



Hydraulics Research
Wallingford

PARTICULATE POLLUTANTS IN THE NORTH SEA

Mud Properties

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CONTRACT

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ABSTRACT

It is known that contaminants have an affinity for the finest size fractions of the marine sediment, and therefore the processes controlling the erosion, transport and deposition of the clay and mud particles are very important in modelling the movement of contaminants.

This report summarises the collective knowledge about the distribution and behaviour of mud in the North Sea from work conducted at Hydraulics Research and from other researchers. A letter was sent to the main European laboratories asking for information about previous field and laboratory work on North Sea mud.

The report describes the laboratory tests on a sample of mud collected from Marsden Bay, off the South Shields coast. This site was chosen as it was also the site of the North-East Coast Cohesive Sediment Dynamics Study (NECCSDS). Laboratory tests for uni-directional current erosion and wave erosion are reported. The laboratory results at HR are compared with observed sediment resuspension and deposition in the field.

A summary of recent laboratory tests conducted at HR on mud collected from estuaries which discharge into the North Sea has been undertaken. The sedimentological, consolidation and erosion properties are compared for several muds. Representative parameters to describe the behaviour of North Sea Mud are identified.

Field measurements of settling velocity from the NECCSDS group are summarised, and preliminary results of settling velocity from an image processing technique used at HR are given.

The distribution of heavy metals in the North Sea is summarised, and factors which affect the adsorption and desorption are discussed.

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1 INTRODUCTION

The North Sea and the estuaries which discharge into it support a wide range of industrial, commercial and recreational activities. The North Sea itself is recognised as one of the world's most important fishing grounds, and yet it is also a major recipient of the domestic and industrial wastes from the countries which surround it. Increasing awareness of the impact of such waste products on the marine environment has stimulated several studies of different aspects of the North Sea.

It is known that contaminants have an affinity for the finest size fractions of the marine sediment, and therefore the forces controlling the erosion, transport and deposition of the clay and mud particles are very important in modelling the movement of contaminants.

As part of the UK research contribution to the North Sea Task Force, Hydraulics Research (HR) is developing a 3-D model of flow, mud transport and heavy metal transport in the North Sea. This research project is funded by the Department of the Environment. There is a general concern about the lack of knowledge of North Sea mud, with respect to its erosion and deposition properties under the action of waves and tidal currents. It was therefore considered important to investigate the behaviour of a sample of marine mud, taken from the UK coastal zone, using laboratory procedures that have been recently developed through a strategic research programme on cohesive sediments.

This report describes the laboratory tests on a sample of mud collected from Marsden Bay, off the South Shields coast. This site was chosen as it is

also the site of the North-East Coast Cohesive Sediment Dynamics Study (NECCSDS) - a joint study between MAFF Fisheries Laboratory - Lowestoft, the Department of Earth Sciences - University of Cambridge (U Cambridge), the Bullard Laboratory - University of Cambridge, the School of Environmental Sciences - University of East Anglia (UEA) and the NERC Proudman Oceanographic Laboratory (POL).

The NECCSDS group has developed a bed frame and logging system for deployment at sea, and developed and used miniature optical backscatter probes for measuring in-situ suspended sediment concentrations, with specific interest in episodic storm events. The study also involved measurements of particle settling velocities using Quisset bottles and development of a theory for wave-current interaction, to predict the bed shear stress and kinematics of the near-bed flow given good estimates of the bed roughness.

Sedimentological analysis and tests for uni-directional current erosion and wave erosion were carried out. The laboratory results at HR were compared with observed sediment resuspension and deposition behaviour in the field.

In addition to the laboratory tests on the North Sea mud, this report also includes a summary of recent laboratory tests conducted at HR on mud collected from estuaries which discharge into the North Sea, in order to establish whether it is sufficient to model the mud with one set of behavioural parameters.

In order to add to this knowledge a literature search was undertaken and a letter was circulated to

UK and European laboratories, asking about their work on North Sea sediments. The letter, and addresses of the people to whom it was sent are included in Appendix 0. Judging by the responses, of which there were only a few, there has been little work done on the transport properties of the sediment. However, this report includes information on the distribution of sediment (Chapter 2) and heavy metals (Chapter 5), which has been collected from other authors.

Chapter 3 summarises the results on settling velocity from the North-East Coast Cohesive Sediment Dynamics Study and from field measurements made by HR. Chapter 4 contains the HR laboratory results on sediment transport properties; these are compared with results from other studies where known.

2 DISTRIBUTION OF SEDIMENT ON THE BED AND IN SUSPENSION

2.1 Concentration in suspension

The concentration of suspended sediment in the bottom and surface layers of the North Sea is given by Eisma and Kalf (Ref 1) and reproduced in Figure 1. This shows a concentration of less than 1mg l^{-1} in the north west of the North Sea, increasing to around $5\text{-}10\text{mg l}^{-1}$ near the European coast and in the Channel. Eisma and Kalf also refer to measurements which suggest that the suspended sediment concentration increases in the winter, by 50-150% in the central part of the North Sea, and up to 300% in the coastal areas.

Puls also used a numerical model to calculate suspended sediment concentrations (Ref 2), starting

with a North Sea containing no suspended sediment and using input concentrations from rivers, cliff erosion and the sea boundaries. The concentration distribution after one year was very similar to that reported by Eisma and Kalf, although higher concentrations of mud were recorded around the estuary mouths.

In 1988, Hydraulics Research collated near-bed mud concentrations, tide and wave data collected in Tees Bay during a few north-east storms, and examined the correlation between these variables. Based on the observed correlation, long term frequency distributions of sediment concentration were calculated numerically (Ref 3). Suspended mud concentrations were predicted to be about half as high in summer as the rest of the year, at around 70ppm for the period May - September, as opposed to 150ppm for October - April. These concentrations are much higher than has been measured in the main body of the North Sea, but, in agreement with Puls, the concentrations of suspended sediment are higher around the estuary mouths as the main source of sediment into the North Sea is from the estuaries.

2.2 Distribution on the bed

Several authors have published information about the sediment distribution on the bed of the North Sea. Most of the North Sea bed is sand or gravel, with a few isolated patches of mud. The bottom of the Norwegian trough (up to 400m deep) is muddy. Figure 2 shows this distribution of sediment according to the MAFF Atlas of the Seas around the British Isles (Ref 4). There are some muddy coastal zones around the estuaries of Britain and Europe which are not shown on Figure 2.

The predominantly sandy bed is supported by samples taken by the NECCSDS group in the coastal area offshore of Marsden Bay (Ref 5). When averaged, their samples showed a trend of increasing mud content (< 63 microns) offshore. The percentages of mud were, on average, 48% mud at 20-25m depth, increasing to 60% mud at 30-35m depth.

In the modelling of sediment in the North Sea, Puls has calculated distributions of sediment deposited on the bed under tidal current action and also under currents and waves combined. Even without including the effect of waves, the quantity of material deposited in one month over the main body of the North Sea was less than 60gm^{-2} , with 100gm^{-2} exceeded at only a few points including an area in the Norwegian Trough. Assuming a density of the mud of 200kgm^{-3} (this represents a very weak mud deposit), this only amounts to 3mm - 6mm deposition per year. This represents a worst case, since waves will be important over all but the deepest parts of the North Sea. In that case, the deposited material would easily be resuspended anyway. For the purposes of modelling the mud transport, consolidation of the mud deposits is not therefore considered to be important.

3 SETTLING VELOCITY

3.1 North-East Coast Cohesive Sediment Dynamics Study

As part of NECCSDS the University of Cambridge has developed a method of making field measurements of particle settling velocity using Quisset bottles (Ref 5). The calculated value of the median settling velocity, w_{50} , was in the range $0.009\text{-}0.450\text{mms}^{-1}$. The results indicate the probable

existence of two predominant sub-ranges of settling velocity (giving two modal values of settling velocity), $0.018-0.032\text{mms}^{-1}$ and $0.100-0.320\text{mms}^{-1}$. This may indicate two predominant sub-ranges of particle size. There is no indication of the concentration for which these settling velocities were determined.

3.2 HR Field Measurements

In the past, HR has made measurements of settling velocity in the field using Owen tubes. This works on a bottom withdrawal method, and the current analysis procedure only gives reliable results for concentrations above 100ppm. In general the analysis results in a relationship between median settling velocity and concentration. In the North Sea, the concentrations are considerably lower than it is currently possible to determine settling velocities with Owen Tube analysis. Future developments by HR may remove this limitation.

For low concentrations it is sometimes possible to calculate a relationship for w_{90} with concentration (90% of the particles have a settling velocity less than w_{90}) from the Owen tube results. A median settling velocity may then be calculated using a relationship between w_{50} and w_{90} which has been calculated from previous settling velocity results. Such a relationship is given in the Estuarine Muds Manual (Ref 6) for the Thames.

The median settling velocity (as a function of concentration) has been measured (for concentrations above 100ppm) in a number of estuaries which feed into the North Sea. These are presented in Figure 3. At 0.1kgm^{-3} the settling velocities range

from approximately 0.1mms^{-1} in the Thames, to approximately 0.4mms^{-1} for the Grangemouth estuary.

Recent developments at HR include an image processing system for measuring settling velocities. This involves aligning an open-ended sampling cylinder with the flow, to collect a sample of the flocculated sediment. On removal, the cylinder is turned vertically and individual flocs are recorded on a video system. Many different flocs are recorded during the first ten minutes after collection. After this time, it is felt that the particles will begin to show the effect of the stationary water column, and that measurements made after this may not reflect the real situation in the field.

Some preliminary tests were done in the Tees estuary (Ref 7), where the concentration of sediment in suspension is also very low. The values of settling velocity measured by this method were rather higher than would be suggested from extrapolation of the Owen tube measurements. The results are also shown on Figure 3, as three data points representing the three samples. A dotted line through the points indicates a median settling velocity of 0.3mms^{-1} (6ppm) to 0.5mms^{-1} (10ppm). This value supports the measurements made in Marsden Bay by the NECCSDS group.

For the purposes of modelling, a value of the median settling velocity in the range $0.3 - 0.5\text{mms}^{-1}$ is suggested for the low concentrations present in the North Sea. The relatively high settling velocity indicates that the dilute suspension consists entirely of a few large mud flocs.

4 LABORATORY TESTS

4.1 Characterisation

Several muds from British estuaries feeding into the North Sea have been tested in the Sedimentology Laboratory at HR. Bulk density, size grading, organic content and cation exchange capacity are regularly measured. Table 1 shows a comparison of these parameters for muds from Grangemouth, Harwich, Medway, Tees, Thames and Marsden Bay. It is noticeable that the sediment from Marsden Bay has a very low mud/silt content (≤ 63 microns) and a corresponding high bulk density because of the high sand content. Figure 4 shows the size grading of two samples from Marsden Bay. Only 25-30% of the material is less than 63 microns in diameter, and the median particle size is 110-130 microns. The Marsden Bay sample also has a low cation exchange capacity (13meq/100g on $<20\mu\text{m}$ fraction); this is also shown by the sample from Tees Seal Sands, which also has a lower silt content (and higher bulk density) than the other muds (Table 1).

Mud is usually transported as a suspended load, whereas sand may be transported as bed load or suspended load, depending on the velocity. Therefore when modelling the suspended load it is reasonable to consider a sediment with a high silt content. Suggested values of the parameters chosen on this basis are given at the bottom of Table 1.

4.2 Consolidation

The consolidation of several north-east coast muds has been measured at HR in settling columns. The procedure and analysis is described in detail in Appendix 1. The objectives of the tests are to

determine relationships between the effective stress and density and between permeability and density. Effective stress, σ' , is the stress between individual mud particles, holding them together (total stress minus excess pore pressure). Permeability, k , is a measure of how easily pore water is expelled. The relationships which were determined for some recently tested north-east coast muds are given in Table 2.

However, earlier in this report it was concluded that because of the very small thickness of temporary mud deposits in the North Sea, consolidation was not one of the important processes in this case. Therefore no generalisation has been made for a "North Sea Mud" from the data given in Table 2.

4.3 Uni-directional current erosion

4.3.1 Carousel tests

The erosion properties of several muds from the north-east coast estuaries have been tested in the HR Carousel. This apparatus is used to test under-consolidated mud beds, which have been deposited from suspension. The dry density of the material which is eroded is generally between 80kgm^{-3} and 250kgm^{-3} . The beds are generally tested after 2 - 6 days consolidation. A detailed description of the annular flume is given in Appendix 2, along with the method of analysis. The objectives of a test are to determine a relationship between erosion shear strength and density, and an erosion constant in the frequently used erosion equation:

$$dm/dt = m_e (\tau - \tau_e), \quad \tau \geq \tau_e \quad (1)$$

where

dm/dt = erosion rate per unit area

m_e = erosion constant ($kgN^{-1}s^{-1}$)

τ = applied shear stress (Nm^{-2})

τ_e = shear strength of exposed mud (Nm^{-2})

The relationships determined for a number of muds are given in Table 3. The shear strength with density relationship for all these muds is shown in Figure 5, which indicates that in the density range $50 - 200kgm^{-3}$ at least, a typical relationship can be chosen for erosion of a weak deposit of "North Sea Mud". This relationship is

$$\tau_e = 0.0005\rho^{1.4} \quad (2)$$

A typical value of the erosion constant from Table 3 is $0.0008kgN^{-1}s^{-1}$.

4.3.2 Marsden Bay erosion tests

Preliminary tests on erosion of the mud from Marsden Bay were conducted in the Carousel, using the methodology of Appendix 2. Because of the high sand content, mass erosion of the bed occurred early in the tests - large chunks of the bed were eroded rather than even erosion over the whole surface. The concentration and depth of erosion are then no longer representative of the whole sample.

An alternative method was used for testing the Marsden Bay sample. A reversing flume, 27m long, 600mm wide with a maximum depth of 200mm was used. Flow was produced by the rotation of an impeller driven by a constant discharge $0.14m^3s^{-1}$ axial pump. Altering the impeller-vane pitch caused the flow velocity to increase or decrease. The vane angle was changed at a constant rate in order to produce a

constant acceleration over given periods of time. Measurements were also carried out when the flow was steady. Flows from -1 to 1 m s^{-1} could be produced although flows were only used in one direction. Screens and guide vanes were placed at the entrance to provide a uniform flow. Uniform flow was confirmed by velocity profiles measured at the working section, which was 19m downstream from the entrance. The flume had a removable sample box in the working section. A false bottom allowed the sample box to be placed below the flume floor with the sample surface flush with the flume bed.

The depth of flow (150mm) was kept constant for all experiments. Velocity profiles were measured using a propeller current meter of diameter 10mm. Shear stresses were then calculated from these profiles, by fitting a semi-logarithmic curve (to at least the bottom part of each profile) to calculate the shear velocity, u_* , according to an empirical relationship:

$$u/u_* = 1/k \ln(z/z_0) \quad (3)$$

where

u = horizontal velocity component

u_* = shear velocity

z = distance above the bed

z_0 = the intercept of the semi log profile on the height axis

k = von Karman's constant (0.4)

The bed shear stress, τ , is then

$$\tau = \rho u_*^2 \quad (4)$$

where

ρ is the density of the fluid.

Three tests were run on Marsden mud; the mud was used directly from the container in which it was collected, moulded by hand to fit into the flume sample box and the surface smoothed. The surface of the sample was approximately 0.46m by 0.42m. A propeller current meter at a fixed height above the bed measured a reference velocity throughout the test. An ultrasonic probe mounted above the bed measured the depth of erosion.

The flow was accelerated at a constant rate from zero until erosion was observed visually (within 10-15 minutes). The acceleration was stopped and the flow held constant for another 15-20 minutes to assess the effects of erosion and measure a velocity profile.

Figure 6 shows the reference velocity and the depth of erosion (from the ultrasonic probe) as measured in the three tests. The current meter measuring the reference velocity was placed at 80mm (tests 1 and 2) or 75mm (test 3) above the bed, which was generally above the part of the profile used to calculate the bed shear stress.

