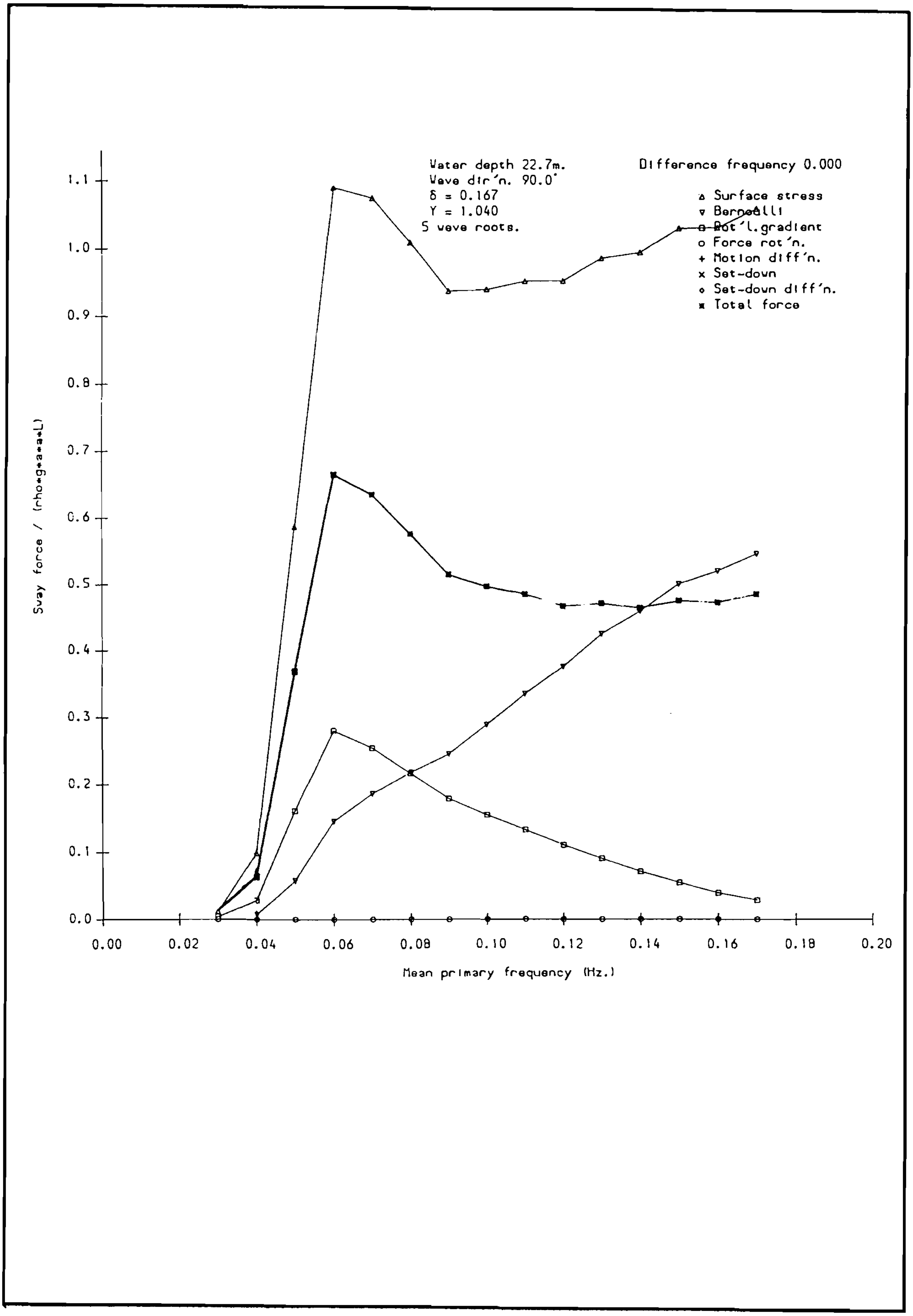

Fig 4 Sway steady drift force - DRIFTKEEL , no roll , beam sea.

