Estuary Regime

Part 1: Numerical Modelling

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Report SR 366
September 1993

HR Wallingford

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Contract

This report describes work funded by the Department of the Environment under Research Contract PECD 7/6/234, for which the DoE nominated officer was Mr P Woodhead and by the Energy Technology Support Unit for the Department of Trade and Industry under Agreement E/5A/4109/2589, for which the ETSU project manager was Dr L M Davies. It is published on behalf of the Department of the Environment, but any opinions expressed in this report are not necessarily those of the funding Department. The work was carried out at HR Wallingford by Mr M N H Waller and Dr M P Dearnaley in the Ports and Harbours Group under the management of Dr J V Smallman

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Date 3/12/93

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1 Introduction

1.1 Background

The morphological regime state of an estuary is a dynamic equilibrium between accretion and erosion where the balance tends to occur over a number of cyclic periods, such as tidal cycles, seasons or even a number of years.

A complete numerical representation of estuarine behaviour can only be achieved by the use of a three-dimensional approach. However, there are a large number of problems associated with attempting to model all the complex phenomena inherent in tidal waterways both from a theoretical point of view and a computational point of view. Therefore, in order to progress towards a greater understanding of the long term regime state of estuaries, it is necessary to make a number of assumptions, generalisations and simplifications. This means that a purely theoretical approach is not sufficient but needs to be complimented by a wise choice of empirical simplifications.

1.2 Objectives

The primary aim of the research into estuary regime is to provide engineers with an improved method of predicting the long term evolutionary effects of significant engineering changes in estuaries. This objective is to be met by:-

(i) Refining the regime model developed in the previous studies (Reference 1), and extending the modelling undertaken as part of the earlier work to incorporate such effects as non-uniform tides, salinity and varying freshwater flow.

(ii) Using regime algorithms to identify the key factors that influence estuary morphology. This analysis is to be undertaken within the framework of a knowledge based system.

(iii) Exploring the validity or practicality of applying chaos theory to an estuarine situation.

The results of this study are presented in three separate reports. Item (i) is addressed here whilst items (ii) and (iii) are reported in References 2 and 3 respectively.

1.3 Programme

The study directed towards item (i) above was divided into three main areas which evolved through the study:

- Analytical approach
- Iterative approach
- REGIME modelling development

The approach used in this work was to start by considering a one-dimensional flow model as in the earlier study (Reference 1) and then to incorporate sedimentation processes and bathymetry evolution.

Two approaches have been used for the flow models. The first is the analytical approach which has lead to the production of two models specifically for this work. The second approach was to use an iterative numerical model.

The REGIME modelling technique was then developed to predict the long term regime state of an estuary. This method incorporates a one-dimensional flow model into a system which represents the sedimentation processes. REGIME simulates the evolution of the cross sectional bed profiles of an estuary given a series of input climatic conditions (in the form of river discharges into the estuary and tidal oscillations at the mouth) which are representative of a given period of time and any bathymetric restrictions that may be imposed by natural conditions or by engineering constructions.

1.4 Report structure

The remainder of this report is in six chapters. Chapter 2 describes the estuaries and available field data which have been examined in this study. Chapter 3 describes the theory behind the analytical Reflected Wave Model and Chapter 4 presents the results obtained from it. Chapter 5 briefly describes the iterative numerical model SALMON-Q and the results of the work with this model. The REGIME modelling technique is detailed in Chapter 6. The conclusions and recommendations arising from this report are summarised in Chapter 7.

2 *Estuaries studied*

Five UK estuaries have been studied in this report: the Thames, the Nene, the Conwy, the Parrett and the Dee. They were chosen so that they could be adequately represented by a one-dimensional hydrodynamic model while at the same time presenting a variety of different characteristics from a tidal point of view.

2.1 The Thames

The Thames is considered to be close to an 'ideal' estuary. It displays a very regular cross-sectional evolution. The width can be remarkably well described by an exponential function of distance. The depths vary very little from the mouth to Upper Pool (70km upstream). The tidal oscillations are practically a sinusoidal function of time.

For this study, the Thames is modelled from its mouth at Southend up to the tidal limit at Teddington almost 100km upstream. The weir at Teddington prevents tidal propagation further upstream. The model bathymetry is defined by surveyed cross-sections taken from 13 fairly evenly spaced positions along the estuary, including Southend and Teddington as listed in Table 2.1.

Table 2.1 Thames Estuary Section/Chainage

Cross-section name	u/s chainage from mouth
Southend	0 km
Thameshaven	15 km
Gravesend Reach	26 km
Long Reach	38 km
Halfway Reach	47 km
Barking Reach	51 km
Woolwich	56 km
Limehouse Reach	65 km
Upper Pool	70 km
Chelsea Reach	77 km
Corney Reach	87 km
Syon Reach	93 km
Teddington Weir	99 km

There is a large amount of simultaneous recorded field data available for the Thames. For the purposes of this study, four tides (NT, ST, NP, SP) have been drawn from Reference 4.

NT - is the neap tide which occurred on the 30/9/68 during a period of high river discharge (130 m^3/s);

ST - is the corresponding spring tide which occurred on the 24/9/68 with a river discharge of 84 m^3/s;

NP - is the neap tide which occurred on the 6/10/69 during a period of low river flow (11 m^3/s);

SP - is the corresponding spring tide which occurred on the 26/9/69 with a river discharge of 13 m^3/s.

Each set of observations consists of depths and velocities recorded at half hour intervals throughout the tidal cycle at the 13 positions where the surveyed cross-sections were measured.

2.2 The Nene

The tidal reach of the river Nene is a narrow, artificial channel about 40km long with very few bends which leads to the Wash. The maximum tidal range is about 7m at the mouth and 2.5m at Dog-in-a-Doublet where the tidal limit is imposed by sluices. The cross-sections tend to be very regular with 1:3 side slopes and a flat bed consisting of clean sand.

The data for this study has been drawn from Reference 5. Cross-sections were surveyed at 13 positions along the length of the Nene as listed in Table 2.2. However, water level and velocity measurements were only taken at four positions along the length of the Nene. The names and chainages of these positions are given in the table below.

Table 2.2 Nene Estuary Section/Chainage

Cross-section name	u/s chainage from mouth
Twin Lighthouses	0 km
South Holland Drain	6.5 km
Wisbech Quay	16 km
Guyhirne Road Bridge	26 km

Simultaneous measurements were recorded on two separate occasions: the first set, corresponding to a neap tide, was on 17 September 1964, and the second set is from 23 September 1964 and corresponds to a spring tide.

2.3 The Conwy

The Conwy estuary displays a number of complications compared to the other examples. Its shape is considerably constrained: geologically by the Deganwy narrows at the mouth, and by a number of engineering constructions further upstream. As well as this, the tidal ranges are very large compared to the low water depths.

The tidal limit on the Conwy estuary is about 20km upstream from the Deganwy narrows at the Tan-lan road bridge. Over this distance, 9 cross-sections have been used in the analytical modelling. Since the tidal cross sections for the Conwy are not shown here since the Conwy Estuary was not used for iterative modelling.

There is only one complete set of depth observations over a whole tide. This is for the spring tide of 21 July 1978. There is also some data for an average tide.

Simultaneous velocity observations have only been made at 4 of the sections at hourly intervals for a river flow of 27.1 cumecs.

2.4 The Parrett

The Parrett estuary downstream of Bridgwater follows a meandering course through reclaimed land to the mouth at Stert Point. The estuary tends to be very dynamic for a few hours either side of high water and then relatively tranquil for a long period as the ebb tide runs out. The sedimentary regime is extremely dynamic, but the mechanisms of change are broadly balanced so that on average over a period of time the dimensions of the estuary will remain approximately constant.

Being situated within the Bristol Channel, Stert Point is subject to extremely large tidal ranges of over 11m during spring tides and about 4m during neap tides. Over most of the 18.8km downstream of Bridgwater, the water at low tide is restricted to small channels within the river bed.

The Parrett is joined at Burrowbridge by a tributary, the Tone, at 28.8km upstream from the mouth. This is downstream of the tidal limit, so the estuary effectively has two tidal limits: one at 38.5km from the mouth on the river Tone at Knapp Bridge, and the other tidal limit at 33km from the mouth on the river Parrett at Oath Lock.

Table 2.3 Parret Estuary Section/Chainage

Cross-section name	u/s chainage from mouth
Stert Point	0 km
Black Rock	2.5 km
Barge	6.3 km
Marchants	9.7 km
Cottages	11.8 km
Pimms	15.8 km
Bridgwater	18.8 km
Pipe Bridge	26.3 km
Burrowbridge	28.8 km
Oath Lock*	33.0 km
Knapp Bridge**	38.5 km

* On Parrett tributary
** On Tone tributary

A comprehensive hydraulic survey was carried out for the Parrett Barrier study in October 1977. This consists of simultaneous mid-channel observations at the first 7 chainages shown in Table 2.3 for one complete neap and one complete spring tide. This unfortunately only covers the stretch of the estuary between the mouth at Stert Point and Bridgwater.

The observations for tidal depths can be assumed to be reasonably accurate, but velocity observations (References 6 and 7) are far more difficult to obtain reliably since the velocities vary throughout the whole width and depth at a given distance from the head of the estuary.

The same 1977 survey also describes the bathymetry with great accuracy, giving cross-sections about every kilometre.

2.5 The Dee

The most extensive survey available which covers the whole Dee estuary was carried out in 1965 by the Hydraulics Research Station (now HR Wallingford) and is described in Reference 8. It was therefore decided to use the 1965 configuration of the estuary and for the sake of consistency to use only measurements taken from this survey for the purposes of this study.

The Dee estuary in Cheshire drains into Liverpool bay, and comprises three sections of quite different character.

Table 2.4 Dee Estuary Section/Chainage

Cross-section name	u/s chainage from mouth
Hilbre Island	0 km
Mostyn Quay	1.4 km
Mostyn	2.8 km
Caldy	4.2 km
Thurstaston	5.6 km
Greenfield	7.0 km
Holywell	8.4 km
Bagillt	9.8 km
Flint	12.6 km
Connah's Quay - John Summer's Jetty	18.5km
Connah's Quay	19.5 km
Hawarden Bridge	21.3 km
Queensferry Bridge	22.7 km
West Saltney	24.0 km
East Saltney	25.3 km
Cop Farm	26.8 km
Saltney Ferry	27.9 km
Saltney	29.4 km
Golf course	30.7 km
Railway Bridge	32.0 km
Chester Weir	33.4 km

The larger, outer portion from Hilbre Island to Flint is of a regular tapering shape and about 16km long. It is free from any man-made restraints on the lateral movement of the tidal channels which are constantly evolving and changing their paths (See Reference 9).

Between Flint and Connah's Quay, the estuary narrows considerably from 5.5km in width to only 0.5km in width. Along this stretch there are a number of man-made restrictions in the form of training banks.

Upstream of Connah's Quay, the flows are restricted to a narrow artificial channel through reclaimed land for about 15km up to the tidal limit which is at Chester Weir for most conditions.

There are a total of 21 available cross-sections (Table 2.4). Most of them are fairly evenly spaced at about 1.5km. But there is a significant gap of almost 6km between Flint and Connah's Quay where no cross-sections were specified. This is unfortunate since it corresponds to an area of significant bathymetry variation between the larger outer section of the estuary and the man-made channel which forms the upper end. The sections are described in Table 2.4.

3 Reflected Wave Model theory

3.1 Reasons for an analytical model

The aim of this work is to establish the long term regime state of an estuary, and thus to be able to determine the effect of varying certain initial conditions on the final results. It is of limited interest to iterate through the whole sequence of events which leads to the final regime state. It is therefore appropriate to investigate the production of an analytical model which estimates the regime state characteristics of an estuary directly from the initial parameters without going through the whole evolution which results in this final state.

From a computational point of view, this would have all the traditional advantages of an analytical solution over an iterative solution. That is to say that it would not be subject to the danger of instabilities or other convergence problems which can be encountered with iterative models. The accuracy of the results would not be dependent on having short timesteps or on carrying out runs over very long periods of time. An analytical solution would hence be expected to be far more economical in terms of computing resources.

3.2 Previous work

A tidal volume model has been developed and described in Reference 1. This model was developed in three stages.

The first stage was to predict low water depths once the tidal behaviour and the cross-sections had been described along the whole length of an estuary. The calculations were based on the observation that the maximum bed shear stress appears to be constant along the whole length of an estuary. Good results were obtained at this stage for the Thames Estuary.

The second stage was designed to avoid specifying the cross-sections along the whole length of the estuary. Instead, the widths were assumed to increase exponentially downstream. This second stage once again led to fairly good results on the Thames. However, this method is not universally applicable, but is restricted to estuaries for which the evolution of width downstream can be approximated by an exponential function. This is a good approximation in the case of the Thames but for most estuaries it is not applicable.

The final stage in this model development was to remove the need to specify the tidal behaviour along the whole length of the estuary, and instead just to prescribe it at the mouth. In the case of the Thames, this is a relatively straightforward operation since the tidal disturbance behaves virtually as a progressive sinusoidal wave travelling upstream in deep, frictionless water.

In summary the previous analytical model gave good results for the Thames, but it had the major disadvantage of being in a very site-specific form. It also failed to incorporate a physical representation of the effects of factors such as friction and bathymetry on the propagation of the tidal wave. Thus the model cannot directly simulate the effects of any changes to the estuary. It is therefore necessary to approach this problem in a more general manner in order to obtain a more satisfactory, widely applicable model.

3.3 Description of the Reflected Wave Model

The new analytical model presented in this report is based on representing a tide as a single sinusoidal wave propagating into an estuary. It has been observed in certain cases that the reflection of the tidal wave off the head of an estuary can be of considerable importance, even leading sometimes, as in the case of the Severn, to a resonance effect, hence increasing tidal ranges considerably. Hence it was decided to incorporate the reflected wave into this analytical model.

The amplitude of the model tidal wave is determined by two factors: a damping coefficient, μ, and any variations in the cross-sectional areas.

The linear damping coefficient, μ, takes into account a number of factors such as friction, turbulence, planform and cross-sectional geometry. It varies along the whole length of the estuary and is calculated automatically by the model.

The effects of variations in cross-sectional areas on tidal amplitudes are incorporated into the model by means of Green's Law. For example a reduction in the channel width in the upstream direction tends to increase the amplitude of the tidal oscillations as the tide progresses up the estuary.

The water level calculations are entirely analytical, but once they have been established, the tidal volume calculations are carried out in the same way as for the previous model (Reference 1) and are essentially an expression of the continuity equation. They therefore depend on a very simple first order finite difference scheme which has no stability or convergence problems.

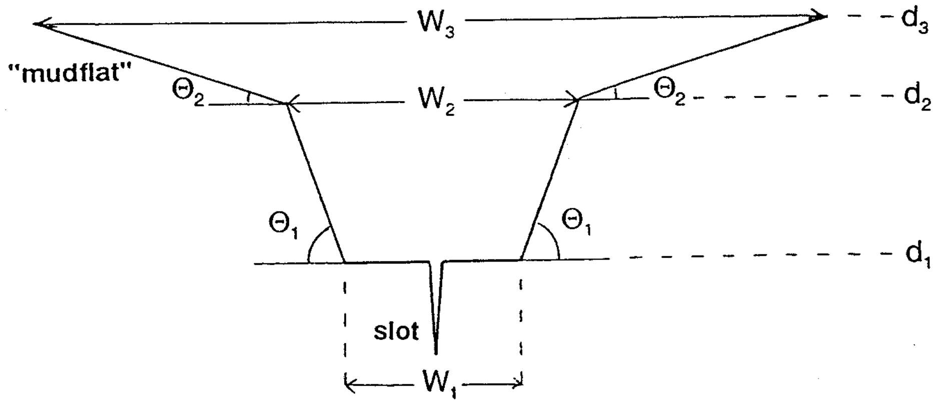
The Reflected Wave Model has a number of advantages over the previous model described in Section 3.2. It incorporates a number of additional physical processes: the reflection of the tidal wave off the estuary head, friction and the effects of the geographical evolution of the cross-sectional area; but the main advantage is that it can be used for a much wider range of estuaries.

3.4 Mathematical formulation

3.4.1 Setting up the model

The horizontal x-axis is taken along the length of the estuary orientated upstream, with the origin (x=0) taken at the head. This means that x is negative between the estuary head and its mouth. The cross-sectional shape of the estuary is specified at N+1 positions x_i along the x-axis, where $x_0=0$ is at the head and x_N at the mouth. The tidal volume calculations assume linear interpolation of the cross-sections between two adjacent positions. Therefore the optimal number of positions with which to describe the cross-sectional geometry depends very much on the regularity of the estuary bathymetry.

Each cross-section is determined by three different width values and the depths at which these occur. So the cross-section is effectively represented by two trapeziums, the base of the upper one being the top of the lower one. This enables the representation of a trapezoidal cross-section with associated 'mud flats'.



Once the estuary has been described, a calibration tide must be defined. This is simply characterised by the tidal ranges and the phase delay for each of the x_i positions.

Under certain conditions at particular locations along the longitudinal profile the model may give negative values for depth since the water levels are constrained into following a sinusoidal curve. In order to overcome this problem, slots can been inserted in the bottom of the bed at these positions. These represent channels on the bed to which the water may be confined if the water level drops exceptionally low. They have very little effect on the results other than insuring that the model runs properly, since the influence of the slots on the tidal volumes is very small. The only circumstances under which the results depend significantly on the depth of the slot are when the water is restricted to one of these slots. Under these conditions, the velocity at the corresponding position and time depend directly on the depth and width of the slot which are arbitrary. This represents however in reality a period in the tide where the water is restricted to shallow channels on the estuary bed, in which case velocity measurements would be very sensitive to the location at which they are taken and hence of lesser value for the purposes of this study.

3.4.2 The depth calculations

The surface deflection from the mean depth is given by the sum of the effects of incident wave and reflected wave:

$$\eta(x,t) = \eta_i(x,t) + \eta_r(x,t)$$

where

$$\eta_i(x,t) = \frac{A_0}{2} H(x) e^{-\mu x} \cos(\sigma t - kx)$$

$$\eta_r(x,t) = \frac{A_0}{2} H(x) e^{\mu x} \cos(\sigma t + kx)$$

where A_0 is the tidal amplitude at the head, H(x) represents the variations of amplitude due to Green's Law, and the damping is given by the exponential term. In the above equations, the expressions *kx* and *μx* are used very loosely, since in fact they are only piecewise linear functions for which the values at x_j should be written:

$$kx_j = \sum_{i=1}^{j} k_i (x_i - x_{i-1})$$

$$\mu x_j = \sum_{i=1}^{j} \mu_i (x_i - x_{i-1})$$

H(x) is given by Green's law:

$$H(x) = \sqrt{\frac{w(0)}{w(x)} \sqrt{\frac{d(0)}{d(x)}}}$$

where w(x) and d(x) are respectively the estuary width and depth at position x.

Assuming that the damping coefficient and the wave propagation velocity remain the same throughout the tidal cycle, they can be calculated from the following results:

$$\mu x = \operatorname{arccosh} \sqrt{\frac{N_x + 1 + \sqrt{\Delta_x}}{2}}$$

$$kx = \arccos\sqrt{\frac{N_x+1-\sqrt{\Delta_x}}{2}}$$

where

$$N_x = \left(\frac{A_x}{A_0}\right)^2 \frac{w(x)}{w(0)}\sqrt{\frac{d(x)}{d(0)}}$$

and

$$\Delta_x = (N_x+1)^2 - 4N_x[\cos(\sigma t)]^2$$

The model begins by calculating values for the damping coefficient and for the phase delay throughout the estuary in order to make its predictions fit the calibration tide data. It is worth noting that the incident wave phase delay is in fact not the same as the observed phase delay because the point of maximum amplitude of the incident wave does not necessarily represent high water. This is because it is necessary to take into account the influence of the reflected wave.

Since the available Admiralty data for the phasing of high water along the length of an estuary is not very accurate it was considered appropriate to approximate the phase delay by a linear (or piecewise linear) function, thus smoothing out the irregularities in the data. However, in contrast, the model damping coefficient is calculated from an accurately measured set of observations: the tidal amplitudes. It was thus decided not to carry out any smoothing on the values for damping given by the model.

3.4.3 The tidal volume calculations

Once the depths have been established, the average velocities for each cross-section can be calculated with the following simple scheme:

$$v(x_j,t) = \frac{\left(\dfrac{vol(x_j,t+\delta t)-vol(x_j,t)}{\delta t}\right) - Q}{A(x_j,t)}$$

where *vol(x,t)* represents the tidal volume upstream of the point *x* at time *t*, *Q* is the river flux and *A(x,t)* is the total cross-sectional area at the position *x* at time *t*. This finite difference scheme is extremely stable, given the smooth nature of the time evolution of the upstream volume. Therefore the time step δt does not have to be very small. All the results looked at in this study have used a timestep, δt, of half an hour.

4 Reflected Wave Model Results

In the following sections results from the analytical model are presented for the Thames, Nene, Conwy and Parrett estuaries. The results are presented in the form of a series of sensitivity tests designed to examine the potential of the analytical model for regime work. References to the different estuaries are made within the individual sections.

4.1 Sensitivity of the Thames model to small bathymetry alterations

There are two reasons for carrying out sensitivity tests of the model to small bathymetry changes. The first reason is simply in order to gain a better understanding of the behaviour of the model, but the second, more interesting reason is that this study is looking at the long term evolution of estuaries which involves a certain amount of bathymetry evolution due to erosion and deposition.

The following tests have all been carried out on the Thames using the tide SP. The original bathymetry has been modified by altering the widths of the 13 sections which define the model bathymetry.

The following five modifications have been used:

modification 1 - a 10% increase to the widths at the mouth and then alternate 10% reductions and increases to the widths further upstream.

modification 2 - 10% on the widths at the mouth and alternately +10%, -10% on the upstream widths.

modification 3 - a 10% increase on the widths over the whole length of the estuary.

modification 4 - a 10% decrease on the widths over the whole length of the estuary.

modification 5 - a 10% reduction to the base width and a 10% increase to the mean water level width at the mouth and then alternate reductions and increases to both the base widths and the mean water level widths upstream of Southend.

4.1.1 *Effect on water levels*

Any cross-sectional inaccuracies have very little effect on the water level predictions since the damping coefficient, μ, is modified accordingly. Modelling shows that 10% modifications to the cross-sectional data lead to less than 1% differences in the depths. The largest difference is likely to be in the ratio of the amplitude of the incident wave over that of the reflected wave.

4.1.2 *Effect on the damping coefficient*

In the cases of bathymetry modifications 3 and 4, the constant alteration in widths over the whole length of the estuary leads to no change in the internally calculated damping coefficient. However, modifications which alternately increase and decrease the widths along the estuary by 10% can lead to over

100% alterations to the value of the damping coefficient μ (see Figure 1a). But this is misleading since the damping coefficient does not come into play as such in the model: it only appears under the form of the accumulated damping μx which is a function of x and which in this case varies by no more than 10% (see Figure 1b).

4.1.3 Effect on velocities

The velocity calculations are affected by small cross-sectional modifications in two ways:

(i) The upstream tidal volume for a position x_j is modified by an amount which averages approximately the percentages of the area alterations upstream of it. And the tidal flux through x_j is hence modified by a similar proportion.

(ii) The velocity through a given cross-section is inversely proportional to its area and it is hence directly affected by any cross-sectional modifications.

Therefore if a 10% error bound on the cross-sectional data is allowed, it is possible to incur up to a 10% error on the upstream tidal volume and also a 10% error on the cross-sectional area. If these two errors were both increases or both reductions in cross-sectional area they would cancel each other out. Therefore the maximal velocity error of about 20% can only happen once throughout an estuary since it means that all the upstream errors have to be in one direction and the error at x_j in the opposite direction. So in the majority of cases one would expect the errors to cancel each other out to some extent, giving velocity errors in the region of 10%. This is in fact what has happened in the Thames model (see Figure 1c).

4.2 The damping coefficient, μ

The main reason for examining the damping coefficient given by this model is to determine whether it has any physical significance. That is to say whether it is possible to establish some kind of relationship between this theoretical value μ and values which can be measured in the field (the bed material, the average grain size of bed particles, whether the bed is smooth or rippled, the shape of the cross-section, the shape of the estuary, river flow etc.)

The coefficient μ seems to behave sensibly with modifications in the estuary cross-sections in the sense that it tends to increase when widths are reduced. There are however two reasons which tend to rule out trying to attribute too much physical significance to the value of the damping coefficient given by this model. The first reason is that μ is too sensitive to any slight changes in the bathymetry data, varying by up to 150% for 10% width changes in the Thames model (see Section 4.1.2). The second and most important reason is that in all but one of the examples that have been looked at, the damping coefficient is actually negative over one or two intervals, and it is hard to justify physically such a negative damping.

One possible explanation for obtaining negative values for the damping coefficient is that they are due to inaccuracies in the cross-sectional data or in the depth observations. This hypothesis is supported by the fact that μ is very sensitive to slight changes in this data. However, the value for μ does not appear explicitly in the model, but rather the value, μx, appears which is defined to be the accumulation of the total damping from the head of the estuary at x_0 to the position x_k. That is to say:

$$\mu x(x_k) = \sum_{i=1}^{k} \mu_i(x_i - x_{i-1})$$

When modelling the Thames, a 10% change in μx leads to a similar change in the depth predictions, however the same cannot be said about modelling the Conwy. An attempt to keep μ positive in this case leads to changes in μx of less than 8%, but it also leads to changes in the modelled tidal range of over 90%. These discrepancies can be explained by looking at the equations for the model. In the tidal range calculations, damping enters in the form of the factor $exp(-\mu x)$. So a small modification to μx of value d is equivalent to multiplying the model tidal range by $exp(-d)$. So it is not the percentage of the modification to μx which is important, but its magnitude. It is then relatively simple to estimate the percentage of the alteration to the tidal range that a given modification to μx will cause, since it is given by 100(1-exp(-d)).

It appears that μ will have to be considered as a purely theoretical value which incorporates the effects of a number of physical elements without being directly dependent on them.

4.3 Flow modelling results

4.3.1 Water levels

The model is calibrated from tidal range and relative phase observations taken from a given tide. The representation of maximum and minimum water levels is therefore accurate. However, the accuracy of the model velocity calculations is limited by how well the tidal curve can be represented by the sum of two sinusoidal waves of the same frequency but in opposite directions. In this respect, the Thames is well suited for the Reflected Wave Model (See Figures 2 and 3), whereas the Parrett and the Nene have tidal curves which cannot accurately be represented by a sinusoidal wave (See for example Figures 4 and 5) and therefore the velocities are less faithfully represented.

It should be noted that whilst modelling the Parret, due to the lack of data upstream of Bridgwater and also due to the added complications of incorporating the confluent river Tone in the model and thus having two tidal limits (one on the Tone and one on the Parrett), it was decided for the purposes of the Reflected Wave Model to restrict the area under study to downstream of the confluence at Burrowbridge. Given the nature of the analytical model, this should not have much effect on the water level calculations, but it does mean that the upstream tidal volume will vary less than it should and hence that the water velocities would not be as large as in reality, especially close to the tidal limit. Due to this approximation, it was decided that it was not necessary to use the full extent of the available data for bathymetry values. Therefore, only 7 cross-sections were used.

4.3.2 Depth averaged velocities

Shortly after low water the flow changes from ebb to flood (the delay tends to grow with the distance from the tidal limit). Therefore this transition is sharper than the transition from flood to ebb which happens close to high tide when all the velocities are less pronounced. This difference between the transitions ebb/flood and flood/ebb will be most marked in the cases where the low water depth is very small. An example of this is given by the Parrett for which observations agree with this characteristic of the model, sometimes in spectacular fashion with the rapid transistion from ebb flow to flood flow synonymous with the production of a tidal bore (see Figure 6).

The lack of accuracy in the model representation of the water levels in the cases of the Parrett and the Nene means that both the instantaneous tidal volumes and cross-sections are not accurately represented and hence the water velocity curves are themselves inaccurate (See Figures 4, 5, 6 and 7). The differences between the modelled and observed velocities are somewhat aggravated by the difficulties in obtaining accurate field velocity measurements.

In the case of the Thames for which the water levels are reasonably well represented in the model, modelled and observed velocities tend to correlate reasonably well at the seaward end (see Figure 2), but the correlation deteriorates very rapidly close to the tidal limit. Figure 3 shows a very poor comparison between observed and predicted velocities in Syon Reach.

Two problems that can be encountered close to the tidal limit are highlighted in Figure 3b. The first is that even though the phase for the surface elevation is correct, the modelled velocities appear completely out of phase with the observations. The duration of the observed flood flow at Syon Reach is also considerably shorter than it is for the model. So it appears that the model assumes that there is a far smaller upstream tidal volume at this point than there really is.

These discrepancies close to the tidal limit are not simply due to inaccuracies in the velocity field data since similar discrepancies occur with the predicted and observed velocities from all four estuaries examined. Furthermore, slight inaccuracies in the cross-sectional data are not likely to produce such major changes in the model results. One must therefore conclude that the model gives an unsatisfactory representation of the interactions between the freshwater input and the tidal waters. At present this is simply modelled as an extra discharge input in the tidal volume calculations, but the freshwater flow does not figure in the water level calculations.

4.4 Model predictions

One of the important characteristics of this analytical model is the facility it has to make predictions of the changes in hydrodynamic behaviour which would be brought on by various changes to the initial boundary conditions.

4.4.1 Mean sea level alterations

It is considered (see Reference 10) that the present climatic trends due to global warming will lead to a rise in the mean sea level of about 20cm around the British Isles by the year 2030. Such a rise could potentially have a considerable effect on estuary regime.

It should be noted that a climatic change which causes a rise in mean sea level is likely to have an influence on the pattern and annual variation of freshwater flow. Hence the two events should not be examined in isolation for the purposes of examining the impact of climate change.

It is straightforward to modify the model to simulate a rise in the mean sea level. The damping coefficient and phase delays established for the calibration tide are adjusted according to the new average depths. The damping coefficient is proportional to water velocity which is itself inversely proportional to the cross-sectional area. Therefore, the damping coefficient is taken to be inversely proportional to any change in depth.

Similarly, the phase delay is inversely proportional to wave celerity and is hence inversely proportional to the square root of depth. This leads to the following modifications:

$$\mu x_{new} = \sum_{i=1}^{j} \mu_i \frac{d_{old}(x_i) + d_{old}(x_{i-1})}{d_{new}(x_i) + d_{new}(x_{i-1})} (x_i - x_{i-1})$$

$$kx_{new} = \sum_{i=1}^{j} k_i \sqrt{\frac{d_{old}(x_i) + d_{old}(x_{i-1})}{d_{new}(x_i) + d_{new}(x_{i-1})}} (x_i - x_{i-1})$$

Hence both the damping and the phase differences are reduced by an increase in the average depth. At the same time, the amplitude modifications due to Green's Law become less pronounced since a fixed variation in depth has a lesser relative importance as the depth is increased.

From the foregoing it can be seen that a rise in the mean sea level can have conflicting effects on the model results: the reduction in the damping coefficient tends to increase the tidal range whereas on a 'typical' estuary where the cross-sections become smaller upstream, the reduced effect of Green's Law's means that the tidal ranges are themselves reduced. The overall prediction with the model when it is run for the Thames estuary is an increase in the tidal range which becomes progressively more and more pronounced in the upstream direction. Hence, in this example the modifications to the damping coefficient have a larger effect than Green's Law (see Figure 8).

These results tend to agree with the expected effects of mean sea level rise (see Reference 11). There is however no observational data with which these results can be quantitatively compared.

4.4.2 Tidal range alterations

The sea level rises mentioned in Section 4.4.1 would themselves generally lead to a tendency for tidal ranges to increase due to the reduction of bed friction. From a practical point of view, it is a simple matter to alter the tidal range at the seaward boundary once the model has been calibrated for a particular tide.

The amplitude of the incident tide can be altered by modifying the constant A_0 in equations (2) and (3). Multiplication by the ratio R of the new tidal range at the mouth over the old one generates a new tidal amplitude. So the model tidal range at each position can be determined by multiplying the original tidal range for this position by the constant R which remains the same for the whole length of the estuary (see Figure 9a).

In order to examine this effect the model has been calibrated on a spring tide and then the tidal range at the mouth has been changed to that of a neap tide. The model predictions for this neap tide have then been compared with the available observational data. This process can give some good results, as shown in Figure 9b for the Thames model calibrated on ST and then modified to simulate NT. For a reduction of 1.37m in the tidal range at the mouth, the average error for the predictions is only 0.34m.

The effect on the model velocities of modifying the tidal range seems to be approximately equivalent to multiplying them by some constant value over the whole length of the estuary. Looking at the tide SP on the Thames (Figure 9c), it is clear that the maximum and minimum velocities of the original calibration tide are both multiplied by approximately the same constant value of 0.8 for a reduction of 1m to the tidal range at the mouth, by about 1.18 for an increase of 1m and about 1.36 for an increase of 2m. These ratios are exactly the same as the ratios of the modified tidal range over the original tidal range (4.49/5.49=0.82, 6.49/5.49=1.18 and 7.49/5.49=1.36).

4.4.3 Freshwater flow alterations

The Reflected Wave Model calculates water levels without taking into account the freshwater discharge. This only comes into play at the level of the discharge (and hence velocity) calculations. This result from the model is however clearly contradicted by observations which indicate that the freshwater discharge can have a considerable effect on the water levels. It is therefore recommended that this model is calibrated for each value of freshwater discharge for which it is to be used.

4.4.4 Creation of a tidal barrier

The model can also be used to look at the effect of imposing a tidal barrier within the estuary. This is achieved by allowing the reflection of the incident wave to take place where the barrier is sited, at the new tidal limit. For this case, it is assumed that the barrier does not affect the tidal wave propagation until the wave reaches it. This means that the damping coefficient and the wave propagation velocity are kept the same as before.

It is worth noting here that in 3 out of 4 of the examples that have been looked at the model calculates a significant increase in the damping coefficient in the upstream stretch before the tidal limit. This leads to a marked contrast between the amplitudes of the incident and reflected waves with the reflected wave being considerably damped. If the section of the estuary with high damping coefficient is removed by installing a tidal barrier further downstream, the reflected wave will have a far greater amplitude than before. This part of the model is therefore not satisfactory: as it stands, the tidal range at the barrier can be almost doubled in certain cases.

There are two possible ways of modifying the model to obtain more realistic results for a tidal barrier. Both of these are methods of simulating an absorption of energy as the incident wave reflects off the barrier. The first would be to increase the damping coefficient in the final section before the barrier. This would make the graph of μ look more similar to pre barrier case for the full unobstructed estuary. The other method is simply to allow for a reflection coefficient on the barrage.

This modelling approach is not sufficient to represent a tidal energy barrage which affects tidal propagation upstream of the barrage. In this case it is necessary to represent the mode of operation of the barrage within the model. This is possible in many cases but inevitably requires some parameterisation of the operating characteristics of the barrage. Given the accuracy of this modelling technique it must be considered that such a representation is inappropriate and that other modelling techniques are to be preferred.

4.4.5 Cross-sectional alterations

The model treats all phenomena which lead to modifications in the cross-sectional data in the same way. The method for modelling these cases is very straightforward.

Once the calibration run of the model has calculated values for the damping coefficient and for the phase delay, the cross-sectional data at the appropriate positions can be altered to represent erosion, deposition, dredging or reclamations. At present it is assumed that the values for kx and for μx remain unchanged by this, however it is likely that including some kind of dependency on the bathymetric data would be beneficial. These factors have been shown to change with modifications to the depth, the same argument could easily be extended to this scenario.

Two reclamation scenarios have been investigated within the Thames model. Both are 1km long and are situated just upstream of Gravesend. Reclamation A is 300m wide at the top and 200m wide close to the low water depth. Reclamation B reduces the channel to a rectangular cross-section with the same width as the original base width.

The model assumption that the damping coefficient is not modified by these changes means that the only way the tidal ranges can be affected is through Green's Law. So the effect on water levels is limited to those sections which have actually seen their bathymetric data altered. This is what the model results show, and Figure 10a, which shows the changes due to the smaller reclamation, demonstrates a marked increase in the tidal range along the stretch of river with the modified bathymetry. In this case, the model predicts that the construction of reclamation B would approximately double the tidal range along the stretch where this modification has been made. This appears to be an unrealistically large alteration to the tidal range, indicating that the model may give more credible results if the damping coefficient values were altered along the stretches of the estuary where the cross-sectional data has been modified.

At all the positions where no cross-sectional alteration has been made, the depths remain unmodified throughout the tidal cycle and accordingly so do the cross-sectional areas. This means that a reclamation has no effect on the upstream tidal volume and hence on the depth averaged velocity at any point upstream of it. This aspect of the model results is somewhat unsatisfactory as in reality one would expect that the reduction in cross-sectional area at the position of the reclamation would lead to a reduction of the discharge through this stretch, potentially leading to a reduction in the tidal volumes further upstream. This drawback of the model could be rectified by allowing the damping coefficient to vary with any changes in the cross-sectional area as noted above.

However, at the points downstream of the cross-sectional alterations, the upstream tidal volume is changed by two factors. The first is the fact that any alteration to the cross-sectional areas means an alteration to the volume of water needed to fill these stretches to a given depth. At the same time though, the depths at the modified cross-sections do not remain the same. These two factors have opposing effects on the tidal volumes and hence on the velocities, so it is important to work out which effect is the most pronounced.

A reduction in width by a factor *f* for a stretch of the estuary leads to a reduction in the cross-sectional areas by this same factor *f*. At the same time Green's Law means an increase in the tidal range proportional to the square root of *f*. Therefore, the overall effect is to reduce the tidal volume in the modified stretches proportional to the square root of *f*, and hence to reduce the velocities downstream of the modified stretch.

The observations from the model run with the two different reclamations do in fact show a very slight reduction in the velocities downstream of the alterations near Gravesend as well as larger increases in the velocities at their actual location. Figure 10b shows this for the smaller of the two reclamations. The modifications to the velocities downstream of the bathymetry changes in this case are so small that they can practically be ignored.

4.5 Summary

There are a number of possible improvements which could be easily tested within the Reflected Wave Model, such as enabling the damping coefficient to vary with depth and velocity throughout the tidal cycle or such as representing the loss of energy caused by reflection of the tidal wave off the head of the estuary or off a structure.

The Reflected Wave model is a useful tool for obtaining PC based calculations for estuary hydrodynamics. However, it is important to know the limitations of this model which are the impossibility to give an accurate representation of the water level oscillations with time in the case of non-sinusoidal tidal oscillations, and also the very simplified modelling of the interaction between freshwater discharge and the tide.

It was decided that the above limitations of the Reflective Wave Model made it desirable to use an iterative numerical model for further investigations. A number of such models exist at HR, but it was decided to use SALMON-Q for the purposes of this study, since this model is the most appropriate 1-D model for estuarine applications. The use of 2-D models was precluded in terms of the resources and time available, accordingly the assumption was made that for those estuaries examined a 1-D approach was not unreasonable. It is accepted that in the case of the Thames and the Dee Estuaries this assumption is questionable.

5 An iterative hydrodynamic model: SALMON-Q

5.1 Description

SALMON-Q is a one-dimensional hydrodynamic model which uses finite difference schemes to solve time dependent partial differential equations governing the conservation of mass and momentum. SALMON-Q is a development of the TIDEWAY 1-D model which incorporates additional process models for predicting salinity, water quality and suspended sediment concentrations. As it is only one-dimensional it is designed to be used in well mixed estuaries where vertical and lateral variations in concentrations are small compared with the longitudinal variations.

Similarly to the analytical model, a number of cross-sections need to be described; but in this case each one is defined by a number of points and can therefore be of a more specific shape without the generalisation needed for the analytical modelling.

As a predictive tool, SALMON-Q is more ambitious than the Reflected Wave Model described in Chapters 3 and 4 because, even though a number of parameters need to be calibrated or estimated, it is not necessary to have through tide water level observations along the whole length of the estuary as input data.

Engineering structures can be incorporated into the model in the form of weirs, sluices or modifications to the cross-sections (reclamations or dredging).

5.2 Mathematical formulation

The estuary to be modelled is divided into a number of reaches with the cross-sectional bathymetry described at the upstream and downstream limits of each reach. Each reach is then subdivided into a number of elements all of a similar length. In the cases examined in this study the length of each element is between 1000m and 5000m long. The water levels are then calculated at both ends of each element whereas the flow velocities are calculated at the mid-point of the element.

The flow through each element is obtained by solving partial differential equations representing the conservation of mass and momentum:

$$w\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} - q = 0$$

$$\frac{\partial Q}{\partial t} + gA\frac{\partial h}{\partial x}Q\frac{\partial u}{\partial x} + u\frac{\partial Q}{\partial x} + \frac{\tau_b p}{\rho_0} + \frac{Adg}{2\rho_0}\frac{\partial \rho}{\partial x} = 0$$

where:

w = water surface width (m)
h = water surface height (m)
Q = discharge into the element (cumecs)
u = area mean velocity (m/s)
A = cross-sectional area of the element (m^2)
q = lateral inflow into the element (m^2/s)
τ_b = bed stress (N/m^2)
p = wetted perimeter of the element (m)
ρ_0 = density of freshwater (kg/m^3)
d = depth of flow (m)
g = acceleration due to gravity
ρ = water density (kg/m^3)

The friction term is represented by the equation:

$$\frac{\tau_b}{\rho} = f\frac{|Q|Q}{8RA}$$

where R is the hydraulic radius.

The friction factor, f, is calculated from the Colebrook-White transition law as a function of water velocity, depth and roughness length. SALMON-Q allows for two basic types of bed morphology with respect to the roughness length: one in which the bed roughness does not change significantly with time (a fixed bed) and the other in which the bed shape (and hence roughness) changes with the overlying flow regime (a mobile bed). In the case of the fixed bed model, the roughness length is directly input as data, whereas for a mobile bed it is calculated from the water velocity and the median grain size of the bed material.

The finite difference schemes used for SALMON-Q give the following stability criterion:

$$\Delta t < \frac{\Delta x}{u}$$

In practice, this means that in order to have stability it must be impossible for a water particle to pass through a whole model element within a time step. Since the shortest element that has been used for this study is 1000m long and the velocities stay below 2m/s, the time step of 5 minutes used for this work keeps the calculations comfortably within the limits of stability.

5.3 Practicalities

Before SALMON-Q can be run it is necessary to give values to a large number of parameters. Some of these need to be measured in the field (cross-sections, grain size and settling velocities) and some are calibration parameters (dispersion coefficients) which can be adjusted by the user in order to obtain a better representation of observational data. These parameters make the setting up of an estuary to run with SALMON-Q a lengthy process. This is the disadvantage of SALMON-Q over more simple models like the Reflected Wave Model described in Chapter 3. SALMON-Q is a very good tool once it has been set up and calibrated for a given estuary, but if a large number of estuaries are to be looked at in a short amount of time, the previous analytical model would be preferable.

5.4 Results

SALMON-Q has been set up and run specifically for this study for the Thames, Nene, Parrett and Dee estuaries in the British Isles. SALMON-Q is widely used at HR and has been run for numerous other estuaries and rivers in the UK and overseas for site specific studies.

5.4.1 Thames

Since the Thames is a long, regular estuary, and since only 13 bathymetry sections have been specified, it was decided to use relatively large model elements between 4 and 5 km long.

The resulting modelled water levels are of a similar degree of accuracy as for the analytical model. This result however is considerably more powerful in the sense that it is not dependent on prescribing a whole series of observations of tidal levels, but simply one set at the mouth.

The more significant result however is the accuracy of the water velocity predictions obtained from SALMON-Q. Figure 11 shows that even towards the upstream tidal limit where the analytical model was showing a very poor performance, SALMON-Q produces a velocity profile which is in much better agreement with the observations. The velocity representation does however still remain far less satisfactory towards the tidal limit of the estuary than it is closer to the sea. Figures 12 and 13 show comparisons between SALMON-Q output and observations for both water levels and velocities at four different locations.