The velocity profiles measured at the start of erosion in each test are shown in Figure 7. This also shows the logarithmic fit to the lower part of each profile and, for each test, gives the values of the shear velocity, u_* , the y axis intercept, z_0 , and the corresponding value of the bed shear stress, τ_e , as calculated from the fitted line. This value of shear stress will be slightly higher than the actual critical shear stress for erosion because during the time taken to stop the flume from accelerating, the velocity will have still increased a little. The calculated values of bed shear stress

just above the critical shear stress for erosion were:

	shear vel. u_* (ms^{-1})	intercept z_o (m)	shear stress τ_e (Nm^{-2})
Test 1	0.026	0.0009	0.7
Test 2	0.035	0.0058	1.2
Test 3	0.029	0.0008	0.9

Tests 1 and 3 indicate a similar value of critical shear stress, whereas test 2 resulted in a higher shear stress for erosion. This is reflected in the depth of erosion (Fig 6) which increased relatively quickly for tests 1 and 3 (approximately 6mm erosion in 6 minutes) but rather more slowly for test 2 (5mm in 10minutes). These values of the critical shear stress are applicable to consolidated mud deposits; the density of the samples was around 1670kgm^{-3} , which is similar to the in-situ bulk density (see Table 1). A range of shear stresses is to be expected as the samples were not homogeneous. This range ($0.7 - 1.2\text{Nm}^{-2}$) suggests a typical value for the critical shear stress of a consolidated deposit of mud (including sand) of 1.0Nm^{-2} .

4.4 Wave erosion

Experience with flume experiments has shown that the peak bed shear stress exerted by a wave is one of the most important factors controlling the rate and depth of erosion of a thin deposit of mud.

4.4.1 Typical wave conditions in a muddy coastal zone

Tide and wave measurements taken between February and December 1988 in Tees Bay were examined in a study at HR (Ref 3). Tees Bay is a typical shallow

bay on the east coast, which contains a mixture of mud and sand deposits. Frequency distributions of the significant wave height, H_s , the zero-crossing period, T_z , and the tide level were calculated. These were used to derive frequency distributions for the peak wave-induced bed orbital velocity, U_{max} , and the maximum wave-induced bed shear stress, τ_{max} . A summary of the frequency distributions is given below, giving the value of each variable which is exceeded for 50% (and 10% and 1%) of the time:

	50% >	10% >	1% >
H_s (m)	0.50	1.30	2.04
T_z (s)	4.9	7.0	8.5
Tide (m CD)	3.26	4.90	5.55
U_{max} (m/s)	0.10	0.40	0.73
τ_{max} (Nm^{-2})	0.2	1.5	3.8

The study reports that (as expected) the wave heights and periods are lower in the summer months. The wave heights are considerably lower than would be expected well outside Tees Bay, but the wave periods are quite typical of open sea conditions. This results in a low wave steepness throughout the year, about half to two-thirds of what one would expect for coastal ocean waves. This is because the wave heights are reduced due to the sheltering effects on entry to Tees Bay, but the wave periods are almost unchanged. The water depth used in these calculations was 9 - 15m, based on an average bed level of -9m CD.

4.4.2 Typical wave conditions offshore

For deeper water in the North Sea, the same linear wave theory was used to calculate typical wave-induced bed shear stresses. Typically, wave heights

and wave periods are lower in the southern part of the North Sea, especially near coasts and in shallower water (Ref 8). In summer (June - August), the significant wave height is typically 0.5 - 1.5m, (exceeded 50% of the time) with a wave period of 6 - 7 seconds. In winter (December - February), the significant wave height is typically a little larger at 1.5 - 3.0m (exceeded 50% of the time), with a similar wave period of 5 - 7 seconds. Storms are known to occur on about 30 days a year in the North Sea. In summer, wave heights of 1.5 - 3.0m are exceeded 10% of the time, with wave periods of 7 - 10 seconds. Assuming more storms occur in winter than in summer, wave conditions which are exceeded 20% of the time have been calculated. These indicate wave heights of 1.8 - 4.5m and wave periods of 7 - 11 seconds. Significant bed shear stresses for a number of depths have been calculated for the above conditions, and are given in the table below:

	H_s (m)	T_z (s)	d (m)	τ_{max} (Nm^{-2})
Summer	1.5	7	40	0.08
50% >	1.5	7	50	0.06
	1.5	7	60	0.04
	1.5	7	80	0.02
Summer	3	10	40	0.35
10% >	3	10	50	0.22
	3	10	60	0.16
	3	10	80	0.10
Winter	3	9	40	0.28
50% >	3	9	50	0.18
	3	9	60	0.13
	3	9	80	0.08
Winter	4.5	11	40	0.86
20% >	4.5	11	50	0.57
	4.5	11	60	0.39
	4.5	11	80	0.20
	4.5	11	110	0.11

If a critical shear stress of erosion of 0.1Nm^{-2} is assumed, the wave-induced shear stresses calculated for the North Sea would be sufficient to erode material deposited on the bottom at around 35m depth in typical summer conditions or 60m in typical winter conditions. During storms, the shear stresses would be sufficient to erode material at 80m (summer) or 110m (winter). Thin layers of very soft recent deposits would be even more susceptible to erosion.

4.4.3 Laboratory tests

The sample of mud (and sand) from Marsden Bay was tested in a wave flume at HR to investigate its behaviour under mono-frequency waves. The tests were conducted in a shallow flume (0.5m) with short period waves (1.2s), without currents. Although this flume is unable to generate the conditions of wave height, period and water depth representative of the field, it was considered that the bottom orbital velocities (and hence peak bed shear stresses) which can be generated in the flume would be similar to peak bed shear stresses in the field. The flume was 25m long with a trough for the bed, 10m by 0.2m, in the central part of the flume. A wave generator at one end produced mono-frequency waves; the wave height was increased by altering the stroke length on the generator.

A turbidity sensor was used to measure the suspended sediment concentration at several depths and positions along the flume, at 30 minute intervals throughout each test. An ultrasonic probe was used to monitor the surface level of the bed at several points along the bed, at 30 minute intervals.

Two wave tests were run on Marsden Bay mud. For each test, a high density slurry (mean dry density $530 - 540\text{kgm}^{-3}$) was made up and placed in the test section. The depth of water in the flume was 0.3m in each test. The bed was subjected to mono-frequency waves, with each wave height (and hence bed shear stress) being applied for an hour before being increased. The wave period was constant at 1.2s, the wave height was in the range 0.05 - 0.10m, with corresponding bed shear stress in the range $0.25 - 0.48\text{Nm}^{-2}$.

The bed levels measured by the ultrasonic probe were not very reliable for these tests because the probe is very sensitive to small changes which can be caused merely by the movement of a sand grain. Figure 8 shows the suspended sediment concentration at 0.05m above the bed during test 1. The test section is in the part of the flume from 0.5m to 10.5m. This figure shows a progressive increase in the suspended concentration, and also that it is higher at the end of the flume furthest from the wave generator. The ultrasonic probe measurements did not suggest that one part of the bed was eroding faster than any other, so it was assumed that the material was eroded evenly from the bed but collected towards one end of the flume due to the pattern of induced currents.

Figure 8 also shows the calculated erosion rate per unit area ($\text{kgm}^{-2}\text{s}^{-1}$) with time for test 1. This shows a decreasing erosion rate with time, even though the shear stress was increased at the end of each hour. If it is assumed that the erosion rate is proportional to the excess shear stress, then in order to observe a decrease in erosion rate with an increase in applied shear stress (as observed here), the bed itself would have to have increased in shear

strength. This would be possible if consolidation had occurred. It was not possible to determine density profiles of the bed accurately during the tests and so it is not known whether the bed had eroded down to material of a higher density (and possibly higher shear strength). It is possible to make an upper bound estimate of an erosion constant, based on an equation for erosion under waves of

$$dm/dt = m_w (\tau - \tau_{cw}) \quad (5)$$

where

dm/dt = rate of erosion per unit area ($\text{kgm}^{-2}\text{s}^{-1}$)

m_w = erosion constant for wave erosion

τ = applied bed shear stress

τ_{cw} = critical shear stress for wave erosion

If one assumes that after each hour, the mud has eroded to a point where it can withstand further erosion (this gives a high estimate for the shear strength, as the mud has not actually stopped eroding), then the excess shear stress can be calculated (low estimate). The erosion constant can then be calculated from the measured erosion rate - this is a high estimate because of the low estimate for excess shear. On test 1, the value of the erosion constant calculated by this method was $0.0003\text{kgN}^{-1}\text{s}^{-1}$. The critical shear stress appeared to increase during the test, but was in the range $0.1 - 0.3\text{Nm}^{-2}$ for a bed with an initial dry density of around 540kgm^{-3} .

Figure 9 shows the corresponding suspended sediment concentrations along the flume and erosion rates for each 30 minute period for test 2. A different pattern is observed, in that the erosion rate appears to increase with an increase in shear stress (the shear stress is increased at the end of each

hour). The mean density of the bed in test 2 was around 530kgm^{-3} . As density profiles of the bed were not recorded during these tests, the density of the surface layer is unknown. Previous wave tests on other muds have indicated that there is little or no change in density profile during a test, when starting with such a high density slurry.

The erosion rate during each test is shown against applied shear stress in Figure 10. It is not easy from this graph to relate the erosion rate to the excess shear stress according to equation 5, because it appears that the shear strength of the bed is changing during the test. However, for these two tests, in which the dry density of the bed was around 530kgm^{-3} , the shear strength of the bed was in the range $0.1 - 0.4\text{Nm}^{-2}$ and the erosion rate was around $3 \cdot 10^{-5} \text{kgm}^{-2}\text{s}^{-1}$.

5 HEAVY METALS

5.1 Distribution

It is well documented that heavy metals have an affinity for the finer fractions (clays and silt, < 63 microns in diameter) of a given sediment (Ref 9). To model the transport of heavy metals, it is therefore important to understand the transport processes of the fine sediments.

The distribution of heavy metals dissolved in the water and attached to the sediment has been studied by a number of researchers. Kersten et al. (Ref 10) report on the pathways of heavy metals in the marine environment, including the point sources such as estuaries and dredged spoil disposal and the diffuse sources such as the atmosphere. Kersten et al. also summarise the input quantities from each of the

sources, although there are still great uncertainties in the determination of atmospheric input. In general, the concentration (metal/mud) of heavy metals in the particulate phase, attached to the sediment, is much higher than that of the same metal in the dissolved phase (metal/water).

The distributions of cadmium and lead in the particulate phase, as determined by a survey by Dicke in winter 1987, are shown in Figure 11 (from Kersten) over the whole North Sea. The samples of suspended sediment were collected from 10m water depth. The metals show quite different patterns, with cadmium showing higher concentrations in the central and northern part of the North Sea. In contrast, lead shows higher concentrations around the British and Dutch coasts, decreasing towards the central part. Other distributions are found for metals in the bed sediments and dissolved in the water.

Distributions of copper, zinc, cadmium, nickel, iron and manganese in both the dissolved phase and the particulate phase were measured by Nolting (Ref 11) in a survey in 1984. In general the concentrations of metals in both phases decreased from the coastal zones towards the central bight of the Southern North Sea. Relationships between metal concentrations and salinity and suspended sediment concentrations were also investigated. The distributions of zinc, iron and manganese in the particulate form was shown to be dependent on the suspended sediment concentration.

Taylor (Ref 12) reports on heavy metal distributions in sediments along the East coast of Britain, in order to assess the environmental impact of waste material which is discharged from the east coast

estuaries. The author concluded that the geology of the area appeared to be a more important factor than the industrial input in the resulting metal content of the marine sediments.

5.2 Adsorption/desorption

The processes of adsorption and desorption of heavy metals to and from cohesive sediment are the subject of much controversy between authors (Ref 9).

Several studies report a close association between heavy metals and organic matter. As the organic content of North Sea sediment appears to show seasonal changes, this may also affect the concentrations of heavy metals adsorbed on the sediment.

The affinity of heavy metals for the finer grained sediments is well known. The size grading of the muds in the North Sea is therefore important in assessing whether the metals are likely to be desorbed as the sediment is transported over areas of coarse grained sediment (with the subsequent exchange of suspended sediment). In addition, the mineralogical composition of the sediment is important: clay minerals are more readily available for ion exchange processes.

The ambient conditions of salinity, redox potential (Eh), acidity (pH), turbidity and temperature also control the adsorption/desorption processes.

Increased turbidity appears to increase the level of adsorption - with a large surface area providing more potential sites for adsorption to occur.

Similarly, temperature is generally seen to act as a catalyst, increasing reaction rates.

A study by Odd and Murray (Ref 13) gives desorption rates of metals from sewage sludge, based on analysis of experimental data from other researchers. Desorption is attributed to the oxidisation of organic matter or metal sulphides, the dilution ratio and pH of the seawater and the complexation of metals to form soluble complexes of inorganic and organic ligands. The experiments were carried out at high sludge levels (1:50 to 1:200 dilution) and the desorption rate constants determined may be useful for modelling the initial stages following disposal. However there is a strong dependence of the partition coefficients (mass of dissolved metal per unit mass of water/mass of adsorbed metal per unit mass of suspended particulate matter) on the dilution factor. In reality the disposal of sewage sludge in the open sea produces much higher dilutions than those achieved in the experiments, so more data is needed in order to model the adsorption/desorption of heavy metals in low suspended sediment concentrations.

6 CONCLUSIONS

1. The concentration of suspended sediment in the North Sea is less than 1mg l^{-1} in the north west of the North Sea, increasing to $5\text{-}10\text{mg l}^{-1}$ near the European coast and in the Channel (Fig 1). Measurements suggest that the suspended sediment concentration increases in the winter, by 50-150% in the central part of the North Sea, and up to 300% in the coastal areas. The bed sediment over most of the North Sea is sand or gravel, with a few isolated patches of mud. The bottom of the Norwegian trough (up to 400m deep) is muddy (Fig 2). There are some muddy coastal zones around the estuaries of Britain and Europe which are not shown on Figure 2.

2. The NECCSDS group measured field settling velocities using Quisset bottles. The calculated value of the median settling velocity, w_{50} , was in the range $0.009 - 0.450\text{mms}^{-1}$. Preliminary tests were conducted by HR in the Tees estuary using an image processing technique. The values of settling velocity measured by this method indicated a median settling velocity of 0.3mms^{-1} (6ppm) to 0.5mms^{-1} (10ppm).
3. The sedimentological properties of several muds from estuaries on the north-east coast of Britain were compared (Table 1). Values of the parameters chosen for a typical "North Sea mud" were: 85% silt (< 63 microns), 5% organic content, cation exchange capacity 20meq/100g (<20 μm fraction). The consolidation properties of several muds from north-east coast estuaries were compared (Table 2). Because very thin layers of mud are only temporarily deposited in the North Sea, consolidation was not considered to be one of the important processes. The erosion properties of several north-east coast muds have been tested in the HR Carousel, and were compared (Table 3). A shear strength with density relationship for North Sea mud was chosen to be:
- $$\tau_e = 0.0005\rho^{1.4}$$
- A typical value of the erosion constant is $0.0008\text{kgN}^{-1}\text{s}^{-1}$.
4. A sample of sediment collected from Marsden Bay was tested for uni-directional current erosion in the reversing flume at HR. The shear stress needed for erosion was calculated from a velocity profile measured at the onset of

erosion. The critical shear stress for erosion was in the range $0.7-1.2\text{Nm}^{-2}$ for mud at a bulk density of around 1670kgm^{-3} . The sample of reconstituted mud (and sand) from Marsden Bay was also tested in a wave flume at HR to investigate its behaviour under monofrequency waves. For these two tests, the mean dry density of the bed was around 530kgm^{-3} (no density profiles were measured but little consolidation was expected), the critical shear stress for erosion of the bed was in the range $0.1 - 0.4\text{Nm}^{-2}$ and the erosion rate was around $3 \cdot 10^{-5} \text{kgm}^{-2}\text{s}^{-1}$.