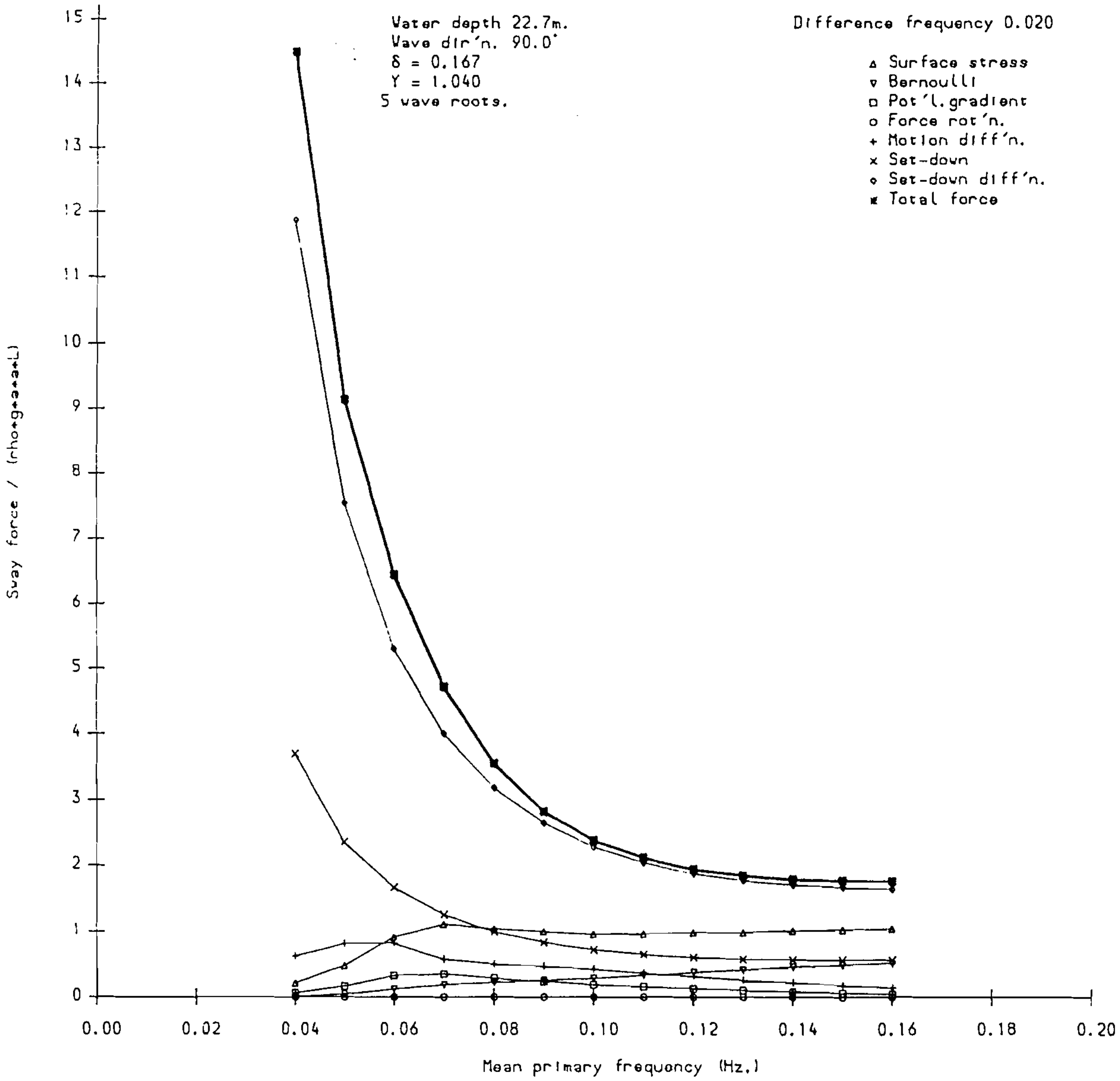

Fig 5 Sway force , 50 second period - DRIFTKEEL , no roll , beam sea.

