Alluvial Friction in the Transition Zone

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Report SR 283
November 1991

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This report describes work funded by the Department of the Environment under contract PECD 7/6/160, Alluvial resistance.

It is published on behalf of the Department of the Environment, but any opinions expressed in this report are not necessarily those of the funding Department. The nominated officers were Mr P Woodhead for DoE and Mr W R White for HR.

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ABSTRACT

Laboratory experiments carried out in the transition zone between upper and lower regime flows are described. The results of these experiments imply that the transition zone consists of a family of curves that fill the area and that the trajectory of a flow through the transition zone depends upon the constraints applied. To parameterise the transition zone it is suggested that use is made of the parameter U_e. This parameterisation should be based on experimental data, were available, but in the absence of such data an approximation is presented. This should enable the calculation of flows within the transition zone.

CONTENTS

1. INTRODUCTION

The calculation of flow in an alluvial channel requires an estimate to be made of the frictional losses on the boundary of the channel. A method of determining the friction in an alluvial channel is also required for the determination of sediment transport rates related to the design of irrigation channels and river improvement works. The determination of these frictional losses is difficult as bed forms and hence frictional resistance vary with the flow and the sediment transport rate.

In the case, for example of a laboratory channel with an initially plane sediment bed, down which the discharge is gradually increased, the bed features will change as the discharge changes. Provided that the sediment is sufficiently fine, ripples will develop first. At a higher discharge, dunes will appear and increase in size. The dunes then diminish as the discharge increases to be replaced by an approximately smooth bed. Anti-dunes will then develop.

Previous work, (Engelund, 1966;Brownlie, 1981, and White et al, 1987) has suggested that there is one frictional relationship for dune and ripple beds, described as the lower flow regime, and an essentially different relationship for upper plane beds, standing waves and anti-dunes, described as the upper flow regime. In 1987 White et al published work giving relationships to describe both lower and upper flow regimes. These relationships consisted of two families of curves, one for upper regime and one for lower regime. In each family there were different curves for different sediment sizes. As the two families of curves did not intersect the issue was raised of the transition between lower and upper regimes. This report describes work carried out to predict the transition between the two regimes. The work is based on the analysis of laboratory experiments carried out at HR.

2. PREVIOUS WORK

2.1 Yalin

Yalin argues that the alluvial friction must depend on:

(1) the granular roughness of the bed;
(2) the geometry of any bed features; and
(3) the amount of sediment in suspension

On the basis of this he argues that alluvial friction must be expressible in terms of the functions

X, Y and Z for subcritical flow

and X, Y, Z and F for supercritical flow,

where $X = \frac{DV_*}{\upsilon}$, $Y = \frac{\rho V_*^2}{\gamma_s D}$, $Z = \frac{d}{D}$,

and F is a Froude number.

2.2 Engelund

Engelund, in his analysis of alluvial friction, plotted the dimensionless bed shear due to skin friction against the total dimensionless bed shear for a sequence of flume data. The relationship that he obtained and data from Guy et al (1966) are shown in Figure 1. The lower curve applies for dunes; ripples being excluded from Engelund's analysis, while the upper curve applies to flat beds, standing waves and anti-dunes.

2.3 Brownlie

Brownlie (1981) carried out a dimensional analysis to determine a set of non-dimensional parameters to describe sediment transport and alluvial friction. By assuming that flow resistance is related to the largest scale of bed roughness, Brownlie proposes an equation of the form

$$\left(\frac{\rho_i - \rho}{\rho}\right) \tau \,/\sigma_g^z = W\ (9\ S^{1+y/x})x$$

Two sets of multiple regression analysis were carried out on data from lower and upper regime to determine the parameters w, x, y and z. To consider the problem of determining the flow regime Brownlie utilised four non-dimensional groups;

$$F_g,\ \frac{D_{50}}{\delta},\ S \text{ and } \sigma_g$$

where F_g, is the grain Froude number defined by

$\sqrt{\rho}\bar{v}/\sqrt{(\bar{\rho}_s-\rho)g\,D_{50}}$

From an analysis of the data the following boundaries between the regimes were proposed:

Lower limit of the upper flow regime

$$\log F_g/F_g^1 = \begin{cases} -0.02469+0.1517\log(D_{so}/\partial)+0.8381(\log(D_{so}/\partial))^2 \text{ for } D_{so}/\partial<2 \\ \log 1.25 \quad \text{for } D_{so}/\partial\geq 2 \end{cases}$$

and for the upper limit of the lower regime

$$\log F_g/F_g^1 = \begin{cases} -0.02026+0.07026\log(D_{so}/\partial)+0.9330(\log(D_{so}/\partial))^2 \text{ for } D_{so}/\partial<2 \\ \log 0.8 \quad \text{for } D_{so}/\partial\geq 2 \end{cases}$$

where $F_g{}^1 = 1.74\ S^{-1/3}$

Between these values lies the transition regime.

Brownlie asserts that for slopes greater than 0.006 only upper regime flows occur.

Brownlie argues that the depth of flow at the lower limit of the upper flow regime, and the depth at the upper limit of the lower regime will be approximately the same. A reasonable estimate of flow depth during a gradual transition may be the average of the two depths. Alternatively, he argues that one might suspect that transition will take place along a line of constant depth. In this case, during a gradual rise in discharge, the depth would reach the upper limit of the lower regime and remain constant during transition, and for a gradual decrease in discharge the depth would reach the lower limit of the upper regime and remain constant.

The experimental data described below, collected during this study, would suggest that the depth is not constant during the transition but varies. It seems unlikely therefore that the method proposed by Brownlie will provide an adequate description of the transition zone.

