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Representation of directional spreading in harbour wave disturbance models.

S. R. Bryant BSc PhD

N. P. Tozer BSc MSc

J. V. Smallman BSc PhD

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Address: **Hydraulics Research Ltd,** Wallingford, Oxfordshire OX10 8BA, United Kingdom.
Telephone: 0491 35381 International + 44 491 35381 Telex: 848552 HRSWAL G.
Facsimile: 0491 32233 International + 44 491 32233 Registered in England No. 1622174

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ABSTRACT

This report describes the implementation and validation of the inclusion of directional spreading in two existing wave disturbance models. The purpose of this work was to improve the accuracy of representation of wave propagation within harbours. The report describes the theoretical background to the procedure used, and the method of implementation. Two types of numerical ray models are used which cover the range of frequently occurring entrance types. The inclusion of directional spreading is validated by comparison with simple analytical test cases, and physical model and field data.

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1. INTRODUCTION

1.1 Background

Numerical models of harbour wave disturbance have been in use in studies for many years. They are frequently used to examine the feasibility of a proposed development, and are an efficient and effective tool. Most of the models which are available use as input a single wave frequency and direction component at the boundary. Wave heights in the harbour area are thus calculated for a single specified incident wave period and direction per model run. To represent spectral input therefore requires a series of runs for a number of discrete period and direction components, the results being combined using linear superposition. Typically this would require 25 model runs (5 periods and 5 directions) for each incident spectrum, which would be both expensive and time consuming.

Practical experience has shown that in most cases a single selected period and direction combination can be used as a reasonable representation of the incident wave spectrum. This gives an approximation to wave conditions in the harbour resulting from an incident spectrum. However, the model accuracy would be improved and the physical situation more realistically represented, if calculations for a spread of frequency and direction components could be included economically.

The mathematical models which were developed and are frequently used in wave disturbance studies at Hydraulics Research are PORTRAY and PORTGAP. These are forward-tracking ray models dealing with a semi-infinite breakwater and a breakwater gap at the harbour entrance respectively. On reviewing the underlying physics of ray methods it is obvious that the ray path is strongly dependent on frequency. Therefore models of this type will certainly need to consider each frequency component separately. However, the wave direction only influences the initial ray paths and the proportion of energy which is carried through the calculation procedure. Therefore it is possible for a range of directions to be modelled, without the need to re-run the models for each direction component, by consideration of the redistribution of energy resulting from a change in the incident direction. This procedure will go some way towards a more accurate spectral representation whilst maintaining the models advantages of being fast and inexpensive tools for harbour design.

1.2 Terms of Reference

Previous reports (References 1 and 2) describe the modification of the PORTRAY harbour wave disturbance model to handle diffraction by a breakwater gap at the harbour entrance, the resulting program being PORTGAP. These two models can only simulate a single incident wave direction and period.

This report describes work carried out to extend the existing harbour wave disturbance models PORTRAY and PORTGAP to include directional spreading. The extended models have been verified using

(i) linear superposition of the existing models
(ii) simple analytical test data for a flat bed and for a concave bed
(iii) physical model data for Dover Harbour Eastern Docks (PORTGAP) and field data for Venice (PORTRAY)

Chapter 2 describes the methods used to incorporate directional spreading into the models. The results of the validation tests are discussed in Chapter 3, and the conclusions drawn are given in Chapter 4.

2. DESCRIPTION OF MATHEMATICAL MODELS

2.1 The Existing Models

The work described here covers two harbour wave disturbance models. These models track wave rays (lines perpendicular to wave crests) inshore from a harbour entrance to enable the effects of refraction, reflection and diffraction with the harbour to be examined. For each set of conditions at the harbour entrance the models give wave heights, phases and directions at all points on a user specified grid.

The PORTRAY model deals with the case of a semi-infinite breakwater at the harbour entrance. The energy transmitted into the harbour is carried directly by the incident rays, and indirectly by rays diffracted from the breakwater tip. The PORTGAP model deals with the case of a breakwater gap at the harbour entrance. A 'gap' in this context is typically less than about three or four wavelengths. In this case the energy is transmitted into the harbour indirectly by the diffracted rays, via far field coefficients which are dependent on the gap width and alignment and on the incident wave conditions. For convenience in PORTGAP these coefficients are calculated separately from the ray tracking procedure.

A distinction is drawn between modelling diffraction at a gap, and by two closely aligned semi-infinite breakwaters on the basis of gap width. This is because the use of the diffracted fields from two semi-infinite breakwaters to represent diffraction at a gap assumes that there is no interaction between the diffracted waves from the two breakwater tips. This is a reasonable assumption provided that the gap between the breakwaters is of the order of three to four wavelengths. For narrower gaps this interaction needs to be modelled, and this can be achieved by considering diffraction by a breakwater gap.

2.2 Inclusion of Directional Spreading

2.2.1 PORTGAP

This model deals with a harbour entrance formed by a small breakwater gap, typically less than four wavelengths. At a sufficient distance from the gap the diffracted field appears to be due to a point energy source in the gap, and can thus be represented as a fan of rays radiating from the gap centre. The trajectory of each ray is dependent only on the wave celerity which is a function of the frequency and the local water depth. Thus for a specified wave period (frequency is the inverse of period) and harbour bathymetry, the ray path is independent of both the incident direction and the wave energy associated with the ray. That is, the energy flux between rays does not affect the ray paths. To calculate refraction and shoaling the energy flux is assumed constant between rays. These give rise to an equation describing the conservation of energy between neighbouring rays (see Ref 4),

$$A^2 c_g b = E$$

where A is the wave amplitude, c_g the group velocity and b the ray separation; E is the energy flux between rays. For a fan of rays on a flat bed $b = r\Delta\Theta$ hence

$$A = (E/c_g r\Delta\Theta)^{1/2} \qquad (1)$$

In the far field limit the amplitude of the diffracted field for waves pressing through a gap in an infinite straight breakwater can be written in the form:

$$A = F(\Theta,\Theta_0,\lambda)/r^{1/2} \qquad (2)$$

Here $F(\Theta,\Theta_0,\lambda)$ is the far field coefficient, which is dependent upon the angle of observation Θ, the incident angle Θ_0 and the wavelength λ, and r is the radial distance from the gap centre to the point of interest in the wave field. The far field coefficient provides a description of the diffracted field at large distances from the breakwater gap, and is derived from solving the problem of diffraction of water waves by a breakwater gap (see Ref 5). Combining equations (1) and (2) relates $F(\Theta,\Theta_0,\lambda)$ to the energy flux E;

$$E = F^2(\Theta,\Theta_0,\lambda)c_{go}\Delta\Theta \qquad (3)$$

c_{go} being the group velocity at the gap, and hence substituting in (1),

$$A = F(\Theta,\Theta_0,\lambda)(c_{go}\Delta\Theta/c_g b)^{1/2} \qquad (4)$$

c_g and b being the group velocity and ray separation at the point of observation respectively. It should be observed from (3) that the dependence of the energy flux on the incident direction is solely via the far field coefficients, $F(\Theta,\Theta_0,\lambda)$.

