Sediment Transport: The Ackers and White Theory Updated

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ABSTRACT

In 1973 Ackers and White published a sediment transport theory to predict the movement of non-cohesive sediments. The theory contains four empirical parameters which were determined by fitting them to observed transport data. These equations are widely used to calculate sediment transport, morphological change in channels, the size of stable alluvial channels and to design physical models. While in general providing reliable predictions, application of the equations to fine and coarse sediments has raised uncertainties about the confidence that can be placed on predictions in these ranges. When deriving the empirical parameter little data had been available in this range. Since the development of the theory much more sediment transport data has become available. This report describes the re-derivation of the empirical parameters on an extended data set. The new equations give lower predictions of sediment transport for fine sediments and coarse sediments.

CONTENTS

1 INTRODUCTION

In 1973 Ackers and White published a theory to predict the transport of non-cohesive sediments. These equations are widely used to calculate sediment transport, morphological change in channels, the size of stable alluvial channels and to design physical models. While in general providing reliable predictions, application of the equations to fine and coarse sediments has raised uncertainties about the confidence that can be placed on predictions in these ranges. When deriving the empirical parameters little data has been available in these ranges.

A feature of the theory was that it assumed different modes of transport for coarse and fine sediments. It was assumed that coarse sediments are moved predominantly on or adjacent to the bed and the transport rate depends upon the shear stress exerted on the bed. Fine sediment was assumed to be distributed throughout the flow and the transport rate to be a function of the total energy loss in the channel. Transition sizes between fine and coarse were assumed to be transported by a combination of the two processes. The theory is thus capable of dealing with a wide range of sediment sizes and has been successfully applied in a large number of situations (White, Milli, Crabbe 1973) In common with all presently used sediment transport theories the Ackers and White theory contains a number of parameters which must be determined empirically. A notable feature of the original development of the theory was the use of an extensive set of laboratory data on which to determine the empirical parameters. This undoubtedly contributed to the wide applicability of the theory.

An optimisation procedure was used to select values of the parameters which minimised the difference between the observed and predicted sediment transport rates. As the parameters are assumed to be functions of the sediment size the optimisation procedure is used to determine values of the parameters for different sediment sizes. A function then has to be fitted to these values. The form and precise nature of these functions depends upon the observed data on which the optimisation is carried out.

Since the original derivation of the theory in 1973 more sediment transport data has become available. The opportunity has, therefore, developed to use this extra data to improve the values of the pararmeters determined originally. There was particular interest in the values of the parameters for the fine and coarse sediments as, in the original derivation, they were obtained on limited data. This report describes a study to derive new parameters for the Ackers and White sediment transport theory on a wide range of data.

2 SEDIMENT TRANSPORT DATA

The original data set collected by Ackers and White (1972) was supplemented by data from Brownlie (1981), Wang Shiqiang and Zhang Ren, (1990), Jopling and Forbes (1979) and Mantz (1983).

This data comprised 2098 sets of observations which were arranged in 104 data sets, each data set representing the results for a given sediment, that is, fixed sediment size. The average number of observations in a data set was thus approximately 20. Sediment sizes ranged from 0.04mm to 28mm. See Table 1 for details of the data sets.

3. ACKERS & WHITE SEDIMENT TRANSPORT THEORY

Ackers and White expressed the sediment transport rate in terms of a dimensionless sediment transport rate G_{gr} defined by

$$G_{gr} = \frac{Xd}{sD} \left(\frac{v*}{V} \right)^n \qquad (1)$$

This dimensionless sediment transport rate was related to the sediment mobility F_{gr}, defined by

$$F_{gr} = \frac{v_*}{\sqrt{gD(s-1)}} \left(\frac{V}{\sqrt{32} \log 10d/D} \right) \qquad (2)$$

The form of F_{gr} was selected to ensure that for fine sediments it depended upon the total energy degradation or total bed shear stress and for coarse sediments upon the net grain resistance. For the transition sizes between these two it depends upon a combination of the two.

It was assumed that there is a critical value of F_{gr}, denoted by A, below which no sediment movement takes place. For F_{gr} greater than A it was assumed that

$$G_{gr} = C \left(F_{gr}/A - 1\right)^{m} \qquad (3)$$

Where C, A and m are parameters.

The parameters n, A, m and C were assumed to vary with the sediment size. This was expressed in terms of the dimensionless sediment diameter of D_{gr} defined by

$$D_{gr} = D \; \frac{g\,(s-1)}{\nu^{2}} \qquad (4)$$

The form of the functions

$$n = n \quad D_{gr} \qquad (5)$$

$$A = A \quad D_{gr} \qquad (6)$$

$$m = m \quad D_{gr} \qquad (7)$$

$$C = C \quad D_{gr} \qquad (8)$$

were determined using observed sediment transport data and an optimisation procedure.

4 DETERMINATION OF PARAMETERS n, A, m and C

4.1 Original procedure

In the optimisation procedure adopted the objective function selected was

$$\sum_i \left[\log^2 \frac{F_{gr_i}}{A\,(1+ (G_{gr_i}/C)^{1/m})} \right] \qquad (9)$$

where F_{gr_i} and G_{gr_i} were computed from equations (2) and (1) respectively using the observed values of V, d, s, D and X.

