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A TECHNIQUE FOR THE GENERATION OF SHORT CRESTED WAVES IN WAVE BASINS

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Abstract

This paper describes a technique for generating short crested waves in a laboratory using a multi element array of paddles. The control signal applied to each paddle in order to create a realistic sea is developed by using the technique of filtering white noise, building upon the preferred method previously used for wave generation in flumes. By using the side walls of the wave basin as reflectors a larger working area can be achieved than by using other techniques, and the need to calculate correlations between paddles in the array avoided.

Introduction

Interest in the hydrodynamic loading of structures in offshore locations has increased the need to take account of the multidirectional (short crested) nature of sea states. Whereas previously with most wave loading problems occurring in coastal waters, it could be argued that the multidirectional effects were small, so that the sea would be well represented in wave basins by a unidirectional (long crested) spectrum, this no longer seemed obvious from offshore locations. In particular the advent of rapid offshore oil development in the inhospitable climate of the North Sea created the need to develop physical model facilities which could generate directional sea states to enable their effects to be studied in detail.

The first facility constructed primarily for this task was the Complex Sea Basin built at Hydraulics Research in the early 1970's (Huntington 1978). In this basin the short crested sea was formed by superposing ten independent long crested random wave trains all directed towards the focal area from generators subtending 50° either side of the centre line of the wave basin. The physical requirements governing the construction ofsuch generators gave a working area over which the multidirectional sea was sufficiently homogeneous of about 3 m square. This area was quite adequate for the first generation of structures used in the North Sea such as gravity platforms, jackets, moored semi-submersibles, etc. and studies were carried out to investigate the significance of the short crested aspects of the waves in the design of such structures (Huntington 1981).

Such a facility is also quite adequate for some of the later types of structures such as tethered buoyant platforms and articulated towers. Nevertheless, there is a class of structure where the relatively small working area is restrictive: the floating structure with at least one large horizontal dimension. Examples of this are ships on floating terminals, either for transportation purposes, or more recently, permanently moored for early production facilities, dynamically positioned large crude carriers and also ships moored in tandem as part of a production/transportation system. In addition the majority of wave energy devices are in this form of long horizontal floating structures.

Consequently, in the late 1970's a means of generating a short crested sea which is homogeneous over a much larger working area was required. To study the new problems described above a working area of at least 12 m square was desirable.

Increasing the size of the components of the wave basin already constructed at HR was ruled out due to the inefficient use of space - a 18 m square basin was required to generate a 3 m square working area. An alternative possibility was to use a multi-element paddle. Such wave generation devices had been in existence at HR and Delft for many years to generate oblique regular waves but were restricted in their use by their mechanical drive system. Each paddle was driven from a common shaft with a phase lag set by a mechanically adjusted cam. Since different frequencies require different phase lags to give waves generated in a particular direction, it is not possible to mix frequencies in this device. However, once it became feasible to use independent servo controlled actuators for the paddle elements any pattern of paddle movement could be created simply by computing appropriate signals, and the control problem reduces to a question of sensible signal generation.

The problem then was to design a wave basin with a multi-element wave generator along one side together with a method of signal generation so that a specified directional spectrum could be reproduced over a working area which is a substantial percentage of the area of the basin.

Generation of oblique waves

An array of multi-element wave makers is shown in Fig. 1 which also shows the coordinates, dimensions and parameters used to describe the waves. If each element is controlled to have a sinusoidal motion of frequency f and there is a constant delay τ between adjacent paddles then an oblique wave at angle θ will be produced where

$$f\tau = Kw \sin\theta \qquad (1)$$

where K is the circular wave number (i.e. K = l/wavelength).

However, there is a limit on the waves that can be generated by a wave maker assembled from paddles each of finite width (Gilbert 1976). Beyond this limit unwanted waves will be generated in addition to the required ones (Sand and Mynett, 1987). To avoid these spurious secondary waves being generated the primary waves must satisfy