In the case where directional spreading of the incident wave is required each direction component of the incident wave train will contribute to the total energy of each followed ray, the ray paths being independent of the incident angle. Hence combination of the far field coefficients for each component in the incident spectrum will enable a modified far field coefficient to be used in the equation above, thus representing a spread of incident wave angles. Previous experience has shown that a linear weighted average of single direction results models the multi-directional case adequately, hence from the above equation the modified far field coefficient, F', will be of the form

$$F'(\Theta,\lambda) = (\textstyle\sum_i [w_i F_i(\Theta,\Theta_i,\lambda)^2])^{1/2} \qquad (5)$$

where the sum is over all incident angles being considered. The weights used are typically of the form $a_i \cos^n \Theta_i$, where Θ_i is the divergence from the mean incident direction, a_i is the normalising factor for the i th component. The value of n varies according to the incident conditions at the site being modelled, typical values being 6 and 30.

Therefore, to model directional spreading in the PORTGAP model, the modified far field coefficient F' needs to be calculated from the individual direction far field coefficients F_i for each ray being sent out from the gap. One set of rays are used in the model with a modified far field coefficient defining the energy associated with each ray. This will obviously lead to significant time saving in the calculation procedure, as it avoids the use of a set of rays for each incident direction.

2.2.2. PORTRAY

The PORTRAY model represents diffraction at a semi infinite breakwater by using a set of rays whose initial paths are determined from the analytical solution of diffraction by a breakwater derived by Sommerfeld, see Ref 4 for further details. This requires a number of ray types to be considered. Firstly, there are the incident rays which carry the majority of the energy into the harbour. Secondly, there are the three sets of rays representing diffraction at the breakwater tip, their initial energy and phases being determined from the far field solution to the problem. Two sets of rays, the u-rays are sent out at angular intervals from the line of the shadow boundary ray. This has its initial direction corresponding to the incident wave direction at the breakwater tip. The third set of rays are the v-rays which emanate radially from the breakwater tip on the sheltered side.

The ray paths of all but the v-rays are dependent upon the incident direction and hence must be tracked for each component in the incident spectrum and the energies summed over each grid square. The v-rays however can be treated in a similar manner to the diffracted rays in the PORTGAP program; the v-ray trajectories only need to be calculated once and the energy contributions from all the incident directions summed. However, consideration of the implementation of this approach concluded that while attractive in theory this was not a practical technique to use, and the v-rays are in fact tracked for each incident direction in the same way as all the other ray types.

The single direction PORTRAY program calculates a wave height, direction and phase for each grid cell. Once multiple incident directions are considered the wave activity in a cell will be composed of several waves travelling with different directions and phases. The modified model therefore gives a wave height calculated from the total energy, the direction and phases of the component with the greatest energy, plus the range of directions and phases over all components present within the cell.

3. MODEL VERIFICATION

3.1 Outline of approach

As discussed earlier it is possible to represent directional spreading in both models by running separate direction components, and then combining the results from individual runs for different directions about the mean direction. The results obtained using this method should be identical, for the same incident conditions, to those derived using the modified models which include directional spreading using the methods described in the previous section. This produces a basic verification for both modified models, and it was undertaken for two simple bed geometries. These represent the case of uniform water depth, and a concave bed profile (see Figure 1) in the lee of the breakwater. The outcome of this verification is discussed in section 3.2.

This procedure establishes that the method for including directional spreading in PORTGAP and PORTRAY is operating correctly. It does not however indicate the effect that directional spreading will have on the accuracy of the representation of the physical processes occurring in a harbour. This is examined here by comparing the model results with those from a physical model and field data. For the PORTGAP model a comparison was made with the results of a physical model investigation of Dover Eastern Docks. In the case of PORTRAY, field data from the Chioggia Inlet in Venice was used. In both cases the data sets were not ideal to fully examine the models' capabilities, but they were used provide an insight into the improvement of the physical representation which can be achieved using directional spreading. A discussion of the results for each of these cases is given in section 3.3.

3.2 Comparison with existing results

3.2.1 PORTGAP

As described in section 2.2.1, the calculations within the PORTGAP model are done in two stages. First, the far field coefficient for each incident wave direction is calculated, then the ray tracking calculation is performed using the far field coefficients as input. There are therefore two points in the calculation process at which comparisons can be made between the original model and the version which has been modified to include directional spreading. Firstly, the modified far field coefficients can be compared with repeated runs of the single direction program which have been combined after output using expression (5). These are shown in Table 1 for the simple test configurations.

Secondly the results of the PORTGAP phase of the modified model can be compared with those of the original PORTGAP model. These comparisons have been made for a straight breakwater gap with both a uniform depth bed and for a concave bed. When identical far field coefficient data is used the original PORTGAP and the modified version give identical results. When the two sets of values shown in Table 1 are used the results are, not surprisingly, a little different. Although, as can be seen from Figures 2 and 3 for a flat bed and a concave bed respectively, these differences are very small. It should be noted that the previous combination of far field coefficients was carried out on values output to two decimal places, whereas the new program uses the calculated values to machine precision, implying that the new values are more accurate than the previous estimates.

For these simple test cases the level of agreement achieved between repeat running of the PORTGAP model, and PORTGAP modified to include directional spreading is good. This establishes that the modified model is operating correctly, and so can be used to examine the effect of directional spreading on wave propagation in a real harbour.

3.2.2 PORTRAY

The basic verification of PORTRAY was carried out for waves approaching a semi-infinite breakwater with both a uniform depth and a concave bed profile in its lee. Unlike PORTGAP, there is no intermediate point at which comparisons can be made. Results for the two versions of the model were identical for both cases, and are not reproduced here.

3.3 Comparison with physical model and field data

3.3.1 PORTGAP

Having completed the basic tests of the modified PORTGAP model it was then set up to represent wave propagation in Dover Harbour. Here it was possible to make comparisons between the computational model results, and those from physical model tests which were carried out for Dover Harbour Board in 1985. The approach used was to set up the modified PORTGAP for the Dover layout, the area represented is shown in Figure 4. The model was then run assuming that the western harbour entrance is a straight gap between two breakwaters. The gap width was taken as the distance between the entrance breakwaters along a line normal to the incident wave direction. For the wave conditions used in the test the equivalent gap width is approximately 0.9 of a wavelength. The modified PORTGAP model was then run for a single incident wave direction and for two directionally spread conditions with a cos square distribution applied to 30° either side of the mean direction at 5° and 10° intervals. The results of these calculations, together with those from the physical model, for the probe positions shown in Figure 5 are given in Table 2.