5.4.2 Nene

The field data that is available for the Nene is described in Section 2.2. The field measurements were carried out during a period while the sluices at Dog-in-Doublet were closed. However, the sluices were fully opened to let accumulated water out shortly beforehand. There is no available data concerning flow velocities during that time. Unfortunately though, it is highly probable that these high discharges would have affected the observations even after the closing of the sluices (Reference 5). Therefore it seems likely that the most realistic results from the SALMON-Q model would be obtained using a river discharge which is higher than that measured during the period of field observations.

SALMON-Q was run for the Nene with 1km long elements and a mobile bed. This means that the average grain size within the bed is specified instead of an abstract friction value. The Nene study described in Reference 5, gives field data for this value along with a large number of other important parameters needed to run the SALMON-Q model. The only parameter remaining which was used as the calibration parameter for the model from a hydrodynamic point of view is the freshwater discharge. The final calibration gave it a value of 2.5 cumecs which is consistent with the comments made above.

The accuracy of the hydrodynamic fit for the spring tide observations at the four survey stations is shown in Figures 14 and 15. There is a considerable improvement over the results given by the Reflected Wave Model shown in Figures 4 and 5.

Once again there is a considerable reduction in the accuracy of the representation of velocities towards Guyhirne Road Bridge. This seems to indicate that, in the same way as for the Thames model, the accuracy of the velocity representation deteriorates towards the tidal limit.

There are two methods of defining the boundary data at the mouth for the evolution of sea levels through the tidal cycle. The first method uses a time series of water levels taken from a set of observations. This method is good for calibrating the model with further observations taken over the same tide, but its relevance in more general terms is very much dependent on how "typical" the observed tide happens to be. For looking at the regime state of the estuary however, it is useful to use a more generally applicable tide which is not dependent on the randomness of a given day's field measurements of water level. For this reason it was decided to use the sea level boundary data drawn from tidal harmonics given in the Admiralty Tide Tables for further investigations.

5.4.3 Parrett

The Parrett is a difficult estuary to represent by a computational model because of the tidal bore, the very small depths at low water and the large tidal ranges, and also because of its dual tidal limits (See Section 2.4). However, there is a large amount of cross-sectional data available from the survey carried out by HR in 1977. It is therefore possible to obtain a far more accurate representation of this estuary than was used for either the Thames or Nene. It is for this reason that the use of short 1km model elements has been adopted.

In the case of the Parrett, the observations for sediment grain size seem to indicate that it is highly variable both along the length and across the width of the estuary. What's more, there also appears to be a time variability due to the fact that large quantities of silt seem to settle around high water and are then flushed away by the ebb flow to reveal a sandy bed (See Reference 6). It was therefore decided to use SALMON-Q as a fixed bed model rather than a mobile bed model since even though the latter would allow for a certain time variability in the bed friction, this would be due to the wrong process: changes in the bed shape rather than a variation in the average grain size of the material in the upper layer of the bed.

The downstream boundary data was given by a time series of observed water depths at Stert corresponding to the spring tide of 13 October 1977. Unfortunately the survey only measured these depths relative to the bed, without specifying the level of the bed with respect to Chart Datum. It was therefore assumed that the reference zero depth is the lowest point in the corresponding bed cross-section given in the bathymetry survey.

The actual freshwater discharges for that day are not available. Instead, the average daily mean is used in the model: 4.18 m^3/s at Knapp Bridge, 8.32 m^3/s at Oath Lock, giving 12.5 m^3/s at Bridgwater (See Reference 7).

The bed friction calibration was carried out using six further sets of depth observations at positions between Stert and Bridgwater which is 19km upstream. Unfortunately, once again the depth observations are taken relative to the bed without the level of the bed being specified. At first this was dealt with in the same manner as for the boundary data at Stert. But this was shown to lead to a highly unlikely sequence of water levels, and therefore the reference bed levels were used as 'free' parameters which could be varied within a specified range in order to obtain the best possible correlation between the model results and the observations. This was not considered to be a problem since from past experience the average depths tend to be fairly accurate before calibration, the effects of calibration being most noticeable in terms of high water phase or of tidal ranges.

The bed friction was assumed to be constant along the whole length of the estuary because the water level observations are only restricted to downstream of Bridgwater and could therefore not be used for calibrating the friction any further upstream. A good match for the water levels was found with a bed friction value of k_s=0.3 as is shown in Figures 16 and 17.

5.4.4 *Dee*

The SALMON-Q model of the Dee was set up with a mobile bed and with an average bed material grain diameter which can vary along the length of the estuary. The length of each model element was taken to be 1km.

The model was calibrated on the spring tide of 6th April 1966 for which the river flow at Chester Weir was 56 cumecs. Figure 18 shows a good correlation between the model output and the observations in the case of this spring tide and also in the case of the neap tide recorded on 1st February 1966 with a river discharge of 38 cumecs.

5.5 Summary

SALMON-Q shows a satisfactory representation of the hydrodynamics of the four estuaries which have been studied, even in the regions where there is strong interaction between the freshwater fluvial input and the saltwater tidal input. The same four estuaries have been studied in more depth with SALMON-Q in Ref 11. This study looked at both the hydrodynamics and water quality from the point of view of climate change.

6 The regime modelling technique

6.1 Introduction

Any changes to the hydrodynamic regime of an estuary are likely to cause modifications to the patterns of siltation and erosion, leading to modifications in the cross-sections. Modification of the morphology of an estuary will lead to a new hydrodynamic behaviour, and consequently changes in the morphology and so on in an iterative manner. This is the regime problem, the dynamic interaction between flow and sediment transport processes.

The previous chapters have presented some aspects of the tidal flow processes and the impact of modifications to the morphology, mean sea level and freshwater flow on the tidal flows. This has demonstrated that simplification of these processes for representation within an analytical model is not appropriate. The simplification loses important aspects of the physical processes. Hence the use of an iterative numerical model incorporating more of the physical processes was proposed.

Having established a suitable flow model for use in a regime modelling approach it remains to choose a suitable sediment process model in order that the morphological evolution of an estuary can be predicted. The aim is to develop a technique which incorporates interactively both the hydrodynamic behaviour and the deposition/erosion properties of the estuary. Based upon the experience gained from the flow modelling aspects, where it was found that at this stage of a regime research programme it was important to keep as many of the physical processes within the model, it was decided to start first with a fairly complex tried and tested siltation model prior to looking for simplifying assumptions.

The SAP (Siltation at a Point) model is a suitable tried and tested cohesive sediment model available at HR (See Reference 12). The model incorporates the results of laboratory and field experiments in predicting the siltation, erosion and consolidation of a cohesive material at a point. In order to interface the SAP model with a flow model a shell was required. The development of this shell is described in part of this chapter.

Regime modelling is not just the simple matter of interfacing a sediment model with a flow model. If this were the case then the numerous 2 and 3-D models of flow and sand/mud transport that already exist would be entirely appropriate for this type of work. Regime modelling using an iterative approach requires a new approach to long term predictive modelling. If an analytical approach were possible then there would be no need for iteration.

Regime modelling must be capable of examining a number of different time scales: the tidal cycle, the spring-neap cycle, the seasonal cycle, the annual cycle and secular trends. The seasonal and annual cycles are generally dominated by variations in freshwater flow, although in certain cases other effects may be important. Secular trends are associated with longer time scales and may be associated with natural or manmade influences. In this chapter a methodology to address this problem is proposed. We have called the methodology the REGIME technique. In the application presented here the SALMON-Q and SAP models are used but this need not be the case and the merits/demerits of these particular models shall not be addressed here but will be commented upon in Chapter 7.

The aim of the REGIME technique is to provide a shell programming structure from which it is possible to develop a model which can simulate the evolution of the bed profile of an estuary under various different tide and freshwater flow conditions. In order for this to be achieved, a one-dimensional hydrodynamic model of the estuary under study as well as a siltation-erosion model must be inserted into the REGIME shell. The interface between the flow and siltation models is provided within REGIME by means of a model called X-flow which transforms the one-dimensional hydrodynamic output into quasi 2-D flows. The output from a 2 or 3-D flow model could be used with a little restructuring of the X-flow model.

6.2 The SAP model

The SAP (Siltation at a point) model is a well established computational model which simulates erosion, deposition and consolidation at a point on a cohesive sediment bed over a number of tidal cycles. Consolidation is represented by having 10 discrete homogeneous layers of cohesive sediment within the bed. Material transfer between the water column and the uppermost bed layer is due to erosion and deposition. Material transfer between the different bed layers is caused by consolidation due to their own self-weight and the dissipation of excess pore pressure within the bed. The various physical processes included in the model are now briefly described.

6.2.1 Erosion

The erosion of a cohesive sediment bed may be assumed to occur when the applied bed shear stress τ exceeds the erosion strength τ_e. The rate of erosion may be expressed as

$$\frac{dm}{dt} = m_e(\tau-\tau_e) \qquad \tau > \tau_e$$

where the erosion shear strength τ_e and the rate of erosion m_e may be found experimentally.

6.2.2 *Deposition*

Deposition occurs only when the applied shear stress τ is less than a critical value τ_d. The rate of deposition can be expressed as a function of the near-bed concentration of suspended sediment c and the median floc settling velocity w_{50} which must be determined in the field:

$$\frac{dm}{dt} = c\ w_{50}\ \frac{\tau_d - \tau}{\tau_d} \qquad \tau < \tau_d$$

6.2.3 *Consolidation*

Consolidation of the cohesive sediment bed is modelled on the basis of three principal assumptions. The first is that the bed can be represented as discrete layers each having a particular density and thickness. The second is that there exists an engineering relationship between the effective stress σ_v' and the dry density ρ_d of the cohesive sediment of the form

$$\sigma_v' = F + G\rho_d + H\rho_d^2$$

where F, G and H are constants. In addition, it is necessary to know the dry density ρ_{d0} of the sediment immediately on deposition to the bed. This by definition is the density at which the effective stress is zero. The third assumption is that there also exists an engineering relationship between the permeability of the cohesive sediment k and its dry density ρ_d of the form

$$\log(k) = J + K\rho_d$$

where J and K are constants.

Numerical values for the constants F, G, H, J and K can be obtained by laboratory experiments conducted in columns on deposited beds.

6.3 X-flow

X-flow is a model developed specifically for the purpose of being integrated into the REGIME model system. It is necessary in order to link the one-dimensional output from the hydrodynamic model with the two-dimensional data needed in order to run the siltation-erosion model at points on a two-dimensional grid.

X-flow takes the average cross-sectional velocity output from the hydrodynamic model together with the bathymetric data that was used and gives quasi-2D output in the form of depth averaged velocities for various thin vertical columns of water across the width of the estuary.

The theory used is based on the Manning Formula:

$$V = \frac{R^{\frac{2}{3}} S^{\frac{1}{2}}}{n}$$

where:

V = the depth averaged velocity;
R = the hydraulic radius;
n = surface roughness coefficient;
S = longitudinal gradient.

For a given cross-section, the slope of the river and the Manning's coefficient can be assumed to be constant, thus giving:

$$V = k \left(\frac{A}{P}\right)^{\frac{2}{3}}$$

where:

A = the area of the channel;
P = the wetted perimeter.

The assumption made above means that the parameter k remains constant across the whole width of a given cross-section. This means that k can be calculated using the values for V, A and P corresponding to the whole cross-section. The same value of k can then be used with V, A and P restricted to a narrow column of water. In this case, A represents the cross-sectional area of this column and P the length of the portion of bed along the bottom of the column. This calculation can give the depth averaged velocity for the column of water above any given point on the bed.

6.4 The REGIME modelling technique

The REGIME modelling technique is here illustrated by an example application using SALMON-Q and SAP. This example is summarised in the flow chart of Figure 19.

It is first necessary to decide upon a series of discrete freshwater flow conditions to be run along with the duration of each freshwater condition and the order in which they occur. Once this has been done, the hydrodynamic model can be run with the first freshwater condition in order to determine the corresponding hydrodynamic behaviour of the estuary. As an example the input freshwater flow data might be as simple as considering eight months of low flows, three months of average flows, one month of high flows and 3 days of flows equivalent to the annual flood.

The output from SALMON-Q is a series of sets of time dependent flow data, one set for each cross-section. This one-dimensional output is effectively transformed into two-dimensional data by the program X-flow which gives flow data for various pre- determined points across each cross-section. For each of these points, the third vertical dimension within the bed is effectively simulated by the SAP model in the form of distinct layers with different densities. Each individual run of SAP calculates a new bed level as well as the new characteristics of the ten bed layers. Combining all these sets of results gives a new bathymetry profile for the estuary. If the new cross-sections obtained in this manner are substantially different, there is a possibility that the differences would affect the hydrodynamics. It is therefore necessary to feed the new cross-sections back into SALMON-Q to obtain a new set of flow data. The model cycle thus described repeats itself until the end of the simulation is reached.

Through this simulation of the evolution of the estuary bathymetry by the REGIME modelling technique it should be possible to recognise a convergent trend either towards a particular bathymetry profile or towards a cyclic variation in bathymetry.

Advection effects could be included in the model by the development of another process model. This is an important process for all timescales. Changes in the flow patterns which result in erosion of parts of the bed will lead to increases in the concentration profiles at particular locations. Increases in concentrations will lead to greater rates of deposition during slack water periods (when the applied shear stress τ is less than a critical value τ_d). Such increases may also have significant effects on the morphology of the estuary.

Another scenario where the advection of material is important is when the introduction of engineering works modifies the flows significantly causing short term erosion of a particular location. When this happens the eroded material is not necessarily lost from the estuarine system but may accumulate at another location within the estuary and have a consequent impact on the hydrodynamic regime. In dynamic estuaries, for example the tributaries of the Severn Estuary, with naturally high suspended sediment loads this impact is likely to be less significant than in estuaries with lower suspended sediment loads such as the Thames.

Additional input requirements for the modelling are details of the density structure of the bed at the longitudinal cross sections. Estimates and generalisations can be made based on measurements from within the UK and overseas but in order to represent the impact of a flood event on the cross sectional profiles in the upper reaches of an estuary site specific data is required. This particular problem is not specific to the approach taken here. In all cases if absolute changes due to a particular scenario are to be investigated then the bed density profile will be required.

The SAP model can be used to estimate rates of consolidation of material but generally this consolidation is applicable for fresh material rather than historical deposits, which reflect a time history of events rather like rock strata from a geological point of view. However, if relative effects are to be examined comparing an existing situation with one or more proposed schemes then an estimate of a baseline density profile would be sufficient for the purpose of comparisons.

In the tests that have been undertaken it was noted that an accurate representation of the cross sections is required in order that changes due to erosion, etc., can be distinguished from background uncertainity. Additionally in an area where engineering works were proposed then there is a requirement for sufficient resolution in the longitudinal location of the cross sectional profiles so that all morphological changes can be represented. In practice this does not represent a significant problem, since in most scenarios geotechnical work is undertaken at the site of a proposed scheme, the data from such survey work would normally provide adequate information.

7 Conclusions and Recommendations

Two one-dimensional numerical models have been used in order to attempt to give an adequate representation of the regime state of estuaries. Various UK estuaries have been used in order to validate the models with as wide a range of different types of estuaries as possible.

The first model used is an analytical model written specifically for this work: the Reflected Wave Model. This model is very easy to use, requiring minimal computing resources and it appears to give fairly good results. It does however have two major restrictions: it needs to be calibrated with tidal range data for points along the whole length of the estuary, and the interaction between the tide and the freshwater flow is poorly represented.

Because of the drawbacks of RWM, a more elaborate numerical model has been used: SALMON-Q. This model appears to work equally well for each of the different estuaries examined. It has been used for looking at the effects of a number of modifications to the input conditions.

In order to take the long term regime state modelling further, it is necessary to include morphological modifications due to siltation and erosion. A new methodology for long term morphological changes was developed. This new method was simply called the REGIME technique.

The REGIME modelling technique involves using an iterative cycle of 1-D flow modelling and of siltation/consolidation/erosion modelling. The method has been used to design a test example, but extensive testing has not yet been undertaken because such an application is outside the scope of this study. The resolution required for the bathymetrical cross sections in order to distinguish changes due to erosion, deposition and consolidation from background uncertainty is very high.

The indication from those tests undertaken is that whilst the modelling technique works the main limitation of the technique is its general unwieldiness. Having initially attempted to examine regime problems with a simplistic micro based analytical model it has been clearly demonstrated that the point has not yet been reached whereby sufficient assumptions can be made and incorporated into such a generally applicable model. The output from this part of the regime studies is thus the proposed REGIME modelling technique which is a working, useable, long term prediction model but a far cry from a simple, micro based system.

8 Acknowledgements

The majority of this study was carried out by Mr M N H Waller under the supervision of Dr M P Dearnaley. Mr T N Burt also gave valuable advice throughout the study. Advice on the use of SALMON-Q was given by Mr E T Jones. The implementation of the REGIME modelling technique was carried out by Mrs C M L Rainey. This report was written by Mr M N H Waller and Dr M P Dearnaley.

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Figures

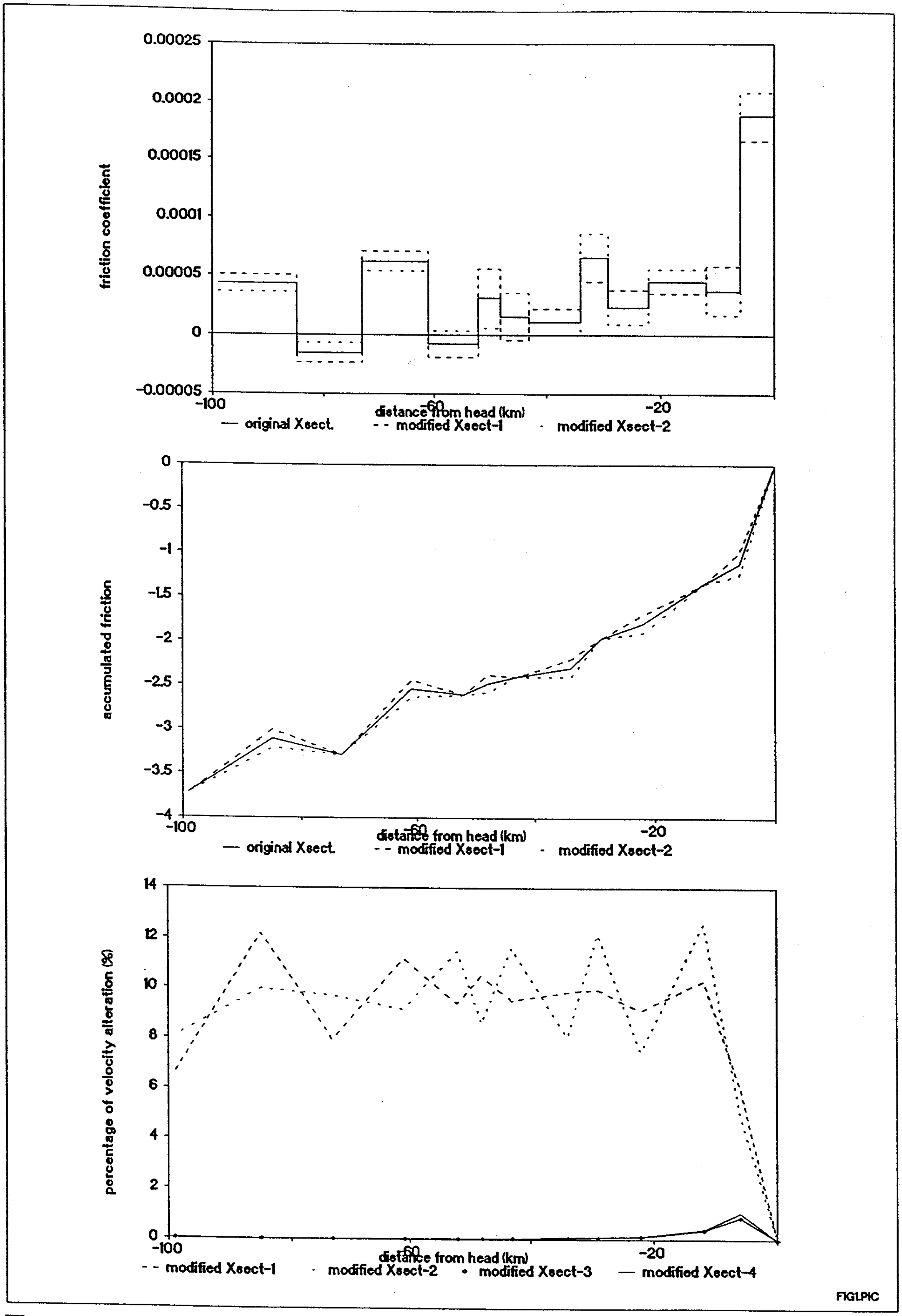


Figure 1 Reflected Wave Model of Thames: Sensitivity to cross-sectional alterations.

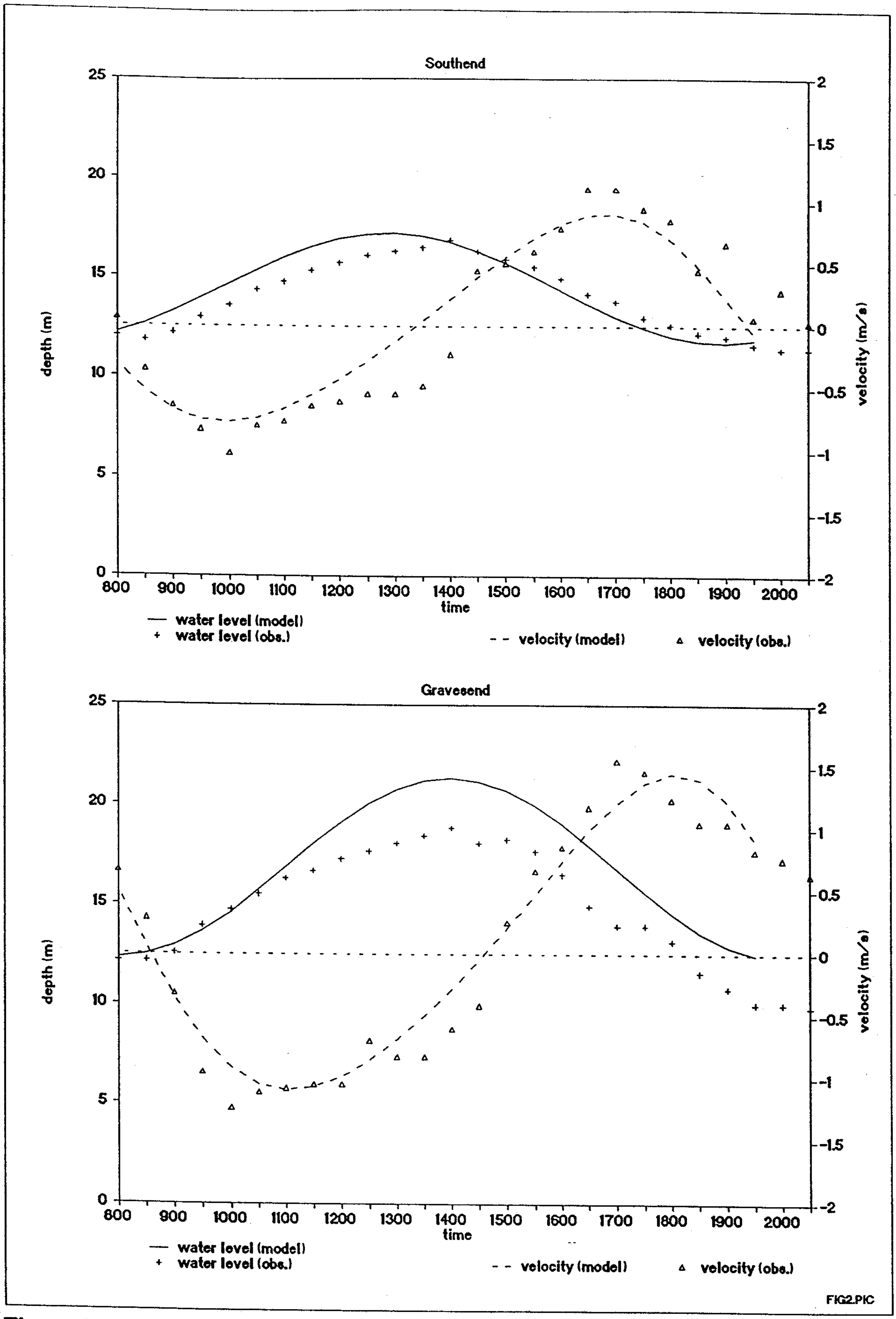


Figure 2 Reflected Wave Model of Thames: Comparison of results with observations at downstream end.

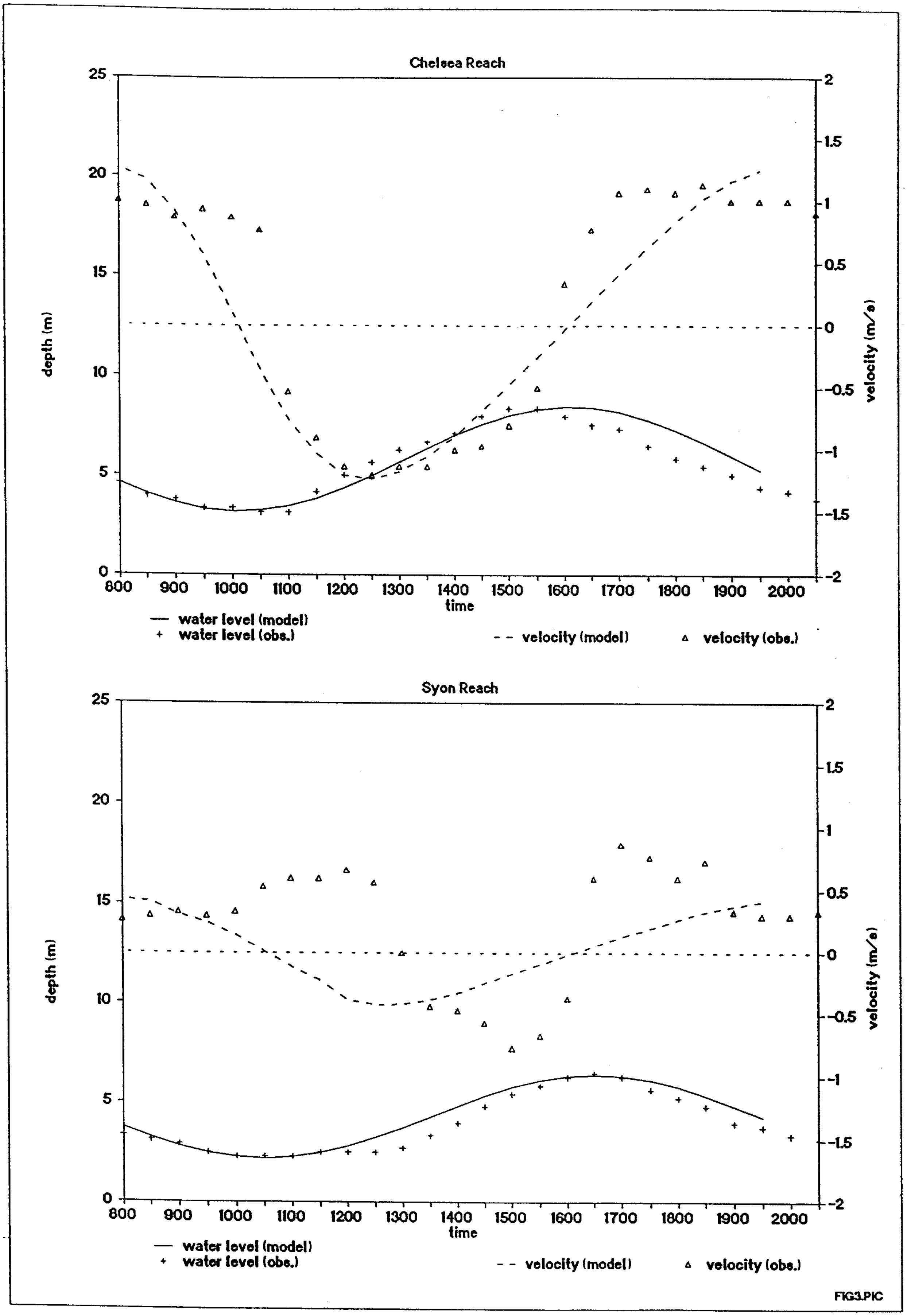


Figure 3 Reflected Wave Model of Thames: Comparison of results with observations at upstream end.

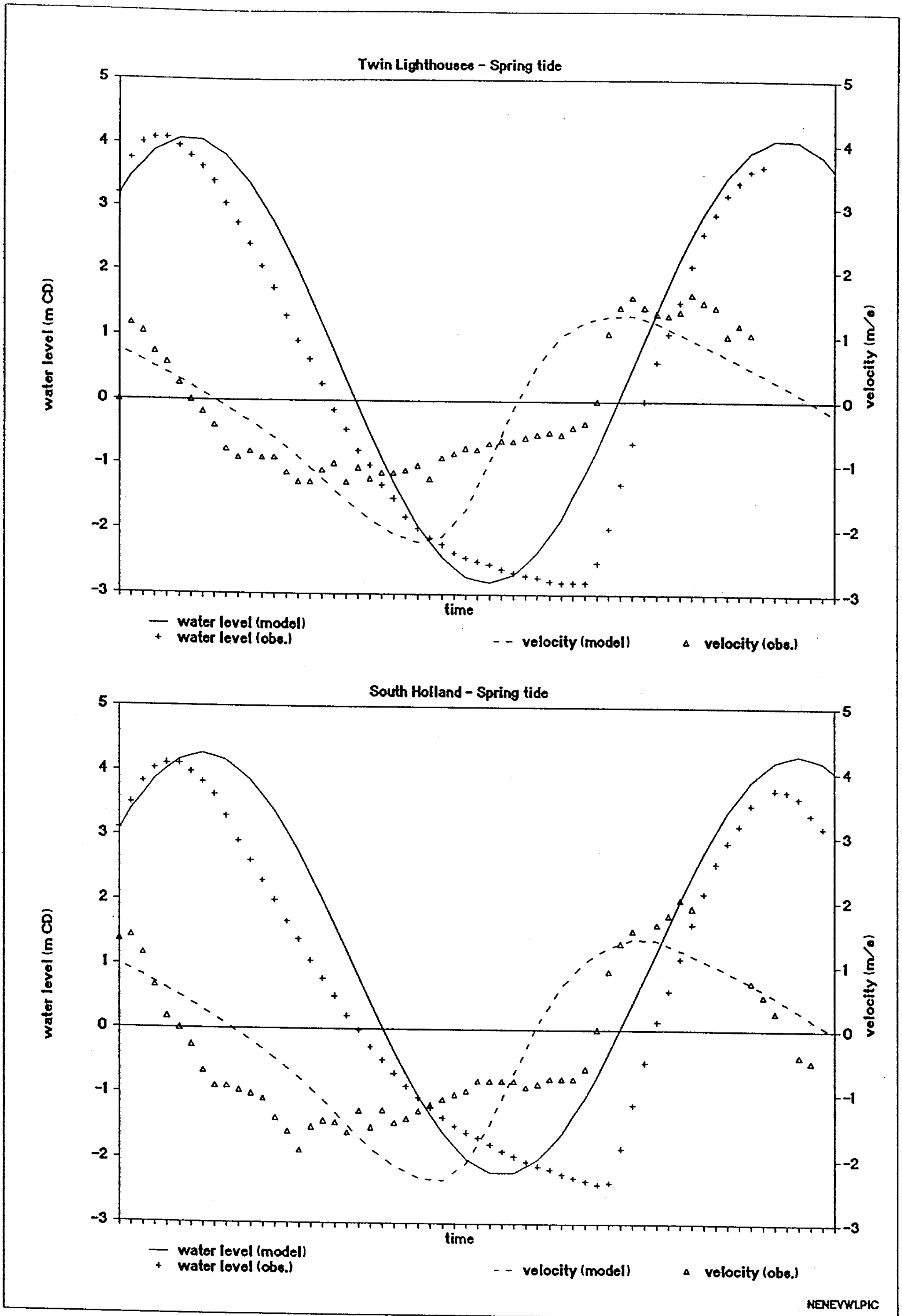


Figure 4 Reflected Wave Model of Nene:
Comparison of results with observations at downstream end.

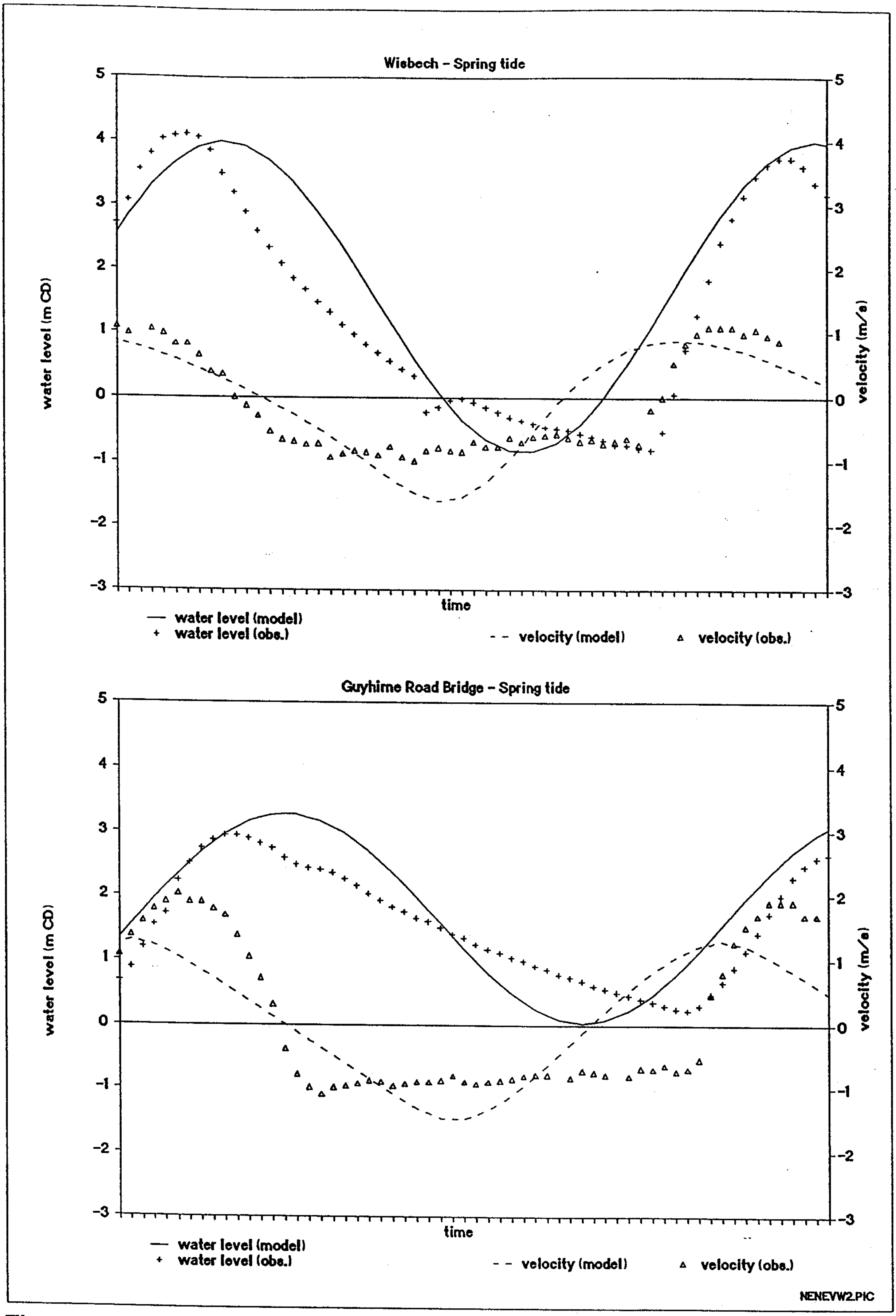


Figure 5 **Reflected Wave Model of Nene: Comparison of results with observations at upstream end.**

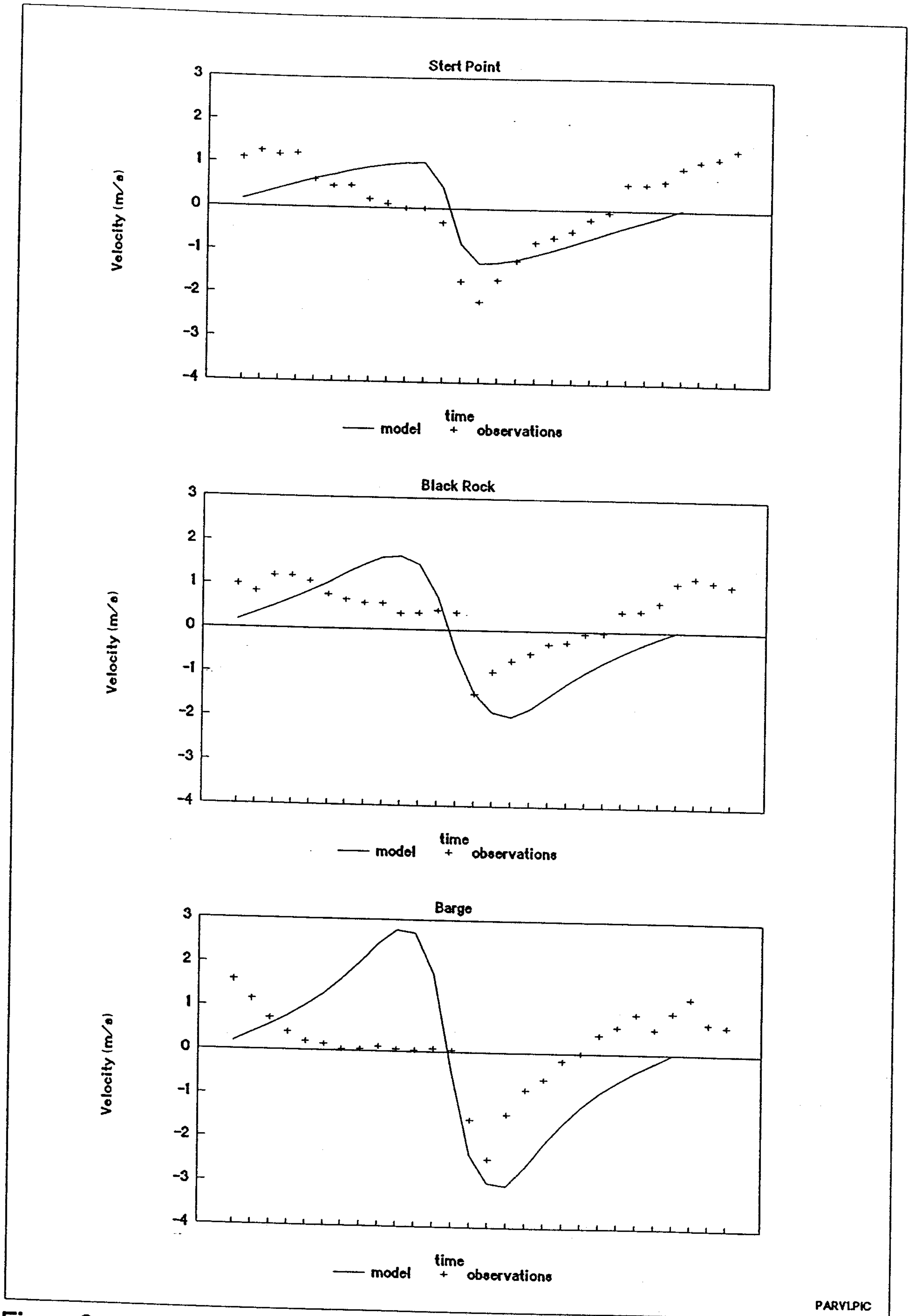


Figure 6 Reflected Wave Model of Parrett: Comparison of velocities with observations at downstream end.

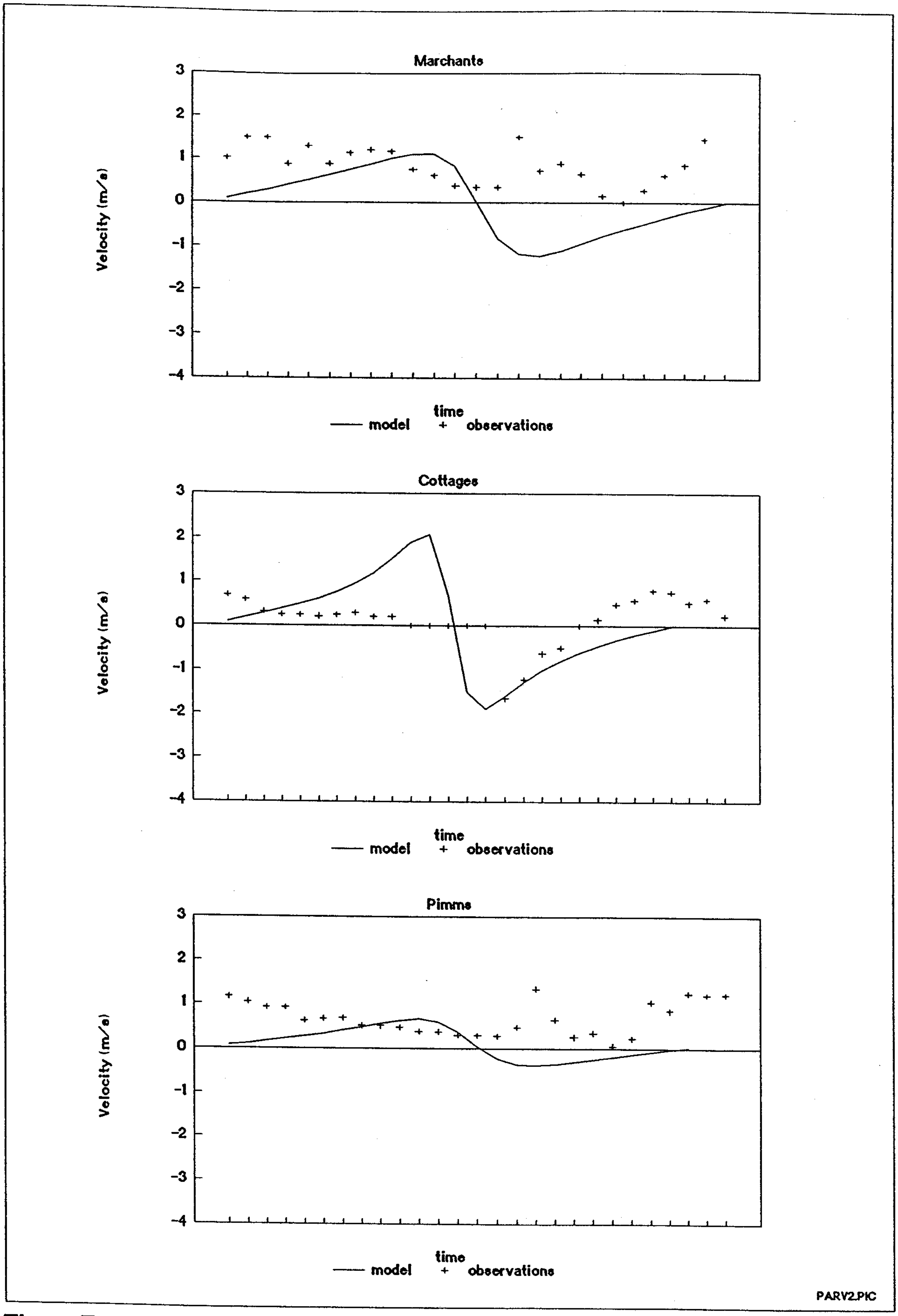
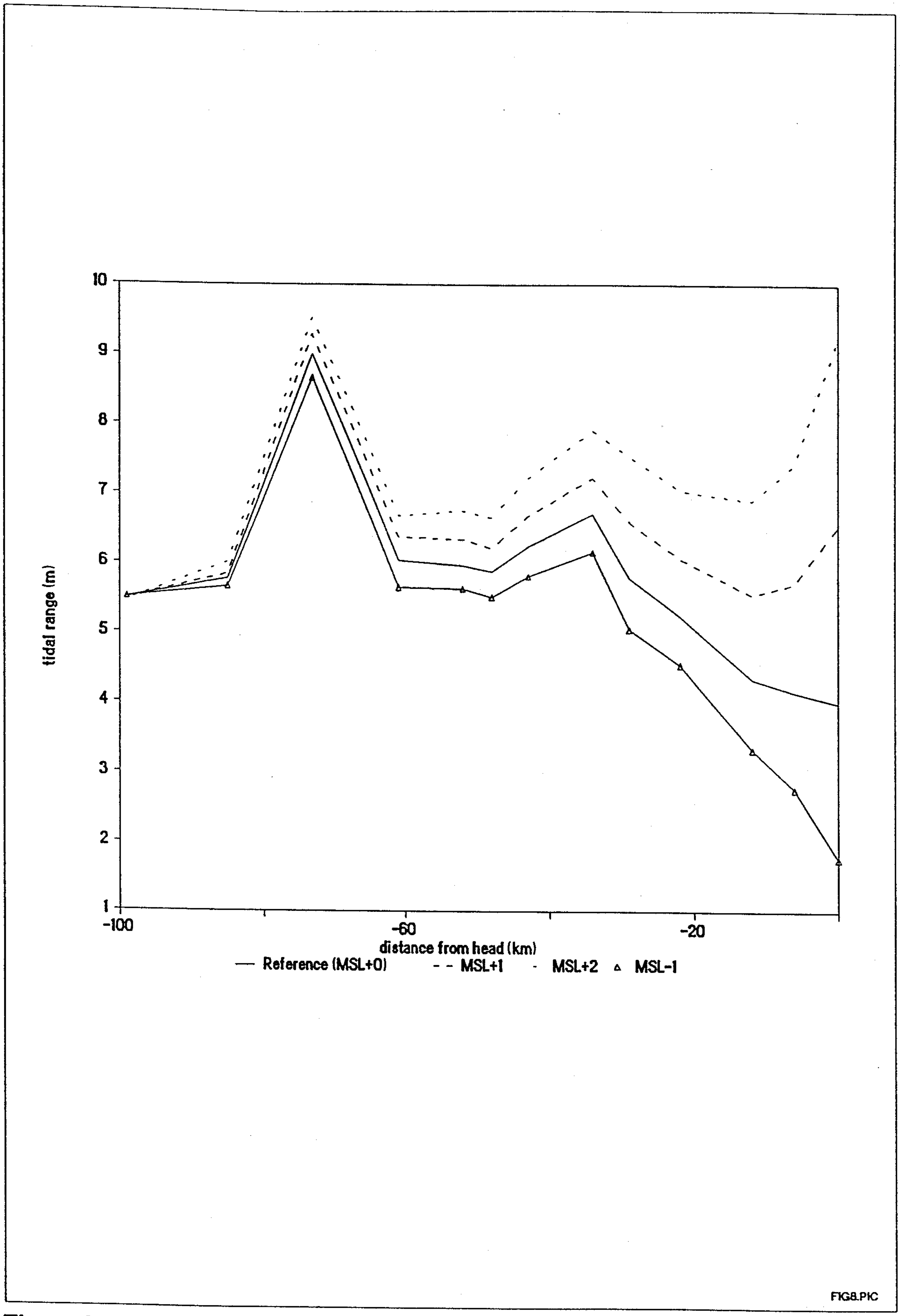


Figure 7 Reflected Wave Model of Parrett:
Comparison of velocities with observations at upstream end.