5. Tide and wave measurements taken between February and December 1988 in Tees Bay (representative bed level -9m CD) were examined in a study at HR. The wave heights and periods are lower in the summer months. The wave heights are considerably lower than would be expected well outside Tees Bay, but the wave periods are quite typical of open sea conditions. A peak bed shear stress of 0.2Nm^{-2} is exceeded 50% of the time. The modal summer wave conditions in the North Sea result in a shear stress of around 0.1Nm^{-2} at 50m; winter conditions give 0.1Nm^{-2} at around 60m. Much higher stresses would be experienced during storms.
6. A number of researchers have measured distributions of metals in the North Sea. In general, the concentration (metal/mud) of heavy metals in the particulate phase, is much higher than that of the same metal in the dissolved phase (metal/water). In most cases the concentrations of metals in both phases decreased from the coastal zones towards the

central bight of the Southern North Sea. The rates of adsorption and desorption are affected by the organic content of the suspended sediment, the grain size, the mineral composition and the ambient conditions of salinity, redox potential, turbidity and temperature.

7 ACKNOWLEDGEMENTS

The replies from the European laboratories, responding to the request for information about North Sea Mud Properties, are gratefully acknowledged. Helpful comments were received from Mr N V M Odd.

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TABLES

TABLE 1 Comparison of muds recently tested at HR
Sedimentology analysis

MUD TYPE	%SILT (≤ 63 microns)	ORGANIC CONTENT	CATION EXCHANGE	MINERALOGY		BULK DENSITY
				quartz	clays	
	(%)	(%)	(meq/100g) *	(%) *	(%)	(kgm ⁻³)
GRANGEMOUTH	80-90	5	20	18	82	1370
HARWICH	88-95	2.5	23	24	76	1250
MEDWAY	80	3	22	21	79	1220
TEES Seal Sands	75	4	15	19	81	1550
TEES Dredged	75	11	18	22	78	1430
THAMES	90-98	5	24			
MARSDEN BAY	20-30	6	13			1800

Suggested values for North Sea mud

NORTH SEA	85	5	20			
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* = analysis on less than 20 micron fraction

TABLE 2 Comparison of muds recently tested at HR
Consolidation properties

MUD TYPE	PERMEABILITY (k ms ⁻¹)	EFFECTIVE STRESS (σ' Nm ⁻²)
GRANGEMOUTH	log(k)= -0.010ρ - 5.5	σ' = 0.0005ρ ² - 0.01ρ + 0.05
HARWICH	log(k)= -0.0115ρ - 5.3	
TEES Dredged	log(k)= -0.011ρ - 5.0	σ' = 0.0008ρ ² - 0.016ρ + 0.08

TABLE 3 Comparison of muds recently tested at HR
Erosion properties of weak under-consolidated beds

MUD TYPE	EROSION CONSTANT ($m_e \text{ kgN}^{-1}\text{s}^{-1}$)	EROSION SHEAR STRENGTH as a function of ρ ($\tau_e \text{ Nm}^{-2}$)	EROSION SHEAR STRENGTH AT $\rho = 70\text{kgm}^{-3}$ ($\tau_e \text{ Nm}^{-2}$)
GRANGEMOUTH	0.0005 - 0.0014	$\tau_e = 0.0045\rho^{0.9}$	0.28
HARWICH	0.0007	$\tau_e = 0.00035\rho^{1.43}$	0.15
IPSWICH	0.0009 - 0.0030	$\tau_e = 0.0003\rho^{1.55}$	0.22
MEDWAY	0.0007	$\tau_e = 0.0007\rho^{1.3}$	0.18
TEES Seal Sands	0.0002 - 0.0014	$\tau_e = 0.0025\rho^{1.0}$	0.18
TEES Dredged	0.0005 - 0.0018	$\tau_e = 0.00014\rho^{1.7}$	0.19

Suggested values for North Sea mud

NORTH SEA	0.0008	$\tau_e = 0.0005\rho^{1.4}$	0.19
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FIGURES

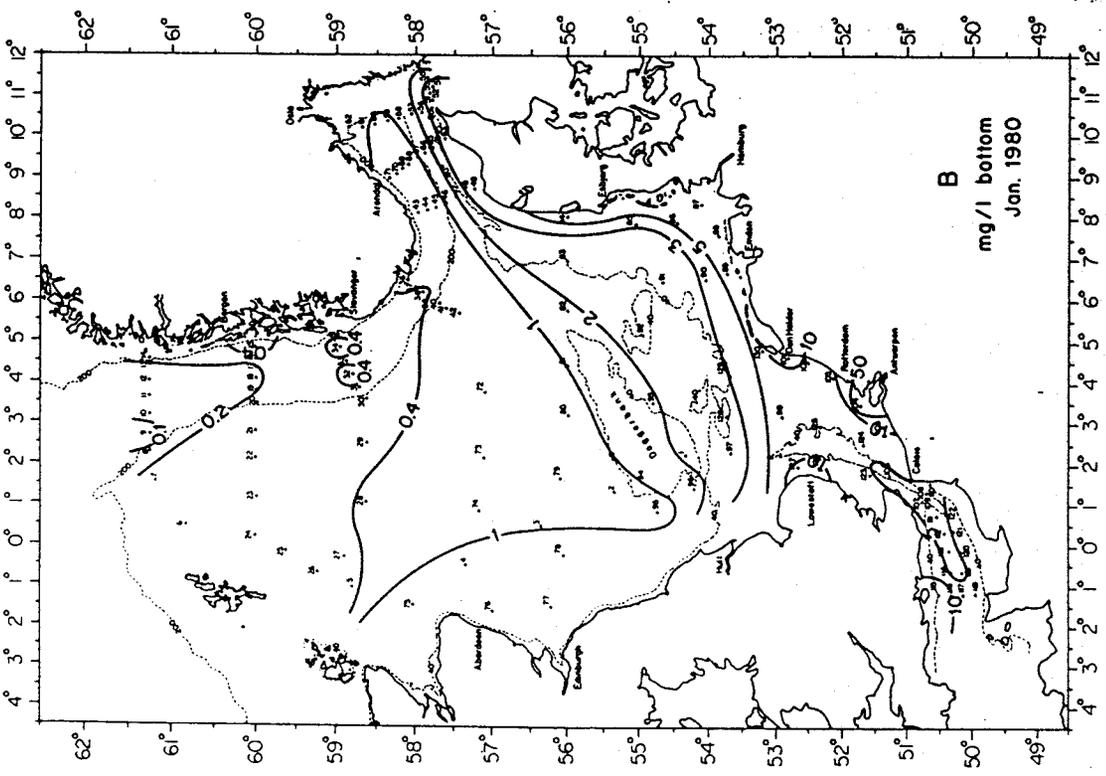
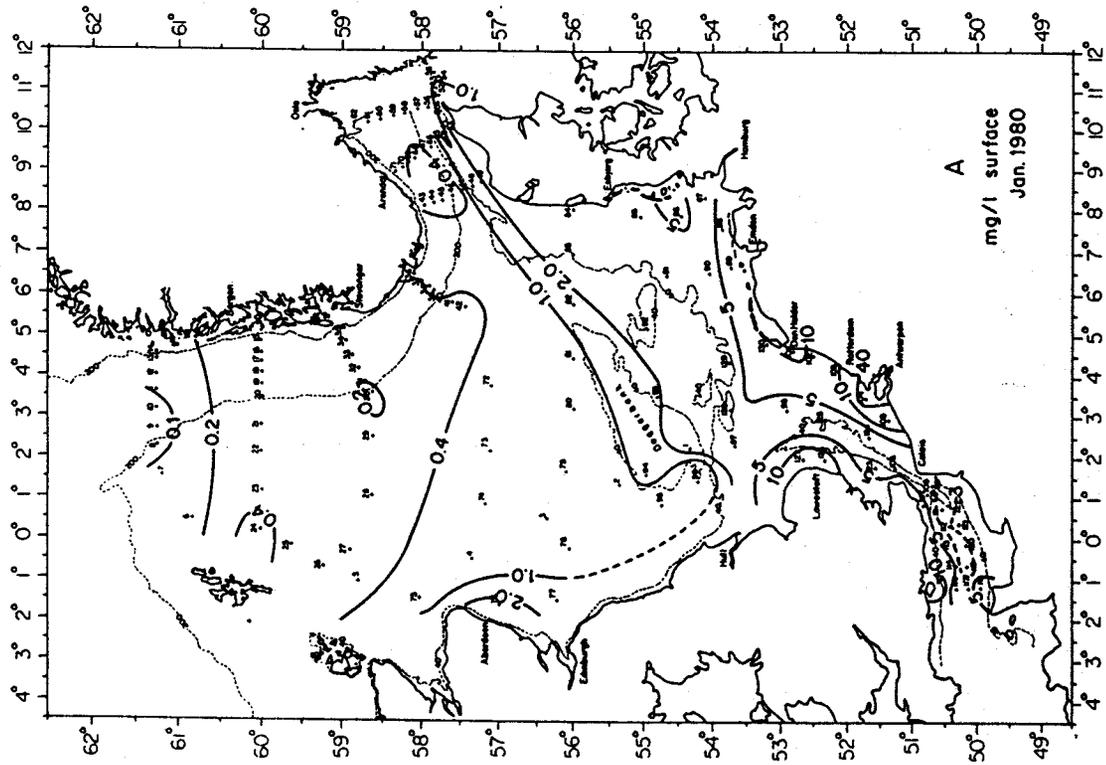


Fig 1 Concentration of suspended sediment in the surface (a) and bottom (b) layers of the North Sea (Eisma and Kalf)

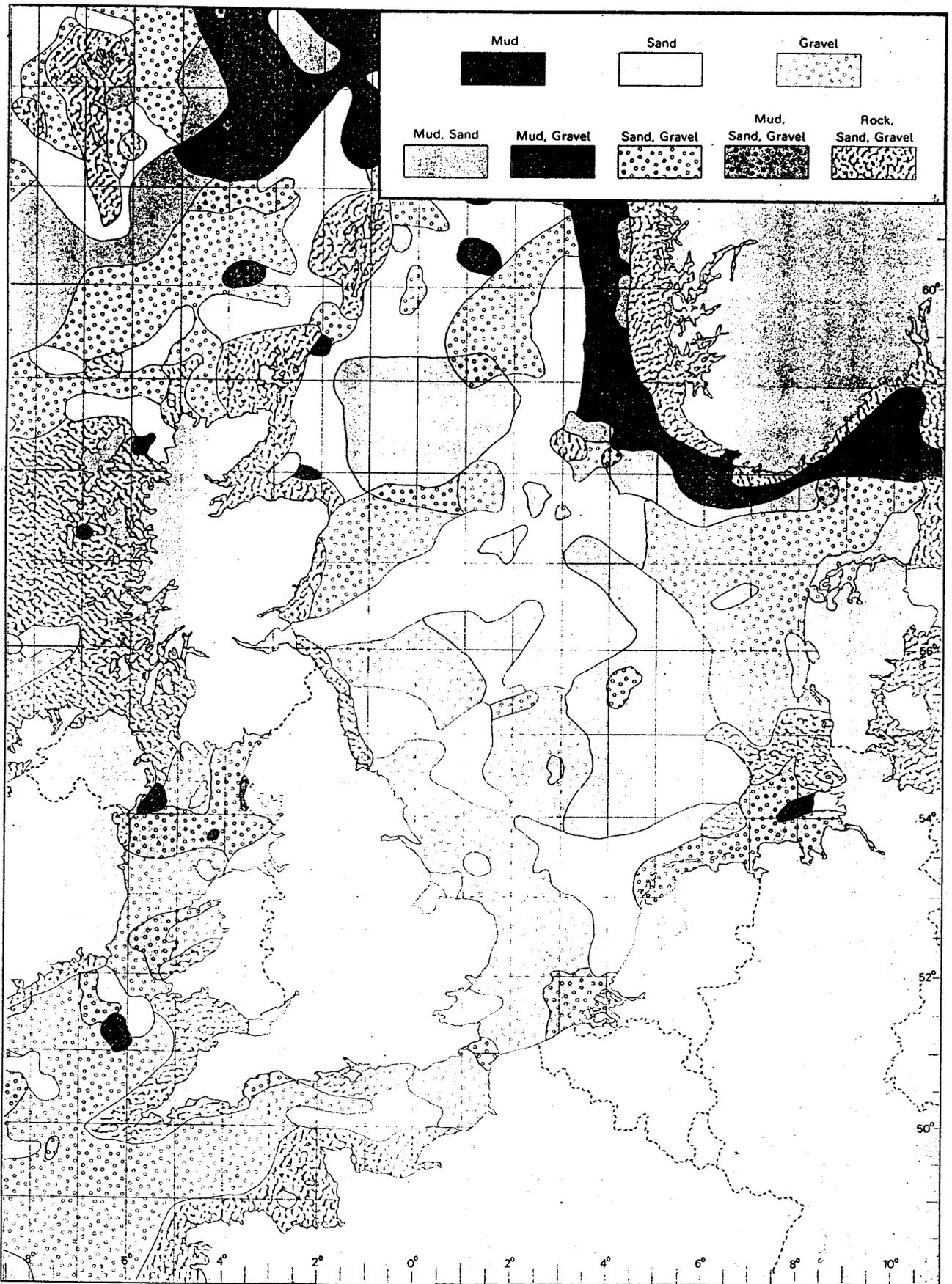


Fig 2 Distribution of bed sediment type in the North Sea (MAFF)

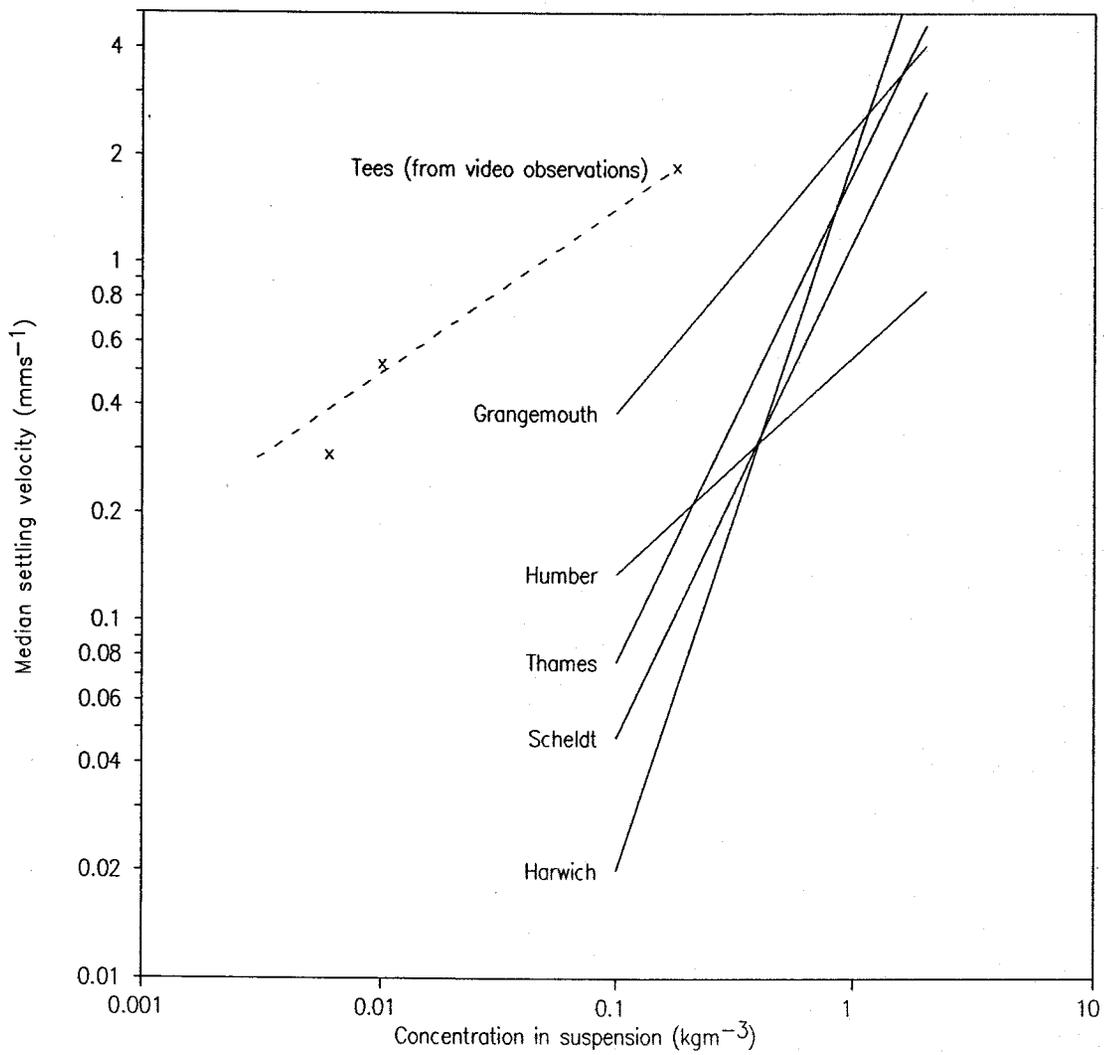


Fig 3 Settling velocity against concentration for sediment from North Sea estuaries