From Table 2 it can be seen that single direction results are in good agreement with those from the physical model; the maximum error being of the order of 10%. It should be anticipated that for this case the directionally spread versions will give similar results to those for the single direction. This is because the single direction far field coefficients for the ray directions penetrating the Eastern Docks do not differ substantially from those for the combined directions. This is a feature of the particular geometry of the layout examined. This feature is evident from Table 2 where the results for all three mathematical model runs are similar, and all are in good agreement with those from the physical model. The best overall agreement is achieved for the 30° spread at 10° increments, but on average all of the sets of results are within 6% overall of the physical model results. This demonstrates that the modified PORTGAP model is operating well for a realistic harbour. In addition to the accuracy of the model, a comparison was also made of the computer run times of the original model calculating the far field coefficients and the PORTGAP simulation with the new version of this program. The runs were carried out for Dover using three incident directions. The details are given in Table 3. As can be seen the new program version takes about half the time needed by the original program for the data set used. Indeed for each additional direction added to the spectrum a further saving of approximately five minutes would be made using the new version.

3.3.2 PORTRAY

To test the modified PORTRAY model with directional spreading for a realistic situation, comparisons were made with tidal data collected at the Chioggia Inlet to Venice Lagoon. At this site waves approach an inlet to the lagoon which has breakwaters on either side. Wave measurements were made at several locations for an extended period during 1988. Published results are available for two locations within the inlet, see Figure 6, for a storm which occurred in March 1988. These are in the form of wave height coefficients at locations B and C. The incident wave conditions for PORTRAY were derived from a mathematical wave refraction model, whose results at the boundary of the PORTRAY model were in good agreement with field measurements at this location for the March 1988 storm. The advantage of using a mathematical wave refraction model to define the incident wave conditions for PORTRAY, is that the directional spread is calculated by the refraction model. This therefore provides a good test of the ability of the model to represent directional spreading effects for a realistic situation.

The procedure which was used to test PORTRAY was to set it up for the layout shown in Figure 6. Incident conditions were applied along the eastern model boundary, with the wave period and mean direction being taken from those for the storm. The predicted incident directional spread was $\cos^{18}$ about the mean. Tests were carried out applying the weightings corresponding to this relatively narrow spread within $\pm 30°$ and $\pm 15°$ of the mean direction with increments of 10° and 5° respectively. In addition $\cos^{2}$ and $\cos^{6}$ were also used to define the weightings of the directional spread in order to assess its sensitivity.

The results from these tests are shown in Table 4 in the form of wave height coefficients at the positions shown on Figure 6. Measured data was available at two points B and C, the specification of C was unclear so two points C1 and C2 were used in the analysis. Results are also shown of eight further locations to provide an indication of the overall distribution of wave height within the area. If we first consider the results at positions B, C1 and C2, it can be seen that the best agreement is achieved for the spreading functions with a narrow angular range, ie those at $\pm 15°$ to the mean with 5° steps. This is because the incident spectrum is known to be narrowly banded ($\cos^{18}$), and therefore a time increment is required around the mean direction to resolve the energy spread accurately. For the more coarsely spaced angular ranges, ie those at $\pm 30°$ to the mean with 10° spacing, the agreement with measured data is poor even for the narrow spreading functions. In these cases the resolution is insufficient to represent the physical situation accurately. From the results at other locations within the inlet it is clear that the choice of spreading function, and the accuracy of its resolution was a significant effect on the predicted wave heights.

A comparison of run times for repeat runs of the original PORTRAY model, and for the modified version with directional spreading is shown in Table 5. The modified version takes marginally longer than the original for the example shown, but as described earlier increased direction and phase information is provided. Where a greater number of directions are considered the modified version is faster than running and combining the separate directions, and it is also
AR more economical in its use of filesto~~n~~re.

4. CONCLUSIONS AND RECOMMENDATIONS

1. An efficient method for including directional spreading in two existing mathematical models, PORTRAY and PORTGAP, has been implemented.

2. The modified models were first validated using results from simple test data sets. It was found that there was good agreement between the results from directionally spread versions of the models, and those from combined single direction runs.

3. The modified models were then set up for two realistic harbour layouts, and comparisons made with the results from physical models and field measurements. Agreement for these situations was good, demonstrating that the modified models provide an accurate representation of wave propagation for realistic harbour layouts.

4. The modified models were found to provide a more efficient method for representing directional spreading than the direction combination technique previously used.

5. As more directional wave data sets become available it is recommended that further verification of the models is carried out. A suitable data set for this purpose is presently being collected at the Port of Shoreham. It is anticipated that comparisons of the modified PORTRAY model with this data can be made later this year.

5. REFERENCES

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4. Southgate H N. A ray model of wave action in harbours. Proc. Int. Conf. of Numerical and Hydraulic modelling of Ports and Harbours, Birmingham, England. BHRA publications 1985.

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TABLES.

TABLE 1 Comparison of far field coefficient values

Observation angle (degrees)	Combined Values[1]	New Values[2]
181.0	3.29	3.29
196.0	3.51	3.51
211.0	4.04	4.04
226.0	4.59	4.59
241.0	4.65	4.65
256.0	3.86	3.86
271.0	2.45	2.45
286.0	1.34	1.34
301.0	1.23	1.24
316.0	1.30	1.30
331.0	1.21	1.22
346.0	1.12	1.12

Wave period	5.0s
Breakwater gap	36.5m
Incident angle	45° ±15°
Depth at gap	10.0m

Notes:

1. Values obtained by generating coefficients for each of the three incident directions and manually combining them.

2. These values generated by the modified version of the model.

TABLE 2 Significant wave heights, H_s(m), for Dover Harbour Eastern Docks

Incident conditions : H_s = 4.9m, T_p = 8.5s, Direction = 203°N

Position	Physical model	Mathematical model single direction	$\pm$30°, 10° steps	$\pm$30°, 5° steps
A	0.51	0.47	0.48	0.52
B	0.74	0.74	0.73	0.75
C	1.08	1.12	1.09	1.12
D	0.97	1.04	1.00	1.04
E	1.13	1.08	1.06	1.10
F	0.69	0.78	0.76	0.79
G	0.71	0.79	0.78	0.80
H	1.00	1.09	1.09	1.11

TABLE 3 PORTGAP runtime comparison

Three directions considered for Dover Eastern Docks model

Original Programs:

One set of far field coefficients	210.0 secs
One run of PORTGAP	339.0 secs
Total time for three sets of far field coefficients and PORTGAP runs	1647.0 secs

Directional Spreading included:

One run including three sets of far field coefficients	802.0 secs

All runs carried out on a SUN 3/50 workstation and including the generation of ray plots.