In the original optimisation a multi-stage method was adopted using a one-dimensional optimisation procedure in which each variable was considered in turn. In the first optimisation all the parameters were allowed to vary. The values of n and A determined were then plotted against D_{gr} and an appropriate curve fitted. The values of n and A were then assumed to be given by these curves and a second optimisation was performed varying only m and C. The values of m were then plotted against D_{gr} and an appropriate curve fitted. The values of n, A and m were then assumed to be given by these curves and a third optimisation was carried out. The values of C were then plotted against D_{gr} and an appropriate curve fitted.

The resulting equations were:

for the transitional sediment sizes $1 \leq D_{gr} < 60$

$$n = 1.00 - 0.56 \log D_{gr} \qquad (10)$$

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \qquad (11)$$

$$m = \frac{9.66}{D_{gr}} + 1.34 \qquad (12)$$

and

$$\log C = 2.86 \log D_{gr} - (\log D_{gr})^2 - 3.53 \qquad (13)$$

For coarse sediments, $D_{gr} \geq 60$

$$n = 0.0 \qquad (14)$$

$$A = 0.17 \qquad (15)$$

$$m = 1.5 \qquad (16)$$

and $$C = 0.025 \qquad (17)$$

These functions are shown in Figures 3.1.

4.2 New procedure

The computer software used in the original optimisation was no longer available and so new software was prepared. It was decided to follow the same sequence of successive optimisations but the optimisations using four and two variables were carried out using a downhill simplex method. It was considered that this would provide a more robust algorithm for these cases. For the one variable optimisations a Newton-Raphson technique was used.

The initial stage of the study was the use of the original Ackers and White data set to reproduce the original results, so that one could have confidence that any changes introduced when considering a larger data set, were due to the data set and not to the optimisation procedure adopted. The original Ackers and White results were reproduced to within the tolerances of the calculations.

4.3 First optimisation

The optimisation procedure was then run on a combination of the original and new data sets. The values of n and A derived are shown in Figures 4.1 and 4.2 respectively.

4.3.1 A as a function of D_{gr}

Figure 4.2 shows the values of A derived from the data together with equation (11). There are strong theoretical arguments to suggest that for coarse sediments the initiation of motion should be independent of viscosity.

This implies that for coarse sediments the value of A should be independent of D_{gr}. For fine sediments initiation of motion should be independent of sediment diameter which implies that for fine sediments A should be proportional to $\frac{1}{\sqrt{D_{gr}}}$ Within these constraints a number of options were considered for the function A but none provided a sufficiently significant improvement on the original relation:

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \qquad \text{for } 1 \le D_{gr} \le 60 \qquad (18)$$

$$A = 0.17 \qquad \text{for } D_{gr} \ge 60$$

to warrant any change.

4.3.2 n as a function of D_{gr}

Figure 4.1 shows the values of n derived from the data together with equation (10). The theoretical constraints upon n are that it should take the value of 0 for large D_{gr} and 1 for small D_{gr}. The results clearly indicate that the previous assumption that n achieves the value 0 for D_{gr} = 60 is substantiated by the new data. The evidence for the value of D_{gr} at which n achieves the value 1 is more equivocal. A number of relationships were considered for which n achieved the value 1 for D_{gr} values greater than 1 but none provided any improvement on the relationship

$$n = 1.0 - 0.56 \log_{10} D_{gr} \qquad (19)$$

4.4 Second Optimisation

The equations (10) and (11) were used for n and A respectively and the optimisation procedure was used to determine m and C.

4.4.1 m as a function of D_{gr}

Figure 4.3 shows the calculated values of m and the original equation for m, equation (7). The results for fine sediments suggest that the value of m determined by equation (7) for small values of D_{gr} is too large. For coarse sediment the results also suggest that equation (7) gives values of m which are too low. Equation (7) is of the form

$$m = \frac{A}{D_{gr}} + B \qquad (20)$$

Equations of the form $m = \frac{A}{D_{gr}}\alpha + B$ (21)

were fitted to the data. The most satisfactory fit was achieved by the function

$$m = \frac{6.83}{D_{gr}} + 1.67 \qquad 1 \leq D_{gr} \leq 60$$

$$m = 1.78 \qquad D_{gr} \geq 60 \qquad (22)$$

4.5 Third Optimisation

The values of n, A and m were determined using equations (10), (11) and (22). A third optimisation was carried out to determine the values of C.

4.5.1 C as a function of D_{gr}

The values of C are shown in Figure 4.4 together with equation (13). Equation (13) is of the form

$$\log C = A + B \log D_{gr} + C (\log D_{gr})^2 \qquad (23)$$

The best fit equations of this form are

$$\log C = 2.79 \log D_{gr} - 0.98 (\log D_{gr})^2 - 3.46, \qquad (24)$$

$$\text{for } 1 \leq D_{gr} \leq 60$$

$$\text{and } C = 0.025 \qquad \text{for } D_{gr} \geq 60 \qquad (25)$$

5 THE REVISED ACKERS & WHITE SEDIMENT TRANSPORT THEORY

5.1 Revised equations

The revised equations for the parameters n, A, m and C are

$$n = 1.0 - 0.56 \log_{10} D_{gr} \qquad \text{for } 1 \leq D_{gr} \leq 60$$

$$n = 0.0 \qquad \text{for } D_{gr} \geq 60 \qquad (26)$$

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \qquad \text{for } 1 \leq D_{gr} \leq 60 \qquad (27)$$

$$A = 0.17 \qquad \text{for } D_{gr} \geq 60$$

$$m = \frac{6.83}{D_{gr}} + 1.67 \quad \text{for } 1 \le D_{gr} \le 60 \tag{28}$$