$$Kw + K_I w \leqq 1 \qquad 2)$$

or equivalently

$$\sin\theta \leqq \frac{1}{Kw} - 1 \qquad (3)$$

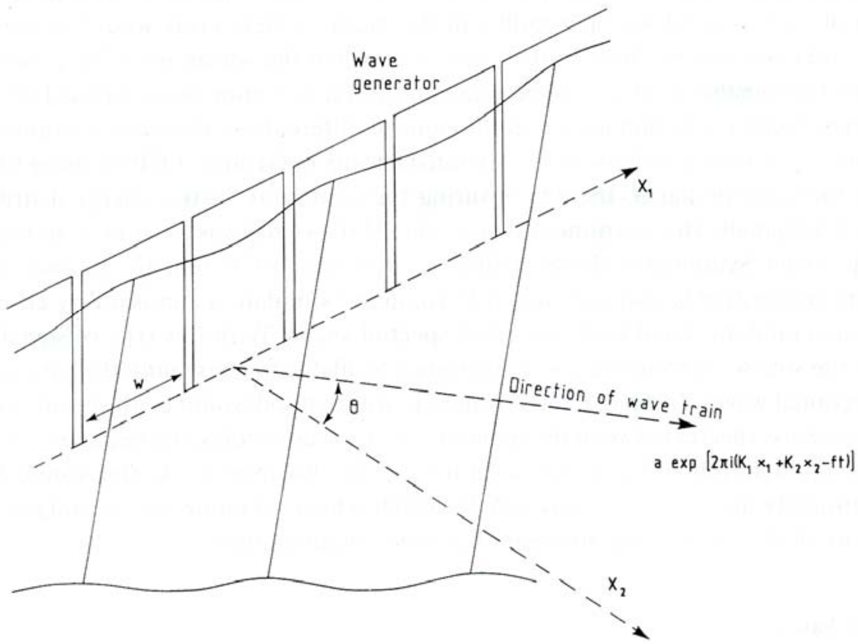

Fig. 1. Generation of oblique wave

Clearly for *Kw* < ½ waves can be generated over the complete range up to 90º without secondary waves, but for increasing values of *Kw* (higher frequency waves) the above constraint puts a progressively more severe limit on the angles which can be cleanly generated.

A first approach to the problem of generating a directional spectrum is to regard it as a sum of a large number of sine waves and to generate each of these by phasing along the paddles in this way to give the mixed frequency and direction content of the sea. This approach though adopted for most multi-element wave generators was thought at HR to be best avoided if possible due to previous experience with ordinary random wave generation in one direction.

Goda (1970) has shown that at least fifty sine waves are required in one direction to give a wave sea state whose characteristics match reasonably the continuous unidirectional spectrum that is to be modelled. Moreover a structure introduces the further constraint that the frequency component lines must be sufficiently densely packed to ensure that any resonances exhibited by the structure are excited. Recent investigations into the second order responses of floating structures with their low frequency resonances have shown how the problem of discrete frequency lines in the spectrum is even more acute than previously thought. A more detailed discussion of these problems is given by Huntington (1983).

It is quite clear then from previous work that to generate a multi-directional sea using a super-position of sine waves would require a very large number of components and for each component a phase must be computed for each paddle at every time step in the generation procedure. Computation of such a signal for each paddle in the multi-element array would be very time consuming, and could not be done in real time) (Le., when the waves are being generated). Consequently, the signals would have to be generated and stored prior to any particular run. This can be an unnecessary restriction if long simulations of different sea states are commonly required. In one direction these problems were very satisfactorily overcome at HR by using filtered white noise as the random signal, thereby ensuring the continuity of the energy distribution with frequency. Originally the instrument that produced the signal was a hardwire device called the Wave Spectrum Synthesizer (Fryer, Gilbert and Wilkie, 1973) but the method of the Wave Spectrum Synthesizer is also well suited to computer simulation and is a very efficient way of generating a random signal to a prescribed spectral shape. With this type of signal generation however the sine wave components are no longer available to phase onto the various paddles to give directional waves. Instead, the equivalent in this method would be to specify correlation in the form of cross spectra between the signals to the various paddles, the relative sizes of the cross spectra being determined by the required directional distribution. As this would be awkward computationally an alternative was sought which whilst retaining the advantage of the synthesizer method bypasses the problem of correlation altogether.

