# Guidelines for the use of computational models in coastal and estuarial studies

## Wave transformation and wave disturbance models

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Report SR 450
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## *Summary*

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Computational models are frequently used to assess the impact of engineering works in coastal and estuarial waters. With the increasing use of computational hydraulic software by civil engineers and scientists not involved in model development it is essential that comprehensive guidelines on the limitations and assumptions of such models are widely available. In selecting a model for a particular application it is important that the engineer be aware of the range of models available, the processes they can represent, the underlying assumptions on which the models are based, their limitations and the solution method used. In order to address this issue, HR Wallingford was commissioned in 1994 by DOE to develop guidelines for engineers on the selection and application of computational models for estuarial engineering studies. The guidelines incorporate wave transformation and disturbance models and flow and sediment transport models.

In the first stage of this project, completed in March 1994, a review of computational models in engineering use for hydraulic studies in the UK was made. This review covered models representing wave transformation, harbour wave disturbance, flow, sediment transport and ship manoeuvring, movement and mooring (HR Wallingford 1994). During this first stage it became evident from industry contacts that very few ship manoeuvring and movement models are used by non-specialists and that many such models are still under development. As a consequence, the production of guidelines for such models would be premature and so they were not included in the second stage of the project.

The guidelines for the computational models produced in the second stage of this project are based on the results obtained from applying computational models to a series of benchmark tests. This report contains the guidelines for wave transformation and wave disturbance models, together with details of the benchmark tests and results. The guidelines for flow and sediment models are presented in the companion report, HR Wallingford 1996.

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# 1 Introduction

## 1.1 Aims

Computational models are frequently used to assess the impact of engineering works in coastal and estuarial waters. With the increasing use of computational hydraulic software by civil engineers and scientists not involved in model development it is essential that comprehensive guidelines on the limitations and assumptions of such models are widely available. In selecting a model for a particular application it is important that the engineer be aware of the range of models available, the processes they can represent, the underlying assumptions on which the models are based, their limitations and the solution method used. In order to address this issue, HR Wallingford was commissioned in 1993 by the DOE to develop guidelines for engineers on the selection and application of computational models for estuarial engineering studies. The guidelines will incorporate wave transformation and disturbance models and flow and sediment transport models.

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## 1.2 Methodology

Many Consulting Engineers in the UK use computational wave disturbance and wave transformation models. Some of these models have been developed by the Consulting Engineers themselves, although, more commonly, they use models that were developed at one of the major international hydraulics laboratories or at UK Universities. Following the survey of computational wave transformation and wave disturbance models described in HR Wallingford (1994), a number of UK Consulting Engineers were contacted, and asked to take part on this project by applying their models to one or more of a series of benchmark tests.

The tests were selected at HR Wallingford with the intention that from the results, the ability of the models to represent different physical processes be assessed. The test cases range from simple tests with analytic solutions, to real situations for which field measurements exist. In addition, a number of the tests are based on physical models, for which data exists. Since the analytic, physical model and field data were used to assess the performance of the models, this information was not supplied to the participants. It should be noted that the participating Consultants ran the tests at their own expense.

The guidelines incorporated within this report are based on the results supplied to HR Wallingford by the participants. They refer to models in use between February 1993 and December 1995. It should be noted when using this report that existing computational models are periodically updated and that new models are continually being developed.

## 1.3 Notes on the guidelines

The guidelines on the use of wave transformation and wave disturbance models in coastal and estuarial studies, given in Chapters 2 and 3, are based on the results obtained when applying a number of models to a series of benchmark tests. The guidelines concentrate on the model types discussed in Chapters 4 and 5, rather than on individual models. The models applied to the benchmark tests are representative of those currently in use in the UK. It is important to note, however, that existing models are periodically updated and that new models continue to be developed.

The results from the benchmark tests indicate that no one model type can be considered best for all coastal sites, in terms of producing accurate solutions efficiently. For some benchmark tests, the same model was applied by different engineers. The results obtained demonstrate how the interpretation of the problem and the application of a model by a particular engineer can have a considerable impact on the results obtained. In addition, the results also show how, for a particular bathymetry, the performance of a model may depend on the incident wave conditions.

When selecting a suitable model type, or model, for a partcular application, various considerations must be taken into account. These may include:

- What are the important physical processes at the site?
- What information is required?
- What will the wave data be used for?
- Are there financial and duration constraints on the project?

This report presents guidelines on choosing suitable wave transformation and wave disturbance models for particular studies. While emphasis is given to the physical processes represented by the model types, information on other factors, such as where information is required and whether the site being modelled is large, is also given.

The guidelines in Chapters 2 and 3 are given in two parts. The first concentrates on model types and gives an indication of what sort of study to which each model type is particularly applicable. Following this, short, general, descriptions of typical coastal sites to which wave transformation and wave disturbance models may be used is given. The types of models are given in a possible order of preference, taking into account both accuracy and efficiency. More detailed information on the types of models and the models actually tested in this project are given in other chapters of this report and in the Appendices.

## 1.4 Organisation of the report

The remainder of this report is organised as follows. The guidelines for the application of wave transformation and wave disturbance models to estuarial and coastal studies are presented in Chapters 2 and 3 respectively. The models applied to the benchmark tests are briefly described in Chapters 4 and 5. Further details of the models are given in Appendices 1 and 2 for wave transformation and wave disturbance models respectively. The benchmark tests are described in Chapter 6 and the results discussed in Chapter 7. Finally, conclusions from the project are given in Chapter 8.

It should be noted that this report supercedes HR Wallingford Report SR 450 which contains information on the benchmark tests.

# 2 *Guidelines for wave transformation modelling*

## 2.1 Introduction

In this chapter guidelines on the use of wave transformation models in coastal engineering projects are given. The guidelines are based on the results of the benchmark tests presented and discussed in Chapters 6 and 7. The models applied to the tests are representative of the main types of wave transformation models currently in use in the UK. It is worth noting once more that most existing models are continually being updated and extended, and that new models are developed. In this project, none of the participating Consulting Engineers used nomographs or models based on hyperbolic approximations to the mild slope equation.

The guidelines given concentrate on the range of physical processes represented by types of wave transformation models. It should be noted that linear wave transformation models can be used to model spectral wave conditions. This is achieved by running the model repeatedly for the different spectral components and using linear superposition to combine the results. In some models, wave breaking is not explicitly represented. In such cases, it is possible to use an empirical representation of wave breaking and simply limit the predicted significant wave height to be a fraction of the water depth. In some models which claim to represent wave breaking, this is how it is modelled. When applying models which use forward marching solution methods, for example models based on the parabolic approximation to the mild slope equation, care should be taken in selecting the orientation of the model grid. Such models generally give more accurate results if the x-axis of the grid system is aligned with the main forward propagation direction. That is, the incident waves should be less than approximately 60° to the x-axis.

As was noted in Section 1.3, a number of factors will usually be taken into account when selecting a suitable model for a particular application. These include:

- What are the main physical processes affecting the wave conditions at the site?
- Need all the processes identified be represented, or can some simplifying assumptions be made?
- What information about the wave conditions is required, for example, significant wave heights, periods, directions, spectral components?

- Is the area being modelled, large? Are wave conditions over the whole of the area required or just at single specified locations?
- Will the results be used for design purposes? In which case, as accurate results as possible will be required. Otherwise, a less comprehensive assessment may be appropriate to give an initial indication of wave activity at the site.
- Are there duration and financial constraints on the project?

For some studies, it may be appropriate to use more than one wave transformation model. For example, a refraction model may be used to predict the wave conditions over a large area. A more detailed analysis of a smaller area may then be carried out using a refraction/diffraction model. In this case, results from the refraction model can be used as input to the refraction/diffraction model. There may, in some projects, be calibration data available. This is usually in the form of limited wave recordings in the area being studied. If such data is available, then this can be used to validate or calibrate the model being used, for example, by selection of a suitable sea-bed friction coefficient.

The guidelines are given in the following two sections. In the first, types of coastal sites to which each of the main categories of wave transformation models is particularly suited, are given. In the second, a number of coastal sites, typical of engineering projects likely to arise, are given along with the types of wave models which would be suitable.

## 2.2 Wave transformation models

### *2.2.1 Ray tracking refraction models*

**Back tracking models**

Models in this category are particularly suited to studies in which spectral representations of the wave conditions at single locations are required. These models are generally computationally efficient and so can be used to represent wave transformation over very large areas. Shoaling and refraction are the main processes represented. Back tracking ray models can be used to give approximate predictions in areas where diffraction due to the bathymetry is important, particularly if the bathymetry is relatively simple and short period waves are being modelled. In areas where sea-bed friction is likely to be significant, these models are likely to over-estimate the wave heights.

**Forward tracking models**

Models which are based on forward tracking ray methods can be used to predict monochromatic wave conditions throughout large coastal sites. Shoaling and refraction are the main physical processes modelled, although energy dissipation due to sea-bed friction and wave breaking may also represented. Such models may also be used to give approximate predictions in areas where diffraction due to the bathymetry is important. However, very irregular bathymetries may lead to caustics which in turn lead to such models predicting very large wave heights in some areas and very low wave heights in others. Forward tracking ray models represent short period waves more accurately than long period waves.

### 2.2.2 *Finite difference refraction models*

Refraction models which are based on finite difference methods can be used to model shoaling and refraction over relatively large coastal areas. Wave conditions throughout the site can be predicted. The size of the finite difference grid used will affect both the accuracy and efficiency of the model (a fine grid leading to more accurate solutions, but is more computationally expensive). Generally such models will represent energy dissipation due to both sea-bed friction and wave breaking. Finite difference refraction models can be used in areas where diffraction due to the bathymetry occurs to give approximate predictions of the wave conditions. This is particularly so when the bathymetry is relatively smooth.

### 2.2.3 *Refraction/diffraction models*

Wave transformation models which represent both refraction and diffraction are generally based on the solution of a governing equation (or system of equations) using finite difference or finite element methods. In areas where diffraction due to the bathymetry is important, refraction/diffraction models will generally give more accurate predictions of the wave conditions than refraction only models. This is particularly so when there are very rapid changes in the bathymetry, for example, due to a harbour approach channel. Models which are based on a parabolic approximation to the mild slope equation may give poor predictions if the incident wave is at a large angle to the principal wave propagation direction.

Such models can also be used in areas where shoaling and refraction are the dominant processes, to predict wave conditions over the whole site. Using such models over very large coastal areas may be computationally expensive, compared to refraction only models, without a corresponding increase in accuracy. Energy dissipation due to both sea-bed friction and wave breaking is usually represented.

## 2.3 Typical coastal sites

In this section a number of typical coastal sites where wave transformation modelling may be required are listed. For each site, an indication of the types of models which may be applied is given. The suggested models are given in a possible order of preference, taking into account both accuracy and efficiency. In the following sections a large area is considered to be of the order 20km x 20km and a small area 3km x 3km (these should be considered only as very approximate indications of sizes).

**Wave conditions are required over a very large area where refraction is the dominant process.**

- Forward tracking ray models.
- Finite difference refraction models.
- Refraction/diffraction models.

**Wave conditions are required over a small area where refraction is the dominant process.**

- Finite difference refraction models.
- Forward tracking ray models.
- Refraction/diffraction models.

**Wave conditions are required at few locations, although the area being modelled is large and refraction is the dominant process.**

- Back tracking ray models, particularly if spectral wave conditions are required.
- Forward tracking ray models.
- Finite difference refraction models.
- Refraction/diffraction models.

**Wave conditions are required at few locations within a small coastal area where refraction is the dominant process.**

- Finite difference refraction models.
- Back tracking ray models.
- Forward tracking ray models.
- Refraction/diffraction models.

**Wave conditions are required in a large coastal area where refraction and energy dissipation, due to sea-bed friction (for example if the sea-bed is rocky) and wave breaking, are important.**

- If suitable data is available, then forward tracking ray models can be calibrated to give accurate predictions efficiently.
- Refraction/diffraction models - particularly those which use forward marching solution methods or employ acceleration techniques such as multigrid methods or conjugate gradient methods.
- Finite difference refraction models.
- Back tracking ray models.

**Wave conditions are required in a small coastal area where refraction and energy dissipation, due to sea-bed friction and wave breaking are important.**

- Refraction/diffraction models.
- Finite difference refraction models.
- Forward tracking ray models.
- Back tracking ray models.

**In the following examples of coastal areas where wave transformation models may be applied, it is assumed that the sea-bed is not flat, so that both shoaling and refraction of the waves will occur.**

**Wave conditions are required in large coastal areas where mild diffraction effects, for example due to small berms or shoals, may occur.**

- If initial estimates of the wave conditions area required, or conditions near the berm or shoal, then finite difference refraction models may be suitable.
- Refraction/diffraction models, particularly those employing acceleration techniques or which use forward marching solution methods.
- Possibly back tracking ray models, if conditions near the shoal or berm are required.

**Wave conditions are required in a small coastal area where mild diffraction effects may occur.**

- Refraction/diffraction models.
- Finite difference refraction models, particularly if conditions near the berm or shoal are required.

**Wave conditions in areas where there are rapid changes in the bathymetry, for example, due to the presence of large berms or shoals.**

- Refraction/diffraction models. If the area is very large, then models which use forward marching solution techniques will be the most efficient.

**Wave conditions in areas where there are rapid changes in the bathymetry, for example due to dredged channels.**

- Refraction/diffraction models based on the parabolic approximation to the mild slope equation, particularly if the incident wave has a long period and the area being modelled is large.
- Other refraction/diffraction models.

# 3 Guidelines for wave disturbance modelling

## 3.1 Introduction

Guidelines on the application of wave disturbance models to coastal engineering projects are given in this chapter. The guidelines are mainly based on the results of the benchmark tests presented and discussed in Chapters 6 and 7. As with wave transformation models, it is important to note that existing wave disturbance models are updated and extended periodically and new models are continually being developed.

When considering engineering projects at coastal sites where natural or man-made structures are present, it will be necessary to apply a wave disturbance model rather than a wave transformation model. Wave disturbance models, as well as representing refraction and shoaling, also represent reflection and diffraction due to surface piercing structures. Such coastal sites will often involve waves propagating through shallower water and so energy dissipation processes such as wave breaking and seabed friction may also be represented. In regularly shaped harbours, where the harbour boundaries are mainly vertical, concrete walls, harbour resonance may also be a problem. The representation of these different physical processes and their interaction mean that wave disturbance problems are generally more complicated than wave transformation projects.

As is evident from the numbers of models tested in this project, there are far fewer wave disturbance models in common engineering use than wave transformation models. At present, physical wave disturbance models are often used for many harbour development projects and are often the preferred option when design wave conditions are required. This is particularly so when there is no data available for calibrating computational wave disturbance models.

The wave disturbance models applied to the benchmark tests fall into two main categories; ray models and models based on the solution of the mild slope equation. Models in the former category are well-established and have been used on very many engineering projects. In contrast, models based on the mild slope equation models are relatively new to application to coastal engineering studies. This is, to some extent, illustrated by the results discussed in Chapter 7 where the ray models tend to give more accurate solutions than the finite difference and finite element models. This is partially deemed to be a feature of their more frequent use.

At present, the most appropriate use for wave disturbance models is in comparing possible harbour layouts. If calibration data is available and the performance of the computational wave disturbance model validated, the results from a wave disturbance model could be used for engineering design. If such data is not available, then computational wave disturbance models can be effectively used alongside physical models of wave disturbance. Most often, the computational model will be used to compare a number of possible harbour developments, with the preferred layout subsequently tested using a physical model. The physical model will be used to provide design parameters and to further optimise the selected layout.

As noted in earlier Chapters, a number of factors will usually be taken into account when selecting a suitable model for a particular application. These include:

- What are the main physical processes affecting the wave conditions at the site?
- Need all the processes identified be represented, or can some simplifying assumptions be made?
- What information about the wave conditions is required, for example, significant wave heights, periods, directions, spectral components?
- Is the area being modelled, large? Are wave conditions over the whole of the area required or just at single specified locations?
- Will the results be used for design purposes? In which case, results as accurate as possible will be required. Otherwise, a less comprehensive assessment may be appropriate to give an initial indication of wave activity at the site.
- Are there duration and financial constraints on the project?

As with computational wave transformation models, the guidelines are given in two sections. In the first, types of coastal sites to which each of the main categories of wave disturbance models is particularly suited, are given. In the second a number of coastal sites, typical of engineering projects likely to arise, are given along with the types of wave models which would be suitable.

## 3.2 Wave disturbance models

### *3.2.1 Ray tracking models*

Back tracking ray models

The use of back tracking ray models in wave disturbance projects is limited to sites where refraction, shoaling and diffraction due to a single natural or man-made structure are the important processes. Such models can be efficiently used to represent spectral wave conditions at individual locations at fairly open coastal sites, near, for example, a natural headland. Back tracking wave transformation models are often used to provide incident wave conditions for computational wave disturbance models by, for example, predicting wave conditions near a harbour entrance.

**Forward tracking ray models**

Forward tracking wave disturbance models predict wave conditions throughout the model area and can be used to represent both small and large harbour areas. These models can generally represent a fairly complicated layout of harbour boundaries and can represent both total and partial reflections from harbour walls. Forward tracking ray models are most suited to projects where the incident wave period is in the range 5 - 15s, although they can be used with care to represent wave periods outside this range. If the bathymetry in the harbour area is complicated, for example, due to an approach channel, these models will tend to over-estimate wave heights due to caustics caused by the crossing of wave rays.

### *3.2.2 Finite difference / finite element models*

**Helmholtz equation**

The use of models based on the finite difference or finite element solution of the Helmholtz equation is restricted to fairly idealised harbour geometries, with a constant depth of water. Such models can be used in preliminary studies to provide an estimate of wave conditions within or near a harbour.

**Mild slope equation**

Models based on the solution of the mild slope equation are suited to small or medium sized harbours, particularly where diffraction due to variations in the seabed may be important. For example, harbours in which there is an approach or navigation channel, or where there are irregular mounds in the seabed, perhaps due to dumped material or natural rock. Finite difference models may be restricted to fairly simple geometries in which the harbour boundaries can be represented along the grids lines. The use of variable sized triangular grids, however, makes finite element models suitable for fairly complicated harbour geometries. When using finite element models to compare layouts, some care must be taken to ensure that differences in results are due to differences in the harbour layout, rather than differences in the finite element grid. Models based on the mild slope equation can be used to represent incident wave conditions with periods longer than the 15s quoted for forward tracking ray models.

### *3.2.3 Non-linear wave disturbance models*

One category of computational wave disturbance models not represented in this project is non-linear wave disturbance models such as those based on the solution of the Boussinesq equations. At present, such models are expensive to use in terms of computer memory and processing time and tend to be rarely used by Consulting Engineers in the UK. It is likely, however, that with the advent of increasingly powerful machines and efficient solution methods, such models will become more widely used in the future. Models based on the solution of the Boussinesq equations are particularly suited to coastal projects where non-linear wave-wave interaction is likely to be important and where harbour resonance is likely to occur, for example, in regularly shaped harbours exposed to long period incident waves. These models can be used to represent set-down which can also result in harbour resonance which in turn can cause major problems for vessels within a harbour.

## 3.3 Typical coastal sites

In this section a number of typical coastal sites where wave disturbance odelling may be required are listed. For each site, an indication of the types of model which may be applied is given. The suggested models are given in a possible order of preference, taking into account both accuracy and efficiency. In the following sections a large harbour area is considered to be of the order 5km x 5km and a small area 1km x 1km (these are very approximate indications of size).

**Wave conditions are required at individual locations at sites where refraction and diffraction due to a surface piercing structure are the dominant processes.**

- Back tracking ray models.
- Forward tracking rays models.
- Finite difference / finite element models.

**Wave conditions are required throughout a coastal area where refraction and diffraction due to a surface piercing structure are the dominant processes.**

- Forward tracking ray models.
- Finite difference / finite element models.

**Wave conditions are required throughout a small, regularly shaped harbour with constant water depth and incident wave period less than 15s.**

- Foward tracking ray models.
- Finite difference / finite element models based on the Helmholtz equation.
- Models based on mild slope equation.

**Wave conditions are required throughout a small, regularly shaped harbour with constant water depth and incident wave period greater than 15s.**

- Models based on the Helmholtz equation.
- Models based on the mild slope equation.
- Forward tracking ray models.

**Wave conditions are required throughout a small harbour area with complicated bathymetry.**

- Models based on the mild slope equation.
- Forward tracking ray models.

**Wave conditions are required throughout a regularly shaped harbour exposed to long period waves where resonance may be a problem.**

- Models based on the Boussinesq equations.
- Models based on the mild slope equation.

**Wave conditions are required throughout a large harbour area with simple bathymetry and harbour structures.**

- Forward tracking ray models.
- Models based on the mild slope equation.

**Wave conditions are required throughout a large harbour area with simple bathymetry but complicated harbour layout.**

- Forward tracking ray models.
- Models based on the mild slope equation.

**Wave conditions are required throughout a large harbour area with complicated bathymetry and harbour layout.**

- Finite element models based on the mild slope equation.
- Forward tracking ray models.
- Finite difference models based on the mild slope equation.

**When designing harbours it is important that as well as assessing wave disturbance, possible resonance problems, navigation issues and sedimentation are also considered.**

# *4 Wave transformation models*

## 4.1 Classification of wave transformation models

In this chapter each of the wave transformation models tested during this project is briefly described. Further details about the models is given in Appendix 1, including information on the input required and the output produced by the models, together with details on the physical processes represented and governing assumptions made. The descriptions of the models are based on information supplied by the participating Consulting Engineers. The physical processes represented should be reviewed in the light of the benchmark test results.

A wide variety of wave transformation models exist and it is not always obvious which model or even what type of model is most suited to a particular problem. Some models only represent a limited range of physical processes, but are usually computationally efficient and easy to use. In some situations, they will give adequate assessments of the wave conditions. More complicated models can be expensive in terms of computer memory and processing time, but can, potentially, give a more comprehensive representation of wave transformation.

The models considered in this study are all currently in use in the UK. They have been developed at Universities, Hydraulic Laboratories and by Consulting Engineers both in the UK and overseas. Each model represents a number of physical processes and is based on either a ray tracking method or finite difference or finite element solution of a governing equation. Each model requires a description of the bathymetry in the area being modelled, together with information on the incident wave conditions. Some of the models are "spectral", that is, wave transformation processes are applied to a range of frequencies and directions. It should be noted that the linear models which currently only consider monochromatic waves can be used to represent random waves. This is achieved by repeatedly running the monochromatic versions for incident waves with different frequencies and directions, and using linear superposition to combine the results. Most of the models included here have been validated against field data or physical model data.

In this report, wave transformation models are classified firstly according to the processes represented and secondly according to the solution technique used. Computational refraction models are based on the tracking of wave rays or on the finite difference solution of refraction equations. Models representing both refraction and diffraction are based on the finite difference or finite element solution of a governing equation which represents wave transformation processes. Each class of wave transformation model is described in the following sections.

## 4.2 Nomographs

In the context of this project, nomographs are charts or diagrams which can be used for facilitating calculations of wave refraction and diffraction. Diagrams for representing wave refraction are constructed using Snell's Law to track wave orthogonals (wave rays in the absence of currents) from deep water to nearshore. For straightforward bathymetries, this method is similar to that used by a forward tracking ray model. Diffraction due to surface piercing structures, for example, breakwaters, can be modelled using the diffraction diagrams presented in Wiegel (1964).

For relatively simple bathymetries, nomographs can be used to obtain approximate estimates of wave conditions, cheaply and fairly quickly. However, nomographs have not been used in this project.

## 4.3 Computational wave refraction models

### *4.3.1 Forward tracking ray models*

The effects of refraction and shoaling on waves can be modelled using a ray technique which comes from the theory of light. A wave ray, in the absence of currents, is a line which is perpendicular to the crest of a wave. The modification of a ray path, in response to changes in the bathymetry as a wave propagates shorewards, is governed by Snell's law.

In forward tracking models, the wave rays are tracked shorewards, from an offshore boundary, in the direction of wave propagation. The wave rays provide information on how wave energy is re-distributed and so the wave height and direction of a particular frequency component can be calculated. Using a forward tracking ray method, the wave conditions over a large area can be predicted relatively quickly. A limitation is that diffraction, particularly due to depth variations, is not represented in the governing equations. In some models, diffraction is included by other means.

### *4.3.2 Back tracking ray models*

Back tracking ray methods can be used to predict wave conditions at specific sites, where energy dissipation is not significant. In back tracking methods, the wave rays are tracked seawards from an inshore point at which the wave conditions are required. These rays give information on how energy travels between the seaward edge of the area of interest and the inshore point. The results can be used to calculate the refraction of a large variety of offshore wave conditions fairly simply. The wave height and direction for each frequency component of a wave energy spectrum is calculated, rather than for a single representative frequency component as in other models.

### *4.3.3 Finite difference models*

In a finite difference wave refraction model, the refraction equations are solved numerically at each point on a grid, using a finite difference method. For a given incident wave condition, the wave height, period and direction can be predicted at each point in the grid covering the area being modelled. The effects of sea-bed friction and wave breaking may be incorporated into this type of model. Examples of the equations on which this type of model may be based are given below.

The propagation of wave energy may be based on the solution of an energy balance equation. This equation may be modified to include terms for modelling wave growth due to wind action and energy dissipation due to sea-bed friction and wave breaking. An example of an energy balance equations is:

$$\frac{\partial}{\partial x}(C_x{}^*A_0) + \frac{\partial}{\partial y}(C_y{}^*A_0) + \frac{\partial}{\partial \theta}(C_\theta{}^*A_0) = T_0$$

$$\frac{\partial}{\partial x}(C_x{}^{**}A_1) + \frac{\partial}{\partial y}(C_y{}^{**}A_1) + \frac{\partial}{\partial \theta}(C_\theta{}^{**}A_1) = T_1$$

where $C_i^*$ are propagation speeds of action and $C_i^{**}$ are propagation speeds of mean frequency. $A_0$ and $A_1$ are the zero-th and first order moments of the

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action density spectrum in each spectral direction. The generation and dissipation of $A_0$ and $A_1$ are represented by $T_0$ and $T_1$.

## 4.4 Computational models of wave refraction and diffraction

Models which represent both refraction and diffraction are usually based on the solution of governing equations which describe wave propagation, using finite difference or finite element methods. Although these models may, potentially, be able to give a better representation of the physics of propagating waves, they do suffer from one major drawback compared to ray tracking methods. In order that the governing equation be solved accurately, a minimum number of grid points per wavelength is usually required in the finite difference or finite element method. This may mean the use of very fine grids when solving over large areas or when the wavelength is small. This in turn makes the equation expensive to solve, in terms of both computer storage and processing time. The development of efficient finite difference and finite element models is currently a very active area of research, especially as computing power continues to increase.

### 4.4.1 *Mild slope equation*

The propagation of small amplitude surface gravity waves over a sea-bed of mild slope can be described by the mild slope equation, Berkhoff (1972),

$$\nabla.(CC_g\nabla\Phi) + \omega^2\frac{C_g}{C}\Phi = 0 \tag{1}$$

where $\Phi(x,y)$ is the complex wave potential function, $C$ is the phase velocity, $C_g$ is the group velocity and $\omega$ is the angular frequency.

This equation takes account of the combined effects of refraction, shoaling and diffraction, but in the form given above, the influences of sea-bed friction, wave breaking, current and wind are ignored. The mild slope equation can be used to represent wave propagation in coastal areas, as well as propagation into harbours with reflecting boundaries.

Usually a finite difference or finite element method is used to solve the mild slope equation. These methods solve the equation at each point of a grid which covers the area being modelled. The grid should be fine enough to represent the bathymetry accurately and to adequately resolve the wavelength of the propagating waves. This, together with the elliptic nature of the equation, means that the mild slope equation can be expensive to solve in terms of both computer memory and processing time. This is particularly so when the wavelength is small and the area being modelled is large. Some models use acceleration techniques, such as the multigrid and conjugate gradient methods, to solve the mild slope equation efficiently.