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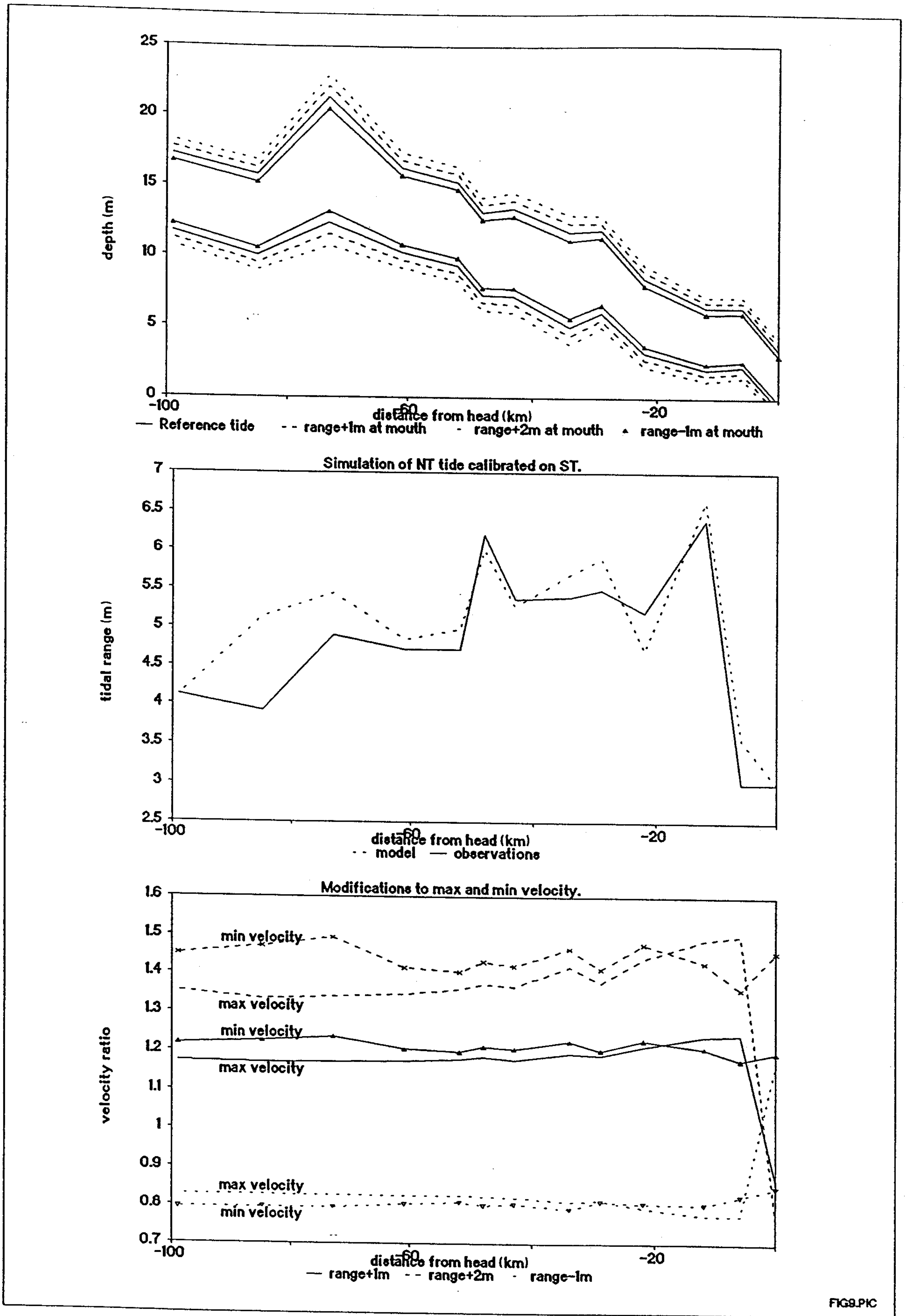


Figure 9 Reflected Wave Model of Thames: Effects of modifying tidal range at mouth.

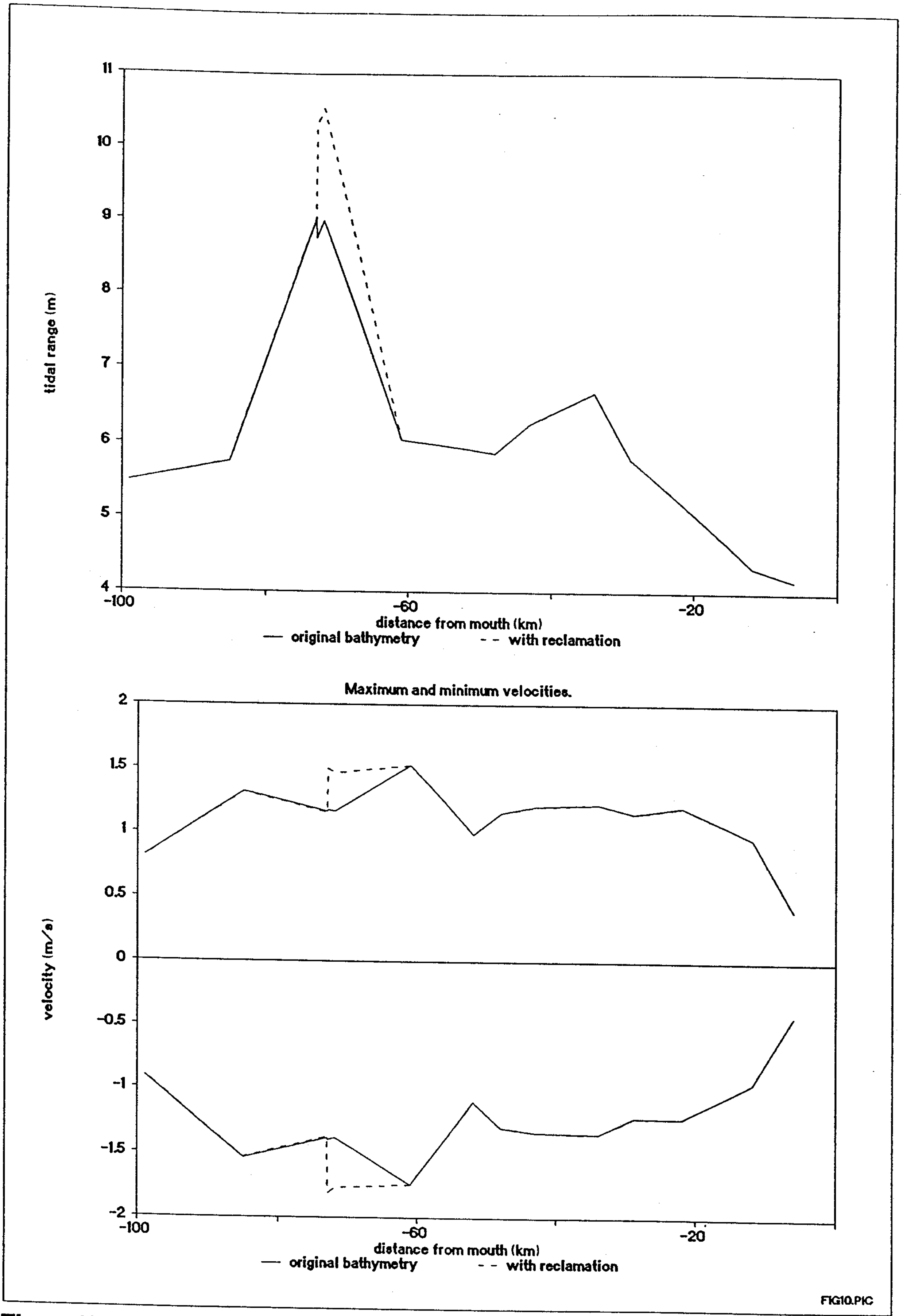
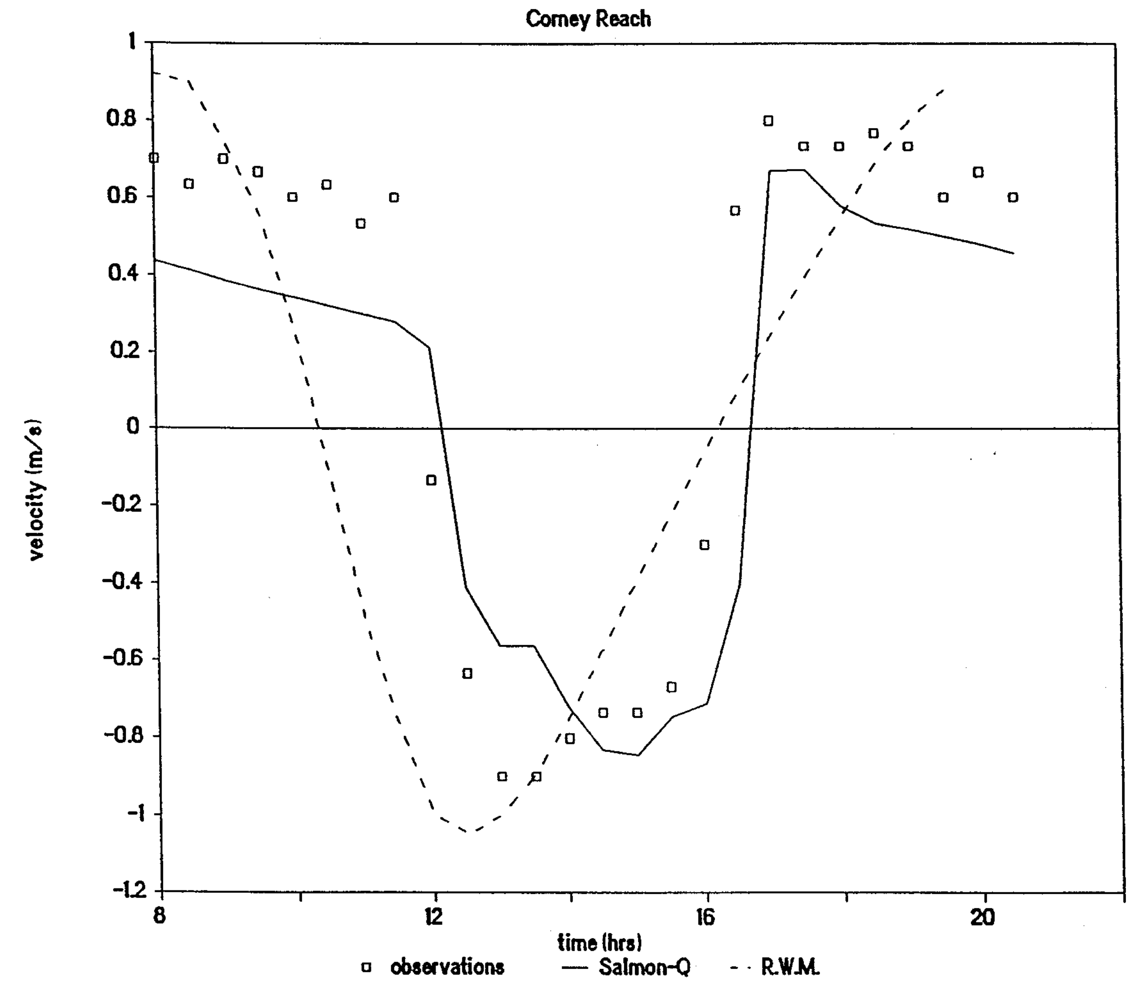
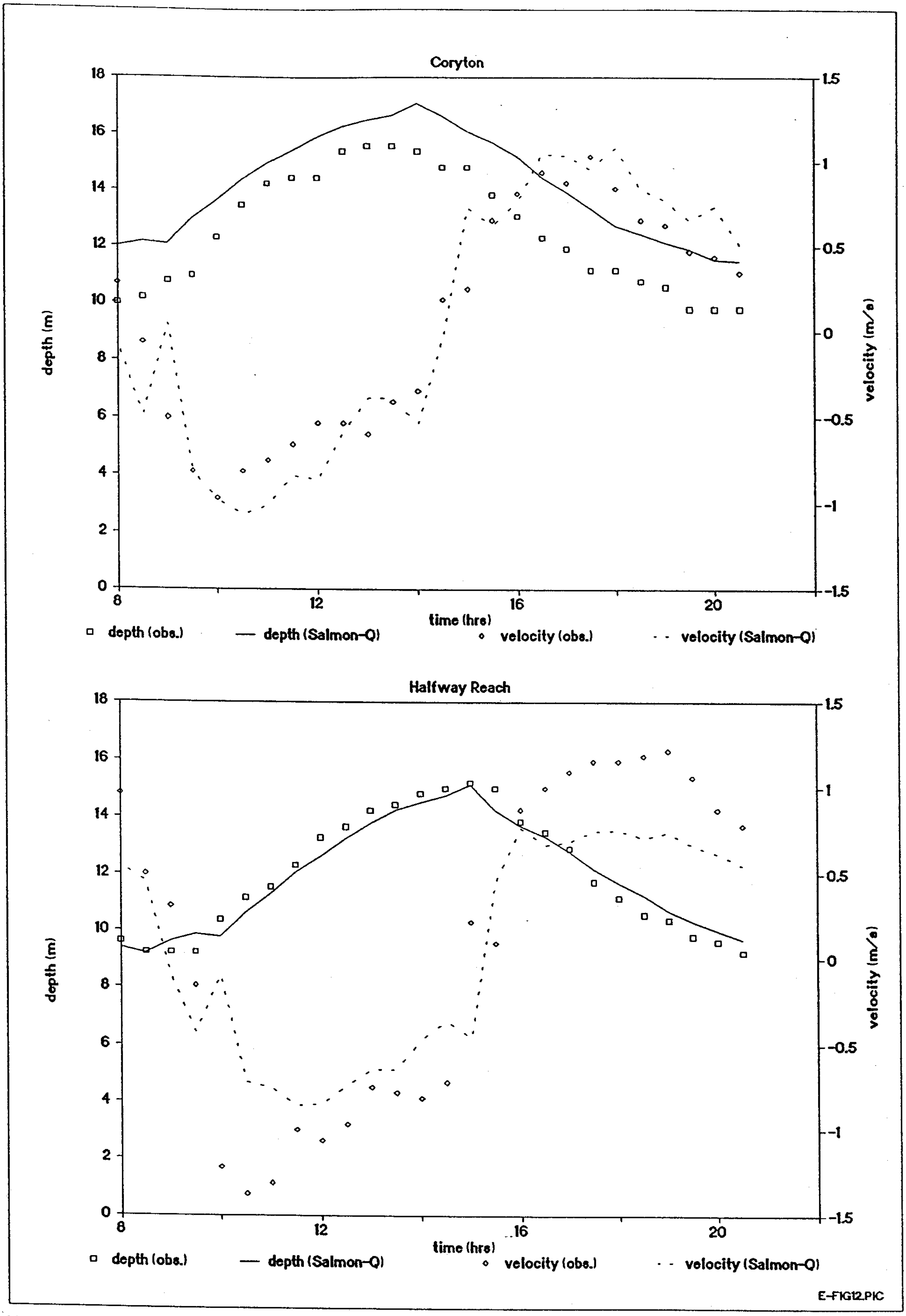


Figure 10 Reflected Wave Model of Thames:
Effect of reclamation on hydrodynamics.



E-FIG11.PIC

Figure 11 SALMON-Q and Reflected Wave Model velocities.



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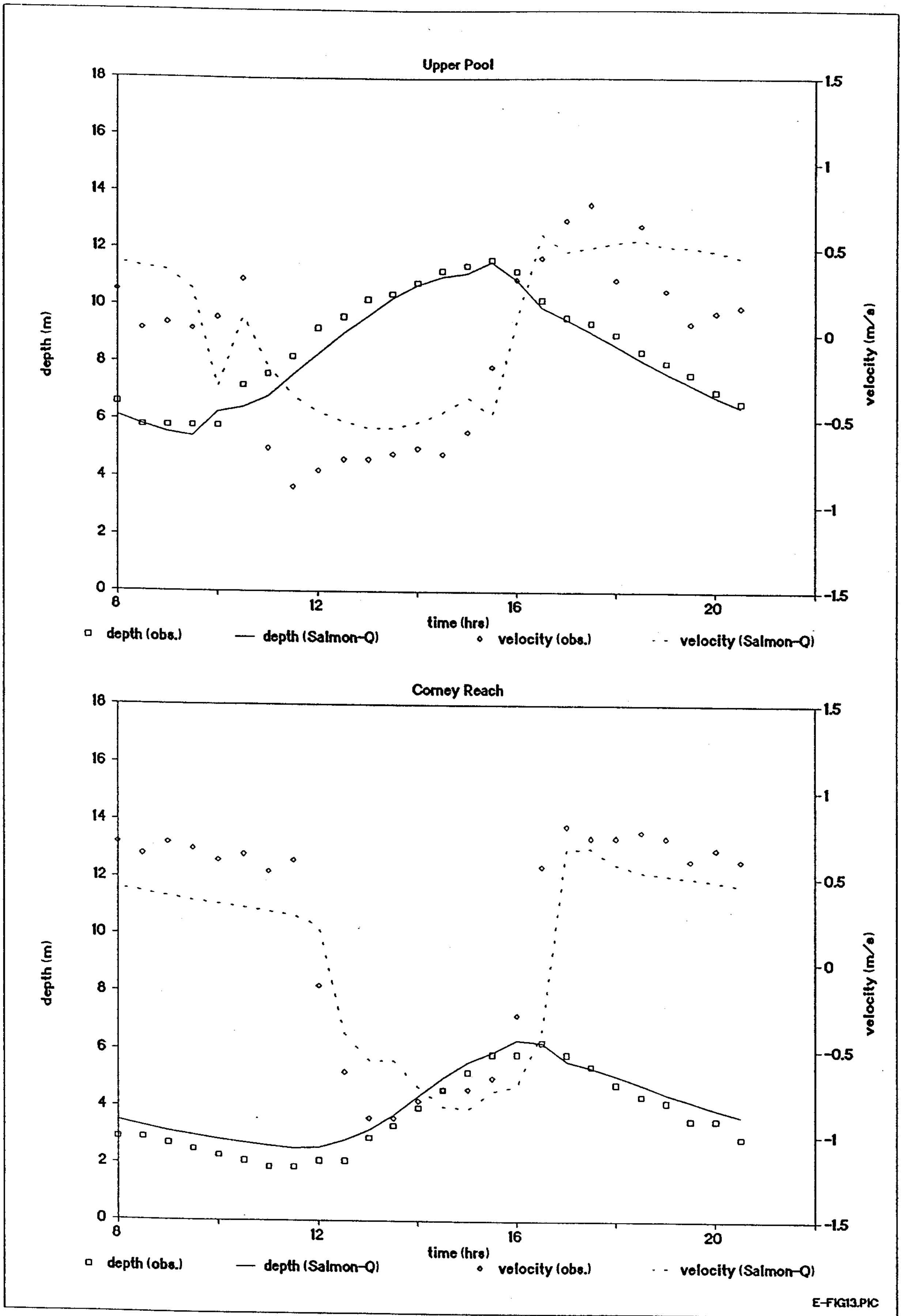
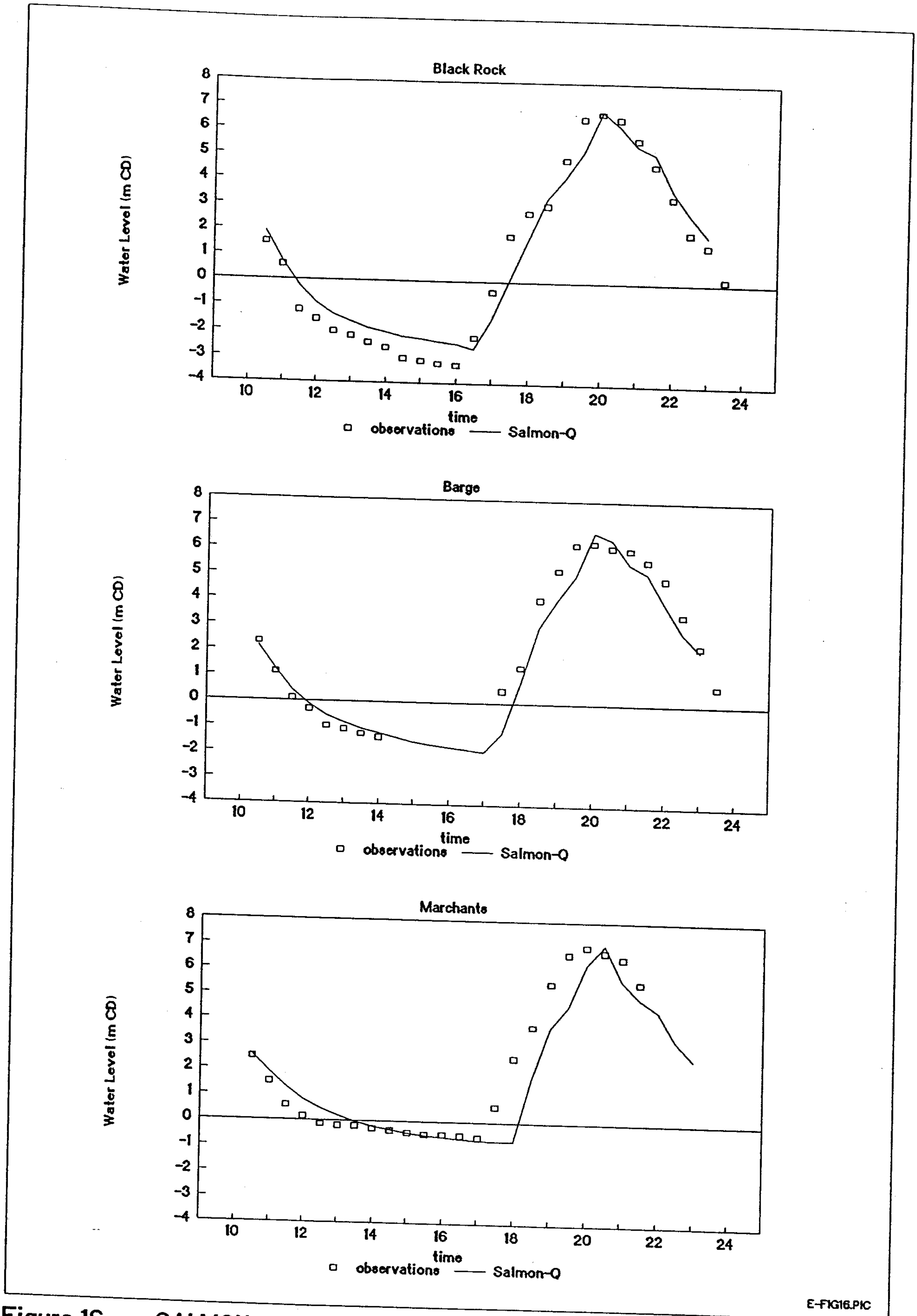


Figure 13 SALMON-Q model of Thames: Hydrodynamic results at upstream end.



E-FIG16.PIC

Figure 16 SALMON-Q model of Parrett: Water level results at downstream end.

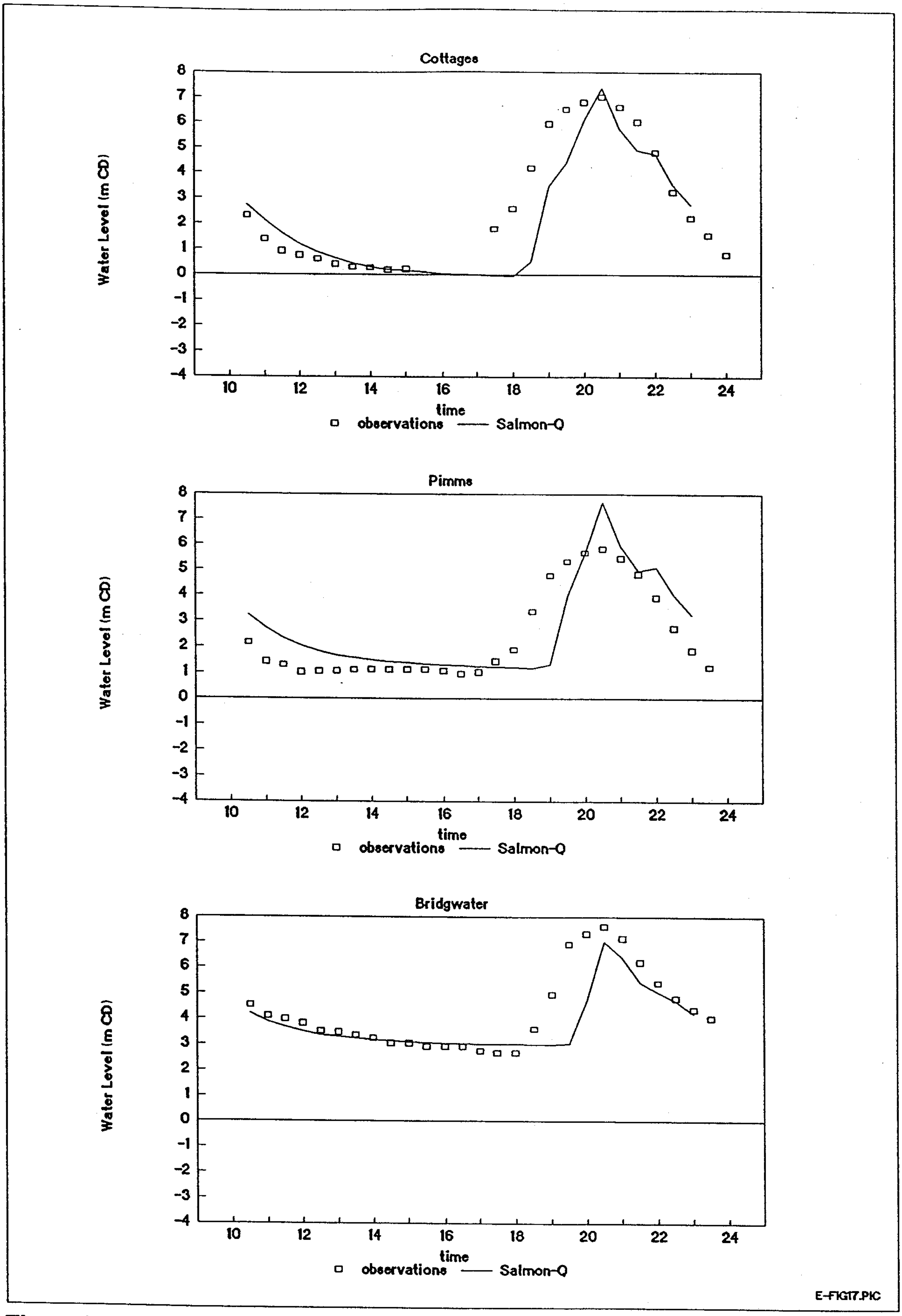


Figure 17 SALMON-Q model of Parrett: Water level results at upstream end.

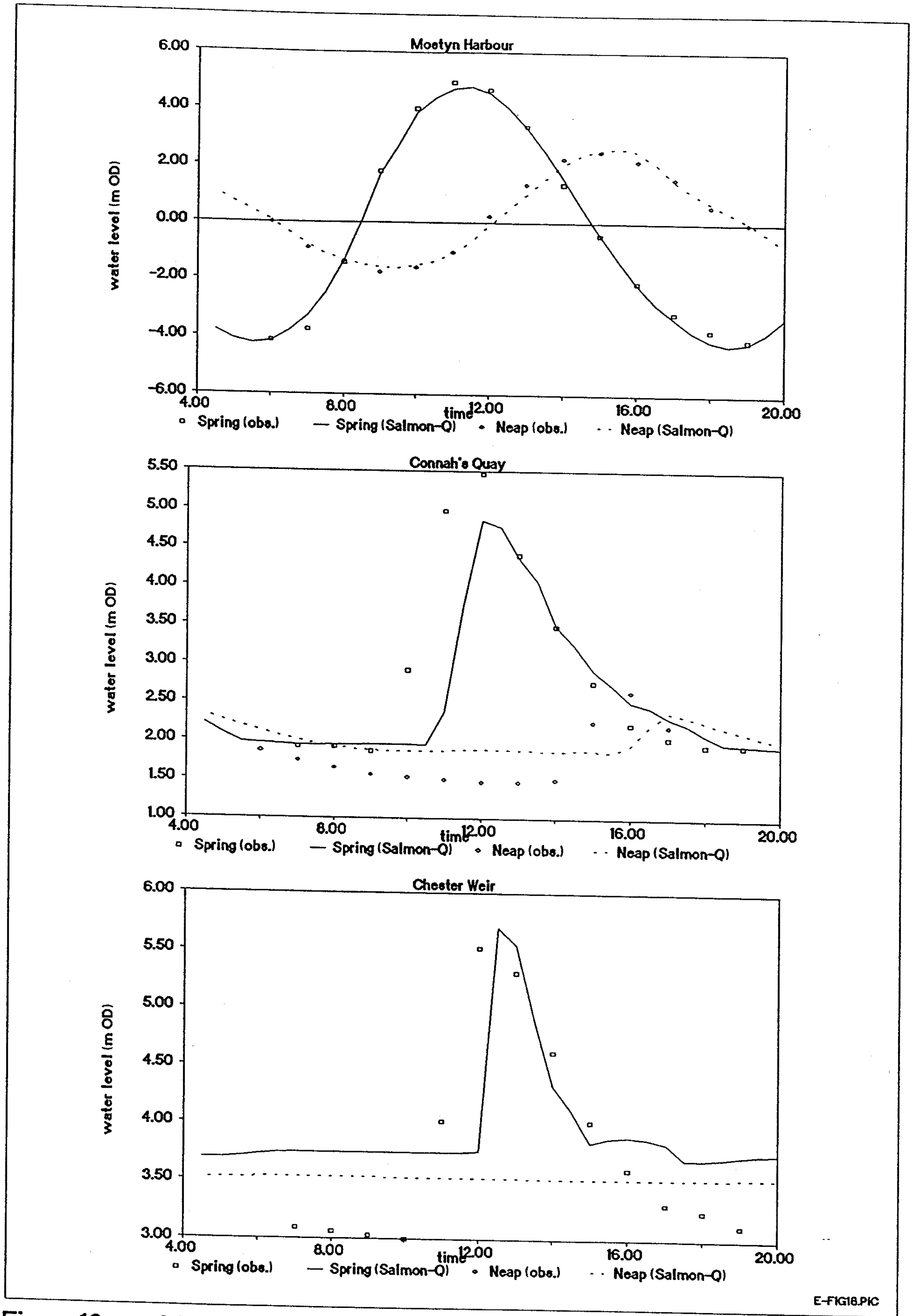


Figure 18 SALMON-Q model of Dee: Hydrodynamic results.

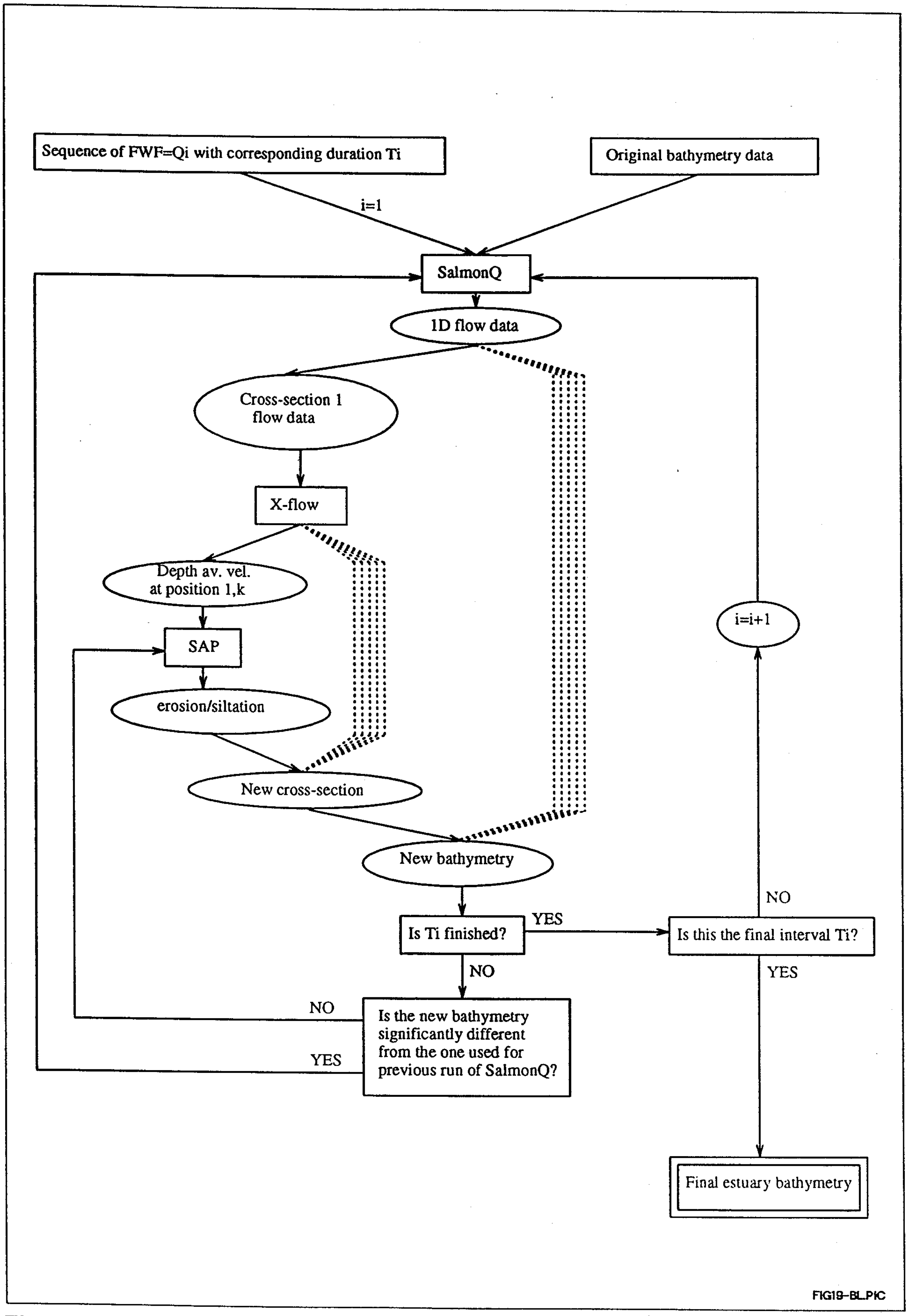


Figure 19 Schematic summary of REGIME modelling technique

Estuary Regime

Part 2: Classification and Regime Algorithms

J R Spearman

Report SR 366
July 1993

HR Wallingford

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Contract

This report describes work funded by the Department of the Environment under Research Contract PECD 7/6/234, for which the DoE nominated officer was Mr P Woodhead and by the Energy Technology Support Unit for the Department of Trade and Industry under Agreement E/5A/4109/2589, for which the ETSU project manager was Dr L M Davies. It is published on behalf of the Department of the Environment, but any opinions expressed in this report are not necessarily those of the funding Department. The work was carried out at HR Wallingford and Oxford Brookes University by Mr J R Spearman under the supervision of Mr J M Dennis, Division of Civil Engineering and Building, Oxford Brookes University and Dr M P Dearnaley of the Ports and Harbours Group at HR Wallingford.

Prepared by ....
(Job title)

Approved by John M Dennis ....

Date 3/12/93

Summary

Estuary Regime
Part 2: Classification and Regime Algorithms

J R Spearman

Report SR 366
July 1993

This report details the work undertaken in the period January 1992 to March 1993 for HR Wallingford with funding by ETSU on behalf of the DTI, and DoE. The work is part of a PhD programme and is the product of a collaboration between Oxford Brookes University and HR. It is part of a larger programme aimed at updating the current state of knowledge of estuary regime processes.

The report includes a literature review of the subject covering a period of about forty years, a brief review of case histories of civil engineering schemes constructed in estuaries, and consideration of the applicability of extremal hypotheses to estuary regimes.

An attempt is made to promote an estuary classification scheme based on fundamental parameters describing the regime state. Those parameters are identified from research and literature and using criteria of measurability and homogeneity, the appropriate parameters are found to be tidal prism, peak discharge, river discharge, salinity and wave action.

A correlation study is made to evaluate the ability of various widely used empirical formulae to describe the regime state. Relationships based on peak discharge and area seem to give the best correlation and describe the regime state very well. An attempt to improve the fit of such relationships by considering salinity and wave action effects is undertaken with some success.

An outline of a method for using a flow model with an empirical relationship is suggested and such a method is utilised in a simple prediction test and compared with a numerical approach. There is significant discrepancy between the predictions of the two methods used.

Finally further work is outlined for future study.

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1 Introduction

This report details the work undertaken by J Spearman in the period January 1992 to March 1993 for HR Wallingford with funding by ETSU on behalf of the DTI, and DoE.

The work is part of a PhD programme under the supervision of Mr J M Dennis (Oxford Brookes University), Dr M P Dearnaley (HR Wallingford) and Dr W R White (HR Wallingford). It is a product of collaboration between Oxford Brookes University and HR.

1.1 Objectives

This report is part of a larger programme aimed at updating the current state of knowledge of estuary regime processes. The ultimate goal of the research is to produce a computer-based model, based on regime morphological relationships, which can be usefully employed in a long-term predictive manner. This report is an attempt to proceed towards this goal by identifying the main processes affecting estuary regime, by assessing the validity of the morphological relationships which are currently proposed in literature; this information is then applied in taking some initial steps towards examining the usefulness of such a method for long-term prediction.

1.2 An Introduction to Regime Theory

This investigation is concerned with the viability of Regime Theory for the purpose of long-term prediction in estuaries. Regime Theory is the name given to an approach to channel theory based on empirial relationships between morphological variables, such as depth, width and slope, and hydraulic variables, such as discharge and velocity. This approach was first postulated in 1895 by Kennedy as a result of investigations into the stability of irrigation canals in India [Dennis 1988]. Since then the theory has been improved and extended to rivers, for a range of bed sediments. The use of regime in estuaries is much less well described. Most of the work in this area has not strictly been on estuaries but on the allied topic of tidal inlets. O'Brien, Bruun and Gerritsen, and Inglis and Allen stand out as the main contributors but recently there has been a minor resurgence in interest in this field with work by Eysink (1990) and O'Connor (1990). The reasons for the poor development of Regime Theory in estuaries is firstly that the effect of an oscillating tide makes conclusions on the relationships between morphological variables much more difficult. Whereas rivers or canals generally pass a similar discharge from one day to the next, with change occurring on a seasonal basis, estuaries experience dramatic change in discharges over an hourly basis. This dynamicism creates a much greater degree of 'noise', that is, spatially and temporally heterogeneous variation of properties within the estuary. A second reason is that whereas rivers and canals are primarily associated with non-cohesive sediment, estuaries abound in mud and silt. The flocculative processes resulting from complex physico-chemical interactions are not well described by science and quantifying sediment transport for this type of sediment cannot give anything like the reliability of the sediment transport theories developed for the non-cohesive sediment in river channels.

1.3 Why the Regime Approach?

There are many types of engineering works carried out in estuarine environments - dredging schemes, training schemes, jetties, bridge building and tidal barrage schemes are examples. The basic effect on estuary hydraulics is to reduce (or enlarge in the case of dredging) the area through which the tide can flow and to increase the friction forces in the vicinity of the works. This has the effect of reducing the energy available for sediment transportation in the estuary and as a result deposition occurs. The time scale over which this siltation occurs can be up to a century, and thus it is very common for estuaries to take fifty years or more to re-establish an equilibrium after the execution of civil engineering works.

With the changes in public awareness of the environment that have happened over the last twenty years, it is now necessary to be able to predict the final ecological consequences of a proposed scheme and this means making a long-term prediction for the future. Even with careful planning and investigation it can be extremely difficult to foresee the possible consequences of a proposed scheme. With today's generation of fast computers and related measuring techniques it is now much easier to predict possible outcomes by using physical and numerical models but research into large time-scale effects is still inhibited by cost and by uncertainty in observed measurements and in the description of sediment processes. For long-term predictions such uncertainties propagate through iterative numerical models rendering the final results questionable and at best qualitative.

Regime Theory asserts that an estuary readily assumes an equilibrium state, if left to its own devices, and seeks to identify the nature of this equilibrium in terms of morphology. Some authors are sceptical about this and conclude that an estuary never actually achieves a balance. This is plausible given the constantly varying tides, and the even more widely spatially varying freshwater flow, together with the variation in geology that is experienced by the majority of estuaries. However, it has been established that the regime approach works satisfactorily in rivers and canals and the work of O'Brien, de Jong and Gerritsen, Eysink, and others, has shown that a very definite correlation exists between estuary tidal discharge and/or tidal prism and cross sectional area, which is highly suggestive of equilibrium attenuation.