$$m = 1.78 \quad \text{for } D_{gr} \ge 60$$

$$\log C = -3.46 + 2.79 \log D_{gr} - 0.98 (\log D_{gr})^2$$

$$\text{for } 1 \le D_{gr} \le 60$$

$$C = 0.025 \quad \text{for } D_{gr} \ge 60 \tag{29}$$

5.2 Alteration to predictions

To indicate the differences between the revised and the original equations the ratio of the calculated to the observed G_{gr}, denoted by λ, was determined for a range of values of F_{gr} and D_{gr}, see Figure 5.1. The figure indicates that for fine sediments the concentrations derived with the new equations are increased for a range of mobilities just exceeding the threshold value but are decreased for larger mobilities. For coarse sediments with large mobilities, the sediment concentrations are reduced. For intermediate sizes the change to the predicted sediment concentration is small.

5.3 Comparison with observations

The effect of the new and old expressions for the parameters on the predictions of the theory were tested on two data sets : one set was field data from the Lower Yellow River and the other was from the North Saskatchewan and Elbow Rivers. The range of parameters for the data from the Lower Yellow River was:

Sediment size	0.049 to 0.067mm
Depth	1.44 to 5.02mm
Slope	0.10×10^{-3} to 0.19×10^{-3}
Concentration	9,000 to 92,000ppm

With the original formulation the mean discrepancy ratio, the ratio of predicted to observed concentration, was with no predictions within a factor of 2 of the observations 5.97. The new expression for mean discrepancy ratio was 3.22 with 70% of the predictions being within a factor of 2 of the observations.

The data from the gravel rivers in Canada had the following range of parameters:

Sediment diameter	10.51 to 40.8mm
Depth	0.73 to 2.74mm
Slope	1.58×10^{-3} to 7.45×10^{-3}
Concentration	4 to 524ppm

The mean discrepancy ratio with the original formulation is 3.84, with 33% of the predictions within a factor of 2. With the new values of the parameters the mean discrepancy ratio is 2.94 with 33% of the predictions within a factor of 2.

The comparison of predictions with observations on both these data sets is rather disappointing and they do not approach the agreement obtained by White, Milli and Crabbe. The new expressions for the parameters do provide, however, a significant improvement in the predictions.

6 CONCLUSIONS

1. An optimisation procedure was developed that reproduced the original results obtained by Ackers and White on their original data set to the tolerances of numerical accuracy.

2. An increased data set was obtained including, in particular, more data for fine and coarse sediments that had been used originally.

3. The derivation of the parameters n, A, m and C was repeated on this extended data set.

4. It was not possible to find a significant improvment in the relationships for n and A.

5. A revised function for m was derived :

$$m = \frac{6.83}{D_{gr}} + 1.67 \qquad \text{for } 1 \le D_{gr} \le 60,$$

$$m = 1.78 \qquad \text{for } D_{gr} \ge 60$$

6. A revised function for C was derived

$$\log C = 2.79 \log D_{gr} - 0.98 (\log D_{gr})^2 - 3.46$$

$$\text{for } 1 \le D_{gr} \le 60$$

$$C = 0.025 \text{ for } D_{gr} \ge 60$$

7 The alterations to these parameters change the predictions provided by the theory. For fine sediments the concentrations derived with the new equations are increased for a range of mobilities just exceeding the threshold value but are decreased for larger mobilities. For coarse sediments, with large mobilities, the sediment concentrations are reduced. For intermediate sizes the change to the sediment concentrations predictions is small.

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Mantz PA, 1983, Semi-empirical correlations for fine and coarse cohesionless sediment transport, Proc (CE 75 Part 2)

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TABLES

TABLE 1 : Summary of data sets

No of set	Investigators	Date	Sediment specific gravity	Sediment size (mm)	Sediment D_{gr}	No of data in set
1	Laurson	1957	2.65	0.040	1.10	8
2	Mantz	1983	2.65	0.042	1.10	18
3	Jepling	1979	2.65	0.045	1.31	16
4	Wang Shiqiang	1990	2.65	0.076	1.93	27
5	Laurson	1957	2.65	0.100	2.50	15
6	Willis et al	1972	2.65	0.100	2.67	33
7	Willis et al	1972	2.65	0.100	2.67	33
8	Willis et al	1972	2.65	0.100	2.67	30
9	Brooks	1955	2.65	0.090	2.70	10
10	Abdel-Aal	1969	2.65	0.105	2.77	10
11	Kennedy and Brooks	1965	2.65	0.140	3.70	8
12	Vanoni and Brooks	1957	2.65	0.140	3.70	14
13	Straub	1958	2.65	0.163	3.77	6
14	Davies	1971	2.65	0.150	3.82	39
15	Davies	1971	2.65	0.150	3.82	40
16	Nomics	1956	2.65	0.150	4.10	11
17	Brooks	1955	2.65	0.160	4.30	8
18	E Pakistan	1967	2.66	0.150	4.32	19
19	Guy, Simons & Richardson	1966	2.65	0.190	4.60	23
20	Barton and Lin	1955	2.65	0.180	4.70	29
21	Straub	1954	2.65	0.191	4.72	18
22	USWES	1935	2.65	0.210	5.10	14
23	Hill River	1959	2.65	0.210	5.28	11
24	Vanomi and Hwang	1965	2.65	0.230	5.98	7
25	Kennedy	1961	2.65	0.233	6.39	27
26	Nordin	1976	2.65	0.250	6.40	45
27	Guy et al	1966	2.65	0.270	6.50	13
28	Guy et al	1966	2.65	0.280	6.50	24
29	E Pakistan	1968	2.64	0.250	6.53	10
30	Onishi Jain & Kennedy	1972	2.65	0.25	6.61	14
31	Taylor	1971	2.65	0.228	6.71	12
32	E Pakistan	1968	2.64	0.25	6.81	14
33	Gilbert & Murphy	1914	2.65	0.30	7.00	6
34	Foley	1975	2.65	0.29	7.56	12
35	Guy et al	1966	2.65	0.32	8.30	16
36	Guy et al	1966	2.65	0.33	8.30	6
37	Gilbert et al	1914	2.65	0.37	8.60	28
38	Guy et al	1966	2.65	0.33	8.80	10
39	USWES	1935	2.65	0.31	8.80	11
40	USWES	1935	2.65	0.35	8.90	26
41	O'Brien	1936	2.57	0.36	8.90	41
42	Soni	1980	2.65	0.32	9.19	23
43	Guy et al	1966	2.65	0.45	10.10	19
44	Stein	1965	2.65	0.40	10.50	32
45	USWES	1935	2.65	0.48	11.20	19
46	Guy et al	1966	2.65	0.47	11.30	32
47	Gilbert et al	1914	2.65	0.51	11.70	25