The wave basin

The size of the area of homogeneous sea and the method of signal generation are tied into the question of the amount of wave absorption to be allowed in the basin. The front wall of the wave basin will be occupied by the multi-element wave generator and the back wall by a beach to absorb the waves. This leaves the two side walls which may be made either absorbing or renecting. If they are absorbing, suppose a directional spectrum is generated and consider the components travelling at ± 45° from the normal. Then the working area for these components will be restricted to the triangle shown in Fig. 2, even assuming a geometrical transmission of waves. But when diffraction effects are taken into account this area is further restricted, for the wave height along lhe boundaries of the triangle will be down to 0.5 of the nominal wave height. Some further parabolic boundaries have been drawn in Fig. 2 where the diffraction effect is down to 0.2 of the nominal wave height. These have been drawn for various ratios of wavelength to wavemaker length. Along these boundaries, depending on the interference belween the main and diffracted wave, the spectrum will vary with position from 1.44 to 0.64 of the nominal value. Even allowing this amount of variation the working area is hopelessly restricted. For other directional components the working area, which may be imagined by rotating the boundary lines with their parabolas attached around to the appropriate angle, will still be very restricted at quite modest angles such as ± 30° to the normal.

Now suppose that the side walls are reflecting. Then the situation, although complicated by the reflections or the beams of waves from the side walls, is not essentially different from before. A point may lie within a direct beam or its various reflections for some directions of wave travel and for other directions lie in a shadow or within the diffraction zone near the boundary of a beam. Hence the spectrum at the point will vary in a complicated manner with position and it is difficult to see quite what is the working area. Eventually, further down the basin, after multiple reflections and when diffraction has spread the beams, the wave system will be sorted into the modes of the basin which travel down its length unchanged.

Modes

This suggests that a better method is to have reflecting side walls and to actually generate the waves in the modes of the basin so that the total wave system remains unchanged as it moves down the basin. At a particular frequency a mode may be thought of as a pair of waves at angles ± 8 generated simultaneously, the angles being chosen so that an integral number of half wave lengths fits across the basin width with both walls being anti-nodes. The reflection condition is then satisfied at the basin sides and the wave pair propagates unchanged down the basin. The number of half waves across the basin is called the mode number (*m*) and the across basin shape of the lowest few modes is shown in Fig. 3. The maximum number of modes that can be generated will be equal to the number of paddles in the generator.