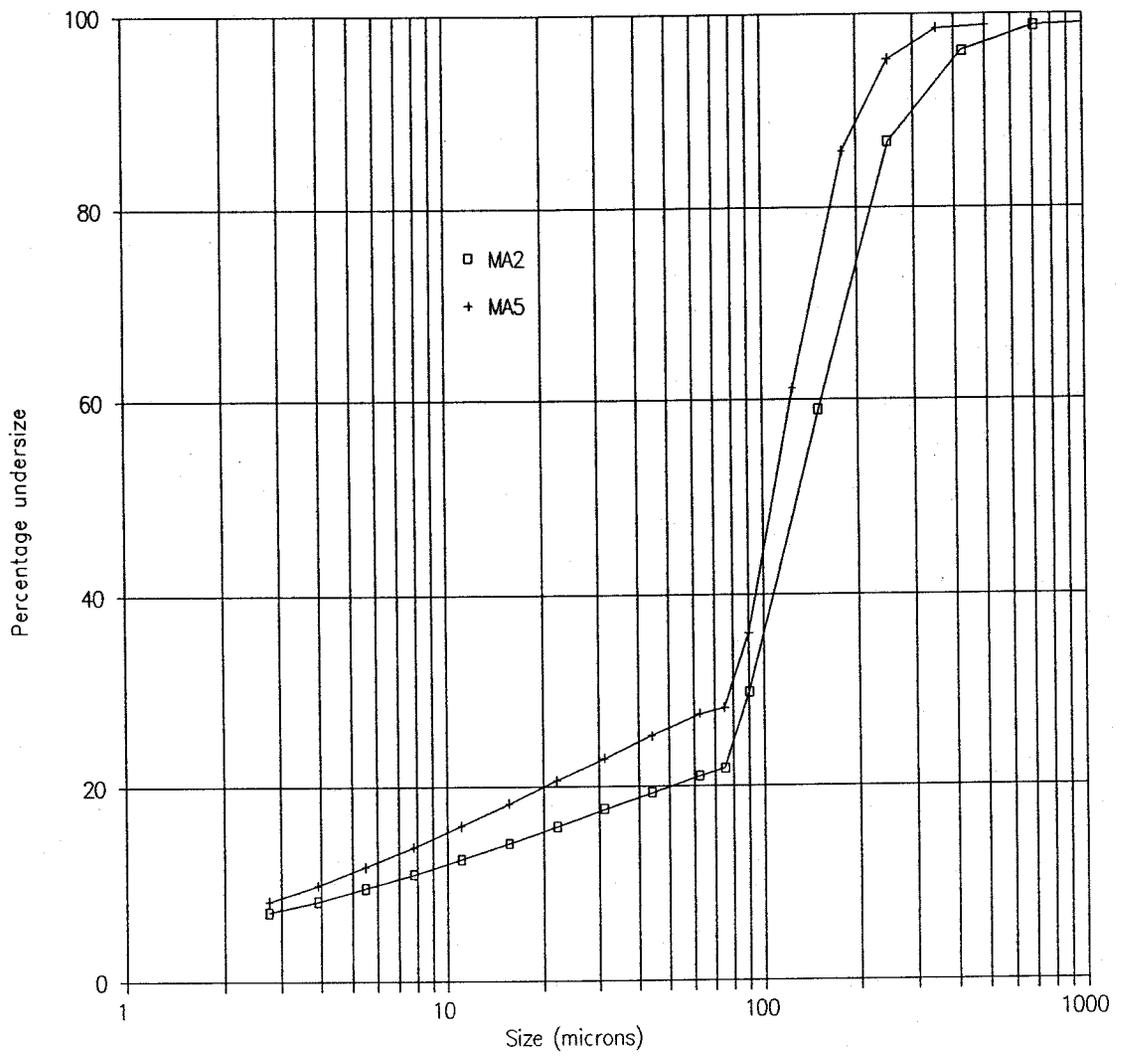


Fig 4 Size grading of Marsden Bay mud

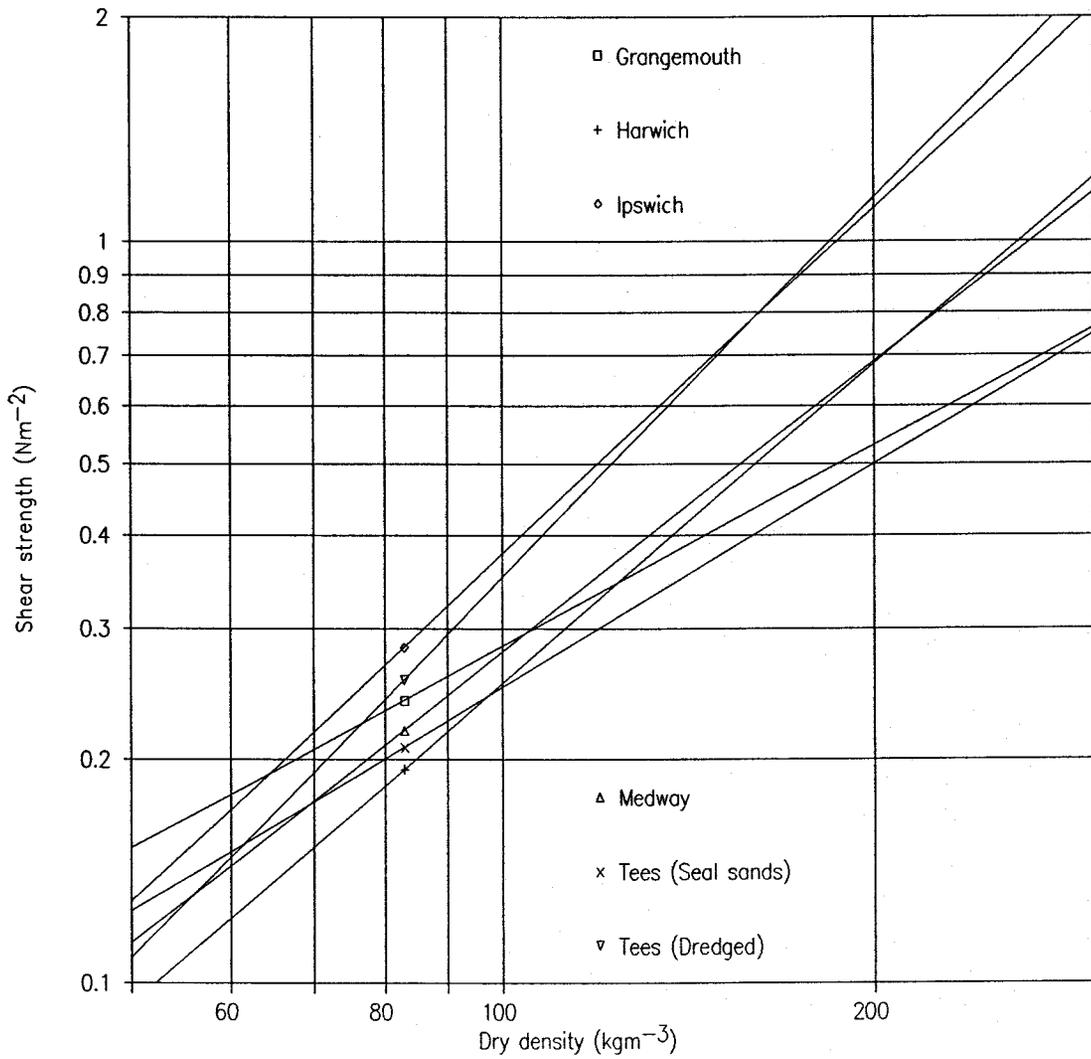


Fig 5 Shear strength with density relationships for sediment from north-east coast estuaries

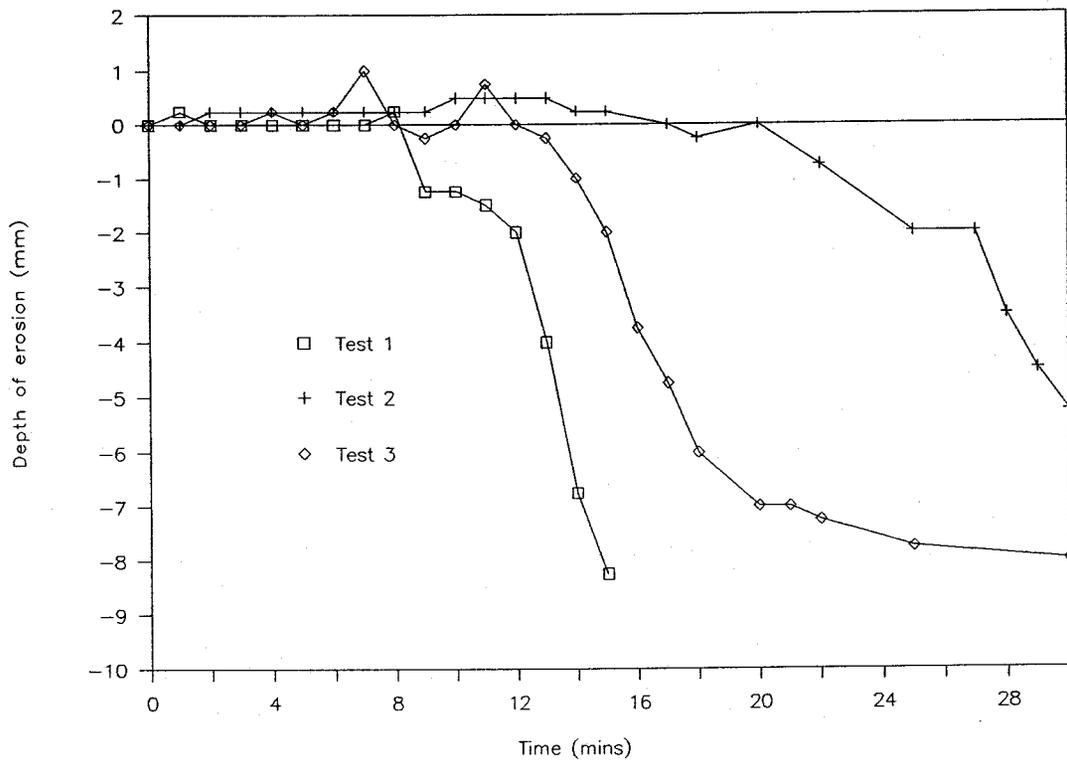
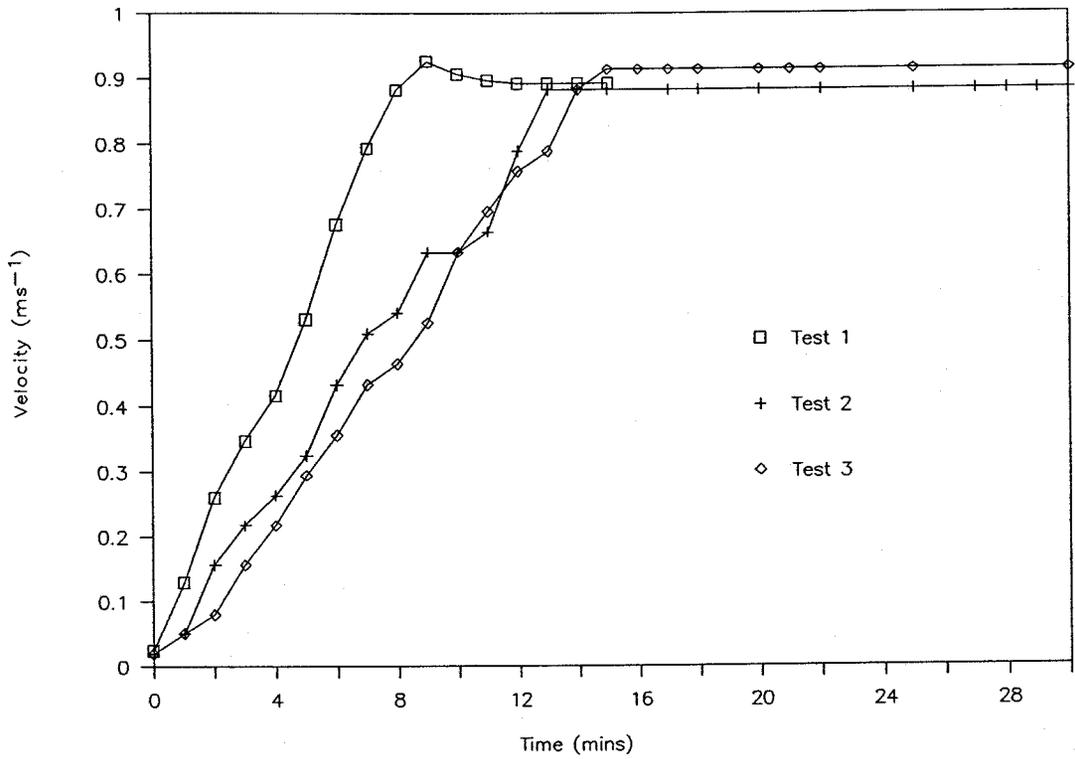


Fig 6 Velocity at reference point and depth of erosion with time, erosion tests 1, 2 and 3