Other models are based on approximations to the mild slope equation which can be solved more efficiently.

### 4.4.2 *Parabolic mild slope equation*

The parabolic version of equation (1), as derived by Radder (1979), is given by:

$$\frac{\partial\phi}{\partial x} = \frac{i}{2k}\frac{\partial^2\phi}{\partial y^2} + (ik - \frac{1}{2k}\frac{\partial k}{\partial x})\phi \tag{2}$$

where $\phi = \Phi(CC_g)^{1/2}$, $x$ is the main direction of propagation, $y$ is the transverse direction and $k$ is the wave number.

The derivation of this equation assumes that the reflected wave field is negligibly small, so that only forward travelling waves are considered. This means that equation (2) can be solved more efficiently than the mild slope equation using, for example, forward marching finite difference methods. However, since it does not include reflection effects it may lead to poorer results in areas where reflection is significant. In addition, equation (2) assumes that the main effects are in the direction of propagation, which means that incident waves at large angles to the $x$ axis (taken to be the main direction of propagation) may also lead to poor results. Recent work involving the parabolic approximation has included modifications to allow incident waves at large angles to be modelled successfully.

### 4.4.3 *Hyperbolic mild slope equation*

The mild slope equation can also be expressed as a hyperbolic system of equations. As with the parabolic approximation, the hyperbolic system can be solved more efficiently than the elliptic form of the equation. In addition, the reflected wave component is also represented. The system of first order equations as given by Copeland (1985) is:

$$\nabla Q + \frac{C_g}{C}\frac{\partial \eta}{\partial t} = 0 \qquad (3)$$

$$\frac{\partial Q}{\partial t} + CC_g \nabla \eta = 0 \qquad (4)$$

where *Q* is a vertically integrated function of particle velocity and $\eta$ is the surface elevation. A forward marching finite difference method can be used to solve these equations. However, there may be difficulties in converging to the steady state solution. Since only the steady state solution is a solution to the mild slope equation, this method may not be as reliable as solving the parabolic and elliptic forms of the mild slope equation.

## 4.5 Models applied to the benchmark tests

### 4.5.1 *Ray tracking refraction models*

**ORCAWAVE**

ORCAWAVE was developed at Orcina Ltd. (1990) and is based on a forward tracking ray method. Wave rays which are tracked shorewards from an offshore boundary give information on wave conditions over the whole area. ORCAWAVE is most suited to coastal sites where shoaling, refraction, sea-bed friction and wave breaking are the dominant processes, but where diffraction effects are negligible. Since ORCAWAVE is based on a ray tracing technique, it can be used to model very large areas efficiently. Significant wave heights can be predicted at each point on the grid used in the model and plots showing the wave ray refraction pattern can be obtained with breaking wave crests shown in a different colour.

The physical processes represented in ORCAWAVE are:

- shoaling.
- refraction.
- sea-bed friction.
- wave breaking.

**OUTRAY / OUTURAY**

OUTRAY (HR Wallingford 1989) was developed at HR Wallingford and is based on a back tracking ray method. Wave rays, which are tracked seawards from an inshore point of interest to the offshore boundary, give information on the transfer of wave energy. OUTRAY is most suited to studies where spectral wave conditions are required at single inshore points, in areas where shoaling and refraction are the dominant processes. Since OUTRAY is based on ray tracking techniques, it can be used to model very large areas efficiently. Wave-current interaction can be modelled, but diffraction, sea-bed friction and wave breaking are not included within the model. However, the computed wave heights can be limited to being a fraction of the total water depth.

The physical processes represented in OUTRAY are:

- shoaling.
- refraction.
- directional and frequency spreading.
- wave-current interaction.

**PORTRAY**
PORTRAY (HR Wallingford 1988) was also developed at HR Wallingford and is based on a forward tracking ray method. This model can be used to predict wave conditions throughout very large coastal areas where shoaling and refraction are the dominant processes. Rays are tracked from the offshore edge of the area being modelled, in the direction of propagation, inshore. Significant wave heights, periods and directions at each grid point can be predicted using PORTRAY. Reflections and wave-current interaction, as well as energy dissipation due to sea-bed friction and wave breaking are modelled in PORTRAY. Although diffraction due to surface piercing structures is included in the model, diffraction due to variations in the bathymetry is not.

The physical processes modelled in PORTRAY are:

- shoaling.
- refraction.
- diffraction due to surface piercing structures.
- sea-bed friction.
- wave breaking.
- wave-current interaction.
- reflection.

**REFRAC**
REFRAC was developed at Delft Hydraulics (1990) and is based on ray tracking methods. The wave rays may either be tracked forwards, that is from offshore to inshore, or backwards, from inshore to offshore. This model can be used to estimate the wave conditions in open coastal areas where shoaling and refraction are the dominant physical processes. Diffraction, and energy dissipation due to sea-bed friction and wave breaking are not included in REFRAC, although the wave heights can be limited to be a fraction of the water depth. Output from REFRAC includes tables of directions, shoaling and refraction coefficients and amplitudes.

The physical processes modelled in REFRAC are:

- shoaling.
- refraction.

**W-RAY**
W-RAY (Scott Wilson Kirkpatrick 1993) is currently being developed by Scott Wilson Kirkpatrick. It is based on a back tracking ray method and so is suited to studies where spectral wave conditions at single inshore points are required. The model can be used to efficiently model wave transformation over very large areas, where shoaling and refraction are the dominant processes, but where sea-bed friction and diffraction are not significant. W-RAY predicts the significant wave height, zero crossing period and wave energy distribution at the specified inshore point. Wave breaking is represented by limiting the wave height to be a fraction of the total water depth.

The physical processes represented in W-RAY are:

- shoaling.
- refraction.
- directional and frequency spreading.

### *4.5.2 Finite difference refraction models*

**ENDEC**

ENDEC was developed at Delft Hydraulics (1990) and can be used to predict wave conditions at single locations at coastal sites where the offshore bathymetry is relatively simple. The model is based on the solution of the wave action and radiation stress equations along a user defined wave ray. The bathymetry is specified along the wave ray and it is assumed that the depth contours are straight and parallel. Shoaling, refraction and energy dissipation are represented in ENDEC, but diffraction is not. Wave-current interaction and wave growth due to wind are also modelled. Wave heights, directions and the water depths along the wave ray are output from ENDEC.

The physical processes represented in ENDEC are:

- shoaling.
- refraction.
- sea-bed friction.
- wave breaking.
- wave-current interaction.
- energy gain due to winds.

**HISWA**

HISWA was developed at the Delft University of Technology (1992). It can be used to predict wave conditions at large coastal sites where the bathymetry may be complicated and where the conditions at multiple points along the shore are required. HISWA is based on the solution of the energy balance equation, adapted to include wave growth by wind action and energy dissipation due to sea-bed friction and wave breaking. Data output from HISWA includes significant wave heights, peak periods and directions at any user defined point. Shoaling and refraction are represented, but diffraction is not modelled in HISWA and only waves propagating in a forward direction are represented.

The physical processes modelled include:

- shoaling.
- refraction.
- sea-bed friction.
- wave breaking.
- directional spreading.
- wave-current interaction.
- energy gain due to winds.
- energy dissipation due to currents.

**LINDAL**

LINDAL was developed at Applied Wave Research and is based on a model developed by Dalrymple (1988). This model uses a finite difference method to solve equations based on the irrotationality of the wave number and the conservation of wave action. Refraction coefficients, wave directions and an indication of where wave breaking occurs are output by the model. Since LINDAL uses a forward marching finite difference technique it can be used to model fairly large coastal areas where shoaling and refraction are the dominant processes and where diffraction can be ignored.

The wave transformation processes represented in LINDAL are:

- shoaling.
- refraction.
- sea-bed friction.
- wave breaking.
- wave-current interaction.

**MIKE 21 NSW**
This model was developed at the Danish Hydraulic Institute (1991) and represents both wave transformation and wave generation. MIKE 21 NSW is based on the finite difference solution to conservation equations for the zero-th and first moments of the wave action spectrum. A forward marching finite difference scheme is used. Due to stability criteria the angle of wave incidence should be less than 60° and the step sizes limited. This model is suited to open coastal regions where diffraction and reflection are not significant. Output from MIKE 21 NSW consists of significant wave heights, mean periods, mean wave directions and directional standard deviations throughout the area being modelled.

The physical processes represented in MIKE 21 NSW are:

- shoaling.
- refraction.
- sea-bed friction.
- wave breaking.
- directional spreading.
- wave-current interaction.
- energy gain due to winds.

### *4.5.3 Finite difference refraction/diffraction models*

**ARMADA**
ARMADA is based on the multigrid model developed by Li and Anastasiou (1992). It uses the multigrid method developed by Brandt (1977), together with a finite difference scheme, to solve the mild slope equation derived by Berkhoff (1972). ARMADA is particularly suited to coastal sites where shoaling, refraction and diffraction, due to changes in the bathymetry, are important, but where reflections are not significant. Wave heights at each node of a finite difference grid covering the area being modelled are predicted. Since ARMADA uses the multigrid method to solve a modified version of the mild slope equation, large coastal areas can be modelled relatively efficiently.

The wave transformation processes represented in ARMADA are:

- shoaling.
- refraction.
- diffraction.
- sea-bed friction.
- wave breaking.

- directional and frequency spreading.
- wave-current interaction.

**MIKE 21 PMS**

MIKE 21 PMS (1993) was developed at the Danish Hydraulic Institute and is based on the solution of parabolic approximations to the mild slope equation. The model is suited to limited coastal areas where shoaling, refraction and diffraction effects are important. Energy dissipation due to sea-bed friction and wave breaking is also represented. Although an efficient forward marching finite difference technique is used, the finite difference grid should be fine enough to ensure that the wavelength is adequately resolved. Significant wave heights, mean periods, mean wave directions and radiation stresses are output at each grid point.

The physical processes included within MIKE 21 PMS are:

- shoaling.
- refraction.
- diffraction.
- sea-bed friction.
- wave breaking.
- directional and frequency spreading.

**MULTIGRID**

MULTIGRID is based on the multigrid solution of the mild slope equation, developed by Li and Anastasiou (1992). The multigrid technique used to solve the finite difference equations, together with the modified form of the mild slope equation, enable MULTIGRID to be used on relatively large areas, efficiently. The model is suitable for sites where shoaling, refraction and diffraction are important, but where reflection is not significant. Wave heights and directions at each point of the finite difference grid covering the area being modelled are output.

The physical process represented in MULTIGRID are:

- shoaling.
- refraction.
- diffraction.
- sea-bed friction.
- wave breaking.
- wave-current interaction.

**PARAB**

PARAB was developed by Dodd (1988) and HR Wallingford (1992), and is based on the solution of a parabolic approximation to the mild slope equation. This model is suited to sites where shoaling, refraction and diffraction are important, but where reflections are not significant. An efficient forward marching finite difference method is used to solve the equation. In order that the waves be adequately resolved, the finite difference grid should contain at least eight grid points per wavelength. Wave heights throughout the area being modelled are output from PARAB.

The physical processes represented in PARAB are:

- shoaling.
- refraction.
- diffraction.
- sea-bed friction.
- wave breaking.
- directional and frequency spreading.

**WC2D**

WC2D was developed at the University of Liverpool (1990), and is based on the solution of a number of equations using explicit finite difference methods. The use of explicit finite difference schemes enforces a restriction on the time step used due to stability criteria. However, the schemes account for full interaction between waves, currents and turbulent motions. WC2D is suited to areas where refraction, shoaling and diffraction are the dominant physical processes and where reflection is not significant. Wave vectors, currents and water surface elevations throughout the area being modelled are output from this model.

WC2D represents the following physical processes:

- shoaling.
- refraction.
- diffraction.
- sea-bed friction.
- wave breaking.
- wave-current interaction.
- turbulence and eddy viscosity.

### *4.5.4 Finite element refraction/diffraction models*

**CGWAVE**

CGWAVE was developed by Panchang et al. (1991) at the University of Maine (1993) and is based on the solution of the mild slope equation. In this model, a finite element method is used to solve the equation, with the resulting system of equations being solved using a conjugate gradient method. The use of this iterative solution method enables large coastal areas to be modelled relatively efficiently. One of the advantages of finite element methods is that unstructured, for example triangular, grids can be used to represent the area being modelled. Better representation of the boundaries can usually be obtained using unstructured grids, compared to the more usual structured grids used in finite difference methods. CGWAVE can be used to represent wave transformation at coastal sites, bounded by land and open sea boundaries, where shoaling, refraction, diffraction and reflection are the dominating processes. Wave heights throughout the area being modelled are predicted. In order that good resolution of the waves be obtained, the grid should contain at least five grid points per wavelength.

The wave transformation processes represented in CGWAVE are:

- shoaling.
- refraction.
- diffraction.
- sea-bed friction.
- reflection.

# 5 Wave disturbance models

## 5.1 Classification of wave disturbance models

Types of wave disturbance models fall into the same categories as those for wave transformation models described in Chapter 4. The additional feature for wave disturbance models is the representation of boundaries and the corresponding physical processes of reflections and diffraction around surfacing piercing structures. In ray models, the boundaries are represented as a series of straight lines in the appropriate locations. In finite difference models, boundaries are usually represented as a series of stepped lines following the edges of grid cells. In models based on the finite element method, very good resolution of the boundary can be achieved if unstructured (triangular) grids are used. In both finite element and finite difference models, grid cells adjacent to the boundaries are designated as boundary cells and are treated differently compared to the interior cells.

The reflection of wave energy from boundaries is usually represented through assigning appropriate boundary conditions in the model. It is important that both total reflection and partial reflection, which depends on the structure of the boundary and the incident wave, can be represented in a wave disturbance model.

In addition to reflected wave energy it is important that diffraction due to surface piercing structures is also represented in wave disturbance models. In some models, particularly ray models, the representation of external diffraction is based on the Sommerfeld solution for diffraction of light waves.

Brief descriptions of the wave disturbance models applied to the benchmark tests are given in this chapter. Further details are given in Appendix 2.

## 5.2 Models applied to the benchmark tests

### *5.2.1 Ray models*

**PORTRAY**

PORTRAY (HR Wallingford 1988) was developed at HR Wallingford and is based on a forward tracking ray method. This model can be used to predict wave conditions throughout very large coastal areas where shoaling and refraction are the dominant processes. Rays are tracked from the offshore edge of the area being modelled, in the direction of propagation, inshore. Significant wave heights, periods and directions at each grid point can be predicted using PORTRAY. Reflections and wave-current interaction, as well as energy dissipation due to sea-bed friction and wave breaking are modelled in PORTRAY. Although diffraction due to surface piercing structures is included in the model, diffraction due to variations in the bathymetry is not.

The physical processes modelled in PORTRAY are:

- shoaling.
- refraction.
- diffraction due to surface piercing structures.
- sea-bed friction.
- wave breaking.
- wave-current interaction.
- reflection.

**OUTDIF**

OUTDIF (HR Wallingford 1989) was developed at HR Wallingford and is an extended version of the HR Wallingford OUTRAY wave refraction model. OUTDIF includes additionally the effects of diffraction by a semi-infinite breakwater. Wave rays, which are tracked from an inshore point of interest to the offshore boundary, give information on the transfer of wave energy, including the effects of a semi-infinite breakwater. OUTDIF is most suited to studies where spectral wave conditions are required at inshore points, in areas which are sheltered from wave action by a structure and refraction and shoaling are dominant processes. Since OUTDIF is based on a ray tracking technique, it can be used to model very large areas efficiently.

The physical processes represented in OUTDIF are:

- shoaling.
- refraction.
- diffraction.
- directional and frequency spreading.

### *5.2.2 Finite difference/Finite element models*

**DIFFRAC**

DIFFRAC was developed at Delft Hydraulics (1992) and is based on the boundary element method. This model can be used to estimate wave conditions in sheltered coastal sites such as harbours, where the dominant physical processes are diffraction and reflections. Refraction and energy dissipation due to sea-bed friction and wave breaking are not included in DIFFRAC. Output from DIFFRAC includes wave heights at individual locations, or contour plots of wave heights.

The physical processes represented in DIFFRAC are:

- diffraction due to surface piercing structures.
- reflection.

**PORTCGS**

PORTCGS was developed at HR Wallingford (1994) and is based on the solution of the mild slope equation. In this model, a finite difference method is used with an iterative pre-conditioned conjugate gradient method to solve the resulting equations. In order that the waves be adequately resolved, the finite difference grid should contain at least eight grid points per wavelength. The model is suited to coastal areas where shoaling, refraction, diffraction and reflections are the dominant processes. Energy dissipation processes such as sea-bed friction and wave breaking are not included in the present version of the model. Wave amplitude, velocity potential and wave phase are output from PORTCGS.

PORTCGS represents the following physical processes:

- shoaling.
- refraction.
- diffraction.
- reflection.

**ARTEMIS 2.0**

ARTEMIS 2.0 was developed at LNH (1992) and is based on the solution of the mild slope equation. In this model, a finite element method is used to solve the equation, with the resulting system of equations being solved using an iterative pre-conditioned conjugate gradient-like method. In order that the waves be accurately resolved, the finite element grid should contain at least eight grid points per wavelength.The model is suited to coastal areas where shoaling, refraction, diffraction and reflections are the dominant processes. Good representation of boundaries can be obtained using the unstructured finite element grid, which makes the model particularly suited for representing harbours, where the grid can be made to fit the shape of the harbour basin. Energy dissipation processes such as seabed friction and wave breaking are not included in this model.

ARTEMIS 2.0 represents the following physical processes:

- shoaling.
- refraction.
- diffraction.
- reflection.

# 6 Benchmark tests

## 6.1 Selection of the test cases

The guidelines on the use of wave transformation and wave disturbance models, given in Chapters 2 and 3, are based on the application of different types of models to a series of test bathymetries. By undertaking these tests, the strengths and weaknesses of wave transformation and disturbance methods was identified and so the suitability of types of models to particular situations assessed. Brief descriptions of the test cases are given in this chapter. Further information describing the tests, which was supplied to the participating Consulting Engineers, is given in Appendix 3.

When selecting suitable test cases for this type of project, the following points should be considered. The test cases should be representative of the studies, likely to arise in practice, to which wave transformation and disturbance models will be applied. So that the performance of the models can be assessed, data with which the model results can be compared should be available. Ideally, field measurements should be used to provide offshore and inshore wave conditions. However, since such information does not always exist, test cases based on physical model tests may be used. Idealised test cases, to which analytic solutions exist, can be used to test a model's ability to represent certain physical processes.

When applying computational wave transformation and disturbance models to real situations, there may be some field data with which to calibrate or validate the model. Often, however, this data is either limited or of poor quality, or such data may not exist. Thus, the participants in this project were not supplied with calibration data for the tests described below. The data supplied to the participants included details of the bathymetry and the incident wave conditions. It should be noted that when using most wave transformation and disturbance models, the results obtained will depend on the user's interpretation of the problem and the available data.

## 6.2 Test A - Linear Beach

This simple example tests how well a computational model represents shoaling and refraction. The bathymetry consists of a plane slope rising from a region of constant depth, as shown in Figure 6.1. The incident wave condition consists of a single period, uni-directional wave train approaching the beach at an angle to the slope. The analytic solution to this example can be computed using Snell's law.

The wave height coefficients predicted by the computational models, at fifteen analysis points, were compared to the analytic solution. Any model which does not predict good approximations to the wave conditions for this example is likely to be a poor candidate for any field study where shoaling and refraction may occur.

## 6.3 Test B - Elliptic Shoal

The elliptic shoal on a slope is a widely used test case in the literature for wave transformation models. The bathymetry is similar to that used in the linear beach test, but there is an elliptic shoal on the slope. Physical model results exist for this example, with which computational model results can be compared. Refraction and shoaling, as well as diffraction due to the varying bathymetry, are the physical processes that occur. This can be a difficult test for models in which diffraction is not represented.

The incident wave condition consists of a monochromatic, uni-directional wave train approaching at an angle to the slope. Wave height coefficients predicted at fifteen analysis points were compared with the physical model results. The bathymetry and positions of the analysis points are shown in Figure 6.2.

## 6.4 Test C - Harbour Approach Bathymetry

This test, which is also based on a physical model, represents a bathymetry typical of a dredged harbour approach channel. The results from the physical model at ten analysis points were used for comparison with the computational models. The physical model was based on an actual harbour approach and so the test represents a realistic situation. The dredged channel results in refraction and diffraction effects. In the physical model tests, a significant amount of wave breaking was observed near the wave paddle. Depending on the channel depth, wave period and wave direction relative to the channel, waves may be reflected from the side of the channel. Often, deep channels and long period waves result in reflections, otherwise waves are transmitted across the channel.

Four different incident wave conditions were specified, these are given in Table 6.1, each represented by a uni-directional wave spectrum. The specified wave conditions cover a range of incident wave heights and periods, incident along the channel and at an angle of 25° to it. Significant wave height coefficients predicted at the ten analysis points, for each incident wave condition, were compared to the physical model results. Details of the bathymetry and the analysis points are shown in Figure 6.3.

## 6.5 Test D - Perranporth

This test is based on a site near Perranporth, which lies on the north coast of Cornwall. Wave measurements from an offshore waverider buoy were used as the incident wave conditions. The wave transformation models were used to predict the wave conditions near the position of an inshore waverider buoy. Wave measurements from this buoy were used for comparison. Wave spectra, with both directional and frequency components, for ten storms were used. At this site, refraction and shoaling will be encountered, although sea-bed friction is not likely to be significant. The bathymetry, together with the positions of the offshore and inshore waverider buoys, is shown in Figure 6.4. The storms used are given in Table 6.2.

## 6.6 Test E - South Uist

Test E is based on a site west of South Uist in the Outer Hebrides. Wave measurements from both an offshore and inshore waverider buoy are available. Wave spectra corresponding to ten storms were used as the incident wave conditions. Significant wave height coefficients predicted by the computational models at the inshore buoy were compared to the recorded data. At this site, the non-linear effects of sea-bed friction are likely to be significant, as well as refraction and shoaling of the incoming waves. The analysis point and bathymetry are shown in Figure 6.5, the incident wave conditions are summarised in Table 6.3.

## 6.7 Test F - Elliptic shoal with currents

The elliptic shoal is a widely used test case in the literature for computational wave models. The bathymetry used in this case is similar to that used in Test B, however, in this test currents are imposed on the beach. Physical model results exist for this example, with which computational model results can be compared. Refraction, shoaling and diffraction, due to the varying bathymetry and currents are the physical processes which occur. This can be a difficult test for models in which diffraction and refraction due to the effects of currents are not represented.

The two incident wave conditions consist of spectral uni-directional wave trains approaching at 90° to the beach. The incident wave conditions used are shown in Table 6.4 and for models which can be run in a spectral mode a JONSWAP spectrum was fitted to each of these wave conditions. Wave heights at fourteen analysis points were compared with the physical model results. The bathymetry and the locations of the analysis points are shown in Figure 6.6.

## 6.8 Test G - Semi-Infinite Breakwater

This simple example tests how well a wave disturbance model represents diffraction around a structure. The bathymetry consists of a bed with a constant depth of six metres and a semi-infinite breakwater which extends eastwards from the centre of the southern boundary of the grid system, as shown in Figure 6.7. The five incident wave conditions consist of single period, uni-directional waves of height 1.0m approaching the breakwater from angles either side of and including 90° to the breakwater. Wave heights in the region north of the breakwater can be calculated using the Sommerfeld solution.

The wave height coefficients predicted by the computational models at twelve analysis points were compared to the analytic solution. Any model which does not predict good approximations to the wave conditions for this example is likely to be a poor candidate for modelling situations where diffraction around structures is important.

## 6.9 Test H - Idealised Harbour Entrance

This test, which is based on a random wave physical model, represents a typical harbour entrance. The bathymetry for this test case is shown in Figure 6.8 and is designed to test the ability of computational models to represent the effects of reflections off structures, diffraction around structures, refraction and shoaling as well as energy dissipation due to wave breaking and bed friction. Results from the physical model exist, with which the computational model results can be compared.

The incident wave conditions used are shown in Table 6.5 and for models which can be run in a spectral mode a JONSWAP spectrum was fitted to each of these wave conditions. Wave heights predicted at fourteen analysis points both inside and outside the harbour, the locations of which are shown in Figure 6.8, were compared with the physical model results.

## 6.10 Test I - Pittenweem Harbour

This case is based on Pittenweem Harbour, which is located on the east coast of Scotland in the Firth of Forth. Wave conditions are known at a point (Beacon Rock) near the harbour entrance from a previous physical model. Two incident wave conditions were used as input to the computational wave models tested and the resulting predicted wave conditions at nine analysis points in the harbour compared with results from the physical model. The incident wave conditions used are shown in Table 6.6 and for models which can be run in a spectral mode a JONSWAP spectrum was fitted to each of the incident wave conditions. At this site refraction, shoaling, diffraction and reflections off harbour boundaries will be encountered as well as energy dissipation due to wave breaking and bed friction. The bathymetry, together with the locations of the analysis points and the location of Beacon Rock, is shown in Figure 6.9.

## 6.11 Test J - Aberdeen Harbour Entrance

This case is based on Aberdeen Harbour, which is located on the east coast of Scotland. Results from a random wave physical model are available and incident wave conditions are known at two paddles, the locations of which are shown in Figure 6.10. Two of the incident wave conditions at each paddle were used as input to the computational wave models tested. The corresponding wave conditions predicted at seven analysis points in the harbour entrance were compared with results from the physical model tests. The incident wave conditions used are shown in Table 6.7 and for models which can be run in a spectral mode a JONSWAP spectrum was fitted to each of these incident wave conditions. As with Pittenweem Harbour, the effects of shoaling, refraction, diffraction and reflections off harbour boundaries will be encountered as well as energy dissipation due to wave breaking and bed friction. However, in this case both full and partial reflections from harbour boundaries occur. The bathymetry as well as the locations of the analysis points and the wave paddles are shown in Figure 6.10.

# 7 Benchmark test results

## 7.1 Introduction

In this chapter the results from the application of computational models to the benchmark tests are presented and discussed. For each test case, the participant was supplied with the summary given in Appendix 3, bathymetric data and where appropriate, information on the wave spectrum. Exactly how the data was interpreted and used depended on the participant and the model applied. The participants were asked to supply wave height coefficients or wave heights at the appropriate analysis points. These values are shown in the appropriate tables. The wave heights and coefficients are quoted to the same degree of accuracy as the field, physical model or analytic data. However, the percentage errors were computed using the results supplied by the participants. Most participants did not run their models for all of the test cases. This was either due to time and financial constraints or because their model was not suited to a particular test. Lists of the participants, the wave transformation and wave disturbance models applied and the test cases used are shown below:

**Test programme for wave transformation benchmark tests**

| Participant | Model | Test case | | | | | |
|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | F |
| ABP Research and Consultancy | MULTIGRID | | ✓ | ✓ | ✓ | ✓ | |
| Acer Consultants Ltd | OUTRAY | | | | ✓ | | |
| Applied Wave Research | LINDAL | ✓ | ✓ | | ✓ | ✓ | |
| Binnie & Partners | WC2D | | | ✓ | ✓ | ✓ | |
| Halcrow | ARMADA | ✓ | ✓ | | | | |
| HR Wallingford | OUTRAY | ✓ | ✓ | ✓ | ✓ | ✓ | |
| | OUTURAY | | | | | | ✓ |
| | PORTRAY | ✓ | ✓ | ✓ | ✓ | ✓ | |
| | PARAB | ✓ | ✓ | ✓ | ✓ | ✓ | |
| Kirk McClure Morton | CGWAVE | | ✓ | | | | |
| | OUTRAY | | | | | ✓ | |
| Laboratoire d'Hydraulique de France | ARTEMIS | ✓ | ✓ | | | | |
| Orcina | ORCAWAVE | ✓ | ✓ | ✓ | ✓ | ✓ | |
| Posford Duvivier | ENDEC | ✓ | ✓ | | | | |
| | HISWA | ✓ | ✓ | | ✓ | ✓ | |
| | REFRAC | ✓ | ✓ | | | | |
| Scott Wilson Kirkpatrick | MULTIGRID | ✓ | ✓ | ✓ | | | |
| | W-RAY | | | ✓ | ✓ | | |
| WS Atkins* | MIKE21NSW | | | | ✓ | | |
| | MIKE21PMS | ✓ | ✓ | ✓ | | | |

✓ Indicates that this test was carried out.
* The Danish Hydraulic Institute ran the models MIKE21 NSW and MIKE21 PMS. In the UK WS Atkins act as agents for and use these models, which were developed at the Danish Hydraulic Institute.