2.4 White Paris and Bettess

In a series of papers White et al (1980, 1987) have analysed alluvial friction in terms of parameters first introduced by Ackers and White (1973) in the context of sediment transport. Following Ackers and White the sediment size was characterised using a dimensionless sediment size D_{gr} defined by

$$D_{gr} = \left(g\frac{(s-1)}{v^2}\right)^{1/3} D$$

They introduced a general sediment mobility, F_{gr}, defined by

$$F_{gr} = \frac{v*^n}{\sqrt{(gD\ (s-1)}} \left\{\frac{V}{\sqrt{(32)}\ \log_{10}\ (10d/D)}\right\}^{1-n}$$

n is an exponent which varies from 1.0 for fine sediments to 0.0 for coarse sediments. Thus for fine sediments

$$F_{fg} = \frac{V_*}{\sqrt{(gD(s-1))}}$$

and for coarse sediments

$$F_{cg} = \frac{V}{\sqrt{(gD\ (s-1))}\ \sqrt{(32)}\ \log_{10}\ (10d/D)}$$

White et al (1980) demonstrated that alluvial friction in the lower regime may be described by a sequence of straight lines of F_{gr} against F_{fg} with a different linear relationship for each sediment size see Fig 3. White et al (1987) demonstrated that for bed-features in upper regime the alluvial friction could be described also in terms of a relationship between F_{gr} and F_{fg} but that in this case the relationship was represented by a curve for each sediment size rather than a straight line. For a given sediment size the curves for lower and upper regimes do not intersect and so a transition region is required between these two regimes see Fig 4.

3. EXPERIMENTAL INVESTIGATION

To provide data within the transition region a series of experiments was carried out at HR, Wallingford. The experiments were performed using a 24m long, tilting, re-circulating flume. The width of the flume was 0.917m and the length of the measuring section was 10.7m. The sediment used in the flume was a medium coarse sand with a D_{50} value of 0.76mm, the value of $\frac{D_{84}}{D_{16}}$ was 1.29. A sandlayer

150mm thick was placed on the flume bed. Each test was run to equilibrium which, depending upon the flow conditions, took from 0.5 to 5.5 hours. A flat-V weir was used to measure the discharge through the flume. The water surface level was measured using five tapping points connected to stilling pots and using vernier point-gauges reading to an accuracy of 0.02mm.

The experimental results are summarised in Table 1. The discharge varied between 22.5 and 135 l/s, the depth from 0.08 to 0.14m and the slope from 0.11 x 10^{-3} to 1.07 x 10^{-2}. A total of 44 tests were performed out of which 20 were in the transition region.

One suggestion had been that the transition zone comprised an essentially unsteady domain in that flows in the transition zone would not be stable and would progress in time either to the lower or upper regime solution. The experiments indicated, however, that flows in the transition zone are indeed stable and can be maintained, without change, for a considerable period of time.

4. DESCRIPTION OF THE TRANSITION REGION

All the experimental evidence suggests that, for a given sediment size, in both the upper and lower regimes there is a unique relationship between F_{gf} and F_{fg} so that the relationship can be described in the form $f(F_{gr}, F_{fg}, D_{gr}) = 0$

The experiments indicated that it was possible to obtain stable flows which lay between the two curves. Thus the transition flows do not represent an unstable transitory regime but flows which can persist through time. The transition flows do not appear to be restricted to particular paths between the lower and upper regime curves. This suggests that, in the transistion zone, instead of having a single curve, the transition seems to consist of a family of curves filling an area.

Experimental evidence provides information on flows within this transition region but this must be interpreted with care as the way in which the experiments were performed may influence the path taken to cross the transition region. For example, if experiments are carried out in a flume with a closed flow, re-circulation system then there is a strong constraint on the depth of flow. The results therefore traverse a path of almost constant depth. If the system is constrained in another way, however, for example as may occur in a natural river, then presumably the transition zone will be traversed by a different path. How the transition zone is crossed in particular circumstance, therefore, will depend upon the constraints imposed on the system.

The existence of a family of curves which fill a region suggest that an extra parameter is involved in this area and that a unique path through the transition zone will only be defined by specifying a further constraint in terms of the extra parameter. It is therefore suggested that in the transition zone the alluvial friction relationship must be of the form

$f\ (F_{gr},\ F_{fg},\ D_{gr},\ P) = 0,$

where P is some non-dimensional parameter. The definition of P will not be unique and there are a number of possible suitable formulations.

There would appear to be a number of possible choices for the parameter P.

a) d/D : Some authors have done their analysis using the parameter d/D. Curves are provided for values of d/D upto 2,000 and, for example, different curves are given for values of d/D of 1,500 and 2,000. It seems unlikely that the macro features of the flow and physical processes are indeed sensitive to relatively small changes in large values of d/D. It would seem, therefore, that the parameter d/D is acting as a surrogate for some other move physically relevant parameter. Contours of constant d/D values for the transition zone for the experimental results from Section 3 are plotted in Figure 5.

b) $V/\sqrt{gD}$: This parameter varies over a range of approximately 5 to 40 for the available data. It provides a parametresation for the transition zone. Contours of constant $V/\sqrt{gD}$ values for the transition zone for the experimental results from Section 3 are plotted in Figure 6.

c) White et al (1987) characterised the transition region using the parameter U . They argued that the transition region could be defined by the parameter U_E, where

$$U_E = \frac{VS}{(gv)^{1/3} D_{gr}}$$

If $U_E < 0.011$ then the flow is in lower regime. If $U_E > 0.025$ then the flow is in upper regime and the transition zone is given by

$$0.011 < U_E < 0.025$$

Having used the parameter U_E to define the transition region then it seems most natural to use this parameter to describe the transition region. Thus this parameter is used in the rest of this report Contours of constant U_E for the transition zone for the experimental results from Section 3 are plotted in Figure 7.