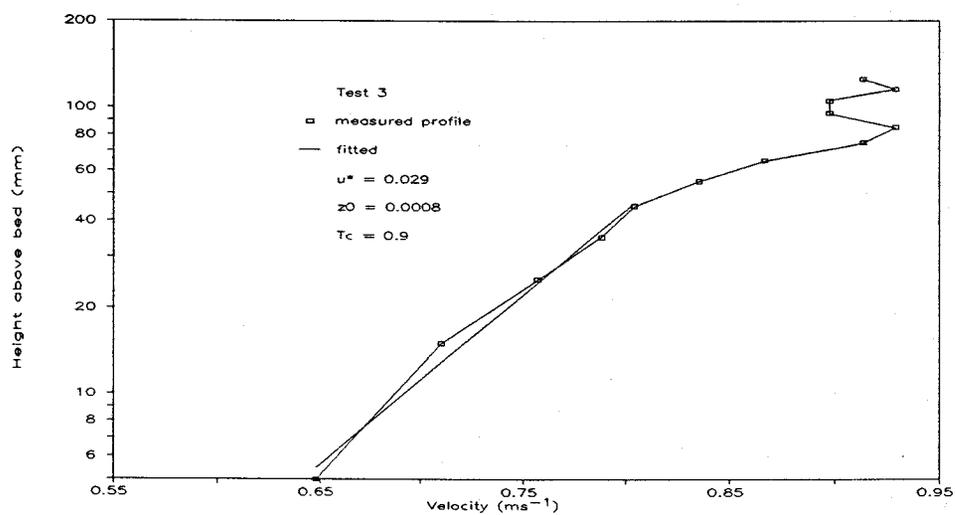
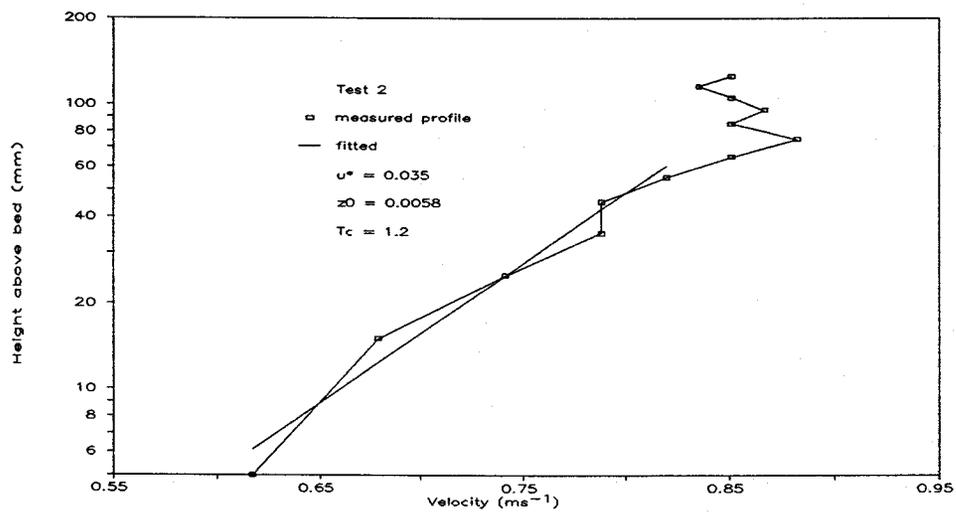
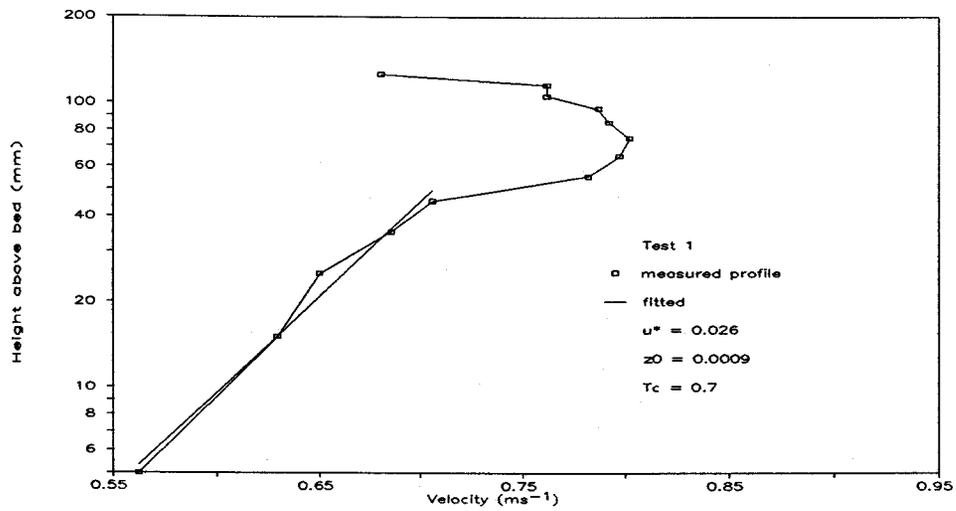


Fig 7 Velocity profiles at the onset of erosion, erosion test 1, 2 and 3

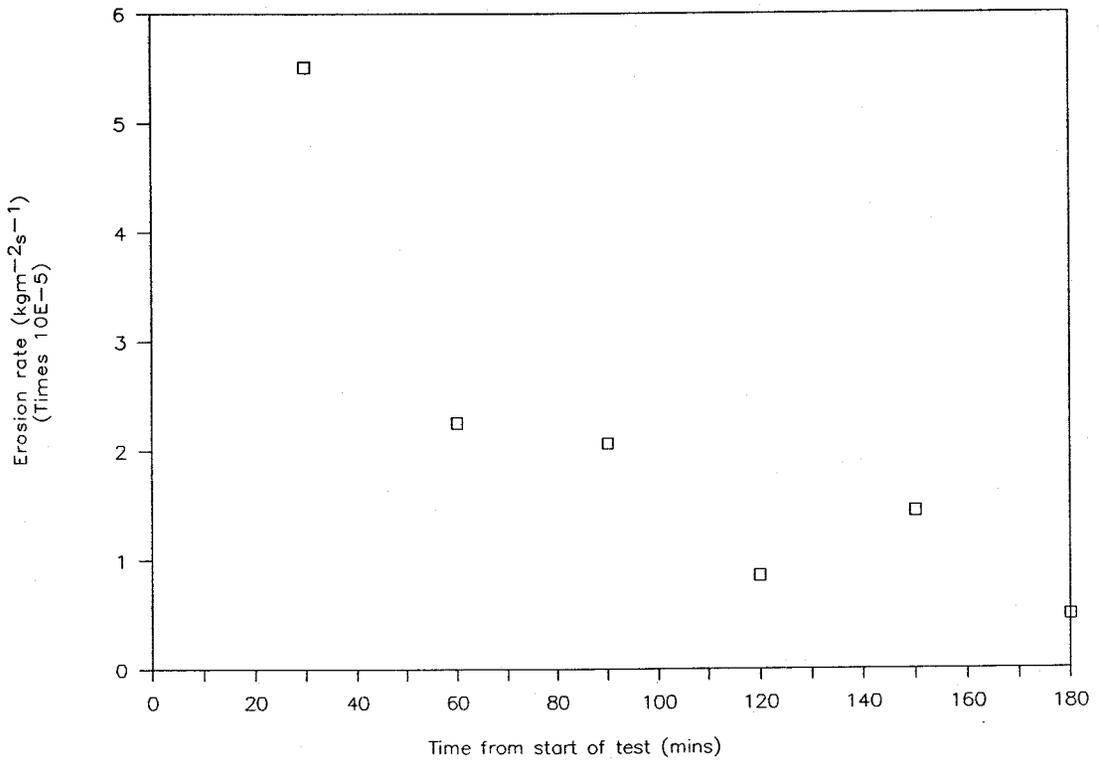
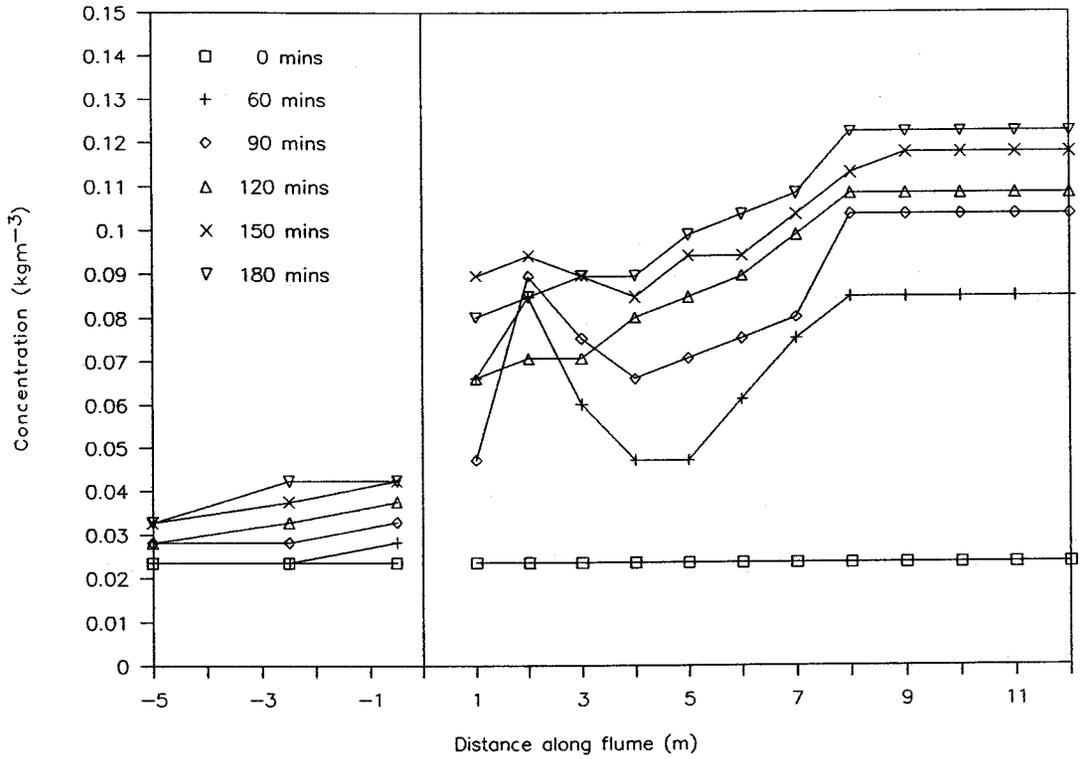


Fig 8 Concentration along the flume with time; erosion rate against time, wave test 1

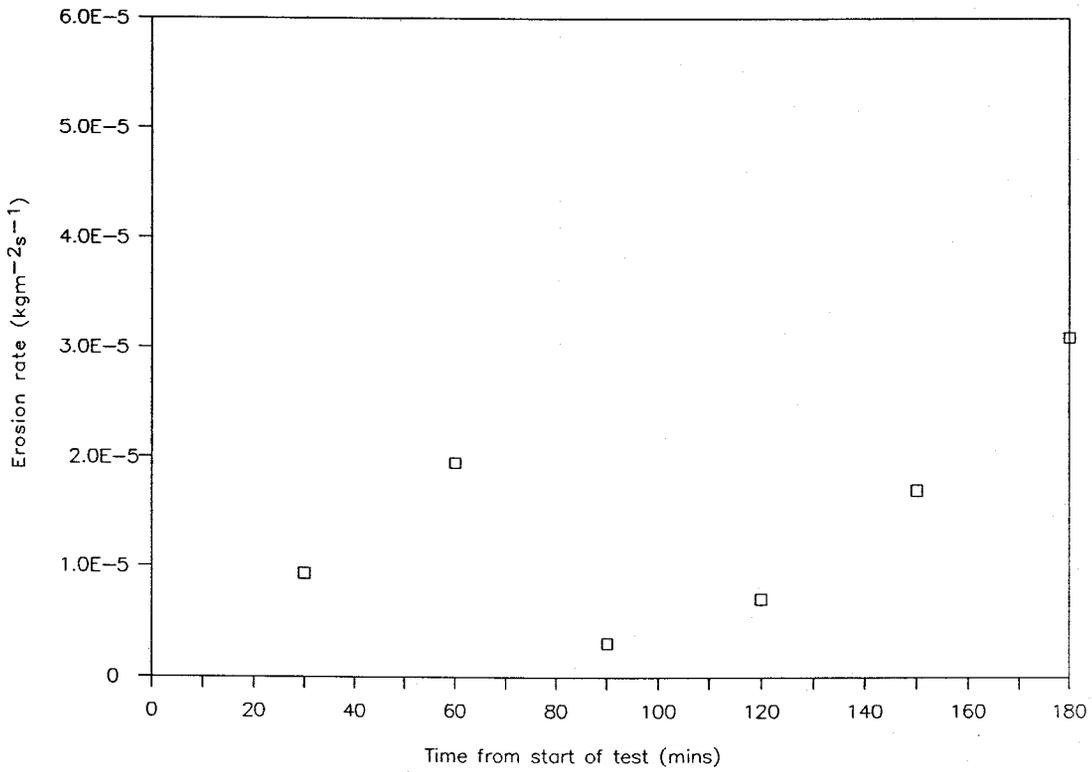
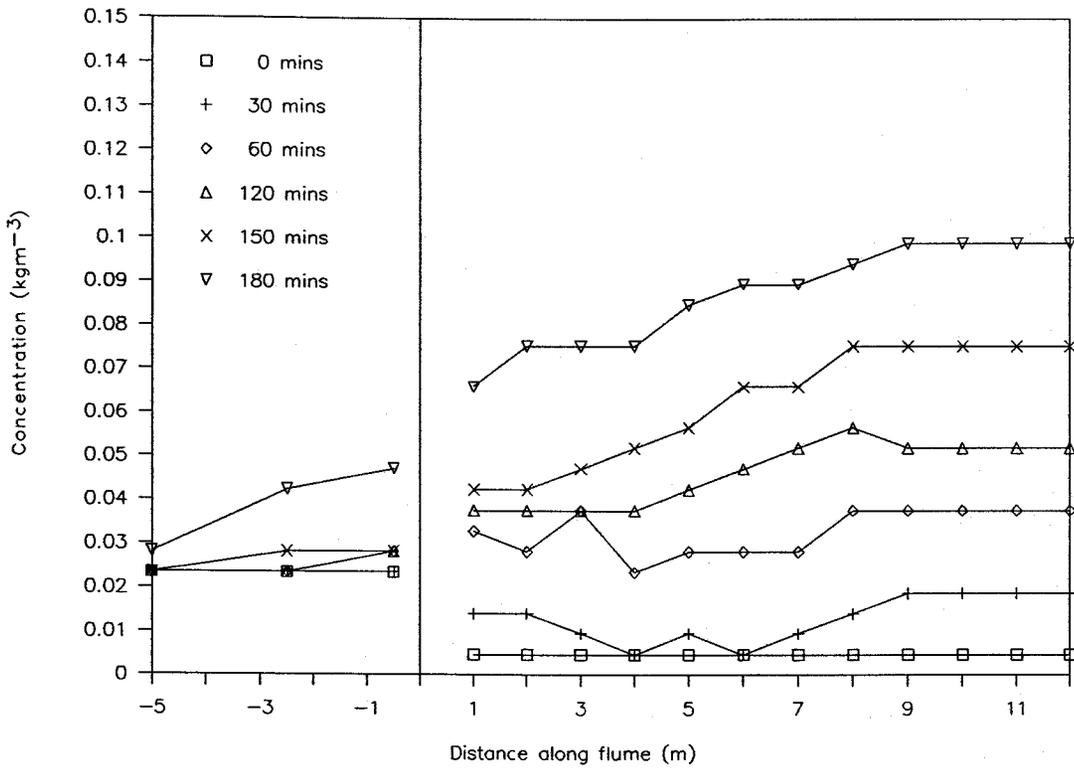


Fig 9 Concentration along the flume with time; erosion rate against time, wave test 2