**Test programme for wave disturbance benchmark tests**

| Participant | Model | Test case | | | |
|---|---|---|---|---|---|
| | | G | H | I | J |
| Sir Alexander Gibb & Partners | PORTRAY | ✓ | ✓ | | |
| HR Wallingford | OUTDIF | ✓ | | | |
| | PORTRAY | ✓ | ✓ | ✓ | ✓ |
| | PORTCG | ✓ | ✓ | ✓ | |
| Laboratoire d'Hydraulique de France | ARTEMIS | ✓ | ✓ | | |
| Posford Duvivier | ENDEC | ✓ | | | |

✓ Indicates that this test was carried out.

The results for each test case are discussed in the following sections. For each test case, the computed results are presented, together with the percentage errors as compared to the field, physical model or analytic data. Predicted wave heights and coefficients which lie within 10% of the measured data are generally considered to be good approximations.

## 7.2 Test A - Linear Beach

The wave height coefficients at the fifteen analysis points are shown in Table 7.1a, the percentage errors in the wave height coefficients are shown in Table 7.1b. This is a relatively straightforward test in which a monochromatic wave is modified by the physical processes of shoaling and refraction due to the bathymetry. The analytic solution can be computed at each of the analysis points using Snell's law and is also shown in Table 7.1a. The results given in the tables show that all the models applied accurately represent a monochromatic wave propagating over a linear slope.

### *7.2.1 Wave ray tracking refraction models*

The results in Table 7.1b indicate that models based on both forward and back tracking ray methods represent shoaling and refraction over a linear beach accurately. The wave height coefficients predicted at the analysis points are within 7% of the analytic solution. OUTRAY, which is a spectral and back tracking model, and REFRAC tend to over-estimate, while PORTRAY underestimates the wave height coefficients.

### 7.2.2 Finite difference refraction models

The wave height coefficients predicted using LINDAL are within 1% of the analytic solution. ENDEC, which is a spectral model, over-estimates the wave heights by up to 5% of the analytic solution, with the errors increasing as the wave propagates forwards. Due to directional spreading, HISWA tends to underestimate the wave height coefficients, again the errors increase as the wave propagates forwards. However, most of the predicted wave heights lie within 10% of the analytic solution.

### 7.2.3 Refraction/diffraction finite difference models

Both ARMADA and MULTIGRID are based on a multigrid solution of the mild slope equation. As the results in Table 7.1b show, both models accurately represent shoaling and refraction over a linear beach, with the predicted wave heights lying within 2% of the analytic solution. ARTEMIS uses a finite element method to solve the mild slope equation and the predicted wave heights lie within 10% of the analytic solution. The models based on the parabolic approximation to the mild slope equation, M21PMS and PARAB, also give good predictions of the wave height coefficients. PARAB over-estimates the wave heights by up to 4% of the analytic solution, whereas M21PMS underestimates the wave heights by up to 16%. In the latter case, the errors tend to increase as the wave propagates forwards.

### 7.2.4 General comments

The results for this test case indicate that each type of model tested represents shoaling and refraction over a linear slope. Most of the predicted wave height coefficients lie within 10% of the analytic solution. As has already been noted, this is a relatively simple test case and any model not performing well would be a poor candidate for more realistic studies.

## 7.3 Test B - Elliptic Shoal

The bathymetry for the second test case, Test B, is similar to that used in Test A except for an elliptic shoal on the linear slope. As a result, the monochromatic incident wave will be subject to the physical processes of shoaling, refraction and diffraction as it propagates through the domain. The wave height coefficients predicted at the fifteen analysis points are shown in Table 7.2a. The percentage errors in the computed coefficients, compared to the physical model data quoted in the literature, are presented in Table 7.2b. As the results indicate, this is a more difficult test case for wave transformation models, than Test A.

### 7.3.1 Wave ray tracking refraction models

Although the back tracking model, OUTRAY, is usually used to represent wave spectra, the results in Table 7.2b indicate that it has represented the propagation of the monochromatic wave in Test B fairly accurately. The percentage errors in the wave height coefficients at over half the analysis points lie within 10% of the physical model data. The results deteriorate as the wave propagates over and away from the shoal, where the diffraction effects are more significant. Forward tracking ray models are not expected to perform well, due to the focusing effects, resulting in caustics, as waves propagate over the shoal. This results in over-estimation of the wave heights behind the shoal, since wave energy is not transferred laterally as diffraction is not represented. This is illustrated by the results for PORTRAY and REFRAC quoted in Table 7.2b, although ORCAWAVE gives reasonable results at most of the analysis points.

### *7.3.2 Finite difference refraction models*
Similarly to the models based on ray tracking methods, ENDEC, HISWA and, in particular, LINDAL give good predictions of the wave height coefficients over and immediately behind the shoal. However, as the wave propagates away from the shoal, the results deteriorate as the diffraction effects become relatively more important. Over half the wave height coefficients predicted using LINDAL lie within 10% of the physical model data.

### *7.3.3 Refraction/diffraction models*
Generally, the models which represent both refraction and diffraction give good predictions of the wave height coefficients. In particular, most of the coefficients predicted using the models based on the multigrid solution to the mild slope equation (using finite difference methods) lie within 20% of the physical model data. CGWAVE, which is also based on the mild slope equation, does not give as accurate predictions, but the version used in this project is designed for closed bay areas. Most of the wave height coefficients predicted using M21PMS and PARAB lie within 20% of the physical model data. However, there appears to be a deterioration in the results as the wave propagates forwards away from the shoal.

### *7.3.4 General comments*
For Test B better representation is generally obtained when using a refraction/diffraction model. However, although Test B involves the physical process of diffraction, as well as shoaling and refraction, the results in Tables 7.2a and 7.2b indicate that refraction only models give fairly accurate predictions of the wave conditions. In particular, the refraction only models show a marked deterioration in accuracy as the wave propagates away from the shoal. Such a deterioration is also encountered in models based on the parabolic approximation to the mild slope equation.

## 7.4 Test C - Harbour Approach Bathymetry
The third test case, Test C, is based on a physical model which in turn was based on a real harbour approach bathymetry. Four incident wave conditions were specified and the corresponding wave height coefficients at ten analysis points are shown in Tables 7.3a - 7.6a. The percentage errors in the wave height coefficients, as compared to the physical model data, are shown in Tables 7.3b - 7.6b. In Test C diffraction due to the channel, as well as shoaling and refraction, are the important physical processes. In addition, energy dissipation due to sea-bed friction and, particularly, wave breaking are important. Each incident wave condition comprised several frequency components.

### *7.4.1 Wave ray tracking refraction models*
Four refraction models based on ray tracking techniques were applied to Test C - two back tracking models and two forward tracking model. As the results in Tables 7.3a - 7.6b show, none of these refraction models represented wave propagation over the channel accurately.

The back tracking models, OUTRAY and W-RAY, tend to over-estimate the wave height coefficients, especially when the incident waves have long periods, as in cases 1 and 3. OUTRAY, in particular, predicts the wave heights more accurately for shorter period incident waves, with most of the coefficients for case 4 lying within 20% of the physical data. Neither of the models tested include energy dissipation due to sea-bed friction and OUTRAY does not model wave breaking. This partly accounts for the over-estimation of the wave heights.

Forward tracking models are not expected to perform well near channels. Thus, even though the forward tracking ray models tested, PORTRAY and ORCAWAVE, represent energy dissipation due to sea-bed friction and wave breaking, there is not a significant improvement in the accuracy of the predicted wave height coefficients. Better representation of shorter period incident waves, compared with longer period waves, is illustrated by the smaller percentage errors for cases 2 and 4. In the latter case, over half the coefficients predicted using PORTRAY lie within 20% of the physical model data. Reflections from the sides of the channel lead to caustics, which in turn lead to areas with very high predicted wave heights and areas where the predictions are very low. Since PORTRAY and ORCAWAVE do not represent diffraction, there is no lateral spreading of the wave energy due to diffraction at the channel.

### *7.4.2 Refraction/diffraction models*

Five models which represent diffraction, as well as shoaling and refraction, were applied to Test C. As the results in Tables 7.3a - 7.6b show, these models also represent shorter period incident waves better than waves with longer periods. The results for the models based on the mild slope equation, MULTIGRID, illustrate how different interpretations of the input data can lead to different results. Generally, the models based on the parabolic approximation to the mild slope equation lead to better predictions of the wave height coefficients compared to the other diffraction models applied to Test C.

For case 1, a longer period wave incident along the channel, the spectral version of M21PMS gives the most accurate results, with most of the coefficients lying within 20% of the physical model data. The parabolic models, M21PMS and PARAB, tend to underestimate, while MULTIGRID and WC2D over-estimate the wave heights. Although, each model over-estimates the wave height at point 10, which is near to the entrance to the outer harbour. The shorter period wave specified in case 2 is also incident along the channel. All models give better predictions of the wave height coefficients, compared to case 1. In particular, half of the predictions made by the monochromatic version of M21PMS are within 10% of the physical model data. Each model tends to over-estimate the wave heights, particularly at points 3 and 10 which lie in the channel.

The longer period wave incident at an angle of 25° to the channel leads to poorer results from the refraction/diffraction models. While half, or more, of the wave heights predicted by the parabolic models lie within 20% of the physical data, the remaining models over-estimate the coefficients considerably. Again, none of the models accurately predict the wave heights in the channel, particularly near the entrance to the harbour. The results for case 4, shown in Table 7.6b, indicate that it was the longer period, rather than the angle of incidence which led to the poor results for case 3. Most of the wave heights predicted by M21PMS and WC2D lie within 20% of the physical data, as do approximately half of the wave heights predicted by MULTIGRID and PARAB. Once again, none of the models accurately predict the wave height coefficient at point 10.

### 7.4.3 *General comments*

Due to the diffraction effects of the harbour approach channel, the refraction/diffraction models generally give a better representation of the wave conditions than the refraction only models. The models tested seem to be able to represent shorter period waves, cases 2 and 4, better than longer period waves, cases 1 and 3. The increased angle of incidence of the incoming waves in cases 3 and 4 does not seem to lead to a deterioration in the performance of the parabolic models. Generally, the models tend to over-estimate the wave heights, particularly at the points which lie in the channel.

## 7.5 Test D - Perranporth

Measured wave data at both an offshore and inshore location are available for the fourth test case, Test D. At Perranporth the coastline is relatively straight and the nearshore depth contours are approximately parallel to the coastline. Thus, shoaling and refraction are the dominant physical processes. The sea-bed appears to be smooth and sandy, so energy dissipation due to sea-bed friction is not likely to be significant. In this project, ten offshore storms were selected as the incident wave conditions and the participants supplied wave height coefficients at a point near the location of the inshore buoy. These are shown in Table 7.7a and the percentage errors in the wave height coefficients are given in Table 7.7b. The incident conditions cover a range of wave periods and heights. Since the waveriders did not record wave direction, the measurements were augmented with directional spreading derived from mathematical modelling.

### 7.5.1 *Wave ray tracking refraction models*

As the results in Table 7.7b show, the ray tracking models give good predictions of the inshore wave conditions. Most of the predictions lie within 20% of the recorded wave height coefficients, with over half of those predicted using OUTRAY lying within 10% of the waverider data. The forward tracking ray models, PORTRAY and ORCAWAVE, tend to over-estimate the wave heights and seem to give less accurate results for incident waves with higher periods, for example storm 3. The performance of the back tracking ray models does not seem to be similarly affected.

### 7.5.2 *Finite difference refraction models*

The finite difference refraction models applied to Test D also give accurate predictions of the wave height coefficients. Most of the predictions lie within 20% of the measured values, with 80% of those predicted using LINDAL lying within 10% of the measured data.

### 7.5.3 *Refraction/diffraction models*

Three refraction/diffraction models were applied to Test D - MULTIGRID, PARAB and WC2D. These models also give good predictions of the wave height coefficients, with no increase in accuracy as compared to the refraction only models. Most of the predictions lie within 20% of the measured data. Storm 3, with the longest wave period, did not present any problems to the models. However, both MULTIGRID and PARAB over-estimated, by 50%, the inshore wave height for storm 4. The mean wave direction for this storm is at a relatively large angle to the x axis (which is assumed to be in the main forward propagation direction).

### 7.5.4 General comments

The results quoted in Tables 7.7a and 7.7b show that all the models applied to Test D gave good predictions of the inshore wave heights. Since shoaling and refraction are the dominant processes, the refraction only models predict the inshore wave heights as accurately as the refraction/diffraction models. The predictions made by MULTIGRID and PARAB for storm 4 were 50% larger than the recorded data. For PARAB, particularly, this is mainly because the incident wave angle to the x axis is large.

## 7.6 Test E - South Uist

Measured wave data is also available for the fifth test case, Test E. Recordings at both offshore and inshore locations were made by waverider buoys near South Uist in the Outer Hebrides. Here the coastline is relatively straight and open to the sea, and waves approaching the coast are subject to shoaling and refraction. Hydraulic studies carried out at South Uist indicated that energy dissipation due to sea-bed friction is significant. The Monarch Islands, to the north-west of South Uist, provide some shelter from waves approaching from northerly directions. Although these Islands are not included in the grid supplied for Test E, some account of this shelter can be made by selecting suitable offshore boundaries. The wave height coefficients near the inshore waverider location predicted by the participants are shown in Table 7.8a and the percentage errors in the coefficients, as compared to the waverider data, are shown in Table 7.8b. Ten measured storms were used as the incident conditions for Test E, each storm having a specific tidal level. Since the waveriders did not record wave direction, the measurements were augmented with directional spreading derived from mathematical modelling.

### 7.6.1 Wave ray tracking refraction models

The results in Table 7.8b indicate that Test E is a difficult problem for wave transformation models based on ray tracking methods. Most of the wave height coefficients predicted by OUTRAY over-estimate the recorded data considerably, with less than a third of the predicted wave heights lying within 20% of the recorded wave heights. Although the forward tracking ray model, PORTRAY, represents dissipation due to sea-bed friction, the errors in most of the predicted wave heights are greater than 20%. For most of the storms, PORTRAY underestimates the wave heights, suggesting that the friction coefficient used was too large. The worst prediction made is for the storm with the longest peak period, storm 6.

ORCAWAVE was run with two different values for the friction coefficient, 0.05 and 0.3. The results obtained for both values are given in Tables 7.8a and 7.8b. The smaller friction coefficient leads to over-estimation of the wave heights. The higher friction coefficient leads to more accurate results, although only half the values lie within 20% of the measured data.

### 7.6.2 Finite difference refraction models

Although HISWA, which is based on the solution of the energy balance equation, represents dissipation due to sea-bed friction, it over-estimates the wave height coefficients. Of the refraction models applied to Test E, LINDAL gives the most accurate predictions, with over half of the wave height coefficients lying within 20% of the recorded data.

### *7.6.3 Refraction/diffraction models*

Test E also proved to be a difficult test case for the refraction/diffraction models, all of which represent dissipation due to sea-bed friction. Approximately half of the predicted wave height coefficients computed using MULTIGRID and PARAB lie within 20% of the measured values. The worst predictions made by the parabolic model, PARAB, are for those storms in which the mean wave direction was at a large angle of incidence to the x-axis, storms 6, 7 and 10. WC2D tends to over-estimate the wave heights considerably.

### *7.6.4 General comments*

As the results show, Test E proved to be a difficult test case for all the wave transformation models applied. The finite difference refraction model, LINDAL, predicted the wave heights most accurately, while the models which do not include dissipation due to sea-bed friction over-estimated the wave heights considerably. For most models, the choice of a suitable friction coefficient without measured data available for calibration is extremely difficult.

## 7.7 Test F - Elliptic shoal with currents

This test is based on a physical model for which data is available at the analysis points shown on Figure 6.6. Only the back tracking ray model OUTURAY was applied to this test which includes the effect of current refraction on the propagating waves. The computed wave height coefficients and percentage errors are presented in Tables 7.9 and 7.10 for the two incident wave conditions specified in Table 6.4.

From Tables 7.9 and 7.10 it can be seen that generally OUTURAY over-estimates the wave heights at most of the analysis points. This is a more difficult test case than, for example, Test B and this is reflected in the accuracy of the computational results, particularly for Case 2 where most of the predicted significant wave heights are 50% larger than the measured values.

## 7.8 Test G - Semi-Infinite Breakwater

Test G is based on a single semi-infinite breakwater on a flat bed, with incident monochromatic waves from five directions. The wave height coefficients and percentage errors at the analysis points shown on Figure 6.7 are presented in Tables 7.11 - 7.15 for incident wave directions 120°N, 150°N, 180°N, 210°N and 240°N respectively. The wave height coefficients can be computed from the Sommerfeld solution for the diffraction of light. These are also presented in the tables.

From these tables it can be seen that each model applied to this test gives reasonable approximations to the wave height coefficients at the twelve analysis points. The errors in the coefficients computed using PORTRAY (HR) are less than 10% for all five incident wave conditions. This is expected as diffraction within PORTRAY is based on the Sommerfeld solution. However, the results obtained with PORTRAY (Gibb) illustrate once again how the user of the model has a considerable impact on the results obtained.

DIFFRAC, which is based on the Helmholtz equation, also gives good results, except for the monochromatic wave incident from 120°N when DIFFRAC underestimates the wave height coefficients by up to 40%. Both PORTCGS and ARTEMIS are based on the mild slope equation. The results in the tables indicate that both models represent diffraction due to surface piercing structures, but better results are obtained for incident wave directions close to perpendicular to the breakwater.

### 7.9 Test H - Idealised Harbour Entrance

This test, which is based on a random wave physical model, represents a typical harbour entrance. The significant wave heights measured during the physical model tests at the analysis points shown on Figure 6.8 are shown in Tables 7.16 and 7.17, together with the computed significant wave heights and the percentage errors in the computed values.

From the tables it can be seen that PORTRAY gives good results at all the analysis points. The computed significant wave heights are within 20% of the recorded values at all the analysis points except points 3 and 8 where PORTRAY over-estimates the wave height. For this test, the models based on the mild slope equation, PORTCGS and ARTEMIS, give less accurate results particularly for the longer period incident wave, Case 2.

### 7.10 Test I - Pittenweem Harbour

Test I is based on a physical model test of Pittenweem Harbour. The significant wave heights recorded during the physical model test are shown in Tables 7.18 and 7.19 together with the results computed using PORTRAY and PORTCGS. This test, being based on a real harbour, is a more stringent test case for the computational models, which is reflected in the results. Neither of the models give good results, compared to the physical model results, particularly in the inner basin where both models underestimate the wave activity considerably.

### 7.11 Test J - Aberdeen Harbour Entrance

Only PORTRAY was applied to the final test case, which is also based on a physical model of a real harbour. The recorded and predicted significant wave heights at the analysis points shown on Figure 6.10 are presented in Tables 7.20 - 7.23. From these tables it can be seen that for waves approaching the harbour from 85°N, PORTRAY underestimates the wave activity in the harbour, but for the more northerly incident wave, PORTRAY over-estimates the significant wave heights considerably. This test, as with Test I, is a stringent tests case being based on a real harbour.

## *8 Conclusions*

### 8.1 Summary

The aim of this study was to produce guidelines for the selection of computational models for application to coastal and estuarial engineering projects. This report covers guidelines for wave transformation and wave disturbance models. The guidelines for flow and sediment models are reported separately.

In the first stage of this project, a literature survey of wave transformation and wave disturbance models was carried out to identify which models are regularly used in the UK. Consulting Engineers and Universities were then asked to take part in the second part of this project by applying their wave transformation and wave disturbance models to a number of benchmark bathymetries. The models tested are representative of several categories of wave models, ranging from simple refraction models to finite element models of wave disturbance. It should be noted that wave disturbance models are presently used less frequently by non-specialists than wave transformation models. This is partly because there are fewer wave disturbance models available and partly because they are generally more difficult to apply since they must represent solid boundaries and the associated physical processes. The results from the benchmark tests showed few surprises, with no one model appearing to be "best" overall when considering both accuracy and efficiency. The accuracy achieved by the computational models is very dependent on both the interpretation of the problem and how the model is actually applied by the user.

The guidelines for the selection of wave transformation and wave disturbance models are given in Chapters 2 and 3, respectively. As was indicated in these chapters, several factors must be considered when selecting models for coastal projects. These factors range from the important physical processes occurring at the site, to time and financial constraints on the project. The guidelines given in this report concentrate on physical processes. Firstly, each model type is considered and a summary of the types of application to which they are most suited is given. Secondly, a series of coastal sites typical of coastal and estuarial projects likely to arise in the UK is given. For each site, categories of appropriate wave transformation or wave disturbance models are listed, in a possible order of preference.

## 8.2 Accuracy of computational wave models

A number of engineers involved with coastal engineering projects have indicated that predicted wave conditions within 10% of the true values are desirable. As the benchmark tests have showed, this level of accuracy cannot, at present, be guaranteed unless the area being modelled is particularly straightforward or unless there is good calibration and validation data available. It is also important to note that the results obtained from the computational models can only be, at best, as accurate as the input data, which includes bathymetric information, harbour layout information and incident wave conditions.

When applied intelligently, wave transformation models can be considered to give a good representation of the transformation of offshore waves to the nearshore zone. Existing wave disturbance models are best used as a comparative tool to compare a number of alternative layouts or to assess the effect of a development on the neighbouring area. Computational wave models can only be used confidently to provide design wave data when there is good quality calibration data available.

## 8.3 Future wave transformation and wave disturbance models

Until fairly recently, most commercially used wave transformation models were based on ray tracking techniques due to their computational efficiency. With the advent of more powerful computers with both increased memory and processing capability, models based on the solution of governing equations are becoming more popular. These models generally represent more of the physical processes affecting the propagation of waves. Models such as those based on the solution of the mild slope equation can also be used to represent wave disturbance, particularly if a finite element method is used to solve the equations when good resolution of the boundaries can be achieved when using unstructured grids. Current areas of research are directed towards developing efficient solution methods for such equations.

At present, energy dissipation effects due to seabed friction and particularly wave breaking are not well represented in wave transformation and wave disturbance models. This is particularly the case when wave conditions are represented spectrally.

Most of the models regularly used at present are based on linear wave theory. The development of non-linear wave models is currently a very active area of research. Non-linear models based, for example, on the solution of the Boussinesq equations can be computationally very expensive. The standard versions of the equations are only applicable in a limited range of water depth and so equations which can be applied in a range of water depths are being developed.

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# Tables

## *Table 6.1 Incident wave conditions for Test C*

| Case number | Case name | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Direction (°) |
|---|---|---|---|---|
| 1 | storm 0° | 4.3 | 8.6 | 0 |
| 2 | typical 0° | 1.9 | 6.0 | 0 |
| 3 | storm 25° | 6.0 | 10.0 | 25 |
| 4 | typical 25° | 3.2 | 7.5 | 25 |

## *Table 6.2 Storm conditions recorded by offshore waverider buoy at Perranporth*

| Storm number | Offshore wave conditions (site 2) | | |
|---|---|---|---|
| | Significant wave height, $H_s$ (m) | Zero crossing period, $T_m$ (s) | Predicted mean wave direction (°N) |
| 1 | 4.0 | 7.0 | 259 |
| 2 | 3.2 | 7.3 | 253 |
| 3 | 7.1 | 10.5 | 253 |
| 4 | 3.2 | 6.1 | 11 |
| 5 | 3.4 | 8.0 | 255 |
| 6 | 4.2 | 7.3 | 271 |
| 7 | 4.5 | 7.2 | 273 |
| 8 | 5.8 | 7.7 | 343 |
| 9 | 3.6 | 6.7 | 276 |
| 10 | 3.4 | 6.2 | 279 |

## Table 6.3 Storm conditions recorded by offshore waverider buoy to the west of South Uist

| Storm number | Offshore wave conditions | | |
|---|---|---|---|
| | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Predicted mean wave direction (°N) |
| 1 | 3.5 | 9.0 | 240 |
| 2 | 6.0 | 10.0 | 255 |
| 3 | 4.0 | 10.0 | 260 |
| 4 | 7.5 | 14.1 | 280 |
| 5 | 5.7 | 11.7 | 265 |
| 6 | 3.7 | 14.5 | 225 |
| 7 | 4.8 | 9.5 | 350 |
| 8 | 4.8 | 13.6 | 265 |
| 9 | 5.3 | 11.7 | 265 |
| 10 | 3.5 | 8.3 | 330 |

## *Table 6.4 Incident wave conditions for Test F*

| Case number | Significant wave height $H_s$(m) | Peak period $T_p$(s) | Direction (°N) |
|---|---|---|---|
| 1 | 2.5 | 5.0 | 180 |
| 2 | 3.1 | 6.0 | 180 |

## *Table 6.5 Wave conditions at the paddle for Test H*

| Case number | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Direction (°N) |
|---|---|---|---|
| 1 | 2.5 | 6.6 | 180 |
| 2 | 4.2 | 7.3 | 180 |

## *Table 6.6 Wave conditions at Beacon Rock used as input conditions for Test I*

| Case number | Water level (m above OD) | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Mean period, $T_m$ (s) | Direction (°N) |
|---|---|---|---|---|---|
| 1 | +2.1 | 2.8 | 10.4 | 6.5 | 139 |
| 2 | +1.2 | 1.5 | 5.3 | 3.8 | 186 |

## *Table 6.7 Incident wave conditions at the paddles for Test J*

| Case number | Water level (m above CD) | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Mean period, $T_m$ (s) | Direction (°N) |
|---|---|---|---|---|---|
| 1 | 3.4 | 4.5 | 9.2 | 7.0 | 85 |
| 2 | 3.9 | 5.4 | 10.1 | 7.6 | 85 |
| 3 | 3.5 | 2.6 | 8.4 | 6.0 | 56 |
| 4 | 3.8 | 4.2 | 11.2 | 7.6 | 56 |