This list of parameters is by no means exclusive and it may be that there are other, more suitable parameters, to describe the transition zone.

It would appear that the transition zone can be parameterised using U_E as a parameter. What this implies is that in the lower or upper regime it is sufficient to specify D and V_* in order to determine the velocity V. In the transition zone the values of D and V_* only determine a range of possible values of V and in order to determine V a further parameter must be specified. This can be done explicitly using U_E, or by using the definition of U_E it can be done implicitly through a value of S or d. Thus by formulating any constraint on the system then the path of the system through the transition zone may be determined.

5. PARAMETERISATION OF TRANSITION REGION

It has been suggested that the transition zone be parameterised using U_E as a parameter. As the transition zone varies with D_{gr} it follows that this parameterisation must vary with D_{gr}. For those values of D_{gr} for which there is experimental data available it is possible to fit equations to the data and so to parameterise the region. For those values of D_{gr} for which no data is available, however, it is possible to develop an approximate set of appropriate curves.

If the lower regime curves are denoted by

$$\frac{(F_{gr} - A)}{(F_{fg} - A)} = \phi_L \ (D_{gr}),$$

where $\phi_L = 1 - 0.76 \{1 - \frac{1}{\exp[(\log_{10} D_{gr})^{1.7}]}\}$

and the upper regime curves are denoted by

$$\frac{(F_{gr} - A) + 0.07 \ (F_{gr} - A)^4}{(F_{fg} - A)} = \phi_u (D_{gr})$$

where $\phi_u = 1.07 - 0.18 \log_{10} D_{gr}$. Then a set of curves which cover the transition area between the lower and upper regimes is

$$\frac{(F_{gr} - A) + (\frac{U_E - 0.011}{0.14})(F_{gr} - A)^4}{(F_{fg} - A)} = \frac{0.025 - U_E}{0.014}\phi_L + \frac{U_E - 0.011}{0.014}\phi_u$$

In the absence of experimental data these curves will enable approximate calculations to be carried out in the transition zone.

6. CONCLUSIONS

1) Experiments have been carried out which cover the transition zone between lower and upper flow regime.

2) The experimental data suggests that the transition zone consists of a family of curves that fill the area and that the trajectory of a flow through the transition zone depends upon the constraints applied.

3) To parameterise the transition zone it is suggested that use is made of the parameter U_E. This parameterisation should be based on experimental data but in the absence of such data an approximate parameterisation is provided. This should enable the calculation of flows within the transition zone.

4. Equations to describe the transition zone are given to enable calculations to be performed in this zone.

7. REFERENCES

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Tables

Table 1 Experimental results (D_{15} = 0.68mm, D_{50} = 0.76mm)

Run	Discharge	Depth	Slope	Temperature	duration of test	bed forms*	bed features		
	l/s	m	$x10^3$	°C	hr		Length (cm)	Height (cm)	Speeds (cm)
1	22.5	0.115	0.111	16.4	2.5	1			
2	30.5	0.125	0.165	16.7	1.5	1			
3	39.8	0.127	0.235	16.5	3.0	1			
4	39.8	0.113	0.422	17.0	3.0	3	60	3.5	1.01
5	41.8	0.109	1.373	16.5	5.2	3	80	4.0	1.33
6	44.8	0.113	1.669	17.9	3.5	3	100	4.5	1.83
7	50.3	0.118	1.993	17.3	3.0	3	100	5.0	3.88
8	53.3	0.113	2.883	17.7	3.8	3	100	5.0	7.67
9	61.3	0.122	3.224	17.7	3.5	3	100	5.5	11.00
10	51.5	0.100	4.005	17.8	2.0	3	100	5.5	11.50
11	68.3	0.127	3.325	17.1	3.0	3	100	6.0	12.50
12	72.3	0.127	4.122	18.0	2.5	3	110	6.5	15.67
13	77.3	0.132	4.446	17.5	3.0	3	120	7.0	17.67
14	82.3	0.135	4.767	16.4	3.0	3	120	7.0	19.17
15	86.8	0.135	5.101	16.5	2.0	3	140	7.0	21.67
16	92.7	0.140	5.243	16.3	3.0	4	100	5.0	25.00
17	96.3	0.140	5.513	16.7	1.3	4	100	5.0	27.50
18	101.3	0.143	5.701	17.1	1.2	4	90	3.5	45.00

Run	Discharge	Depth	Slope	Temperature	duration of test	bed forms*	bed features		
	l/s	m	$x10^3$	°C	hr		Length (cm)	Height (cm)	Speeds (cm)
19	106.8	0.139	5.990	16.7	1.0	4			
20	113.3	0.139	6.310	17.0	1.0	4			
21	115.3	0.136	6.069	16.3	1.0	4			
22	126.3	0.131	6.285	16.0	2.0	4			
23	80.0	0.124	5.473	15.8	1.5	4			
24	80.0	0.120	5.900	15.7	2.7	4			
25	96.3	0.121	5.753	15.2	2.0	4			
26	102.3	0.120	5.858	14.8	1.7	4			
27	108.3	0.124	5.689	14.7	2.3	4			
28	117.3	0.122	6.070	16.3	1.2	4			
29	118.3	0.118	5.951	15.3	1.5	4			
30	56.3	0.085	7.110	18.6	3.0	4			
31	64.5	0.090	6.921	17.8	2.0	4			
32	82.3	0.095	6.423	17.1	2.6	4			
33	78.9	0.085	7.143	18.1	2.5	4			
34	82.3	0.083	7.497	17.1	1.5	4			
35	85.0	0.087	7.270	15.7	1.3	4			
36	89.0	0.087	7.280	15.9	1.3	5			