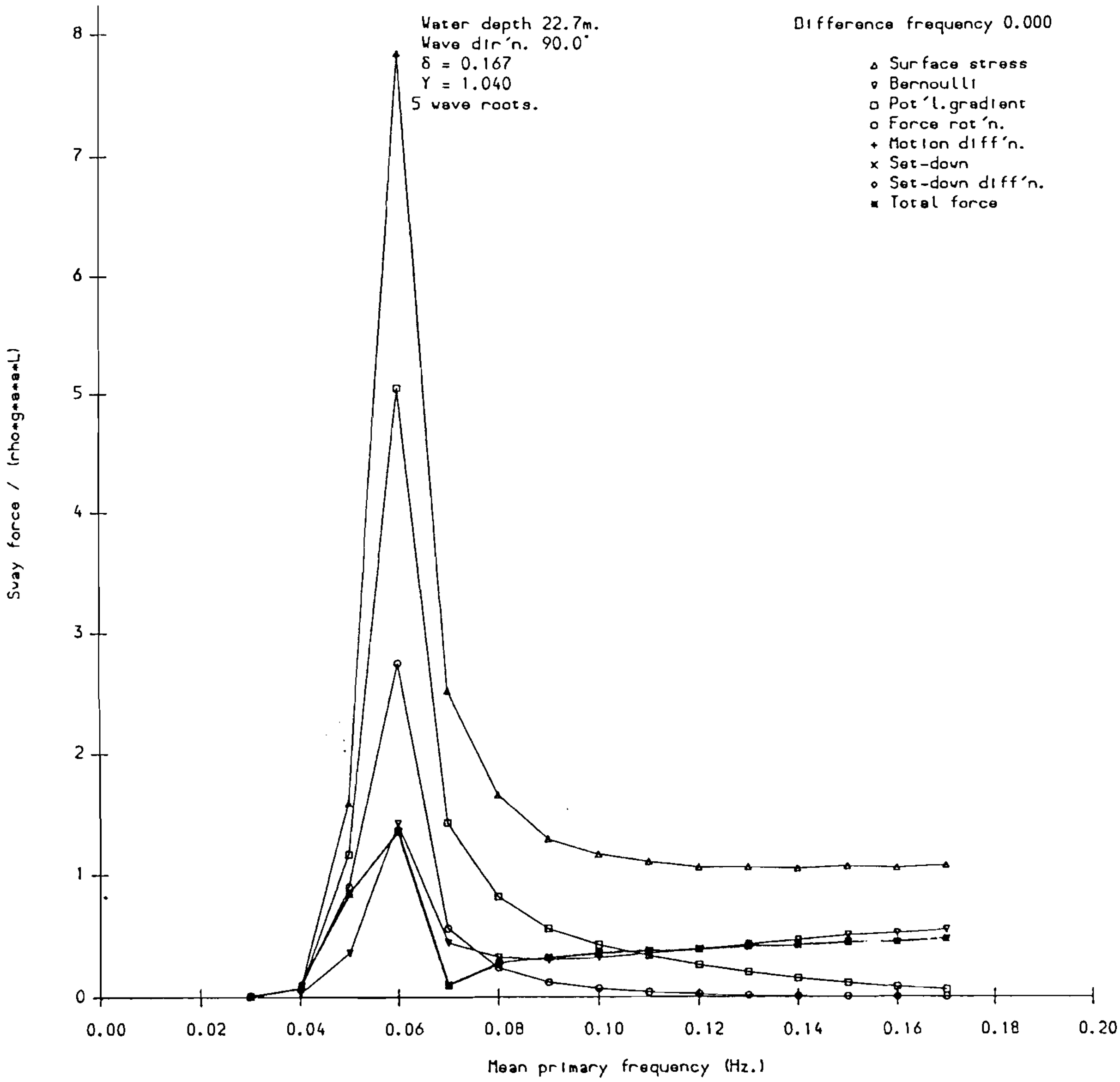

Fig 6 Sway steady drift force - DRIFTKEEL , beam sea.

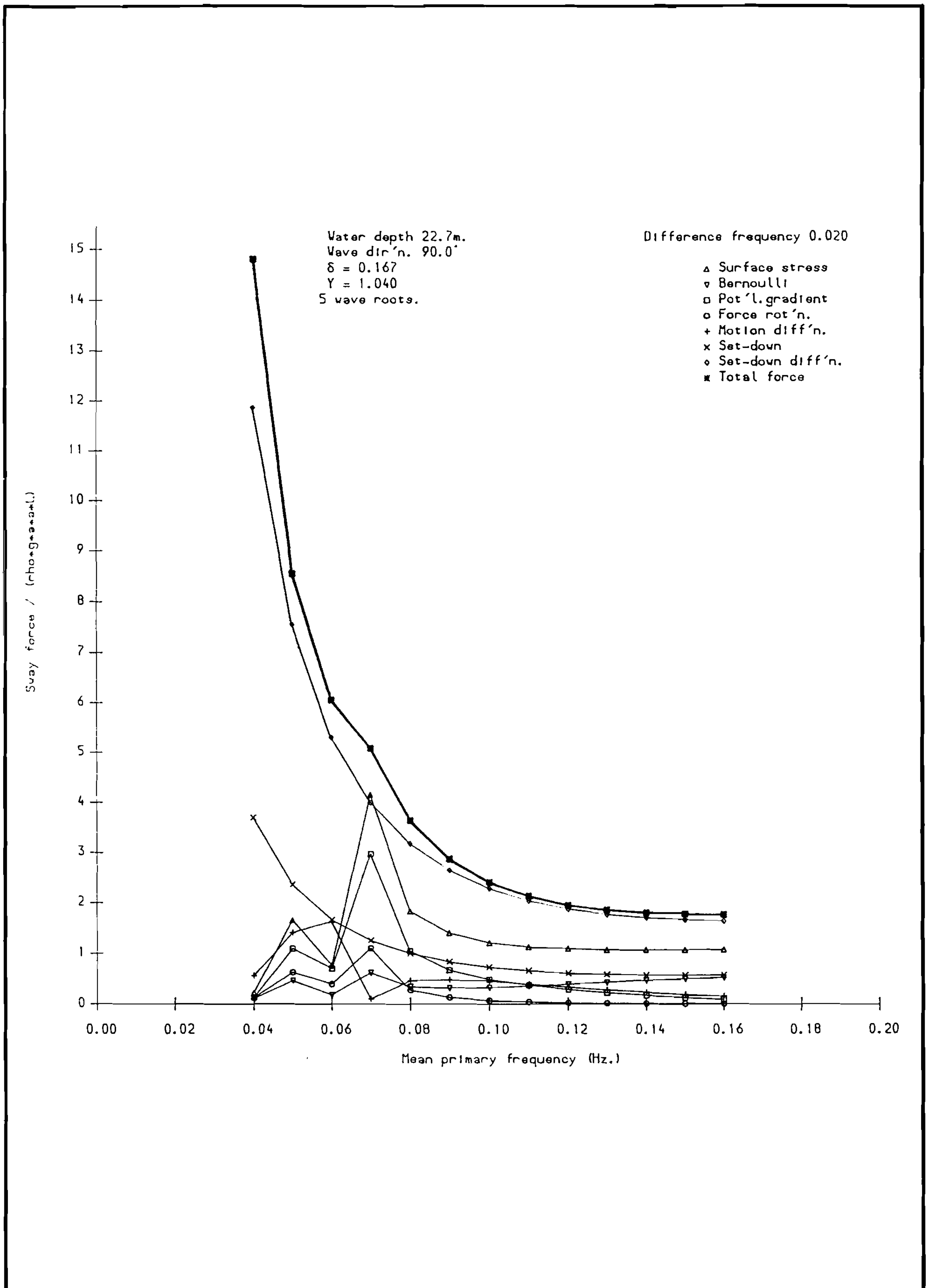

Fig 7 Sway force , 50 second period - DRIFTKEEL , beam sea.