If this idea of an estuary in balance can be assumed, then it could prove to be a most valuable tool for long-term predictive purposes. The need for representation of all the physical processes and for quantifying highly spatially and temporally variable parameters will go, greatly simplifying the modelling procedure and greatly reducing its cost. The results gained will never be strictly precise, but will however give a much more accurate picture of future events.

1.4 Knowledge-Based Systems, Databases and Long-Term Prediction

One of the original aims of the project included a proposal to initiate the setting up of a knowledge-based system which could be used as a predictive tool and/or as a data base for regime information which could then be utilised for further research. This original idea for the direction in which the research should progress was found to be ill-defined as the investigation into the field of regime proceeded. The reasons for this are given in the following paragraphs.

The idea of a database to store regime information was found to be an over-ambitious one in the context of an all-round investigation into regime. Before the data base could be designed important questions need to be answered, such as,

'What exactly is the database to be used for?',
'What is to be stored on the data base?',
'What format should this storage take?'.

These are not trivial questions. Failure to consider these questions in depth would render the database ineffective and ultimately a costly white elephant. To illustrate this point, the second question is discussed below.

The area which is ultimately under investigation is that of the applicability of regime theory in estuaries. This discipline is in its infancy and much is not known about the major factors that influence estuaries, let alone the less important factors. To attempt to construct a database that will be effective in the future either requires taking the risk that information needed for future regime work, as the sophistication of the methodology improves, is not included in the database, or requires a catch-all storage of data that might be necessary in the future, which is both costly and inefficient. Indeed it may turn out in the future that the regime approach is unfruitful for estuary purposes and until the basic research has been undertaken, the answer will not become apparent. Hence the creation of a database is premature, given the current status of regime knowledge.

The development of site specific Geographical Information Systems (GIS) is becoming more common at some sites around the UK. Such systems sometimes allow for simple algebraic operations on the data contained within them. Setting up and maintaining these systems is known to be an intensive and expensive operation. At this initial stage, given the volume and type of data that exists for some UK estuaries, it would be hard not to recommend the use of such systems. However, the use of such a database as part of a GIS system for other estuaries where far fewer records are available - possibly only such items as geometry and an estimate of catchment size - would be inappropriate. Accordingly, the choice of database, should a knowledge based system, be appropriate, must be left until the approach to the regime prediction problem is defined.

Furthermore, consultation has established that the concept of using a knowledge-based system to solve the long-term predictive problems involved in civil-engineering schemes in estuaries might also be flawed due to the nature of the problems that knowledge-based systems are designed to solve. Such systems are created to store knowledge in the form of a large set of 'rules' or truths. An example of this is,

'A symptom of appendicitis is stomach pain',

which could be one of the rules in a diagnostic medical program. If the patient did have stomach ache, the program would investigate the possibility of appendicitis, among other possible causes, and would ask other questions which might lead to the conclusion that the patient had appendicitis if they were answered appropriately. The power of such a system is in situations where knowledge is already 'known' and can be put in the form of 'rules' where a layman wants an 'expert' opinion given a full set of information.

This differs from the long-term predictive problem since in the latter case there is a set of information known, which is the state of the estuary at present, and the nature of the proposed disturbance to it, but there are no rules. The expertise on the problem is given by using a model, in the widest sense, to take the data available and convert it to a new set of data detailing the future state of the estuary. The 'system' would therefore be composed of a method for storing the data of the present estuary state, and the effects of a scheme, together with the 'model', and a means for presenting the new information about the future estuary state. The expertise is contained only in the 'model' and this may be composed of one or more numerical, physical or empirical models and these are based on a set of algorithms. The algorithms are implemented and a new set of information is derived.

A proposal for such a 'model' is outlined in Chapter 8.

1.5 Report Structure

The remainder of this report is split into eight chapters. Chapter 2 is a literature review. Chapter 3 is a qualitative review of the effect of civil engineering works on estuaries. Chapter 4 is concerned with the identification of the main processes affecting estuary regimes. Chapter 5 is an assessment of the applicability of extremal hypothesis theory to estuary regimes. Chapter 6 assesses the performance of several morphological regime relationships, while Chapter 7 takes one of the better performing morphological relationships and examines the effect of adjusting the relationship to allow for wave action and salinity. Chapter 8 describes a method for use of a regime relationship in a long-term predictive model and tests such a model using a projected scenario from the Avon estuary. The conclusions of the report and proposals for further work are given in Chapter 9.

2 Literature survey

This chapter consists of a historical summary of the work found in the literature which is relevant to a study of regime theory in the estuarine environment.

The following definitions are used in this document, except where specified otherwise:

P	- Tidal Prism at a mean Spring tide (m^3),
A	- Cross-sectional Area (m^2),
A_{max}	- Maximum cross-sectional area (m^2),
T	- Period of Tide (s^{-1}),
Q	- Discharge (m^3s^{-1}),
Q_{max}	- Peak Discharge (m^3s^{-1}),
D	- Depth of channel (m),
R	- Hydraulic Depth (m).

O'Brien [1931]:
An empirical relationship was derived between the tidal prism and the cross-sectional area of an inlet entrance, from studies on the western coast of the U.S.A.,

$A = 1000.P^{0.85}$ (Imperial units).

Keulegan [1951]:
Keulegan analysed flows from tidal inlets and produced the formula,

$Q_{max} = \pi CP/T$

where C is a coefficient which ranges from 0.81 to 1.0, averaging about 0.86.

He went on to study further the hydraulics of tidal inlets and improve on the work of

Brown [1928].
Brown had analyzed the flow through a tidal inlet by making the following assumptions:-

* The tidal variations in both ocean and bay are sinusoidal.
* The flow area of the inlet channel (below MSL) is constant (prismatic) from ocean to bay.
* The surface area of the bay does not vary with the tidal level in the bay.
* The surface of the bay remains horizontal throughout the tidal cycle.
* The depth of the inlet channel is large as compared with the range of tide.
* The difference in head necessary to accelerate the mass of water in the inlet channel is neglected.
* There is no surface runoff into the bay.

Keulegan deduced that the behaviour of the water level in the bay of a tidal inlet was not sinusoidal. This is because the flow through the inlet channel is proportional to the square root of the difference in head. He further deduced that the ratio of the tide in the bay to that in the ocean, the slack water lag in the inlet channel, and the maximum velocity in the channel could be represented as functions of K, where K is given by,

$$K = \frac{T}{2\pi a_0} \frac{A_C}{A_B} \sqrt{\frac{2ga_0}{F}}$$

where

a_0 is the ocean amplitude;
A_C is the cross-sectional area of inlet channel;
A_B is the area of the bay;
F is the 'impedance' of the inlet channel given by,

$F = \Sigma k + fL_c/4R$,

where

k is the coefficient of velocity head loss;
f is the friction coefficient;
L_c is the length of hypothetical channel of area A having same friction loss as real channel;
R is the hydraulic radius.

Keulegan compared his method to that of Brown and found that in situations where the difference in results should be at its greatest, there was a 10-15% difference between the two sets of results. O'Brien [1980], commenting on the usefulness of the work of Keulegan, points out that such a difference could be all accounted for by data error, and since Brown's method is much simpler in application it may be the more efficient of the two. However, he points to the usefulness of the Keulegan method as a means of ordering and analysing flow data.

Leopold and Maddock [1953]:
Leopold and Maddock postulated that channel parameters velocity, width, depth and slope were related by a power law to the mean discharge.

$$V \alpha Q^{m} \ , \ B \alpha Q^{b} \ , \ D \alpha Q^{f} \ , \ S \alpha Q^{z}$$

where

B is the width,
D is the depth,
S is the slope,
V is the velocity of the channel.

The exponents were found to take the values m=0.1, f=0.4, b=0.5, with z varying between -0.5 and 1.0.

Langbein [1963]:
Pillsbury [1939], defined an ideal estuary as one in which the tidal range,depth and current are uniform through the length of the channel. He showed that in such an estuary the width decreases exponentially with the distance up the estuary and

$$B = B_0 e^{-(n \cot\varphi + k)x}$$

where

φ is the lag between maximum slope and maximum velocity,
n is a parameter dependent on $\sqrt{D}$,
k is a constant.

He also showed, using Leopold and Maddock's definitions, that $S \alpha D^{-1/2}$, which implies from Leopold and Maddock's equations that $z=-f/2$, and that $V \alpha D^{1/5}$, implying $m=f/5$.

With the continuity relation that Q=VDB, or m+b+f=1.0, only one more equation is needed to solve this set of equations.

Langbein proposed that "an analogy with entropy production in a steady state system leads to a statement that the geometry of natural waterways is governed by two opposing influences; equal work/unit area of bed and minimum work done in the system as a whole". By allowing V and D to become variables (unlike in Pillsbury's work where they were constants), he showed approximately that, P_w = 1.7b-0.7, where b is the width exponent and P^2 is the relative work performed during a tidal cycle in an estuary. Langbein went on to show, that by including principles of entropy and the logic developed in a previous paper [Leopold and Langbein 1962], b must take an average value of b=0.71 whence substitution in the Leopold and Maddock equations gives m=0.05, f=0.24, z=-0.12.

Myrick and Leopold [1963]:
Noting that the shape and characteristics of a river channel were believed to be determined by the 'dominant discharge', the effective discharge most influential in river morphology, i.e. the bankfull discharge, these authors adapted the power functions of Leopold and Maddock, by using the bankfull discharge, Q, for tidal channels. The values of m, b, and f were deduced for six cross-sections in a tidal channel in a salt marsh during the flood tide for increasing and decreasing velocity situations. The results gained were poor. However, the work was extended by measuring hydraulic parameters in a downstream direction or bankfull discharge at all cross sections and by comparing two other tidal channels for which data was available. The results obtained were comparable to the theory developed by Langbein above,
i.e., b=0.77, f=0.23, m=0.00 .

Colby [1964]:
Colby investigated the relationship between the average bed material discharge and the mean cross-sectional velocity. He computed an expression for the total sediment transport in a river for a combination of mean velocities, depths, grain sizes and temperatures. The interesting result is that at a velocity of 3ft/s, the sediment transport is independent of depth. This paper may be the origin of the commonly held rule of thumb amongst engineers that the maximum velocity in an estuary or tidal channel is roughly 1 m/s. Bruun [1968] verified his observations of this tendency by referring to the work of Colby. Unfortunately the results of Colby do not stand up if more accurate sediment transport equations than appear in the paper are used [Skou 1990]. Then, there is little, if any, evidence of an equilibrium velocity, and certainly not for speeds of approximately 1m/s.

O'Brien [1969]:
A study of inlets in equilibrium under effects of tidal currents from the North West coast of the U.S.A. gave rise to another similar relationship,

$A = 4.072\times10^{-2}.P^{0.85}$ (S.I. units).

This report contained inlets with groynes/breakwaters, etc, and the exclusion of these data sets gave a relationship of,

$A = 6.562\times10^{-2}.P$ (S.I. units).

O'Brien concluded that:

* The equilibrium minimum flow area of a tidal inlet is controlled by its tidal prism.
* If the tidal area is connected by two/more channels, closure of one or more channels will enlarge the flow area of the others.
* The equilibrium flow area of a tidal inlet depends hardly at all on bed material size.
* Tractive force is not a meaningful criterion for the equilibrium conditions of tidal inlets.

* Estuaries of large rivers follow same flow area/tidal prism relationship as tidal lagoons and bays.

Nayak [1971]:
With the data obtained by O'Brien and further data obtained from physical modelling, Nayak found similar results to O'Brien,

$A = 3.58 \times 10^{-2} P^{0.85}$ for inlets with jetties(S.I.) and

$A = 6.562 \times 10^{-2} P$ for inlets without jetties(S.I.)

Johnson J.W. [1973]:
Johnson found similar relationships to Nayak and O'Brien but used mean instead of spring tidal prism.

$A = 5.784 \times 10^{-2} P^{0.9}$ for inlets with jetties (S.I.) and,

$A = 5.971 \times 10^{-2} P$ for inlets without jetties.

Mason [1973]:
Mason investigated various regime equations in the case of reversing tidal flow through inlets (Lacey, Blench and Simons/Albertson) and found the latter to be the most accurate, namely,

$A = 1.1\ K_1 K_2 Q^{0.86}$ $R \leq 7'$ (Imperial units)

$A = 1.8\ K_1 Q^{0.5} + 0.84\ K_1 K_2 Q^{0.86}$ $R > 7'$

Mason stated that these equations hold when

* The inlet is located in sand with a median grain size of 0.2mm.
* The ocean tidal period is semi-diurnal.
* The discharge through the inlet is over a tidal cycle.

O'Brien [1976]:
The author cast doubt on the validity of Mason's relationships for their over-simplified assumptions (e.g. sinusoidal tide variation, neglection of friction losses in flow channel). To remedy this he classified tidal inlets into six types:

* short frictionless channels;
* small, deep lagoons;
* long lagoon, estuary;
* narrow, deep inlet channel;
* short, deep inlet channel;
* long inlet channel.

For each of these simplified assumptions were made about the character of friction loss and tide variation.

Chantler [1974]:
Chantler investigated the validity of the power function approach on seven different estuaries and noticed a marked tendency for the values of b and f to increase and decrease respectively towards the estuary mouth. That is, the estuary widens and shallows out in a seaward direction. He ascribed this

effect to the removal of lateral restraint at the estuary mouth, and noted the need for a function to describe this effect. He also investigated the relationship between cross sectional area (i.e. the product of depth and width) and maximum discharge and found a much better correlation than in the case of the power functions. His findings were that Q_{max} was roughly proportional to area, suggesting that maximum velocity is roughly constant along an estuary length. He went on to investigate the relationship between breadth and depth along the estuary. Although a trend was identified to a particular relationship between Q_{max} and B/D, nothing was conclusive.

Renger and Partenscky [1974]:
The authors approached the problem using horizontal layers to describe the volume capacity of the tidal channel instead of the vertical cross section approach. Relationships between the tidal basin area, E, and the low water volume and the low water area were derived semi-empirically together with a relationship between the elevation and the relative volume of the prism.

$V_{MLW} = 8.10^{-3}E^2$ $\qquad E_{MLW} = 2.5\ .10^{-2}E^{3/2}$

$z^* = \log_a (V/V_{MLW})$ $\qquad$ where $a = 5.E^{-0.272}$

The above relationships were studied for 22 different tidal basins on the North Sea Coast of Germany and gave good results.

They can be combined to give,

$V = 8.10^{-3}\ E^2(5E^{-0.272})^{z^*}$.

Jarrett J.T. [1976]:
Jarrett studied three sets of sandy tidal inlets along the American coast and found a different prism/area relationship for each.

Atlantic: $A = 8.654x10^{-2}P^{1.05}$ (S.I.)

Gulf Coast: $A = 3.381x10^{-2}P^{0.84}$ (S.I.)

Pacific: $A = 4.387x10^{-2}P^{0.91}$ (S.I.)

Shigemura [1976]:
The author carried out a statistical multiple regression analysis between ten parameters and the throat area and throat width of tidal inlets on the pacific coast of Japan. The parameters used in the regressions were:

* wind energy off the tidal inlet,
* direction of wind energy off the tidal inlet,
* wind energy penetrating into throat section,
* direction of wind energy penetrating into throat section,
* wave energy off the tidal inlet,
* direction of wave energy off the tidal inlet,
* direction of wave energy penetrating into throat section,
* wave energy penetrating into throat section,
* direction energy penetrating into throat section,
* volume of tidal prism for mean tidal range,
* mean rate of tidal flow at the throat section.

The results of this experiment were poor, but tidal prism was found to be the most dominant parameter, followed by discharge, relating to throat area.

The author then went on to classify the inlets in terms of a dimensionless parameter, r_{as}, which was the ratio of throat area to backed bay. For each class the same regression on throat area above was carried out and very good correlations were obtained. Similarly when estuaries were classified according to a dimensionless parameter, r_{hxb}, the ratio of maximum depth at throat to throat width, very good correlations were found for throat width with the parameters listed above.

Krishnamurthy [1977]:
Starting from considerations of sinusoidal flow and of a logarithmic velocity distribution, the author derived a formula for tidal prism as follows,

$$P = 1.25(By_0)V_{fc}T(1+\frac{2a_0}{\pi y_0})(\ln\frac{10.93y_0}{k})$$

where

B is the width;
y_0 is the depth;
a_0 is the tidal range;
V_{fc} is the friction velocity corresponding to the critical shear stress;
k is the roughness coefficient.

The implications of this result are that the prism/area relationship depends very little on the roughness coefficient k, although tidal range (for meso and macrotidal estuaries), shear stress, and tidal period are major factors influencing the estuary. The relationship outlined above was tested on a score of estuaries. The results were disappointing but possibly this was due to assumptions made about values of input parameters due to the lack of real data.

Byrne et al [1980]:
O'Brien's and Keulegan's works were investigated on fourteen inlets in the lower Chesapeake bay in the period 1978-1979. The results from this study were compared with the results of studies by Jarrett [1976] and Mayor-Mora [1977].

Best fit lines in the style of O'Brien were derived,

Jarrett data:	$A=9.954 \times 10^{-6} P^{1.14}$	r=0.97
Chesapeake Bay:	$A=9.902 \times 10^{-3} P^{0.61}$	r=0.87
Mayor-Mora:	$A=7.61 \times 10^{-3} P^{0.68}$	r=0.95

The conclusions of the authors were as follows:

* Departure from inlet throat/tidal prism relationships derived from ocean inlets for $A<100$ m^2;
* The transition zone from ocean inlet geometry to smaller throat geometry is at $100<A<500$ m^2;

* In smaller inlets, maximum velocity is much less than for ocean inlets;
* In general tidal phase lag and tidal range ratio were equal.

de Jong and Gerritsen [1984]
The authors studied the Western Scheldt, a large estuary in the Netherlands and carried out various regression analyses between morphological and hydraulic parameters for each different cross-section. Very high correlations were found for the regressions listed below, for all cross-sections except those near the estuary mouth.

* Q_{max} against mean cross-sectional area, A;
* P against A;
* $Q_{max}/C(t_s/\rho g)^{0.5}$ against maximum cross-sectional area.

This latter regression had the best fit, and moreover, when shear stresses near the estuary mouth were adjusted to allow for wave effects, it was found that the cross-sections near the estuary mouth also fitted into the relation above. The relation is similar to that put forward by Krishnamurthy [1977] with a Chezy roughness coefficient instead of the hydraulic roughness of flow. Here the data used in the regression is much more detailed and the fit correspondingly much better.

The authors also put forward a criterion governing the stability of a tidal channel, namely that a channel is stable if,

$$\int_{t_1}^{t_2} \frac{\partial q}{\partial x} dt + \Delta q_y + \Delta C_t = 0$$

where,

Δq_y is the lateral transport in time t_2-t_1,
ΔC_t is the change in suspended sediment concentration.

This equation is useful however only in a qualitative sense since sediment transport cannot be precisely evaluated.

di Silvio [1989]
di Silvio proposed a zero-dimensional model of a tidal lagoon to model morphological evolution. The lagoon was schematized into three components -tidal flats or shoals, littoral and tidal channels. His model was based on the idea that although at a particular location, at any one time, sediment transportation is in a non-equilibrium state, if an average is made of hydrodynamic quantities both spatially and over a long period of time a relationship exists between these quantities and the sediment concentrations in the area.

The following balance equations were composed by considering the possible fluxes of sediment:

$$T_x - T_y - I_x = D_x$$

$$T_y - I_y = D_y$$

where T_x and T_y are the annual net transport from the tidal channels into the sea and from the shallow areas into the tidal channels;

I_x and I_y are the annual sediment input in the channels and the shallow areas (e.g. sediment transported by the rivers and the wind into the lagoon);

D_x and D_y are the volumes of sediments respectively removed each year from the bottom of the channels and the shallow areas.

If V is the volume of water that is exchanged each year between shallow areas and channels, then it follows,

$$T_y = V.(y - x)$$

where y and x are the concentrations of sediment in the shallow areas and the channels averaged over space and time.

Then, if V_m is the water volume exchanged between the channels and the sea one may write:

$$T_x = V_m.(x - z)$$

where z is the time-averaged concentration of sediments in the sea, near to the inlet.

Finally, since the shallow area is roughly the average surface,

$$V \approx V_m \approx 700\ a_m.s_m$$

where a_m is the annual average tidal range, s_m is the average estuary surface area and 700 is the average no. of tides per year.

Combining all the above equations into two simple coupled differential equations produced the following solutions as $t \rightarrow \infty$, (assuming no sea-level rise and no soil subsidence),

$$x \rightarrow z + (I_x + I_y)/V \qquad y \rightarrow z + (I_x + 2I_y)/V$$

Further simple equations can be formed.
In the channels the following equation applies,

$$x = f_x\ (s_m.a_m)^5\ A^{-5}$$

where f_x is assumed to be constant for a certain lagoon and is dependent on the nature of the sediment and the space-time distribution of velocity in the channel.

In the shallow areas,

$$y = f_y/h$$

where h is the average depth of the shoals, and f_y is assumed constant for a certain lagoon.

Substitution of the model results gives,

$A_\infty = V\,(f_x/x_\infty)^{1/5}$ $\qquad$ $h_\infty = f_y/y_\infty$

Skou [1990]:
Skou published, in an exhaustive paper, the work that had been undertaken on the subject of tidal inlet morphology. In this literature survey Skou also included some of her own investigations, notably on response mechanisms and flushing characteristics of tidal inlet channels.

In the discipline of river regime study much work has been carried out on the subject of extremal hypotheses. Extremal theory postulates that a channel will adjust its morphology so as to maximise or minimise some property such as stream power or sediment transport. The suggestion is that if the extreme of these properties can be deduced then it will correspond to the equilibrium state of the channel. Work by White et al [1982] has shown, that maximising sediment transport is equivalent to minimising the stream power in a channel, and therefore either of these properties will lead to a representative extremal solution. A good summary of extremal theory is given in Bettess and White [1987].

Skou investigated the variance of net sediment transport for different morphological characteristics using a 1D sediment transport model of a particular tidal inlet for which there is observational data. The depths and widths of the inlet channel for which net sediment transport is a maximum can be clearly distinguished and correspond to those observed in the inlet. The observation of cross-sectional area, however, was much smaller than the maximum net transport value by a factor of 2/3. This discrepancy may be due to fallacies in the extremal hypothesis or due to errors caused by modelling the inlet using a rectangular channel of constant width and depth throughout its course. Skou came to the conclusion that tidal inlet morphology was independent of net sediment transport.

Gerritsen et al [1990]:
The authors summarised the regime work carried out previously on the Western Scheldt estuary by de Jong and Gerritsen [1984] and then investigated further results gained for a correlation of tidal prism against cross sectional area. They suggested that hydraulic radius (or depth) has a significant effect on the velocity and therefore on Q_{max} and P, and based this idea on a regression of channel velocity against R. When this relationship was substituted into O'Brien's formula (for a semi-diurnal tide) the relationship became as follows,

$$A_c = (7.64\times10^{-4})\,p_w^{0.2}\,P^{0.8}$$

where p_w is the wetted perimeter, A/R.

The authors went on to formulate a dimensionless parameter relating the major factors involved in channel stability. The parameter investigated was derived from work by Bijker [1967], and is given by,

$$\frac{v}{\left(\frac{\Delta D}{\mu}\right)^{\frac{1}{2}} C} = \text{constant} \qquad \text{where,} \quad \mu = \left(\frac{C_{d90}}{C}\right)^{\frac{3}{2}}$$

where C is the Chezy coefficient, Cd_{90} is the corresponding Chezy coefficient relating to a particle size of d_{90}.

The parameter was investigated for the Wadden Sea Inlets and was found to vary between values of 30 and 45 (SI units). Whether this scatter was due to the uncertainty involved in determining C or to the unsuitability of the parameter was not clear.

The authors then proposed a system for using morphological regime relationships with a 1D model. The model was used to establish the new discharge/tidal characteristics under the new regime and the regime relationships predicted the resulting equilibrium, which formed the boundary conditions for the next run of the model, and so forth in an iterative fashion. It was suggested that the effects of a sudden major change on the regime, such as engineering works, develop in an exponential way, the rate of change decreasing with time, i.e.,

$$\frac{\partial A}{\partial t} = -\lambda(A - A_0)$$

where

A_0 is the equilibrium cross-sectional area,
λ is a constant controlling the rate of change.

Kondo [1990]:
The author studied the effects of sea level rise on tidal inlets. In a previous paper he proposed the relationship,

$$\frac{P}{A} = K_S \sqrt{a_s g}\,.T$$

where a_s is the tidal amplitude, and K_s is a parameter depending on the littoral drift/tidal discharge ratio.

In his 1990 paper Kondo developed the above relationship by quantifying the effect of littoral drift on cross-sectional area. The resulting relationship was,

$$A = 1.02 Q_{max}^{1.1} a_s^{-0.5} M_l^{-0.1}$$

where M_l is the annual littoral drift at sea.

O'Connor et al [1990]:
A method for using Jarrett's prism/area relationship in a long-term predictive role was proposed and illustrated using the example of the River Usk. This method is expanded in Chapter 7.

Eysink [1991]:
The author described various empirical relationships useful for engineering purposes in complex estuarine systems. Among those put forward were,

$$A = C_A P\left(1+\frac{1}{2}\left(\xi\frac{U_0}{V}\right)^2\right)^{\frac{1}{2}}$$

where

ξ is a coefficient,
U_0 is the max. orbital bed velocity,
V is the tidal flow velocity,
C_A is an empirical constant,

$A = C_A'P^n + a$ where C_A', n, a, are empirical constants.

Eysink suggested these constants depended on the irregularity of the tidal system, the cohesiveness of the sediment and the level of stratification of the estuary.

$V = C'P^{1.5}$

where V is the volume of tidal channels below mean sea level and C' is an empirical constant, given by

$C' = C_A'.C_s(L/B\Delta h)$,

where L is the estuary length, B is the estuary width, h is the tidal range and C_s is a constant dependent on the estuary geometry.

This latter relationship showed very good agreement with observed results.

Hume and Herdendorf (1992)
Sixteen tidal inlets on New Zealand's North-East coast were examined in a study of morphology, stability and empirical relationships of inlet throats.

The parameters examined in the empirical investigation were throat width (W_{mtl}), maximum (D_{mx}) and minimum (D_{mn}) depths and throat area (A_{mtl}), measured at mean tide level, and also tidal prism, peak discharge and maximum cross-sectionally averaged velocity, measured at mean spring tide.

The correlations between throat width and maximum and minimum depths were found to be r^2=0.62 and r^2=0.68 respectively. The correlations between mean maximum velocity and maximum throat depth and maximum throat area were found to be r^2=0.25 and r^2=0.01 respectively.

The relationships between throat area, tidal prism and peak discharge were found to be,

$A_{mtl} = 0.769xQ^{1.069} \qquad r^2=0.97$

$A_{mtl} = 1.49x10^{-4} P^{0.953} \qquad r^2=0.98$

Since the exponents in the equations were very close to unity, the equations above were linearised to give,

$A_{mtl} = 1.01 Q \qquad r^2=0.95$

$A_{mtl} = 7.78x10^{-4} P \qquad r^2=0.97$

These figures demonstrate a 1:1 relationship between mean tide cross-sectional area and peak discharge.

There was a weaker correlation between tidal prism and depth (r^2=0.82) and width (r^2=0.63). The authors cited Bruun [1990, p843] who states that whilst the cross-sectional area is governed by the tidal prism, the inlet channel width and depth are free to adjust to the wave climate and littoral drift.

Falconer et al [1992]:
The Clyde Port Authority is considering the cessation of dredging in the Clyde, and allowing the estuary to return to regime state. Falconer et al investigated the possible effects of such a decision. The method used was to examine the upstream reaches which were already in regime, to deduce the nature of this regime state in the form of morphological relationships, and to extrapolate these relationships downstream to give the regime state of the whole estuary. The morphological relationships observed in the upper estuary were,

* A consistent rate of increase in both LW and HW cross-sectional areas,
* A constant ratio of HW to LW cross-sectional area,
* A constant ratio of total upstream tidal storage volume to volume of flow passing the section.

These relationships were used to produce new-regime cross-sections, and the resulting profiles were entered into a 1D flow model of the river, to give the flow characteristics of the new estuary. The profiles were then adjusted until a near uniform bed-velocity resulted throughout the estuary, since it was assumed that the bed-material at regime would be fairly uniform. The resulting cross-sections were then assumed to be the regime state of the estuary.

A simple numerical model was developed, based on average sediment input rates, to derive the time taken for the regime state to be attained.

3 A qualitative review of effects of civil engineering works on estuaries

3.1 Introduction

There are many types of civil engineering works carried out in estuarine environments - dredging schemes, training schemes, jetties, bridge building, and tidal barrier schemes are examples. The basic effect on estuary hydraulics is to reduce (or enlarge in the case of dredging) the area through which the tide can flow and to increase the friction forces in the vicinity of the works. This will have the effect of reducing the energy available for sediment transportation in the estuary and as a result deposition will occur. The time scale over which this siltation occurs can span up to a century, as in the example of the removal of old London Bridge below, and certainly it is very common for estuaries to take fifty years to re-establish an equilibrium after civil engineering works.

In the following sections are some examples of engineering schemes and their documented/observed effects on the local estuary regime.

3.2 Training Wall Construction - Mersey Training Scheme 1909-1936 [McDowell and O'Connor 1977, Price and Kendrick 1963]

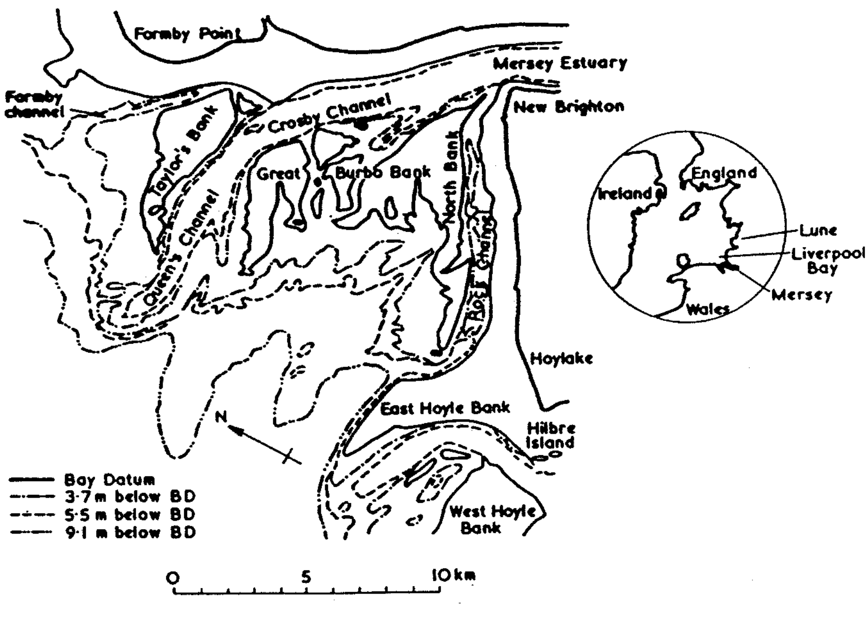


Figure 3.1 *Liverpool Bay in 1890 (after McDowell and O'Connor 1977)*

In 1909-10 a 3.6 km length of training wall was constructed along the face of Taylor's bank on the outside of the Crosby Channel bend (See Figure 3.1). The intention was to prevent the continued northward movement of the channel and to prevent a partially-formed channel from breaking through

Taylor's Bank and establishing a new system of channels, as occurred frequently prior to 1890.

This had little or no effect upon the shoals at the southern end of the Crosby Channel. Consequently in 1914, the work started on extending Taylors Bank training wall to the west, and on walls on both sides of the Crosby channel. This work was completed by 1939.

The training walls succeeded in increasing the ebb flow in the navigation channel. Consequently the ebb flow reduced in the Rock channel and the flood-dominated zone extended and moved inshore. As a result this channel, at one time the main approach to Liverpool Port with depths of over 30ft, was reduced to depths of 2-3ft. The reduction of ebb flow over Great Burbo Bank gave the strengthened flood tide a longer period in which to move sediment inshore and the bank increased in height leading to over-topping. The tide/wave action in the shallow water on Burbo bank then increased the supply of sediment to the mouth of the Mersey where density currents progressively moved it into the inner estuary.

3.3 Low Water Channel Training Scheme - Lune Estuary 1847-1891 [McDowell and O'Connor 1977]

The Lune estuary in Lancashire has a Spring tidal range of 8m and a steep longitudinal gradient which causes considerable drying out of a large part of the estuary at low tides. Freshwater flows are described by McDowell and O'Connor as "small to moderate".

The training scheme was implemented because the low water channel in the upstream part of the estuary moved about considerably from year to year. It started with the dredging of hard areas in the estuary, deepening the channel by about a metre. Then a 3km rubble wall was constructed on the western side of the estuary to a height of a metre above MLW. Some 45 years later the wall was extended by creating a wall on the eastern side of the channel and by building a 1.5 km wall just above the estuary mouth.

Comparison of Stevenson's survey from 1838 and HR's survey of 1955 show that a massive accretion had occurred (8.35×10^6 yd^3 Kestner [1966]) and that the tidal volume had reduced by some 47%. In the middle and upstream reaches of the estuary the inter-tidal banks behind the training walls had risen by some 5m and some narrowing of the low water channel had also occurred where the salt-marsh topped the walls. In the downstream part of the estuary the accretion was less severe, but the main channels at the mouth altered considerably.

Water levels rose throughout the estuary. The period of the flood tide at the mouth remained the same, although ratios of flood-ebb discharges had evened out in favour of ebb discharges.

McDowell and O'Connor state that the reasons for the effects on the Lune were the suppressing of lateral movement of the low water channel and the modification of tidal velocities in an estuary with a high sediment load. The flood-ebb circulation changed so that an ebb-dominant channel occurred between the trained walls and the area behind the walls increased in flood dominance. Material was eroded from the downstream part of the estuary and carried upstream to fill the space behind the training walls. Similarly increased velocities in the ebb-channel caused scouring. Stability was reached when upstream transport on the flood tide was balanced by downstream transport on the ebb-tide. Maximum flood tide discharges at the estuary mouth were found to decrease in direct proportion to mean tide cross sectional areas.

3.4 Dredging - Eastham Channel, Mersey 1953 [McDowell and O'Connor 1977, Price and Kendrick 1963]

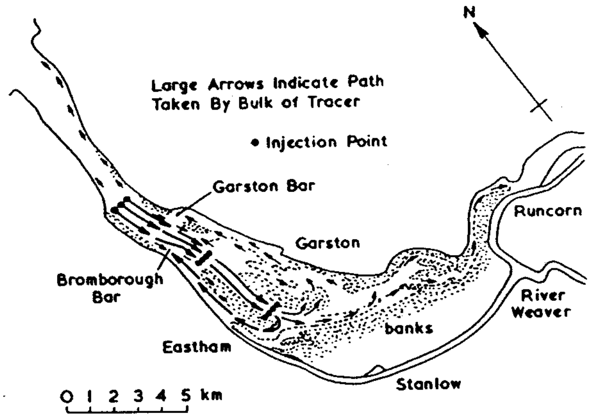


Figure 3.2 *Mersey Upper Estuary and Tidal Currents (after McDowell and O'Connor 1977)*

Eastham Channel is the main approach route in the Mersey Estuary for ships using the Queen Elizabeth II Oil Dock and the Manchester Ship Canal (See Figure 3.2). The approach channel was deepened in 1953 in order to provide access for large tankers. Nearly 1M tonnes of sediment was removed by bucket dredgers from the downstream end of the channel (Bromborough Bar) in 1953/1954. Depths increased by 1.2m. The dredging rate was then increased to 3M tonnes/annum. Unfortunately channel conditions deteriorated considerably as a result. The reason for the lack of success of the dredging programme is explained by consideration of the residual circulation pattern. This pattern has been demonstrated by Price's and Kendrick's model tests [1963]. Sediment entering the estuary through the narrows, moves progressively landwards up to the Middle Deep Channel and then seawards on the ebb tide down the Eastham and Garston Channels. Deepening of Bromborough Bar would initially increase the tidally averaged flow of water through the Eastham/Middle Deep Channels leading to erosion in the upstream ends of both these channels. This sediment is carried predominantly down Eastham Channel to the bar area.

Once dredging rates were reduced, as proposed by Price and Kendrick, the approach channel depths increased almost immediately. This deepening continued further until the late 1960's, levels having improved by 3.5 m at that time.

3.5 Dredging - Mississippi [Simmons and Herrman 1972]

Dredging and other improvements were carried out on the Mississippi to provide deep draft navigation channels through the entrance bars. Two locations were involved, South Pass and South West Passage. Depths were increased from 3 or 4 metres to 9 metres, in the case of South Pass and 12 metres in the case of South West Passage.

The deepening of the channel made it possible for saltwater to intrude progressively further upstream - as much as 225 km above the estuary entrances during very low freshwater flow. This had major implications for New Orleans' water supply.

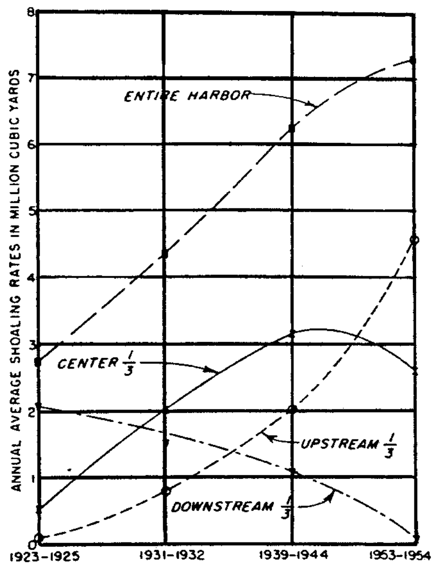


Figure 3.3 *Past and present shoaling in Savannah Harbour (after Simmons and Herrman 1972)*

With the salt intrusion came the shoaling that occurs at the salt/freshwater interface. This shoaling also moved upstream. Simmons and Herrman investigated the shoaling rates at Savannah Harbour in the South Pass channel of the Estuary Delta. In this channel, improvements such as removing snags and wrecks and dredging to enlarge navigation channels had started as early as 1889. Since then the navigation channels in the Harbour area have slowly been extended and deepened. Simmons and Herrman split the Harbour area into three parts and investigated the mean shoaling rates for each of these parts for the periods 1923-25, 1931-32, 1939-44, and 1953-54. (These periods were chosen as no major civil engineering programmes were carried out during these times). The results are shown in Figure 3.3. Shoaling moves further upstream and the Harbour starts to fill up.

3.6 Dredging in the Weser 1887-1945 [Kestner 1966]

Up to 1887 the Weser was only able to take ships up to a 10ft draught. For the next 50 or so years there was a programme of removal of obstacles and of deepening and widening of the shipping channel through dredging. The effect of the scheme was to increase the tidal range of the upstream estuary dramatically. Bremen, located some 42 miles upstream of the estuary mouth, had its range increased from 8 inches to 10.2 ft. The dredging schemes were assisted by the greater tidal velocities that occurred due to the increase in tidal volume, itself caused by the lowering of low water and the removal of obstacles to the tidal flow. The increase in estuary volume became greater than the volume of material removed by dredging, and the increase in velocities was able to maintain the increase in volume. Kestner makes the point that this phenomenon is only possible where there is considerable scope for improvement in an estuary and further dredging in the Weser would be unproductive - the tidal volume cannot increase any further. He also points out that dredging has to be carried out on a major scale for tidal volume to increase which was not the case for the Mersey, described above. (Section 3.4)

3.7 Re-routing Freshwater Flow - Charleston Harbour 1942 [Simmons and Herrman 1972]

Charleston Harbour is an estuary located in South Carolina. Prior to 1942, the estuary was of the well-mixed type, and required very little in the way of maintenance dredging. The entrance channel to Charleston City was 9m deep. Until this time Charleston Harbour had very little freshwater flow, roughly about 4 cumecs. Net flow was predominantly seawards and the harbour seemed to be gradually deepening. However in 1942, the Santee-Cooper project of the South Carolina Public Service Authority diverted an annual average flow of 425 cumecs from the nearby Santee river, via the Cooper river into Charleston Harbour. Subsequently the shoaling rate in the harbour increased by a factor of sixty over the pre-1942 conditions.

This effect was caused by the increase in river-borne sediment routed in to the harbour and also the change of the estuary from well-mixed to partially-mixed. This change was marked by the emergence of strong upstream density currents which in turn led to bedload sediment travelling in an upstream direction. The Harbour, then, had developed into a sediment trap. The effect is similar to that seen in the dredging cases described above but the cause is dissimilar, being manifoldly the change in estuary type.

3.8 Jetty Construction - An example from the Thames 1873-1966 [Kendrick and Derbyshire 1976]

At the downstream extremity of Long Reach, some 35 km above the Thames mouth, construction and extension of two jetties was carried out, as described by Kendrick and Derbyshire [1976]. Work on the first jetty started in 1873, further work was carried out in 1959 and a second jetty was built 500m downstream from the first in 1966.

The first jetty caused a local riverward movement of the low tide mark of 80m. Its extension caused further deposition. The second jetty caused a further bankline advance of about 50m. The reason for the deposition was that the jetty construction disturbed the long-term balance of the sediment circulation in this part of the estuary. Sediment was carried into the area on the flood tide, deposited around slack high water, re-entrained around the ebb-tide and then transported seawards. The civil engineering works reduced the current velocities near the bank and provided a longer period for deposition at high water. Furthermore the reduced velocities meant that the ebb current was less efficient in re-entraining sediment and as a result there was a build up of sediment. This continued until a new balance was set up.

3.9 Tidal Barrage Scheme - Proposed Mersey Barrage [HR 1988]

Tidal barrage schemes seek to use natural tidal energy to drive turbines set in barrage structures across a cross-section of an estuary. The effect they can cause on estuaries is very major since tidal flow along the estuary is impeded by the barrage, causing a reduction in tidal prism and therefore leading to deposition. HR carried out an investigation into the suitability of the Mersey for a tidal barrage. The results of this investigation for a barrage across the Narrows are summarised below.

The main effect of the barrage was to reduce the tidal prism of the estuary by 20%. This figure was derived from physical modelling. The figures derived from mathematical modelling were between 25% and 40% depending on the type of barrage used. Upstream of the barrage, velocities are reduced in the main, peak flood velocities reducing by up to 1m/s, and mostly by 15%-50%. Peak ebb velocities fall by 50%-75% except above Runcorn where there is little change. Flood tidal periods were in the main increased slightly upstream of the barrage, while ebb tide periods were reduced by an hour except upstream above Runcorn.

Water levels just downstream of the barrage increase by 0.4m. Levels upstream are reduced on Springs and increased on Neaps.

Maximum salinity levels were reduced slightly downstream of the barrage and greatly reduced upstream. Minimum salinities were reduced for the first 17km (Spring) or 5 km (Neap) from the estuary mouth and increased thereafter, saline water being trapped by the barrage operations.

Siltation is judged to be 9.7×10^6 m^3 in the first year after construction, and the estuary is predicted to take at least 40 years to achieve equilibrium. The shoaling rate over this time is mainly dependent on availability of sediment from outside the estuary.

3.10 Tidal Barrage - The Eider 1936 [Kestner 1966]

The Eider discharges into the North Sea just north of the Elbe. In 1936 a barrage was completed to prevent salt water flooding into the neighbouring marshlands. At first the barrage appeared to be very successful, but soon accretion began to occur, not just mud but also sand. By 1948 the low-water cross-section at the barrage had reduced from 8200 ft^2 to less than 22 ft^2 and this was repeated downstream.