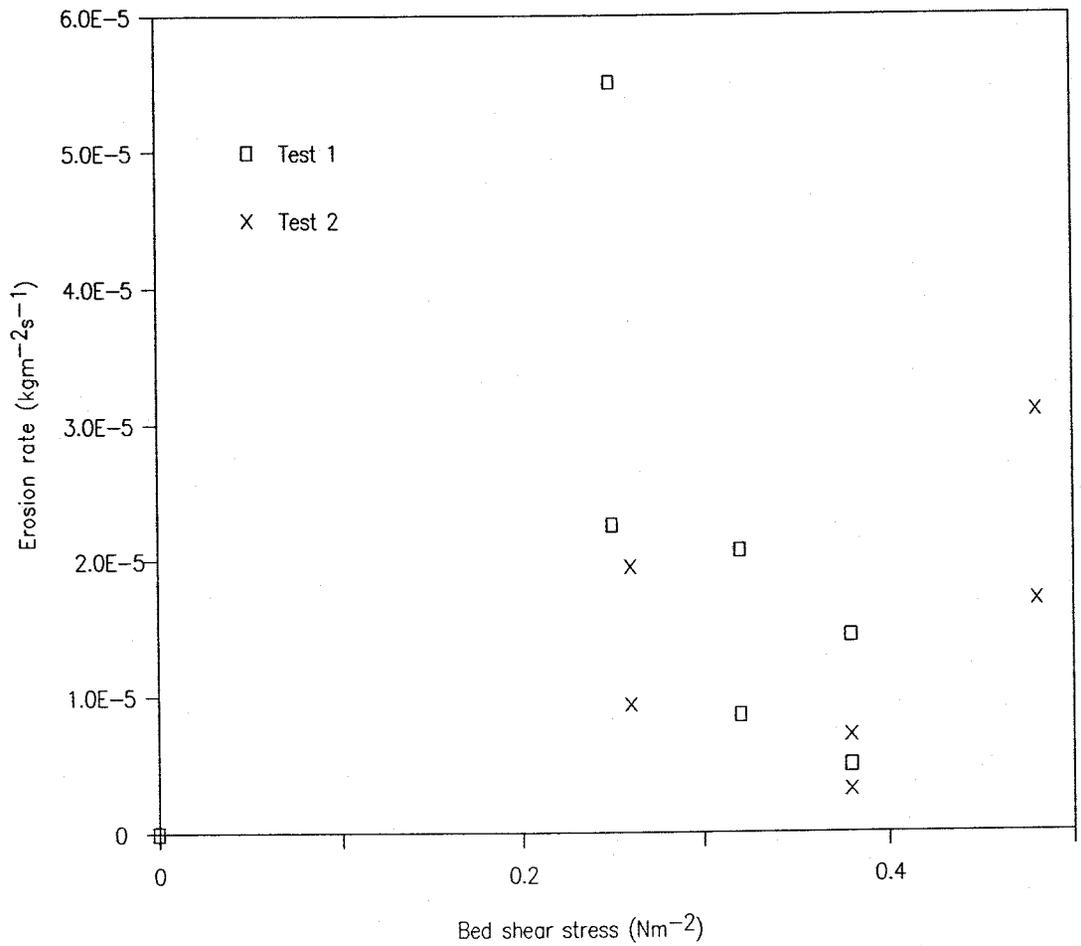


Fig 10 Erosion rate against shear stress, wave tests 1 and 2

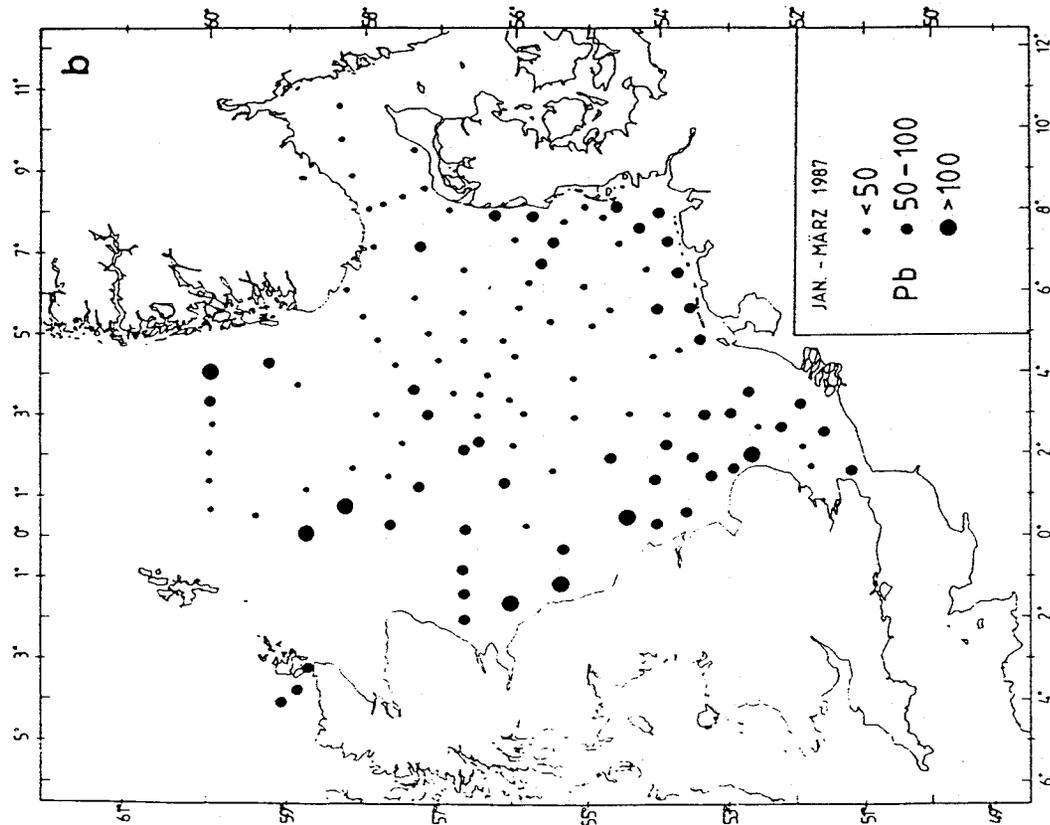
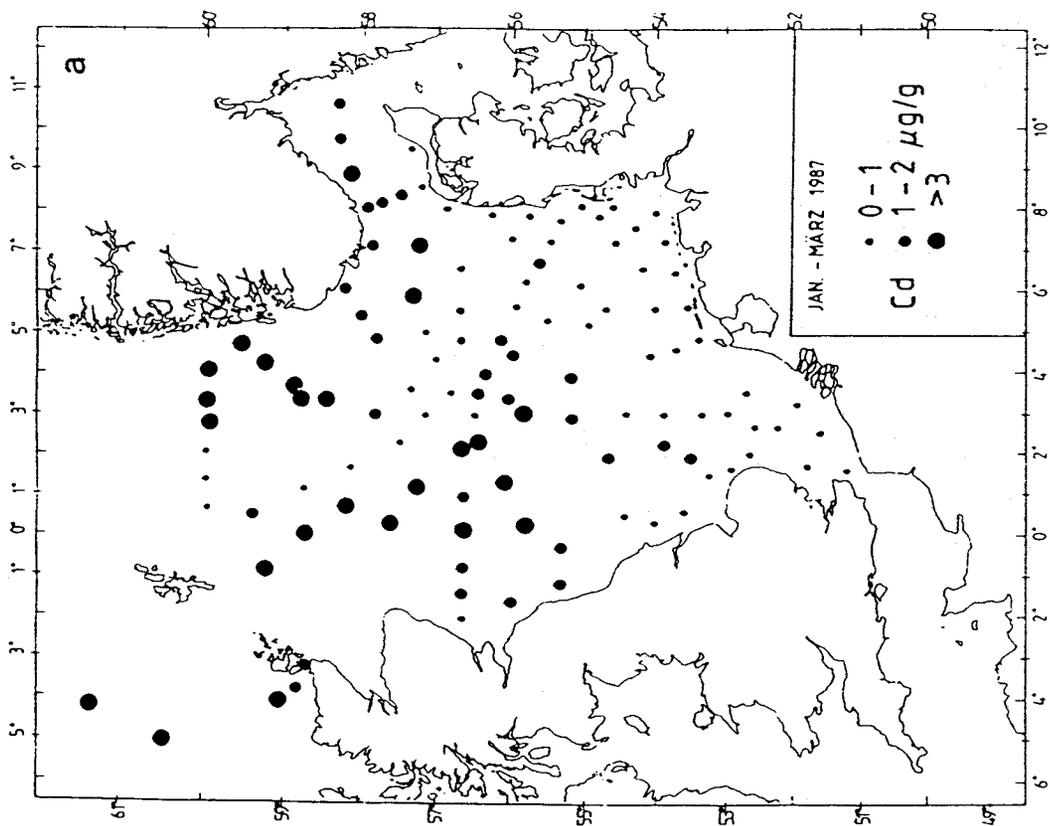


Fig 11 Distribution patterns of cadmium (a) and lead (b) in the North Sea (Kersten et al)

APPENDICES

APPENDIX 0 - LETTER TO EUROPEAN LABORATORIES AND ADDRESS LIST

Our Ref: TE/SR/32

17 April 1990

NORTH SEA TASK FORCE - MUD PROPERTIES

Dear

1. As part of the United Kingdom contribution to the North Sea Task Force I am currently reviewing observations made on North Sea mud properties. I have completed a review of relevant literature known in the United Kingdom.

2. I wish to extend the review to cover work outside the United Kingdom. I would be grateful if you could provide me with either relevant reports or a list of references known to you. I am particularly interested in laboratory or field measurements on North Sea mud, with respect to its erosional behaviour and extent (mapping). Hydraulics Research will be able to provide contributors with a copy of the final report.

3. Thank you for your anticipated co-operation.

Yours sincerely

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APPENDIX 1

CONSOLIDATION TESTS : TEST PROCEDURE AND ANALYSIS

The objective of the consolidation tests is to determine two relationships, specific to the mud under investigation, which can be used in a computer model to simulate the consolidation of a bed of that particular mud. The first relationship is effective stress as a function of dry density. Effective stress is the stress between the individual mud bed particles (total stress minus pore pressure). The second is a relationship between permeability and dry density.

Description of apparatus

The tests are carried out in a 2 metre settling column of 0.092m internal diameter, constructed of perspex (Fig A1.1). At a height of 0.5m above the base of the column there is a sampling port for extracting small volumes of the suspension for concentration and salinity analysis. At the same height a steady stream of a mixed suspension is injected into the column, which is initially full of saline water. An outlet at 1.75m above the base of the column is used to extract the excess clear water above the settling bed. The suspension to be injected is kept constantly mixed by a recirculating pump. It is then extracted from the bottom of the mixing tank by a variable speed peristaltic pump and injected into the column.

Density profiles are obtained for the bed in the settling column by measuring the transmission of emissions from a Ba¹³³ source (having a 7.5 year half life) over a 30 second time period. This is measured at 3mm, 5mm or 10mm vertical intervals throughout the depth of the bed. The interval used

is dependent on the thickness of the bed and is chosen to give approximately 10 readings for each bed.

It is assumed that there is no significant change in density over the time period taken to read the complete profile. The standard deviation of a reading was found in a previous investigation to be approximately $\pm 1\%$, which results in a dry density with an accuracy of approximately $\pm 10\text{kgm}^{-3}$. The vertical height of the transmission probe can be read to the nearest 0.5mm. The transmission probe is calibrated regularly by measuring the count rate in saline solutions of known density. The calibration indicates a linear relationship over the density range applicable in the tests of the form

$$\rho_d = K_1 r + K_2 \quad (\text{A1.1})$$

where

ρ_d = dry density (kgm^{-3})

r = count rate per minute

K_1, K_2 = constants

Excess pore pressures are determined at the same time as the density profiles. These pressures are measured at set distances above the column base, giving measurements throughout the settled bed and overlying fluid throughout the test.

Test procedure

The input conditions for the column are chosen to be representative of conditions at the site of interest. Different final bed thicknesses and resulting densities are obtained by changes in the input rate and duration of input. A constant rate of input over a set period of time is chosen for each test.

At the start of the test the peristaltic pump is switched on to allow the suspension into the column. The speed of the pump is chosen according to the thickness of bed required and is left running for 2 or 4 hours. To prevent the suspension from mixing upwards in the column the inlet is angled towards the column base, thereby allowing clearer fluid to be drawn off at the overflow outlet.

Samples of the suspension being injected by the peristaltic pump are taken at regular intervals during the input phase. These are analysed for suspended sediment concentration.

The total quantity of sediment put into the column during each test is estimated from the input variables, so integration of the density profiles indicates the distribution of mass through the column and checks the total mass in the column.

Density profiles, excess pore pressures and bed thicknesses are recorded regularly during the first day of the test. Subsequent readings are made at approximately 24 hour intervals until the excess pore pressures have dissipated and the decrease in bed thickness has stabilised.

Analysis

The results of each test are entered into a spreadsheet in the form of density profiles with time and pore pressures at several fixed heights above the base with time. Analysis of the data is as follows:

1. From each density profile, calculate the cumulative mass from the bottom of the bed and the square of

the density. Use linear regression to fit a straight line of the form

$$M = k_1 \rho^2 + k_2 \quad (\text{A1.2})$$

for each time that a density profile was taken

where

M = cumulative mass from the bottom of the bed
(kgm^{-2})

ρ = dry density (kgm^{-3})

k_1, k_2 = constants

2. For each set of pore pressure readings, calculate the square root of excess pore pressure. Use linear regression to fit a straight line relating the square root of pore pressure to the depth from the surface of the bed, for each time that pore pressure readings were taken. If this relationship is not adequate, a better one may sometimes be found with a different power of the pore pressure.
3. Using the cumulative mass calculated in step 1, interpolate between the data points to calculate the height in the bed of the points representing 20% of the mass (ie the height of the point with 20% of the mass below it), 40%, 60%, 80% and 100% of the mass. (For the purposes of the analysis, these percentage points are arbitrary; this is merely a convenient number of points, conveniently spaced.) Calculate these heights for each time the density profile was taken. Plot the height of each mass point against the inverse square root of time. Use linear regression to fit a straight line of the form

$$h_{20} = k_3 t^{-1/2} + k_4 \quad (\text{A1.3})$$

for each mass point

where

h_{20} = height in millimetres of the 20% mass point

(similarly for 40%, 60%, 80%, 100%)

t = time since the start of the test (minutes)

k_3, k_4 = constants

4. Use the fitted curve from step 1 to calculate the density at the 20%, 40%, 60%, 80%, 100% mass points for each time that a profile was taken. For each mass point, plot the density against the inverse square root of time and fit a straight line of the form

$$\rho_{20} = k_5 t^{-1/2} + k_6 \quad (\text{A1.4})$$

where

ρ_{20} = density at the 20% mass point (kgm^{-3})

k_5, k_6 = constants

5. For each time, for each mass point, calculate the effective stress from

$$\sigma' = \sigma - u \quad (\text{A1.5})$$

where

σ' = effective stress

σ = total stress

u = excess pore pressure

Calculate the total stress from the total mass above that mass point. This total stress at any mass point remains constant with time. Calculate the pore pressure at that mass point and time from the fitted lines of height vs time (step 3) and height vs pore pressure (step 2). Use the lines fitted in step 4 to give the density at a mass point at a given time, and plot this density against the effective stress. Use lines rather than symbols to indicate the time series curve for each mass point. The sets of line segments should indicate a

relationship between the effective stress and dry density of the form

$$\sigma' = a_0 + a_1\rho + a_2\rho^2 \quad (\text{A1.6})$$

with $\sigma' = 0$ at $\rho = \rho_0$ for some $\rho_0 \geq 0$

where

σ' = effective stress (Nm^{-2})

ρ_0 = density at the surface of the bed (kgm^{-3})

a_0, a_1, a_2 = constants

Fit a curve of this form by eye.

6. Differentiate the fitted curves of pore pressure against height (step 2) to get the hydraulic gradient, i , at each time. Plot the hydraulic gradient against $\text{time}^{0.3}$, and fit a straight line. If this relationship is not adequate, choose a different power for the time which results in a better straight line relationship.
7. Assuming that the velocity, v , of the water moving upwards at a particular mass point, is equal to the rate of change of the bed height at that mass point with time, calculate v from

$$v = dh/dt \quad (\text{A1.7})$$

using the lines fitted in step 3 for each mass point.