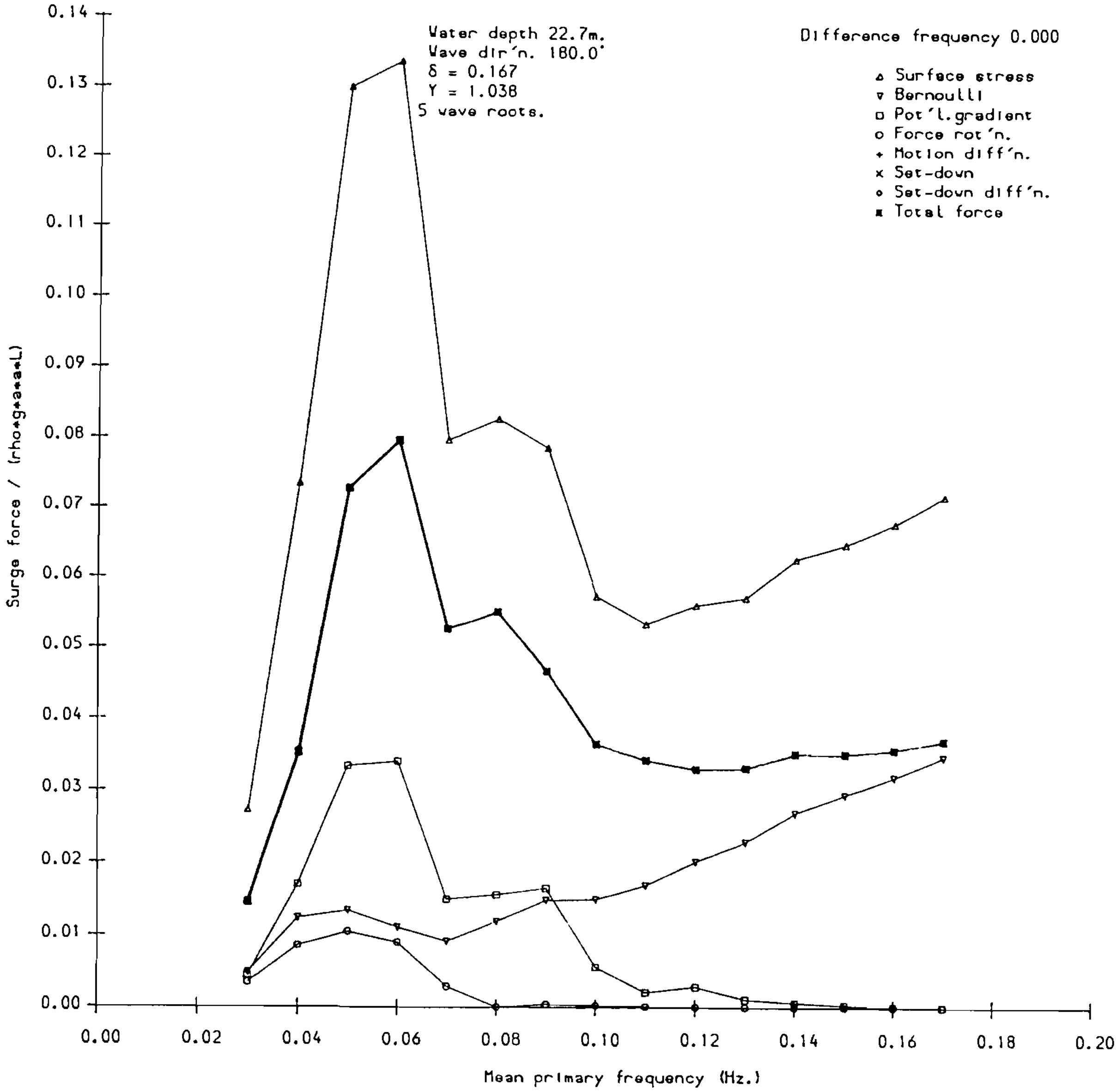

Fig 8 Surge steady drift force - DRIFTKEEL , bow sea.