TABLE 1 (Cont'd)

No of set	Investigators	Date	Sediment specific gravity	Sediment size (mm)	Sediment D_{gr}	No of data in set
48	USWES	1935	2.65	0.51	11.90	14
49	Pratt	1970	2.65	0.49	12.40	16
50	USWES	1935	2.65	0.52	12.60	22
51	USWES	1935	2.65	0.50	13.10	23
52	Guy et al	1966	2.65	0.54	13.70	17
53	Pratt	1970	2.65	0.49	13.90	18
54	Willis	1979	2.65	0.54	14.30	31
55	Singh	1960	2.64	0.62	14.57	30
56	Singh	1960	2.64	0.62	14.57	30
57	Singh	1960	2.64	0.62	14.57	30
58	Singh	1960	2.64	0.62	14.57	30
88	Casey	1935	2.70	2.45	58.33	28
89	Liu	1937	2.65	2.30	66.28	23
90	Liu	1937	2.65	3.41	98.52	15
91	USWES	1935	2.65	4.08	100.35	10
92	Sato Kikkawa & Ashida	1958	2.65	4.58	126.95	28
93	Meyer-peter & Muller	1948	1.25	5.20	132.10	9
94	Meyer-peter & Muller	1948	2.68	5.21	132.30	15
95	Meyer-peter & Muller	1936	2.68	28.65	124.44	25
96	USWES	1936	1.07	0.90	6.60	15
97	USWES	1936	1.35	1.10	14.40	17
98	USWES	1936	1.85	0.91	20.10	28
99	USWES	1936	1.32	1.48	21.90	18
100	USWES	1936	1.85	1.07	23.50	25
101	USWES	1936	1.74	1.33	26.10	27
102	USWES	1936	1.07	3.55	27.60	14
103	USWES	1936	1.32	3.20	48.00	15
104	USWES	1936	1.35	3.30	53.80	14