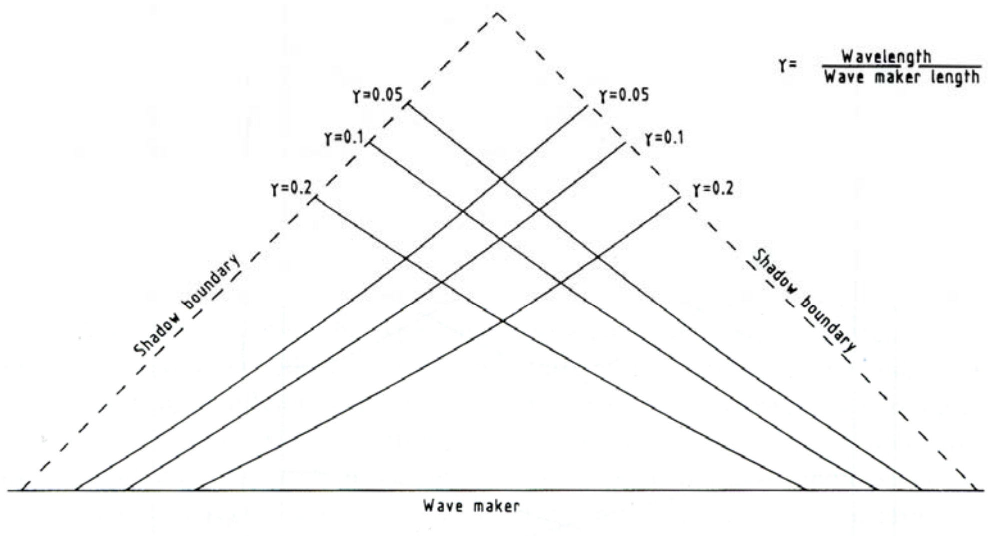

Fig. 2. Diffraction in the working area

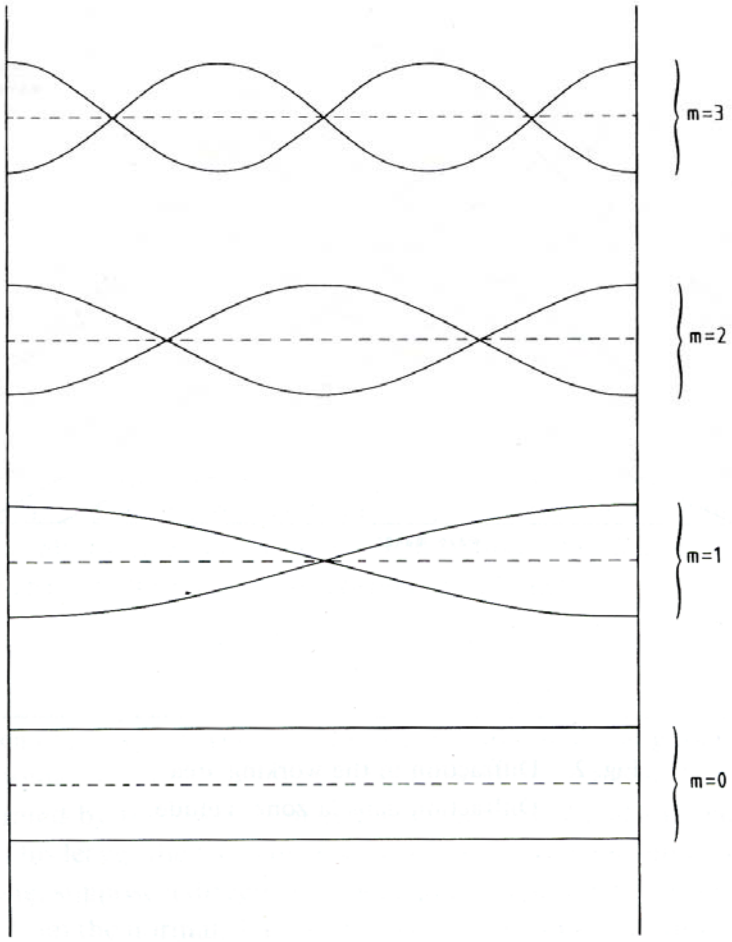

Fig. 3. Paddle movement envelopes for modes *m* = 0,1,2,3

If a mode is fed with a random signal with a specified spectrum then this complete spectral shape will propagate unchanged down the basin. If the spectrum is thought of as a superposition of sine waves over a continuous frequency band then the angles of the resulting component wave trains will vary continuously with the frequency. For mode (*m*) the constant quantity over the spectrum is the cross basin wave number given by:

$$K_1 = \frac{m}{2b} \qquad (4)$$

where *b* is the basin width.

This wave system generated by feeding a mode with a spectrum may therefore be considered as a slice of a directional spectrum $S(K_1, K_2)$ taken at constant cross basin wave number K_1 or rather as two slices since the system represents waves at $\pm K_1$. By feeding all the different modes with independent random signals each with an appropriate spectrum, the total directional spectrum can therefore be built up in the basin. This representation of the directional spectrum will be continuous in frequency since potentially infinite time is available but discrete in cross basin wave number since the basin can only be of finite width. Equation (4) shows that the available wave numbers become more densely packed as the basin is made wider. The directional spectrum so generated must necessarily be symmetric about the normal direction down the basin, but this is always the case for the type of testing envisaged.

This method of representation of the spectrum also solves the signal generation problem discussed earlier. Now the synthesizer method can be used to generate the various mode spectra signals and each is applied to a particular paddle simply by multiplying by the value at that paddle of the appropriate mode shape factor. The first four such mode shape factors are shown diagrammatically on Fig. 3. Thus each paddle receives a contribution from each of the independent random mode signals with a weighting depending on the mode and paddle position. Hence no frequency dependent phases need be considered. In terms of the correlation of neighbouring paddles mentioned earlier, it will be seen that this will automatically come about because near paddles will have similar mode shape factors for all but the highest modes.

An obvious objection to this method of modes is that the wave condition of a mode is not homogeneous across the basin; indeed the mth mode has m nodal lines running down the basin along which the wave elevation is zero. These arise from the strict correlation of the wave trains at positive and negative angles due to the reflection at the sidewalls. Now if many modes are present, then the correlation will be expected still to occur at the side walls since all modes have anti-nodes there, but to be reduced towards the centre of the basin where the nodal lines of the various modes occur in different positions. To demonstrate that a substantial working area does indeed exist across which wave conditions are approximately homogeneous it is sufficient to consider first of all the effect of taking just the five neighbouring modes: $m + r$ where r = - 2, ... + 2. Let the spectra on the five modes in any frequency band be in the ratios: α_r where $\alpha_{-r} = \alpha_r$. Since these are fed by independent signals the variation across the basin of the total disturbance in any frequency band is obtained by taking the sum of the squares of the mode shapes. Thus the wave disturbance is proportional to

$$\sum_{r=-2}^{+2} \alpha_r \cos^2 (m+r) \frac{\pi x}{b} = (\tfrac{1}{2}\alpha_0 + \alpha_1 + \alpha_2) +$$

$$\left(\tfrac{1}{2}\alpha_0 + \alpha_1 \cos\frac{2\pi x}{b} + \alpha_2 \cos\frac{2\pi x}{b} \right) \cos \frac{2m\pi x}{b}$$