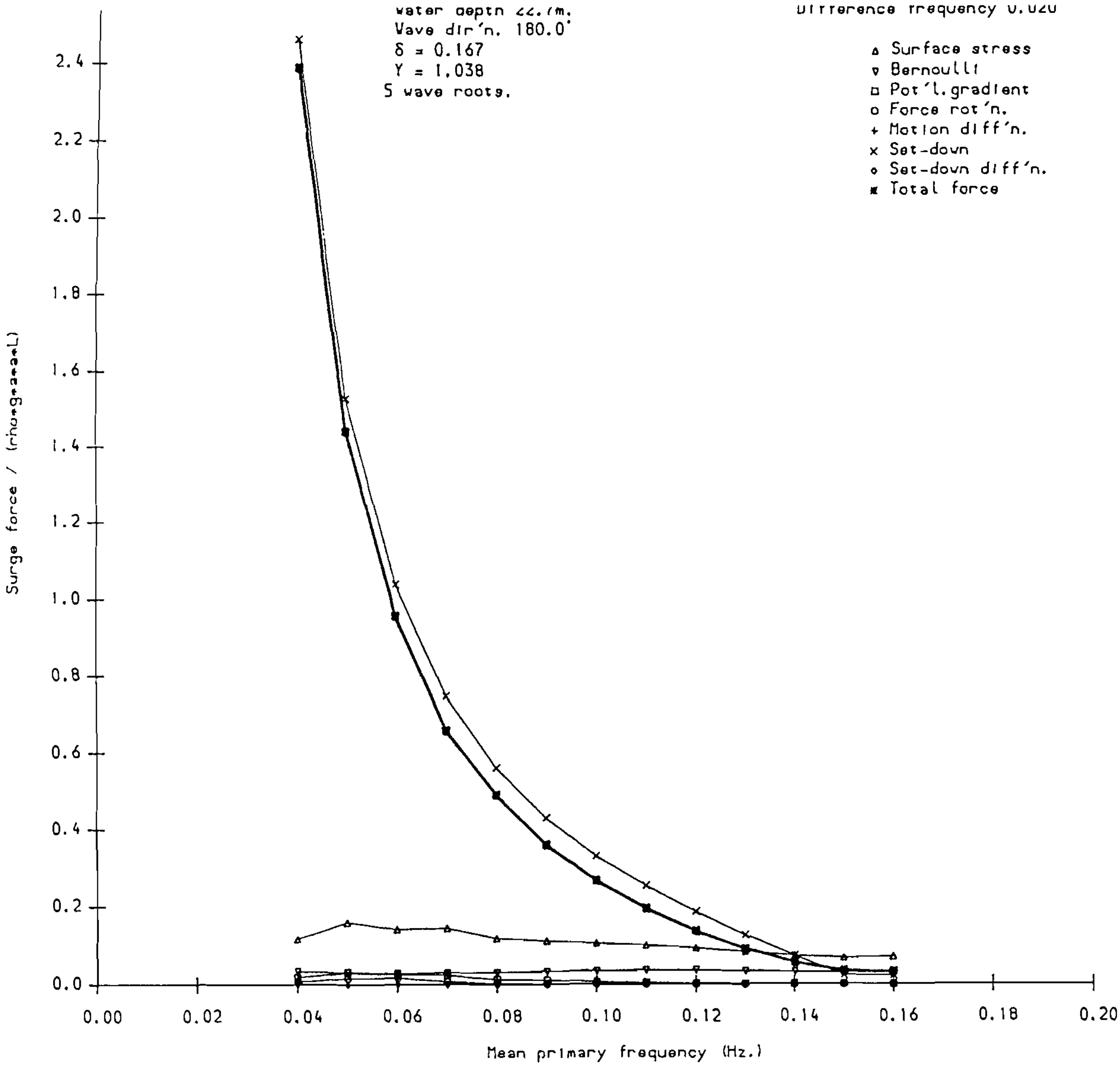

Fig 9 Surge force , 50 second period - DRIFTKEEL , bow sea.

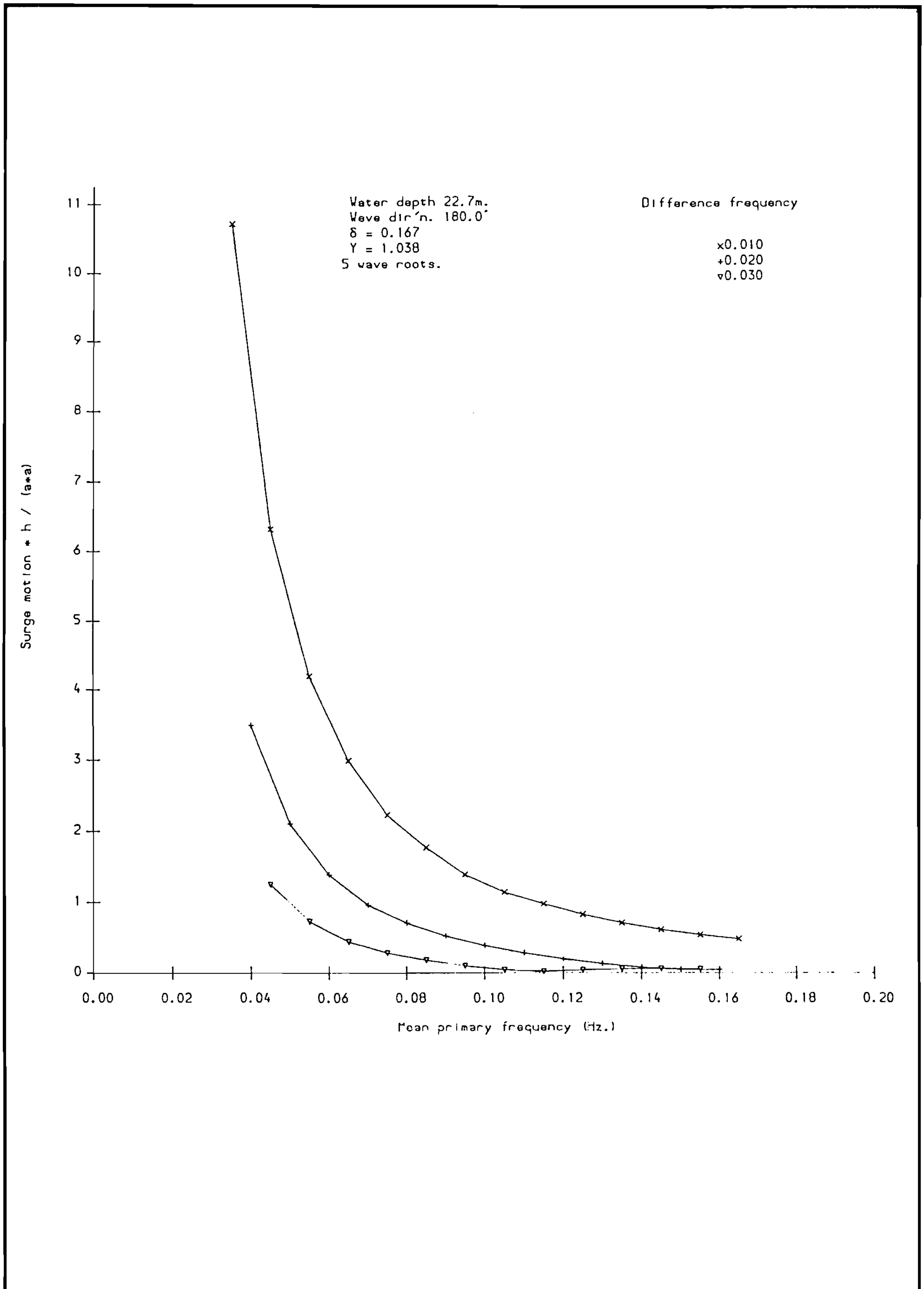

Fig 10 Surge long period responses - bow sea.