The construction of the barrage had altered the duration of the flood and ebb tides. Flood tides became shorter and ebb tides longer. As a result flood velocities increased while ebb velocities decreased. Assuming that there is a threshold velocity below which no sediment is transported, then a plot of velocities (Figure 3.4) shows that after barrage construction there was a marked imbalance in the amount of sediment moved by the flood and ebb tides. The estuary began to fill up with sediment from the sea. The estuary continued to fill up until the slope had been increased up to a point where the flood tides had been sufficiently reduced and the sediment transport was balanced again. Some sedimentation will also have been caused by the reduction in velocities, silt that had previously been carried in suspension being accumulated on the estuary bed.

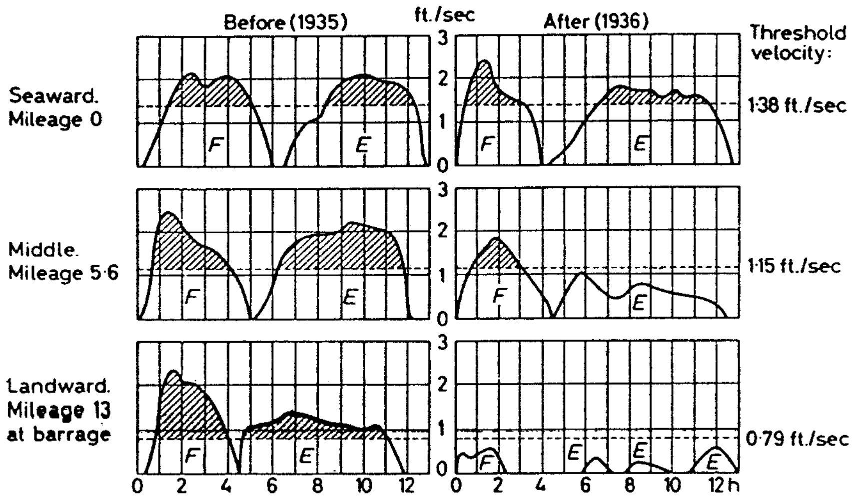


Figure 3.4 *Velocities, before and after barrage construction (after Kestner 1966)*

3.11 Tidal Barrage - Thames Flood Relief Barrier [Kendrick and Derbyshire 1978]

The Thames Barrier has been in operation for many years, and so far does not yet seem to have suffered the problems of the barrage described above. This can be explained by the crucial difference in design between the Eider barrage and the Thames Barrage - namely that the Thames barrage was designed in such a way that the tidal propagation, i.e. the water levels, and the discharges of the flood and ebb tides, were virtually unaffected by the structure of the barrier, [Kestner, and Price and Kendrick in discussion of the paper]. Physical model studies undertaken by the HRS between 1968 and 1975, showed that this was indeed the case. As a result physical sedimentation models showed that any accretion occurring was less than normal annual variation.

3.12 Bridge Building - Removal of Old London Bridge 1830's [Inglis and Allen 1957]

The influence of bridge building can be illustrated by the effect of the removal of Old London Bridge. The result of the removal was a tidal range increase of 25%, with a similarly large increase in discharge. The difference in water level across the bridge became as much as 5 ft at LWS. Upstream of the old bridge a very rapid deepening of the channel took place due to the increase in tidal velocities, to such an extent that three other bridges were endangered. Downstream the effects will have tapered off but were perceptible as far as the 'Mud Reaches'.

Inglis and Allen reported that at the time of writing, a new equilibrium had not then been attained in the upper reaches, over a century after the event took place.

3.13 Summary

The evidence of the case histories summarised above shows that the long-term effects of civil-engineering schemes can be dramatic and can take decades to realize. It also illustrates strongly the correspondence between reductions in the tidal propagation through an estuary and siltation, and conversely, the relationship between increases in the tidal strength and erosion. The details of such siltation/erosion processes are often difficult to elucidate without careful modelling, but the overall picture can be described quite adequately using the basic rule that reductions in tidal volume leads to sedimentation and vice versa. The conclusion is that the use of a reliable regime theory would be of great use in a long-term predictive role.

The future effects of dredging schemes can be particularly hard to deduce, as the example of the Mersey shows. Here dredging activity was actually contributing to sedimentation, a very puzzling result that needed physical modelling to understand.

The Charleston Harbour example is very interesting, and illustrates the effect that a change in the freshwater input to an estuary can have dramatic consequences. The dynamics for this catastrophe are explained by Simmons and Herrman in terms of bed currents and plumes, and no information on the tidal system is given. This is unfortunate as it is not possible to say whether regime theory would add further explanation on the experience of Charleston Harbour.

4 Main factors in estuary regime

Since regime theory depends on empirical relationships between the main driving forces in estuaries, it is important to study literature and glean what knowledge has already been brought to light on the fundamental factors working within the estuary system. Such a review is given below and includes the methods of classification that have been introduced to explain the difference in estuary systems.

4.1 A Review of Classification Systems

Several attempts have been made at classifying estuaries, and in a number of different ways. Methods vary from topographical classification to classification by residual sediment-flow direction.

Possibly the simplest type of classification is that of a topographical nature e.g. **Pritchard** [1952]. He proposed three types of estuaries:

* Drowned river valleys (coastal plain estuaries) with extensive mud-flats, a large width/depth ratio which increases towards the mouth, and often sinuous channels.
* Fjords with very deep channels with very thin veneers of sediment, a low tidal prism relative to the river discharge and virtual rectangular channels.
* Bar-built estuaries with characteristic bars across their mouths, depositional coasts, a large seasonably variable river and rapidly varying channels over time.

In further work **Pritchard** [1955] proposed a straightforward classification of estuaries into four groups:

* Salt-wedge estuaries - highly stratified, large ratio of freshwater to tidal flow, small depth to width ratio.
* Fjords - highly stratified but very deep estuaries, again large freshwater flow.
* Partially-mixed estuaries - an intermediate stage between the above and below.
* Well-mixed estuaries - Densities and salinities virtually homogeneous, small freshwater to tidal flow ratio, large width to depth ratio.

Two reports by **Abbott** [1960a,b] divide estuaries into 'convective' and 'salinity' types. These types can both be sub-divided according to whether sediment flow is predominantly landward or seaward.

In the case of 'convective' estuaries, the surface velocity can be written as,

$$U(x,t)=U_0(x)[\cos wt-\psi(x)]$$

where U_0 is the amplitude of the tidal current on the surface, $\psi(x)$ the time lag. Drift above the seabed can then be shown to be seawards or landwards if,

$$\frac{d}{dx}(U_0 e^{\Psi})<>0$$

Similarly for a 'salinity' estuary drift above the bed is landwards/seawards if,

$$\frac{h}{2}\left(-\frac{\partial\rho}{\partial x}\right)<>\rho\frac{\partial S(x)}{\partial x}$$

where h is the depth, ρ the water density and S(x) the mean tidal level up the estuary.

Bruun and Gerritsen [1960a] found that the stability of inlets could be related to the degree of littoral drift experienced by the inlet. They divided inlets into three classes:

P/M > 300	⇔ stable
100 < P/M < 300	⇔ intermediate
P/M < 100	⇔ unstable

where M is the yearly average value of littoral drift to the inlet (m^3/yr). The authors went on to suggest that the value P/M or Q/M influences the stability values of shear stress in the inlet. They suggested a set of average values of stability shear stresses for each of the above classes. Bruun [1978] went on to modify these classes later, the new classes being P/M>150 (stable), 100<P/M<150, 50<P/M<100, 20<P/M<50 and P/M<20 (very unstable).

Ippen and Harleman [1961] suggested a method of classification using the ratio of energy dissipation in a channel, G, to the rate of gain of potential energy per unit mass, J. Here G is given by,

$$G=(Px_1-Px_2)/\rho bh(x_1-x_2)$$

and J is given by,

$$J=g(\frac{\Delta\rho}{\rho})hu_f/L$$

where Px is the rate of transport of tidal energy across any section,

h is the depth,
b is the breadth,
u_f is the mean freshwater velocity over the estuary,
ρ is the density,
$\Delta\rho$ is the density difference between fresh and seawater
L is the length of the estuary.

The stratification number G/J is a measure of the amount of energy lost by the tidal wave relative to that used in mixing the water column. The ratio is analogous to the inverse of the Richardson number. Increasing values of the stratification number indicate well-mixed estuaries, and low numbers, highly stratified conditions.

Harleman and Abraham [1966] showed in flume tests that,

$$PF_0^2/Q_fT = \text{constant} \times G/J$$

where Q_f is the freshwater flow and F_0 is the Froude number.

This latter estuary number is readily calculable in real estuaries. Essentially this number is comparing the stabilising effect of density increase with the turbulent energy needed to mix the layers.

One of the most important attempts at classification was carried out by **Hansen and Rattray** [1966]. Here two dimensionless parameters were used in estuary characterization - a stratification parameter dS/S_0, (defined as the ratio of salinity difference from top to bottom of the estuary to the mean cross-sectional salinity), and a circulation parameter u_s/u_f, (the ratio of surface velocity to mean cross-sectional fresh water flow velocity). Using these two parameters estuaries can be classified into seven distinct types:

1. The net flow is seaward at all depths and the upstream salt transfer is effected by diffusion. The estuary is well-mixed and salinity stratification is slight.
2. As 1 above except that appreciable stratification occurs.
3. The net flow reverses at depth and both advection and diffusion contributes importantly to the upstream salt flux. The estuary is well-mixed and salinity stratification is slight.

4. As 3 above except that appreciable stratification occurs.
5. Essentially as for 3 above except that there is a dominance of advection which accounts for over 99% of the upstream salt transfer, and stratification is small.
6. As for 5 above except that the lower salinity layer is so deep that the salinity gradient and circulation do not extend to the bottom. Fjord estuaries are generally of this type, at least initially; subsequently they may be classified as in 5 above.
7. Salt wedge estuaries in which the stratification is greater and the flow grades from a thick upper layer flowing over and little influenced by a thin lower layer, to a shallow surface layer flowing with little influence over a deeper lower layer.

O'Brien's [1976] contribution to classification of tidal inlets is of note. He proposed a classification system based on his simple A/P relationship. Inlets were classified as one of six types and different equations for A and P were proposed according to assumptions made about each inlet type. The six inlet types were:

* short, frictionless channels
* small, deep lagoons
* long lagoons/estuaries
* narrow, deep inlet channels
* short, deep inlet channels
* long inlet channels

Nummedal and Fischer [1978] classify tidal inlets in a simple way, based on thoughts about their shoal geometry. An inlet can be classified as,

* wave dominated - characterized by shoals on the inlet side of the gorge, only, which is comparatively stable, and a low tidal range.
* mixed - similar to the above but with a slightly greater tidal range.
* tide dominated - with a deep and straight ebb channel, sand bodies only occurring to the seaward side of the inlet.

Aubrey and Spear [1985] classify tidal inlets/estuaries as either flood or ebb dominated and then further, as shallow or deep, in terms of the ratio of sea-level fluctuations, a, to water depth, h. Thus,

$$a/h > 0.3 \Leftrightarrow \text{shallow system}$$
$$0.1 < a/h < 0.2 \Leftrightarrow \text{deep system}$$

Other classifications of interest are included here, which don't relate strictly to estuaries/inlets but to river channels in general.

Chang published a detailed analysis of river meanders in 1984. Using an energy approach the relationships between river geometry, sediment load, flow resistance, bank stability and circulation at bends in channels was examined. It was suggested that rivers could be classified in terms of the ratio of valley to channel slope - a value of less than 1.5 indicating a 'straight' channel - and that meanders could be classified in terms of the ratio of the radius of curvature to the channel width, r_c/B.

4.2 Discussion on Classification

For the purposes of an investigation into estuary regime classification must be aligned towards the geometrical or sediment nature of estuaries. Freshwater flow/tidal discharge ratio, tidal prism, sediment type/cohesive quality, etc, may be more important factors than salinity stratification or the mean to surface flow ratio (Hansen and Rattray [1966]), etc, (although even these latter properties may have minor effects on regime relationships and so will also need to be investigated). Therefore many of the classification systems proposed above, although being helpful in over-view, are not useful for a study of regime theory and more suitable property classes must be devised. The new property classes need not be based on complicated computations such as the Hansen or Ippen methods, but may be simply interval classes for estuarial properties, such as the division of estuaries into macro (>4m), meso (2-4m) or micro-tidal (0-2m) ranges. It may be that investigation will reveal possible classification properties not considered a priori and so the classification system should not be thought of as fixed but rather as an evolutionary process.

4.3 Towards an Initial Classification System - Important Factors in Estuary Regime

A first attempt at classification will generally correspond to the properties thought to be important in the relationship between discharge and channel cross sectional area. The most fundamental factors listed in literature are discussed below.

4.3.1 Tidal Discharge/tidal prism

Authors in the field have differed about whether tidal discharge (mean or maximum) or tidal prism is the most fundamental parameter in deciding channel cross sectional area (or estuary regime in general). O'Brien [1969], Nayak [1971], Johnson [1973], Jarrett [1976], and Eysink [1991], for instance, have used tidal prism while Leopold and Maddock [1953], Bretting [1958], O'Brien [1980], and Kondo [1990] have suggested maximum discharge as the main parameter. There has not been a great deal published of comparisons between the two but de Jong and Gerritsen [1984] found a better correlation using the discharge rather than the prism. Shigemura [1976], however, found that tidal prism correlated consistently higher than mean tidal discharge in his statistical studies on Japanese inlets. Hume and Herdendorf [1992] found that for the sixteen tidal inlets investigated in their paper, tidal prism correlated more highly than peak discharge against mean tidal cross-sectional area. The argument is made partly specious by the fact that the two are heavily dependent on each other. Indeed, for simple harmonic tides,

$$Q_{max}=\pi P/T$$

Moreover, for real inlets, Keulegan [1951] found that,

$$Q_{max}=\pi CP/T$$

where C is a constant of proportionality depending on the cross sectional shape, ranging between 0.81 and 1.00.

Chapter 6 is concerned with correlations of Maximum Discharge and Tidal Prism with cross-sectional area, using data sets from the Thames, Avon, Nene, Parrett, Loughor and Mersey Estuaries using observed results or 1D model output, against cross-sectional area (at maximum discharge, and at mean tide level). The respective correlation coefficients are presented later in full but the highest average correlation overall with area came from discharge corresponding to maximum velocity, r=0.9898. The best average prism correlation was against mean discharge area with 0.9754. However correlating volume at mean tide in the estuary against prism$^{1.5}$ (as suggested by Eysink [1991]) gave the best result overall with an average value of r=0.9913.

Using these data sets, tidal prism was plotted against Qmax, and the average correlation value was r=0.9796. This is a relatively high correlation and in part validates the results of Keulegan. However, Keulegan's coefficient C, mentioned above, took values between 1.0 and 2.3, figures which are very different to the value range C=0.81-1.0 which Keulegan gives.

4.3.2 River Discharge

As a first approximation Bruun and Gerritsen [1960b] suggest that the tidal discharge used in discharge/area relationships should be adjusted as follows:

$$Q'_{max} = Q_{max} + Q_0$$

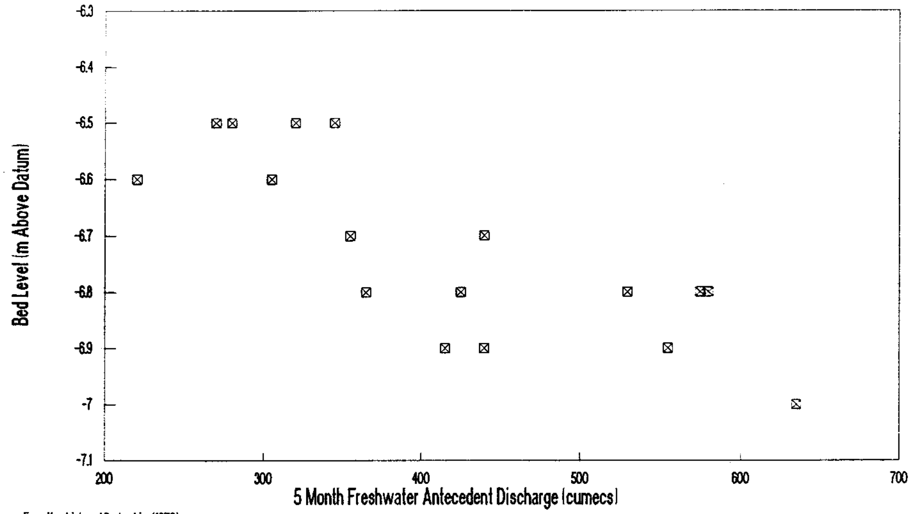
where Q_0 is the freshwater discharge occurring over the same period.

This approach is flawed for the simple reason that the contribution of freshwater to the overall discharge is, apart from the extreme upstream estuary, negligible and cannot account for the seasonal effects of erosion/accretion noted below.

Inglis and Allen [1957] noted the significance of freshwater flow rates on the estuary regime. They observed in their Thames study that high winterborne flows lead to dramatically different values of salinity and channel depth in the upstream estuary reaches compared to the low flows of summer and gave three reasons for this: firstly that the freshwater is clear and 'silt-hungry', secondly that ebb shear stresses increase and thirdly that the increased freshwater acts as a deflocculating agent on the sedimentary mud, reducing the tractive force needed for sediment movement.

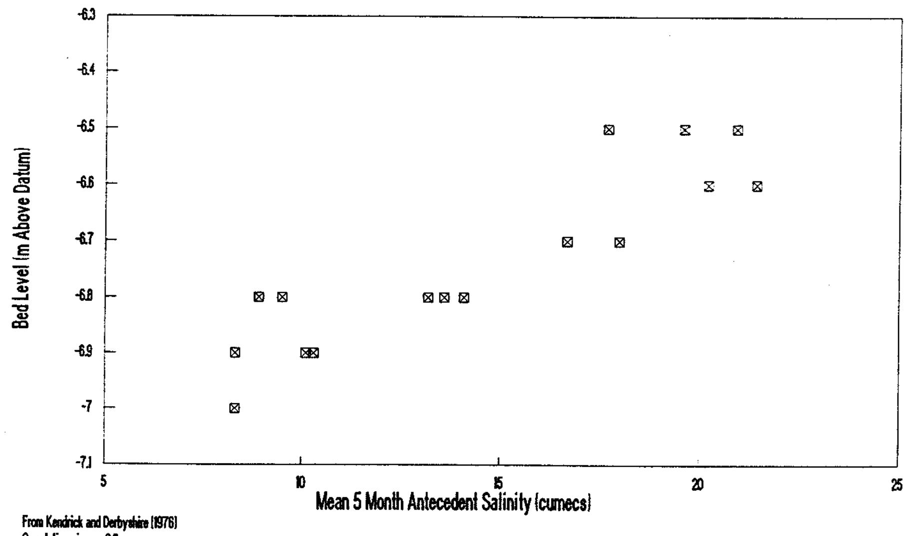
Kendrick and Derbyshire [1976] gave the most fundamental factors affecting estuary regime as those that produce the natural rhythm of diurnal (or semi-diurnal), bi-monthly and seasonal fluctuations due to predictable changes in tides and weather, citing tidal discharge, freshwater flow and the resultant distribution of saline water as the main influences. The paper went on to confirm the influence of these factors by studying the Thames at two sample points over different time scales. It was found that high freshwater flow was associated with erosion in the upper reaches of the estuary and with accretion in the lower reaches, and vice versa in the summer low freshwater flow period.

Using their observations, for a site roughly mid-length along the Thames estuary, a correlation of 5 month average antecedent freshwater discharge against bed level gave a correlation coefficient of r=0.8 (Figure 4.1), in spite of measurement errors associated with bed level measurement. A similar correlation using 5 month average salinity gave a correlation of r=0.9 (Figure 4.2) suggesting possibly that it was the effect of salinity on cation bonding in the estuarial mud rather than increased ebb velocities that had the more dominant effect.



From Kendrick and Derbyshire (1976)
Correlation gives r=-0.8

Figure 4.1 Correlation between bed levels and antecedent freshwater discharge for the Thames



From Kendrick and Derbyshire (1976)
Correlation gives r=0.9

Figure 4.2 Correlation between bed levels and antecedent salinity for the Thames

A similar exercise for a site downstream, gave a poor inverse correlation (r=-0.3) with antecedent freshwater discharge and an inverse correlation (r=-0.85) with 5 month antecedent average salinity, reflecting the accretion of the sediment downstream of the sediment eroded at the upstream end of the estuary. This accretion would occur at a point where salinity levels were sufficiently high to encourage re-flocculation [Derbyshire and Kendrick 1985].

Further investigations into the effects of salinity are included in the salinity section below. The results of work by some authors have disputed whether salinity is indeed a significant factor influencing regime.

Odd, in a report on the effect of fluvial flow on bed levels in the Great Ouse estuary [1992] found that there was a good correlation of antecedent fresh water flow (best correlation being r^2=0.804) with changes in bed level, the degree of correlation decreasing with increasing closeness to the estuary mouth, there being very negligible correlation at the mouth.

Eysink in his paper [1991] suggested a modification of O'Brien's relationship,

$A = C.P^n + a$,

where C,a and n are constants,and postulates that for estuaries with large freshwater flow rates, Eysink proposed that this should be taken into account by redefining the tidal prism, P, as P'=P+QT/2, where Q is the bed-shaping river discharge. Again this proposal, like Bruun and Gerritsen's above, suffers from the fact that the effects of scouring are not accounted for by increased discharge alone.

The seasonal effects of freshwater flow/salinity on estuary have been noted above, but the effect of discrete storm/surge events must be mentioned. In some estuaries more sediment can be delivered into the estuary in one high return event than in years of normal flow.

There have been few studies into this subject though the impact of hurricanes on estuaries has been dealt with by Hayes [1978]. The impact of a tropical storm called Agnes in 1972 on the Chesapeake region was reported by Schubel [1969] and Nichols [1972]. As a result of 30cm of rain in two days the Susquehanna river discharged more sediment in one week than the previous half century. Nichols concluded that 90% of the sediment was trapped within the estuary. In the bay hundreds of acres of new intertidal flats were formed. Post-storm coring showed that an average of 17cm of sediment was deposited in the upper reaches of the bay [Zabawa and Schubel 1974].

In Britain however the impact of storms is not generally as dramatic as that described above. High freshwater discharges seem to cause changes that are reversible - that is bed levels revert to their previous levels as discharges decrease. This is certainly true of the Thames [Inglis and Allen 1957].

The division by Pritchard [1955] of estuaries into salt-wedge, partially mixed and well-mixed estuaries is an appropriate classification for regime. The density-currents driven by large salinity differences are often significant in affecting regime parameters. The change from a well to a partially-mixed estuary occurred in 1942 at Charleston harbour, South Carolina [Simmons and Herrman 1972]. It resulted in a change from a gradually deepening estuary to a rapidly filling one. Upstream density currents, produced as a result of a vast increase in freshwater flow through the estuary, began to carry large amounts of bedload sediment upstream. The regime change was dramatic. It seems plausible therefore that partially-mixed estuaries will have different regime characteristics to well-mixed ones.

Most estuaries in Britain are of the well-mixed type, but some like the Mersey, which has been included in this study, are partially mixed.

4.3.3 Velocity

Given that discharge and tidal prism are clearly dominant factors in shaping an estuary, one would expect velocity to be important also. It is linked closely with bed shear stress and as such deserves a place in a list of parameters affecting estuary regime.

However velocity has not been a favoured tool with which to define estuary equilibria, except indirectly. The reason for this is the large amount of data needed to gain a representative value. One of the few uses of this parameter is by Chantler [1974] who concluded that the cross sectional velocities in an estuary in equilibrium will tend to a constant dependent on the bed material properties of the estuary. This idea was also pointed to by Allen [1958], and is a rule of thumb commonly held amongst hydraulics engineers.

Bruun [1968] put forward the idea that maximum mean velocity in a tidal inlet is roughly 1m/s, from observations and work by Colby [1964]. Colby investigated the relationship between the average bed material discharge and the mean velocity in rivers. He found that for a velocity of 3ft/s, or about 0.9m/s, the sediment discharge remains the same independently of the value of the depth. So this speed defined a stable condition for the river as no erosion or deposition could occur. Unfortunately the procedure Colby used to calculate the suspended load was really only satisfactory for flow over dune beds, and is not universally applicable. Use of other more accurate sediment transport formula in his method did not give the result that a speed of 3ft/s, or any other speed, makes sediment discharge independent of depth [Skou 1990].

Gerritsen et al [1990] suggested a dimensionless stability parameter which would describe the state of equilibrium in an estuary or a tidal inlet. This parameter is given by,

$$\frac{V}{\left(\frac{\Delta d_{50}}{\mu}\right)^{\frac{1}{2}} C} = \mathrm{CONST}$$

where,

V is a representative velocity, eg. maximum velocity;
Δ is the relative density of the sediment;
d_{50} is the median particle size;
C is the Chezy coefficient;

and where μ is given by,

$$\mu = \left(\frac{C_{d_{90}}}{C}\right)^{\frac{3}{2}}$$

C_{d90} being the roughness of a cross-section corresponding to the grain-size d_{90}.

The computation of this parameter for 35 tidal inlets in the Netherlands, gave a range of values for the parameter varying from 28 to 45.

A brief investigation of this parameter will suggest to the reader that it is very dependent on the extremely heterogeneous quantities d_{50} and d_{90}. Regime theories require a much more stable quantity that is representative for a given stretch of river. An investigation using sediment grab samples from the Avon [HR 1992a] and 1D computer model results from different cross sections of the river, gave widely varying results for the parameter (more than a magnitude) due to the scatter of values of d_{50}. The parameter also required a considerable amount of data to derive and this raises questions about its usefulness as well as its validity.

4.3.4 Bed Shear Stress

Extremely good correlations were found by de Jong and Gerritsen [1984] for a parameter based on bed shear stress, tidal discharge, Chezy coefficient and fluid density. This same parameter was suggested earlier by Bruun and Gerritsen [1960c]. Bretting [1958] in his derivation of relationships for channel profile and cross-sectional area, also found that area was linked directly to critical shear stress. Since shear stress, or more correctly, the stability shear stress of the bed governs sediment movement it is intuitive that it will be a major factor in deciding estuary equilibria.

Bed shear stress is affected amongst other things by wave action, the grain size/density of the sediment material, sediment and salinity concentrations in the estuary and the bed form.

4.3.5 Wave action

The influence of wave action on bed shear stress was shown dramatically in Mehta [1988] and was pointed towards by Chantler [1974] who noted an increase in cross sectional area towards the mouth of an estuary as a result of the extra turbulence caused by the orbital wave velocity. Due to this turbulence a much smaller current was needed to cause erosion than for a current without wave action. Mehta [1988] described waves as weakening and fluidizing the bed. Thus the shear stress needed for erosion became much lower, and as a result, estuary channels near their mouths tended to increase rapidly in cross sectional area. This phenomenon is shown in Figure 4.3 in the example of the Thames [Data from HR 1971].

The effect of waves on shear stress has received attention from several authors who have tried to quantify it. The most widely used formula is that of Bijker,

$$\tau_{wc}=\tau_c[1+\frac{1}{2}(\xi\frac{U_0}{V})^2]$$

where

τ_{wc} is the combined shear stress;
τ_c is the shear stress due to current;
U_0 is the orbital velocity amplitude;
V is the mean current velocity;

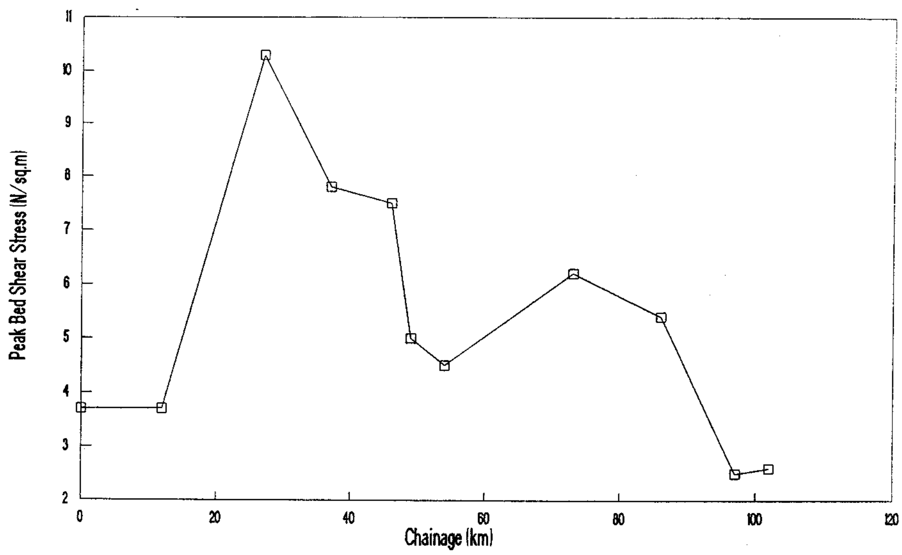


Figure 4.3 *Peak bed shear stress in the Thames Estuary*

and where ξ is given by,

$$\xi = C_h\left(\frac{f_w}{2g}\right)^{\frac{1}{2}}$$

f_w is the bottom friction coefficient;
C_h is the Chezy coefficient.

This equation can be shown to be a simple sum of shear stresses,

$$\tau_{wc} = \tau_c + \tau_w$$

Other formulae attempting to improve on the work of Bijker have been put forward. The G8M Coastal Morphodynamics MAST Project attempted to improve the state of knowledge about wave-current interaction. The idea was to fix a method for determining wave-current interaction such that the performance of the commonly used theories could be examined and compared [Soulsby et al 1993]. The theories examined were Fredsøe [1984], Myrhaug and Slaattelid [1990], Huynh-Thanh and Temperville [1991], Grant and Madsen [1979], Davies et al [1988] and Bijker's theory. R.Soulsby, a contributor to the project suggests the use of Fredsøe as the best formula [pers comm.].

The use of the effects of wave action in estuary regime formulae was been demonstrated by de Jong and Gerritsen [1984] and Eysink [1991].

4.3.6 Grain size/density

For non-cohesive beds it has been demonstrated that the critical shear stress, the point at which sediment will start to move, is directly related to the particle size by Shields [e.g. Vanoni et al, 1960]. Moore [1972] found that the value of the ratio P/A for tidal inlets on the American coast was generally bigger where the sediment grain size was small, and smaller for larger sized sediment, although there was a lot of scatter which implied other factors at work.

For cohesive beds the situation is much less clear. Critical shear stress may be dependent on salinity (below), mineral concentration in the estuary, the time history of the estuarial mud, the clay composition and also temperature. However Villaret and Paulic [1986] found an approximate relationship between density and critical shear stress for San Francisco Bay,

$$\tau_c=\zeta(\rho_b-1)$$

where ρ_b is the wet bulk density (gcm^{-3}), τ_c is the critical shear stress (Nm^{-2}) and $\zeta = 1.0$

This is backed by research from Migniot [1968], Owen [1970], and Thorn and Parsons [1980]. The latter found approximate relationships between density and critical shear stress, namely,

$$\tau_c = 5.42\text{x}10^{-6}\rho_s^{2.28}$$

where ρ_s is the bed surface density (gcm^{-3}) and the units of τ_c are (Nm^{-2}).

It is very common for a power law such as the above to be used in describing the relationship between erosive strength and sediment density. The exact nature of the relationship is normally derived from a number of observations from tests on grab samples in the location and a best fit line taken from the corresponding scatter plot. There tends to be a great degree of scatter in such plots.

The rate of erosion of sediment from a bed can be described as follows,

$$dm/dt=M(\tau - \tau_c) \qquad \text{[Owen 1975]}$$

where m is the mass eroded and M is a coefficient for a particular sediment.

A similar equation also used widely is,

$$dm/dt=M'(\tau - \tau_c)/\tau_c$$

where M' is a coefficient. [Task Committee Report 1983]

It is clear that the critical shear stress, which can be crudely related to density, is not related in any clear way to the stability shear stress, which is closely related to the regime state of a channel. In regime, the channel will be constantly eroded and left to resettle whereas τ_c relates to the state of the channel where no erosion occurs at all. A dense bed will require a greater shear stress from the water velocity for erosion to occur, and so the velocities at regime will tend to be higher.

The amount of data necessary to deduce the correct effect for any estuary, even if this effect could be quantified, does not make sediment density a convenient parameter for a use predictively in regime study.

4.3.7 Sediment concentration

Parchure and Mehta [1985] in laboratory studies on cohesive beds showed that a higher suspended sediment concentration increased the bed shear stress. This was also the finding of Russian experiments cited in Bruun and Gerritsen [1960d]. These results implied that larger concentrations of fine sediment increased bed shear stress by 100%-300% for sediment in the range 0.1-1mm.

Ackers [1991] in his review of the Ackers-White transport formula for open-channel flow and for non-cohesive sediment gave formulae for velocity in a channel in terms of Manning's coefficient, depth and volumetric concentration of suspended sediment. The range of formulae given indicated that as sediment particle size decreased, the effect of sediment concentration also decreased.

Findings by Stevenson and Burt [1985] established a clear relationship between settling velocity (a measure of the degree of flocculation) and suspended sediment concentration for in-situ measurements on the Thames Estuary. Since degree of flocculation is intuitively related to bed shear strength, these prototype observations would lend credence to the laboratory-derived observations above.

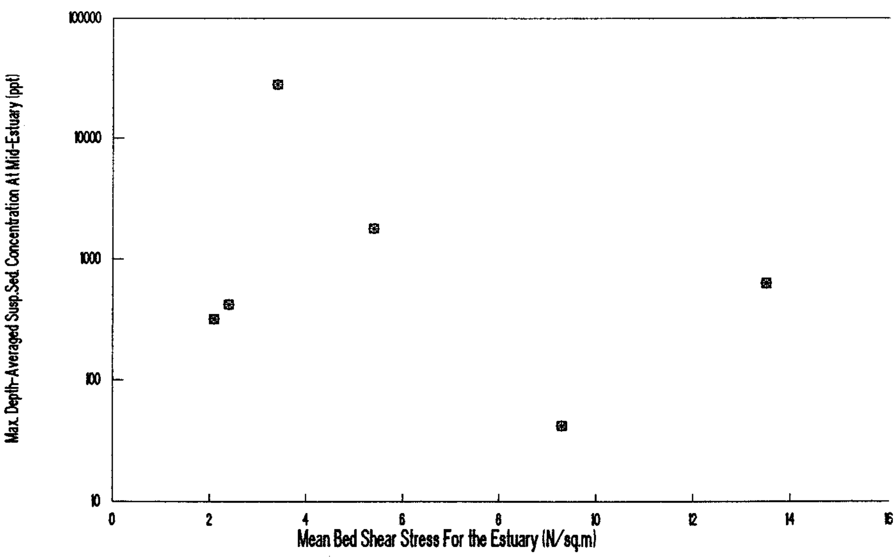


Figure 4.4 *Negligible correlation between the "average" suspended sediment concentration and the "average" bed shear stress for various estuaries*

Figure 4.4 shows a plot of 'average' suspended concentration against 'average' bed shear stress for six different estuaries. (Data is from the sources given in Chapter 6 and Chapter 7). The suspended sediment measure used was the maximum depth-averaged (or mid-depth if this was not available) suspended sediment concentration taken at a point roughly mid-way between the estuary mouth and the tidal limit, on an average spring tide. The shear stress measure used was the shear stress deduced from

$\tau = \rho g v^2 / C^2$

where C, the Chezy coefficient is given by C=30+5log(A), A being the cross-sectional area of flow [Bruun and Gerritsen 1960h]; v is the cross-sectionally-averaged velocity.

The bed shear stress shown is the average stability bed shear stress, the mean of the maximum spring tide values throughout the estuary. This gives figures which seem high but is adequate as a means of comparison. The figure shows negligible correlation between the two parameters.

It is difficult to deduce the effect that suspended sediment concentration has on estuary regime since there is a complicated feed-back mechanism between shear stress and suspended sediment concentration. Conclusions are severely hindered by having to rely on measured data which cannot fully represent this extremely heterogeneous quantity. Use of this property in regime will therefore cause a great deal of error in the regime formula.

4.3.8 Salinity

The investigations of Inglis and Allen [1957] and Kendrick and Derbyshire [1976] showed the relationship between increased freshwater flow (decreased salinity) and deflocculation (and therefore reduced stability shear stress). Parchure and Mehta [1985] in their laboratory experiments on erosion of cohesive sediments (lake mud) found that shear stress was greatly affected by salinity for levels up to 2 ppt, affected roughly linearly for 2-10 ppt, and very little affected by further increases (Figure 4.5).

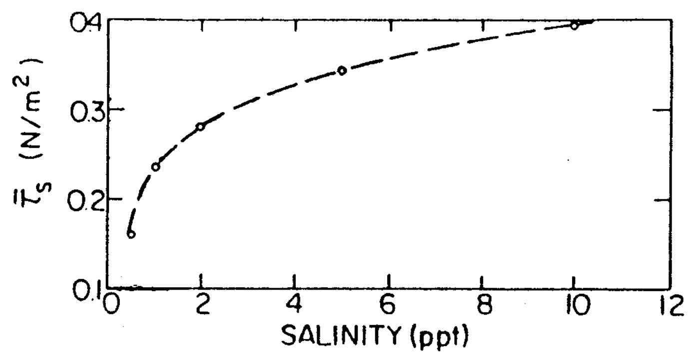


Figure 4.5 *Variation of Mean Bed Shear Strength on the Surface of a Lake Mud (after Parchure and Mehta 1985)*

Older results by Rosenqvist [1961] found that a salinity of 2.15 ppt in seawater was needed for complete flocculation to occur.

Other experiments have been performed on the relationship between settling velocity, (a measure of the degree of flocculation of a sediment) and salinity. Owen [1970] found a roughly linear relationship between the two. Krone [1962] and Allersma et al [1967] also found clear relationships between settling velocity and salinity, both finding that for low salinities settling velocity increases rapidly while there is a negligible increase for high salinities.

However work by Stevenson and Burt [1985] which was based on in-situ measurements of settling velocity and salinity produced evidence to show that there was no significant effect of salinity on settling velocity. The authors concluded that this departure from the consensus of laboratory results was due to the unrealistic conditions that laboratory sediment experiments are carried out under. Laboratory samples are reconstituted by mechanical agitation and the experiments do not hold flocculating material long enough in suspension to simulate real conditions. To support this the authors point out that in-situ settling velocities are generally a magnitude greater than laboratory-derived values. However, the scatter from the values of median settling velocity produced by the in-situ experiments was also about an order of magnitude higher and this scatter may have served to cloud the contribution of salinity. The laboratory results do not represent the prototype as well as in-situ experiments but they are less prone to heterogeneous scatter.

To gain a picture of how salinity affects shear stress on an estuary-wide basis, an attempt was made to plot "average" salinity against "average" shear stress for a number of estuaries. This is shown in Figure 4.6 (Data is from the sources given in Chapter 6 and Chapter 7 and Wallace Evans Ltd [1990).]. The salinity measure used was the maximum depth-averaged (or mid-depth if this was not available) salinity taken at a point roughly mid-way between the estuary mouth and the tidal limit. The shear stress measure used was the shear stress deduced from τ=ρgv^2/C^2 as before, averaged from all available results of the estuary concerned. The results indicated a clear trend showing that shear stress increases with decreasing salinity. The section on 'Freshwater Discharge' explained that, within an estuary, an increase in salinity leads to flocculation, deposition, a lower cross-sectional area, and therefore increased velocities and shear stresses. Figure 4.6 indicates a result suggesting that an estuary with high 'average' salinity will have relatively low velocities and shear stresses.

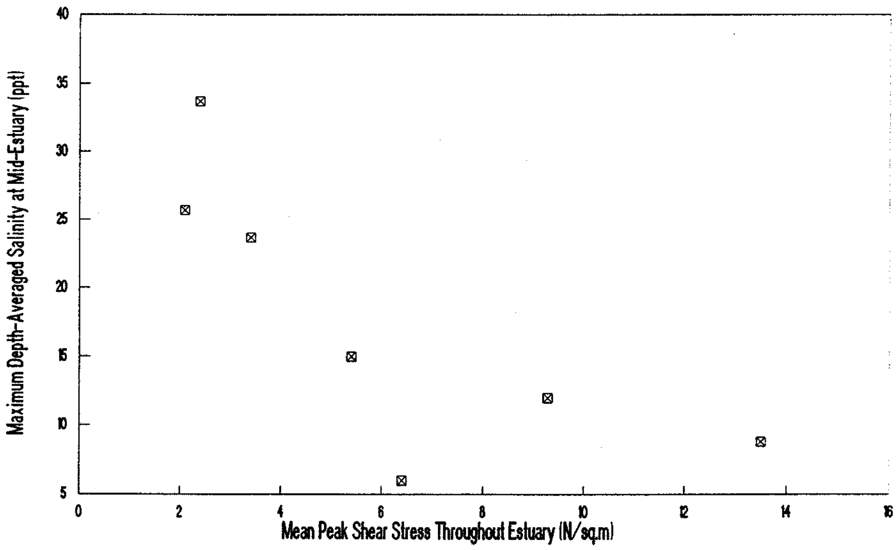


Figure 4.6 *"Average" salinity in terms of "average" shear stress for a number of estuaries*

The reason for this apparent disparity is that Figure 4.6 is based on data for estuaries under low flow conditions. This means that the flocculating 'salinity effects' have in effect been taken out. The figure therefore shows the relationship between the proportion of freshwater flow in the estuary (which is dependent on freshwater discharge and tidal volume) and shear stresses. The suggestion is that estuaries with a higher proportion of freshwater flow have higher shear stresses.

The scatter plot seems to indicate a definite trend between the two parameters, and it would be interesting to supplement this data with other estuary data sets.

4.3.9 Bedform

The bed of a channel can exhibit varying types of ripples or dunes which add to the friction effects on channel flow. Though the bedforms have been studied by many authors (a summary is given in Dyer [1977]) the quantitative effects on shear stress are not nearly so well defined. Attempts at quantifying the effects of rippling on the estuary roughness have been made by various authors and one example which deals with the effect of duning on bed roughness is van Rijn's formula [1984],

$$C=18\log\left(\frac{12R_b}{k_s'}\right) \qquad k_s'=k_s+1.1\Delta(1-e^{-25\psi})$$

where

C is the Chezy coefficient for the dune bed;
R_b is the hydraulic radius of the bed;
k_s is the equivalent roughness for a flat bed;
k'_s is the equivalent roughness for a duned bed;
Δ is the bedform height;
ψ is the bedform steepness.

Other formulae have been put forward, by Yalin [1964] and Allen [1978] among others. However the scatter from implementation of these methods is large and comparisons with observed data are poor. On the non-tidal Vistula river Suszka [1992] investigated all these bedform describing formulae which gave estimations of dune dimensions consistently too large by at least a factor of about two. It follows that the effects of duning are ill-described and bedform is not a reliable parameter for inclusion in estuary regime. It is also something of a negligible influence on a tidal estuary (Suszka [1992]).

4.3.10 Roughness

By analogy with unidirectional open channel flow, estuary regime may be dependent on the 'roughness' of the estuary channel. There are many ways of describing roughness and many have been tried in the context of estuary regime.

The Chezy coefficient was originally suggested in a method proposed by Bruun and Gerritsen [1960e]. This method was used by de Jong and Gerritsen [1984] with some success.

Ackers [1991] in his review of the Ackers-White Sediment transport equations for non-cohesive sediment derives a range of equations for channel velocity in terms of Manning's roughness coefficient and depth and sediment concentration. As sediment size decreases the equations show the effect of roughness becoming more prominent.

Roughness is itself affected by various factors such as bedform which has been referred to previously.

4.3.11 Littoral Drift

As noted before Bruun and Gerritsen [1960f] suggested that littoral drift has an effect on inlet stability, by classifying inlets into three groups based on the tidal prism/ann.littoral drift ratio. Bruun modified these classes later and showed that there was indeed a relation between this ratio and the stability of inlets by classifying a sample of 20-30 different inlets. This result however suffers from the uncertainty about quantitative measures of littoral drift.