Run	Discharge	Depth	Slope	Temperature	duration of test	bed forms*	bed features		
	l/s	m	x10^3	°C	hr		Length (cm)	Height (cm)	Speeds (cm)
37	85.3	0.080	8.675	15.1	1.0	7			
38	94.8	0.090	7.870	15.2	1.0	7			
39	125.3	0.108	6.898	16.7	1.0	5			
40	119.3	0.103	7.634	16.5	0.8	7			
41	125.3	0.105	7.884	16.4	1.0	7			
42	125.3	0.103	8.530	16.5	0.5	7			
43	135.0	0.110	8.241	14.8	0.5	7			
44	107.0	0.085	10.730	14.8	0.5	7			

* 1 - Flat bed (lower regime), 3 - dunes, 4 - reduced dunes, unstable (transition regime), 5 - flat bed (upper regime), 7 - antidunes

Figures

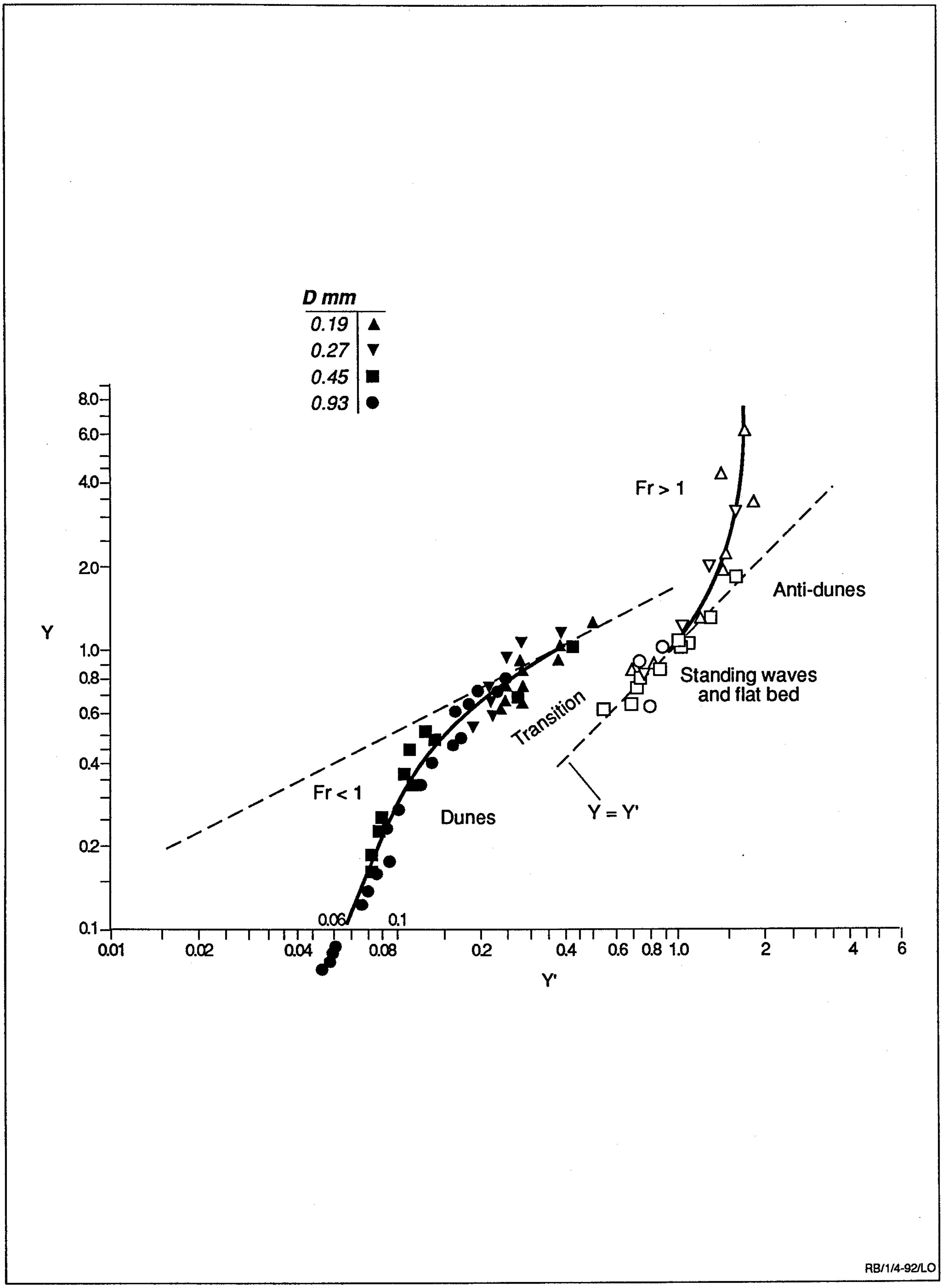

Figure 1 Mobility against effective mobility, after Engelund

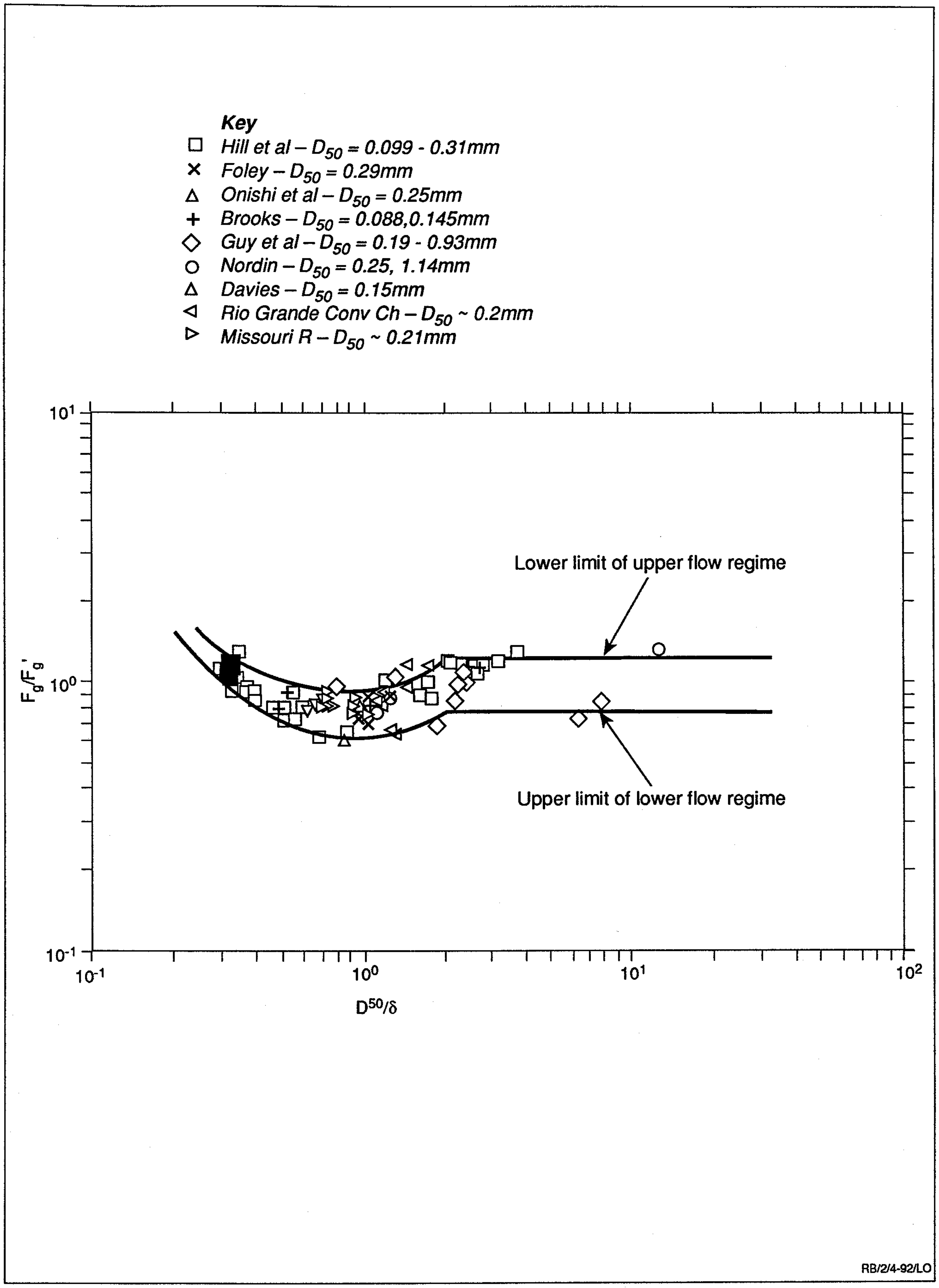

Figure 2 F_g/F_g' against D_{50}/δ, after Brownlie