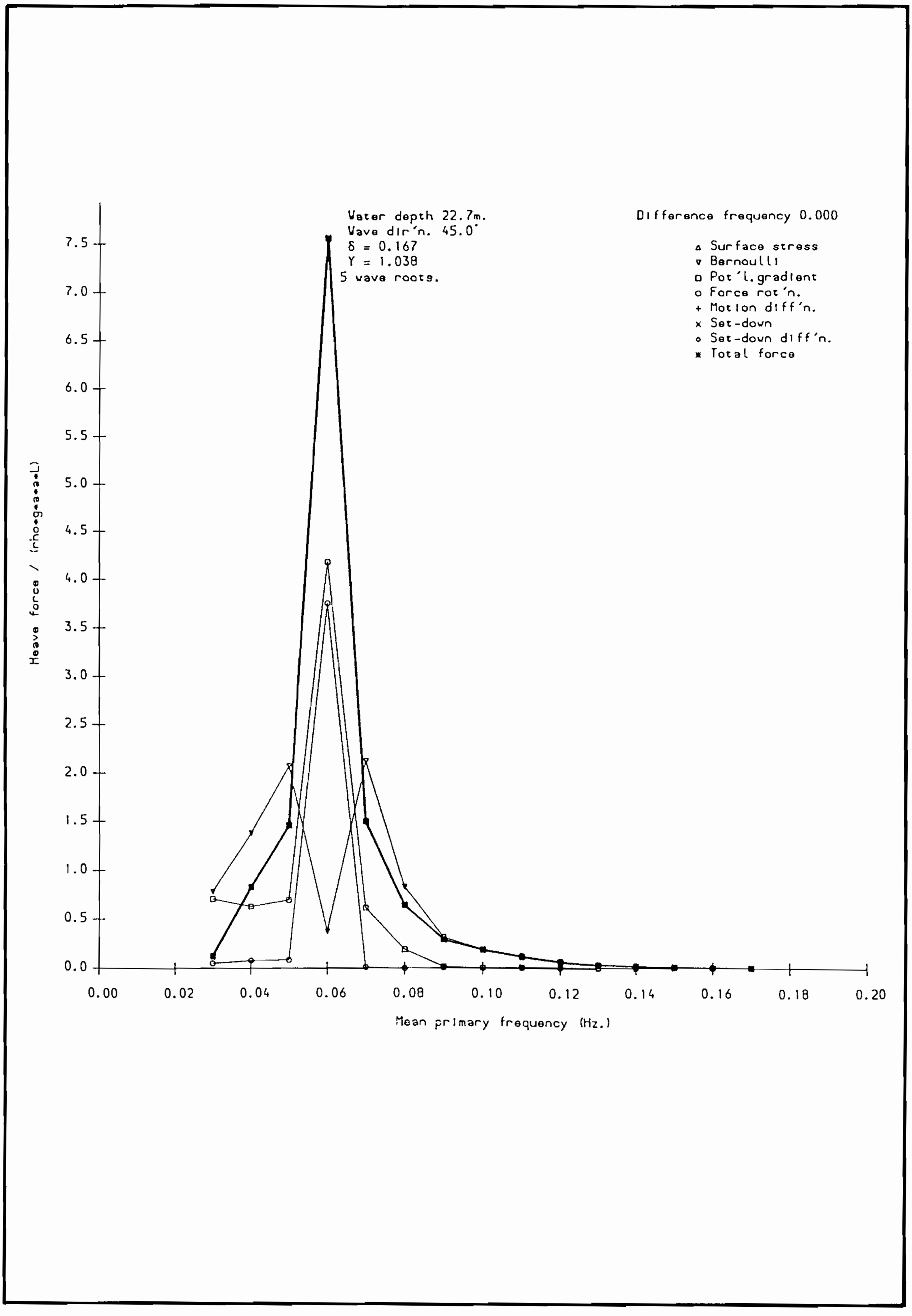

Fig 11 Heave steady force - DRIFTKEEL , stern quarter sea.