Use this velocity, v , and the hydraulic gradient, i , from step 6 to calculate the permeability at a mass point, at a given time from

$$k = v/i \quad (\text{A1.8})$$

where

k = permeability (ms^{-1})

v = velocity of the water moving upwards out of the bed (ms^{-1})

Use the lines fitted in step 4 to calculate the density at a mass point at a given time, plot this against the logarithm of permeability. This should indicate a linear relationship. Fit a line, by eye, of the form

$$\log (k) = c_0 + c_1\rho \quad (\text{A1.9})$$

where

c_0, c_1 = constants

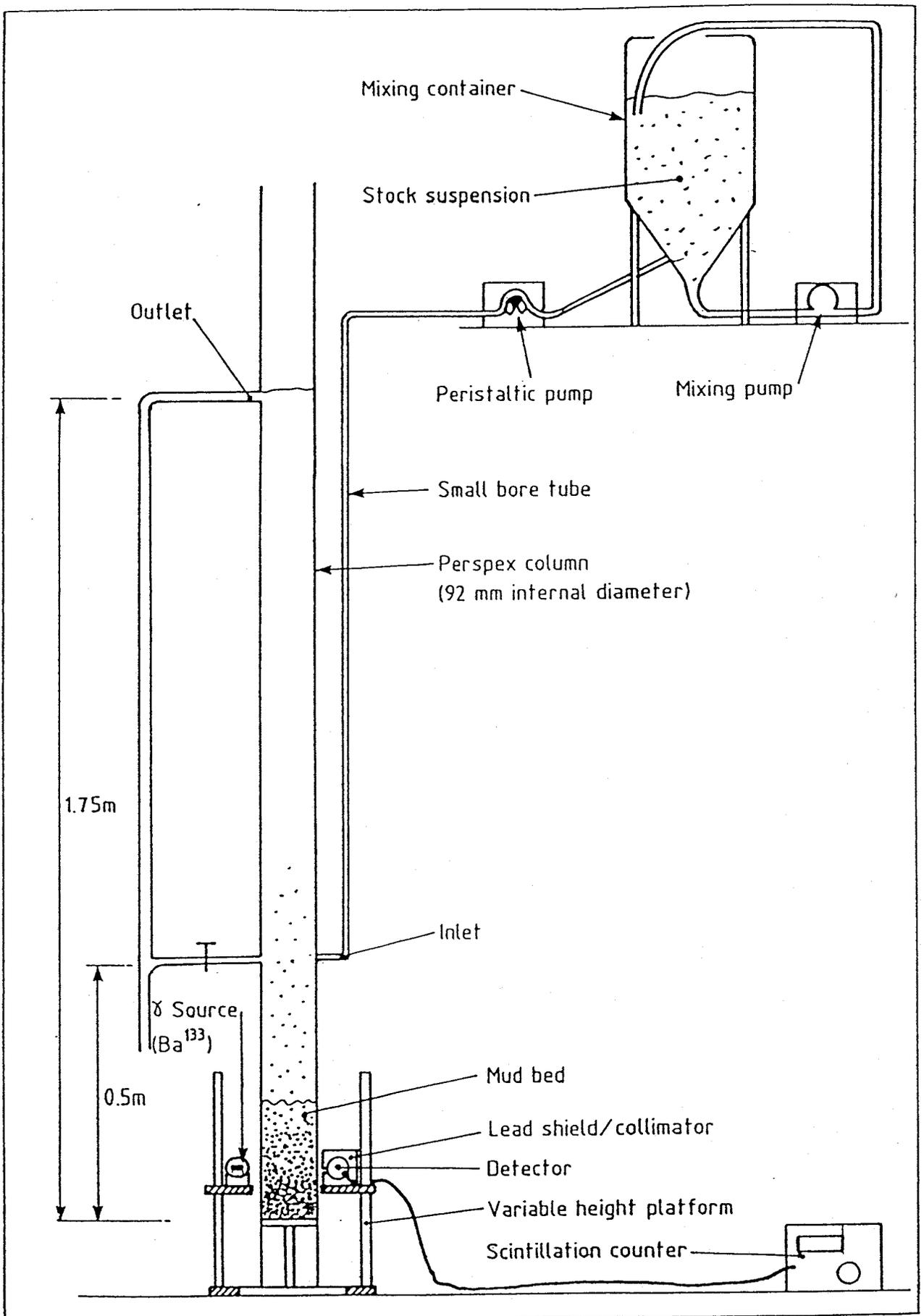


Fig A1.1 Settling column apparatus

APPENDIX 2

EROSION BY UNI-DIRECTIONAL CURRENTS : TEST PROCEDURE AND ANALYSIS

The objectives of the uni-directional current erosion tests are to determine the shear strength of a mud as a function of its dry density, and to determine the rate of erosion with applied shear stress.

Description of apparatus

Uni-directional current erosion tests are conducted in the HR Carousel. The carousel flume (Fig A2.1) is an annular flume, with an outer diameter of 6m, a channel width of 0.4m and depth of 0.35m, and has a detachable roof 0.09m thick. The flume stands approximately 1.1m off the ground, supported by 12 brick pillars. The channel and the roof are constructed of fibre glass, with a 0.12m long perspex section in the channel for viewing. The roof fits into the channel, and floats on the fluid. Fluid motion in the carousel flume is induced and continued by the drag between the roof and the fluid surface as the roof rotates.

The driving mechanism for the roof consists of a DC Torque motor with a drive wheel, which turns a horizontal plate around the central spindle. The drive arm is attached to this horizontal plate at one end and to the roof at the other end.

A strain gauge is used to measure the force applied to the roof of the carousel flume as it rotates. It consists of a spring and displacement transducer arrangement attached to the driving arm at the point of contact with the roof (Fig A2.1). The magnitude of the applied force is determined by the

displacement transducer deflection, which is displayed on a chart recorder. The strain gauge is calibrated by applying known forces via a pulley system.

The speed of the motor, and hence roof speed, is controlled by a micro computer. The motor speed can be set to an accuracy of 0.1% of the maximum speed. This produces a mean water velocity range in the flume from zero to approximately 0.7ms^{-1} , with a corresponding applied shear stress range from zero to approximately 0.7Nm^{-2} .

In the carousel flume the sampling system consists of two port holes, one on each wall of the flume, 80mm above the floor. Through each of these port holes protrudes an 'L' shaped stainless steel sampling tube, which has an internal diameter of 2mm. The outer wall sampling tube has its entrance facing upstream and its elevation can be altered by rotating the outer portion of this tube across a scale corresponding to 0-100mm above the flume floor.

During bed erosion tests fluid is continuously extracted from the carousel flume by a peristaltic pump and passed through a constant temperature water bath and a densiometer before being returned to the carousel flume. The densiometer works on the principle of determining the frequency of a thin vibrating glass tube through which the fluid is pumped and comparing this to the frequency of clean water pumped through a second densiometer. The readings obtained are analysed and displayed on a chart recorder. Bottle samples of the fluid are taken from time to time and analysed gravimetrically to maintain an accurate calibration. In this manner the suspended sediment concentration of the fluid in the carousel flume is measured.

continuously to within a few percent. Previous measurements by Burt and Game (Ref A2.1) have shown that the mean suspended solids concentration of the fluid in the carousel flume is very close to the suspended solids concentration at the centre of flow, certainly less than 5% difference.

The thickness of the bed in the carousel flume is measured from beneath the flume at the perspex viewing section by an ultrasonic transducer. This instrument displays a peak in a signal which indicates the interface between the mud bed and the overlying fluid and enables the thickness of the bed to be determined to within 0.1mm. The transducer is calibrated through a fluid with a salinity similar to that in the mud bed. A movable mounting device holds the transducer in contact with the underside of the perspex section and is used to position the transducer at any point across the 0.4m width of the flume. In this way it is possible to obtain profiles of the bed and determine the depth of erosion at any time during the test.

Bed shear stress measurement

The average shear stress exerted by the fluid on the bed has been measured and calculated in several previous studies (Ref A2.2). Both different roof rotational speeds and different flow depths have been investigated.

The first method was simple and involved direct measurement of the energy input to the roof through the calibrated strain gauge for a number of different speeds of rotation of the roof.

The second and more complex way of determining the bed shear stress was by measurement of the near bed velocity profiles in the flume using laser doppler

anemometry. The operation of the laser is explained in detail in reference A2.2. The friction velocity at the bed was determined from a log-linear plot of height above bed and tangential velocity.

Velocities were determined at three sections across the width of the flume for different speeds of rotation of the roof. The bed shear stresses were then computed from the logarithmic portion of the velocity-depth profiles. An average bed shear stress across the whole width of the flume was calculated for different speeds of roof rotation. However, it must be appreciated that the eroding fluid in an erosion test may have a high concentration of suspended solids and will not exhibit the same hydrodynamic behaviour as the clean water used in the calibration. Nevertheless, for the concentrations of suspended solids present in the carousel flume during an erosion test ($\leq 4\text{kgm}^{-3}$), it is believed that this factor would not significantly affect the calculation of the erosion properties of the mud.

Thirdly, flush mounted shear stress probes were deployed to measure the shear stress along the base and side walls of the flume. The probes used the constant temperature hot wire anemometry technique. Each probe consisted of two thin electrically connected metallic strips mounted side by side on a perspex base. The strips were heated to approximately 30°C by an electrical current; the relative deflection of the connecting wire under stress produced a variation in output voltage, which was calibrated to shear stress.

In addition, the shear stress distribution along the bed was predicted by a numerical model which was developed by Polytechnic South West to predict the hydrodynamics in the carousel (Refs A2.3 and A2.4).

Figure A2.2 shows the bed shear stress against rotational roof speed as measured by these various methods for a flow depth of 100mm. This shows the shear stress on the wetted perimeter as given by the power input measured through the strain gauge, the average bed shear stress across the width of the flume measured by the laser velocity, the average bed shear stress across the width of the flume measured by the shear stress probes, and the average shear stresses predicted by the numerical model. All methods show an increase in bed shear stress with increasing speed of rotation of the roof, with the shear stress probes and the numerical model showing higher stresses at the higher roof speeds than the other two methods.

For the purpose of estimating the bed shear stress during an erosion test the curve representing the average shear stress as calculated from the numerical model is used. For a flow depth of 100mm in the carousel this curve is described approximately by the relationship

$$\tau_{av} = 0.06(\text{rpm})^{1.77} \quad (\text{A2.1})$$

where

τ_{av} = average shear stress across the width of the flume (Nm^{-2})

rpm = rotational speed of carousel roof

Test procedure

To prepare a mud bed in the carousel flume, the mud is first mixed homogeneously in a mixing tank with a recirculating pump. The suspension is then pumped into the flume from the tank until the required depth of suspension in the flume is reached. The roof is lowered onto the suspension surface and the mud in suspension is allowed to deposit and

consolidate. The period of consolidation is usually in the range of 2-10 days and the resulting bed has a thickness of 10-25mm. The depth of fluid above the bed is adjusted to be close to 100mm which corresponds to the depth of flow for which the bed shear stress measurements were made.

An erosion test in the carousel flume comprises a number of discrete runs during which the speed of rotation of the roof (and hence the bed shear stress) is held constant. In a test there may be between 2-5 runs each lasting 60-200 minutes. The speed of the carousel flume is systematically increased for each successive run.

A run commences when the concentration of suspended solids is constant in the previous run. The speed of rotation of the roof is increased over a period of about 30 seconds to its new value. The concentration of suspended solids as measured continuously by the densiometer will at first increase rapidly (indicating a strong erosion), then more smoothly (modest erosion) and finally the concentration will remain nearly constant (no erosion). This pattern is reflected by the readings from the ultrasonic transducer which is mounted on the underside of the flume mid-way across its width. The change in the readings is directly proportional to the depth of erosion.

At the end of a run when erosion has stopped the actual depths of erosion at 20mm intervals across the width of the flume are determined using the ultrasonic transducer. The typical depth of erosion which is normally attained at the end of the test is about 5mm. If more mud is eroded then the high concentrations of suspended sediment begin to prevent the densiometer and ultrasonic depth transducer from functioning correctly. Furthermore,

at the higher speeds of rotation the effects of secondary currents are greater and the differential depths of erosion across the flume become more pronounced.

Analysis

The basic data obtained from a test are the suspended solids concentration with time, the depth of erosion at the mid-section of the flume with time and the depths of erosion across the flume at the equilibrium point in each run.

The shear strength with density relationship is described by the discrete values, at the equilibrium point in each run, of the speed of rotation of the roof, the suspended solids concentration of the fluid and the average depth of erosion across the flume. Using the results presented in figure A2.2 the average bed shear stress is then estimated for the prescribed speed of rotation of each run. At equilibrium in each run, the shear strength of the exposed surface of the bed is equal to the applied shear stress. Therefore, the shear strength against depth relationship of the eroded portion of the bed can be described by these points and expressed in a functional form

$$\tau_e = f_1(h) \quad (A2.2)$$

where

τ_e = erosion shear strength of bed (Nm^{-2})

h = average depth of erosion below original surface (m)

f_1 = function

It is also necessary to calculate the variation in density of the mud bed with depth. The density may be expressed as

$$\rho_h = (dm/dh)A^{-1} \quad (A2.3)$$

where

ρ_h = dry density of mud at a depth h (kgm^{-3})

m = mass of solids in suspension (kg)

A = area of erosion (m^2)

Expressing the mass of suspended solids eroded from the bed in terms of the concentration of suspended solids, equation (A2.3) can be rewritten as:

$$\rho_h = (dc/dh)(V/A) \quad (A2.4)$$

where

c = concentration of suspended solids (kgm^{-3})

V = volume of suspension (m^3)

The ratio of V/A in equation (A2.4) will be nearly constant during a test and is the depth of flow, d . By plotting the concentration of suspended solids against the average depth of erosion, a quadratic function (f_2) may be approximated, giving

$$c = f_2(h) \quad (A2.5)$$

Differentiating equation (A2.5) and substituting into equation (A2.4) leads to the relationship

$$\rho_h = K_3 + K_4h \quad (A2.6)$$

Combining equations (A2.2) and (A2.6) leads to an expression which relates the erosion shear strength to the density of the exposed mud, and which can be approximated to the form

$$\tau_e = K_5 \rho_h^{K_6} \quad (A2.7)$$

Intuitively, one would expect erosion to start when the stress exerted by the flow exceeded the shear strength of the exposed bed and the erosion rate to depend on the excess shear. If the erosive power of the stream is low not much erosion would be expected to take place. There will be times when a burst of turbulence slightly higher than average hits a slightly weaker part of the bed causing untypical erosion, but for practical purposes this can be ignored and it may be assumed that there is a cut off for erosion. The most common representation of erosion is

$$\begin{aligned} dm/dt &= Am_e(\tau - \tau_e) \quad , \quad \text{for } \tau \geq \tau_e \\ dm/dt &= 0 \quad \quad \quad , \quad \text{for } \tau < \tau_e \end{aligned} \quad (A2.8)$$

where

$$\begin{aligned} m_e &= \text{erosion constant (kgN}^{-1}\text{s}^{-1}\text{)} \\ \tau &= \text{applied shear stress (Nm}^{-2}\text{)} \end{aligned}$$

This means that erosion is gradual which is not necessarily the case for certain types of newly formed slack water deposits. Although there is no physical reason for assuming erosion rate to be directly proportional to the excess shear, Delo and Burt (Ref A2.5) showed that this is a better variable for describing mud erosion than others. In any event it is not critically important in tidal conditions to know the erosion rate precisely because the erosion process is self correcting in the sense that if the erosion constant (m_e) is too high then too much erosion occurs in the early stages, but this exposes stronger bed material and erosion slows down accordingly. The opposite happens if m_e is under-valued. The ultimate result in any case would be erosion down to the bed level, where the strength of the exposed material corresponds to the maximum bed stress of the tidal

cycle or of the spring-neap cycle, if longer periods are being considered.

A useful analysis of the erosion constant can be made by assuming that the shear strength of the bed during any discharge run is proportional to the eroded mass (Ref A2.6). The constant of proportionality for a run is given by

$$\alpha = (\tau_b - \tau_o) / (c_e - c_o) \quad (A2.9)$$

where

τ_b is applied bed stress for the run

τ_o is shear strength of bed at start = equilibrium from previous run

c_e is equilibrium concentration at the end of run

c_o is initial concentration = equilibrium concentration at end of previous run

This does not assume that there is a linear relation between strength of bed and overlying weight for the complete bed. This overall structure is fixed by the equilibrium conditions at the end of each run. It is merely assumed that there is a linear variation from one equilibrium state to the next and α can vary for each run.

If it is further assumed that the erosion rate for the exposed mud surface area is given by equation (A2.8) , then using (A2.9) and replacing m by cV gives

$$dc/dt = Am_e\alpha(c_e - c_o)/V \quad (A2.10)$$

where V is the volume of fluid in the flume and A is the area of erosion. This can be integrated as

$$(c_e - c) = (c_e - c_o) \exp (-Am_e\alpha t/V) \quad (A2.11)$$

This solution exhibits the expected behaviour of concentrations, tending to equilibrium values for large times. Based on the analytic form of this theoretical solution, the carousel erosion flume results for a test can be normalised and plotted using linear (αt) (m^2s^{-1}) and logarithmic $[(c_e - c_0)/(c_e - c)]$ axes to give a representative erosion constant for the test.