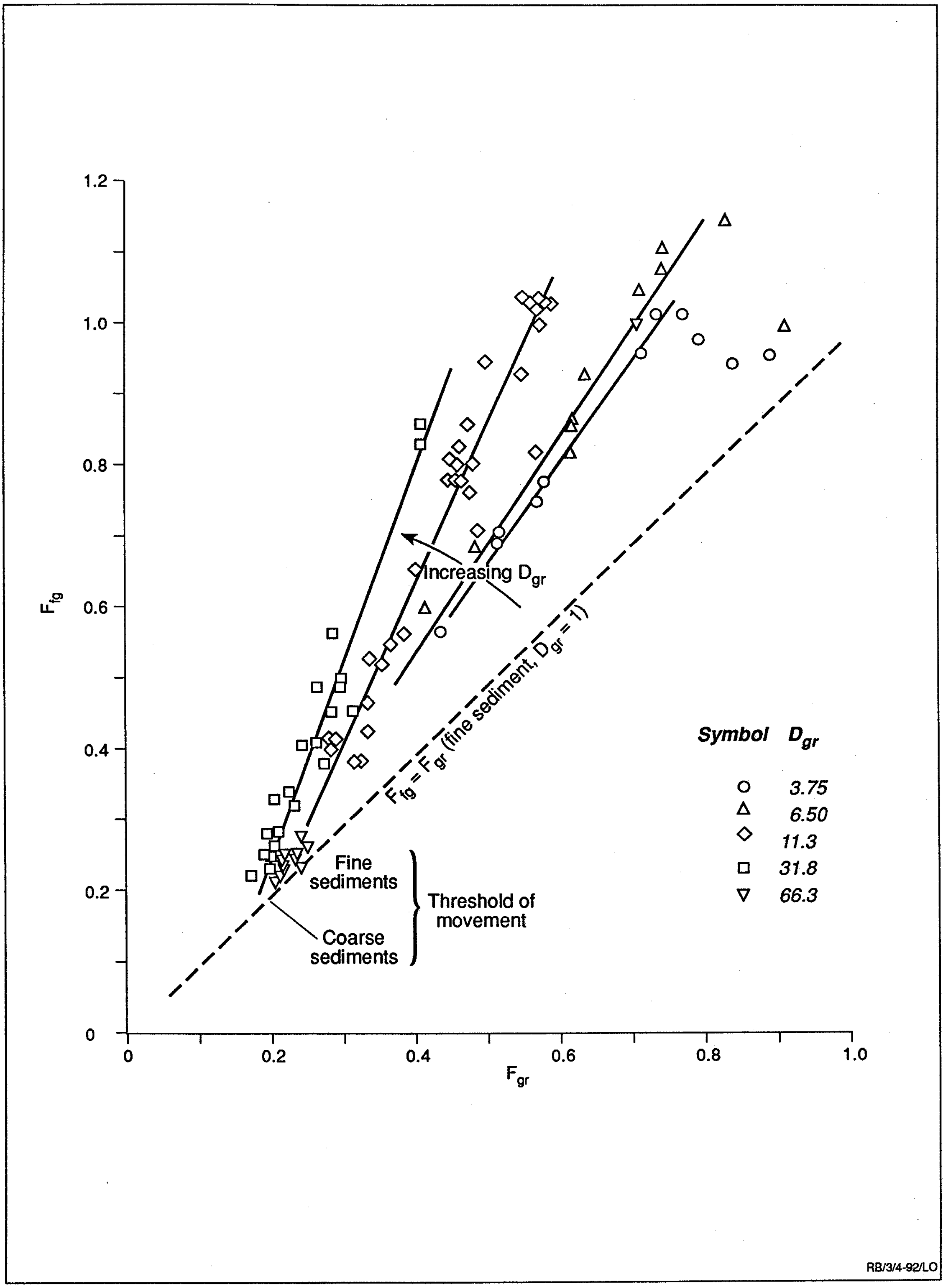

Figure 3 F_{fg} against F_{gr} for lower regime

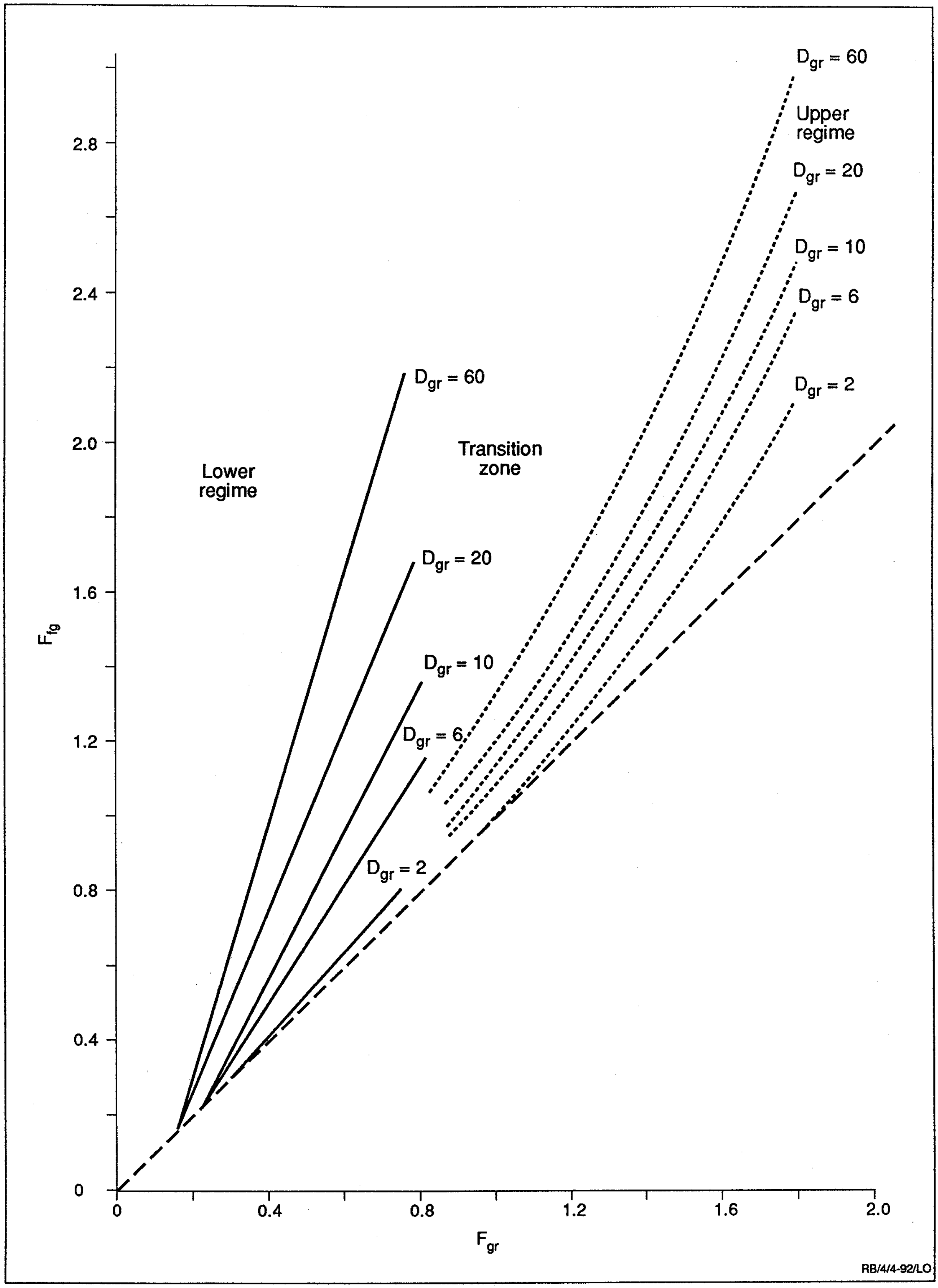

Figure 4 F_{fg} against F_{gr} for upper and lower regimes

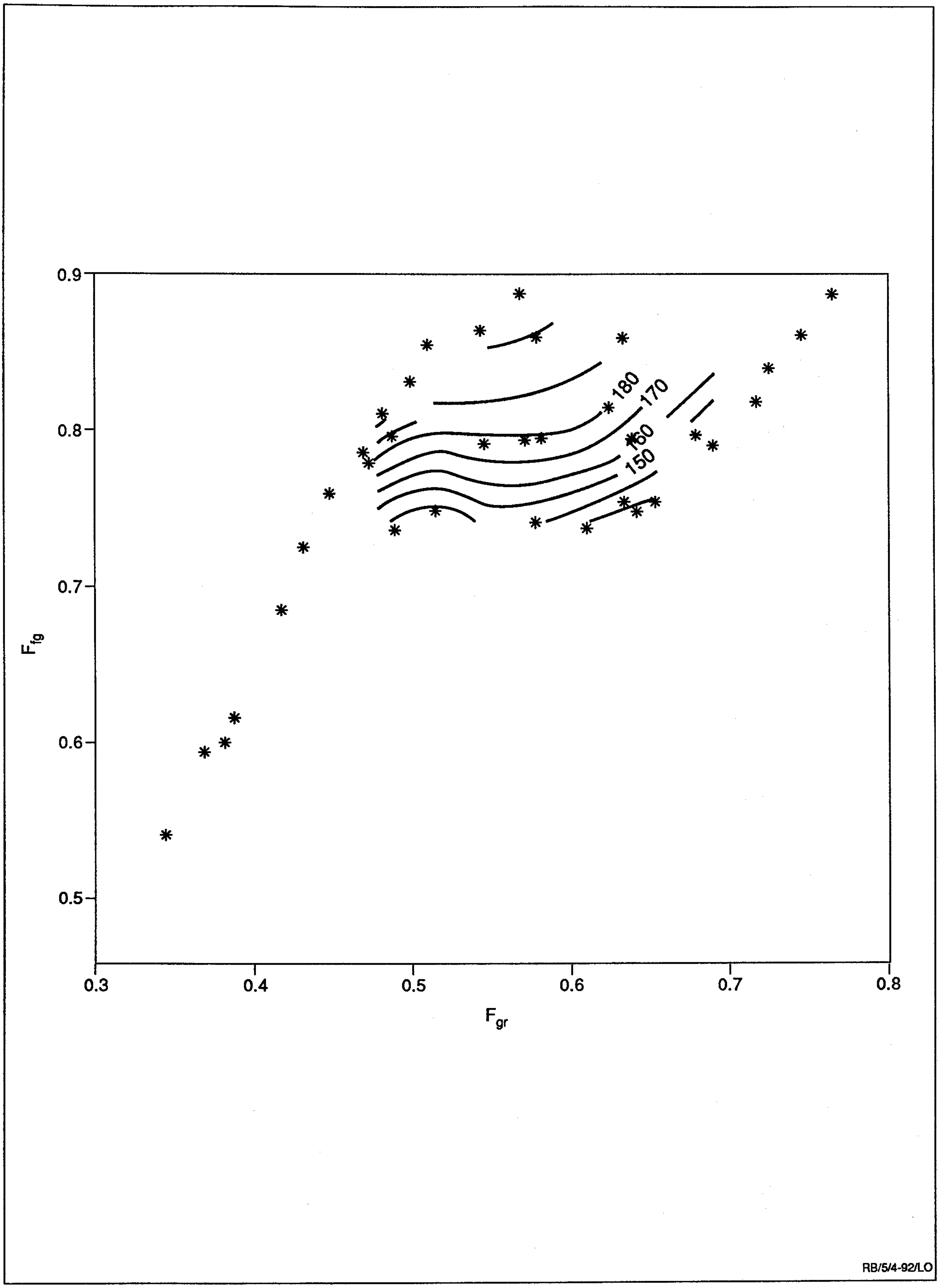