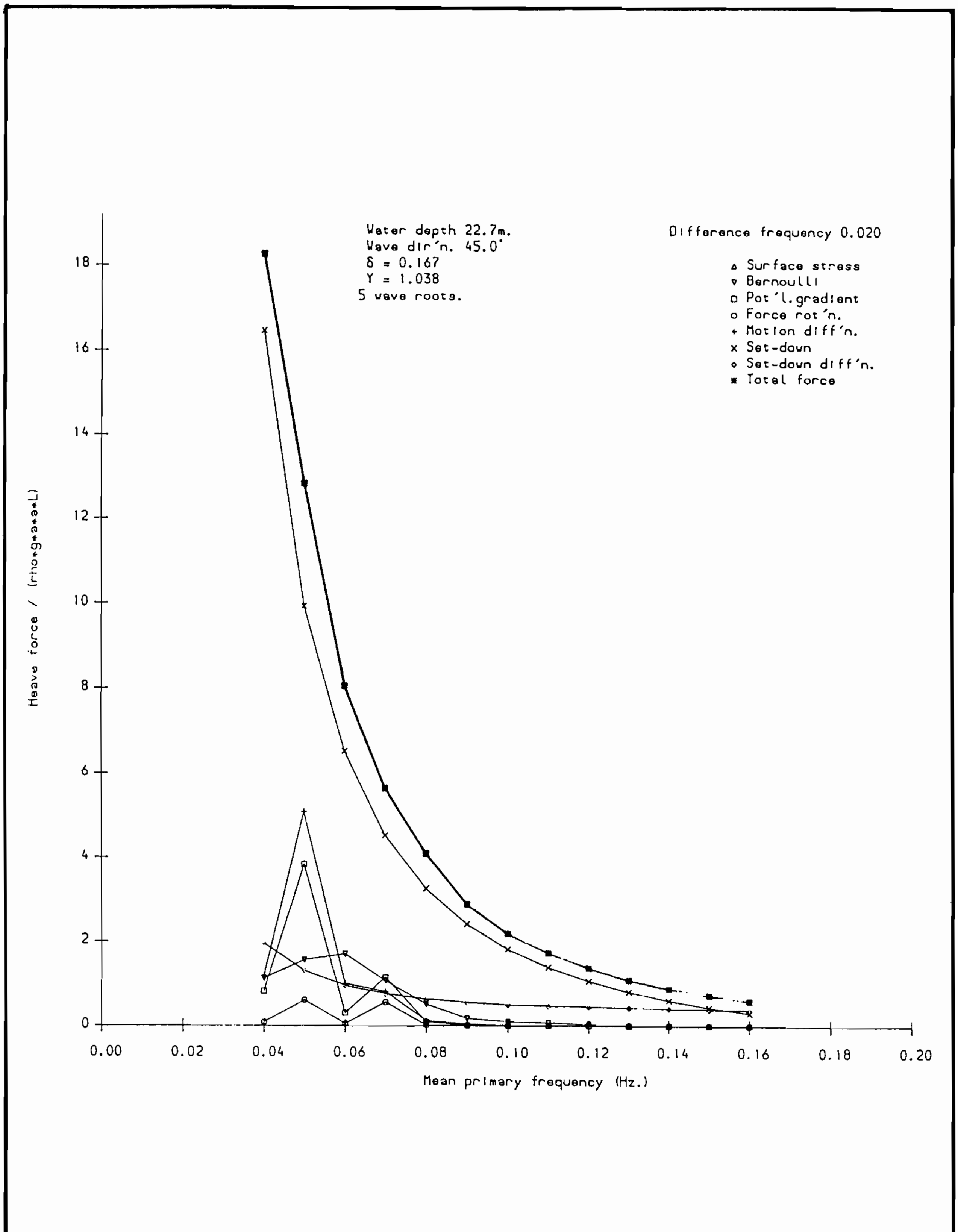

Fig 12 Heave force , 50 second period - DRIFTKEEL , stern quarter sea.