## Table 7.1a Wave height coefficients for Test A - Linear Beach

| Analysis point | Analytic solution | Refraction models | | | | | | | Refraction/diffraction models | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | PORTRAY (HR) | REFRAC (PD) | ENDEC (PD) | HISWA (PD) | LINDAL (AWR) | ORCA WAVE (ORCINA) | ARMADA (HALCROW) | M21PMS (WSA) | MULTIGRID (SWK) | PARAB (HR) | ARTEMIS (EDF) |
| 1 | 0.91 | 0.92 | 0.88 | 0.92 | 0.94 | 0.78 | 0.92 | 0.91 | 0.91 | 0.76 | 0.91 | 0.92 | 0.98 |
| 2 | 0.91 | 0.95 | 0.88 | 0.93 | 0.96 | 0.80 | 0.92 | 0.92 | 0.91 | 0.77 | 0.91 | 0.93 | 1.00 |
| 3 | 0.92 | 0.96 | 0.88 | 0.93 | 0.96 | 0.82 | 0.93 | 0.92 | 0.91 | 0.78 | 0.93 | 0.95 | 0.94 |
| 4 | 0.92 | 0.94 | 0.91 | 0.94 | 0.97 | 0.84 | 0.93 | 0.93 | 0.93 | 0.79 | 0.93 | 0.95 | 0.96 |
| 5 | 0.93 | 0.93 | 0.90 | 0.94 | 0.97 | 0.85 | 0.93 | 0.93 | 0.93 | 0.80 | 0.93 | 0.96 | 0.96 |
| 6 | 0.93 | 0.98 | 0.91 | 0.95 | 0.97 | 0.85 | 0.94 | 0.94 | 0.93 | 0.81 | 0.95 | 0.97 | 0.95 |
| 7 | 0.94 | 0.95 | 0.91 | 0.95 | 0.97 | 0.87 | 0.95 | 0.94 | 0.95 | 0.82 | 0.95 | 0.97 | 0.98 |
| 8 | 0.94 | 0.95 | 0.91 | 0.96 | 0.97 | 0.88 | 0.95 | 0.95 | 0.95 | 0.83 | 0.95 | 0.98 | 0.95 |
| 9 | 0.95 | 1.00 | 0.92 | 0.96 | 0.97 | 0.89 | 0.96 | 0.96 | 0.95 | 0.84 | 0.95 | 0.98 | 0.99 |
| 10 | 0.95 | 0.96 | 0.93 | 0.97 | 0.97 | 0.91 | 0.96 | 0.96 | 0.97 | 0.85 | 0.97 | 0.98 | 1.00 |
| 11 | 0.96 | 0.97 | 0.93 | 0.97 | 0.98 | 0.91 | 0.97 | 0.97 | 0.97 | 0.85 | 0.97 | 0.99 | 0.99 |
| 12 | 0.96 | 1.03 | 0.92 | 0.98 | 0.98 | 0.92 | 0.97 | 0.97 | 0.97 | 0.86 | 0.97 | 0.99 | 1.02 |
| 13 | 0.97 | 0.97 | 0.93 | 0.98 | 0.98 | 0.94 | 0.97 | 0.97 | 0.97 | 0.87 | 0.97 | 0.99 | 0.92 |
| 14 | 0.97 | 0.97 | 0.94 | 0.99 | 0.98 | 0.94 | 0.98 | 0.97 | 0.98 | 0.87 | 0.98 | 0.99 | 1.01 |
| 15 | 0.97 | 0.99 | 0.95 | 0.99 | 0.98 | 0.95 | 0.98 | 0.98 | 0.98 | 0.88 | 0.98 | 0.99 | 1.01 |

Key:
HR : HR Wallingford
AWR : Applied Wave Research
SWK : Scott Wilson Kirkpatrick
PD : Posford Duvivier
WSA : WS Atkins
LNH : L'aboratoire d'Hydraulique de France

## Table 7.1b Percentage errors in wave height coefficients for Test A - Linear Beach

| Analysis point | Refraction models | | | | | | | Refraction/diffraction models | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | PORTRAY (HR) | REFRAC (PD) | ENDEC (PD) | HISWA (PD) | LINDAL (AWR) | ORCA WAVE (ORCINA) | ARMADA (HALCROW) | M21PMS (WSA) | MULTIGRID (SWK) | PARAB (HR) | ARTEMIS (LNH) |
| 1 | +1 | -3 | +1 | +3 | -14 | +1 | 0 | 0 | -16 | 0 | +1 | +8 |
| 2 | +4 | -3 | +2 | +5 | -12 | +1 | +1 | 0 | -15 | 0 | +2 | +10 |
| 3 | +4 | -4 | +1 | +4 | -11 | +1 | 0 | -1 | -15 | +1 | +3 | +2 |
| 4 | +2 | -1 | +2 | +5 | -9 | +1 | +1 | +1 | -14 | +1 | +3 | +4 |
| 5 | 0 | -3 | +1 | +4 | -9 | 0 | 0 | 0 | -14 | 0 | +3 | +3 |
| 6 | +5 | -2 | +2 | +4 | -9 | +1 | +1 | 0 | -13 | +2 | +4 | +2 |
| 7 | +1 | -3 | +1 | +3 | -7 | +1 | 0 | +1 | -13 | +1 | +3 | +4 |
| 8 | +1 | -3 | +2 | +3 | -6 | +1 | +1 | +1 | -12 | +1 | +4 | +1 |
| 9 | +5 | -3 | +1 | +2 | -6 | +1 | +1 | 0 | -12 | 0 | +3 | +4 |
| 10 | +1 | -2 | +2 | +2 | -4 | +1 | +1 | +2 | -11 | +2 | +3 | +5 |
| 11 | +1 | -3 | +1 | +2 | -5 | +1 | +1 | +1 | -11 | +1 | +3 | +3 |
| 12 | +7 | -4 | +2 | +2 | -4 | +1 | +1 | +1 | -10 | +1 | +3 | +6 |
| 13 | 0 | -4 | +1 | +1 | -3 | 0 | 0 | 0 | -10 | 0 | +2 | -5 |
| 14 | 0 | -3 | +2 | +1 | -3 | +1 | 0 | +1 | -10 | +1 | +2 | +4 |
| 15 | +2 | -2 | +2 | +1 | -2 | +1 | +1 | +1 | -9 | +1 | +2 | +4 |

## Table 7.2a Wave height coefficients for Test B - Elliptic Shoal

| Analysis point | Physical Model | Refraction models | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | PORTRAY (HR) | REFRAC (PD) | ENDEC (PD) | HISWA (PD) | LINDAL (AWR) | ORCAWAVE (ORCINA) |
| 1 | 0.30 | 1.53 | 0.77 | 0.70 | 0.84 | 0.87 | 0.46 | 0.36 |
| 2 | 1.65 | 1.48 | 1.06 | 2.37 | 0.84 | 0.86 | 1.21 | 0.83 |
| 3 | 0.90 | 0.49 | 1.34 | 1.90 | 0.83 | 0.85 | 0.52 | 0.43 |
| 4 | 0.40 | 0.48 | 0.28 | 0.69 | 0.83 | 0.89 | 0.50 | 0.46 |
| 5 | 1.85 | 1.62 | 2.19 | 2.18 | 0.81 | 0.88 | 1.94 | 1.16 |
| 6 | 0.68 | 0.53 | 0.41 | 0.78 | 0.81 | 0.86 | 0.56 | 0.56 |
| 7 | 0.65 | 0.60 | 0.13 | 0.85 | 0.80 | 0.91 | 0.60 | 0.61 |
| 8 | 2.00 | 1.13 | 2.97 | 1.34 | 0.80 | 0.94 | 1.85 | 1.28 |
| 9 | 0.70 | 0.65 | 0.71 | 0.80 | 0.80 | 0.88 | 0.69 | 0.72 |
| 10 | 0.95 | 0.89 | 0.28 | 0.99 | 0.79 | 0.87 | 0.95 | 1.00 |
| 11 | 1.25 | 1.31 | 0.24 | 1.15 | 0.82 | 1.01 | 1.27 | 1.30 |
| 12 | 0.90 | 0.90 | 0.90 | 0.90 | 0.86 | 0.89 | 0.90 | 0.90 |
| 13 | 1.10 | 1.02 | 0.84 | 0.95 | 0.89 | 1.00 | 1.03 | 1.02 |
| 14 | 1.10 | 1.07 | 1.27 | 0.97 | 0.96 | 0.97 | 1.08 | 1.08 |
| 15 | 0.87 | 0.93 | 0.98 | 0.92 | 1.03 | 0.96 | 0.95 | 0.95 |

Key: HR : HR Wallingford
KMM : Kirk McClure Morton
SWK : Scott Wilson Kirkpatrick
PD : Posford Duvivier
WSA : W S Atkins
AWR : Applied Wave Research
ABP : ABP Research and Consultancy
LNH : Laboratoire d'Hydraulique de France

## Table 7.2a Continued

| Analysis point | Physical Model | Refraction/diffraction models | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | ARMADA (HALCROW) | CGWAVE (KMM) | M21PMS (WSA) | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | ARTEMIS (LNH) |
| 1 | 0.30 | 0.29 | 1.12 | 0.58 | 0.19 | 0.34 | 0.71 | 0.75 |
| 2 | 1.65 | 1.67 | 1.64 | 0.98 | 1.70 | 1.74 | 1.79 | 1.62 |
| 3 | 0.90 | 0.76 | 0.83 | 0.73 | 0.61 | 0.66 | 0.81 | 0.99 |
| 4 | 0.40 | 0.48 | 0.74 | 0.71 | 0.33 | 0.52 | 0.58 | 0.77 |
| 5 | 1.85 | 1.97 | 2.17 | 1.38 | 1.88 | 1.72 | 2.16 | 2.13 |
| 6 | 0.68 | 0.66 | 0.34 | 0.46 | 0.69 | 0.62 | 0.46 | 0.61 |
| 7 | 0.65 | 0.60 | 0.57 | 0.61 | 0.56 | 0.60 | 0.54 | 0.59 |
| 8 | 2.00 | 2.09 | 2.38 | 1.78 | 2.11 | 2.16 | 2.16 | 2.33 |
| 9 | 0.70 | 0.66 | 0.45 | 0.69 | 0.81 | 0.66 | 0.65 | 0.59 |
| 10 | 0.95 | 0.83 | 0.71 | 0.86 | 0.83 | 0.86 | 0.82 | 0.92 |
| 11 | 1.25 | 1.45 | 1.47 | 1.09 | 1.44 | 1.43 | 1.30 | 1.30 |
| 12 | 0.90 | 0.88 | 1.12 | 0.90 | 0.86 | 1.79 | 0.85 | 0.94 |
| 13 | 1.10 | 1.05 | 0.53 | 1.01 | 1.05 | 1.05 | 0.91 | 1.02 |
| 14 | 1.10 | 1.12 | 1.14 | 0.98 | 1.12 | 1.10 | 0.91 | 1.15 |
| 15 | 0.87 | 0.95 | 0.81 | 0.95 | 0.96 | 0.95 | 0.86 | 0.96 |

## *Table 7.2b Percentage errors in wave height coefficients for Test B - Elliptic Shoal*

| Analysis point | Refraction models | | | | | | |
|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | PORTRAY (HR) | REFRAC (PD) | ENDEC (PD) | HISWA (PD) | LINDAL (AWR) | ORCAWAVE (ORCINA) |
| 1 | +410 | +157 | +133 | +180 | +190 | +53 | +20 |
| 2 | -10 | -36 | +44 | -49 | -48 | -27 | -50 |
| 3 | -46 | +49 | +111 | -8 | -6 | -43 | -52 |
| 4 | +20 | -30 | +73 | +108 | +123 | +26 | +15 |
| 5 | -12 | +18 | +18 | -56 | -52 | +5 | -37 |
| 6 | -22 | -40 | +15 | +19 | +26 | -18 | -18 |
| 7 | -8 | -80 | +31 | +23 | +40 | -7 | -6 |
| 8 | -44 | +49 | -33 | -60 | -53 | -7 | -36 |
| 9 | -7 | +1 | +14 | +14 | +26 | -2 | +3 |
| 10 | -6 | -71 | +4 | -17 | -8 | 0 | +5 |
| 11 | +5 | -81 | -8 | -34 | -19 | +2 | +4 |
| 12 | 0 | 0 | 0 | -4 | -1 | 0 | 0 |
| 13 | -7 | -24 | -14 | -19 | -9 | -7 | -7 |
| 14 | -3 | +15 | -12 | -13 | -12 | -2 | -2 |
| 15 | +7 | +13 | +6 | +18 | +10 | +9 | +9 |

## Table 7.2b Continued

| Analysis point | Refraction/diffraction models | | | | | | |
|---|---|---|---|---|---|---|---|
| | ARMADA (HALCROW) | CGWAVE (KMM) | M21PMS (WSA) | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | ARTEMIS (LNH) |
| 1 | -3 | +273 | +93 | -37 | +13 | +137 | +150 |
| 2 | +1 | -1 | -41 | +3 | +5 | +8 | -2 |
| 3 | -16 | -8 | -19 | -32 | -27 | -10 | +10 |
| 4 | +20 | +85 | +78 | -18 | +30 | +45 | +93 |
| 5 | +6 | +17 | -25 | +2 | -7 | +17 | +15 |
| 6 | -3 | -50 | -32 | +1 | -9 | -32 | -10 |
| 7 | -8 | -12 | -6 | -14 | -8 | -17 | -9 |
| 28 | +5 | +19 | -11 | +6 | +8 | +8 | +17 |
| 9 | -6 | -36 | -1 | +16 | -6 | -7 | -16 |
| 10 | -13 | -25 | -9 | -13 | -9 | -14 | -3 |
| 11 | +16 | +18 | -13 | +15 | +14 | +4 | +4 |
| 12 | -2 | +24 | 0 | -4 | +99 | -6 | +4 |
| 13 | -5 | -52 | -8 | -5 | -5 | -17 | -7 |
| 14 | +2 | +4 | -11 | +2 | 0 | -17 | +5 |
| 15 | +9 | -7 | +9 | +10 | +9 | -1 | +10 |

## Table 7.3a Wave height coefficients for Test C (Case 1) - Harbour Approach Bathymetry

| Analysis point | Physical Model | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | | Reg. | Irreg. | | | | |
| 1 | 0.9 | 0.9 | 0.7 | 1.0 | 0.8 | 0.8 | 0.8 | 1.0 | 1.0 | 0.9 | 1.0 |
| 2 | 0.9 | 1.0 | 1.1 | 1.4 | 1.1 | 1.0 | 0.9 | 1.5 | 1.5 | 0.8 | 1.2 |
| 3 | 0.7 | 0.8 | 0.1 | 0.6 | 0.5 | 0.5 | 0.4 | 0.6 | 0.6 | 0.5 | 0.7 |
| 4 | 0.6 | 0.8 | 0.8 | 1.1 | 1.1 | 0.6 | 0.6 | 0.8 | 0.8 | 0.5 | 0.6 |
| 5 | 0.7 | 1.1 | 0.5 | 1.6 | 1.0 | 0.6 | 0.6 | 1.4 | 1.4 | 0.5 | 1.0 |
| 6 | 0.7 | 0.8 | 0.4 | 0.9 | 0.5 | 0.3 | 0.4 | 0.8 | 0.7 | 0.2 | 1.0 |
| 7 | 0.6 | 0.8 | 0.1 | 0.3 | 0.5 | 0.3 | 0.3 | 0.4 | 0.3 | 0.2 | 0.6 |
| 8 | 0.8 | 1.0 | 0.7 | 1.1 | 1.0 | 0.7 | 0.7 | 1.0 | 1.0 | 0.6 | 1.0 |
| 9 | 0.7 | 1.0 | 0.6 | 1.5 | 1.3 | 0.8 | 0.7 | 1.0 | 1.1 | 0.5 | 1.1 |
| 10 | 0.6 | 0.8 | 1.4 | 1.3 | 0.7 | 0.8 | 0.7 | 1.5 | 1.4 | 0.8 | 1.2 |

Key: HR : HR Wallingford
SWK : Scott Wilson Kirkpatrick
WSA : W S Atkins
ABP : ABP Research and Technology
BIN : Binnie and Partners

## Table 7.3b Percentage errors in wave height coefficients for Test C (Case 1) - Harbour Approach Bathymetry

| Analysis point | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | Reg. | Irreg. | | | | |
| 1 | 0 | -22 | +11 | -11 | -11 | -11 | +7 | +11 | 0 | +6 |
| 2 | +11 | +22 | +56 | +22 | +11 | 0 | +64 | +67 | -11 | +37 |
| 3 | +14 | -86 | -14 | -29 | -29 | -43 | -19 | -14 | -29 | -4 |
| 4 | +33 | +33 | +83 | +80 | 0 | 0 | +38 | +33 | -17 | +5 |
| 5 | +57 | -29 | +129 | +43 | -14 | -14 | +103 | +100 | -29 | +37 |
| 6 | +14 | -43 | +29 | -29 | -57 | -43 | +11 | 0 | -71 | +43 |
| 7 | +33 | -83 | -50 | -17 | -50 | -50 | -35 | -50 | -67 | -3 |
| 8 | +25 | -13 | +38 | +25 | -13 | -13 | +25 | +25 | -25 | +23 |
| 9 | +43 | -14 | +114 | +80 | +14 | 0 | +43 | +57 | -29 | +53 |
| 10 | +33 | +133 | +117 | +17 | +33 | +17 | +145 | +133 | +33 | +97 |

## Table 7.4a Wave height coefficients for Test C (Case 2) - Harbour Approach Bathymetry

| Analysis point | Physical Model | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | | Reg. | Irreg. | | | | |
| 1 | 1.1 | 1.0 | 0.8 | 1.0 | 0.9 | 1.0 | 1.0 | 1.0 | 1.0 | 0.9 | 1.0 |
| 2 | 1.4 | 1.0 | 1.0 | 1.7 | 1.0 | 1.6 | 1.5 | 1.2 | 1.4 | 1.5 | 1.5 |
| 3 | 0.3 | 0.9 | 0.1 | 0.6 | 0.6 | 0.5 | 0.6 | 0.8 | 0.6 | 0.5 | 0.6 |
| 4 | 0.6 | 0.8 | 0.8 | 1.1 | 1.1 | 0.8 | 0.8 | 1.0 | 0.9 | 0.9 | 0.8 |
| 5 | 1.0 | 1.0 | 1.0 | 1.5 | 1.0 | 1.1 | 1.0 | 1.0 | 1.0 | 1.4 | 1.0 |
| 6 | 0.8 | 0.9 | 1.8 | 1.2 | 1.1 | 0.8 | 1.0 | 1.0 | 1.0 | 0.8 | 1.2 |
| 7 | 0.4 | 0.8 | 0.2 | 0.4 | 0.4 | 0.4 | 0.5 | 0.7 | 0.5 | 0.4 | 0.5 |
| 8 | 0.7 | 0.9 | 1.1 | 1.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 0.8 | 1.0 |
| 9 | 0.9 | 0.9 | 1.0 | 1.5 | 1.0 | 1.1 | 1.1 | 1.0 | 1.1 | 1.5 | 1.1 |
| 10 | 0.9 | 0.8 | 1.2 | 1.3 | 0.6 | 0.9 | 1.0 | 1.3 | 1.1 | 0.9 | 1.3 |

Key: HR : HR Wallingford
SWK : Scott Wilson Kirkpatrick
WSA : W S Atkins
ABP : ABP Research and Technology
BIN : Binnie and Partners

## Table 7.4b Percentage errors in wave height coefficients for Test C (Case 2) - Harbour Approach Bathymetry

| Analysis point | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | Reg. | Irreg. | | | | |
| 1 | -9 | -27 | -9 | -18 | -9 | -9 | -12 | -9 | -18 | -9 |
| 2 | -29 | -29 | +21 | -28 | +14 | +7 | -13 | 0 | +7 | +4 |
| 3 | +200 | -67 | +100 | +100 | +67 | +100 | +170 | +100 | +67 | +90 |
| 4 | +33 | +33 | +83 | +83 | +33 | +33 | +60 | +50 | +50 | +37 |
| 5 | 0 | 0 | +50 | 0 | +10 | 0 | +3 | 0 | +40 | -1 |
| 6 | +13 | +125 | +50 | +38 | 0 | +25 | +26 | +25 | 0 | +50 |
| 7 | +100 | -50 | 0 | 0 | 0 | +25 | +78 | +25 | 0 | +20 |
| 8 | +29 | +57 | +57 | +43 | +43 | +43 | +40 | +43 | +14 | +40 |
| 9 | 0 | +11 | +67 | +11 | +22 | +22 | +16 | +22 | +67 | +18 |
| 10 | -11 | +33 | +44 | -33 | 0 | +11 | +40 | +22 | 0 | +40 |

## Table 7.5a Wave height coefficients for Test C (Case 3) - Harbour Approach Bathymetry

| Analysis point | Physical Model | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCAWAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | | Reg. | Irreg. | | | | |
| 1 | 0.6 | 1.0 | 1.0 | 1.1 | 1.0 | 0.7 | 0.6 | 0.9 | 1.0 | 0.6 | 1.0 |
| 2 | 0.5 | 1.1 | 0.5 | 1.2 | 1.0 | 0.6 | 0.6 | 0.5 | 1.1 | 0.5 | 0.9 |
| 3 | 0.4 | 0.9 | 0.9 | 1.4 | 1.2 | 0.7 | 0.7 | 1.0 | 1.4 | 0.7 | 1.0 |
| 4 | 0.6 | 0.9 | 0.3 | 0.8 | 0.5 | 0.5 | 0.4 | 0.9 | 0.8 | 0.3 | 0.8 |
| 5 | 0.6 | 1.0 | 0.5 | 1.1 | 1.0 | 0.6 | 0.5 | 0.4 | 1.0 | 0.5 | 0.9 |
| 6 | 0.5 | 0.9 | 0.4 | 1.1 | 1.0 | 0.6 | 0.5 | 0.6 | 1.2 | 0.5 | 0.9 |
| 7 | 0.4 | 0.8 | 0.2 | 0.9 | 0.6 | 0.4 | 0.4 | 1.1 | 0.8 | 0.4 | 0.9 |
| 8 | 0.6 | 0.9 | 0.5 | 1.1 | 1.0 | 0.5 | 0.5 | 1.4 | 1.0 | 0.4 | 0.9 |
| 9 | 0.6 | 0.9 | 0.4 | 1.1 | 1.0 | 0.5 | 0.5 | 0.5 | 0.9 | 0.4 | 0.9 |
| 10 | 0.4 | 0.8 | 0.8 | 1.4 | 0.9 | 0.7 | 0.6 | 0.2 | 1.2 | 0.6 | 1.0 |

Key: HR : HR Wallingford
SWK : Scott Wilson Kirkpatrick
WSA : W S Atkins
ABP : ABP Research and Technology
BIN : Binnie and Partners

## Table 7.5b Percentage errors in wave height coefficients for Test C (Case 3) - Harbour Approach Bathymetry

| Analysis point | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | Reg. | Irreg. | | | | |
| 1 | +67 | +67 | +83 | +67 | +17 | 0 | +53 | +67 | 0 | +63 |
| 2 | +120 | +0 | +140 | +100 | +20 | +20 | +6 | +120 | 0 | +82 |
| 3 | +125 | +125 | +250 | +200 | +75 | +75 | +160 | +250 | +75 | +155 |
| 4 | +50 | -50 | +33 | -17 | -17 | -33 | +55 | +33 | -50 | +37 |
| 5 | +67 | -17 | +83 | +67 | +0 | -17 | -27 | +67 | -17 | +47 |
| 6 | +80 | -20 | +120 | +100 | +20 | 0 | +12 | +140 | 0 | +74 |
| 7 | +100 | -50 | +125 | +50 | +0 | 0 | +183 | +100 | 0 | +113 |
| 8 | +50 | -17 | +83 | +67 | -17 | -17 | +127 | +67 | -33 | +50 |
| 9 | +50 | -33 | +83 | +67 | -17 | -17 | -20 | +50 | -33 | +47 |
| 10 | +100 | +100 | +250 | +125 | +75 | +50 | -58 | +200 | +50 | +140 |

## Table 7.6a Wave height coefficients for Test C (Case 4) - Harbour Approach Bathymetry

| Analysis point | Physical Model | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | | Reg. | Irreg. | | | | |
| 1 | 0.9 | 1.0 | 1.0 | 1.1 | 0.9 | 1.0 | 0.9 | 1.0 | 1.0 | 1.0 | 1.0 |
| 2 | 0.9 | 1.1 | 0.9 | 1.2 | 1.0 | 0.9 | 0.9 | 0.6 | 1.1 | 0.9 | 1.0 |
| 3 | 1.1 | 0.9 | 1.3 | 1.6 | 0.9 | 1.2 | 1.1 | 1.2 | 1.4 | 1.3 | 1.2 |
| 4 | 0.9 | 0.9 | 0.7 | 1.0 | 0.7 | 0.7 | 0.7 | 1.3 | 0.8 | 0.6 | 0.9 |
| 5 | 0.8 | 1.0 | 0.9 | 1.2 | 1.0 | 0.9 | 0.8 | 0.6 | 1.0 | 0.8 | 1.0 |
| 6 | 0.8 | 0.9 | 0.8 | 1.1 | 1.0 | 0.9 | 0.9 | 0.9 | 1.2 | 0.9 | 1.0 |
| 7 | 0.8 | 0.9 | 0.3 | 1.0 | 0.6 | 0.8 | 0.7 | 1.70 | 0.8 | 0.6 | 0.9 |
| 8 | 0.9 | 0.8 | 0.8 | 1.1 | 1.0 | 0.9 | 0.8 | 1.2 | 1.0 | 0.8 | 1.0 |
| 9 | 0.9 | 0.9 | 0.7 | 1.1 | 1.0 | 0.9 | 0.8 | 0.6 | 1.0 | 0.7 | 1.0 |
| 10 | 0.6 | 0.8 | 1.1 | 1.4 | 1.0 | 1.1 | 1.0 | 0.3 | 1.2 | 1.1 | 1.1 |

Key: HR : HR Wallingford
SWK : Scott Wilson Kirkpatrick
WSA : W S Atkins
ABP : ABP Research and Technology
BIN : Binnie and Partners

## Table 7.6b Percentage errors in wave height coefficients for Test C (Case 4) - Harbour Approach Bathymetry

| Analysis point | Refraction models | | | | Refraction/diffraction models | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | ORCA WAVE (ORCINA) | M21PMS (WSA) | | MULTIGRID (ABP) | MULTIGRID (SWK) | PARAB (HR) | WC2D (BIN) |
| | | | | | Reg. | Irreg. | | | | |
| 1 | +11 | +11 | +22 | 0 | +11 | 0 | +7 | +11 | +11 | +9 |
| 2 | +22 | 0 | +33 | +11 | 0 | 0 | -31 | +22 | 0 | +13 |
| 3 | -18 | +18 | +45 | -18 | +9 | 0 | +10 | +27 | +18 | +13 |
| 4 | 0 | -22 | +11 | -22 | -22 | -22 | +47 | -11 | -33 | +1 |
| 5 | +25 | +13 | +50 | +25 | +13 | 0 | -20 | +25 | 0 | +24 |
| 6 | +13 | 0 | +38 | +25 | +13 | +13 | +11 | +50 | +13 | +21 |
| 7 | +13 | -63 | +25 | -25 | 0 | -13 | +111 | 0 | -25 | +11 |
| 8 | -11 | -11 | +22 | +11 | 0 | -11 | +34 | +11 | -11 | +10 |
| 9 | 0 | -22 | +22 | +11 | 0 | -11 | -29 | +11 | -22 | +8 |
| 10 | +33 | +83 | +133 | +66 | +83 | +67 | -52 | +100 | +83 | +88 |