Kondo [1990] found an empirical dependence of inlet area on littoral drift in his study on the effects of sea level rise on Japanese coastal inlets, namely,

$$A = 1.02\ Q_{max}{}^{1.1}.a_s{}^{-0.5}.M_l{}^{-0.1}$$

where A is the area at mean sea level, a_s is the tidal range at sea, M_l is the littoral drift.

Note however that the dependence on littoral drift is very small. Calculations of representative values of sediment transport are notoriously difficult and would require a great deal of time and effort to produce. Therefore littoral drift is not a useful parameter for regime especially as its effect seems relatively small.

4.3.12 Others

Bruun and Gerritsen have suggested in their list of important parameters for regime channel cross-sectional shape; where there are two channels for instance, friction forces are increased, making the system less 'efficient'. Keulegan [1951] proposed a 'repletion coefficient', K, to measure this effect and the constant of proportionality between P and Q_{max} was dependent on K. K itself was deduced from tidal period, hydraulic radius, cross sectional area, tidal area of bay, and Manning's coefficient of roughness. The larger the value of K, the quicker currents will flow through the cross section [Bruun and Gerritsen, 1960g].

Eysink [1991] suggested that the empirical constants he used to modify O'Brien's relationship (Section 4.3.2) will be dependent on irregularity of the tidal system, the extent to which the bed sediment is 'muddy' (cohesive) and the level of stratification of the estuary.

Shigemura [1976], from work on tidal inlets in Japan, found very good correlations for a multi-variable regression analysis for throat area based on the following factors: wind energy, wave energy, tidal prism, mean rate of tidal flow, and notably the ratio of throat area to mean surface area of backed bay.

4.4 Summary

The discussions above suggest that the main factors quoted in literature affecting estuaries which can be quantified in a satisfactory way, are:

* tidal prism,
* peak discharge,
* river discharge,
* wave action,
* salinity.

The other factors mentioned are either unquantifiable in a simple enough way for regime purposes, or seem to have a negligible effect on the regime state.

The interaction between the influence of freshwater and salinity on the velocities and shear stress in estuaries deserves further attention, as a definite relationship appears to exist (Figure 4.6)

5 *Extremal hypothesis theory*

In the field of river channel studies, where regime theory has been developed to a much higher degree than for estuaries, extremal hypotheses have been used with some success to describe morphological change. Extremal hypothesis theory came about in a response to the seeming lack of universality of the previous regime theories and the suggestion that other factors controlling the regime system existed. With the improvement in understanding of sediment transport dynamics, it became possible to use these dynamics to try and identify some of the unknown regime factors.

For a given water and sediment discharge, the corresponding alluvial unidirectional channel can be described by its width, depth and slope. The system has therefore 3 unknowns, which mathematically need 3 equations to solve. The equations available are the sediment transport equation, and a friction equation (such as Chezy, or Manning-Strickler), but another is needed. This is where extremal hypotheses come in. These hypotheses state that there is some important controlling quantity, like sediment discharge, or stream power, that nature seeks to maximise, or minimise, in a channel, and that this feature adds the extra equation needed to solve the system mathematically.

A number of different controlling factors, or hypotheses have been put forward, but there are three main ones, the maximum sediment transport hypothesis, the minimum stream power hypothesis and the minimum unit stream power hypothesis (Bettess and White [1987]). The first two have been shown by White et al [1982] to be equivalent, assuming such maxima/minima exist uniquely. Bettess and White [1987] compared these theoretical predictive formulae, using firstly the Ackers-White relationships and secondly the Yang-Parker transport equation with the Keulegan Friction Law, together with various empirical formulae, Simons and Albertson, Kellerhals, and Charlton et al. The authors found that while the Ackers-White relationships compared well with the empirical formulae, the Yang-Parker-Keulegan combination gave width and slope results differing by more than an order of magnitude. Their conclusion was that an arbitrary selection of sediment and friction relationships combined with an extremal hypothesis did not provide a satisfactory regime theory and that as a result it was not clear whether extremal hypothesis theory was a sound basis on which to pursue regime theory.

To investigate the universality of this theory, another respected sediment transport theory, that of van Rijn [1984], and the Chezy Roughness equation were used, and these equations were solved for maximum sediment transport using the procedure outlined by White et al [1982]. The results gave inadmissible answers, adding to the uncertainty over this theory.

Skou [1990] investigated the effect of channel morphology on sediment transport in a tidal inlet, for which observational data was available. The inlet was modelled using a 1D numerical model and sediment transport was simulated using the Engelund-Hansen formula. The net sediment transport through the channel over a tide was determined for different widths and depths of the channel which was modelled as a rectangular prism. The channel dimensions for which net sediment transport was at a maximum corresponded reasonably to those observed in the prototype. However the cross-sectional area at which transport became a maximum was half as big again compared to that observed in nature. This latter discrepancy may be due to the crude approximation of the channel as of constant depth and width and rectangular cross-section but Skou's conclusion was that channel morphology is independent of net sediment transport.

This modelling approach seems to be more applicable to investigating extremum hypotheses in estuaries than the mathematical approach above since due to tidal fluctuations the sensitivity of the system to the arbitrary equations chosen is heightened. Reducing the number of equations used by modelling the system gives less chance of a wildly inaccurate answer. There is still no simple way to model the transport of cohesive sediment, unless a complicated computer model is utilised, and so a non-cohesive transport equation will have to be used. Certainly the extremum approach deserves further research, although such research is hampered without further improvement in the knowledge of sediment dynamics.

6 Correlation study

6.1 Introduction

This part of the investigation considers some of the more plausible regime theories and seeks to see how well they perform on data from a number of estuary data sets. The reliability of each of these regime theories was examined and compared by calculating the correlation coefficients of the relevant parameters from each theory using data from various estuaries.

Estuaries often do not satisfy regime theories due to dredging activity or geological constraint. The graphical results from each method were examined to see if any such features were reproduced.

The relationships studied were as follows:

* $P = CA^b$ log (Prism) vs log (Area at mean tide level);
* $P = CA^b$ log (Prism) vs log (Area at LW) (O'Brien [1931], [1969], etc);
* $P = CA^b$ log (Prism) vs log (Area at HW);
* $Q = CA$ Peak discharge vs Area at peak discharge;
* $Q = CA$ Discharge at peak velocity vs Area at peak velocity;
* $P = CA^b$ log (Prism) vs Area at peak discharge;

* $CP^{1.5} = V$ Eysink's relationship [1991];

* $\dfrac{Q}{C\sqrt{\tau_s/\zeta g}} = A$ de Jong and Gerritsen's relationship [1984];

The Eysink and de Jong relationships have been mentioned previously in the literature review. Also of note is Bretting's relationship [1958, see Chapter 2] but this is very similar to the de Jong theory and so has not been included here.

6.2 Data Sources

The data for the investigation came from two main sources, survey data (published and unpublished) and mathematical model data from previous modelling studies. All the data quoted is for spring tide conditions, and for low freshwater flows, except for the Mersey data which is for a roughly average freshwater flow.

The full list of data sources is given below:

Thames - 'Thames Flood Prevention - Field Survey Data', HR Reports, EX 543-554, 1971.

Avon - Results of a FLUCOMP mathematical model of the Avon. The model was used in 'Weir on the River Avon', Downstream Siltation, HR Report, EX 2494, July, 1992, and calibrated using data from 'River Avon Barrage: An appraisal of Siltation at Projected Sites', EX 711, 1975, and 'Avon Weir Project - First Meeting of the Liaison Committee on Siltation in the Tidal Reaches', Bristol Port Company, Sept., 1991.

Nene - 'The Effects of Proposed Extraction of Water on Siltation in the Nene Estuary', HR Report, EX 307, Feb, 1966. 'River Nene Improvement Scheme - Initial Appraisal of the Effects of Moving the Tidal Limit', HR Report, EX 1387, Dec., 1985.

Parrett - 1977 Field Survey of the R.Parrett, Ports and Harbours Group HR, Wallingford.

Mersey - Results of a 2D TIDEWAY mathematical model of the Mersey. The model was used in ' Mersey Barrage - Feasibility Study Stage III. Mathematical Modelling of Tidal Flows and Sedimentation', HR Report, EX 2303, March, 1991.

Loughor - Results of a 2D TIDEWAY mathematical model of the Loughor. The model is described in 'Loughor Estuary Environmental Models - Phase I: Assessment of Field Data', HR Report, EX 2279, 1991, and 'Loughor Estuary Environmental Models - Phase II: Calibration and Verification', HR Report EX 2354, June, 1991.

The data from the mathematical models was very detailed and cross-sectional-averaged hydraulic quantities were available, or easily computed. The accuracy of the models in terms of being able to reproduce the behaviour of the estuaries they represent, was in all cases very good, and within the margin of error associated with the measurements of the estuary observations themselves.

In the case of the Thames, Parrett, and Nene, velocity observations were made at one point, roughly in the middle of the channel. The Thames observations had sufficient data for depth-averaged velocities to be deduced, the Nene and the Parrett had one mid-depth observation. It is desirable to reduce the error caused by making such single observations and assuming them to be representative of the velocity through the channel. The observations from the Thames Nene and Parrett have therefore been corrected to give an approximation of cross-sectionally-averaged velocities. This correction uses the relationship,

$$V'(t)=V(t)\sqrt{\frac{h'}{h}}$$

where V(t) is the measured velocity at time t, h(t) is the depth at the same point at time t, V'(t) is the velocity at a different position in the cross-section, h'(t) is the depth at this new point.

The relationship comes from a combination of the equations,

$\tau_0=\rho ghs$, and $\tau=\rho kU^2$, giving $U=\sqrt{h}.\sqrt{(gs/k)}$,

where h is the depth, s the energy slope, k is a constant and U is the velocity.

It is used to compute the discharge through a cross-section in the following way:

The cross-section is split into a number of vertical strips (See Figure 6.1). For each strip the average velocity through it is calculated using the above equation and mean values of depth for that strip. The discharge through the strip is found by multiplying by its area, and then the discharges of all the strips are summed to give the discharge through the section.

The data on which the correlations have been performed has been used in a way which improves the results but which is consistent with objectivity. Unreliable data has been removed from the cross-section, e.g. where numerical model cross-sections are not at right angles to the direction of flow, or where cross-section bathymetry is not complete enough to give a reasonable estimate of area of flow. Data for tidal prism correlations has excluded in some cases points upstream where the river interaction creates a departure from the standard power-law, e.g. O'Connor et al [1990]. In the case of the Loughor the constriction at the mouth has been excluded from the correlations.

Except where stipulated the Mersey data set used for the correlations is not the full estuary but excludes the first of 20km from the mouth. The Mersey data set taken in full, produces very poor correlations overall due to the well-known constriction of 'The Narrows', a stretch within which there is a rocky bed. The Mersey actually widens further for a considerable distance here, until the bed becomes alluvium again.

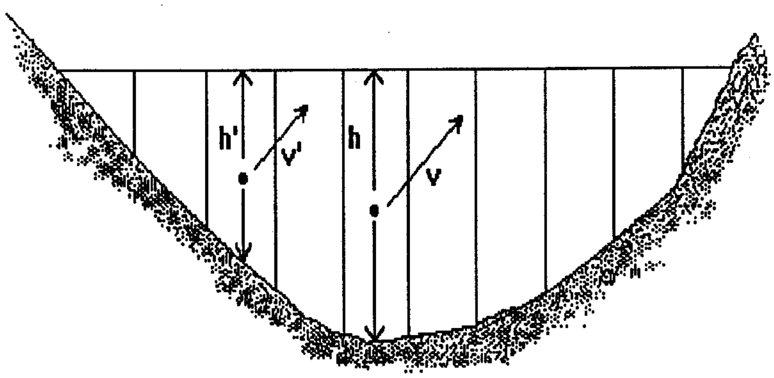


Figure 6.1 *Diagram Showing Method For Correction of Observations*

6.3 Correlation Results

The regime relationships as a whole indicate a very strong linear relationship between tidal discharge and cross-sectional volume. The correlations taken together give a ranking of "regime-ness" appropriate with observations of the estuaries in question. This gives a lot of strength to the regime argument as the relationships considered are very simple.

The regime correlations show that the Eysink relationship is the most highly correlated closely followed by the Q(Vmax)/Area Relationship, with the Qmax/Area relationship performing less well and the prism/area relation correlated least well.

Table 6.1 Regime Correlations (r values)

	Qmax/Area $_{qmax}$	Q_{vmax}/Area $_{vmax}$	Prism/Area $_{msl}$	Eysink	Prism/Area $_{qmax}$
Thames (low)	0.9977	0.9977	0.9940	0.9984	0.9879
Avon	0.9823	0.9864	0.9818	0.9964	0.9812
Mersey*	0.9960	0.9971	0.9783	0.9870	0.9965
Loughor	0.9956	0.9969	0.9822	0.9982	0.9946
Parrett	0.9682	0.9774	0.9764	0.9818	0.9184
Nene	0.9641	0.9746	0.9732	0.9860	******
Thames (high)	0.9988	0.9984	******	******	******
Average	0.9861	0.9898	0.9810	0.9913	0.9714

(***** indicates that there was insufficient data for the correlation).
(* The first 20km from the estuary mouth was excluded from this data set)

Table 6.2 Average Correlations For Each Estuary (r values)

Thames (low)	0.9951
Avon	0.9838
Mersey*	0.9910
Loughor	0.9935
Parrett	0.9644
Nene	0.9745

(* The first 20km from the estuary mouth was excluded from this data set)

Table 6.3 Further Regime Correlations (r values)

	Prism/HW area	Prism/LW area	de Jong
Thames	******	0.9958	******
Avon	0.9778	******	0.9434
Mersey*	0.9884	0.8861	0.9820
Nene	0.9376	0.8571	******
Parrett	0.9979	0.95	******
Average	0.9754	0.9223	0.9627

(***** indicates there was insufficient data for the correlation)
(* The first 20km from the estuary mouth was excluded from this data set)

6.4 Discharge/Area Correlations

The Q(Vmax)/area relationship correlated much more highly than the Qmax/area relationship although they gave similar correlation values for each estuary apart from the Avon and the Nene. Very often the values used for the plots would be the same, especially for the downstream cross-sections of estuaries with large tidal ranges. Intuitively, the Q(Vmax)/area relationship is a better estimate of the equilibrium condition as it is more closely connected with the maximum value of bed shear stress, and so more closely connected with the limiting condition for scour. Perhaps it is for this reason that the Q(Vmax)/area relationship correlates more highly with area than the Qmax/area relationship.

The Qmax/area and Q(Vmax)/area correlations are both sensitive to geological constraints and dredging activities and so describe the regime state well.

The Thames has a very high correlation with both relationships and doesn't show as much variation from regime in the same way as the other rivers above. The main diversion from regime according to the discharge relations is in Long Reach. There is little evidence of any restriction in the prototype and this error can be put down to a problem in estimating its cross-section. (Namely that the cross-section given was incomplete and the area was correspondingly under-estimated). The mud reaches occurring in Halfway reach, which are dredged regularly, do not show up as unusually large in cross-sectional area. This may be due to a deficiency in the regime algorithms or due to the large size of the Thames making even large dredging operations negligible due to the great cross-sectional area.

In the Avon there appear to be geological constraints for most of the 1-10km stretch [HR 1975]. The report states, that the most constricted stretches are around 4.5-6km, with an enlarged cross-section around 7.4km and enlargement at 10-11km. The constriction in the 4.5-6km stretch coincides with outcrops of Keuper Marl [O.S. 1:50,000 Geological Survey] and the results of the Qmax/Q(Vmax) correlations agree with this. The enlarged cross-sections between km's 10 and 11 are due to dredging by the Port of Bristol Authority and again are represented in the regime plots. The enlargement at km 7.4 is well represented. Upstream of 10.5km, the geology becomes alluvium again and outcropping reduces. The Q(Vmax) parameter seems to bunch the data near the origin, peak velocities often occurring near low water, sometimes even for large cross-sections. As a result it is much more difficult to decipher the regime from the Q(Vmax)/area scatter plot.

The Mersey, with its "Narrows" is well represented by the correlations, and gives a very unusual distribution of discharge and area. Further upstream, however, the river reverts to a regime state.

The Loughor has a narrow mouth due to geological constriction and also a narrow cross-section at the railway bridge (cross-section 9), seemingly caused by man-made construction. In both discharge relationships these constraints are displayed.

For the Nene the data was sparse, and although little can be derived from its plot, the dredging occurring near Wisbech is picked up by the relatively high cross-sectional area of the Qmax/area and Q(Vmax)/area points relating to this stretch of the estuary.

Both the Qmax and Q(Vmax) relationship need a lot of input data. This is, on the face of it, a deterrent to their use. However, in most engineering studies nowadays, and it is in this context that a regime relationship is useful, a considerable amount of numerical modelling is carried out. This is in order to gauge the short-term effects of a scheme, such as a barrage, or dredging, etc, and so the data is often available and waiting to be utilised for regime purposes.

6.5 Eysink's Relationship Correlation

The Eysink plot is the most highly correlating of the regime relationships explored, with a mean correlation coefficient value for the six-estuary data set of r=0.9913. The relationship does not rank the estuaries in order of "regime-ness" in the same way as the discharge relationships do, however. Some estuaries which do not correlate highly as Qmax/Area plots, correlate highly with the Eysink plot, e.g. the full Mersey data set (i.e. the data set including the first 20km from the estuary mouth) has a relatively poor correlation of r=0.8883 for the discharge plots but achieves r=0.9944 for the Eysink plot.

The Eysink relationship is more direct in terms of giving information about the volume of the estuary. If the relationship could be believed without question, then no flow model would be needed to produce information about erosion/accretion.

The method suffers from the lack of knowledge of an estuary's volume in the first place. At best, the estuary volume is interpolated into homogeneous cells from partial knowledge given by surveying. At worst, the estuary volume has to be averaged from knowledge of a few cross-sections and the separation between them. Averaging does have one good effect that the relationship is not over-sensitive to local disturbances at one particular cross-section and it gives a better idea of the over-view. On the other hand it is possible that important features may be 'ironed-out' in the averaging process leading to loss of detail.In all the figures derived for this investigation, the volume between two cross-sections was taken to be the geometric average of the two adjoining cross-sections multiplied by the length of their separation.

In the cases of the Avon, Nene, Parrett and the upstream part of the Mersey data set, the Eysink plot does not demonstrate a linear variation between $(\text{prism})^{1.5}$ and Volume_{MSL}. To explore this the best fit relationships between prism and Volume_{MSL} were calculated for the geometric relationship,

$\text{Vol}_{MSL}=C.P^n$

The best fit relationships are shown below. It can be seen that there is a lot of divergence from Eysink's theory.

Relationship between prism and Vol_{MSL}, where $\text{Vol}_{MSL}=C.P^n$

Estuary	Thames	Nene	Parrett	Mersey	Avon	Loughor
n	1.5	2.12	$n \approx 3$	exp	1.88	1.5

The non-linearity displayed by the Nene, Parrett and Avon can be explained by the fact that the Nene and Parrett are not rivers correlating highly in "regime-ness" and the Avon suffers from geological constriction and so is less in regime than the Thames and the Loughor. The Mersey upstream data set, which correlated highly in the discharge plots, gives an exponential relationship between P and V_{msl}, rather than an geometric one, is harder to explain.

The Eysink [1991] relationship makes three assumptions, namely the following:

* A $= C_a.P$ where A is the mean tide cross-sectional area
* ${}_0\int^L P(x)dx$ $= C.P(L)$
* $P(L)$ $= \text{const}.L.B.R$

where B,R are the mean width and range over $0 \le x \le L$

(i) $A = C_a.P$ The original work by O'Brien [1931] proposed a relationship between area at low water and tidal prism for the mouths of tidal inlets. Figures 6.2-6.4 show that assumption (i) is broadly true but by no means reliable.

(ii) ${}_0\int^L P(x)dx = C.P(L)$ This was a reasonable assumption for the Thames and for the Avon where values of C were restricted to ranges 0.35-0.5 and 0.3-0.4 respectively. For the other estuaries the value of C varied much more:

Nene	C=0.28-0.5
Mersey	C=0.38-0.7
Parrett	C=0.2-0.5
Loughor	C=0.22-0.45

So this approximation is also unreliable.

(iii) P(L)=const.L.B.R Ratios of the RHS of the equation to the LHS produce the following results:

Thames	0.83-1.04
Avon	0.59-0.81
Nene	1.0

This assumption does not seem to be generally true in a real estuary.

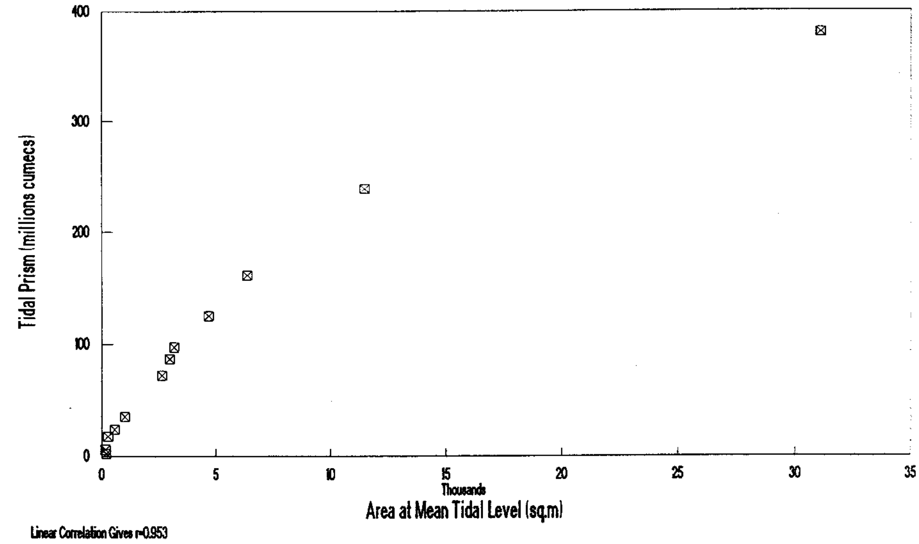


Figure 6.2 *Correlation between the tidal prism and the mean tidal area for the Thames estuary*

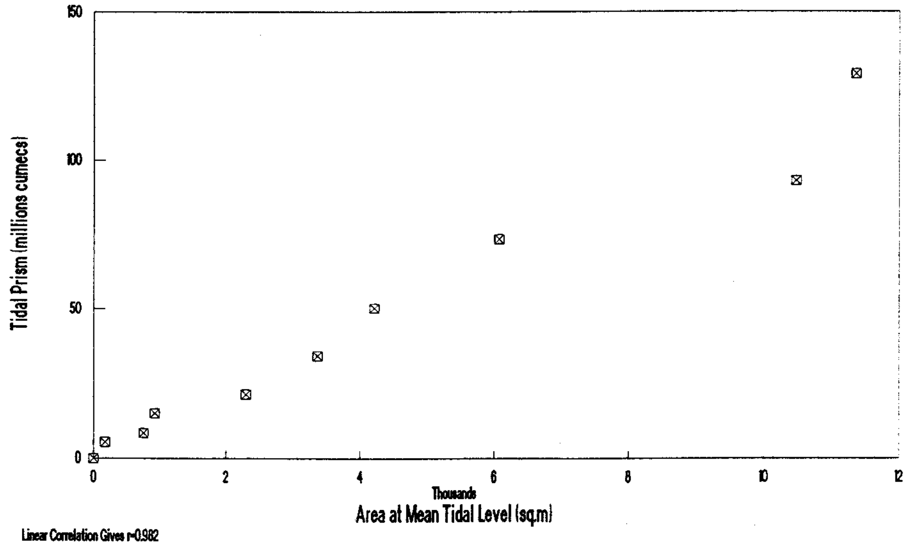


Figure 6.3 *Correlation between the tidal prism and the mean tidal area for the Loughor estuary*

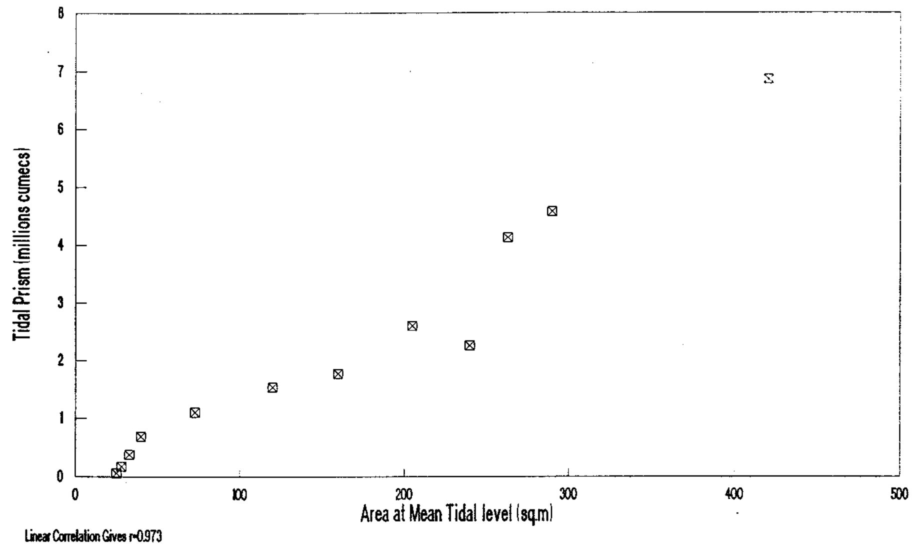


Figure 6.4 *Correlation between the tidal prism and the mean tidal area for the Nene estuary*

Eysink's relationship makes some unreliable assumptions and this must make one sceptical about its applicability. The results of comparing the observed values of C_a from the prism/area correlations (Figures 6.2-6.4) with those derived from the gradients of the Eysink relationship plots reflect this unreliability.

Estuary	C_a (observed)	C_a=gradientx$(BR/L)^{0.5}$
Thames	$1.05x10^{-4}$	$4.0x10^{-5}$
Avon	$6.35x10^{-5}$	$1.4x10^{-5}$
Nene	$1.1x10^{-4}$	$6.3x10^{-5}$

The two sets of results bear no resemblance to each other.

The Eysink relationship does give regime information, and this is explored below.

The Thames plot displays no regime detail, which is similar to the case of the discharge relationships. The Avon plot is non-linear. For upstream points the line has a low gradient (little change in tidal prism for change in volume). For downstream points the gradient steepens. This reflects the regime behaviour of the Avon. The upstream points correspond to cross-sections which have been dredged and there will be a rapid increase in volume up to the point corresponding to km 9 where geological constriction occurs and the gradient steepens as volume increases less rapidly.

The Parrett plot displayed a relatively poor linear fit, and varied according to a quadratic distribution of $P^{1.5}$. The variation seems to portray enlargement in the upper estuary and constriction in the lower part. Geological investigations give no real evidence of this, however.

The Mersey has already been mentioned, but the lower part of the estuary does not display a linear relationship. The implication is that the estuary becomes more and more constricted as you get nearer the narrows.

6.6 de Jong and Gerritsen's Relationship

Consider,

$$A = \frac{Q_{max}}{C\sqrt{\frac{\tau_s}{\rho g}}} \qquad (*)$$

If the formula $\tau = V^2 C^{-2} \rho g$ is used, where V is given by Qmax=VA, then the RHS of (*) becomes,

$$RHS = \frac{Q_{max}}{C\sqrt{\frac{Q_{max}^2}{A^2 C^2}}} = \frac{Q_{max}}{C(\frac{Q_{max}}{AC})} = A$$

Therefore equation (*) becomes a tautology.

So $\tau = \rho gRS$, where R is the hydraulic radius and S the slope, has to be used. The problem with this equation is that the slope is notoriously hard to derive or even estimate correctly due to lack of detailed survey data. Only where a computer model has been run of the estuary will there be enough data to produce a good estimate of the slope. Substitution of this equation for shear stress in (*) yields,

$$A = \frac{Q_{max}}{C\sqrt{RS}} \Rightarrow C\sqrt{RS} = V$$

and so the de Jong and Gerritsen relationship in effect tests for whether Chezy's law is satisfied at the point when discharge is a maximum across a cross-section. This does not tell us about whether an estuary is in regime or not. From the plots there is reasonable satisfaction of the Chezy law, though not without scatter, and this is what one might expect.

6.7 Prism/area Correlations

The prism correlations do not perform as well as those for the relationships based on discharge, although a clear trend between tidal prism and area is borne out by the results. The best overall results seemed to be for prism and area at mean tidal level, followed by prism and area at high water. Results for prism and area at low water were disappointing, half the correlation coefficients being less than 0.9. If the relationships derived for the low water relationships are put into the form $A=a.P^b$ where a,b are constants, as put forward by O'Brien, the value of b ranges from 0.33 to 1.66. Although the values of O'Brien, Jarrett, etc, fall within this range, one is not given any confidence in the applicability of their work to estuaries by this result.

6.8 Summary

The results of this investigation show that there is a clear and very high correlation between cross-sectional area and parameters measuring the size of the tide through the cross-section. The highest correlating relationship is the Eysink relationship which combines tidal prism and estuary volume, but this is unreliable and based on some questionable assumptions. Relationships based on peak discharge and cross-sectional area seem to correlate more highly than those based on tidal prism. Of these Q(Vmax)/area$_{V_{max}}$ correlates highest. Prism/Area relationships still correlate well, especially for estuaries very much in regime, such as the Thames. The degree of correlation for the more successful relationships seems to be a satisfactory measure of the extent to which an estuary is in equilibrium.

The use of regime theory in estuaries has to be sensible. Obviously where there are significant geological constraints the assumptions on which the regime theory works do not apply. Some of the relationships are restricted in use because of the nature of the input data that they need, however in most contemporary civil engineering investigations, this is not a problem.

7 Investigation of effects of waves and salinity on estuary regime correlations

7.1 Introduction

It has been shown that the action of waves on the bed of a channel lowers the current velocity needed to initiate sediment transport, Mehta [1988], and that as a result an estuary widens near its mouth, Chantler [1974]. Attempts to quantify this effect have been made by Bijker [1967], who has produced the most commonly used formula, but also by others who have sought to improve on Bijker's work, e.g. Grant and Masden [1979], Fredsøe [1984].

Similarly the effect of salinity on sedimentation in channels has been noted by Parchure and Mehta [1985] and before them by Rosenqvist [1961] and Peirce and Williams [1966]. These authors suggested there is a salinity, of around 2ppt, before which shear stress increases rapidly with salinity and after which shear stresses increase much less rapidly. Partheniades and Mehta went on to say that after 10ppt there is negligible further increase in shear stress. The effect of salinity on a real estuary is the seasonal erosion of the channel at the upstream end when the large freshwater flows of Winter/Spring occur with corresponding accretion due the low flows of Summer/Autumn. These results are brought into doubt by the findings of Stevenson and Burt [1985] who found no significant effect due to salinity on settling velocities, however the wealth of work supporting the effect of salinity and seasonal variation in bed levels, not fully explained by increased winter flow, suggests that salinity has an effect.

The two effects of wave action and salinity will make some impact on the discharge/area relationships. Cross-sectional areas at the extreme downstream of the estuary will be larger as will cross-sections where the salinity falls to levels of below 2ppt. The impact of this effect and the wave effect is to make the relationship between discharge and area non-linear. The correlations of discharge and area produced in the previous chapter, without correcting for the influence of waves and salinity, already exhibit a linear relationship between discharge and area. What this part of the investigation seeks to do is to remove the effects of salinity and waves from the correlations to see if the correlations improve, that is, to see if the relationship between area and discharge is made even more linear.

If there is a linear relationship between discharge and area, then the effects of waves and salinity disturb this and add 'noise' to the relationship. By estimating the effects of salinity and wave action on the discharge/area observations and adjusting the observed data in such a way that these effects are removed, it is possible to remove some of the 'noise' and gain a better idea of the relationship.

The method and results of the adjustments is given below. It should be noted that the only estuaries affected by waves are the Thames, Loughor and to a negligible extent, the Avon. The Mersey is probably affected but there is a constriction at the 'Narrows' which will obscure the effects. The estuaries for which the data available is affected by low salinities are the Thames, Loughor, Nene, and possibly the Avon although salinities for the upstream 5km are not available.

The relationship used in this investigation is Q_{max}/area since at an early stage of writing this report it looked to be the more reliable than the Q_{Vmax}/area relationship, even though the latter turned out to be the more highly correlating of the two (see chapter 6).

7.2 Wave Corrections

As an initial investigation into the effects of waves on regime correlations the method of Bijker [1967] was used; this seems to be at present the most commonly used equation. Recent work by the G8M project [Soulsby et al 1993] into wave-current interactions has been able to compare the performance of various formulae and any improvement on Bijker's work will be incorporated into the following study in future work. Bijker's formula, assuming for simplification that waves and current are in the same direction, is given by,

$$\tau_{wc} = \tau_c[1 + \frac{1}{2}(\xi \frac{U_0}{V})^2] \qquad (1)$$

where

τ_{wc} is the shear stress due to waves and currents (N/m^2),
τ_c is the shear stress due to currents only (N/m^2),
U_0 is the wave orbital velocity (m/s),
V is the current velocity (m/s),
ξ is a coefficient

Eysink [1991] involves the equation (1) with an equation incorporating area and tidal prism. In a similar way, equation (1) can be combined with the discharge/area equation to give,

$$Q_{max} = M.A_{qmax}[1 + \frac{1}{2}(\xi \frac{U_0}{V})^2]^{-\frac{1}{2}} + C \qquad \text{M,C constants}$$

The orbital velocity can be calculated from the following equations [Delo and Ockenden 1992],

$$U_0 = \frac{\pi H}{T\sinh(\frac{2\pi D}{L})}$$

where

H is the wave height (m),
T is the wave period (s),
D is the water depth (m),
L is the wave length (m).

H and T are deduced from standard wave forecasting curves based on JONSWAP data. It is necessary to know fetch lengths and wind speeds to use these curves, and these were deduced from O.S. 1:50,000 maps and suitable wind rose plots.

L is deduced from the equation below [Delo and Ockenden 1992],

$$\frac{2\pi}{T^2} = \frac{g}{L}\tanh(\frac{2\pi D}{L})$$

which is solved for L using a Newton-Raphson iteration method.

For the purposes of an initial investigation, it is sufficient to take a mean wind speed and fetch corresponding to a particular direction and to deduce the increase in bed shear stress caused by it, and then average all the different shear increases according to the probability of a wind being located in a particular sector.

The results from those estuaries affected are as follows:

<u>Thames (low freshwater flow)</u> -
Increase in shear stress at Southend by a factor of 1.28
Increase at Thameshaven by a factor of 1.27

<u>Thames (high freshwater flow)</u> -
(Flow data for cross-section at Southend not available)
Increase in shear stress at Thameshaven of 1.087

<u>Avon</u> -
Increase in shear stress at 0m chainage of 1.0024, which is negligible.

<u>Loughor</u> -
Increase in shear stress at the mouth of 1.136, but this cross-section is constricted anyway and so has not been included in previous or future correlations.

There is no increase in shear stress at further upstream locations as there is a sand bank near the mouth, located in the middle of the estuary, which it is assumed absorbs the wave energy at the time of peak discharge.

7.3 Salinity Corrections

Data on salinity has been derived form the following sources,

Thames - 'Thames Flood Prevention - Field Survey Data', Hydraulics Research Report, EX 543-544, 1971.

Nene - 'The Effects of Proposed Extraction of Water on Siltation in the Nene Estuary', Hydraulics Research Report, EX 307, Feb, 1966.

Mersey - Results of a 3D TIDEWAY Numerical Model of the Mersey. The model is detailed in 'Mersey Barrage Feasibility Study Stage III. Mathematical Modelling of Tidal Flows and Sedimentation', Hydraulics Research Report, EX 2303, March, 1991.

Loughor - 'Llanelli Sewage Disposal Scheme -A Review of Water Quality in the Loughor Estuary'. A Report to Dwr Cymru -Welsh Water South West Division, 1990.

Parchure and Mehta (1985) give a graph for the distribution of critical bed shear stress with salinity for a lake mud (composed of montmorillite, illite, kaolinite and quartz) in water of pH 8.6. This graph is shown in Chapter 4 (Figure 4.5). As can be seen from the graph there is a dramatic reduction in the gradient at round about 2ppt, and at 10ppt there is 'insignificant' further increase. The method used to correct the cross-sectional areas for current effects alone is to assume that the shape of the graph is true for all cohesive sediments, and for the bed of every estuary. The graph then represents a non-dimensional relationship between salinity and shear stress and shows by what factor shear stress reduces in a channel for different salinities. Some authors, (see the section on salinity in chapter 4) postulate that complete flocculation occurs at 2ppt. In this case only the 0-2ppt portion of the graph is used and the salinity correction factor k, is given by,

$$k = \tau^P(s)/\tau^P_{s=2}$$

where s is the salinity (ppt), τ^P is the shear stress from Parchure and Mehta's graph.

The graph of Parchure and Mehta, though, suggests that flocculation is not fully complete until a salinity of about 10 ppt is reached. In this case,

$$k = \tau^P(s)/\tau^P_{s=10}$$

These two assertions that complete flocculation occurs at 2ppt and 10ppt have both been used to correct the discharge/area values derived in the last section, and so two differing sets of correlations are produced for comparison below.

The values of salinity used are the mean values throughout a spring tide. The adjusted shear stress is given by,

$\tau' = \tau/k$

where τ' is the adjusted shear stress, without salinity effects, τ is the observed shear stress and k is the salinity adjustment factor.

The adjustment discharge/area relationship, including the effects of waves is then given by,

$$Q_{max} = M.A_{qmax}.k^{-\frac{1}{2}}.[1+\frac{1}{2}(\xi\frac{U_0}{V})^2]^{-\frac{1}{2}} + C \tag{30}$$

where k is the salinity shear stress correction factor.

The results of this method are given in the table below:

Estuary	section/ chainage	salinity (ppt)	$k^{0.5}$ (2ppt)	$k^{0.5}$ (10ppt)
Thames (low fwf)	Syon Rch	0.4	0.70	0.59
	Coryton	0.4	0.70	0.59
	Chelsea	1.0	0.77	0.73
	U.Pool	2.1	-	0.85
	L'house	3.3	-	0.89
	Woolwich	7.5	-	0.96
Thames (high fwf)	Syon Rch	0.3	0.59	0.50
	Coryton	0.3	0.59	0.50
	Chelsea	0.4	0.70	0.59
	U.Pool	0.5	0.75	0.62
	L'house	0.5	0.75	0.62
	Woolwich	1.4	0.94	0.79
	Barking	2.2	-	0.85
	Halfway	5.0	-	0.93
Nene	Guyhirne	0.4	0.70	0.59
	Wisbech	4.6	-	0.92
Loughor	13.0 km	9.1	-	0.99
	14.1 km	5.9	-	0.94
	15.4 km	3.6	-	0.90
	16.5 km	2.3	-	0.87
	18.2 km	1.1	0.89	0.83

7.4 Correlation Results

The results of 'correcting' the discharge/area data for waves and salinity, are given in the table below.

Estuary	r (no corrections)	r (2ppt)	r (10ppt)
Thames (low fwf)	0.9977	0.9989	0.9989
Thames (high fwf)	0.9988	0.9992	0.9998
Nene	0.9855	0.9822	0.9885
Loughor	0.9956	0.9958	0.9960
Average	0.9944	0.9940	0.9958

7.5 Summary

The results suggest that the correlations can be improved by the filtering out of the wave and salinity effects but there are too few estuaries tested here to draw any conclusion. The results show that there is very slight decrease in correlation using the 2ppt salinity correction but using the 10ppt salinity correction method gives a much better correlation than the 2ppt correction or the original data. If the salinity graph of Parchure and Mehta is taken at face value, together with the use of it described above, then this exercise has lent support to the idea of a linear relationship between peak discharge and cross-sectional area. Further, it suggests that regime relationships such as this one might give more accurate assessments of the state of an estuary if salinity and wave effects are taken into account in a simple way.

8 A simple predictive test

8.1 Introduction

The biggest problem with testing the effective predictive ability of regime relationships, or indeed all morphological models, is the scarcity of available data. It is very rare to find a site where a survey has been carried out beforehand, an engineering scheme carried out, and then another survey carried out some years after to assess the effects of the scheme. This is lamentable from the point of view of the researcher, but may change as the emerging ethos of 'Duty of Care' takes hold in law and in the minds of the public at large.

Frequently it is necessary therefore, for the regime researcher to compare regime model results with those of mathematical models. At present mathematical models are reliable over the short term, but not over the long term. This is due to the uncertainties inherent in descriptions of cohesive-sediment processes, the possibility of episodic climatic events, the lack of flexibility inherent in many models and the cost, computer power and time needed to simulate complex hydrodynamic processes over the long term [Dennis 1990]. Comparison of the long-term predictive results of a regime model with those of numerical models is therefore unsatisfactory since the regime model is being compared with a model that is possibly unreliable, this unreliability being the reason for the creation of the regime model in the first place. If the models agree then it is not known if they are both incorrect or both predicting correctly. If they disagree it is not known which model is producing the most realistic results. However, even if these questions cannot be answered then such a comparison is still worthwhile in that if the models agree, it adds confidence in the predictive ability of both, and if they disagree then it creates an air of suspicion of both, which can only be a healthy attitude in the area of long-term prediction.

8.2 The Test

The estuary selected as a test was the Avon. It was chosen because flow data is available and also there are predictions for long-term effects of a disturbance on the estuary. It is proposed that a weir, or barrage, be constructed at a chainage of 13.3km, some 3.5 km downstream of the present tidal limit. Flow data for the existing conditions, and those immediately post-construction were available from a numerical FLUCOMP model used by HR to model the Avon. The mathematical model was calibrated from data quoted in Hydraulics Research report EX 711, 'River Avon Barrage: An Appraisal at of Siltation at Projected Sites', and using bathymetries given in 'Avon Weir Project - First Meeting of the Liaison Committee on Siltation in the Tidal Reaches', Bristol Port Company, Sept., 1991.

The task of the models used in this test was to predict the long-term regime morphology of the Avon estuary, after the weir has been constructed.

The Avon is not an estuary that is strictly in regime, due to some geological constriction occurring in the downstream estuary and some dredging having occurred in the upstream estuary [See HR EX 711, above]. As such it is something of a worst case for a regime model, but on the other hand probably represents the average estuary well, and therefore the Avon is a suitable case for a test of a regime formula.

8.3 A Description of the Predictive Models Used in the Test

The regime model used was based on that given by O'Connor et al [1990] where a 'model' was proposed for long-term predictive purposes in estuaries comprising of a 1D numerical flow model and a regime relationship. The regime relationship described in this paper was based on a simple relationship by Jarrett [1976], taking the form,

$A_{mtl} = a.P^b$ where a,b are constants. (*)

The flow model was run for existing conditions to give the data needed to derive the empirical relationship describing the regime (the values of a and b) and to give similar relationships for height and width and side slope of cross-sections in terms of P. The flow model was then run for conditions after the civil engineering scheme, in this case a tidal barrage, to give a new set of tidal prisms from which a new morphology could be calculated using the relationships already established. This new estuary geometry was used as boundary conditions in a new flow-model run. This time the resulting data is checked to see if the prism/area values satisfy equation (*). If they do then the future regime morphology has been calculated. If not then the morphology must be re-calculated and the flow model run again, continuing until (*) is satisfied.

O'Connor et al gave an example for the River Usk, and found that two iterations of this procedure were necessary to derive the regime estuary geometry. Making some simple assumptions about sediment concentration and deposition, they derived a value for the time period over which the morphological changes would take place.

The approach of O'Connor et al is a way forward for empirical relationships which previously only gave information as to whether an estuary was in regime or not, to be used in a predictive capacity. This is an important step forward in regime theory. The technique depends on the relation (*), and the other width, depth, slope relationships describing the regime state well enough and it is here that research needs to be applied. The O'Connor system, seems to be the problem solving method best suited to long-term prediction and represents the direction that this research programme will take in the future. A simple outline of the method is shown in flow-diagram form in Figure 8.1 below.