FIGURES

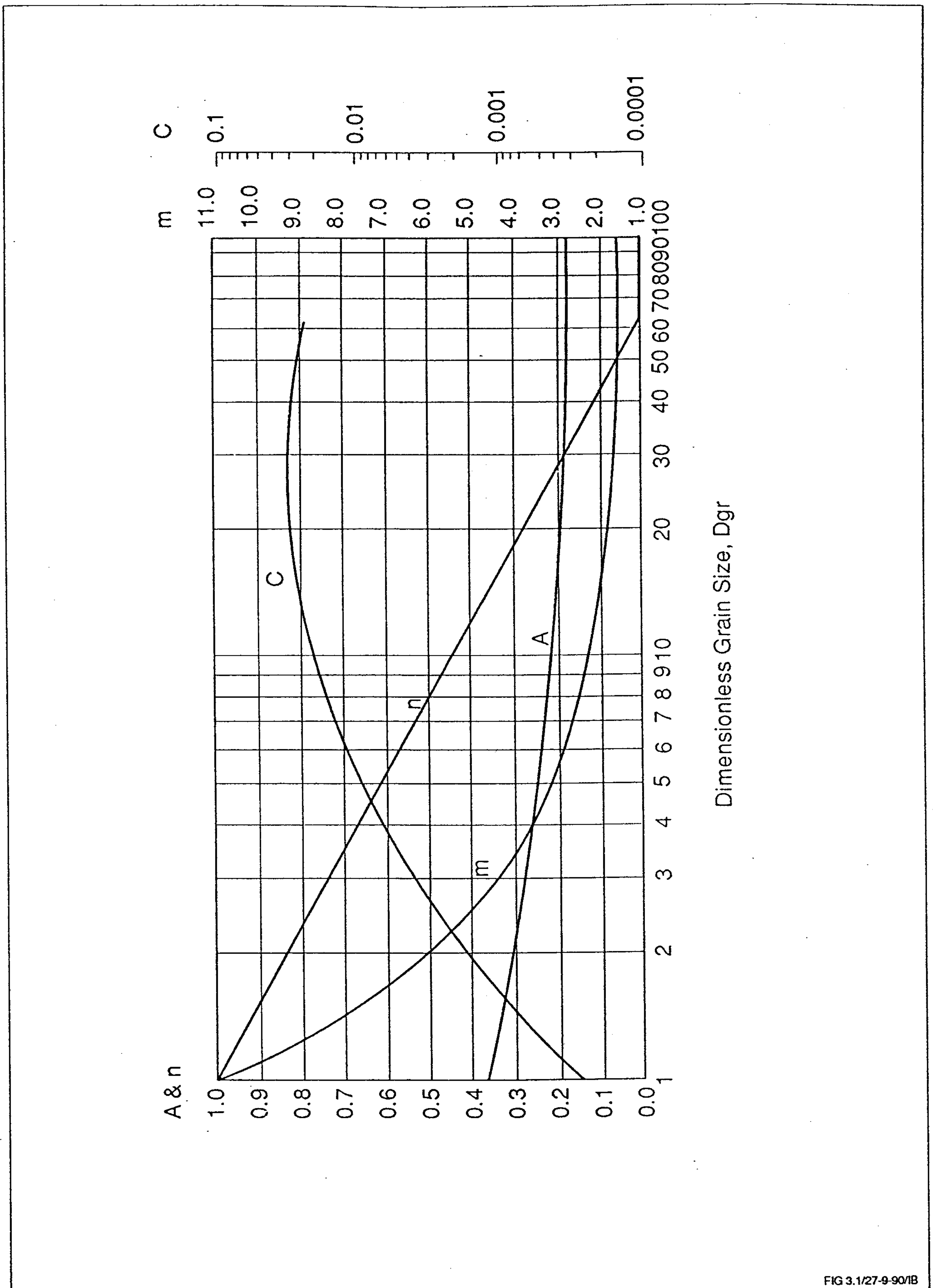

Fig 3.1 n,A,m and C against Dgr, original functions