Choosing the parameters to minimise the variation over the central half of the basin gives $\alpha_2 : \alpha_1 : \alpha_0 = 2 : 4 : 5$. The resulting curve giving the variation in spectral wave conditions across the basin then has the envelope, independent of the particular mode m, shown in Fig. 4. It will be seen that across the central half of the basin the spectral variation is less than ± 6%.

Now look upon this particular optimised five mode distribution as the impulse response of a smoothing filter applied to energy distributions across the modes. Then any required energy distribution which can be regarded as an output of this filter for some appropriate input, will result in a wave condition which is the sum of those produced by the five mode form. Hence the variation across the central half of the basin will be at least as small as that shown in Fig. 4. Since actual cases will require many more than just five modes to be active and since conventional spreading functions vary smoothly with angle it will always be the case that the required mode distribution can be regarded as a filter output in this way. This argument, given here in terms of surface elevation, applies in the same way to other elements of the three-dimensional wave kinematics. Although correlated and hence untypical for one mode they become to a good approximation general and homogeneous when considered over a set of neighbouring modes.

Thus we have a basin with a substantial working area half the width of the basin and running the total basin length except near the paddles, and a signal generation method based on the synthesiser which can be implemented in real time on a computer. It remains to determine the frequency spectra that should be applied to each mode in order to reconstruct any required directional spectrum.

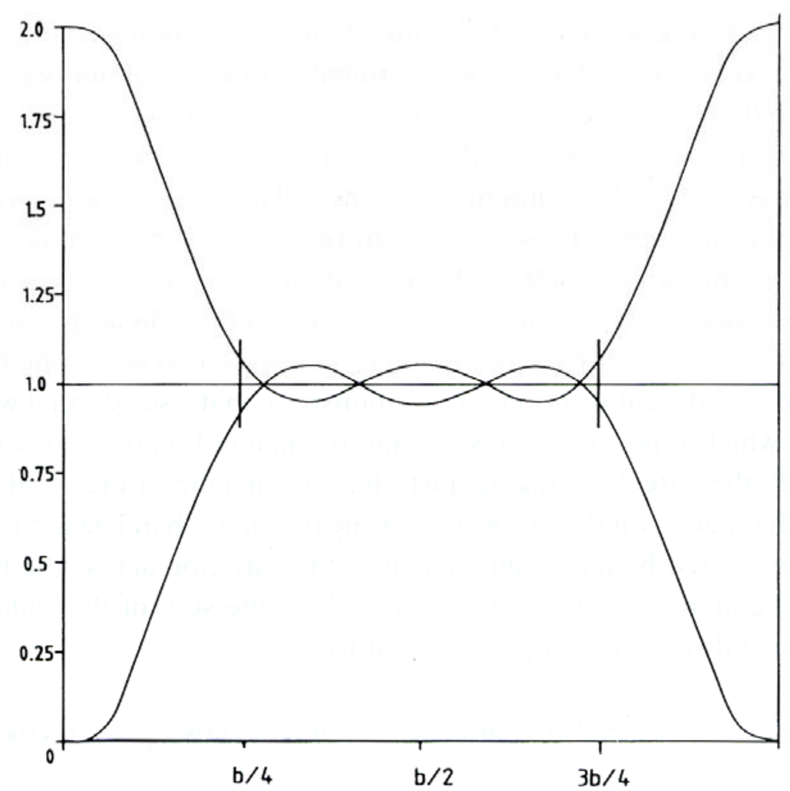

Fig. 4. Variation of spectral density across the basin.