## Table 7.7a Wave height coefficients for Test D - Perranporth

| Storm number | Field data | Refraction models | | | | | | | | Refraction/diffraction models | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (ACER) | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | HISWA (PD) | LINDAL (AWR) | M21NSW (WSA) | ORCA WAVE (ORCINA) | MULTIGRID (ABP) | PARAB (HR) | WC2D (BIN) |
| 1 | 0.9 | 0.8 | 0.9 | 0.9 | 0.7 | 0.9 | 0.9 | 0.8 | 1.1 | 0.7 | 1.0 | 0.9 |
| 2 | 1.0 | 0.8 | 0.8 | 0.6 | 0.7 | 0.8 | 1.0 | 0.8 | 0.8 | 0.7 | 0.8 | 0.9 |
| 3 | 0.8 | 0.8 | 0.8 | 1.9 | 0.7 | 0.8 | 0.8 | 0.8 | 0.5 | 0.9 | 0.9 | 0.7 |
| 4 | 0.6 | 0.5 | 0.6 | 0.7 | 0.9 | - | 0.7 | 0.7 | 0.9 | 0.9 | 0.9 | 0.6 |
| 5 | 0.9 | 0.8 | 0.8 | 1.0 | 0.7 | 0.9 | 1.0 | 0.8 | 0.9 | 0.7 | 0.8 | 0.9 |
| 6 | 0.8 | 0.9 | 0.9 | 0.9 | 0.8 | 0.8 | 1.0 | 0.9 | 0.9 | 0.8 | 0.8 | 0.9 |
| 7 | 0.9 | 0.8 | 0.9 | 0.7 | 0.8 | 0.8 | 0.9 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| 8 | 0.8 | 0.7 | 0.8 | 0.9 | 0.9 | - | 0.8 | 0.8 | 1.0 | 0.9 | 0.9 | 0.8 |
| 9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.8 | 0.8 | 1.0 | 0.9 | 0.9 | 0.8 | 0.9 | 1.0 |
| 10 | 0.7 | 0.9 | 0.9 | 0.9 | 0.8 | 0.9 | 1.0 | 0.9 | 1.0 | 0.8 | 1.0 | 1.0 |

key:
ACER : Acer Consultants Ltd
HR : HR Wallingford
SWK : Scott Wilson Kirkpatrick
PD : Posford Duvivier
AWR : Applied Wave Research
WSA : W S Atkins
ABP : ABP Research and Consultancy
BIN : Binnie and Partners

## Table 7.7b Percentage errors in wave height coefficients for Test D - Perranporth

| Storm number | Refraction models | | | | | | | | Refraction/diffraction models | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (ACER) | OUTRAY (HR) | PORTRAY (HR) | W-RAY (SWK) | HISWA (PD) | LINDAL (AWR) | M21NSW (WSA) | ORCA WAVE (ORCINA) | MULTIGRID (ABP) | PARAB (HR) | WC2D (BIN) |
| 1 | -7 | 0 | 0 | -20 | -2 | -3 | -11 | +23 | -19 | +11 | +2 |
| 2 | -20 | -20 | -40 | -30 | -22 | -3 | -20 | -21 | -28 | -20 | -13 |
| 3 | +5 | 0 | +137 | -16 | -3 | +3 | 0 | -33 | +14 | +13 | -11 |
| 4 | -18 | 0 | +17 | +50 | - | +8 | +17 | +42 | +53 | +50 | -2 |
| 5 | -10 | -11 | +11 | -24 | -12 | +7 | -11 | -3 | -18 | -11 | -6 |
| 6 | +10 | +13 | +13 | -6 | +4 | +19 | +13 | +8 | +1 | 0 | +16 |
| 7 | -6 | 0 | -22 | -17 | -7 | +4 | 0 | +1 | -12 | 0 | +4 |
| 8 | -18 | 0 | +13 | +13 | - | +3 | 0 | +20 | +8 | +13 | +4 |
| 9 | -2 | 0 | 0 | -12 | -7 | +6 | 0 | +4 | -10 | 0 | +6 |
| 10 | +24 | +29 | +29 | +17 | +24 | +39 | +29 | +39 | +19 | +43 | +39 |

## Table 7.8a Wave height coefficients for Test E - South Uist

| Storm number | Field measurement | Refraction models | | | | | | Refraction/diffraction models | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | OUTRAY (HR) | OUTRAY (KMM) | PORTRAY (HR) | HISWA (PD) | LINDAL (AWR) | ORCA WAVE (ORCINA) | MULTIGRID (ABP) | PARAB (HR) | WC2D (BIN) |
| 1 | 0.7 | 0.8 | 0.7 | 0.5 | 0.8 | 0.7 | 0.9/0.8 | 0.7 | 0.7 | 0.9 |
| 2 | 0.6 | 0.9 | 0.8 | 0.2 | 0.8 | 0.6 | 0.9/0.6 | 0.9 | 0.6 | 0.9 |
| 3 | 0.6 | 0.9 | 0.7 | 0.4 | 0.8 | 0.7 | 1.0/0.8 | 0.7 | 0.7 | 0.8 |
| 4 | 0.5 | 1.0 | 0.7 | 0.3 | 0.8 | 0.5 | 0.5/0.3 | 0.4 | 0.3 | 0.7 |
| 5 | 0.5 | 0.9 | 0.7 | 0.6 | 0.9 | 0.6 | 0.8/0.5 | 0.6 | 0.5 | 0.9 |
| 6 | 0.7 | 0.8 | 0.7 | 0.2 | 0.8 | 0.6 | 0.5/0.3 | 0.4 | 0.5 | 0.7 |
| 7 | 0.5 | 0.5 | 0.7 | 0.3 | - | 0.2 | 0.8/0.5 | 0.5 | 0.6 | 0.1/0.2 |
| 8 | 0.6 | 1.0 | 0.7 | 0.7 | 0.9 | 0.7 | 0.8/0.5 | 0.6 | 0.4 | 0.9 |
| 9 | 0.4 | 0.9 | 0.7 | 0.6 | 0.7 | 0.6 | 0.9/0.5 | 0.6 | 0.5 | 0.9 |
| 10 | 0.5 | 0.7 | 0.7 | 0.7 | - | 0.5 | 1.1/0.9 | 0.6 | 0.7 | 0.6 |

Key:
HR : HR Wallingford
KMM : Kirk McClure Morton
PD : Posford Duvivier
AWR : Applied Wave Research
ABP : ABP Research and Consultancy
BIN : Binnie and Partners

## Table 7.8b Percentage errors in wave height coefficients for Test E - South Uist

| Storm number | Refraction models | | | | | | Refraction/diffraction models | | |
|---|---|---|---|---|---|---|---|---|---|
| | OUTRAY (HR) | OUTRAY (KMM) | PORTRAY (HR) | HISWA (PD) | LINDAL (AWR) | ORCA WAVE (ORCINA) | MULTIGRID (ABP) | PARAB (HR) | WC2D (BIN) |
| 1 | +14 | +4 | -29 | +17 | -1 | +34/+16 | +4 | 0 | +21 |
| 2 | +50 | +27 | -67 | +35 | -2 | +47/+2 | +43 | 0 | +50 |
| 3 | +50 | +22 | -33 | +38 | +17 | +60/+25 | +13 | +17 | +40 |
| 4 | +100 | +3 | -40 | +64 | +8 | 0/-46 | -24 | -40 | +44 |
| 5 | +80 | +44 | +20 | +72 | +28 | +60/0 | +14 | 0 | +70 |
| 6 | +14 | -3 | -71 | +19 | -21 | -26/-56 | -41 | -29 | -44 |
| 7 | 0 | +42 | -40 | - | -70 | +52/-4 | -4 | +20 | -78/-62 |
| 8 | +67 | +23 | +17 | +48 | +18 | +35/-17 | +3 | -33 | +57 |
| 9 | +125 | +75 | +50 | +115 | +58 | +115/+30 | +38 | +25 | +115 |
| 10 | +40 | +42 | +40 | - | -10 | +116/+70 | +26 | +40 | +28 |

## *Table 7.9a Wave heights for Test F (Case 1) - Elliptic Shoal with Currents*

| Analysis point | Physical model | OUTURAY (HR) |
|---|---|---|
| 1 | 1.83 | 2.45 |
| 2 | 1.62 | 2.46 |
| 3 | 1.86 | 2.42 |
| 4 | 2.01 | 2.66 |
| 5 | 1.98 | 2.42 |
| 6 | 1.94 | 2.28 |
| 7 | 1.58 | 2.40 |
| 8 | 1.52 | 2.86 |
| 9 | 1.97 | 2.59 |
| 10 | 1.52 | 2.21 |
| 11 | 1.53 | 1.80 |
| 12 | 1.83 | 2.60 |
| 13 | 2.09 | 2.25 |
| 14 | 2.08 | 1.50 |

key: HR : HR Wallingford

**Table 7.9b Percentage errors in wave heights for Test F (Case 1) - Elliptic Shoal with Currents**

| Analysis point | OUTURAY (HR) |
|---|---|
| 1 | +35 |
| 2 | +52 |
| 3 | +30 |
| 4 | +32 |
| 5 | +22 |
| 6 | +18 |
| 7 | +52 |
| 8 | +88 |
| 9 | +45 |
| 10 | +45 |
| 11 | +18 |
| 12 | +42 |
| 13 | +8 |
| 14 | -28 |

## *Table 7.10a Wave heights for Test F (Case 2) - Elliptic Shoal with Currents*

| Analysis point | Physical model | OUTURAY (HR) |
|---|---|---|
| 1 | 2.28 | 3.13 |
| 2 | 1.90 | 3.13 |
| 3 | 2.08 | 3.25 |
| 4 | 2.37 | 3.66 |
| 5 | 2.59 | 3.13 |
| 6 | 1.90 | 2.78 |
| 7 | 1.83 | 2.89 |
| 8 | 1.74 | 3.99 |
| 9 | 2.65 | 3.47 |
| 10 | 1.85 | 2.81 |
| 11 | 1.69 | 2.07 |
| 12 | 2.39 | 3.36 |
| 13 | 2.65 | 2.93 |
| 14 | 2.66 | 1.52 |

key: HR : HR Wallingford

## *Table 7.10b Percentage errors in wave heights for Test F (Case 2) - Elliptic Shoal with Currents*

| Analysis point | OUTURAY (HR) |
|---|---|
| 1 | +37 |
| 2 | +65 |
| 3 | +56 |
| 4 | +54 |
| 5 | +21 |
| 6 | +46 |
| 7 | +58 |
| 8 | +129 |
| 9 | +31 |
| 10 | +52 |
| 11 | +22 |
| 12 | +41 |
| 13 | +11 |
| 14 | -43 |

## Table 7.11a Wave heights for Test G (Case 1) - Semi Infinite Breakwater

| Analysis point | Sommerfeld solution | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|---|
| 1 | 0.24 | 0.25 | 0.54 | 0.15 | 0.16 | 0.23 | 0.27 |
| 2 | 0.18 | 0.18 | 0.25 | 0.11 | 0.12 | 0.19 | 0.23 |
| 3 | 0.11 | 0.11 | 0.13 | 0.08 | 0.09 | 0.08 | 0.12 |
| 4 | 0.08 | 0.08 | 0.08 | 0.06 | 0.05 | 0.22 | 0.09 |
| 5 | 0.43 | 0.45 | 0.81 | 0.19 | 0.32 | 0.46 | 0.49 |
| 6 | 0.35 | 0.36 | 0.58 | 0.15 | 0.27 | 0.39 | 0.37 |
| 7 | 0.25 | 0.24 | 0.33 | 0.12 | 0.22 | 0.38 | 0.32 |
| 8 | 0.19 | 0.18 | 0.22 | 0.10 | 0.17 | 0.22 | 0.22 |
| 9 | 0.15 | 0.15 | 0.18 | 0.04 | 0.09 | 0.17 | 0.19 |
| 10 | 0.11 | 0.11 | 0.12 | 0.04 | 0.07 | 0.12 | 0.14 |
| 11 | 0.07 | 0.07 | 0.07 | 0.03 | 0.04 | 0.09 | 0.08 |
| 12 | 0.05 | 0.05 | 0.05 | 0.02 | 0.03 | 0.06 | 0.07 |

key: HR : HR Wallingford
Gibb : Sir Alexander Gibb & Partners
PD : Posford Duvivier
LNH : Laboratoire d'Hydraulique de France

## *Table 7.11b Percentage errors in wave heights for Test G (Case 1) - Semi Infinite Breakwater*

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|
| 1 | +3 | +125 | -38 | -34 | -4 | +13 |
| 2 | -3 | +39 | -39 | -31 | +6 | +28 |
| 3 | 0 | +18 | -27 | -20 | -27 | +9 |
| 4 | 0 | 0 | -25 | -41 | +175 | +13 |
| 5 | +5 | +88 | -56 | -25 | +7 | +14 |
| 6 | +2 | +66 | -57 | -23 | +11 | +6 |
| 7 | -2 | +32 | -52 | -13 | +52 | +28 |
| 8 | -8 | +16 | -47 | -10 | +16 | +16 |
| 9 | +2 | +20 | -73 | -38 | +13 | +27 |
| 10 | -2 | +9 | -64 | -40 | +9 | +27 |
| 11 | 0 | 0 | -57 | -40 | +29 | +14 |
| 12 | 0 | 0 | -60 | -42 | +20 | +40 |

## *Table 7.12a Wave heights for Test G (Case 2) - Semi Infinite Breakwater*

| Analysis point | Sommerfeld solution | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|---|
| 1 | 0.33 | 0.34 | 0.76 | 0.21 | 0.29 | 0.28 | 0.35 |
| 2 | 0.26 | 0.25 | 0.43 | 0.16 | 0.21 | 0.20 | 0.24 |
| 3 | 0.17 | 0.17 | 0.21 | 0.12 | 0.13 | 0.23 | 0.21 |
| 4 | 0.12 | 0.12 | 0.14 | 0.10 | 0.12 | 0.11 | 0.12 |
| 5 | 0.79 | 0.78 | 1.07 | 0.80 | 0.78 | 0.81 | 0.83 |
| 6 | 0.89 | 0.86 | 1.09 | 0.90 | 0.87 | 0.93 | 0.95 |
| 7 | 1.09 | 1.03 | 1.12 | 1.01 | 1.12 | 1.13 | 1.07 |
| 8 | 1.11 | 1.13 | 1.03 | 1.02 | 1.13 | 0.96 | 1.09 |
| 9 | 0.18 | 0.18 | 0.21 | 0.04 | 0.12 | 0.17 | 0.17 |
| 10 | 0.13 | 0.13 | 0.15 | 0.04 | 0.08 | 0.14 | 0.14 |
| 11 | 0.08 | 0.08 | 0.08 | 0.03 | 0.05 | 0.09 | 0.11 |
| 12 | 0.06 | 0.06 | 0.06 | 0.03 | 0.05 | 0.08 | 0.09 |

key: HR : HR Wallingford
GIBB : Sir Alexander Gibb & Partners
PD : Posford Duvivier
LNH : Laboratoire d'Hydraulique de France

## Table 7.12b Percentage errors in wave heights for Test G (Case 2) - Semi Infinite Breakwater

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|
| 1 | +3 | +130 | -36 | -14 | -15 | +6 |
| 2 | -3 | +65 | -38 | -20 | -23 | -8 |
| 3 | -1 | +24 | -29 | -25 | +35 | +24 |
| 4 | +2 | +17 | -17 | 0 | -8 | 0 |
| 5 | -1 | +35 | +1 | -1 | +3 | +5 |
| 6 | -4 | +22 | +1 | -2 | +4 | +7 |
| 7 | -5 | +3 | -7 | +3 | +4 | -2 |
| 8 | +2 | -7 | -8 | +1 | -14 | -2 |
| 9 | 0 | +17 | -78 | -32 | -6 | -6 |
| 10 | -4 | +15 | -69 | -35 | +8 | +8 |
| 11 | 0 | 0 | -62 | -34 | +13 | +38 |
| 12 | 0 | 0 | -50 | -23 | +33 | +50 |

## *Table 7.13a Wave heights for Test G (Case 3) - Semi Infinite Breakwater*

| Analysis point | Sommerfeld solution | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|---|
| 1 | 0.56 | 0.60 | 0.99 | 0.55 | 0.51 | 0.51 | 0.58 |
| 2 | 0.54 | 0.57 | 0.83 | 0.55 | 0.51 | 0.49 | 0.58 |
| 3 | 0.53 | 0.55 | 0.69 | 0.59 | 0.51 | 0.49 | 0.53 |
| 4 | 0.52 | 0.54 | 0.63 | 0.64 | 0.53 | 0.50 | 0.59 |
| 5 | 1.09 | 1.08 | 0.98 | 1.08 | 1.13 | 1.14 | 1.13 |
| 6 | 0.94 | 0.98 | 1.04 | 0.98 | 0.91 | 0.95 | 0.93 |
| 7 | 0.98 | 0.98 | 1.03 | 1.00 | 0.95 | 0.97 | 1.07 |
| 8 | 1.00 | 0.99 | 1.02 | 1.00 | 1.01 | 1.07 | 0.94 |
| 9 | 0.24 | 0.24 | 0.27 | 0.06 | 0.22 | 0.27 | 0.25 |
| 10 | 0.17 | 0.17 | 0.20 | 0.06 | 0.16 | 0.19 | 0.20 |
| 11 | 0.11 | 0.11 | 0.12 | 0.05 | 0.12 | 0.13 | 0.12 |
| 12 | 0.08 | 0.08 | 0.08 | 0.04 | 0.09 | 0.10 | 0.05 |

key: HR : HR Wallingford
GIBB : Sir Alexander Gibb & Partners
PD : Posford Duvivier
LNH : Laboratoire d'Hydraulique de France

## Table 7.13b Percentage errors in wave heights for Test G (Case 3) - Semi Infinite Breakwater

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|
| 1 | +6 | +77 | -2 | -9 | -9 | +4 |
| 2 | +6 | +54 | +2 | -5 | -9 | +7 |
| 3 | +3 | +30 | +11 | -3 | -8 | 0 |
| 4 | +4 | +21 | +23 | +3 | -4 | +13 |
| 5 | -1 | -10 | -1 | +4 | +5 | +4 |
| 6 | +4 | +11 | +4 | -4 | +1 | -1 |
| 7 | -1 | +5 | +2 | -3 | -1 | +9 |
| 8 | -1 | +2 | 0 | +1 | +7 | -6 |
| 9 | -1 | +13 | -75 | -7 | +13 | +4 |
| 10 | +2 | +18 | -65 | -9 | +12 | +18 |
| 11 | 0 | +9 | -55 | +5 | +18 | +9 |
| 12 | 0 | 0 | -50 | +10 | +25 | -38 |

## Table 7.14a Wave heights for Test G (Case 4) - Semi Infinite Breakwater

| Analysis point | Sommerfeld solution | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|---|
| 1 | 0.95 | 0.92 | 1.06 | 0.95 | 0.96 | 0.98 | 0.93 |
| 2 | 1.09 | 1.04 | 1.09 | 1.06 | 1.11 | 1.11 | 1.16 |
| 3 | 0.99 | 1.04 | 0.94 | 1.02 | 1.01 | 1.04 | 0.99 |
| 4 | 1.07 | 1.05 | 1.06 | 0.99 | 1.07 | 1.11 | 1.31 |
| 5 | 0.98 | 0.98 | 1.00 | 0.98 | 0.96 | 1.04 | 1.04 |
| 6 | 0.99 | 0.99 | 0.99 | 1.00 | 1.00 | 1.03 | 1.05 |
| 7 | 1.01 | 1.01 | 0.99 | 1.00 | 1.04 | 1.00 | 0.98 |
| 8 | 1.01 | 1.00 | - | 1.00 | 1.00 | 1.01 | 0.08 |
| 9 | 0.40 | 0.42 | 0.43 | 0.09 | 0.37 | 0.40 | 0.43 |
| 10 | 0.33 | 0.34 | 0.37 | 0.12 | 0.30 | 0.35 | 0.29 |
| 11 | 0.24 | 0.24 | 0.25 | 0.11 | 0.25 | 0.27 | 0.24 |
| 12 | 0.18 | 0.18 | 0.18 | 0.09 | 0.16 | 0.21 | 0.24 |

key: HR : HR Wallingford
GIBB : Sir Alexander Gibb & Partners
PD : Posford Duvivier
LNH : Laboratoire d'Hydraulique de France

## Table 7.14b Percentage errors in wave heights for Test G (Case 4) - Semi Infinite Breakwater

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|
| 1 | -4 | +12 | 0 | +1 | +3 | -2 |
| 2 | -5 | 0 | -3 | +2 | +2 | +6 |
| 3 | +5 | -5 | +3 | +2 | +5 | 0 |
| 4 | -2 | -1 | -7 | 0 | +4 | +22 |
| 5 | 0 | +2 | 0 | -2 | +6 | +6 |
| 6 | 0 | 0 | +1 | +1 | +4 | +6 |
| 7 | 0 | -2 | -1 | +3 | -1 | -3 |
| 8 | -1 | - | -1 | -1 | 0 | -92 |
| 9 | +5 | +7 | -78 | -8 | 0 | +7 |
| 10 | +2 | +12 | -64 | -8 | +6 | -12 |
| 11 | +1 | +4 | -54 | +3 | +13 | 0 |
| 12 | -1 | 0 | -50 | -9 | +17 | +33 |

## *Table 7.15a Wave heights for Test G (Case 5) - Semi Infinite Breakwater*

| Analysis point | Sommerfeld solution | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|---|
| 1 | 1.05 | 1.05 | 0.98 | 1.05 | 1.05 | 1.09 | 0.97 |
| 2 | 0.96 | 0.97 | 1.00 | 0.98 | 1.02 | 0.93 | 0.69 |
| 3 | 1.02 | 1.03 | 1.01 | 1.00 | 1.04 | 1.24 | 0.31 |
| 4 | 0.99 | 0.99 | 1.00 | 1.00 | 0.89 | 1.20 | 0.11 |
| 5 | 0.99 | 1.01 | 1.00 | 0.99 | 1.07 | 0.87 | 0.81 |
| 6 | 1.01 | 1.00 | 1.00 | 1.00 | 1.10 | 0.98 | 0.51 |
| 7 | 0.99 | 1.00 | 0.28 | 1.00 | 0.98 | 0.90 | 0.12 |
| 8 | 1.00 | 1.00 | - | 1.00 | 0.97 | 1.09 | 0.02 |
| 9 | 0.80 | 0.81 | 0.69 | 0.75 | 0.80 | 0.67 | 0.91 |
| 10 | 0.89 | 0.87 | 0.84 | 0.85 | 0.89 | 0.79 | 0.91 |
| 11 | 1.08 | 1.03 | 1.01 | 1.00 | 1.06 | 1.40 | 0.76 |
| 12 | 1.11 | 1.12 | 1.12 | 1.02 | 1.13 | 1.60 | 0.45 |

key: HR : HR Wallingford
GIBB : Sir Alexander Gibb & Partners
PD : Posford Duvivier
LNH : Laboratoire d'Hydraulique de France

## *Table 7.15b Percentage errors in wave heights for Test G (Case 5) - Semi Infinite Breakwater*

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | OUTDIF (HR) | DIFFRAC (PD) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|---|
| 1 | 0 | -7 | 0 | 0 | +4 | -8 |
| 2 | +1 | +4 | +2 | +7 | -3 | -28 |
| 3 | 0 | -1 | -2 | +2 | +22 | -70 |
| 4 | 0 | +1 | +1 | -10 | +21 | -89 |
| 5 | +2 | +1 | 0 | +8 | -12 | -18 |
| 6 | -1 | -1 | -1 | +9 | - | -50 |
| 7 | +1 | -72 | +1 | -2 | -9 | -88 |
| 8 | 0 | - | 0 | -4 | +9 | -98 |
| 9 | +1 | -14 | -6 | -1 | -16 | +14 |
| 10 | -2 | -6 | -4 | -0 | -11 | +2 |
| 11 | -4 | -6 | -7 | -2 | +30 | -30 |
| 12 | +1 | +1 | -8 | +2 | +44 | -59 |

## *Table 7.16a Wave heights for Test H (Case 1) - Idealised Harbour Entrance*

| Analysis point | Physical model | PORTRAY (HR) | PORTRAY (GIBB) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|
| 1 | 2.63 | 2.50 | 2.50 | 2.75 | 2.68 |
| 2 | 2.45 | 2.50 | 2.50 | 3.48 | 2.60 |
| 3 | 2.59 | 3.40 | 3.42 | 3.10 | 2.33 |
| 4 | 2.34 | 2.36 | 2.36 | 2.90 | 1.95 |
| 5 | 2.41 | 2.41 | 2.42 | 2.53 | 2.82 |
| 6 | 2.44 | 2.29 | 2.30 | 3.35 | 2.78 |
| 7 | 2.52 | 2.41 | 2.42 | 2.13 | 2.25 |
| 8 | 0.59 | 0.84 | 0.62 | 1.05 | 1.58 |
| 9 | 2.23 | 2.32 | 2.34 | 2.85 | 1.98 |
| 10 | 2.45 | 2.32 | 2.39 | 3.78 | 2.10 |
| 11 | 1.03 | 1.06 | 1.13 | 0.90 | 1.23 |
| 12 | 2.82 | 3.41 | 3.43 | 3.90 | 1.75 |
| 13 | 2.96 | 3.41 | 3.43 | 1.25 | 2.50 |
| 14 | 3.06 | 3.41 | 3.43 | 1.93 | 2.15 |

key: HR : HR Wallingford
GIBB : Sir Alexander Gibb & Partners
LNH : Laboratoire d'Hydraulique de France

## Table 7.16b Percentage errors in wave heights for Test H (Case 1) - Idealised Harbour Entrance

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|
| 1 | -5 | -5 | +5 | +2 |
| 2 | +2 | +2 | +42 | +6 |
| 3 | +31 | +32 | +20 | -10 |
| 4 | +1 | +1 | +24 | -17 |
| 5 | 0 | 0 | +5 | +17 |
| 6 | -6 | -6 | +37 | +14 |
| 7 | -4 | -4 | -15 | -11 |
| 8 | +42 | +5 | +78 | +168 |
| 9 | +4 | +5 | +28 | -11 |
| 10 | -5 | -2 | +54 | -14 |
| 11 | +3 | +10 | -13 | +19 |
| 12 | +21 | +22 | +38 | -38 |
| 13 | +15 | +16 | -58 | -16 |
| 14 | +11 | +12 | -37 | -30 |

## Table 7.17a Wave heights for Test H (Case 2) - Idealised Harbour Entrance

| Analysis point | Physical model | PORTRAY (HR) | PORTRAY (GIBB) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|---|
| 1 | 4.51 | 4.20 | 4.20 | 5.08 | 3.32 |
| 2 | 4.19 | 4.20 | 4.20 | 4.66 | 4.07 |
| 3 | 4.45 | 5.77 | 5.83 | 6.97 | 3.23 |
| 4 | 4.03 | 4.01 | 4.04 | 3.40 | 4.24 |
| 5 | 4.35 | 4.17 | 4.15 | 4.12 | 4.87 |
| 6 | 3.98 | 3.89 | 3.92 | 5.71 | 5.00 |
| 7 | 4.20 | 4.10 | 4.13 | 5.38 | 5.04 |
| 8 | 1.09 | 1.43 | 1.09 | 1.51 | 2.39 |
| 9 | 4.04 | 3.95 | 3.86 | 4.87 | 3.02 |
| 10 | 3.84 | 3.92 | 4.07 | 4.62 | 3.49 |
| 11 | 1.71 | 1.84 | 1.98 | 1.55 | 1.39 |
| 12 | 4.93 | 5.79 | 5.84 | 5.08 | 4.37 |
| 13 | 5.03 | 5.78 | 5.84 | 5.12 | 3.32 |
| 14 | 5.13 | 5.79 | 5.84 | 7.31 | 4.41 |

key: HR : HR Wallingford
GIBB : Sir Alexander Gibb & Partners
LNH : Laboratoire d'Hydraulique de France

## Table 7.17b Percentage errors in wave heights for Test H (Case 2) - Idealised Harbour Entrance

| Analysis point | PORTRAY (HR) | PORTRAY (GIBB) | PORTCGS (HR) | ARTEMIS (LNH) |
|---|---|---|---|---|
| 1 | -7 | -7 | +13 | -26 |
| 2 | 0 | 0 | +11 | -3 |
| 3 | +30 | +31 | +57 | -27 |
| 4 | 0 | 0 | -16 | +5 |
| 5 | -4 | -5 | -5 | +12 |
| 6 | -2 | -2 | +43 | +26 |
| 7 | -2 | -2 | +28 | +20 |
| 8 | +31 | 0 | +39 | +119 |
| 9 | -2 | -4 | +21 | -25 |
| 10 | +2 | +6 | +20 | -9 |
| 11 | +8 | +16 | -9 | -19 |
| 12 | +17 | +18 | +3 | -11 |
| 13 | +15 | +16 | +2 | -34 |
| 14 | +13 | +14 | +42 | -14 |