TABLE 4 **Wave height coefficients for Chioggia Inlet**

Position	Field measurement	Mathematical model results*					
		$\cos^{18}$	$\cos^{6}$	$\cos^{2}$	$\cos^{18}$	$\cos^{6}$	Spreading function
		10	10	10	5	5	Angular step (°)
		±30	±30	±30	±15	±15	Angular range from mean (°)
B	0.98	0.89	0.82	0.78	1.09	1.08	
C1 }	1.12	0.56	0.53	0.51	1.12	1.07	
C2		0.65	0.72	0.73	1.09	1.16	
1	-	0.88	0.78	0.74	1.08	1.03	
2	-	0.21	0.20	0.19	0.17	0.17	
3	-	0.99	1.02	1.01	0.80	0.85	
4	-	1.25	1.13	1.08	1.28	1.24	
5	-	0.46	0.51	0.52	0.60	0.62	
6	-	1.71	1.58	1.50	1.65	1.66	
7	-	0.99	0.92	0.88	0.93	0.93	
8	-	0.90	0.91	0.91	0.90	0.90	

* Shown as wave height coefficients

\+ Incident wave period = 6.4s (T_p), mean direction = 110°N

TABLE 5 PORTRAY runtime comparison

Timings given for single direction and for three directions for Chioggia inlet

Original Program	
One run of PORTRAY	100.0 secs
Three runs of PORTRAY plus combination of results	256.0 secs
Directional Spreading Version	
One run of PORTRAY for a single direction	115.0 secs
One run of PORTRAY covering three directions	270.0 secs

Note that the diffraction behaviour is different for the three directions considered, hence the time for calculating results for three directions is less than three times the single mean direction time.

Note also that more information is output from the model for the directionally spread version than for single direction, and that a further allowance for additional time in combining results needs to be made.

All runs carried out on a SPARC IPC workstation and including the generation of ray plots.

FIGURES.