Mode spectra

A picture of the possible wave trains in the basin represented by their wave numbers is given in Fig. 5. This is a plot of the wave numbers (K_1 K_2) made non-dimensional by the basin width, b. Only the trains with positive values of K_1 are shown. The whole picture should be imagined reflected in the vertical axis to give the symmetric negative values. The modes are represented by the vertical lines at integer values of $2K_1$ b. Any point on these lines represents a wave train which can occur. The parabolic curve represents equation (2) giving the limit for no spurious waves in the form of 2Kb + $2K_1$ b = 2N where N is the number of paddle elements in the generator. The curve is drawn for N = 80. Up to 80 modes can then be used but in practice this has been reduced to 60, m = 0, I ... 59 because as can be seen the extra ones would give little extra benefit for most commonly used angular distributions. The radial lines show the angles of the waves and the circular arcs give lines of constant wave number. Given the depth of the basin and the dispersion equation these become arcs of constant frequency. It will be seen how the higher frequencies are restricted in range of angles from the normal by the finite width of paddles. Over the central and lower range of frequencies there is no such restriction but as the frequencies become lower the resolution becomes poorer as the number of modes involved in the complete angular range reduces. Thus the finite paddle width limits the maximum angle at high frequency and the finite basin width limits the angular resolution at low frequency. It should be noted in connection with the discussion of homogeneity what a large number of modes are involved over the important part of the frequency range.

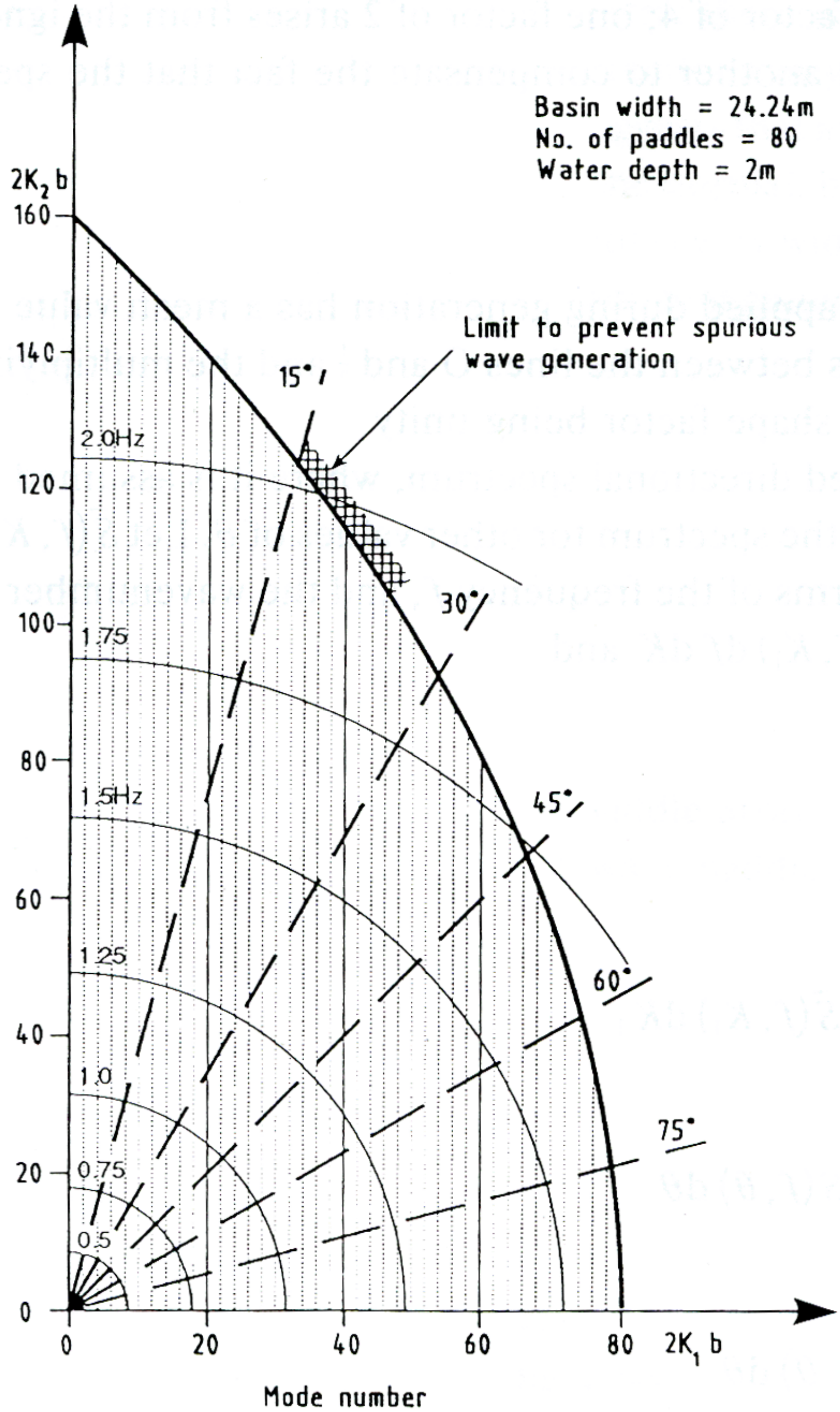

Fig. 5. Generation of multidirectional waves.