Figure 5 F_{fg} against F_{gr} with contours of constant d/D

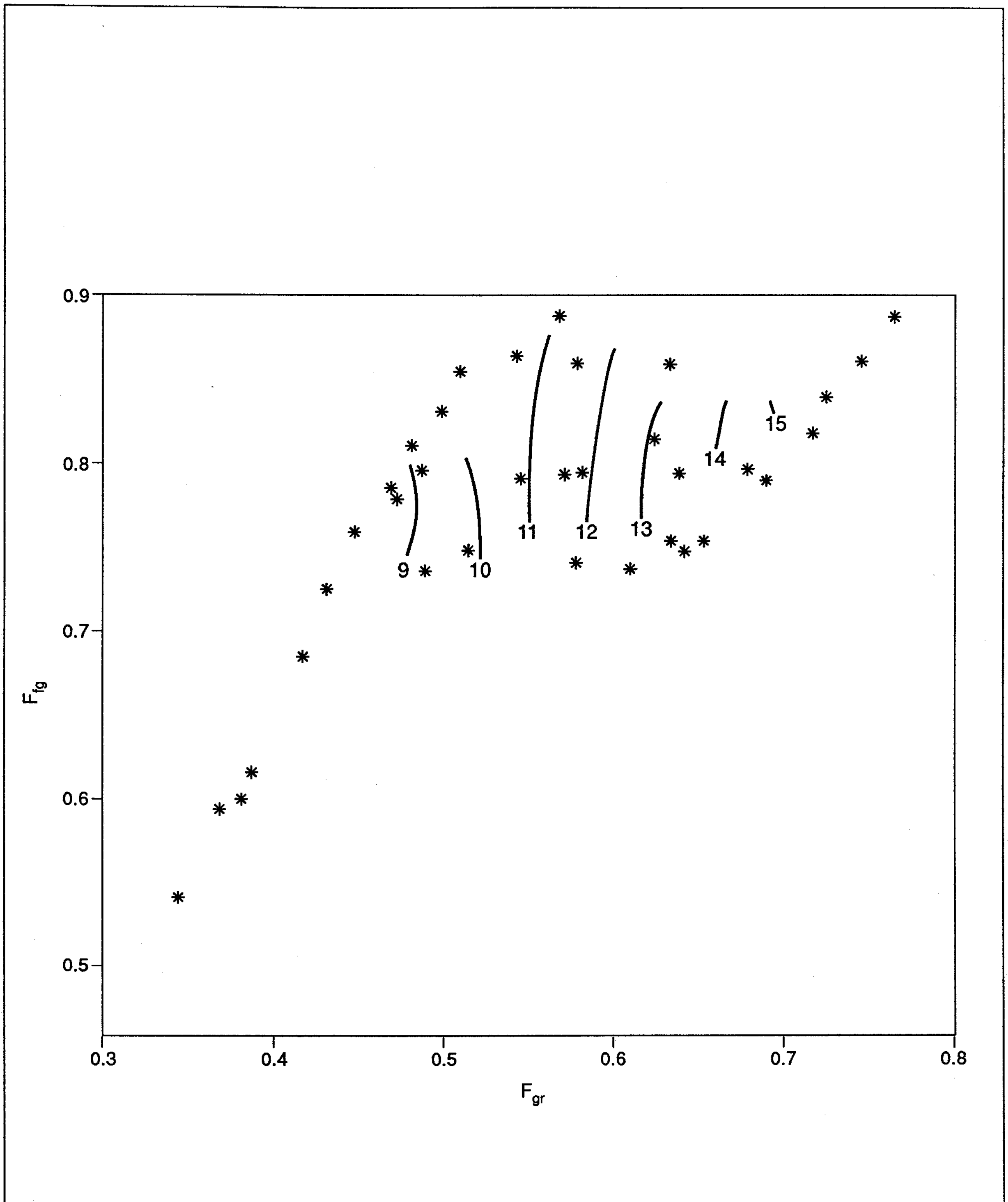

Figure 6 F_{fg} against F_{gr} with contours of constant $V/\sqrt{gD}$

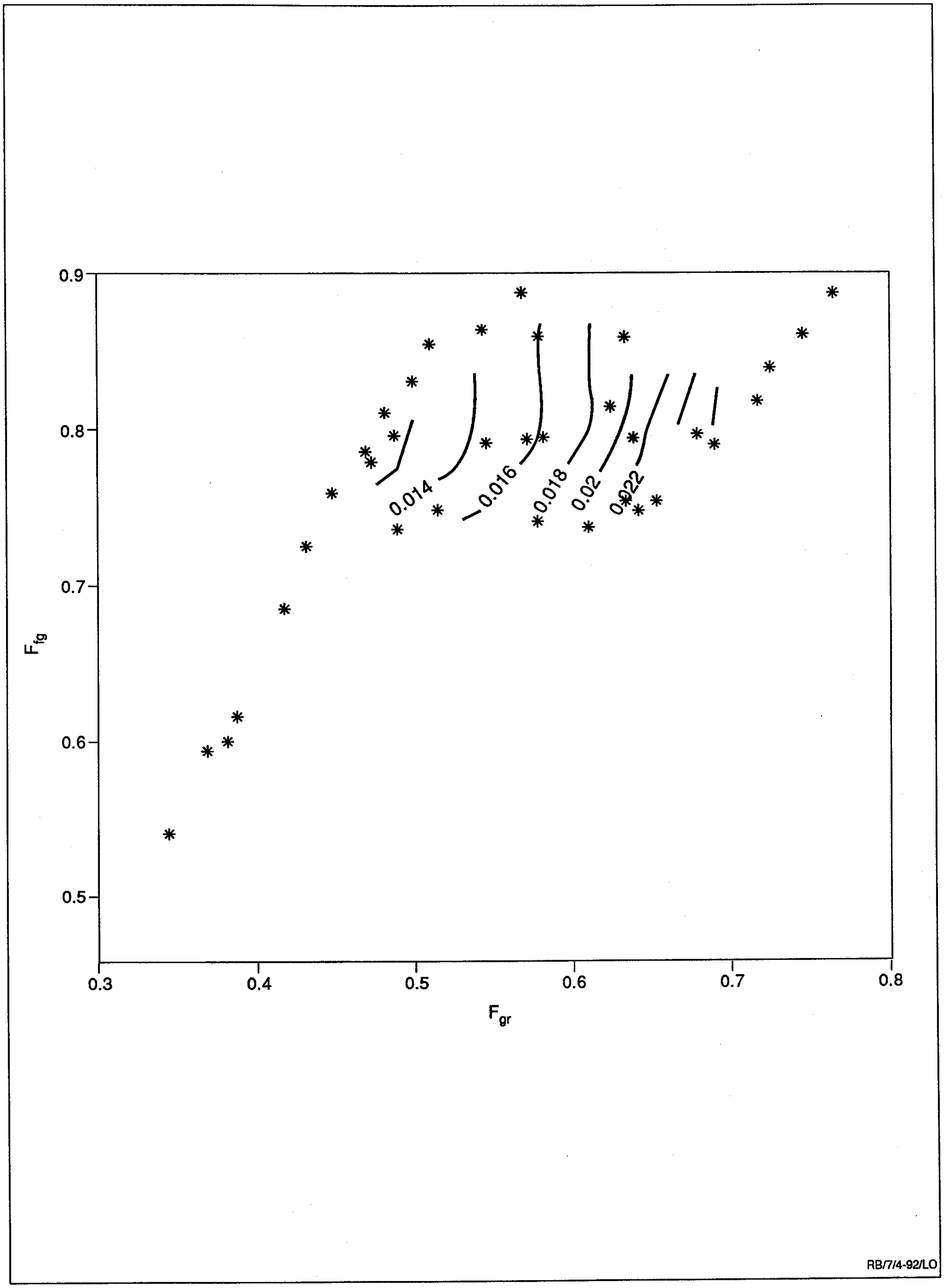

Figure 7 F_{fg} against F_{gr} with contours of constant U_E