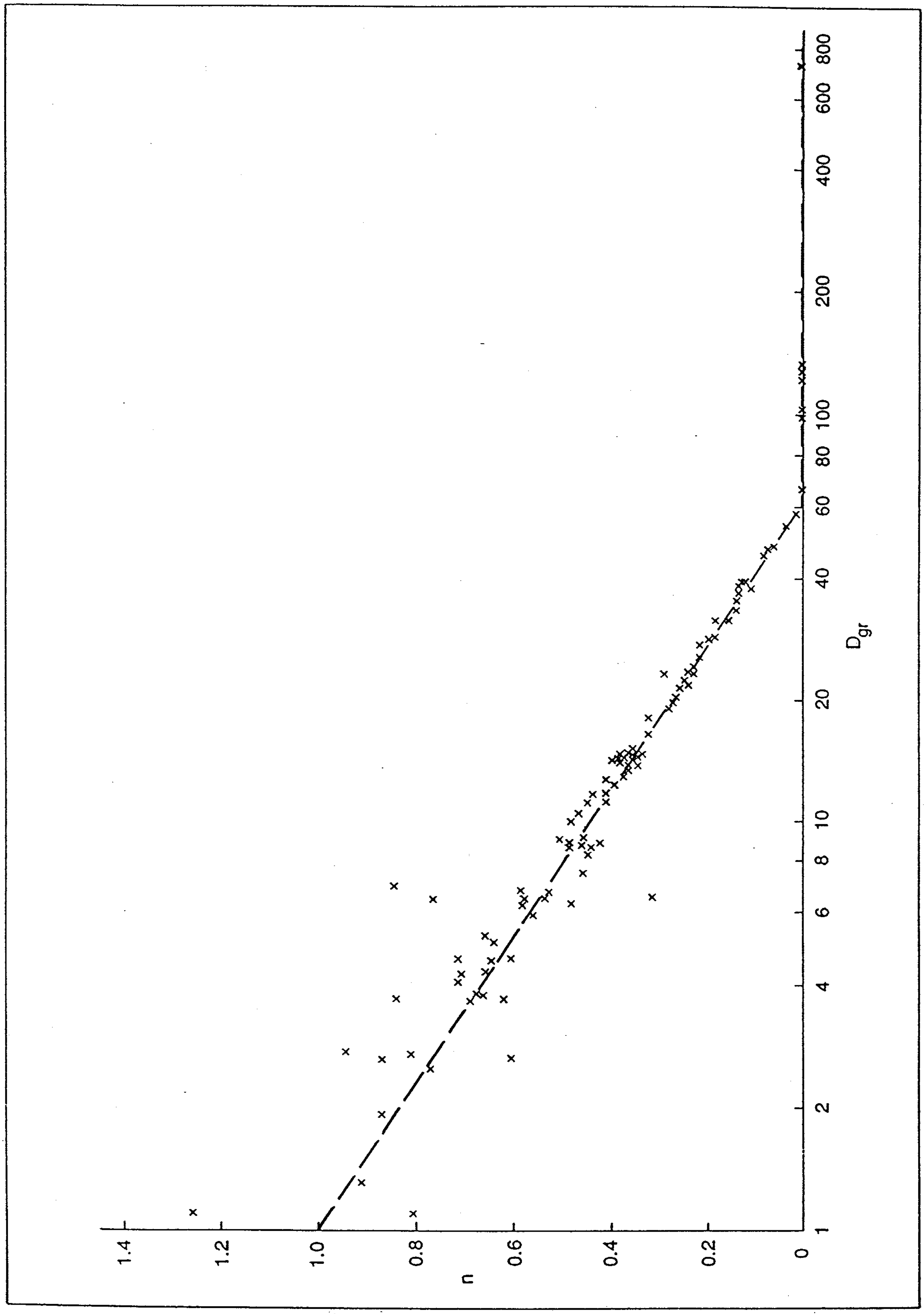

Fig 4.1 n versus D_{gr}

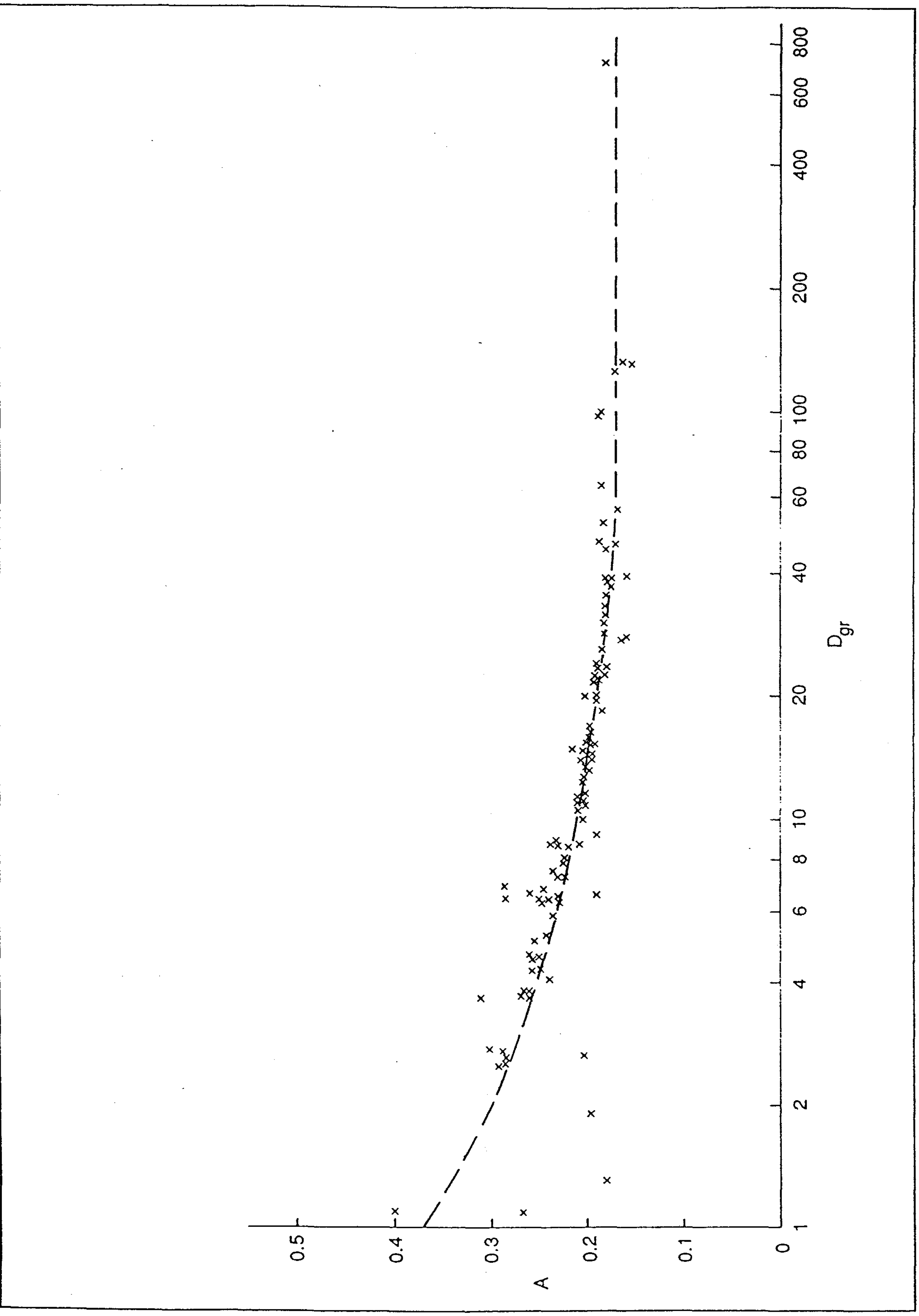