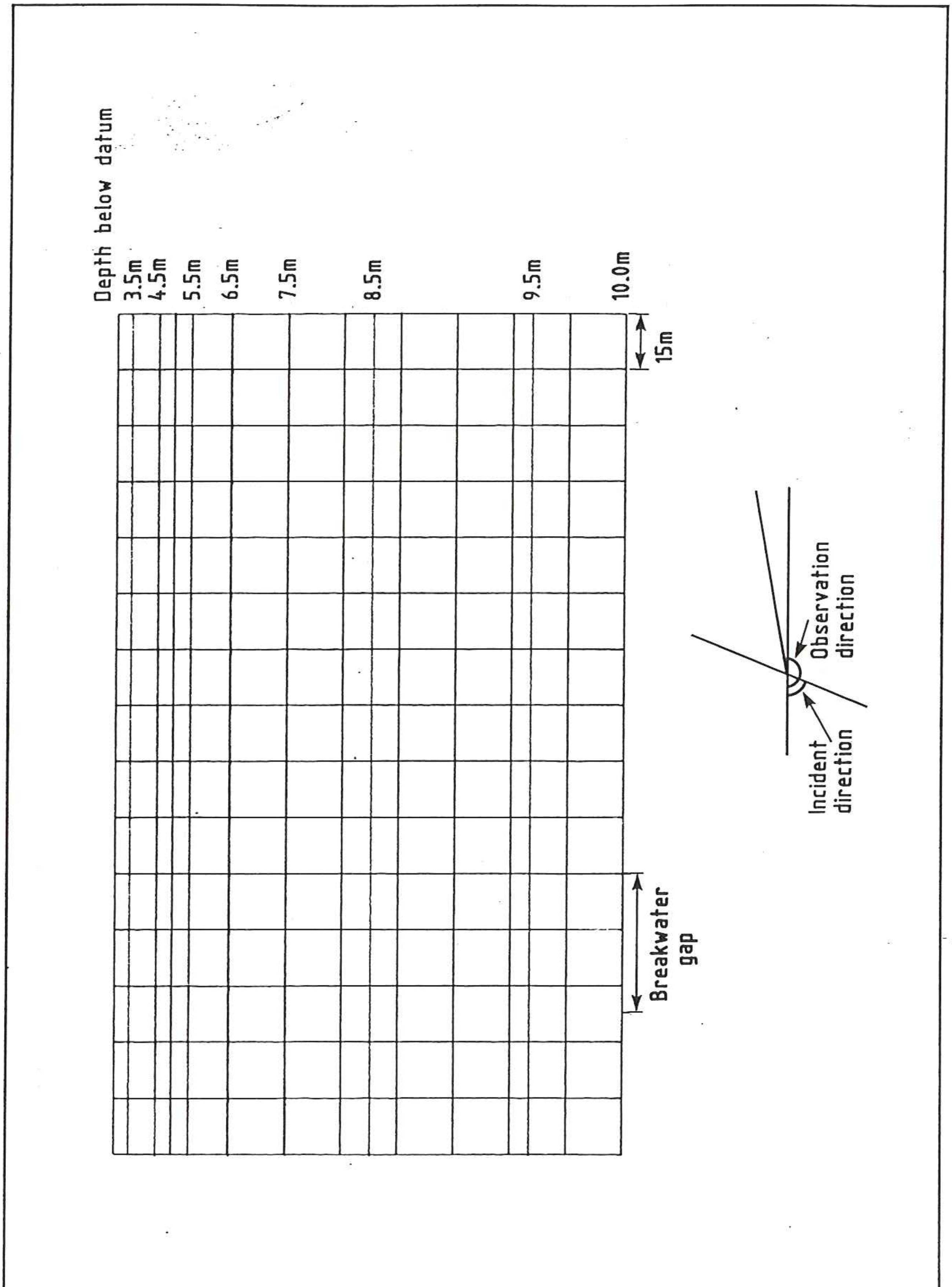
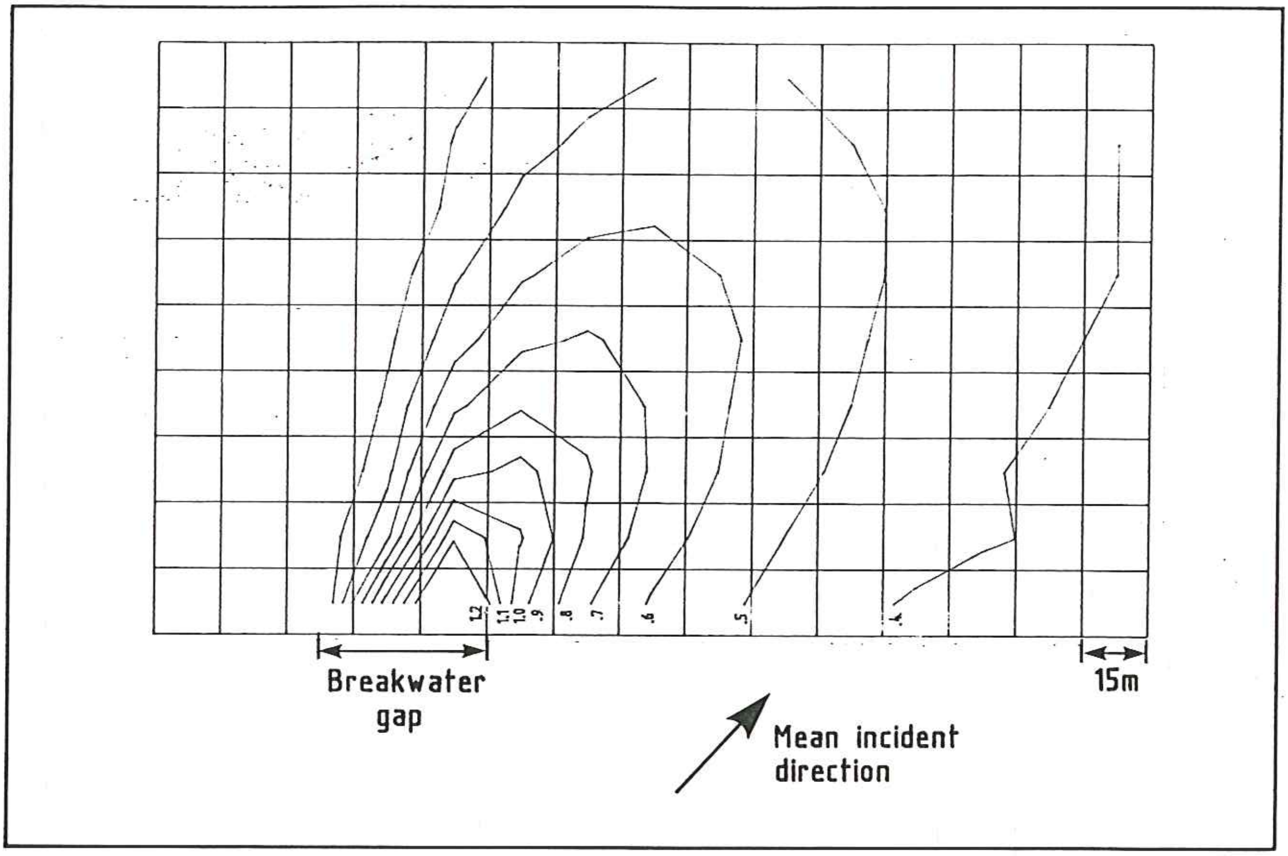
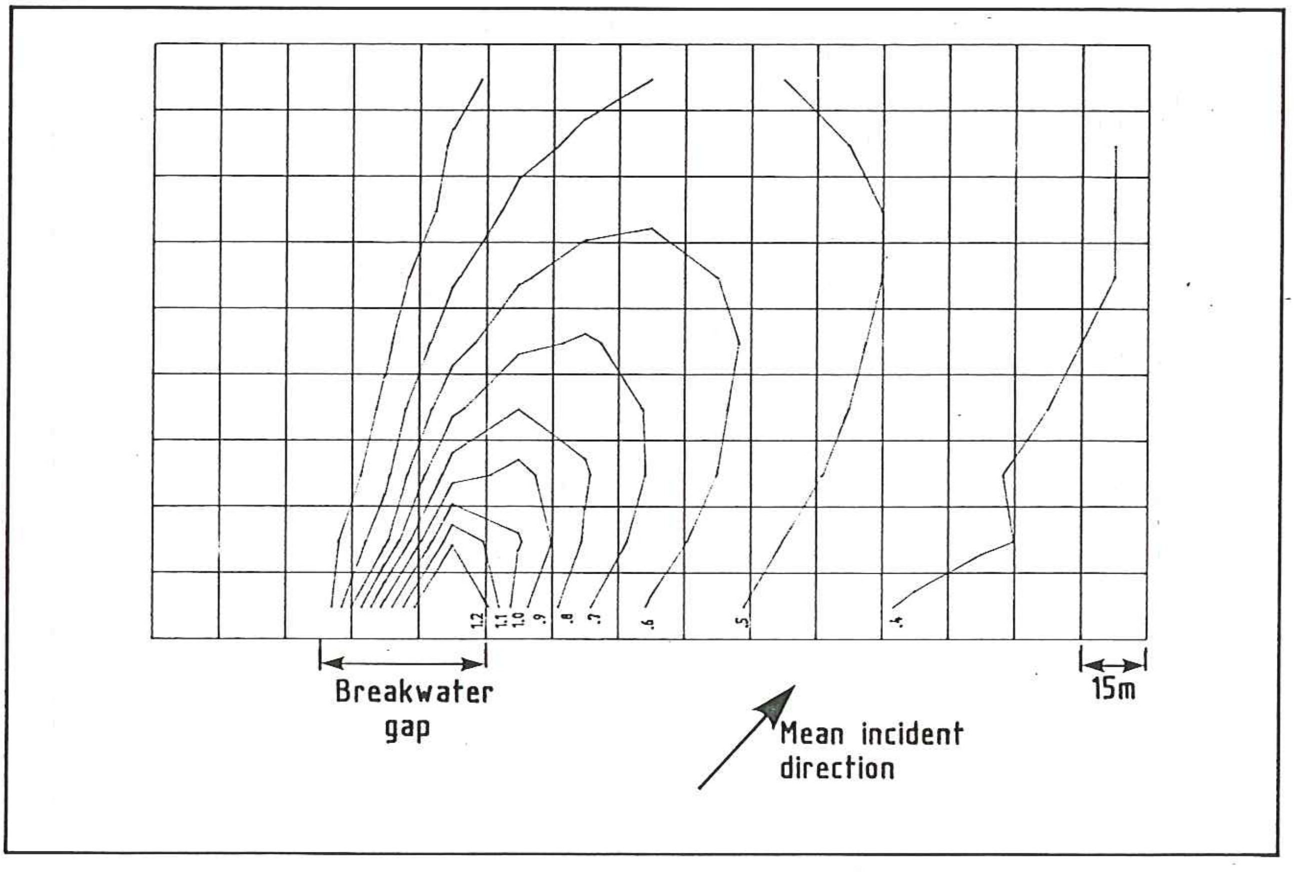


Fig 1 Concave bed model layout.