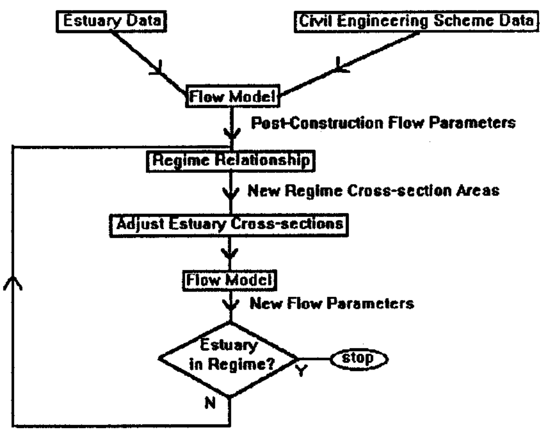


Figure 8.1 *Flow-Diagram of the Proposed Regime Model*

The regime relationship used in the present investigation for the regime mo del is the peak discharge/area relationship for high freshwater flow. A discharge relationship was used as the research of chapter 6 showed that they correlated more highly, and peak discharge was used as in this case, since discharge at peak velocity gave very scattered results, departing from the regime behaviour seen in chapter 6 (See Figures 8.2 and 8.3). Only flow data for high freshwater flow was available for the pre and post-weir cases.

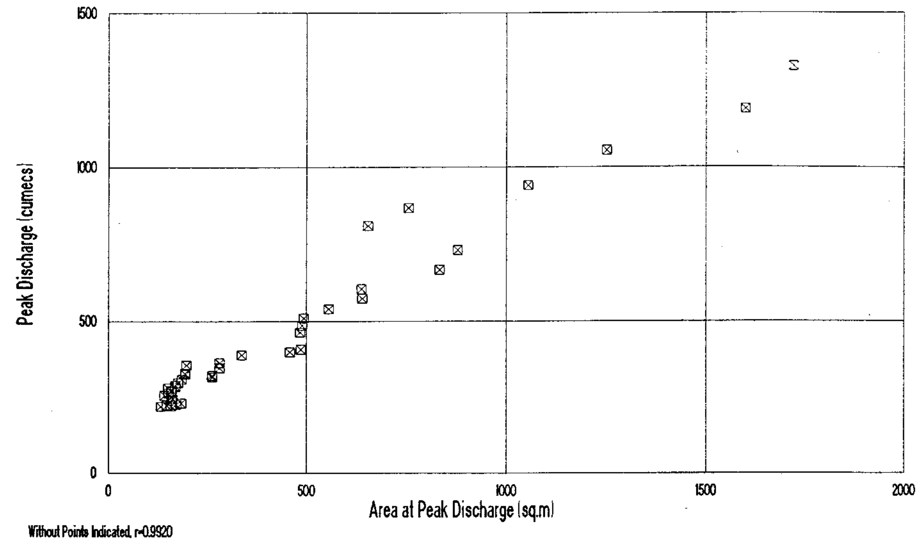


Figure 8.2 *Correlation between discharge and cross-sectional area at peak discharge for the pre-weir conditions*

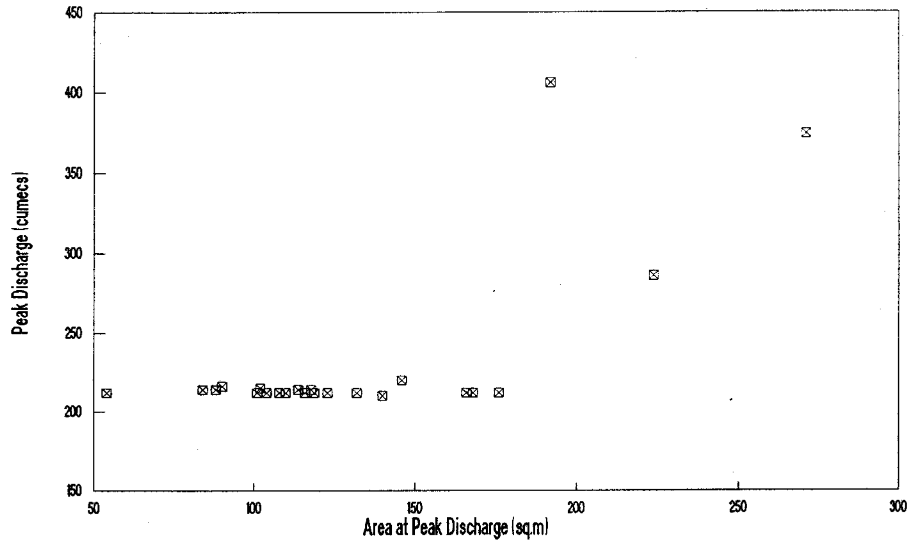


Figure 8.3 *Correlation between discharge and cross-sectional area at peak velocity for the pre-weir conditions*

The correlation between peak discharge and area at peak discharge for the pre-weir conditions is,

$Q_{max} = 0.6772\ A_{qmax} + 153.2 \quad r=0.9920$

This relationship excludes the data from the constricted stretch as it is considered that its cross-sectional area will not reduce with reduced velocity, and so can be treated separately.

At present it has not been possible to run the output iteration of the regime relationship back through a flow model, so the output of the regime model is from one 'iteration' of the model only, and is only a first approximation of the final regime state.

The other model that has been used here is the 'Siltation at a Point' (SAP) model, a zero-dimensional mathematical model which predicts the changes at a point on a cohesive sediment bed, over a period of many tides. Using data derived from flow-models, in this case the results of the flow model mentioned above, and in-situ sediment surveys this model predicts changes in bed level at a cross-section by modelling the whole of the bed-sediment interaction. In order to do this, the model makes simplifying assumptions about the nature of the cohesive sediment on the bed. An explanation of the model is given in HR [1992b].

8.4 Results

Regime Model

The discharge/area plot for the post-weir run is given in Figure 8.4. The discharges are then entered into the pre-weir regime formula to derive the areas (at peak discharge) for the final regime. The regime cross-sectional areas for the points in the constricted section are assumed to remain the same.

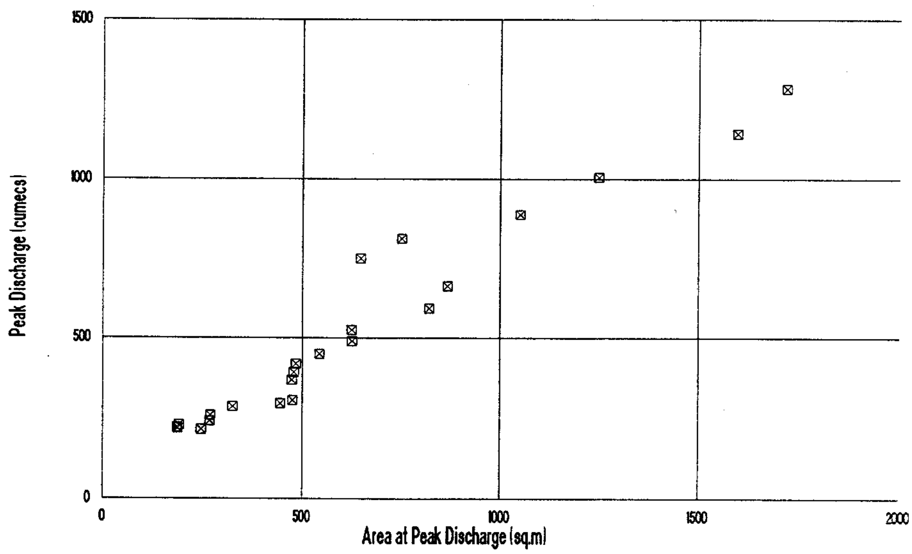


Figure 8.4 *Correlation between discharge and cross-sectional area at peak discharge for the post-weir conditions*

SAP Model

The final regime cross-sections for five of the Avon cross-sections are given in Figure 8.5 [HR 1992b]. The regime cross-sections shown are existing conditions, the best estimate of the final regime state, and a worst case

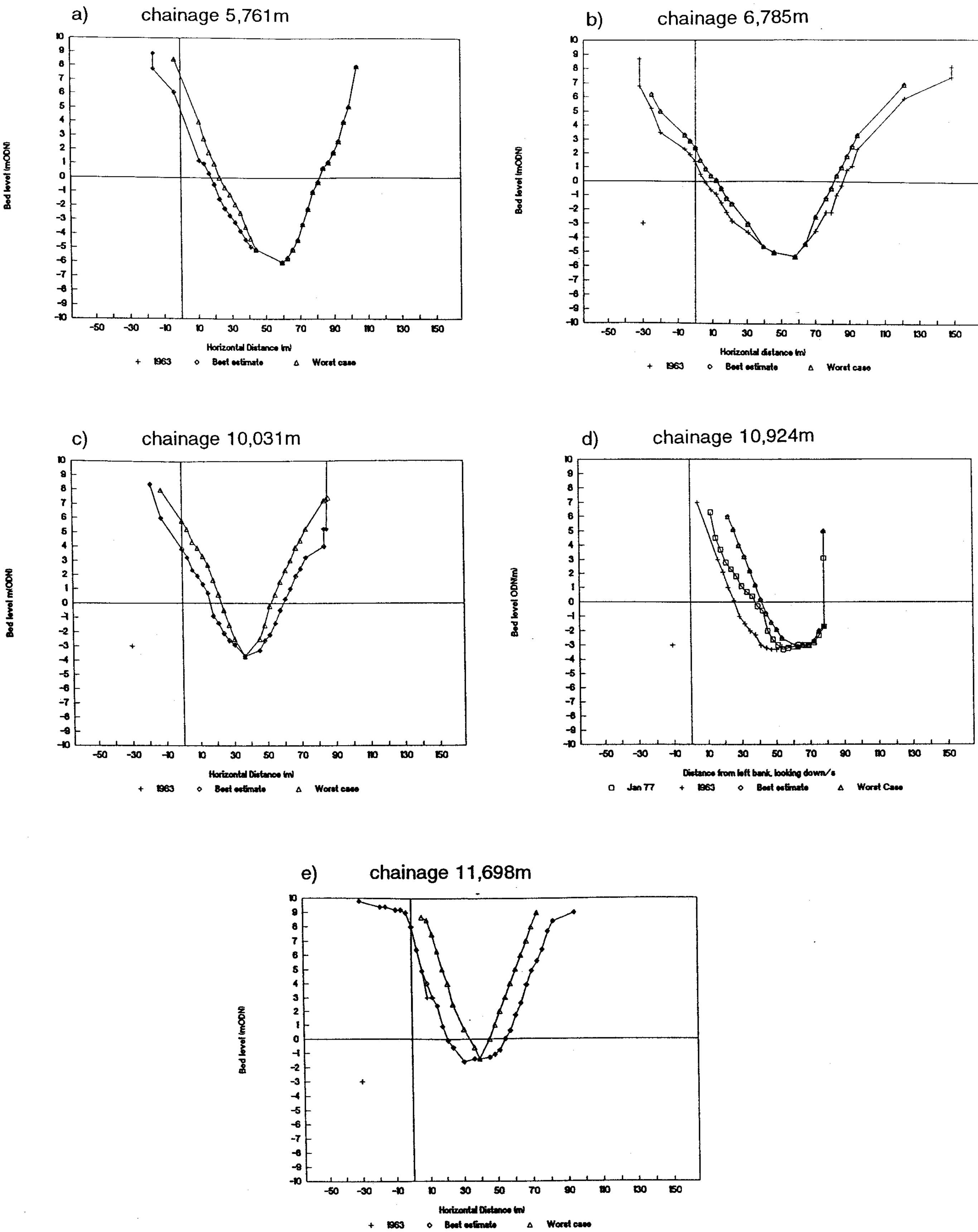


Figure 8.5 *Cross sections on River Avon estimates of post-weir conditions*

scenario. To compare these with the regime results the areas of these cross-sections are computed for the water levels corresponding to the peak discharges of the post-weir run. These may not be the exact levels of the final estuary equilibrium state but serve as an approximation.

The results of both the mathematical SAP model and the 'regime' model are given below. The cross-sectional areas at peak discharge for the five cross-sections considered are shown for both models including the worst case scenario results for the SAP model. They show that the regime model predicts similar cross-sections to the SAP model for the downstream estuary but drastically smaller upstream cross-sections, smaller even than the worst case scenarios. Since the discrepancy is in the upstream part of the estuary, one would immediately suspect that the difference was due to one of the models not taking freshwater flow into account but both models do consider freshwater flow, the regime model being based on discharges from high freshwater flow. The discrepancy may partly be due to the fact that only one iteration of the regime model was used, and therefore it is possible that the regime model had not 'converged', however this is unlikely to account for such a large difference.

Comparison of the Regime and SAP Model Results

Chainage (m)	Regime Model Area (m^2)	SAP Model Area (Best Estimate)	(m^2) (Worst Case)
5719	750	729	729
6785	648	648	581
10031	311	423	335
10924	222	324	324
11698	158	263	263

9 Conclusions and further work

1. The studies undertaken so far have investigated the history of regime theory in estuaries, and have searched literature to find the main factors involved in estuary regime processes.

2. The main factors affecting the estuary regime appear to be tidal and freshwater discharge, wave action, salinity, and also the ratio of freshwater to tidal flow. These factors have been suggested as starting points for an estuary classification.

3. The most promising of the regime relationships have been tested and found to give good correlations, the best correlations coming from relationships involving discharge. Extremum theory has been investigated for relevance to estuary regime theory and although there is success in this area in river regime theory, it appears that at the moment this is not a fruitful way forward for research in estuaries.

4. One of the regime relationships, Q_{max}/area, has been modified to allow for the effects of two main influencing factors, wave action and salinity and correlations are improved as a result, strengthening the regime theory case.

5. A proposal for use of estuary regime relationships in a long-term predictive role has been suggested, and one such relationship has been used in this way to predict future regime cross-sectional areas, after the construction of a weir in the estuary of the Avon. Comparison with the SAP numerical model results was found to be inconclusive.

6. This work has added to the development of regime theory and helped to establish a role in which regime theory can be used predictively. It has pointed towards areas for future improvement of regime relationships but there remains much scope for further work in this field.

Further Work

Firstly, sufficient data to model a before/after scenario must be obtained so that a regime model can be fully tested. To do this, a regime relationship will have to be used in conjunction with a flow model, and a 1D model that is accurate enough but quick to run will have to be obtained. Two possibilities for data sets for test purposes are the Thames, now with a tidal barrier, and the Rance, with a tidal barrage. It is not known at present whether post-construction surveys have been carried out or whether these are obtainable, but it is proposed to explore these avenues.

If such a data set became available then a useful exercise would be to analyse the change in cross-sectional profile through an estuary, to see whether O'Connor et al's [1990] assertion that the profile width and heights are functions of tidal prism, is a dependable relationship that can be used to predict regime morphology. Such an exercise is recommended for future work, but may not be practicable within this research programme.

The relevance of the correlation results in this investigation would be improved if more estuary data sets could be tested. Exploration for extra estuary data sets is a continuous process - the more data the better. Furthermore the effect of correcting these results for waves and salinity would benefit from other estuary data sets. It is proposed therefore to obtain more data sets, where this is practical. Improving on the use of Bijker's formula for wave interaction using Fredsøe's equation, is also to be undertaken.

The interaction between bed shear stress and freshwater flow/salinity is very interesting and deserves further investigation. Certainly the relationship between shear stress and the low water salinity level warrants attention. It is proposed to study this topic further.

It is hoped that these studies will lead towards a reliable regime model that can be universally applied.

10 Acknowledgements

The author would like to acknowledge the help and advice of the following people at HR during the preparation of this report; Mr J V Baugh, Mr T J Chesher, Dr P J Hawkes, Dr D Murphy, Mr R L Soulsby and Mr T N Burt.

Thanks also to Dr N T Crook of Oxford Brookes University.

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Estuary Regime

Part 3: Chaos Theory and Turbulent Flow

John M Dennis

Report SR 366
January 1992

HR Wallingford

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This report describes work funded by the Department of the Environment under Research Contract PECD 7/6/234, for which the DoE nominated officer was Mr P Woodhead and by the Energy Technology Support Unit for the Department of Trade and Industry under Agreement E/5A/4109/2589, for which the ETSU project manager was Dr L M Davies. It is published on behalf of the Department of the Environment, but any opinions expressed in this report are not necessarily those of the funding Department. The work was carried out by Mr J M Dennis, Division of Civil Engineering and *Building*, Oxford Brookes University, during a period of secondment to HR Wallingford. The work was carried out in Mr T N Burt's section of the Ports and Harbours Group headed by Dr J V Smallman.

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Approved by T. N. Burt Principal Engineer

Date 3/12/93

Preface

This report is the result of research carried out by John M Dennis, Department of Civil Engineering, Building and Cartography, Oxford Polytechnic during a period of secondment to HR Wallingford. The work was carried out in Mr T N Burt's Section of the Tidal Engineering Department headed by Mr M F C Thorn.

Summary

Chaos Theory and turbulent flow

By John M Dennis

Report SR 366
January 1992

Chaos theory is a comparatively recent concept which has been subject to considerable and increasing attention and development over the past 30 years. Much of this attention has focused on attempts to explain the transition to turbulent flow in fluids and ultimately to explain the nature of turbulence itself.

The origins and development of Chaos theory are presented in a basic and essentially non-mathematical and descriptive way with the aim of providing an introduction to the new science. The reader is advised to consult the appropriate references in order to fully appreciate the significance of work cited.

The theory of chaotic behaviour is examined in terms of its mechanics, routes to chaos, the concept of attractors and the associated subject of fractal geometry. The nature of turbulent flow is considered and evidence for the existence of chaos in such flows is presented. The limitations of the application of the theory to open flows are discussed and the significance of the theory in the context of predictability outlined.

While there is a considerable amount of evidence to suggest that turbulence is a chaotic process, attempts so far have only resulted in a valid application of the theory to the transition from laminar to turbulent flow. The fundamental nature of turbulence still remains a mystery.

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Appendix

Appendix 1 Chaos theory in fields other than fluid flow

1 Introduction

1.1 Background and Aims

The work described in this report originates from a wider research programme involving a study of regime in estuaries. In this context two particular problems, among many, are the limits of the accuracy of prediction of mathematical models and the nature of turbulent flow.

The approach is in the nature of an academic exercise carried out on the grounds that little is known concerning the validity or practicality of applying Chaos theory to an estuary situation. The justification for investigating the matter is that a negative finding is as valuable as one which is apparently more positive and potentially useful.

There are indicators in the literature which testify to the possible usefulness of Chaos theory in the study of non-linear phenomena in general and turbulence in particular. Cvitanovic (1989) has illustrated that 'large classes of non-linear systems exhibit transitions to chaos which are universal and quantitatively measurable'. Much work on the application of Chaos theory to turbulence has been published and the report includes references to some of this literature. Indeed the Reference section, in addition to containing sources cited in the report, contains other entries of relevance and also of background reading. Two particular references falling into this latter category are to be recommended, ie Gleick (1987) and Stewart (1989b).

The main object of the report is therefore to communicate in as concise and clear manner as possible, the basics of Chaos theory and the associated subject of Fractal Geometry. The concern is not so much with a highly mathematical approach, which a fundamental study demands, but more with a general appraisal of the subject bearing in mind the balance that needs to be maintained between pure scientific curiosity and the more practical engineering need. Thus a simplified view is presented of what is, conceptually, a complex subject.

There is some scepticism among certain parts of the scientific community concerning the significance of the Chaos theory where it is regarded, at least for the present, as a fashionable theory calling for great care in its application. However it has been amply demonstrated that the theory can be used to explain, and in some cases measure, the characteristics of non-linear phenomena; in other instances it has been used to detect and hence avoid potentially catastrophic events.

It is reasonable to state at the outset that little is known concerning the validity of the theory to such a complex situation as an estuary, in terms of either turbulent flow or morphology in relation to fractal geometry.

1.2 Origins of Chaos theory

It is widely accepted that Chaos theory became widely developed from the work of Lorenz in 1959, during his attempts to forecast weather patterns using a mathematical model. It had long been known that such attempts to produce long-range weather forecasts (say > 4 days) using such models had a very limited accuracy; attempts to forecast for periods greater than 10 days were regarded as almost impossible.

The input data for the Lorenz model was accurate to the sixth significant decimal place yet he noted that if he attempted to forecast two weather patterns, using only very slight variations in the initial input data, two totally different end forecasts were produced. Further investigation revealed that deviations between the two weather patterns occurred at a very early stage in the model run. The model had in effect begun to behave in a random or chaotic manner.

The reason for this was the problem of defining the initial conditions for all the variables of the model with sufficient precision. In fact it could be demonstrated that even if the input data could be specified to an accuracy of one hundred decimal places, a wide range of different end results could be expected after only a few iterations within the model.

Modern mathematical models used to forecast weather patterns suffer from the same problems. Typically the basis of such a model would be to represent the atmosphere by a 100 km x 100 km grid horizontally by 1 km vertically; about one million such grids would be required to represent the whole atmosphere. Not only is a large amount of computing power required to run such a model, but also the grid size is so crude as to be unable to represent precisely the behaviour of the atmosphere within it, ie the initial conditions of the model cannot be accurately specified.

In order to appreciate the deficiencies in the type of model referred to above, it is necessary to consider how the model is initially created.

The assumption is that any natural phenomenon will obey physical laws and as such its behaviour should be capable of being described in a mathematical sense eg by Newtonian laws for motion. (It may be of course, that the precise laws governing some natural phenomena have yet to be discovered). Models therefore tend to be set up on the basis that any natural process can be represented by a differential equation derived from a knowledge of Newtonian mechanics. This approach however is not valid since most dynamic processes in nature are non linear and cannot therefore be accurately described using linear equations; in some cases the non-linear effects are removed by schematisation but this normally results in a further decrease in validity. The original thinking that all dynamic systems followed regular patterns ie repetitive or periodic cycles, was clearly demonstrated not to be the case for in many instances the behaviour was apparently random. Prior to the advent of Chaos theory the tendency was to describe the randomness in terms of probability. This is not to imply that chaos is solely responsible for random behaviour, the degree of schematisation used will also have an effect.

As has already been suggested the application of Newtonian laws to a dynamic system at the atomic/molecular level falls down, since for a given moment in time it is virtually impossible to specify the position and momentum of a molecule. This degree of accuracy is necessary to specify precisely the behaviour of a given dynamic system; in many cases this specification means a precision to an infinite number of decimal places - clearly an impossibility. Hence the difficulty in defining identical starting points for the motion of a dynamic system.

In mathematical models the variation of some parameter of a dynamic system with reference to time, is allowed to develop in small finite time steps (ie iteration). This is a form of schematisation of time since in a real system time will flow continuously. If the parameter is allowed to vary in this way, a path will be traced out in space, the shape of which will be dependent on the initial starting conditions of the system; if the starting conditions are changed, even by a small amount, a different path will result. For a given system, the combination of all paths, each corresponding to different starting conditions, will form a phase portrait of the the system. This is an important concept in the study of chaos.

Chaotic effects therefore become apparent when linear equations are used to describe non-linear behaviour in dynamic systems. The effects arise principally because of the inability to specify with sufficient precision the initial conditions in space and time for a given system. Linear equations are incapable of taking account of small random perturbations in a system which, as time progresses either in finite steps or continuously, can multiply in such a way as to cause the system to exhibit completely random and/or chaotic tendencies. This has implications for predictability as exemplified by Cvitanovic (1989) - 'an often repeated statement, that given the initial conditions we know what a deterministic system will do in the future is false'.

The implications within this approach do not point to a limitation of Newtonian mechanics - Newton's laws allow both for chaos within order and for order within chaos.

1.3 Classical experiments

A discussion on the origins of chaos would not be complete without brief reference to two experiments, both involving transitions to turbulence from ordered flow in fluids.

Libchaber and Maurer (1982) described a Rayleigh-Benard type experiment involving convective flow in liquid helium, contained in a small box. The box was capable of being heated at the bottom and cooled at the top. For small temperature gradients the liquid remained static but as the temperature gradient was increased, two convective 'rolls' of liquid developed as shown in figure 1.1a; the onset of this flow occurred at a particular temperature.

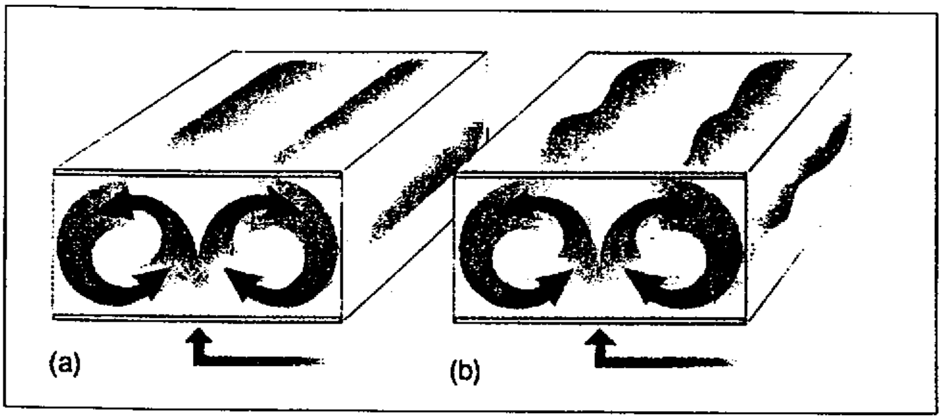

Figure 1.1 *Convective rolls in liquid helium (after Gleick 1987)*

As the temperature difference was increased further instability developed in the rolls characterised by a wave - figure 1.1b of particular period, T . Further increases in temperature resulted in the initial period of the wave successively doubling. However at a specific temperature difference the period doubling showed a considerable increase representing the onset of turbulence at a value of period approaching 2 T ; it was demonstrated that this occurred at a critical value of the Rayleigh number. This process was a cascade process which resulted, for values of period > 2 T , in chaotic behaviour of the system as the temperature difference was further increased.

A second experiment, described by Gollub and Swinney (1975) involved the study of the onset of turbulence in a rotating fluid. The fluid (water) was contained in the space between two concentric cylinders the inner one of which was capable of being rotated at a variable speed. This restricted arrangement produced a characteristic flow pattern usually referred to as Taylor-Couette flow. As the rotational speed of the inner cylinder was gradually increased from zero the development of the flow pattern in the liquid was observed using laser Doppler interferometry. It was observed that three distinct transitions occurred within the fluid as the Reynolds number was increased, each of these transitions being characterised by the addition of a new frequency to the velocity spectrum.

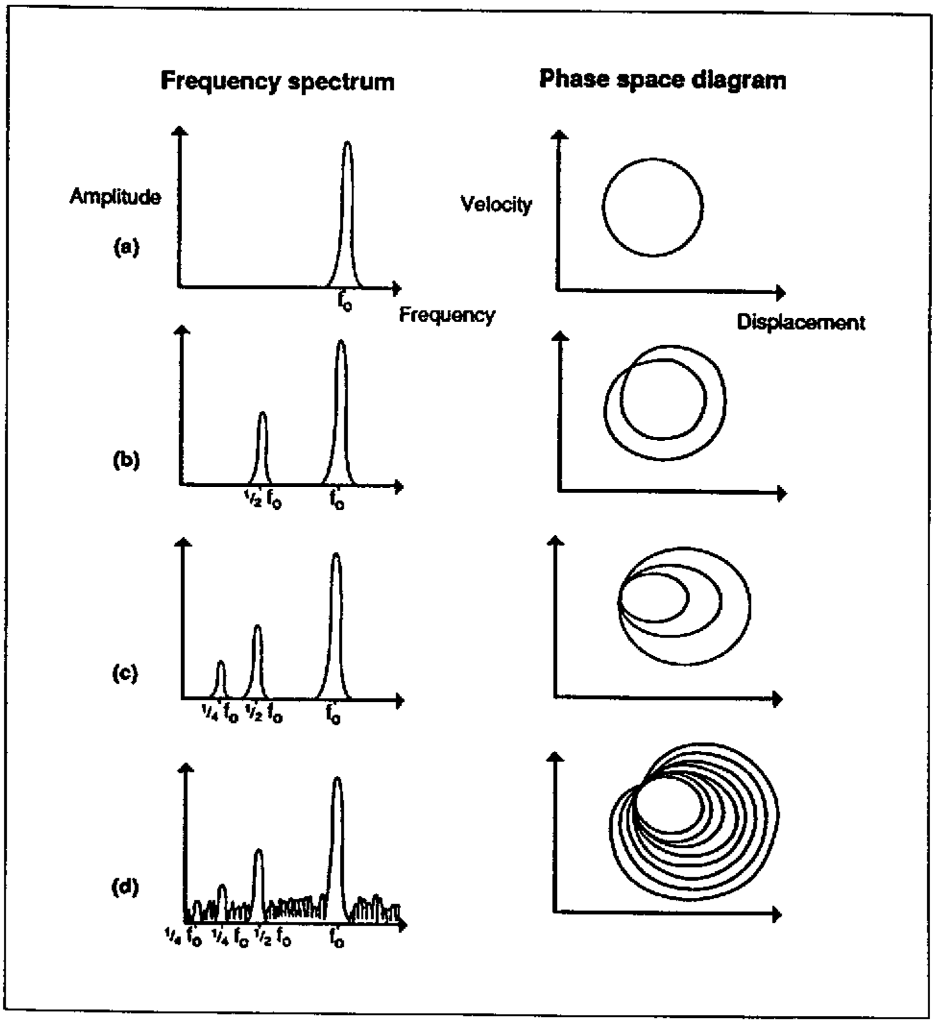

Figure 1.2 *Period doubling and phase space diagrams*

An initial instability in the flow occurred with the appearance in the fluid of stacked 'doughnut' shaped bands; as the speed of rotation was further increased, ripples developed in the 'doughnuts' and a wave developed. Further increase in rotational speed produced further distinct transitions corresponding to successive increases in frequencies of the initial wave. At a higher speed the wave structure disappeared completely at a particular value of Reynolds number, beyond which turbulence set in. In a simplified way this is analogous to the period doubling effect described in the Libchaber and Maurer experiment. Successive period doublings produced a cascade effect leading eventually to chaotic behaviour.

The phenomena observed in the above experiments can be simply represented in Figure 1.2, which illustrates period doubling in the frequency spectra with the corresponding phase space representation.

Figure 1.2d shows presence not just of doubled frequencies but also of other sub-harmonic frequencies.

2 *Chaos*

2.1 Introduction

It has been normal custom in the case of modelling of systems using linear equations to neglect the effect of small perturbations. However if attempts are made to allow for non-linear events by incorporating appropriate functions within these equations, then the systems still show a tendency towards chaotic behaviour.

It has been demonstrated that a body or system which obeys deterministic laws and which moves in a clearly defined region of space, will eventually in time settle down into one of 3 possible states; two of these states can be described in conventional geometric terms. For example a simple pendulum is attracted towards a fixed point; cyclic or periodic motion will be attracted towards a circular or elliptical path. These geometrical descriptions are referred to as attractors. In the case of random or chaotic motion, in the same space region, there is no simple geometric description which can be applied to the path followed - in this case the attractor is referred to as a strange attractor. In fact geometry plays an important part in the appreciation of chaos not in the conventional sense of Euclid, but in the area of topology which is concerned more with the geometrical factors that remain unchanged when a body is subjected to continuous deformation.

An apt and concise description of chaos has been given by Andrews and Read (1989): 'To the mathematician, chaos is a feature of the study of autonomous differential equations where the time-evolution of the solutions is represented by a "flow" in a suitable phase space. As time passes the flow in a dissipative system .... tends asymptotically to an attractor which may be a point in space, cyclic (or quasi-periodic) or chaotic. Simple predictable systems possess point or cyclic attractors. Non-linear systems .... have chaotic or strange attracators .... signalling that the system is quantitatively unpredictable. On the other hand some qualitative predictability may be available if the geometric properties of the chaotic attractor are known.'

It has been demonstrated that patterns exist within chaotic behaviour and furthermore the same patterns have been shown to exist in different systems having chaotic behaviour. In fact it can also be shown that areas of order exist within total chaotic behaviour. Examination of some patterns of chaos has lead to the suggestion by Stewart (1989a) that some of the patterns of chaos might be universal-in other words not specific to individual examples but representative of whole classes of systems. This concept of universality is not new; Fiegenbaum (1975) proposed that 'large classes of non-linear systems exhibit transitions to chaos which are universal and quantitatively measurable.'

Chaos would appear to be complicated behaviour or 'persistent instability' (Percival 1989); however according to Davis (1990) complicated behaviour does not necessarily imply complicated forces or laws. There is evidence to show that chaotic motion frequently follows simple deterministic laws. In some cases the (chaotic) patterns are less sensitive to initial conditions and as a consequence their behaviour is easier to predict.

Stewart (1989a) has suggested that chaos is both exciting and worrying - exciting because it opens up the possibility of simplifying complicated phenomena but worrying because it introduces new doubts about 'traditional model-building procedures of science'. Perhaps the most interesting discovery of chaos is that the theory applies to simple systems having 3 or more degrees of freedom.

2.2 Mechanics of chaos

For a given deterministic system, if the initial conditions are changed slightly, widely differing outcomes can result and prediction as to how the system will behave becomes inaccurate; even a system which is completely deterministic can produce random looking behaviour.

The definition of the transition point from order to chaos in a system is not quite the same as the transition from laminar to turbulent flow in fluids. The transition from order to chaos can be traced through recognisable stages, for a given dynamic system

ordered or steady

⇓

periodic or cyclic

⇓

quasiperiodic

⇓

chaotic or random

These stages can be illustrated by reference to a potential well analogy (McRobie and Thompson 1990). Consider a sphere moving about in a parabolic shaped trough - Figure 2.1.

If the sphere, which is originally stationary in the bottom of the well, is disturbed by an externally applied force it will, after an initial erratic motion, settle into a steady pattern of oscillation determined by the frequency of the external force.

This condition will be stable (ie the sphere will not escape from the well), provided that the motion is subject to linear laws, and will be independent of the initial conditions of the system; this motion will have a conventional attractor. Such periodic motion is represented by the general equation

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F \cos wt$$

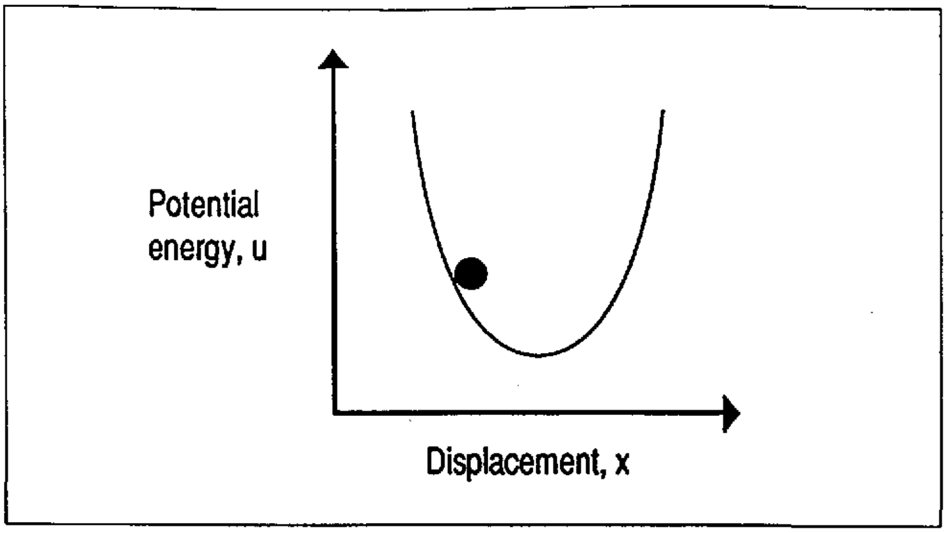

Figure 2.1 *Potential well analogy*

which is the equation of forced damped harmonic motion.

If an element of non-linearity is introduced into the motion several states of steady vibration become possible; in this case the state which the system eventually settles down into will be dependent on the initial conditions. The frequency of the motion may not necessarily be the same as the disturbing force but will normally be some multiple of it.

In these circumstances if the non-linear perturbations are decreased, it is possible that rather than tend towards a steady vibration, the system will oscillate erratically while still remaining within the potential well. This motion has a strange attractor, and is therefore chaotic.

A further possibility is that the non-linear perturbations might increase to the point where the sphere has sufficient energy to escape from the well. This represents total instability and failure of the system.

There are many examples involving simple systems where it is possible to set up equations to represent the transition from order to chaos. These equations, although simple in themselves, can produce complex solutions; a good example of this phenomenon is the iteration of a polynominal, ie consider the equation:

$$x \rightarrow x^2 + c$$

where c is a constant.

This is iterated as follows:

(i) assign an initial value to c
(ii) assign an initial value to x
(iii) determine new value of x
(iv) insert the new value of x into the equation
(v) repeat (iii) and (iv).

The iteration process is then repeated for different values of c.

Stewart (1989b) has demonstrated the effect of this type of iteration using the equation:

$$x \rightarrow kx^2 - 1$$

For example if the iteration was performed for 50 steps for k in the range 1 to 1.75 for x = 0.54321, chaos resulted for k = 1.5 but for k = 1.75 a semblance of order appeared.

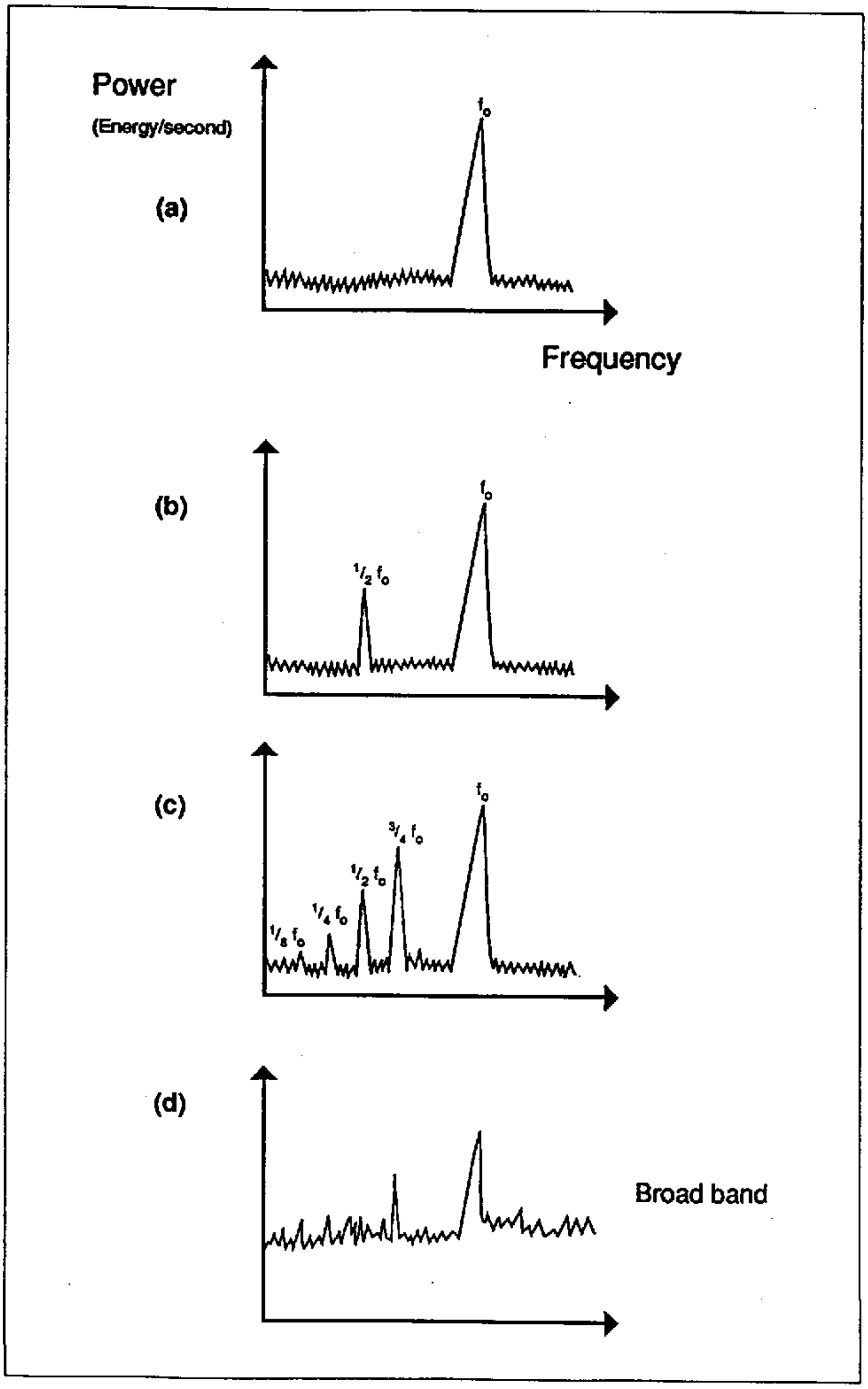

Figure 2.2 *Development of broad band power spectrum*

Apart from observation, there are several ways of identifying the onset of chaos in a system; one of these ways is to plot and examine the power spectrum of the new system.

Essentially a power spectrum is a plot showing the distribution and amplitudes (ie energies) of the different component frequencies which arise during a cyclically varying motion. For a periodic motion the spectrum will contain well defined and regularly spaced peaks; these will be superimposed on the background of any experimental noise present.

A motion which consists of several harmonics of the fundamental frequency, will exhibit frequency peaks at sub-multiples of the fundamental frequency, f_o eg $\frac{1}{2} f_o$, $\frac{3}{4} f_o$, $\frac{1}{4} f_o$ etc. For the case of period doubling the peaks will occur at $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ etc of f_o. As the periodicity of the motion increases, more and more peaks become evident. Eventually after many period doublings the peaks become modulated and indistinguishable from the background noise and a so called broad band spectrum results - the presence of this band is used as an indicator of chaos in the system. The development of this process is shown in Figure 2.2.

Guckenheimer (1986) proposed a need for a criterion (or criteria) to distinguish between non-chaotic and aperiodic flows. According to that author, the operational definition of the latter is that a flow should have a continuous part in its power spectrum. He expressed the reservation that power spectra were limited as tools for the differentiation of different types of aperiodicity.

2.3 Routes to chaos

Many examples from everyday life can be used to illustrate the transition from order to chaos in a system. Consider what happens when the flow of water through a dripping tap is progressively increased.

Initially the drips are uniform with respect to time indicating a periodic flow; eventually the drips come in groups of varying length (quasiperiodic) followed by an entirely random or chaotic motion. In the same way smoke from a cigarette rising into still air has an initially laminar motion which develops into a turbulent state and finally chaotic motion results as the vertical distance from the source increases. In both these cases the chaotic state is reached through a cascade process which is associated with a period-doubling effect.

Period doubling is a well known and studied route to chaos. A good example to illustrate this is an electrical oscillator in which the period doubling effect is achieved by slightly changing the circuit parameters. The output from such an oscillator displayed on the screen of an oscilloscope would appear to change through the following transitions, starting from a point, as the circuit parameter is varied.

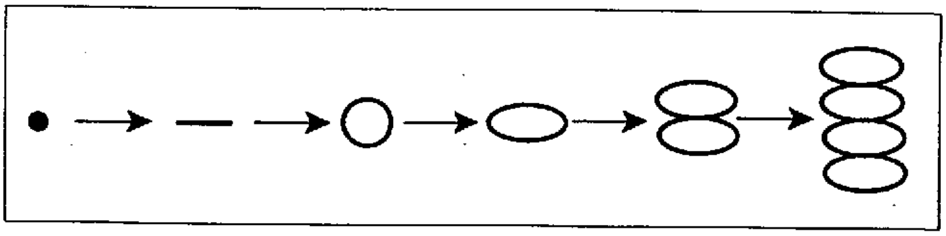

Figure 2.3 *Period doubling in electrical oscillator*

As the period is successively doubled, so the attractor changes from a simple type into a strange attractor.

The period doubling effect can be further illustrated by examining a plot of displacement against force for a damped non-linear oscillating system undergoing forced vibrations, Figure 2.4.