Fig 4.2 A versus D_{gr}

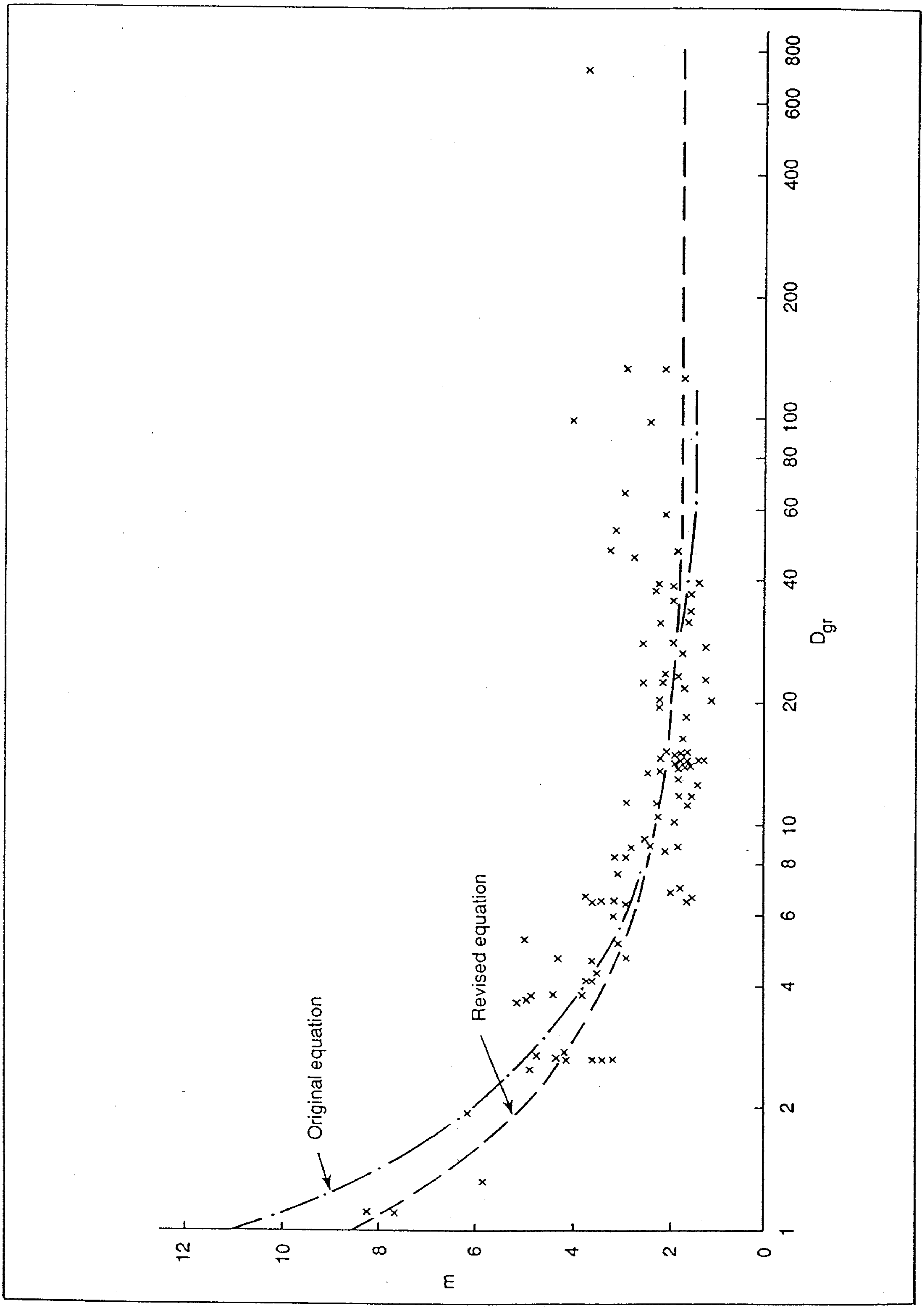

Fig 4.3 m versus D_{gr}

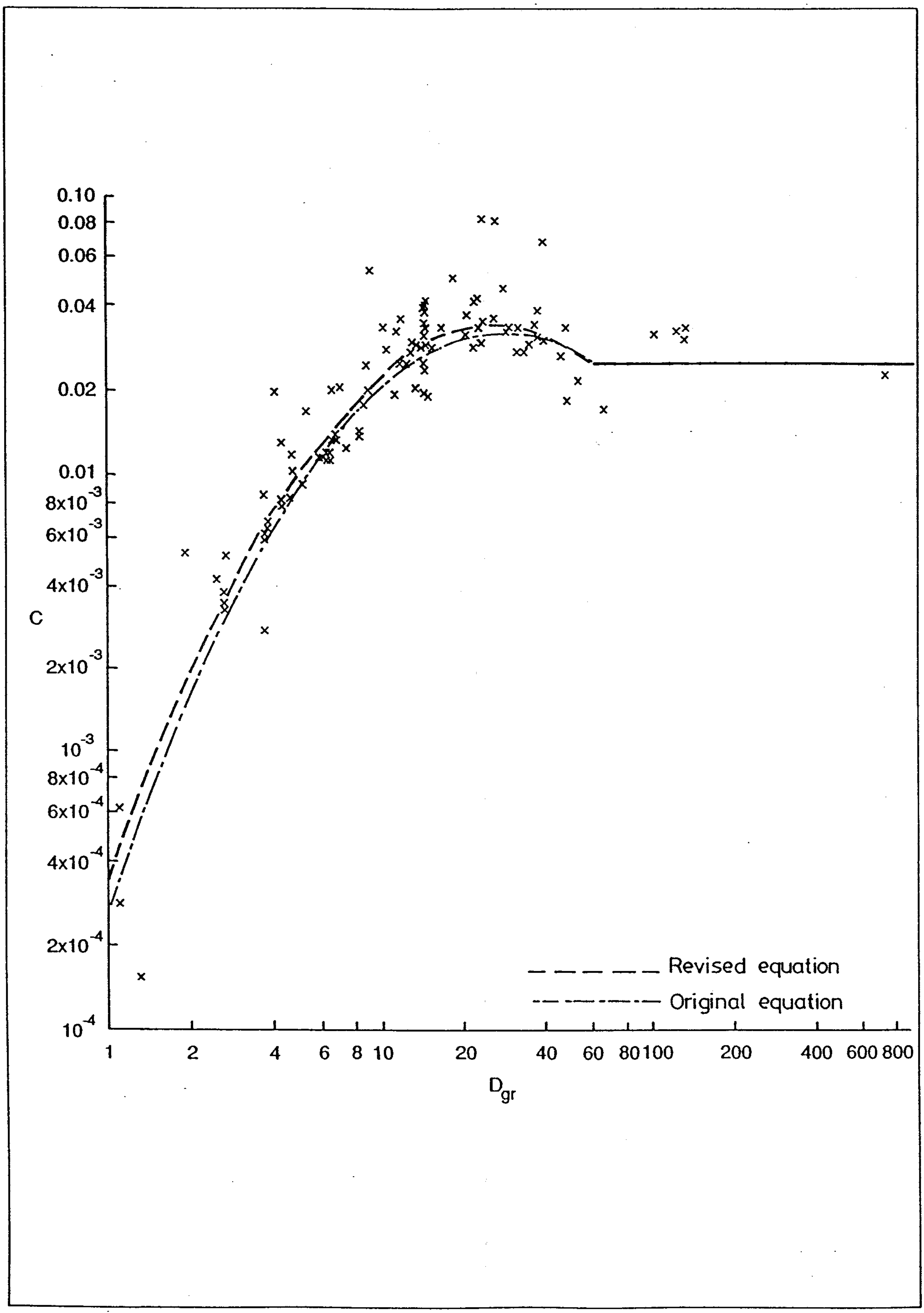