## *Table 7.18a Wave heights for Test I (Case 1) - Pittenweem Harbour*

| Analysis point | Physical model | PORTRAY (HR) | PORTCGS (HR) |
|---|---|---|---|
| 1 | 1.80 | 1.92 | 5.21 |
| 2 | 0.60 | 1.91 | 5.21 |
| 3 | 0.30 | 0.08 | 0.50 |
| 4 | 0.29 | 0.07 | 0.14 |
| 5 | 0.29 | 0.04 | 0.14 |
| 6 | 0.45 | 0.19 | 0.17 |
| 7 | 0.32 | 0.20 | 0.25 |
| 8 | 0.35 | 0.22 | 0.08 |
| 9 | 0.39 | 0.23 | 0.70 |

key: HR : HR Wallingford

## *Table 7.18b Percentage errors in wave heights for Test I (Case 1) - Pittenweem Harbour*

| Analysis point | PORTRAY (HR) | PORTCGS (HR) |
|---|---|---|
| 1 | +7 | +189 |
| 2 | +218 | +768 |
| 3 | -73 | +67 |
| 4 | -76 | -52 |
| 5 | -86 | -52 |
| 6 | -58 | -62 |
| 7 | -38 | -22 |
| 8 | -37 | -77 |
| 9 | -41 | +79 |

## *Table 7.19a Wave heights for Test I (Case 2) - Pittenweem Harbour*

| Analysis point | Physical model | PORTRAY (HR) |
|---|---|---|
| 1 | 1.32 | 1.47 |
| 2 | 1.36 | 1.13 |
| 3 | 0.35 | 0.03 |
| 4 | 0.43 | 0.03 |
| 5 | 0.53 | 0.04 |
| 6 | 0.51 | 0.30 |
| 7 | 0.50 | 0.18 |
| 8 | 0.53 | 0.14 |
| 9 | 0.65 | 0.12 |

key: HR : HR Wallingford

## *Table 7.19b Percentage errors in wave heights for Test I (Case 2) - Pittenweem Harbour*

| Analysis point | PORTRAY (HR) |
|---|---|
| 1 | +11 |
| 2 | -17 |
| 3 | -91 |
| 4 | -93 |
| 5 | -92 |
| 6 | -41 |
| 7 | -64 |
| 8 | -74 |
| 9 | -82 |

## Table 7.20a Wave heights for Test J (Case 1) - Aberdeen Harbour Entrance

| Analysis point | Physical model | PORTRAY (HR) |
|---|---|---|
| 1 | 3.70 | 3.89 |
| 2 | 3.41 | 1.68 |
| 3 | 3.16 | 3.09 |
| 4 | 2.33 | 1.30 |
| 5 | 1.48 | 2.64 |
| 6 | 1.12 | 0.54 |
| 7 | 0.81 | 1.63 |

key: HR : HR Wallingford

## *Table 7.20b Percentage errors in wave heights for Test J (Case 1) - Aberdeen Harbour Entrance*

| Analysis point | PORTRAY (HR) |
|---|---|
| 1 | +5 |
| 2 | -51 |
| 3 | -2 |
| 4 | -44 |
| 5 | +78 |
| 6 | -52 |
| 7 | +101 |

## *Table 7.21a Wave heights for Test J (Case 2) - Aberdeen Harbour Entrance*

| Analysis point | Physical model | PORTRAY (HR) |
|---|---|---|
| 1 | 4.37 | 2.98 |
| 2 | 3.85 | 1.88 |
| 3 | 3.60 | 1.74 |
| 4 | 2.30 | 1.40 |
| 5 | 1.82 | 0.99 |
| 6 | 1.32 | 0.57 |
| 7 | 1.06 | 0.48 |

key: HR : HR Wallingford

## *Table 7.21b Percentage errors in wave heights for Test J (Case 2) - Aberdeen Harbour Entrance*

| Analysis point | PORTRAY (HR) |
|---|---|
| 1 | -32 |
| 2 | -51 |
| 3 | -52 |
| 4 | -39 |
| 5 | -46 |
| 6 | -57 |
| 7 | -55 |

## *Table 7.22a Wave heights for Test J (Case 3) - Aberdeen Harbour Entrance*

| Analysis point | Physical model | PORTRAY (HR) |
|---|---|---|
| 1 | 2.03 | 2.72 |
| 2 | 1.93 | 2.85 |
| 3 | 2.05 | 2.38 |
| 4 | 1.62 | 1.82 |
| 5 | 1.09 | 2.00 |
| 6 | 0.93 | 1.51 |
| 7 | 0.72 | 1.21 |

key: HR : HR Wallingford

## Table 7.22b Percentage errors in wave heights for Test J (Case 3) - Aberdeen Harbour Entrance

| Analysis point | PORTRAY (HR) |
|---|---|
| 1 | +34 |
| 2 | +48 |
| 3 | +16 |
| 4 | +12 |
| 5 | +83 |
| 6 | +62 |
| 7 | +68 |

## *Table 7.23a Wave heights for Test J (Case 4) - Aberdeen Harbour Entrance*

| Analysis point | Physical model | PORTRAY (HR) |
|---|---|---|
| 1 | 4.16 | 4.80 |
| 2 | 3.65 | 5.57 |
| 3 | 3.34 | 3.21 |
| 4 | 2.84 | 5.24 |
| 5 | 2.07 | 4.65 |
| 6 | 1.73 | 2.47 |
| 7 | 1.50 | 2.22 |

key: HR : HR Wallingford

## *Table 7.23b Percentage errors in wave heights for Test J (Case 4) - Aberdeen Harbour Entrance*

| Analysis point | PORTRAY (HR) |
|---|---|
| 1 | +15 |
| 2 | +53 |
| 3 | -4 |
| 4 | +85 |
| 5 | +125 |
| 6 | +43 |
| 7 | +48 |