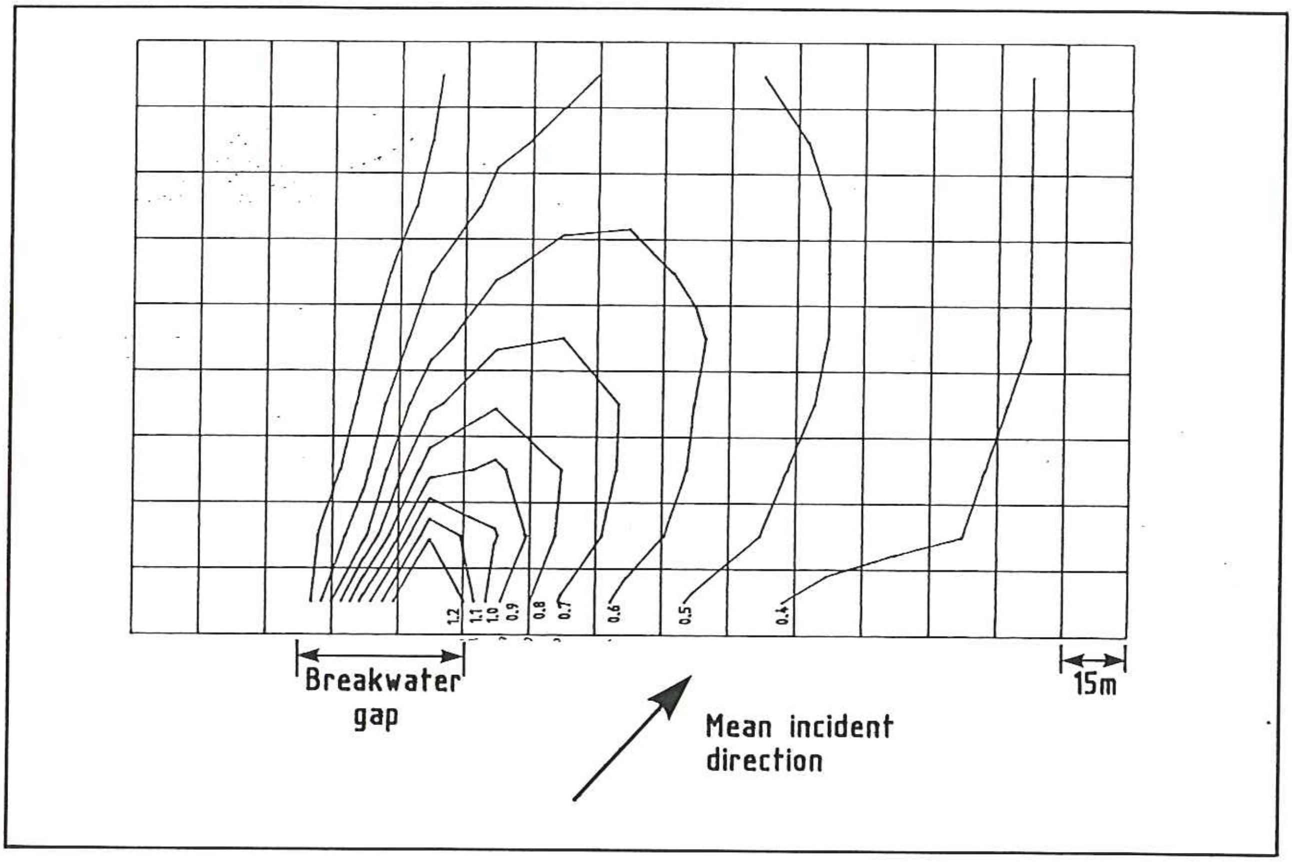


2a Far field coefficients externally calculated and combined.