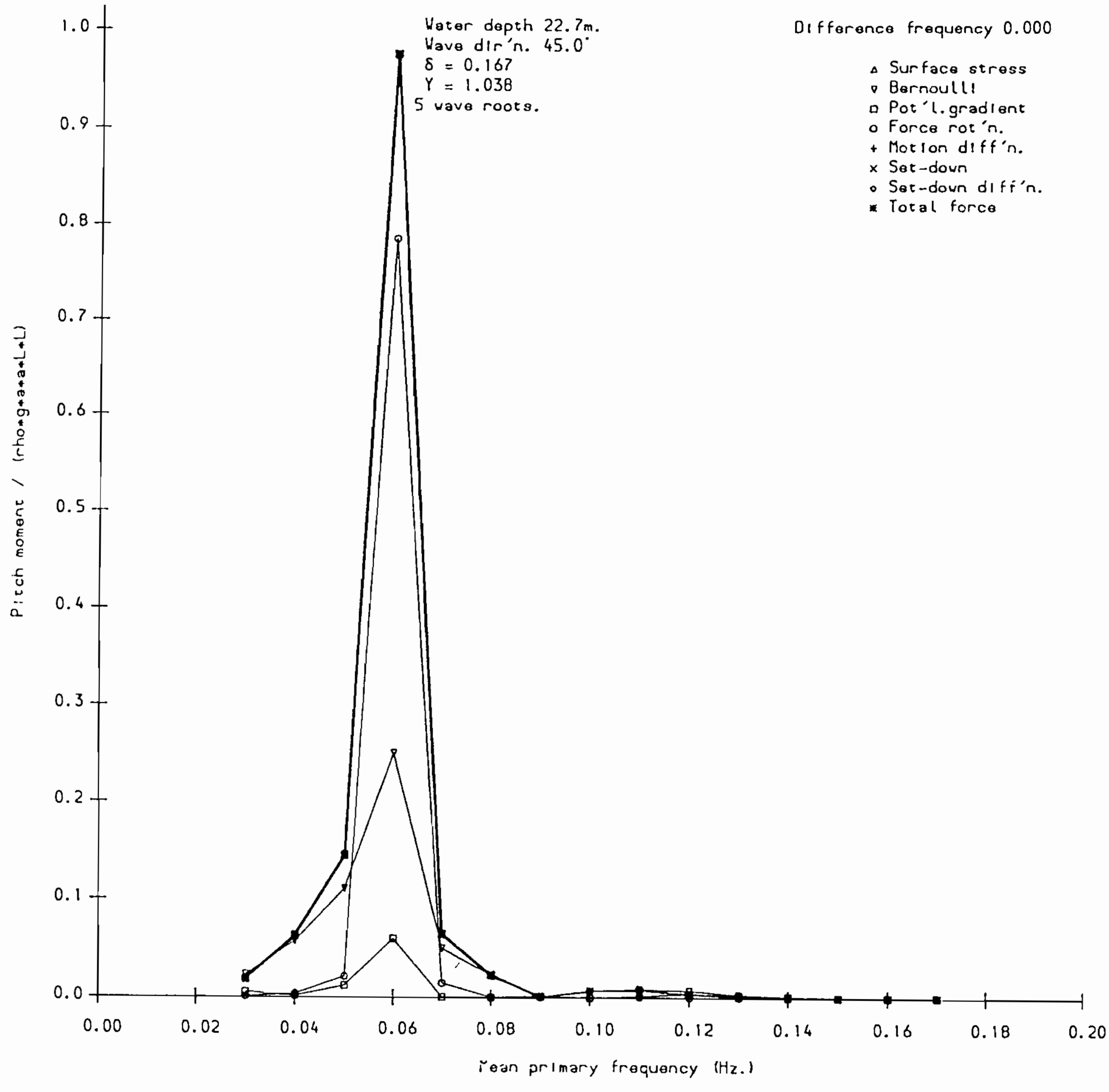

Fig 13 Pitch steady moment - DRIFTKEEL ,stern quarter sea.

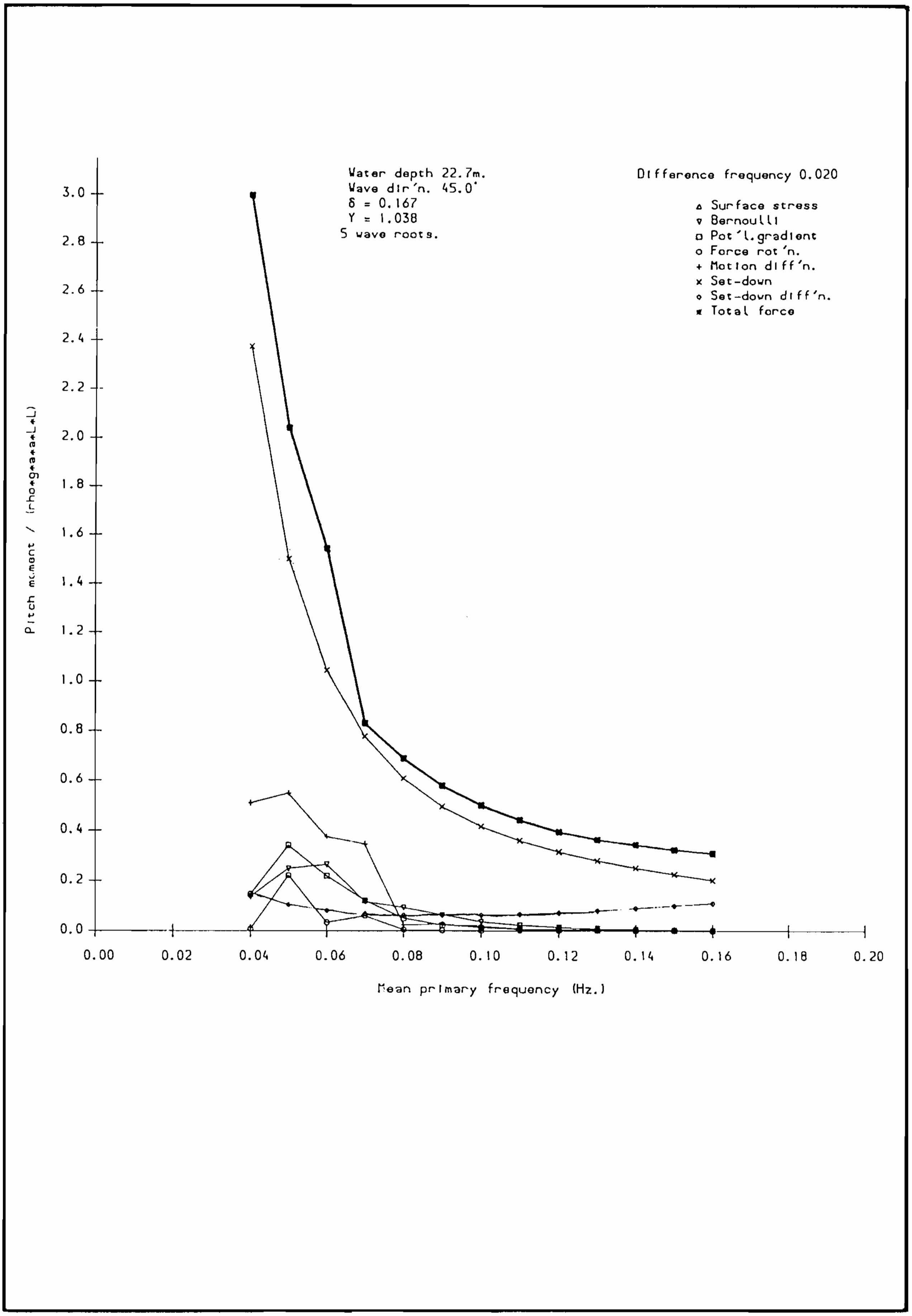

Fig 14 Pitch moment,50 second period - DRIFTKEEL,stern quarter sea.