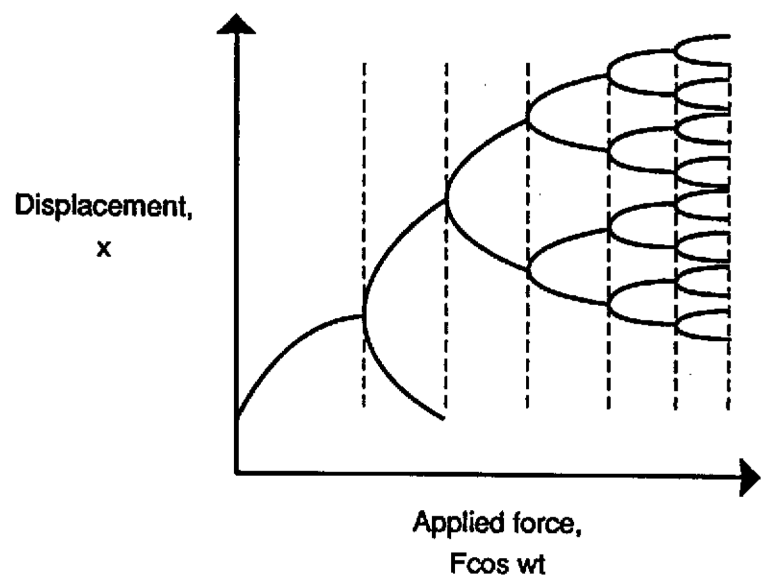

Figure 2.4 *Period doubling by bifurcation*

This type of period doubling involving bifurcations is a generally accepted example of a route to chaos. In certain systems the onset of chaos has been observed to occur abruptly after a small number of bifurcations, sometimes as low as 3. The chaotic state has also been observed to occur as a result of the merging of regions of intermittent chaos on a gradual basis.

The routes to chaos have been intensively studied by many workers. Gollub and Benson (1980) reporting on work to investigate the routes to turbulent convection, identified four sequences of instabilities leading to turbulent convection at low Prandtl numbers (ie 2.5-5.0) in water layers of small horizontal extent. Using a specially designed convection cell and laser-Doppler velocimetry they demonstrated 4 routes to non-periodic motion, viz:

(i) quasi-periodic motion at 2 frequencies leading to phase locking or entrainment. The onset of non-periodicity was associated with loss of entrainment as the Rayleigh number was increased.

(ii) successive sub-harmonic (period -doubling) bifurcations of periodic flow

(iii) a route containing a well defined regime with three incommensurate frequencies and no broad-band noise.

(iv) a process of intermittent non-periodicity where the fluid alternated between quasi-periodic and non-periodic states over a finite range of Rayleigh numbers.

Eckmann (1981) in reviewing the routes to turbulence in dissipative dynamic systems, discussed three scenarios leading to turbulence, (viz Ruelle-Takens-Newhouse, Feigenbaum and Pomeau-Manneville), in both mathematical and in simple terms. Yahata (1984) illustrated routes to chaos resulting from 2 classical experiments (see section 1.3).

(i) Taylor-Couette flow being a quasi-periodic motion with two fundamental frequencies which developed into chaos through a period doubling bifurcation.

(ii) Rayleigh-Benard convection where the fluid density depends non-linearly on temperature and can be shown to undergo a period-doubling cascade to chaos with increase in thermal gradient.

According to Guckenheimer (1986) two things are lacking in studies of routes to chaos. Firstly there are no successful quantitative simulations of the transition to chaotic flow; secondly there is a lack of understanding of the part played by aspect ratio and boundaries.

Andrews and Read (1989) citing the work of T Mullin, University of Oxford, referred to the existence of two distinct routes to chaos in Taylor-Couette flow, in which a totally predictable, periodic flow rapidly gave way to a more complicated flow with finite predictability, as the experimental parameters were varied.

2.4 Attractors and strange attractors

It had always been believed, in terms of classical dynamical theory, that if a point in space was taken as representing some moving dynamic system, at a point in time a solution of the relevant differential equations would predict that the system would settle down to a fixed state, ie a point, a straight line or a closed loop, representing a simple attractor. It was therefore a significant discovery to find that regions in space could exist where the system moved around totally at random, where the attractor was not simple but strange.

Stewart (1987) has defined an attractor as 'some region of phase space such that all nearby points eventually move close to it'.

In this sense 'phase space' is the combination of the sets of co-ordinates which represent all possible states of a given system; a set of co-ordinates will be generated by each variable which is capable of affecting the state of the system. (This approach requires the concept of multidimensional space.)

For the case of a simple pendulum with damping, the attractor will be a point since the system eventually comes to rest. In other words an attractor can be defined as some state of rest or motion to which any nearby state will move towards.

The concept of an attractor can be conveniently illustrated by reference to the motion of a double pendulum such as that shown in Figure 2.5.

The pendulum is fixed at O and hinged at A and rotates in the direction shown. As the rotation proceeds the tip of the pendulum traces out a path and is confined within that path. The path shape is its attractor and while it is circular the motion is periodic. If the motion is speeded up, a complex pattern results and the circular shape changes as the motion begins to show chaotic

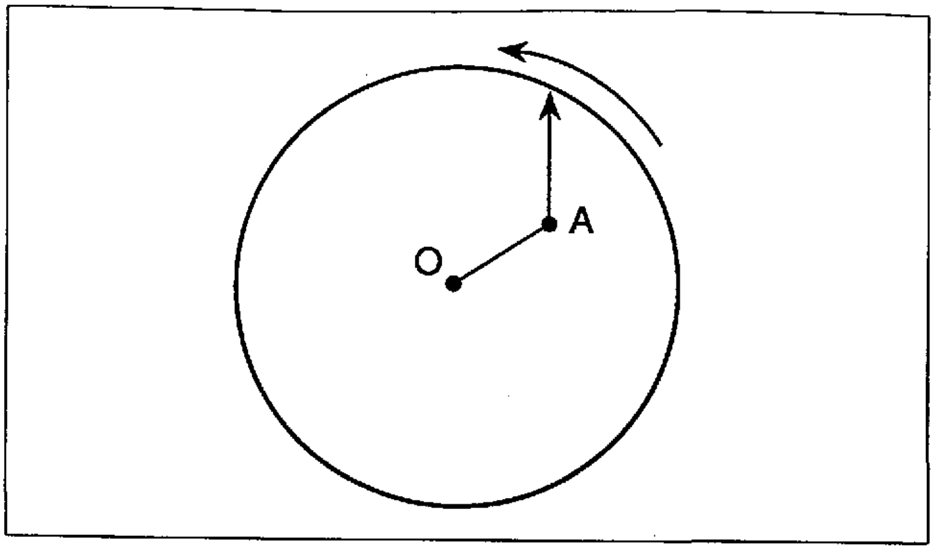

Figure 2.5 *Double pendulum*

tendencies. At still higher rotational speeds the attractor shape becomes complex, with no apparent regular shape - it is now referred to as a strange attractor and the motion is chaotic. However, under these conditions, the pendulum is still subject to the Laws of Motion and it can be shown that the shape of the attractor contains hidden patterns of order ie, there is structure within the chaos. In short, strange attractors result from random behaviour within deterministic systems.

According to Lorenz an attractor represents a set of states in a system which actually occur, as opposed to those which do not occur. Parts of the motion of a system which have a tendency to drift away from the attractor are quickly pulled back into it and are therefore prevented from existing. It is also possible, under certain circumstances, that as the conditions of a dynamic system are slowly changed (sometimes by an insignificant amount), the attractor can suddenly be caused to disappear; what actually happens is a sudden jump or transition from one attractor to another.

Ott (1981) in a review of work related to strange attractors considered two possible routes to a strange attractor; these are presented in Figure 2.6 for interest and for illustrative purposes.

A useful concept in the context of attractors is that of a phase portrait; this is defined by Stewart (1989a) as 'the system of all possible paths along which a system can evolve' shown graphically. Phase portraits can be simple or complex depending on the system to which they are applied. In a simple system all the paths will converge to a point or line (ie a simple attractor). As the motion of the system increases in complexity so the path and hence the attractor becomes more complex. For example, in the case of periodic motion the attractor will be a closed loop; eventually with the onset of quasi periodicity the shape of the attractor becomes more complex. Figure 2.7 shows such a complex phase portrait.

It has been demonstrated that in a cascade resulting from period doubling a scaling ratio can be calculated for successive doubling stages. It has further been shown that this ratio reaches a constant value for a given system and in fact represents the attractor for the system. The Fiegenbaum approach to period doubling leading to chaos produced a scaling factor of 4.669; it can be

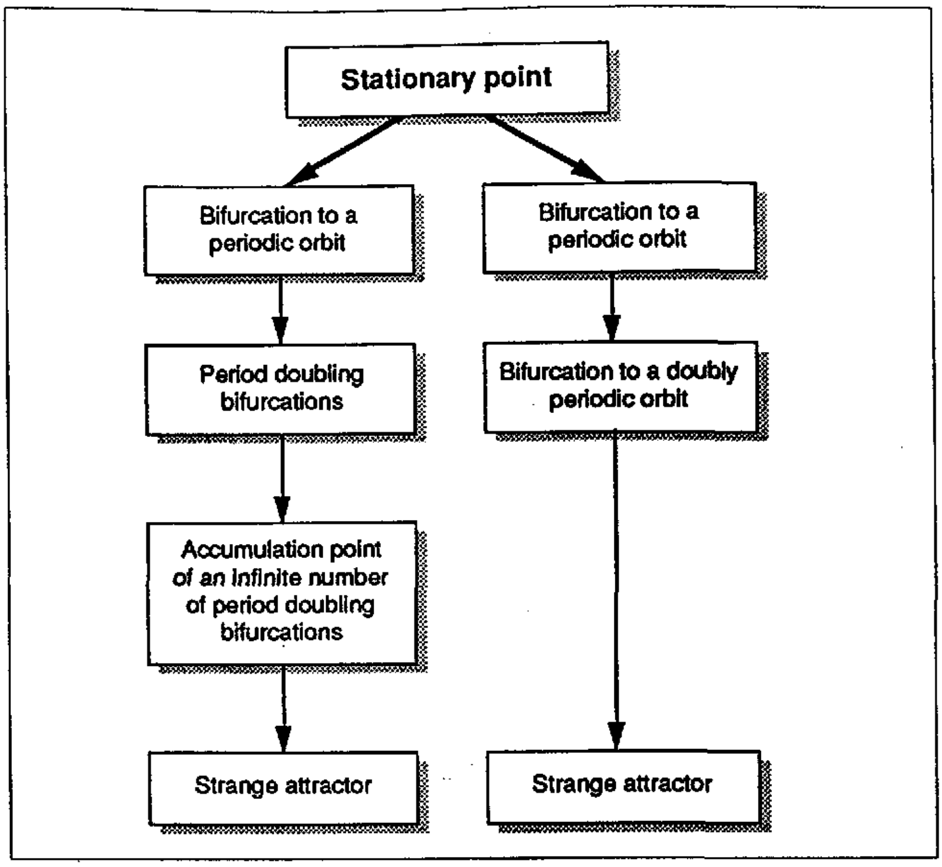

Figure 2.6 *Routes to strange attractor (after Ott 1981)*

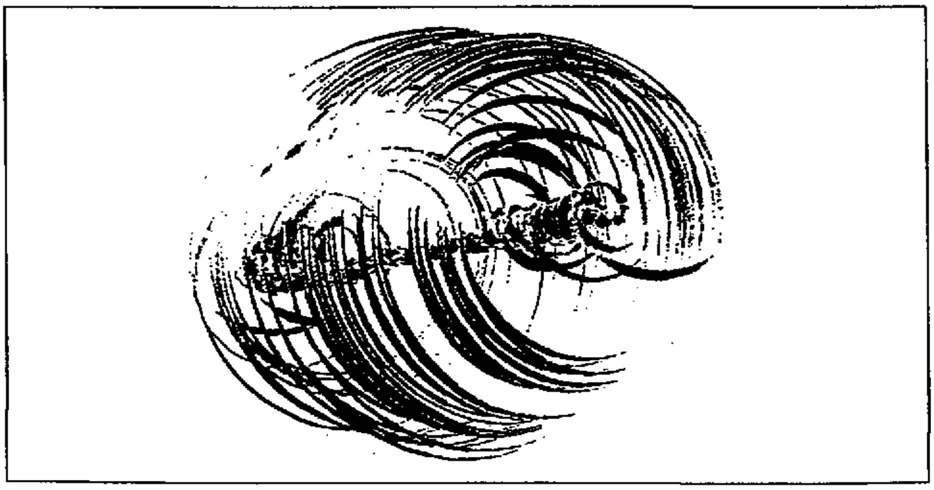

Figure 2.7 *Phase portrait of chaotic flow (after Mullin 1989)*

shown that this represents a geometric convergence, and its significance has been demonstrated by many authors using a variety of examples, eg Libchaber and Maurer (1982), Stewart (1989b), etc.

The standard approach to the analysis of combinations of periodic motions is the Fourier method; where such a combination results in a strange attractor, the classical Fourier approach cannot be used. This problem was partly overcome by Zeeman, of the University of Oxford, (see Andrews and Read, 1989), who reconstructed a chaotic attractor, representing a real physical system, from a finite time series. He was able to demonstrate that the geometry of this attractor was equivalent, at least in a topological sense, to the actual attractor produced by the system.

3 Fractal geometry

3.1 Introduction

Fractal geometry was a concept developed independently from chaos although one of the applications is the describing of chaotic structures. Fractal geometry owes its origin to Mandelbrot (1983a) who proposed fractal geometry as a new branch of mathematics in order to define and describe irregular structures such as those existing in nature. The various patterns in nature consist of apparently formless shapes ie coastlines, trees, rocks, mountains, lakes etc to which standard Euclidean geometry cannot be applied, yet they must be capable of being characterised by some quantitative means since their forms derive from physical laws and rules.

The new geometry stemmed from Mandelbrot's identification of a set of shapes for which the degree of roughness or irregularity was the same at all scales of magnification. He therefore defined a fractal as a shape such that even when magnified an infinite number of times, it still appeared to have the same shape - a snowflake is an example of this type of shape. Perhaps the best known example of this phenomenon is the Mandelbrot set which can be magnified ad infinitum without ever running out of detail. It has been described both as the most beautiful and most complicated object ever seen and yet it obeys a simple mathematical rule, ie:

$$x \rightarrow x^2 + c$$

Fractal geometry is thus involved with finding mathematical descriptions for irregularly shaped systems. One important characteristic of a fractal shape is that it is irregular all over but has the same degree of regularity at all scales of magnification. In other words a fractal object looks the same irrespective of the distance from which it is viewed; on this basis it is said to be self similar or invariant.

Conventional (Euclidean) geometry uses properties such as length, area, angle and curvature, in up to 3 dimensions, in order to define shape and size. Fractal geometry uses numbers, which need not necessarily be integers, to define shape and complexity; illustrations of how this is achieved are given elsewhere.

The significance of fractal geometry within chaos is that strange attractors appear to have a fractal nature in that their structure appears to be the same irrespective of the degree of magnification used. In this sense a strange attractor is an example of a fractal. The development of fractal geometry as a subject is probably far from complete. In answer to the question - 'How important are fractals'? - Mandelbrot (1990) was cautious in reply - '... like for the theory of chaos it is too early to say for sure, but the prospects are favourable ?'

3.2 Nature of fractals

A fractal is a geometric shape having a special property such that as an observer moves closer and closer to it the same shape is always seen. For example a straight line, observed from whatever distance, will always appear to be a straight line. For the case of a circle, provided the viewing distance is large, the shape is clearly identified as a circle. The closer an observer moves in on a part of the circumference, the circle appears to be a straight line and will continue to appear so as the observer moves in still closer. Under these circumstances the shape looks the same irrespective of the distance from which it is viewed. This is the property of invariance which is the main feature of fractals.

A more specific characteristic of fractals is branching on ever decreasing scales - as this happens, minute differences occur in the branching processes giving rise to the different natural shapes. If rules of the branching process are known for a given shape, then these rules can be used to generate a computer model. An example of such a shape is the so-called Koch 'snowflake'.

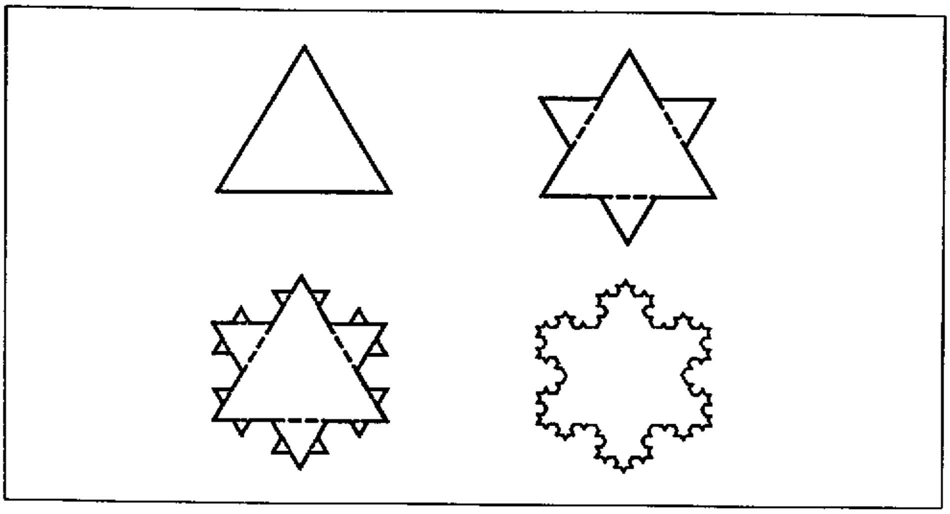

Figure 3.1 *Koch snowflakes*

Figure 3.1 shows the effect of successive branching processes, the rules for which are as follows. Starting from an equilateral triangle with sides of unit length, further triangles one-third the size of size of the original are then constructed at the middle of each side. This process is then repeated, to infinity if required; as each new set of triangles is added the perimeter length increases giving an increasingly detailed outline. Irrespective of the number of times the process is repeated, the total area does not exceed the area of the circle drawn around the centroid of the original triangle.

The increase in perimeter length is given by:

$3 \times {}^4/_3 \times {}^4/_3 \times \ldots$

This type of process leads to the concept of a fractal dimension. Unlike regularly shaped objects which have 1, 2 or 3 dimensions, fractal shapes can have a non-integer number of dimensions; the fractal dimension concept requires an approach different to that for regular shapes. In general the fractal dimension, D, of a shape is given by:

$$D = \frac{\log n}{\log s}$$

where n is the number of copies required to increase the size of the shape by a factor s.

For the case of the Koch snowflake n = 4 and s = 3, since one side is made up of 4 copies of itself each ⅓ size.

$$\therefore D = \frac{\log 4}{\log 3} = 1.261859507$$

A further example of such a fractal is the Cantor set, where starting from a line of unit length the middle third is successively removed, as shown in Figure 3.2.

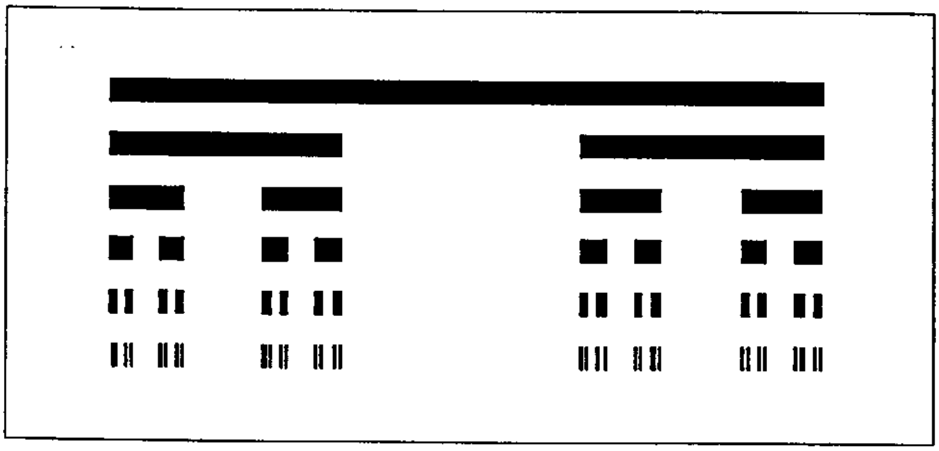

Figure 3.2 *Cantor Set (after Gleick 1987)*

The fractal dimension, D, for this is given by:

$$D = \frac{\log 2}{\log 3} = 0.630929753$$

Mandelbrot (1983a) defined two fractal dimensions, D and D_m, referred to respectively as the similarity and mass dimensions.

The value of D_m for a point, line, square or cube would be 0, 1, 2 or 3 respectively according to physical dimensions. In the case of D_m, if a rod (length R), disc and sphere (both radius R) are considered, then the mass of each will be proportional to R.

ie	rod mass α R	ie	to R^1
	disc mass $\alpha \pi R^2$	ie	to R^2
	sphere mass α $\frac{4}{3}\pi R^3$		to R^3

The exponent of R in these simple cases is the mass dimension, D_m; for fractal masses D_m will not be a whole number.

In summary, a fractal dimension is a means of specifying the difference among various complex shapes by use of a number. This enables the degree of roughness of a given shape to be defined; as the roughness increases, so does the fractal dimension.

Fractal geometry and chaos are closely linked; this is in part demonstrated by the fact that strange attractors are by their nature fractals. (Normal attractors being regular in shape do not come within this definition). The literature contains many examples of fractal attractors.

Fractal concepts have been widely applied with some success to describe a variety of naturally occurring phenomena. According to Stauffer (1987), 'Fractals are concepts unifying different fields. Such concepts are good if used widely .... Thus fractals have passed this democratic examination with flying colours.'

3.3 Fractals and coastlines

Mandelbrot (1983a) has demonstrated the fractal nature of coastlines and shown that the length of a given coastline followed a power-law relationship, the value of the 'power' being related to the fractal dimension. This can be illustrated as follows by reference to the measurement of the length of a given piece of coastline.

Consider a pair of dividers set at a fixed distance x, say; the total length of the coastline can thus be measured in steps of x using the dividers to travel round the length of the coast.

ie total coastline length = Σx

If the setting on the dividers is now reduced to say x^1 and the coastline length remeasured, the total coastline length = $\Sigma x'$.

and $\Sigma x' > \Sigma x$.

If the divider setting is progressively reduced, the coastline length being remeasured for each setting, the measured length will increase progressively.

It can be shown that if the coastline has a fractal dimension D, then

$$\Sigma x = x^{1-D}$$

This is of the same form as Mandelbrot's equation for the same process.

ie $L = cl^{(1-D)}$

where

L is the coastline length
c is a constant of proportionality
l is the length of each measurement step.

This equation has been applied to many coastlines; for the UK the value of D is about 1.25. Thus D is a measure of the roughness of the coastline and its value increases with increasing degree of roughness. D can therefore be used as a parameter to classify a coastline in terms of its degree of irregularity.

4 Chaos and turbulent flow

4.1 Nature of turbulence

Turbulent flow, which has been described as one of the great unsolved problems of science, has always been regarded as a complicated subject involving random variable theory. In spite of recent advances in fluid mechanics the theory of turbulence is not understood with certainty. Part of the reason is that a study of real turbulence in fluids involves the specifying of the non-linear dynamics of the vorticity behaviour in 3 dimensions, and this can be highly complex. It was natural then, that as chaos theory developed, attempts were made to apply this new theory to turbulent flow.

What is certain is that turbulence, like many other natural phenomena, must obey physical laws. At the macroscopic level the phenomenon defies any logical explanation; at the the microscopic level an explanation must be possible but this depends on the time evolution of the system being precisely defined - this is the problem. It is the inability to precisely define the starting point and consequent evolution of a system, which is the basis of chaos. This having been stated, turbulent flow is not without a degree of regularity although this regularity is statistical in nature and is only really apparent when averaging the flow over long periods.

Many contemporary workers have blamed much of the failure to solve the problem of turbulent flow on an over-dependence on the application of the Navier-Stokes equations. This blame is not unreasonable since the equations describe deterministic systems; turbulent flow on the other hand has its origins in the apparent randomness of motion at the atomic and/or molecular level.

Lanford (1982) in discussing the fundamental idea of the strange attractor theory of turbulence suggested that 'turbulence time dependence is not an exceptional feature of particular equations of motion but a property shared by a broad class of typical differential equations.'

The results of work by Rouelle and Takens in 1971 (Ott 1981) demonstrated a possible mechanism by which turbulent solutions to the Navier-Stokes equations could appear as the Reynolds number was increased. Argoul (1989) noted that the central problem of fully developed turbulence was the energy cascading process, which had resisted all attempts at a full physical understanding or mathematical formulation. The reasons given for this apparent failure were:

(i) large hierarchy of scales involved
(ii) non-linear character inherent in the Navier-Stokes equations
(iii) Spatial intermittence of dynamically active regions.

Mullin (1989) was more emphatic concerning the Newtonian and deterministic character of the Navier-Stokes equations. According to this latter author, if attempts are made to describe turbulent fluid flow evolving with time, using these equations, the solutions become chaotic and therefore they cannot be applied to turbulent flow.

This is not to imply the absence of any order in turbulent flow since there is evidence for this. Wallace et al (1977), reporting on structures in bounded turbulent flow, listed this evidence for the existence of what was referred to as coherent structures in turbulent shear flows. Although some of the characteristics of these structures had been determined, much was unknown concerning their nature and origin. Further evidence of such structures is provided by investigations of Taylor-Couette flow in the transition to turbulence under conditions of shear flow; in such experiments clearly identifiable patterns have been shown to result before the onset of recognisable turbulence.

Work by Aref et al (1989) highlighted the role of vorticity and vortices in bringing about chaotic motion in fluid flow, the object of the work being an attempt to develop an understanding of the morphology of the laminar-turbulent interface in a shear flow. The approach in this instance was Lagrangian, rather than Eulerian, to distinguish between laminar and turbulent flows, (ie the stochastic nature of the flow was characterised by considering particle paths rather than velocity fields).

Sirovich (1989) has described further work to investigate the notion of chaotic dynamics of coherent structures in turbulent flow. In this approach turbulence is regarded as having an important underlying structure of its own, an idea which is a departure from the 'probabilistic randomness' approach. The existence of organised structures within turbulent flow has already been demonstrated in a variety of flow situations and there is now a growing body of evidence to justify the consideration of chaotic dynamics in the context of turbulent flow.

Two important issues for discussion arose from the work of Sirovich. Firstly whether the ideas put forward were capable of dealing with rapid temporal effects which were likely to occur in boundary layers and secondly, the effect that the neglecting of low energy modes in a flow system might have on the dynamics of the system overall. It was concluded with regard to the former issue that the application of the Navier-Stokes equations resulted in a 'loss' of these effects; the conclusion in the second case was that provided the flow energy modes involved 'relatively small amounts of energy' then their 'loss' within the dynamics of the system would not give rise to concern.

Caution however must be used when describing flow as turbulent or chaotic as some fluid dynamicists use the words in an interchangeable way, although both words apparently lack precise definition. Writing in this vein, Spiegel (1987) concluded however that 'more than being a metaphor for turbulence, chaos is a basic property of turbulent fluids. Many features of chaos are mirrored in turbulent flows.' A further reference to the use of this descriptive term (ie metaphor) was made by Fiegenbaum in 1975 (Taylor 1984) following the discovery of some 'remarkable' universal properties of non-linear iterations. According to Taylor these have become 'at least a metaphor for, and a possible explanation of, the unsolved problem of classical physics, namely *turbulence.*'

A statement by Cvitanovic (1989) provides an apt summary for the problem under discussion - 'We know that given infinitesimally different starting points we often end up with wildly different outcomes. Even with the simplest conceivable equations of motion, almost any non-linear system will exhibit chaotic behaviour. A familiar example is turbulence.'

4.2 Evidence for chaos in turbulent flow

The classical concept of turbulent flow is of an initially slow flowing stream in which any eddies die away almost immediately after formation; as the flow velocity is increased the eddies persist for increasingly longer times until they eventually combine to produce fully turbulent flow which is apparently chaotic. Perhaps the emphasis here should be on the word 'apparently' since it has been suggested by some workers that the evidence for the existence of chaos in turbulent flow is limited. On the other hand Stewart (1989b) has suggested that the chaotic dynamics of strange attractors is responsible in part for some aspects of turbulent phenomena, although a description of fully developed turbulence may require attractors of enormous dimensions.

One of the classical experiments in the study of the transition from laminar to turbulent flow was that by Taylor and Couette described earlier; this involved the study of fluid motion in the annular gap between two concentric rotating cylinders. As the speed of the inner cylinder was gradually increased from zero with respect to the outer (stationary) cyclinder, well defined transitions were noted. At a given speed waves appeared - higher speeds resulted in an increase in the number of waves until eventually the flow became irregular and chaotic.

The phenomena described above have been extensively investigated by later workers particularly with a view to determining at what point, if any, chaos developed in the system. Rouelle and Takens (see Mullin 1983) attempted an interpretation of the observed phenomena in terms of the existence of a strange attractor. Other workers found that the application of the strange attractor theory was limited to the onset of turbulence but not beyond. Mandelbrot (1984) put forward the view that it was increasingly evident from the studies of turbulence and strange attractors, that it was necessary to apply fractal considerations. He suggested that for the case of mature turbulence 'dissipation occurs in a very small portion of real space, namely on a fractal set of dimensions that equals or slightly exceeds D = 2.5.'

By varying the geometry of the Taylor-Couette experiment, ie rotating an inner cylinder in an outer box of square section, Mullin (1983) showed that the changes in symmetry did not altogether change the previously noted cellular flow. However some changes did occur in time-dependent flows at high values of Reynolds number. Mullin noted that the final state of the motion was apparently chaotic; this occurred for only 'moderate' values of Reynolds number and following a comparatively few noticeable bifurcations. It was suggested that studies using the changed geometry were likely to yield more meaningful results than those produced with the 'idealised' concentric cylinder arrangement.

The changes in flow irregularity with Reynolds number have been noted by Speigel (1987) who suggested that the fully developed turbulent state was characterised by:

(i) a mixing property
(ii) continuous power spectrum of turbulent fluctuation.

The fractal approach to turbulent flow was discussed by Heeger and Pentland (1986) and a classification of fluid motion described leading from the most coherent to the least coherent, eg:

rigid

⇓

elastic

⇓

laminar

⇓

flow with vortices

⇓

fully developed turbulence

The evidence put forward as supporting the use of fractal models to describe turbulence was listed as:

(i) fractal shape of turbulent regions
(ii) intermittency in turbulence was indicative of its fractal nature
(iii) the fractal nature of this intermittency had been exhibited in both computer and theoretical models
(iv) fractal models had been developed which had been used to generate image sequences that closely resembled turbulent flow.

In presenting 'another view' of strange attractors in fluids, Guckenheimer (1986) examined two difference kinds of approach which had predominated in attempts to relate experimental observations with the strange attractor theory of turbulence.

These approaches were:

(i) the examination of observations and data that appeared to describe routes to chaos ie power spectra in which the presence of broadband components is taken to indicate the presence of a strange attractor.

(ii) the investigation of the structure of attractors produced by differing types of flows ie the use of both geometrical and statistical methods. (It was noted that this approach had already been successfully applied to distinguish low-dimensional attractors from other types of aperiodic behaviour).

Turcotte (1988) suggested, (on the basis of the assumption that fractals were a part of chaotic behaviour), that many aspects of turbulence could be shown to have a fractal nature. Taking this point even further, many of the more generally accepted theories of turbulence could be shown to yield fractal behaviour. Mullin (1989) expressed the view that fluid dynamics studies provided a useful test bed for mathematical theories for modelling the transition from order to chaos.

4.3 Recent work and conclusions

Tatsumi (1983) identified a particular problem in the application of chaos theory to turbulence. The turbulent motion of a fluid, by its nature, involves an infinite number of degrees of freedom; chaos as applied to a dynamical system has a finite number of dimensions. This means that fully developed turbulence cannot be precisely described by a model system having a finite number of dimensions and that therefore chaotic behaviour can only be applied to a part of the turbulent phenomenon. A further problem, in practical terms, is that many turbulent effects are caused by the presence of boundaries - at the present, strange attractor theories have yet to be related to the influence of such boundaries.

Sreenivasan and Strykowski (1983) considered the analogies between turbulence in open flows and chaotic dynamical systems, and the question was posed as to whether turbulence was a strange attractor. One of the main properties of both strange attractors and turbulence is their sensitivity to initial conditions, however this is less easy to quantify in the case of turbulence.

Work reported by Sreenivasan (1985) described measurements in the wake of a circular cylinder in a wind tunnel, for Reynolds numbers in the range 30-10000; the object of this work was an attempt to fill gaps in knowledge relating to open, or unconfined fluid flows. The results indicated that the initial stages of transition to turbulence were characterised by narrow 'windows' of chaos interspersed between regions of order.

The sequence of observed events as Reynolds number was increased was as follows:

(i) first appearance of chaos apparently by the route proposed by Rouelle and Takens
(ii) disappearance of chaos with further increase in Reynolds number
(iii) a return to a 3 frequency periodicity
(iv) reappearance of chaos
(v) a return to 4 frequency periodicity (quasi-periodicity)
(vi) reappearance of chaos etc.

Several alternations between order and chaos were observed for values of Re < 200. The results suggested that the dimension of the attractor did not exceed 20, even for values of Re up to 10 . It was found in general that, as Re was increased, the fluid flow varied from periodic through to quasiperiodic and finally chaotic; this chaotic state was not necessarily turbulent within the general meaning of the word. The author (Sreenivasan) was of the belief that the key to both stability and turbulence lay in the Navier-Stokes equations and that no additional hypotheses of a fundamental nature were required.

The author concluded 'not all observations we have made can be understood within the framework of chaos and dynamical systems but we find it amazing that the dynamics of fluid motion which we believe are particularly governed by the Navier-Stokes equations should be at all represented by extremely simple systems'.

A route to chaos and turbulence was investigated by Kida et al (1989) using a series of numerical simulations of the Navier-Stokes equations for motion in an incompressible fluid, with a high symmetry imposed on the velocity field.

From examination of frequency power spectra the following sequence of transitions to chaos, with increasing Reynolds number, was noted.

Steady

⇓

simply periodic

⇓

doubly periodic (or quasiperiodic)

⇓

non-periodic motions (ie chaotic)

⇓

quasiperiodic motions

⇓

non-periodic (ie chaotic)

⇓

turbulent

The chaotic motion was differentiated from turbulent motion in that the former exhibited only temporal irregularity whereas the latter was characterised by both temporal and spatial irregularity. It was noted that the route described above was for discrete and not continuous values of viscosity.

Spiegel (1987) in a presentation to the Royal Society suggested that there were as many different kinds of turbulence as there were ways to stir a fluid. He offered fragmentary evidence for the statement that 'the various forms of turbulence have much in common - they all may have vorticity and I think they are all chaotic'. The overall conclusion is that turbulence is a chaotic process although it remains to be determined whether the 'notion' will be of use.

From observations of chaos arising from the interaction of steady and time - dependent flows, in a Taylor-Couette system, Mullin and Price (1989) demonstrated that for a given set of conditions, a weak form of chaotic motion could be identified. This motion, although not turbulent, could be described ('for an infinite dimensional system in its regular phase') by a relatively simple set of finite-dimensional ordinary differential equations. The authors suggest that the evidence from their results supported the claim for a connection between the Navier-Stokes equations and the properties of finite-dimensional dynamical systems, although this connection could not be proved.

Several authors have suggested that chaos is the manifestation of the apparently unpredictable behaviour of deterministic solutions to non-linear equations. This approach was considered by George (1990) in the context of the Navier-Stokes equations; three particular questions were posed.

(i) Do the Navier-Stokes equations give rise to chaotic solutions, ie solutions which are deterministic but appear random?
(ii) Do the Navier-Stokes equations have strange attractors associated with them?
(iii) If they do, what should we expect them to look like and how should we represent them?'

From a review and analysis of published work George argued that turbulence was a representation of the non-linearity of the Navier-Stokes equations. It was further suggested that the equations had attractors and in the case of turbulent flow the strange attractors were in fact the turbulent flows themselves. In other words turbulence is the counterpart of the strange attractor.

Argoul et al (1989), in work concerned with the application of wavelet analysis to explore the spatial structure of fully developed turbulence, claim to have identified new qualitative features in fully developed turbulence. The work revealed the multifractal nature of the Richardson cascade. The fractal nature of this cascade had been earlier proposed by Mandelbrot (1983a).

A numerical model to study the transition to turbulence in two-dimensional Poiseuille flow in an open channel has been described by Jimenez (1990). The work demonstrated that it was possible to set up a computational model for two-dimensional flow which exhibited, among other phenomena, chaotic behaviour. A whole range of fluid behaviour from laminar to chaotic was apparent such as would be observed in fully developed turbulent three-dimensional flows. According to the author the identification of the many similarities between two and three-dimensional approaches was surprising, and was an indication of the usefulness of adopting the two-dimensional approach in such studies.

5 Conclusions

5.1 Introduction

It is clear that a study of the present state of knowledge of chaos theory poses more questions than it can provide answers for. However, there are some conclusions that can be drawn with certainty:

(i) it is not possible, given the initial conditions of a deterministic system, to accurately predict what it will do in the future.

(ii) the use of linear equations to describe the behaviour of non-linear systems will not necessarily produce valid results.

(iii) there is no satisfactory theory which provides a complete description of turbulence.

Statements such as these require a degree of qualification. Thus, according to Tatsumi (1983), 'the irregular nature of turbulence is now fully understood in terms of the theory of dynamical systems;' this is qualified by the explanation that the irregularity in turbulent motion produces a statistical regularity of a combination of many turbulent motions. Work by Mullin and Price (1989) led to the conclusion that although a complete description of turbulence was still not available, new and interesting ideas continued to emerge. In particular they noted the successes in studies of the evolution of temporal chaos in small scale fluid flows.

Fairbairn (1986) has suggested that 'the apparently chaotic behaviour (of a system) is an indication of some non-linear property of the system, rather than a breakdown of its deterministic behaviour.' Stewart's (1989a) view is that 'the great discovery of chaotic dynamics is that apparently patternless behaviour may become simple and comprehensive if you look at the right picture.'

Clearly strange attractors are central to chaotic behaviour and much work has been carried out to characterise them. Ott, reporting in 1981, posed several questions which required answers in this context:

(i) for a given set of equations, is it possible to predict the occurrence of a strange attractor?

(ii) to what extent can the properties of a strange attractor be predicted?

(iii) what is the distribution function of a strange attractor and what is the most efficient way to find and characterise it?

Complete answers to these questions have yet to be found. In the absence of these answers strange attractors still have useful applications. With particular reference to fluids, Guckenheimer (1986) put forward 3 possible uses:

(i) observation of new aspects of typical dynamical behaviour which in turn would suggest specific experimental observations e.g. routes for transition to chaotic behaviour.

(ii) identification of atypical dynamic behaviour within a given class of systems and the use of this information to predict that such behaviour would not be observed in fluid systems.

(iii) the direct application of systems developed to understand the attractors of differential equations to data from fluids experiments, and comparison of results with computer calculations from a variety of systems.

5.2 Predictability and Chaos theory

The limit of validity of the use of linear equations to describe non-linear processes has already been referred to; if the results produced by these equations are extrapolated for forecasting purposes, then the chances are that considerable errors will occur. An infinitesimally small deviation from linear behaviour, even for a small fraction of time, can result in a massive deviation several time steps later, from which a chaotic state is likely to develop. The same would be true for non-linear equations used to describe non-linear processes since it is unlikely that such equations will represent all possible variables in a system nor will the initial starting conditions be known with the necessary precision.

Citing M Berry of the University of Bristol, McKie (1990) wrote 'chaos theory says that the world is an unpredictable place .... because predictability itself is intrinsically impossible.' This is not to imply that chaos theory has no place in the process of prediction - it might be argued that it has every relevance. This is borne out by McRobie and Thompson (1990) who commented that chaos theory can make detailed predictions about how a system can or cannot behave while at the same time ignoring many of the details of the system. In other words chaos enables the prediction of the limits of predictions to be made. It follows that as a direct result of chaos the accuracy of the long term predictability of the behaviour of a system is likely to be poor.

However, chaos can be utilised in a positive way in this respect. For example in the case of mathematical models being run on computers, it should be possible to determine in advance whether an event is going to be predictable or chaotic over the period of interest and therefore whether any anticipated forecast is likely to be reliable or not. This is achieved by running the same model for two or more occasions, each time using similar but not quite identical initial conditions. If the results develop in similar ways, then it could be concluded that the forecast is in a predictable state and therefore reliable. If the results diverge, then the forecast will be unreliable and in an unstable state. There is an assumption here that no further perturbation beyond the initial starting conditions affects the system. However, a prediction can still be associated with random motion and have validity provided that that motion remains within the attractor and doesn't go outside it.

5.3 Concluding remarks

Much of the work carried out to investigate the part played by chaos in the transition to turbulent flow has involved relatively simple, constrained and closed flow systems. In these cases it appears to be generally accepted that chaos can provide an explanation for the transition stage before the onset of turbulence.

Very little work has been undertaken on large open flow systems such as wakes, jets, boundary and mixing layers, pipe flows etc. A limited investigation was carried out by Sreenivasan and Strykowski (1983) on several different types of flow, although attention was focused on the transition to turbulence in coiled pipes. It was discovered, for the cases of open flow investigated, that in no case did the transition to turbulence occur by any of the generally accepted routes to chaos. It was concluded that turbulence, unless severely constrained, did not behave like a single dynamical system. A redeeming feature of the work, in the case of the coiled pipe, was the discovery that for values of Reynolds number, not too far above the transitional value, the attractor for turbulence was relatively low-dimensional. It was left as an open question as to whether any aspects of chaotic dynamical systems were useful in the description of transition to the turbulence in open flows.

Further work on jet flows was reported by Ng (1990) involving a laboratory study to investigate the chaotic motion of confined jet flows in a cylindrical duct; comparisons were made with pipe flow in the same duct. It was demonstrated that more deterministic features were generated under jet flow conditions than under pipe flow. In other words motion under pipe flow was random or chaotic; however examination of phase portraits indicated an absence of strange attractors. On the other hand for the case of jet flows deterministic and coherent structures were noted and strange attractors identified, ie there was apparently order within chaos in such flows.

It was claimed that this observed behaviour was in agreement with a conclusion reached by Stewart (1989b) that 'chaos gives way to order, which in turn gives rise to new forms of chaos.'

There is a considerable amount of evidence to suggest that turbulence is a chaotic process although the practical use for such a notion has yet to be established. It is reasonably clear that the theory of chaos can be applied to the onset of turbulence but there is little or no evidence to justify its application beyond that stage. Notwithstanding this limitation there is a general consensus among workers in the field that such evidence as currently exists is sufficient to justify further and deeper investigation.

However, the application of chaos theory to a system as complex as the open flow in a tidal estuary having erodible boundaries is clearly not possible given the present state of the art, although the scope for checking the bounds of predictability of the mathematical models for such flows exists (see Section 5.2). There may be limited advantage in investigating further the application of fractal geometry to the classification of coastlines in terms of their degree of irregularity.

6 Acknowledgements

The author acknowledges the help and advice received from members of HR staff during the preparation of this report. In particular the author is grateful to Mr Geoff Gilbert for his helpful comments and suggestions made during the initial stages of the work.

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Appendix

Appendix 1 Chaos theory in fields other than fluid flow

1. Coveney (1990) - Thermodynamics (entropy) - the irreversibility of time.

2. Chaitin (1990) - Mathematics - randomness and unpredictability.

3. Lesurf (1990) - Behaviour of electrical circuits.

4. McRobie and Thompson (1990) - Stability of engineering structures.

5. Murray (1989) - Dynamics of the solar system.

6. May (1989) - Population dynamics.

7. Savit (1990) - Financial and commodities markets.

8. Scott (1989) - Predictability of chemical reactions.

9. Vivaldi (1989) - Feedback effects in iterative processes.