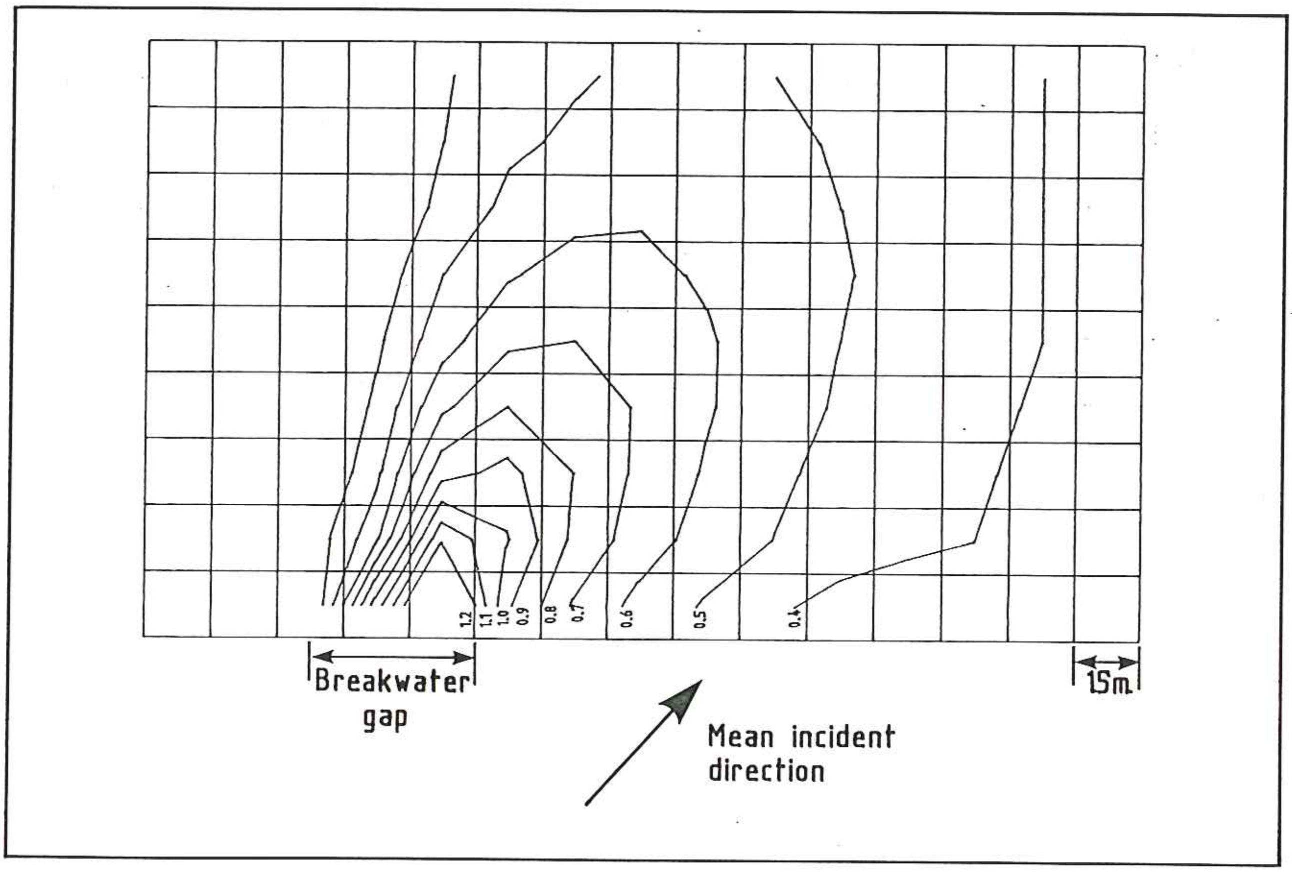


2b Far field coefficients calculated with the modified PORTGAP model.

Fig 2 Contours of wave height coefficient from PORTGAP model for uniform depth test. Incident angle 45°±15°



3a Far field coefficients externally calculated and combined.



3b Far field coefficients calculated within the modified PORTGAP model.

Fig 3 Contours of wave height coefficient from PORTGAP model for concave bed test. Incident angle 45°±15°.

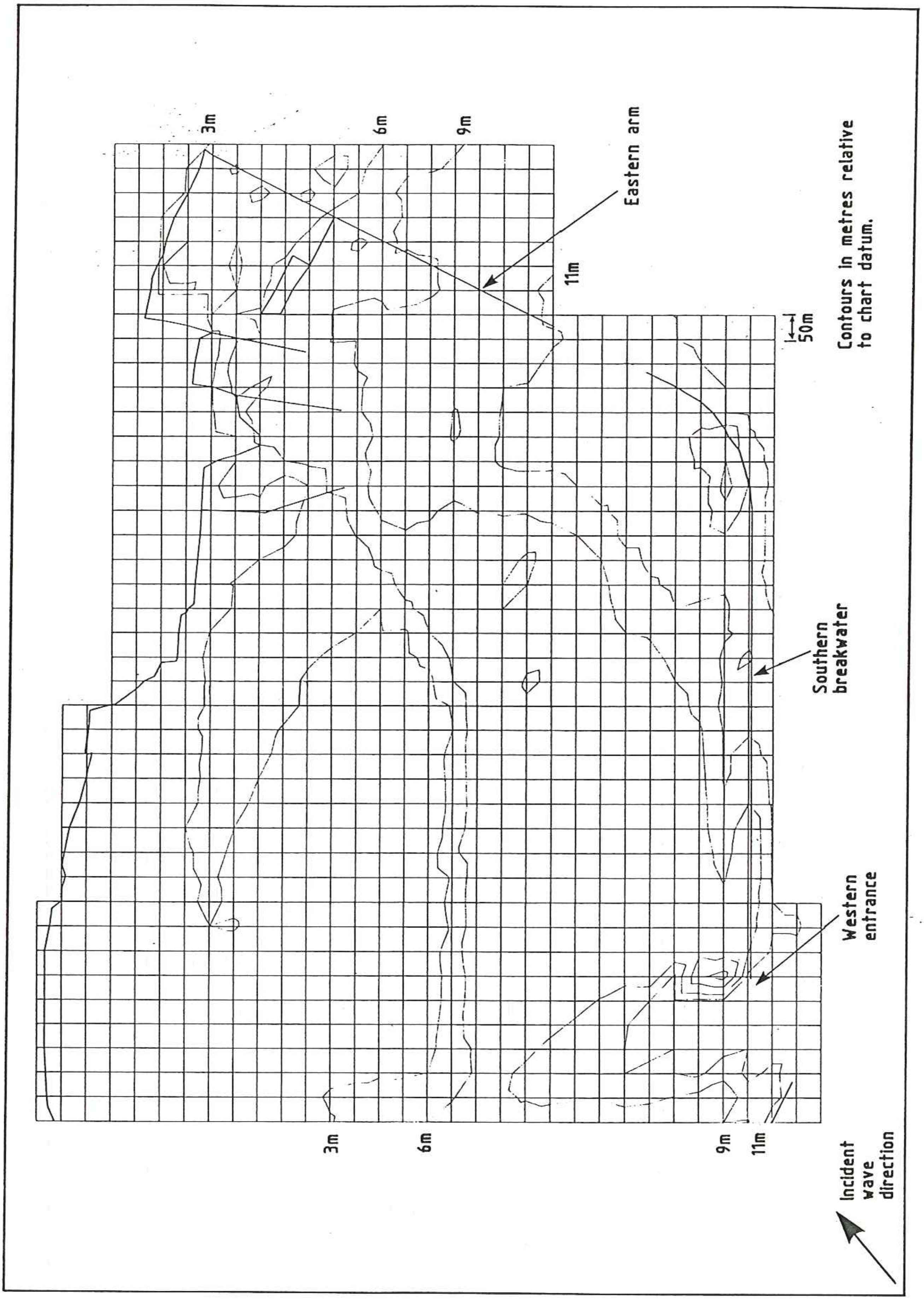


Fig 4 PORTGAP model layout for Dover Eastern Docks.