# Figures

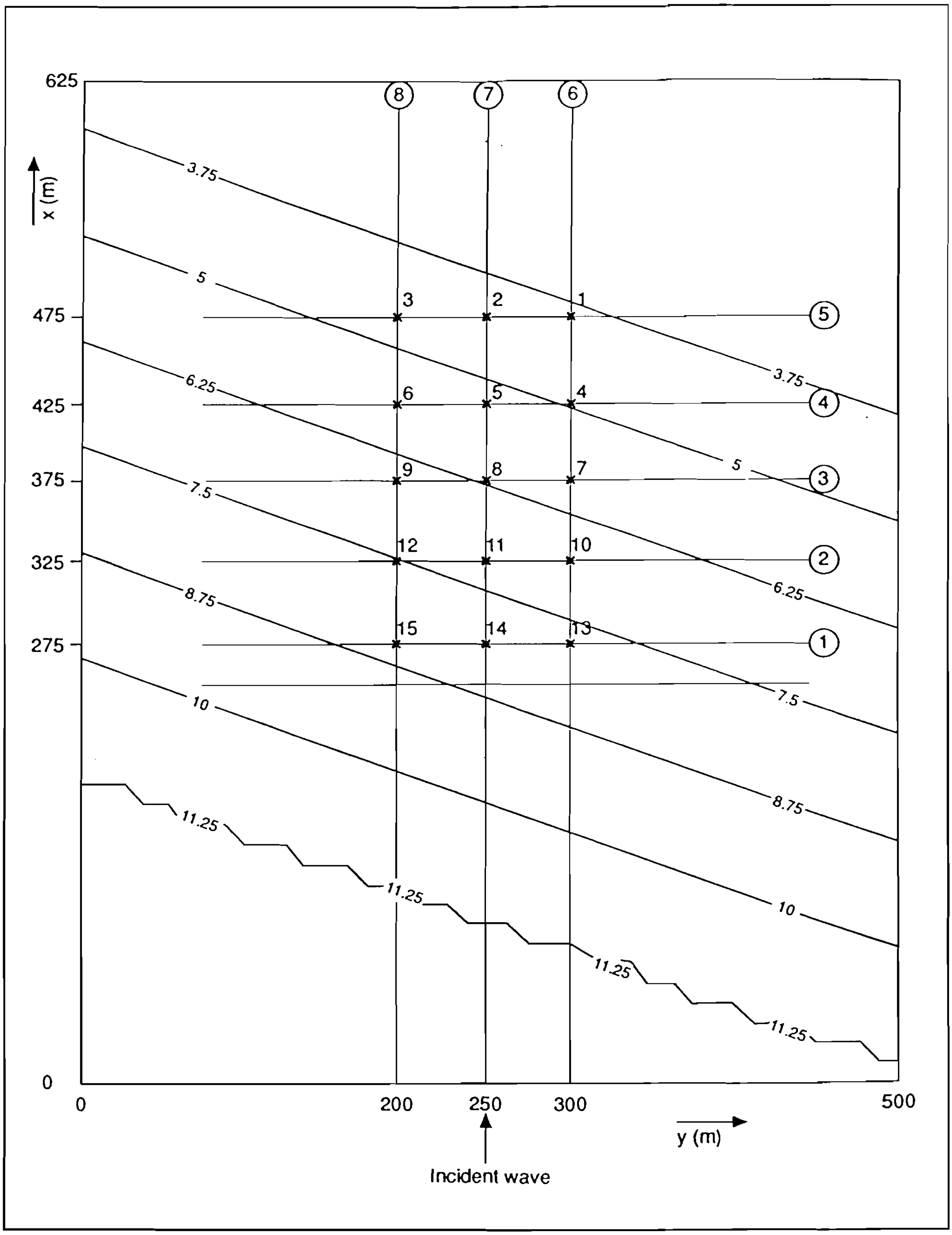



**Figure 6.1 Linear Beach**

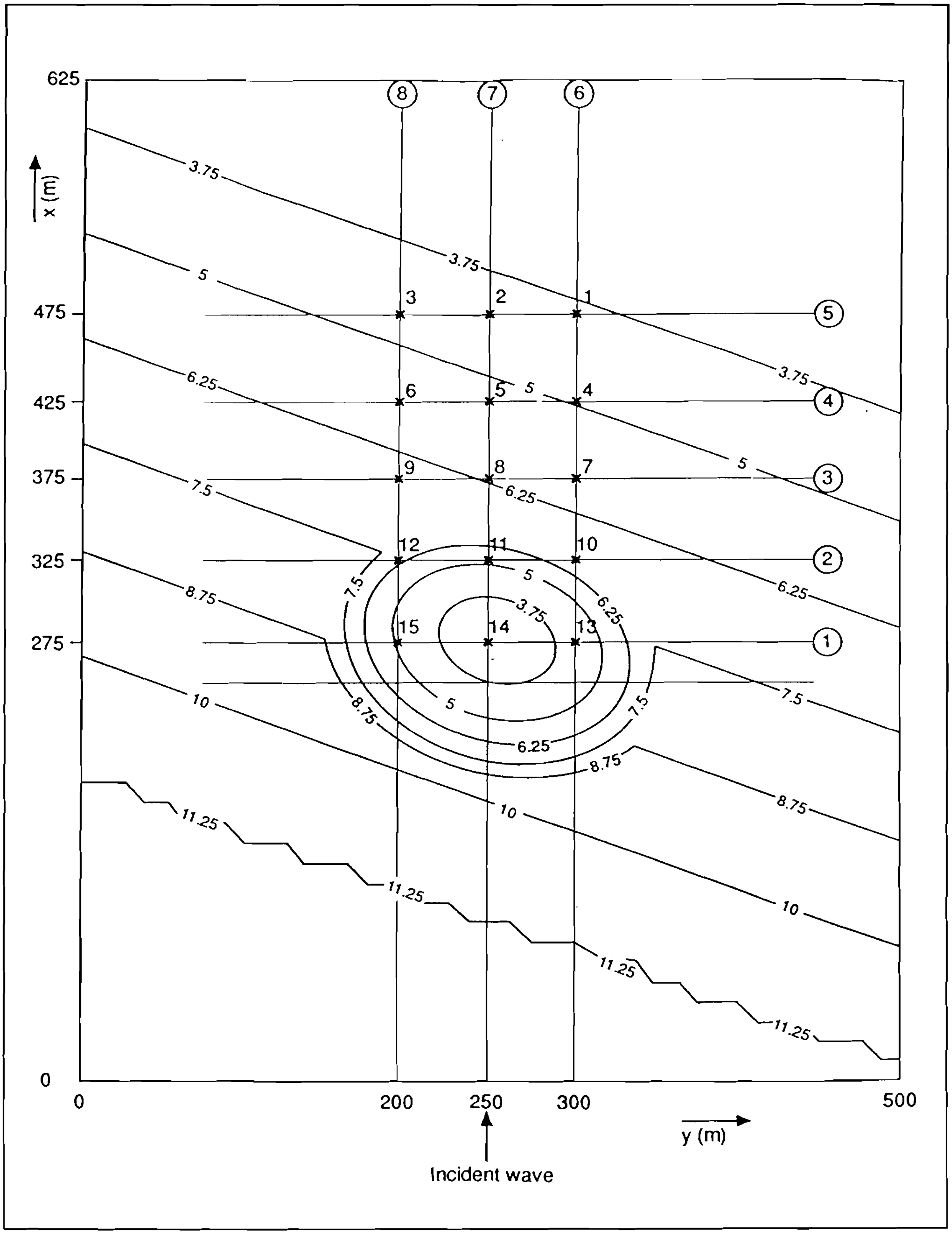



**Figure 6.2 Elliptic Shoal**

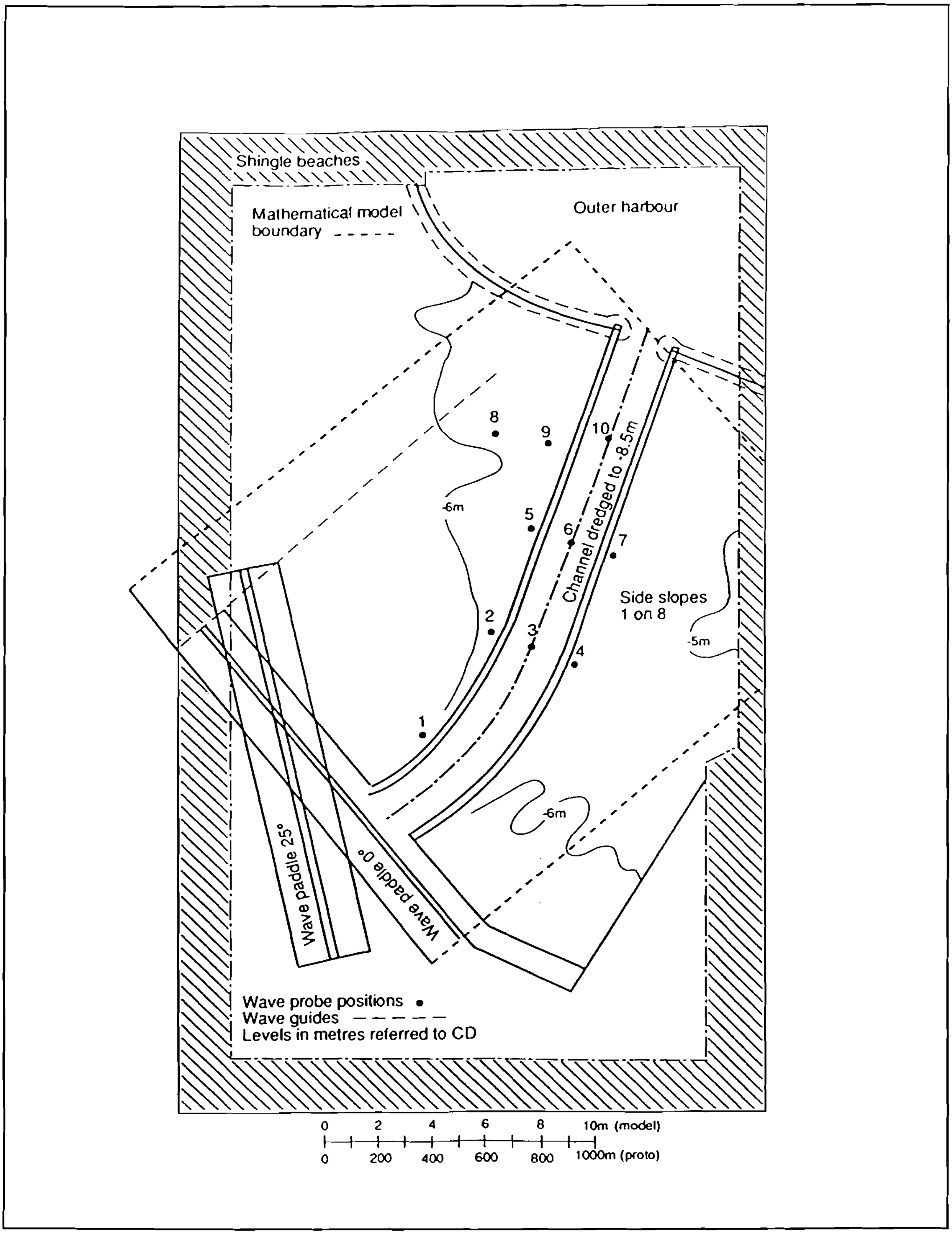



**Figure 6.3 Harbour Approach Bathymetry**

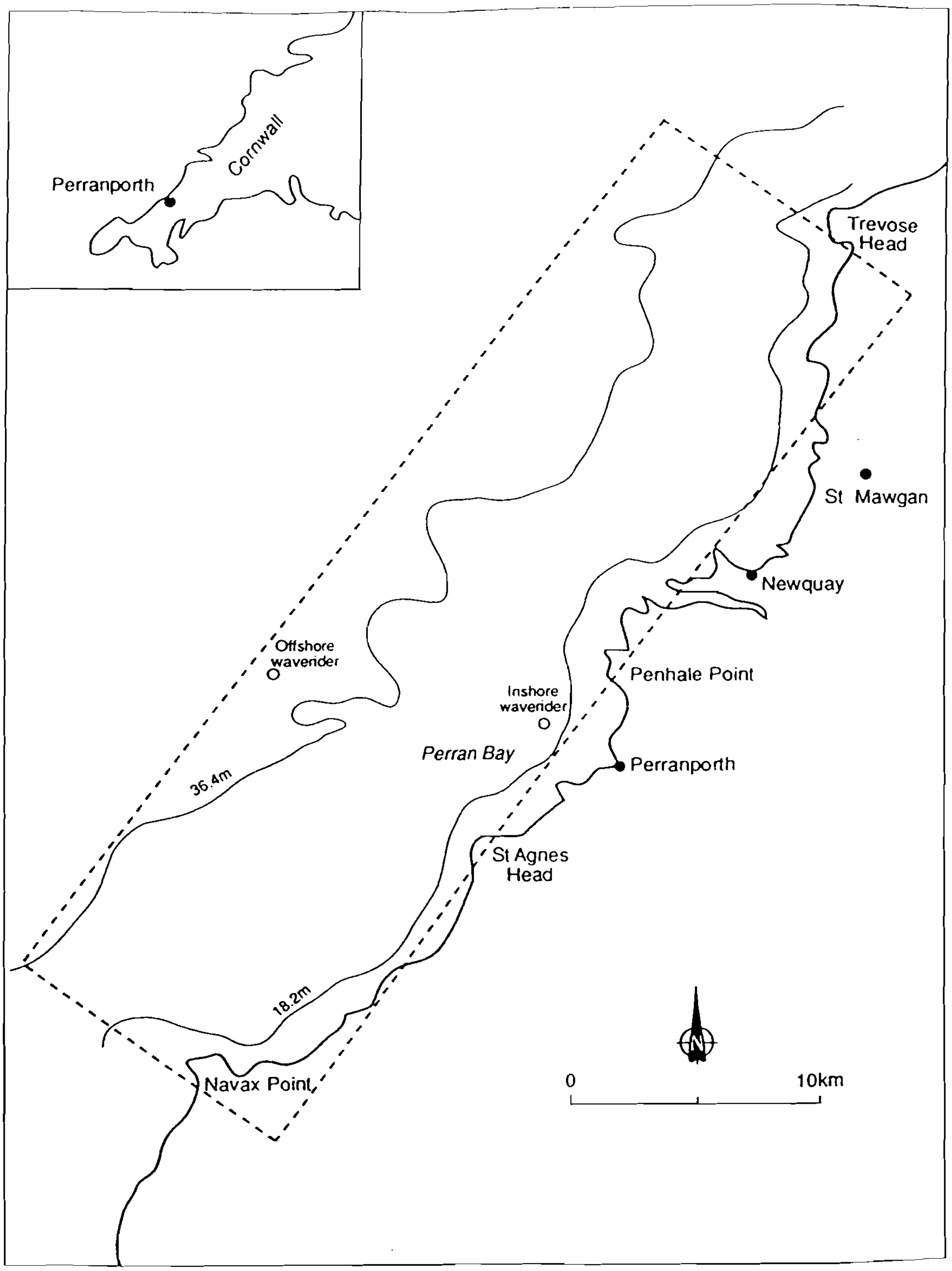



**Figure 6.4 Perranporth**

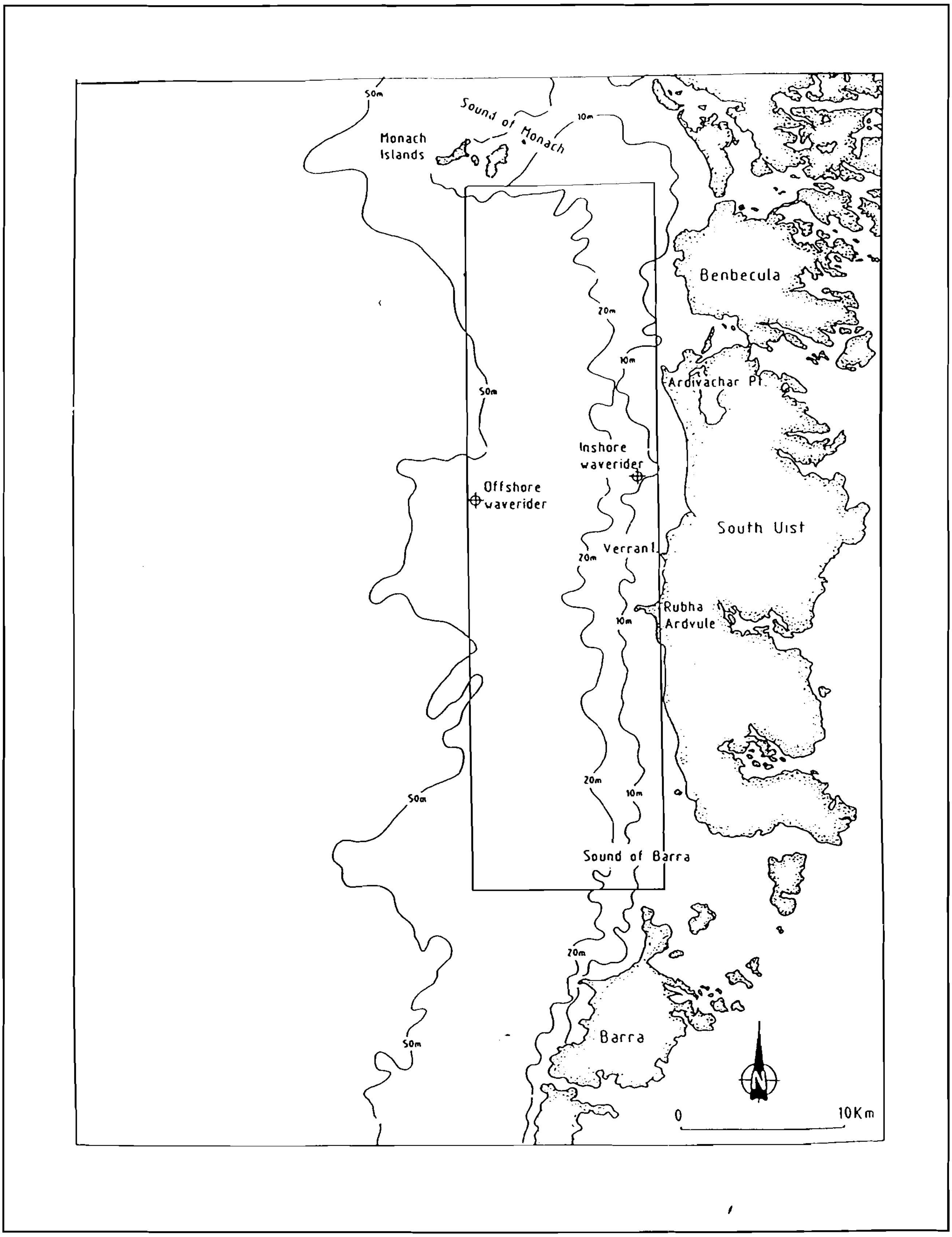



**Figure 6.5 South Uist**

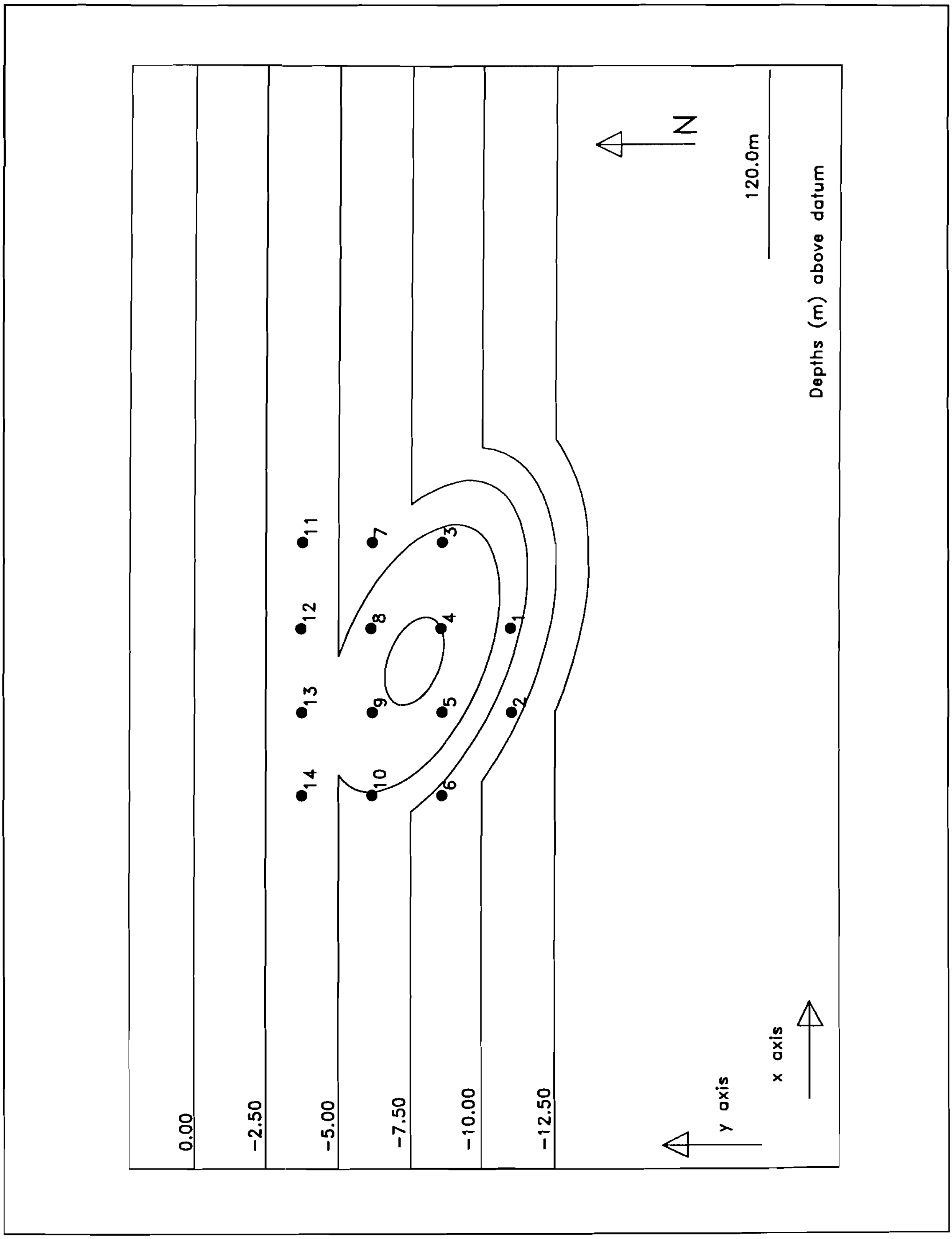



**Figure 6.6 Elliptic Shoal with Currents**

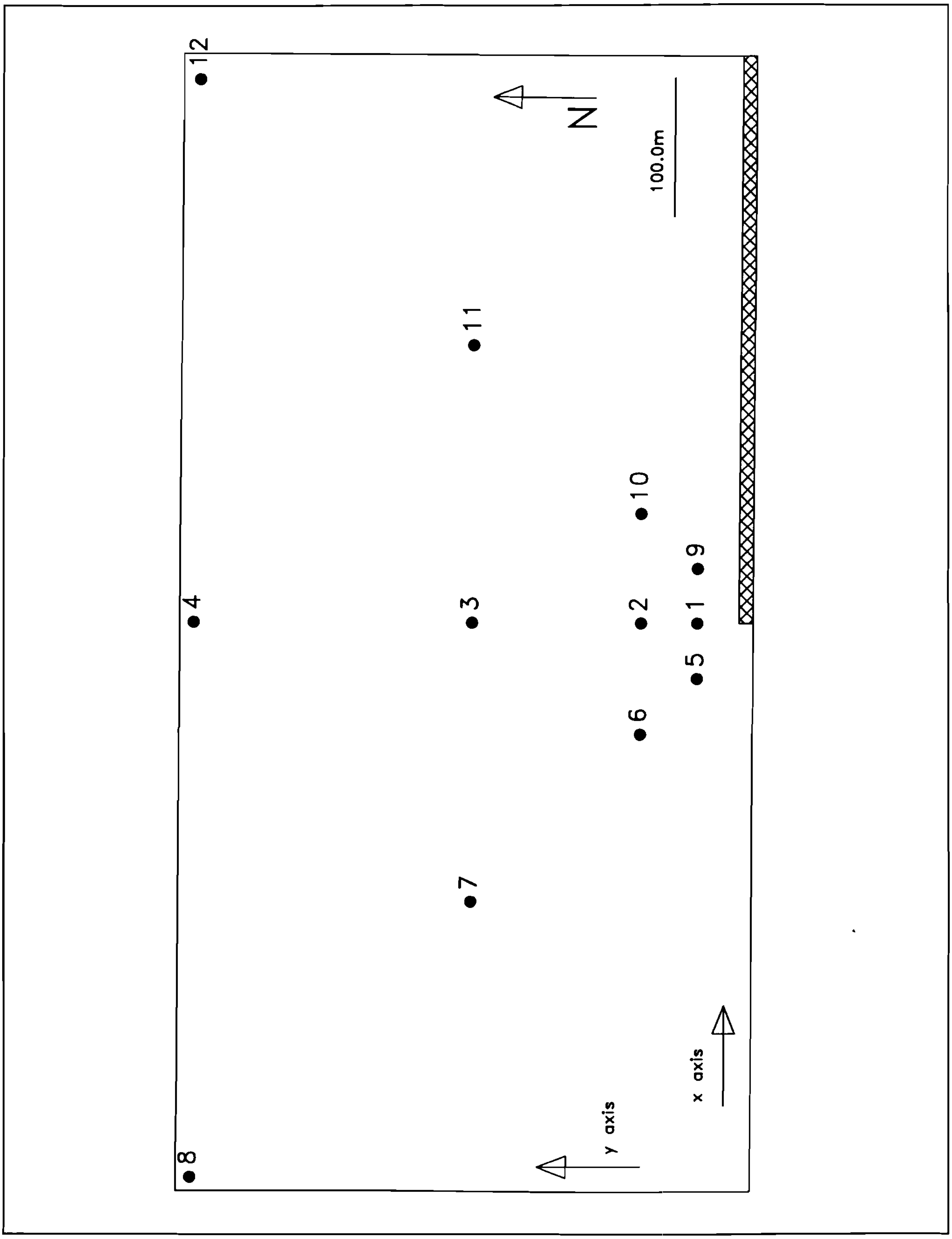



**Figure 6.7** **Semi-Infinite Breakwater**

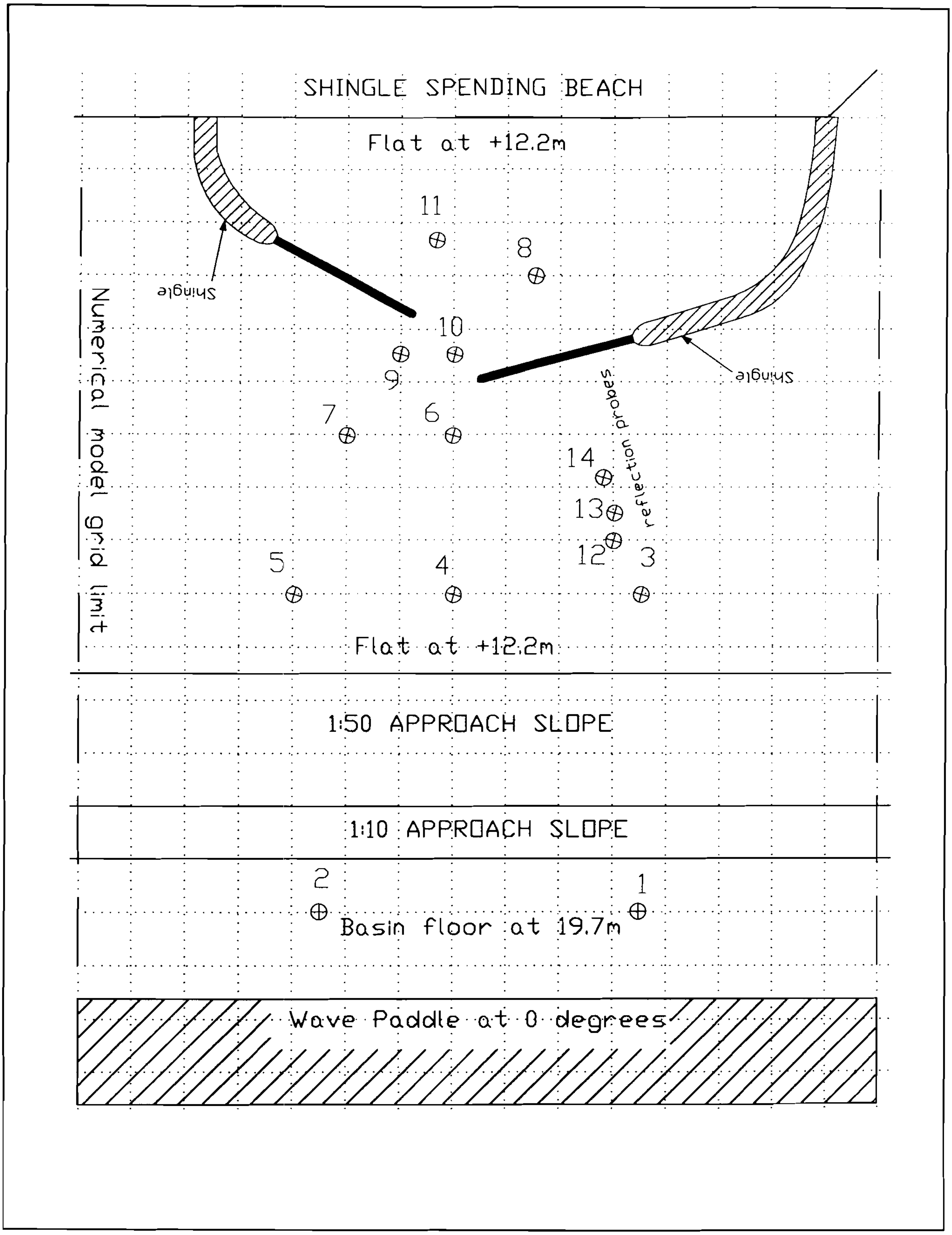



**Figure 6.8 Idealised Harbour Entrance**

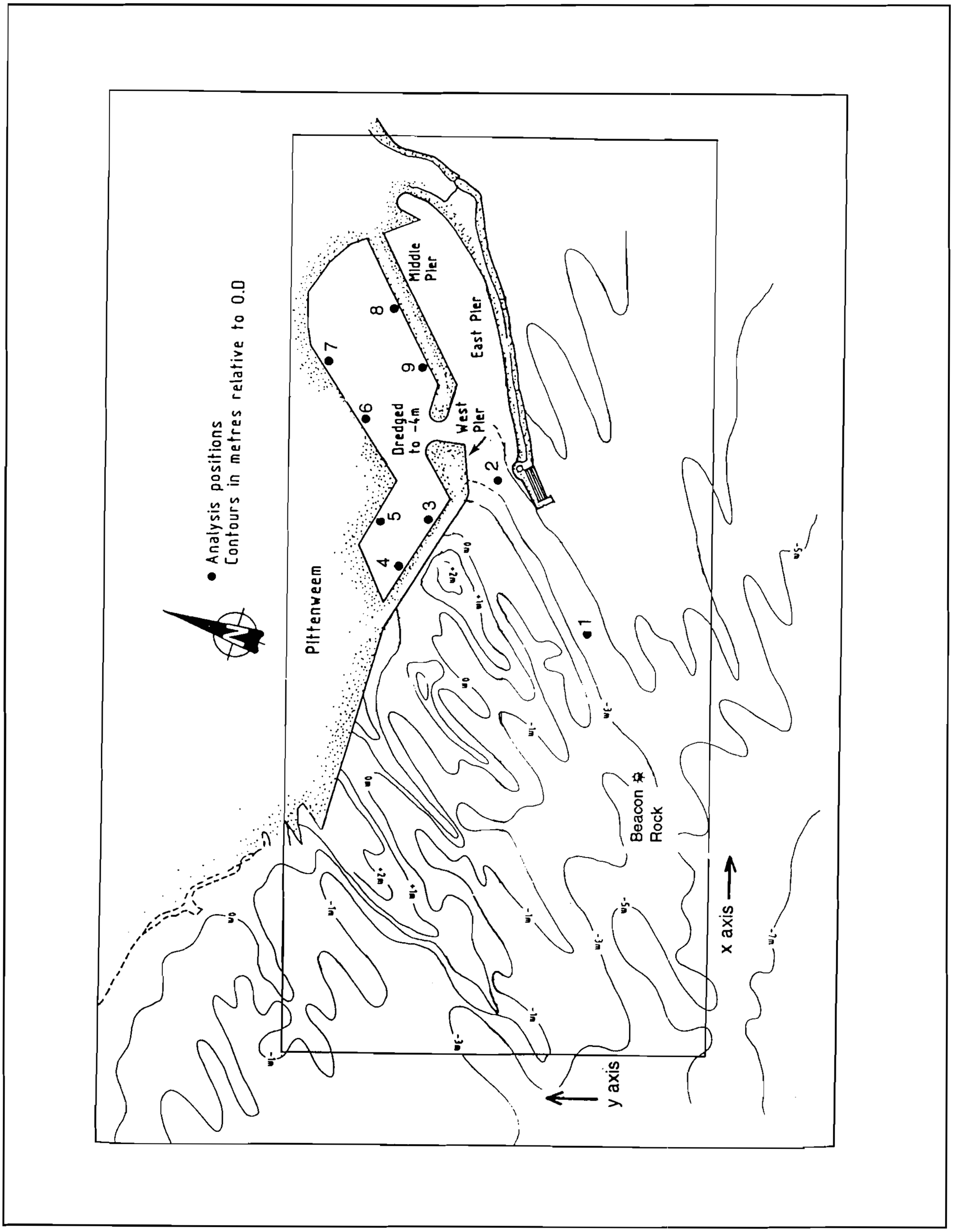



**Figure 6.9 Pittenweem Harbour**

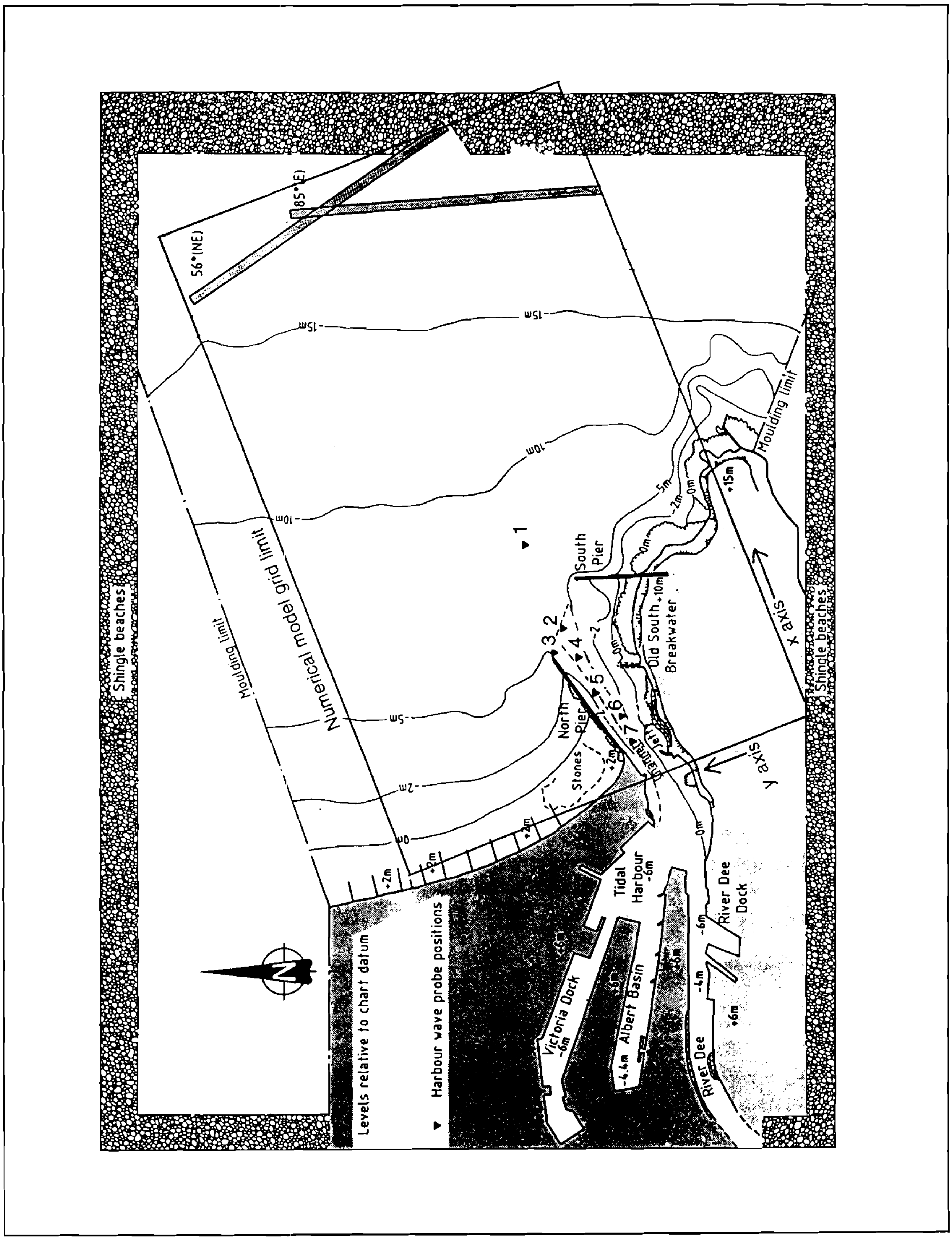



**Figure 6.10 Aberdeen Harbour**

# Appendices

## Appendix 1

Wave transformation model descriptions

## MODEL DESCRIPTION : ARMADA

**Model Name** : ARMADA

**Model type** : Wave transformation

**Authors** : B Li

**Date** : 1990

**Users** : Sir William Halcrow and Partners

**Application Areas :**

This model is suitable for coastal sites where shoaling, refraction and diffraction, due to changes in the bathymetry are important, but where reflections are insignificant.

**Output :**

- Wave heights throughout the area being modelled.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction
- sea-bed friction
- wave breaking
- directional and frequency spreading
- wave-current interaction

**Limitations of Use :**

ARMADA does not model sea-bed friction due to a varying sea-bed material within the model area. Since ARMADA does not represent reflection, it is not suited to applications where reflections are significant.

**Input :**

- A grid of depth values which represents the bathymetry in the area being modelled.
- Details of the computational grids.
- Coefficient of sea-bed friction.
- The offshore wave conditions specified in terms of wave height, period and direction.
- If required, the current velocity throughout the grid system.

**Governing Assumptions** :

- Weak non-linearity since the model is based on linear wave theory.
- The mild slope equation is valid for waves with small amplitude, over a sea-bed of mild slope.
- The reflected wave field is negligibly small.
- A Rayleigh wave height distribution.
- Wave amplitude does not vary rapidly over one wavelength.

**Modelling Technique :**
ARMADA has been developed from the multigrid model developed by Li and Anastasiou at Imperial College, London. Armada can be run in either a spectral or monochromatic mode and is based on the solution of the mild slope equation (mse), derived by Berkhoff (1972). The equation solved by ARMADA is given by

$$\nabla^2\Psi+\nabla\Psi\cdot\nabla\Psi+k_c^2=0 \tag{3}$$

together with suitable boundary conditions, where $\Psi$ is such that the velocity potential $\Phi=e^{\Psi}$ and $k_c$ is the effective wave number. As discussed by Radder (1979), $\Psi$ is a less rapidly varying function than the velocity potential and so if this substitution is made, fewer than eight grid points per wavelength may give a good representation of the wave, for all the dominant processes except reflection. However, there should be sufficient grid points to ensure an adequate resolution of the bathymetry. Within ARMADA, equation (1) is discretised using a standard finite difference scheme. The resulting system of equations is then solved using a modified form of the multigrid method, originally developed by Brandt (1977).

The spectral version of ARMADA consists of repeatedly running the monochromatic version, described above, with different frequencies and directions. The spectral wave field is then constructed by linear superposition of the monochromatic results using a Rayleigh wave height distribution.

Energy dissipation due to sea-bed friction and wave breaking has been included in Armada. Sea-bed friction is modelled using the method described in Dalrymple et al (1984), which can account for several different types of sea-bed material. This method involves the inclusion of a damping term in the governing equation. The effect of wave breaking is represented by limiting the wave height to be a fraction of the water depth.

**Validation :**
ARMADA has been validated against both physical model and field data.

**References :**
Li B. and Anastasiou K. (1992) Efficient elliptic solvers for the mild-slope equation using the multigrid technique, Coastal Engineering, 16 (1992) 245-266.

Al-Mashouk M., Reeve D.E., Li B. and Fleming C.A. (1992) ARMADA : An efficient spectral wave model, In Proceedings of the Conference on Hydraulic and Environmental Modelling in Coastal Waters (Bradford 1992).

## MODEL DESCRIPTION : CGWAVE

**Model Name** : CGWAVE

**Model Type** : Wave transformation and disturbance

**Author** : V G Panchang (University of Maine)

**Date** : 1992

**Users** : Kirk McClure Morton

**Application Areas :**

CGWAVE can be used to model coastal sites, bounded by land and an (artificial) open sea boundary, where refraction, shoaling, diffraction and reflection are the dominant physical processes.

**Output :**

- Wave height and wavelength throughout the model domain.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction
- sea-bed friction
- reflection

**Limitations of Use :**

It is recommended that the finite element grid contains at least five grid points per wavelength, to ensure accurate representation of the waves.

**Input :**

- A representation of the bathymetry in the model area specified in x, y, z form.
- Triangular grid representing the area to be modelled.
- The offshore incident wave condition in terms of wave height, period and direction.
- Reflection boundaries with associated reflection coefficients.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- The mild slope equation is valid for waves of small amplitude over a sea-bed of mild slope.

**Modelling Technique :**

CGWAVE is based on the solution of the elliptic mild slope equation (mse) derived by Berkhoff (1972). The equation solved by CGWAVE is given below:

$$\nabla \cdot (CC_g \nabla \Phi) + \frac{C_g}{C}\sigma^2 \Phi + i\sigma w \Phi = 0 \qquad \textbf{(9)}$$

where $\Phi(x,y)$ is the complex surface elevation function, from which the wave height and direction can be estimated, $\sigma$ is the wave frequency under consideration, $C(x,y) = \frac{\sigma}{k}$ is the phase velocity, $C_g(x,y) = \frac{\partial \sigma}{\partial k}$ is the group velocity, $w$ is the dissipation coefficient, and $k(x,y) = \left( = \frac{2\pi}{L} \right)$ is the wave number, related to the local depth $d(x,y)$ through the dispersion relation $\sigma^2 = gk\tanh(kd)$.

Equation (1) is solved using a finite element method. A system of equations is set up which is solved using an iterative solution scheme based on the conjugate-gradient method. Although elliptic equations can be very expensive to solve, in terms of computing time and storage, the conjugate gradient method used in CGWAVE enables the model to be used on large domains relatively efficiently.

Frictional dissipation is modelled through the dispersion factor, *w*, in equation (1). The incident wave amplitude and a user defined friction coefficient are used to specify this factor.

**Validation :**

CGWAVE has been validated against physical model results

**References :**

University of Maine (1993) CGWAVE: A coastal wave transformation model. General information and User's manual.

Panchang V.G., Pearce B.R., Wei G. and Cushman-Roisin B. (1991) Solution of the mild-slope wave equation by iteration, Applied Ocean Research, Vol. 13, No. 4.

## MODEL DESCRIPTION : ENDEC

**Model Name** : ENDEC v2.11

**Model Type** : Wave transformation and generation

**Author** : Delft Hydraulics

**Date** : 1990

**Users** : Posford Duvivier

**Application Areas :**

At coastal sites where the wave conditions at single locations are required and where the offshore bathymetry is relatively simple.

**Output :**

- Wave height, wave directions and water depths at points along the wave trajectory.

**Physical Processes Modelled :**

- shoaling
- refraction
- sea-bed friction
- wave breaking
- wave-current interaction
- energy gain due to winds

**Limitations of Use :**

ENDEC does not include the physical process of diffraction and should only be used in areas where the bathymetry is such that depth contours are straight and parallel.

**Input :**

- Incident wave conditions specified in terms of significant wave height and peak wave period.
- Bathymetry along a line normal to the shore.
- Angle of the incident wave to the shore normal.
- Friction coefficient.
- Wind velocity.

**Governing Assumptions :**

- The random wave field is linear and can be described by the peak wave period and energy density.
- The depth contours are straight and parallel.

**Modelling Technique :**

ENDEC is based on the solution of two first order differential equations - the wave action equation and the radiation stress equation. From these equations the variation in wave energy and the mean water level can be derived. The development of wave energy is modelled along a user defined wave ray. The equations are solved using a fourth order Runga-Kutta method with a variable step size so that energy decay in shallow water can be modelled correctly and efficiently.

Sea-bed friction is modelled in ENDEC using the method presented by Stive and Dingemans (1984) which also accounts for wave growth due to wind action. Energy dissipation due to wave breaking is modelled using the bore approach presented by Battjes and Janssen (1978).

**Validation :**

ENDEC has been validated by Delft Hydraulics.

**References :**

Delft Hydraulics (1990) ENDEC User Manual.

## MODEL DESCRIPTION : HISWA

**Model Name** : HISWA v920304

**Model Type** : Wave transformation and generation

**Author** : Delft University of Technology

**Date** : 1985 (Revised 1992)

**Users** : Posford Duvivier

**Application Areas :**

Large coastal sites, where the bathymetry may be complicated, particularly when the wave conditions at multiple points along the shore are required.

**Output :**

- Includes data on significant wave heights, peak periods, directions, directional spreading, energy dissipation, wave breaking, orbital bed velocities, wave steepness and wavelength for any user defined point.
- Plots of significant wave heights and peak periods, isolines of other data including energy dissipation, wave breaking and steepness.
- Vectors of energy transport, current velocity, wave induced stress and direction of energy transport.

**Physical Processes Modelled :**

- shoaling
- refraction
- sea-bed friction
- wave breaking
- directional spreading
- wave-current interaction
- energy gain due to winds
- energy dissipation due to currents

**Limitations of Use :**

HISWA does not represent diffraction and since a forward stepping solution method is used, only waves propagating in the forward direction are modelled.

**Input :**

- A grid of depth values which represent the bathymetry of the area being modelled.
- Incident wave conditions specified in terms of significant wave height, wave period and wave direction.
- Wave breaking and sea-bed friction coefficients.
- Wind speed and direction.
- Current blocking factors and current velocities.

**Governing Assumptions :**

- The model is independent of time.
- The energy spectrum is integrated over frequency.

**Modelling Technique :**

Within HISWA, the propagation of wave energy is based on the solution of an energy balance equation. This equation has been adapted to include terms for wave growth by wind action and energy dissipation due to sea-bed friction and wave breaking. The equations are solved using explicit and implicit finite difference schemes. Due to stability criteria for the former scheme, the step sizes used in the finite difference grid are restricted and the incident wave direction should be within 60° either side of the principal wave direction.

Sea-bed friction in HISWA is based on the conventional formulation for periodic waves, that is, the quadratic friction law, with appropriate parameters adapted to suit a random wave field in an ambient current field. The mean frequency for each spectral wave direction is affected by friction, under the assumption that the wave energy dissipation due to bottom friction affects only the energy at low frequencies.

The total energy dissipated through wave breaking is distributed proportionally over the wave directions in HISWA. The dissipation is determined using the bore model presented by Battjes and Janssen (1978) for those waves higher than some threshold value.

**Validation :**

HISWA has been validated against various test cases as reported in the literature, for example, Holthuijsen, Booij and Herbers (1989).

**References :**

Delft University of Technology (1992) HISWA User Manual.

## MODEL DESCRIPTION : LINDAL

**Model Name** : LINDAL (LINear DALrymple)

**Model Type** : Wave transformation

**Author** : E G Pitt

**Date** : 1992-93

**Users** : Applied Wave Research

**Application Areas :**

Areas where refraction and shoaling are the dominant processes and where wave diffraction may be ignored.

**Output :**

- Refraction coefficients throughout the area being modelled. This nears the total wave transformation coefficient.
- A field of grids where the breaking criterion has been met.
- A field of propagation directions (rays and/or orthogonals as required).
- Graphical output.

**Physical Processes Modelled :**

- shoaling
- refraction
- sea-bed friction
- wave breaking
- wave-current interaction

**Limitations of Use :**

LINDAL does not model wave diffraction or the effect of a varying sea-bed material in the area of interest.

**Input :**

- A rectangular grid of depth values which represents the bathymetry of the area being modelled.
- Wave heights and periods at an offshore boundary.
- The general direction of depth contours near the up-wave lateral boundary.
- A global sea-bed friction coefficient.
- Mean currents at all grid points if required.

**Governing Assumptions :**

- Conservation of wave action.
- Irrotationality of the wavenumber.