Fig 4.4 C versus D_{gr}

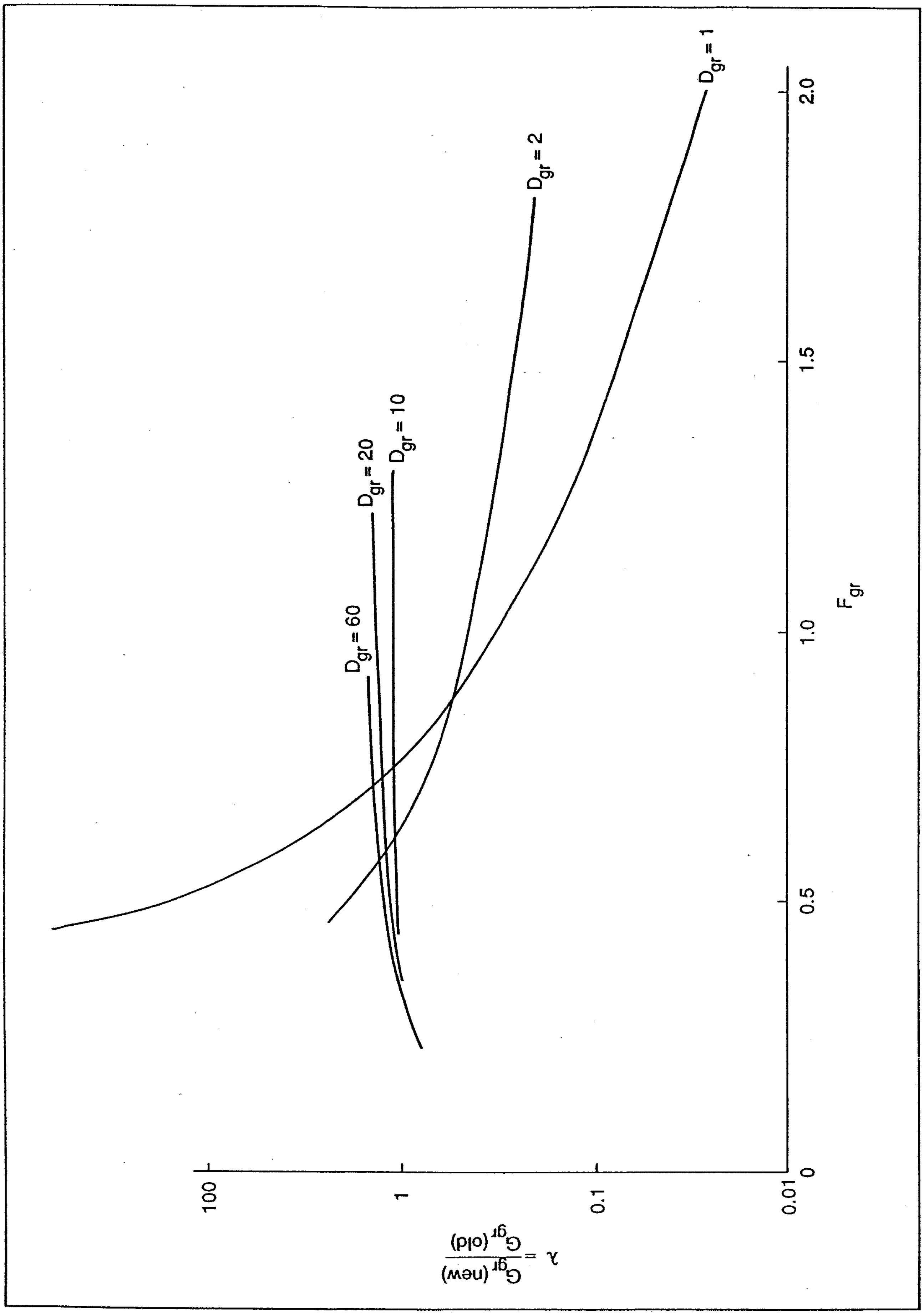

Fig 5.1 λ versus F_{gr}