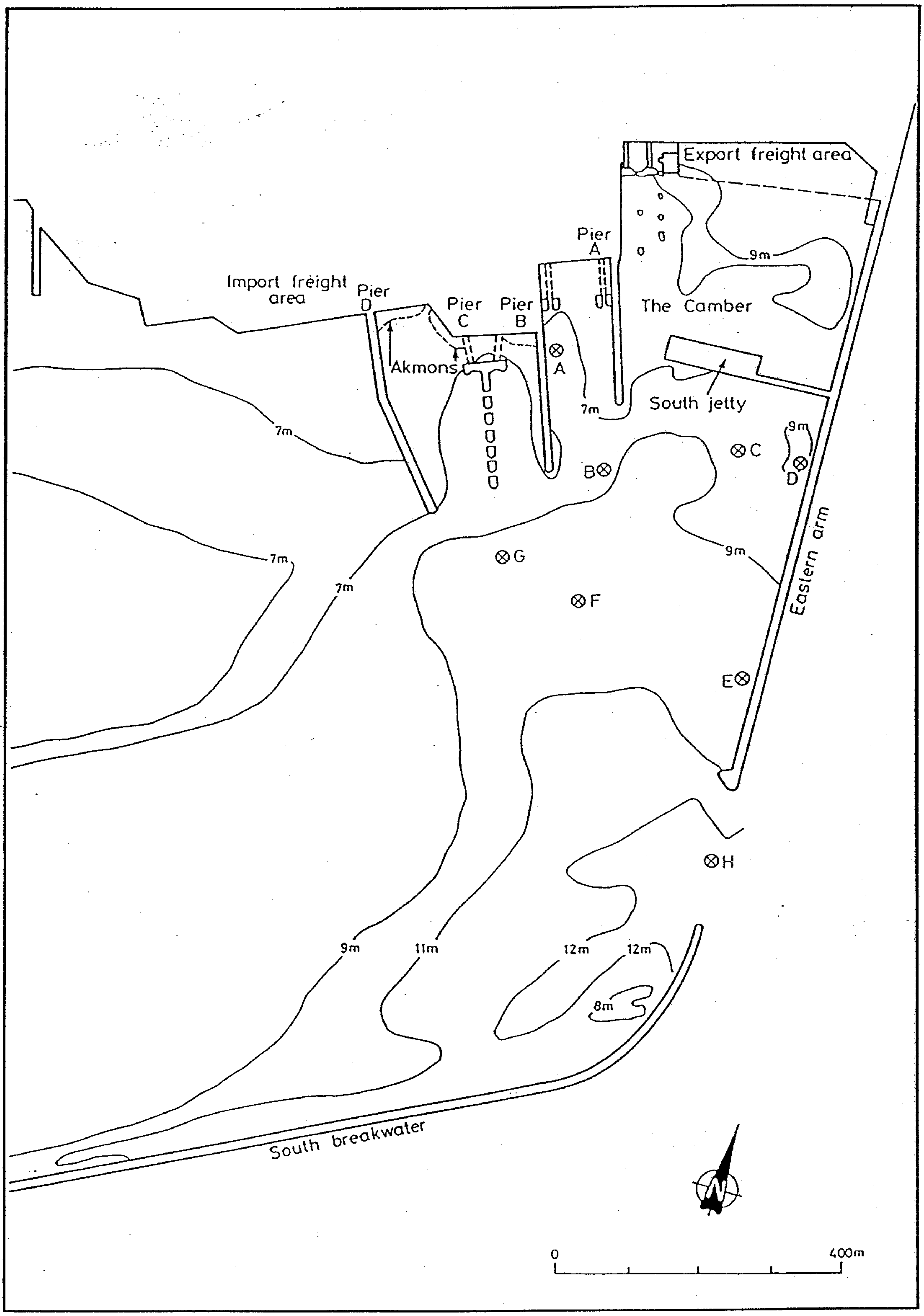


Fig 5 PORTGAP analysis positions in Dover Eastern Docks.

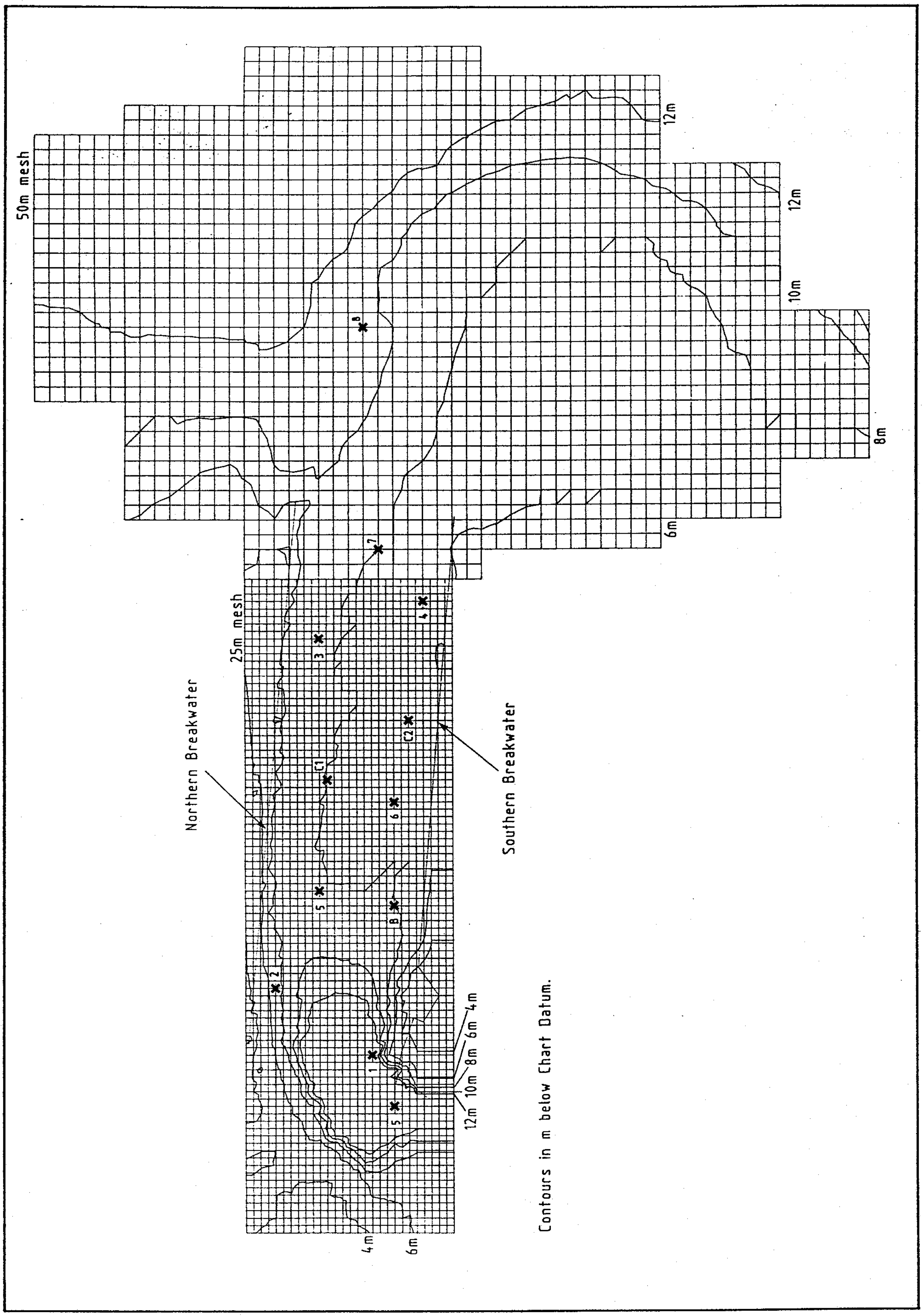


Fig 6 PORTRAY model layout for Chioggia Inlet.