References

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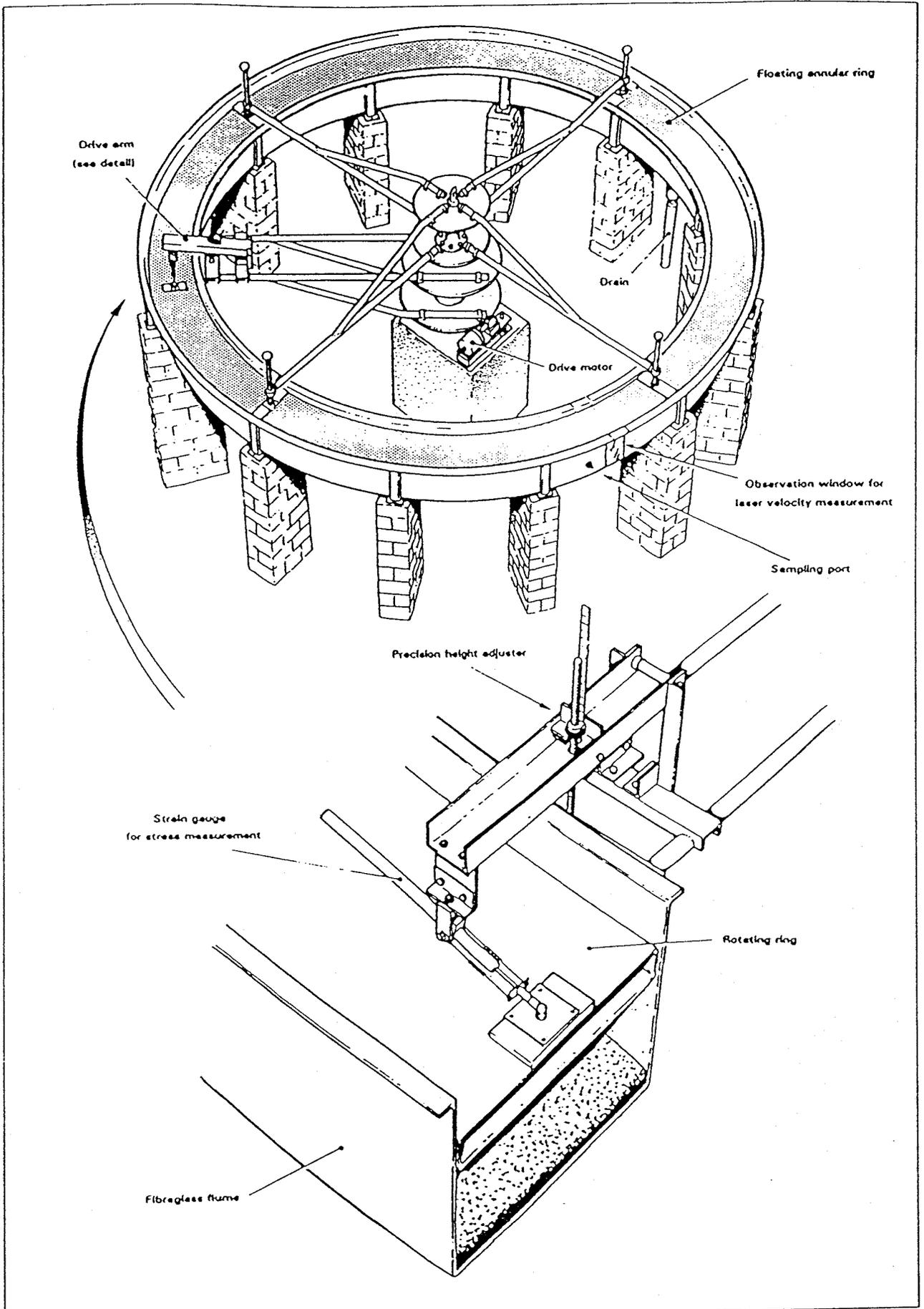


Fig A2.1 The carousel

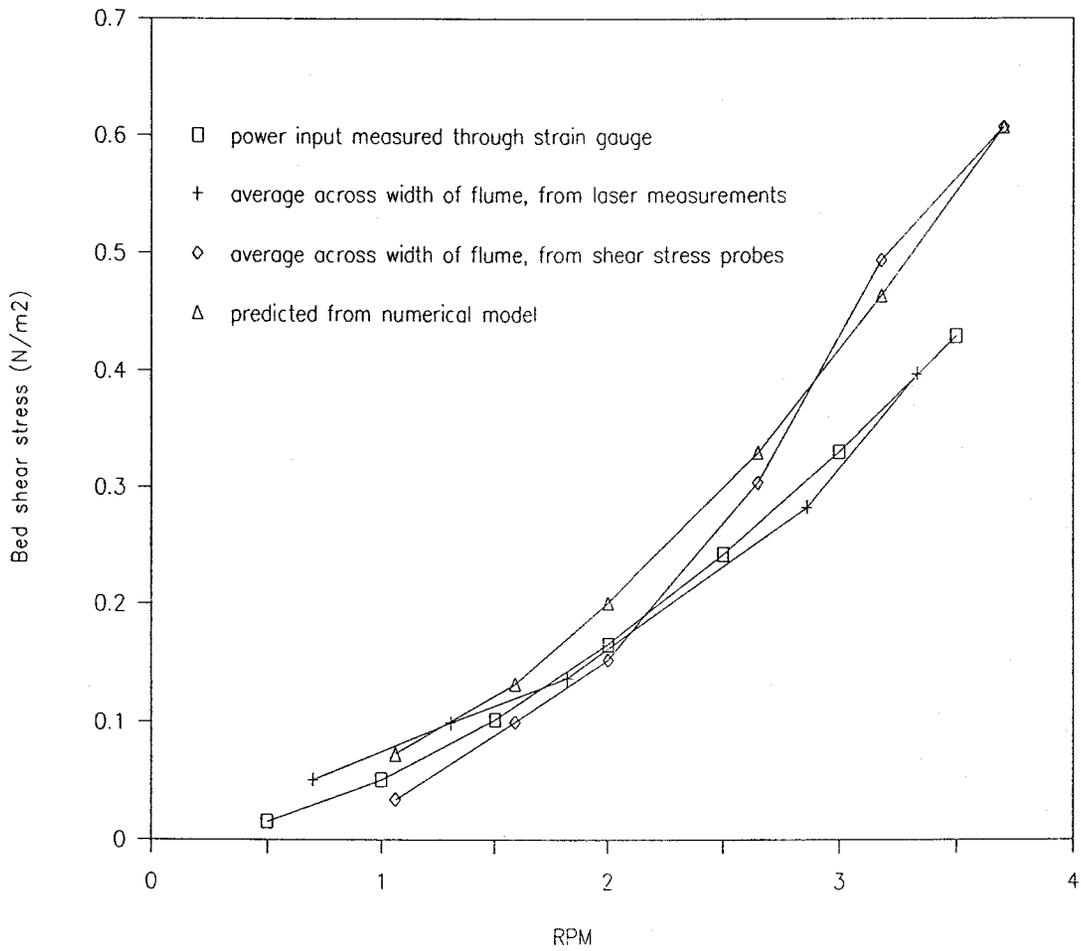


Fig A2.2 Average bed shear stress against roof speed

APPENDIX 3

WAVE EROSION TESTS : TEST PROCEDURE AND ANALYSIS

The objective of the wave erosion tests is to determine a threshold bed shear stress for erosion of mud of a known density and to calculate erosion rates above this critical shear stress. The relationship between density and critical peak bed shear stress is also of interest. This will assist in estimating the amount of mud eroded by waves in the field.

Description of apparatus

The tests are carried out in a wave flume 23m long and 0.3m wide, with a maximum water depth of 0.55m. There is a trough in the floor of the flume to hold the test bed, starting 9.5m from the end of the flume with the wave paddle. The trough is 8m long, 0.3m wide and 0.1m deep. At each end of the test bed section there is a raised lip to prevent mud escaping from the bed as bed load transport.

At one end of the flume are two wave generators, which can be set for monochromatic and random waves. For monochromatic waves, the user controls the wave frequency and wave height manually, by altering the stroke length and frequency on the generator. The random wave generator is controlled by a micro-computer. For random waves, the user inputs a zero crossing period and a significant wave height; the program generates a random wave spectrum which satisfies these input conditions. The wave spectrum generated in this case is the JONSWAP spectrum. At the other end of the flume there is a shingle spending beach to absorb the wave energy and hence minimise wave reflections back into the flume.

An ultrasonic probe mounted above the bed on a computer controlled robot is used to monitor the surface level of the mud bed at intervals along the length of the bed. At positions throughout the test section water pressure transducers are mounted below the water surface and the signals logged onto a computer. The wave spectrum is determined by analysis of the water pressure signals.

A conductivity probe is used to determine the vertical bed density profile in situ. It is assumed that the conductivity of the pore water in the mud bed is constant and, hence, the overall conductivity of the mud is a function of its pore volume ie. its density. The instrument is calibrated using samples of known dry density of the mud under investigation and with saline solutions of known density. The calibration samples have dry densities typically in the range 50kgm^{-3} to 500kgm^{-3} .

A turbidity sensor is used to measure suspended sediment concentrations at various depths and positions along the flume, at time intervals throughout each test. The instrument used is an Analite Nephelometer, which operates on the principle of backscatter of infra red light. The degree of backscatter is proportional to the reflection coefficient and concentration of suspended sediment.

Test procedure

A uniform slurry is made up from the collected mud sample by dilution with saline water. This slurry usually has a dry density in the range 300kgm^{-3} - 500kgm^{-3} . This is then poured into the test section of the flume.

The mud is then subjected to a pattern of waves, either monochromatic or random waves. The wave generating equipment is capable of generating higher peak bed shear stresses with monochromatic waves. The wave period is fixed for the duration of the test. The input parameter of wave height (significant wave height for random waves) is changed up to four times during a test; each wave height is imposed for one hour, then it is changed so that the resulting peak bed shear stress is increased.

The vertical density profile of the bed is determined with the conductivity probe before the start of a test and at half-hourly intervals during a test. This monitors changes in bed density caused by consolidation or wave action.

The bed levels at 0.5m intervals along the test section are measured at the start of the test and at half-hourly intervals with the ultrasonic probe.

The concentration of suspended sediment is measured every 30 minutes. Turbidity readings are taken at several heights in the water column at positions along the entire length of the flume.

Analysis

The objective of the tests is to determine the critical bed shear stress at which erosion of the bed commenced. For monochromatic waves, the peak bed shear stress can be calculated from the water depth and wave characteristics using first order linear wave theory (Ref A3.1) such that:

$$U_m = \frac{\pi H}{T \sinh(2\pi d/L)} \quad (A3.1)$$

where

U_m = maximum bottom orbital velocity (ms^{-1})
 H = wave height (m)
 T = wave period (s)
 L = wave length (m)
 d = water depth (m)

The magnitude of the wave length is determined iteratively from:

$$\omega^2 = gk \tanh(kd) \quad (\text{A3.2})$$

where

$$\omega = 2\pi/T \text{ (s}^{-1}\text{)}$$

g = acceleration due to gravity (ms^{-2})

$$k = 2\pi/L \text{ (m}^{-1}\text{)}$$

The peak bed shear stress is estimated using the following relationship :

$$\tau_m = 0.5 \rho_w f_w U_m^2 \quad (\text{A3.3})$$

where

τ_m = peak bed shear stress (Nm^{-2})

ρ_w = fluid density (kgm^{-3})

f_w = wave friction factor

U_m = maximum bottom orbital velocity (ms^{-1})

The wave friction factor is dependent upon the wave reynolds number and the relative roughness (Ref A3.2).

For random waves, the surface elevation spectrum generated is the JONSWAP spectrum:

$$S(f) = 4.732 \cdot 10^{-4} \exp(-1.25 f_m^4 f^{-4}) f^{-5} 3.3^x \quad (\text{A3.4})$$

with

$$x = \exp(-(f f_m^{-1} - 1)^2 y^{-1})$$

$$y = 0.0162 \text{ if } f > f_m$$

$$= 0.0098 \text{ if } f \leq f_m$$

where

f = frequency of ordinate

f_m = frequency at which spectral peak occurs
 $= 0.87T_z^{-1}$ or $= 0.217H_s^{-0.5}$

T_z = mean zero crossing period (seconds)

H_s = significant wave height (m)

$S(f)$ = value of spectral ordinate at frequency f

Where both T_z and H_s are input, T_z is used as the defining value and the spectrum magnitude is defined with an implied gain. The near-bottom velocity cannot now be described by a single U_m , and is usually described by the standard deviation, U_{rms} , of the time-series of instantaneous velocities. U_{rms} can be approximated as a function of H_s and T_z (Ref A3.1):

$$U_{rms} T_n / H_s = 0.25 / (1 + At^2)^3 \quad (A3.5)$$

where

$$A = (6500 + (0.56 + 15.54t)^6)^{1/6}$$

$$t = T_n / T_z$$

$$T_n = (d/g)^{0.5} \quad \text{scaling period}$$

d = water depth

g = acceleration due to gravity

This fits the JONSWAP curve to an accuracy of better than 1% in the range $0 < t < 0.55$. For equivalent monochromatic waves and random waves with the same variance of bottom orbital velocities

$$U_m = \sqrt{2} U_{rms} \quad (A3.6)$$

The bed shear stress can therefore be calculated according to equation A3.3, using $\sqrt{2} U_{rms}$ in place

of U_m , yielding a representative bed shear stress for the wave spectrum. The equations used to calculate a representative bed shear stress from the bottom orbital velocity are given in Table A3.1.

The total mass of mud eroded per unit area of bed is calculated by two methods for each half hour time interval. Firstly, the difference in bed levels at successive time intervals is calculated, giving a depth of erosion. An allowance is made for consolidation, if this has occurred. Multiplying the depth of erosion by the bed density gives the mass eroded per unit area of bed surface. Secondly, the suspended concentrations at intervals throughout the flume are integrated to give the total mass in suspension for each set of readings. The total mass in suspension is normalised to give mass eroded per unit area of bed surface.

An erosion constant may be found from a relationship between the excess bed shear stress and the rate of entrainment, which is given by:

$$dm/dt = m_{ew} (\tau_{bm} - \tau_{be}) \quad (A3.7)$$

where

dm/dt = rate of erosion ($kgm^{-2}s^{-1}$)

m_{ew} = erosion constant under waves ($kgN^{-1}s^{-1}$)

τ_{bm} = bed shear stress (Nm^{-2})

τ_{be} = critical bed shear stress (Nm^{-2})

References

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TABLE A3.1 Equations used in calculation of bed shear stress from bottom orbital velocity

Input parameters

U_{rms}	Root-mean-square bed orbital velocity for random waves (ms^{-1})
T_z	Mean zero crossing period (s)
ν	Viscosity of fluid (m^2s^{-1})
ρ	Density of fluid (kgm^{-3})
k_s	Nikuradse equivalent grain roughness (m)

Calculated parameters

U_m	Orbital velocity for equivalent monochromatic wave (ms^{-1})
A	Semi-orbital excursion length (m)
R_w	Wave Reynolds number
r	Relative roughness (a/k_s)
F_s	Smooth friction factor
F_r	Rough friction factor
F_w	Wave friction factor
τ	Representative bed shear stress for random wave spectrum (Nm^{-2})

Equations used

$$U_m = \sqrt{2} U_{rms}$$

$$\tau = 0.5 \rho F_w U_m^2$$

$$A = U_m T_z / 2\pi$$

$$R_w = U_m A / \nu$$

$$F_s = 2 R_w^{-0.5} \quad \text{if } R_w < 5 \cdot 10^5 \quad \text{Laminar}$$

$$F_s = 0.0521 R_w^{-0.187} \quad \text{if } R_w > 5 \cdot 10^5 \quad \text{Smooth turbulent}$$

$$F_r = 0.3 \quad \text{if } r < 1.57 \quad \text{Rough turbulent}$$

$$F_r = 0.00251 \exp(5.21 r^{-0.19}) \quad \text{if } r > 1.57 \quad \text{Rough turbulent}$$

$$F_w = \max(F_s, F_r)$$