Given a prescribed directional spectrum S(f, θ) which must be symmetrical about θ = 0, the various mode spectra $S_m(f)$ need to be calculated to drive the mode synthesisers. To picture this calculation the right hand half of S(f, θ) can be imagined as a solid resting on Fig. 5 and represented by contour lines drawn On this figure. Then the contribution to mode m is just the slice of this solid between the lines (m·- ½) and (m + ½). To obtain the total mode spectrum this must be multiplied by a factor of 4; one factor of 2 arises from the ignored left hand half of the directional spectrum and another to compensate the fact that the spectral mode factor

$$\text{Cos}^2\left(m\frac{\pi x_l}{b}\right)$$

which will be effectively applied during generation has a mean value of ½ across the basin. For the zero mode the slice is between the lines O and ½ and the multiplying factor is only 2 for the left hand half, the mode shape factor being unity.

Let S(f, θ) be the required directional spectrum, where it is assumed that - π/2 < θ< π/2 and that there is no energy in the spectrum for other values of θ. Let $\overline{S}(f, K_1)$ be the same directional spectrum expressed in terms of the frequency f, and the wavenumber K_1 along the paddle face.

Then $S(f, \theta)\, df\, d\theta = \overline{S}(f, K_1)\, df\, dK$ and

$$\text{Sin}\,\theta = \frac{K_1}{K}$$

Hence using (4)

$$S_m(f) = 4\int_{\frac{m-1/2}{2b}}^{\frac{m+1/2}{2b}} \overline{S}(f, K_1)dK_1$$

$$= 4\int_{\theta_{2m-1}}^{\theta_{2m+1}} S(f, \theta)d\theta \qquad (5)$$

and

$$S_0(f) = 2\int_{0}^{\theta_1} S(f, \theta)d\theta \qquad (6)$$

Where

$$\text{Sin}\,\theta_n = \frac{n}{4kb}$$

For frequencies which can be generated round to all angles, the top limit in (5) is replaced by π/2 when a value of m is reached where $\theta_{2m+1} > \pi/2$ and then no further modes are calculated. For frequencies where the angle is restricted either by the limit curve or by the maximum mode the mode spectra in (5) and (6) are increased by a constant factor to compensate for the energy which cannot be generated at larger angles.

Conclusions

The ideas presented here have been implemented in a wave basin 24 m wide by 24 m long and 2 m deep. An array of 80 hinged flap type wave makers was installed, each with separate electrical drives. The signal controlling the position of the paddles, and hence the wave conditions in the wave basin, are generated in real time using a dedicated mini computer.

Calibration and thorough measurement of the wave conditions using a multi-probe array show that waves of the correct directional spectrum are created, and that the homogeneity is in the range expected.

An inherent feature of this method of generation is the symmetry of the directional spectrum about the normal to the wave paddles. For ordinary offshore testing where the test object can be rotated and the standard spreading functions are symmetric there is no limitation, but plainly more general cases with skewed distributions or mean directions varying with frequency cannot be generated. However the argument presented earlier suggests that any alternative method is in trouble, when generating waves at substantial angles to the normal, because the working area is severely restricted by diffraction unless the wave generator is very wide. For coastal and harbour problems the method is being adapted to smaller multi element generators which can be moved to give different mean directions.

Notations

a	wave amplitude
b	width of paddle array
f	frequency (Hz)
K	circular wave number (l/wavelength)
K_1	circular wave number in axis parallel to wave paddle array
K_2	circular wave number in axis perpendicular to wave paddle array
m	mode number
N	number of wave paddles in array
n	notational index for modes
r	notational index for adjacent modes
$S(i,j)$	directional wave spectrum
w	paddle element width
x	distance in direction of wave travel
x_1	distance in axis parallel to wave paddle array
x_2	distance in axis perpendicular to wave paddle array
α	spectral ratios
y	ratio wavelength to wavemaker width ($1/Kb$)
θ	angle of obliquity of wave train
τ	time delay between adjacent paddles

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