**Modelling Technique :**

LINDAL is based on a model developed by Dalrymple (1988) which is suitable for a personal computer and represents refraction and shoaling of both linear and non-linear waves. Wave-current interaction can also be modelled. LINDAL uses a finite difference method to solve

$$\nabla_h \times \overline{k} = 0 \tag{1}$$

and

$$\nabla_h \cdot \overline{A} = 0 \tag{2}$$

where $\overline{k}$ is the wave number and $\overline{A}$ is the wave action.

Equation (1) is based on the irrotationality of the wave number and equation (2) is based on the conservation of wave action. LINDAL uses a forward marching finite difference scheme which means that large areas can be modelled efficiently.

Sea-bed friction is modelled in LINDAL using the wave action dissipation method given by Christoffersen and Jonsson (1980). The term $\frac{\varepsilon}{\omega}$, where $\varepsilon$ is the average dissipation per unit area and $\omega$ is the intrinsic frequency, is subtracted from the right hand side of equation (2).

Within LINDAL, the wave height is not allowed to exceed a constant fraction of the water depth. At present this value is 0.78, but work on wave breaking within the model is ongoing.

**Validation :**

LINDAL has been validated against test results in the literature and over simple idealised bathymetries and current distributions.

**References :**

Dalrymple R.A. (1988) Model of refraction of water waves, Journal of Waterway, Port, Coastal and Ocean Engineering, Vol. 114, No 4.

Christoffersen J.B. and Jonsson I.G. (1980) A note on wave action conservation in a dissipative current wave motion, Applied Ocean Research, Vol. 2, No 4.

## MODEL DESCRIPTION : MIKE 21 NSW

**Model Name** : MIKE 21 NSW (Nearshore Spectral Wind-Wave Model)

**Model Type** : Wave transformation and generation

**Author** : Danish Hydraulic Institute

**Date** : 1990-1991

**Users** : DHI, WS Atkins

**Application Areas :**

This model can be used to transform an offshore wave field to a nearshore area in open coastal regions.

**Output :**

- Significant wave heights, mean periods, mean wave directions and the directional standard deviations throughout the area being modelled.
- Radiation stresses which can be used in the calculation of wave-driven currents.

**Physical Processes Modelled :**

- shoaling
- refraction
- sea-bed friction
- wave breaking
- directional spreading
- wave-current interaction
- energy gains due to winds

**Limitations of Use :**

MIKE 21 NSW does not include the effects of reflection or diffraction and so is unsuitable for use in areas where these processes may be significant.

**Input :**

- Bathymetric data.
- Current and wind fields.
- A sea-bed friction coefficient map.
- Offshore boundary condition.

**Governing Assumptions :**

- There is a stationary wind field across the area being modelled.
- The physical processes of diffraction and reflection can be neglected.
- There is a predominant wave direction.

**Modelling Technique :**

MIKE 21 NSW is based on the solution of the conservation equations for the zero-th and first moment of the wave action spectrum.

The effects of wave growth and dissipation due to wind generation, sea-bed dissipation and wave breaking are represented as the source terms in the action balance equation. A forward marching finite difference method, in a direction roughly parallel to the main wave propagation direction, is used to solve the equations. Due to stability considerations, the angle of incidence should typically be <60° and there is a restriction on the step sizes allowed.

MIKE 21 NSW also includes the effect of energy dissipation due to sea-bed friction and wave breaking on the mean wave period. It is assumed that the dissipation of energy is concentrated on the low frequency side of the frequency spectrum. Hence, the energy dissipation has the effect of reducing the mean wave period.

**Validation :**

MIKE 21 NSW has been validated using measured data.

**References :**

Danish Hydraulic Institute (1991), User Guide and Reference Manual, MIKE 21 NSW.

Holthuijsen L.H., Booij N. and Herbers T.H.C. (1989) A Prediction Model for Stationary, Short-crested Waves in Shallow Water with Ambient Current, Coastal Engineering (13), 1989, 23-54.

**MODEL DESCRIPTION : MIKE 21 PMS**

**Model Name** : MIKE 21 PMS (Parabolic Mild-Slope Wave Model)

**Model Type** : Wave transformation

**Author** : Danish Hydraulic Institute

**Date** : 1992-1993

**Users** : DHI
WS Atkins

**Application Areas :**

MIKE 21 PMS is suited for limited open coastal areas, for example, navigation channels, where wave diffraction effects are important, but where reflections can be ignored.

**Output :**

- Significant wave heights, mean periods, mean wave directions and radiation stresses throughout the area being modelled.
- In spectral mode, a frequency energy spectrum at selected points can also be output.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction normal to the principal wave direction
- sea-bed friction
- wave breaking
- directional and frequency spreading

**Limitations of Use :**

MIKE 21 PMS does not include the effects of reflection, diffraction in the principal wave direction or current refraction. For the parabolic approximation method to work well waves should approach at small angles to the principal wave direction (usually less than ±60°). The finite difference grid must be fine enough to ensure the wavelength is adequately resolved.

**Input :**

- A grid of depth values which represents the bathymetry of the area being modelled.
- The wave conditions at the offshore boundary specified in terms of wave height, wave period and wave direction for monochromatic waves. If the spectral mode is required, a directional-frequency energy spectrum must be supplied.
- A sea-bed friction coefficient map.

**Governing Assumptions :**

- Mild bottom slope. The derivation of the mild slope equation assumes that variations in the sea-bed must be small.
- Weak non-linearity since the model is based on linear wave theory.
- The reflected wave component in the negative x-direction is negligibly small and the main effects are in the direction of wave propagation.

**Modelling Technique :**

MIKE 21 PMS is based on parabolic approximations to the mild slope equation, which was originally derived by Berkhoff (1972). The model can be used for both monochromatic and spectral analyses.

Within MIKE 21 PMS, three different parabolic approximations to the mild slope equation may be solved:

(a) Simple approximation
(b) Padé approximation
(c) Minimax approximation (10, 20, 30, ...90°)

The Padé and MINIMAX parabolic approximations were derived to allow for more successful treatment of large angles of wave incidence, while not causing significant distortion to the wave conditions for small angles of incidence. Each of the equations in MIKE 21 PMS is solved using a forward marching finite difference technique.

The energy dissipative processes of sea-bed friction and wave breaking are included in the model. The formulation of sea-bed friction is based on the quadratic friction law. Wave breaking caused by large wave steepness and limiting water depth is based on the formulation of Battjes and Janssen (1978).

**Validation :**

MIKE 21 PMS has been validated using artificial test cases.

**References :**

Danish Hydraulic Institute (1993), User Guide and Reference Manual, MIKE 21 PMS, in preparation.

J Kirby T. (1986) Rational Approximations in the Parabolic Equation Method for Water Waves, Coastal Engineering, 10 (1986), 355-378.

## MODEL DESCRIPTION : MULTIGRID

**Model Name** : MULTIGRID

**Model type** : Wave transformation

**Author** : K Anastasiou and B Li (Imperial College, London)

**Date** : 1991

**Users** : Scott Wilson Kirkpatrick
Associated British Ports Research and Consultancy Ltd

**Application Areas :**

This model is suitable for use in open coastal sites where shoaling, refraction and diffraction, due to sandbanks, channels etc., may be important.

**Output :**

- Wave heights and directions at each point of a grid covering the area being modelled.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction
- sea-bed friction
- wave breaking
- wave-current interaction

**Limitations of Use :**

MULTIGRID does not model sea-bed friction due to a varying sea-bed material within the model area. Since MULTIGRID does not represent reflection, it is not suited to applications where reflections are significant.

**Input :**

- A grid of depth values which represents the bathymetry of the area being modelled.
- The offshore wave conditions specified in terms of wave amplitude, period and direction.
- The bed roughness.
- The current field where appropriate.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- The mild slope equation is valid for waves with small amplitude, over a sea-bed of mild slope.
- The reflected wave field is negligibly small.
- Wave amplitude does not vary rapidly over one wavelength.

**Modelling Technique :**

MULTIGRID has been developed from the multigrid model developed by Li and Anastasiou at Imperial College, London. MULTIGRID can be run in either a spectral or monochromatic mode and is based on the solution of the mild slope equation (mse), derived by Berkhoff (1972). The equation solved by MULTIGRID is given by

$$\nabla^2\Psi+\nabla\Psi\cdot\nabla\Psi+k_c^2=0 \qquad \textbf{(4)}$$

together with suitable boundary conditions, where $\Psi$ is such that the velocity potential $\Phi=e^{\Psi}$ and $k_c$ is the effective wave number. As discussed by Radder (1979), $\Psi$ is a less rapidly varying function than the velocity potential and so if this substitution is made, fewer than eight grid points per wavelength may give a good representation of the wave, for all the dominant processes except reflection. However, there should be sufficient grid points to ensure an adequate resolution of the bathymetry.

Within MULTIGRID, equation (1) is discretised using a standard finite difference scheme. The resulting system of equations is then solved using a modified form of the multigrid method, originally developed by Brandt (1977).

Energy dissipation due to sea-bed friction and wave breaking has been included in Armada. Sea-bed friction is modelled using the method described in Dalrymple et al (1984), which can account for several different types of sea-bed material. This method involves the inclusion of a damping term in the governing equation. The effect of wave breaking is represented by limiting the wave height to be a fraction of the water depth.

**Validation :**

MULTIGRID has been validated against physical model tests.

**References :**

Li B. and Anastasiou K. (1992) Efficient elliptic solvers for the mild-slope equation using the multigrid technique, Coastal Engineering, 16 (1992) 245-266.

**MODEL DESCRIPTION : ORCAWAVE**

**Model Name** : ORCAWAVE

**Model Type** : Wave transformation

**Author** : Orcina Ltd. Consulting Engineers

**Date** : 1990

**Users** : Orcina

**Application Areas :**

ORCAWAVE is suitable for use at coastal sites where refraction and shoaling due to variation in the bathymetry are important, but where reflection and diffraction effects ate negligible.

**Output :**

- Wave height, wave speed, wave length and water depth
- Wave refraction pattern showing paths of wave rays
- Colour plots showing water depth
- Areas where breaking criterion has been met

**Physical Processes Modelled :**

- refraction
- shoaling
- sea-bed friction
- wave breaking

**Limitations of Use :**

ORCAWAVE does not represent diffraction and reflection effects and hence is not suited to modelling harbour areas.

**Input :**

- Sea-bed bathymetry either in contour or grid form.
- Offshore wave period direction and height.
- Water density and tide level.
- Sea-bed friction coefficient.
- Wave breaking constants.

**Governing Assumptions :**

- Linear wave theory and Snell's law
- Slowly varying sea-bed depth

**Modelling Technique :**

ORCAWAVE is a forward tracking ray model, in which wave rays are sent out at equal intervals from a line offshore and are tracked, in the direction of propagation, inshore. The path of each ray is determined by Snell's law and linear wave theroy is assumed. Sea-bed friction is modelled as specified in British Standard BS6349, part 1, 1984. That is, sea-bed friction is assumed to apply a force per unit length of wave crest given by $k\rho u^2$ where k is a friction coefficient defined by the user and u is the water particle velocity at the sea-bed. When intergrated over a wave cicle this gives a power loss per unit area equal to $(2k\rho v^3)/(3\pi)$ where v is the maximum water paarticle velocity at the sea-bed.

**Validation :**

ORCAWAVE has been validated against test cases for which theoretical results can be calculated, such as uniform plane sloping sea beds, where good agreement was obtained. Comparison with calculated results documented in literature has also shown good agreement.

**References :**

ORCAWAVE user manual.

## MODEL DESCRIPTION : OUTRAY

**Model Name** : OUTRAY / OUTURAY

**Model Type** : Wave transformation

**Author** : HR Wallingford

**Date** : 1989

**Users** : HR Wallingford, Acer Consultants Ltd, Kirk McClure Morton

**Application Areas :**

Coastal areas where depth refraction and shoaling, and current refraction, are the dominant processes and where diffraction is relatively unimportant. OUTRAY can be used to transform offshore spectral waves to single inshore points.

**Output :**

- The significant wave height, wave period and mean wave direction at the inshore point.
- The wave energy spectrum, in terms of frequency and direction, at the specified inshore point.
- A plot of the wave ray paths.

**Physical Processes Modelled :**

- shoaling
- refraction
- directional and frequency spreading
- wave-current interaction

**Limitations of Use :**

OUTRAY does not include diffraction, reflection or energy dissipative processes. Since the model calculates the wave conditions at a specified point, OUTRAY is not particularly suitable for studies in which the wave conditions throughout an area need to be predicted.

**Input :**

- A rectangular grid of depth values which represent the bathymetry of the area being modelled.
- The offshore wave conditions may be specified by wave height, period and direction, or in terms of a wave spectrum with frequency and/or direction spreading.
- If current refraction is to be modelled, current velocities at every point in the grid system are required for each stage of the current cycle.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- A wave in water of local depth, d, will behave similarly to a wave in water of constant depth, d.
- Diffraction, reflection and energy dissipation may be neglected.

**Modelling Technique :**

OUTRAY is a back tracking ray model in which wave rays are tracked from an inshore point to the offshore boundary.

The computation is split up into two stages. Firstly fans of rays, at small angular increments, are tracked from an inshore point of interest until they reach deep water. Wave refraction and shoaling are governed by Snell's Law and wave heights are calculated using the principle of conservation of energy between adjacent rays. By considering a large number of such ray paths, a set of matrices (transfer functions) are constructed. The transfer functions describe the transformation of wave energy between the offshore boundary and the inshore point of interest. The second stage of the OUTRAY model uses the transfer functions to modify the offshore spectrum. From this, the significant wave height, wave period and mean wave direction at the inshore point can be calculated.

PCM, the parallel contour model, also developed by HR Wallingford, is a simplified version of OUTRAY. As its name suggests, the model assumes that the depth contours are parallel to the coastline. This simplification greatly reduces the computation time and provides a quick, relatively inexpensive, though less sophisticated analysis.

OUTURAY is another version of the model, which includes the effects of current refraction.

**Validation :**

OUTRAY has been validated against both physical model data and measured field data at HR Wallingford.

**References :**

HR Wallingford (1989) The OUTRAY Wave Refraction Model, Training and User Manual, HR Wallingford Report EX 1914.

## MODEL DESCRIPTION : PARAB

**Model Name** : PARAB

**Model Type** : Wave transformation

**Author** : N Dodd (Bristol University) and HR Wallingford

**Date** : 1992

**Users** : HR Wallingford

**Application Areas :**

PARAB is suitable for use in coastal sites where refraction, shoaling and diffraction, due to variation in the bathymetry, may be important, but where reflections may be ignored.

**Output :**

- A grid of wave heights throughout the area being modelled.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction normal to the principal wave direction
- sea-bed friction
- wave breaking
- directional and frequency spreading

**Limitations of Use :**

PARAB does not include the effects of reflections, diffraction in the principal wave direction or current refraction. It cannot model bed friction for a varying sea-bed material. Due to the assumptions made in deriving this equation, the incident waves should, preferably, be at an angle less than 45° to the principal direction of propagation (the x axis). For an accurate representation of the waves, it is recommended that there should be at least eight grid points per wavelength. This means that a very fine finite difference grid may be required for large model areas and/or small wavelengths.

**Input :**

- A rectangular grid of depth values which represents the bathymetry of the area being modelled.
- Offshore wave conditions specified in terms of wave height, period and direction. If the spectral mode is used, a frequency spectrum and directional spreading function is also needed.
- The size and resolution of the finite difference grid (which can be finer than the resolution of the depth grid).
- Values for the bed friction and wave breaking coefficients.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- The mild slope equation is valid for waves of small amplitude over a sea-bed of mild slope.
- The reflected wave field is negligibly small and the main effects are in the direction of wave propagation.

**Modelling Technique :**

PARAB can be run in either a monochromatic or spectral mode. The model is based on Radder's (1979) parabolic approximation to the mild slope equation (mse) derived by Berkhoff (1972). The equation solved by PARAB is given as:

$$\frac{\partial \phi}{\partial x} = \frac{i}{2k}\frac{\partial^2 \phi}{\partial y^2} + \left[ik - \frac{1}{2k}\frac{\partial k}{\partial x}\right]\phi \qquad (5)$$

where $\phi$ is the wave field, $k$ is the wave number, $x$ is the main wave propagation direction and $y$ is the transverse direction. Within PARAB, equation (1) is solved using the implicit Crank-Nicholson finite difference method. This solution scheme is unconditionally stable.

The parabolic equation, given in (1), does not include non-linear effects such as sea-bed friction and wave breaking. However, these effects have been included in PARAB, using the following methods. Assuming the friction coefficient is independent of wave height and position, the Bretschneider and Reid (1954) formula is used to calculate the proportional change in wave height due to sea-bed friction at each grid point. The energy loss due to wave breaking is calculated whenever the wave height exceeds the breaking wave height. The breaking wave height is either specified as $0.55 \cdot d$, where $d$ is the total water depth, or is calculated using the formula derived by Weggel (1972). The wave heights calculated by solving equation (1) are then modified to take into account the energy lost due to sea-bed friction and wave breaking.

**Validation :**

PARAB has been validated against physical model data at HR Wallingford.

**References :**

HR Wallingford (1992) User guide for PARAB, HR Wallingford Report.

Dodd N. (1988) Parabolic Approximations in Water Wave Refraction and Diffraction, PhD Thesis, Department of Mathematics, University of Bristol.

## MODEL DESCRIPTION : PORTRAY

**Model Name** : PORTRAY

**Model Type** : Wave transformation and wave disturbance

**Author** : HR Wallingford

**Date** : 1988

**Users** : HR Wallingford

**Application Areas :**

PORTRAY may be used to predict wave conditions in the approaches to and within harbours where shoaling and refraction are important but diffraction due to the sea-bed is not significant.

**Output :**

- Significant wave heights, periods and directions defining the wave conditions at each grid point throughout the area being modelled.
- Information on rays used in the model run, including a plot of the ray paths.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction due to surface piercing structures
- sea-bed friction
- wave breaking
- wave-current interaction
- reflection

**Limitations of Use :**

PORTRAY does not model diffraction due to changes in the bathymetry nor does it model bed friction for a varying sea-bed material. The model is monochromatic, but a spectral version is currently being developed at HR.

**Input :**

- A rectangular grid of depth values representing the bathymetry of the area.
- The incident wave condition defined in terms of wave height, period and direction.
- Appropriate values of the friction coefficient and the breaking coefficient.
- Reflecting boundaries can be specified, together with suitable reflection coefficients.
- If required, the current velocity at every node in the grid system.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- A wave in water of local depth, d, will behave similarly to a wave in water of constant depth, d.
- Diffraction due to changes in the bathymetry may be neglected.

**Modelling Technique :**

PORTRAY is a monochromatic wave transformation model which uses a forward tracking ray method. The model may be run in one of two modes:

(1) Harbour mode is used to represent the wave disturbance within a port or a harbour.

(2) Coastal mode is used to model the propagation of waves from further offshore towards and into a harbour.

Only the coastal mode is described here.

Parallel wave rays are sent out at equal intervals from a line offshore and are tracked, in the direction of propagation, inshore. Each ray is tracked until either the grid boundary is reached, the ray runs ashore or the energy associated with the ray falls below a specified minimum. Sufficient rays need to be tracked to ensure that the inshore region of interest is adequately covered. Wave refraction and shoaling are governed by Snell's Law and wave heights are calculated from the principal of conservation of energy between adjacent wave rays. Each ray is tracked through a triangular grid and an energy ratio is calculated for each triangle. This ratio records the proportion of the offshore energy retained by that ray after modifications for shoaling, friction, breaking and reflections have been taken into account. Energy loss due to sea-bed friction and wave breaking is included within PORTRAY. The model uses the non-linear formula of Bretschneider and Reid (1954) to calculate the energy loss due to sea-bed friction. The energy dissipated by wave breaking, assuming a Rayleigh distribution of wave height, is calculated whenever the wave height exceeds the breaking wave height (taken as $0.78 \cdot h$, where $h$ is the total water depth).

The reflection of waves can also be modelled within PORTRAY. The user must specify a reflection coefficient for each reflecting boundary. When a wave ray intersects one of these boundaries, the wave energy is decreased appropriately and the angle of reflection is set equal to the angle of incidence.

An additional model, PORTURAY, has been developed as an extension to PORTRAY. PORTURAY includes the effects of current refraction on waves. As before, wave refraction and shoaling are governed by Snell's law, but are modified by current effects.

**Validation :**

PORTRAY has been validated against both physical model data and measured field data at HR Wallingford.

**References :**

HR Wallingford (1988) The PORTRAY Harbour Wave Disturbance Model Training and User Manual, HR Wallingford Report EX 1774.

## MODEL DESCRIPTION : REFRAC

**Model Name** : REFRAC v1.1

**Model Type** : Wave transformation

**Author** : Delft Hydraulics

**Date** : 1990

**Users** : Posford Duvivier

**Application Areas :**

Open coastal areas where refraction and shoaling are the dominant physical processes and where diffraction and friction are not significant.

**Output :**

- Tables of direction, shoaling and refraction coefficients and amplitudes.
- Plots of wave rays.
- Isolines of wave height.

**Physical Processes Modelled :**

- shoaling
- refraction

**Limitations of Use :**

REFRAC does not include the physical process of diffraction or energy dissipation processes such as sea-bed friction or wave breaking.

**Input :**

- A grid of depth values which represent the bathymetry of the area being modelled.
- Incident wave conditions specified in terms of wave period, wave direction, wave amplitude and wave phase.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- A gradual variation in sea-bed compared to the wavelength.
- There is no loss of energy through, for example, sea-bed friction.
- The vertical motion of free water surface is small.

**Modelling Technique :**

REFRAC is based on wave ray tracking methods. The wave rays can either be tracked from offshore to inshore (forward tracking) or from inshore to offshore (reverse tracking).

**Validation :**

No information supplied.

**References :**

Delft Hydraulics (1990) REFRAC User Manual.

## MODEL DESCRIPTION : W-RAY

**Model Name** : W-RAY

**Model Type** : Wave transformation

**Author** : Scott Wilson Kirkpatrick

**Date** : 1993

**Users** : Scott Wilson Kirkpatrick (SWK)

**Application Areas :**

Open coastal areas where refraction and shoaling are the dominant physical processes and where diffraction and friction are not important. W-RAY can be used to model waves, with both frequency and directional spreading, over very large areas.

**Output :**

- The significant wave height, zero crossing period and wave energy distribution, in terms of frequency and direction, at the specified inshore point.

**Physical Processes Modelled :**

- shoaling
- refraction
- directional and frequency spreading

**Limitations of Use :**

W-RAY does not include the physical processes of diffraction, reflection, current refraction or sea-bed friction. Since the model calculates the wave conditions at a specified point W-RAY is not particularly suitable for studies in which the wave conditions throughout an area are to be predicted.

**Input :**

- A grid of depth values which represent the bathymetry of the area being modelled.
- Incident wave conditions specified in terms of the mean offshore wave direction, a frequency spectrum and the index of the cosine directional spreading function to be applied.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- A slowly varying bed.
- Diffraction and reflection may be neglected.

**Modelling Technique :**

W-RAY is a reverse tracking ray model in which wave rays are tracked from an inshore point to the offshore boundary.

Wave breaking has been included in the model by limiting the wave height to be a fraction of the water depth.

**Validation :**

This model is still in a development stage. It will be validated before use on projects according to Scott Wilson Kirkpatrick's in-house QA requirements.

**References :**

Scott Wilson Kirkpatrick (1993) SWK W-RAY User Manual (draft).

## MODEL DESCRIPTION : WC2D

**Model Name** : WC2D

**Model Type** : Wave transformation

**Author** : D H Yoo, N J MacDonald and B A O'Connor (University of Liverpool)

**Date** : early 1990's

**Users** : Binnie and Partners
Associated British Ports Research and Development

**Application Areas :**
Areas where refraction, shoaling and diffraction are the dominant physical processes and where wave reflections may be ignored (that is, complex bathymetries and/or offshore islands or shoals).

**Output :**
- Wave vectors throughout the model area.
- Wave amplitudes.
- Longshore currents.
- Water surface elevations.

**Physical Processes Modelled :**
- shoaling
- refraction
- diffraction
- sea-bed friction
- wave breaking
- wave-current interaction
- turbulence and eddy viscosity

**Limitations of Use :**
Due to the use of an explicit finite difference scheme, there is a restriction on the time step size in order that the solution scheme remains stable.

**Input :**
- A grid of depth values representing the bathymetry in the area being modelled.
- The offshore incident wave condition specified in terms of wave amplitude, wave period and wave direction.
- A global bottom roughness value.
- If required, a representation of the currents expected in the model area.
- Suitable boundary conditions for the site.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- Variations in the topography are small (<< wavelength L).
- A 2-D depth averaged system is valid.
- The reflected wave field is negligibly small.

**Modelling Technique :**

WC2D is based on the model developed by Yoo, O'Connor and MacDonald at Liverpool University. It is based on the solution of the equations given below. For a plane wave, the wave number vector, $K_i$, and the wave frequency, $\omega$, are given by:

$$\frac{\partial K_i}{\partial t}+\frac{\partial \omega}{\partial x_i}=0 \tag{6}$$

The effect of currents is included using the Doppler relation. Diffraction effects are included using the following relationship from Battjes:

$$K_i^2=k^2+\frac{1}{A}\frac{\partial^2 A}{\partial x_i^2}$$

where $A$ is the wave amplitude. The wave amplitude is derived from the energy conservation equation derived by Phillips:

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x}(EU_i+F_i)+S_{ij}\frac{\partial U_j}{\partial x_i}+\varepsilon = 0$$

Within WC2D the above equations are solved using explicit finite difference schemes, which are considered to be the most convenient to account for the full interaction between waves, currents and turbulent motions. The restriction on time step sizes due to the stability of the explicit finite difference scheme is thought to be preferable to solving the large number of simultaneous equations at each time step, which arise with stable implicit methods.

Sea-bed friction is accounted for by reducing the wave energy in each cell by the total frictional losses within the cell calculated using the wave parameters, bed roughness and local depth. Wave breaking will take place if $A>A_b=\frac{\pi}{7k}\tan\left(\frac{\omega}{\sigma}\left[0.8+\tanh(1.06I_r)kd\right]\right)$ where the Iribarren number $I_r=\frac{m\pi}{\sqrt{(kA)}}$.

**Validation :**

WC2D has been validated against published theoretical and physical model results.

**References :**

University of Liverpool (1990) WC2D manual.

## Appendix 2

Wave disturbance model descriptions

## MODEL DESCRIPTION : ARTEMIS

**Model Name** : ARTEMIS

**Model Type** : Wave disturbance

**Author** : Electricité de France

**Date** : 1992

**Users** : Electricité de France

**Application Areas :**

ARTEMIS may be used to predict wave conditions in the approaches to and within harbours where shoaling, refraction and diffraction due to the sea bed are significant.

**Output :**

- wave heights at grid nodes throughout the area being modelled.
- phase celerity and group celerity.
- components of speed at sea surface.
- wave number.
- real and imaginary potential.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction due to surface piercing structures and varying bathymetry
- reflection

**Limitations of Use :**

ARTEMIS does not model energy dissipating effects such as wave breaking and sea bed friction.

**Input :**

- A finite element, triangular, grid with depths at each node to represent the bathymetry of the area being modelled.
- Boundary conditions including incident wave height, direction and period and reflection coefficients of solid boundaries.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory
- Mild slope

**Modelling Technique :**

ARTEMIS was developed at Electricité De France and is run in monochromatic mode. The model is based on the solution of the mild slope equation:

$$\nabla.(CC_g\nabla\phi) + \omega^2\frac{C_g}{C}\phi = 0$$

where $\phi$ is the complex wave potential, **C** is wave celerity, $C_g$ is the group velocity and $\omega$ is the wave angular frequency. Within ARTEMIS this equation is solved over a finite element grid using an iterative solution scheme based on the conditioned conjugate gradient method.

**Validation :**

**References :**


ARTEMIS Release 2.0 - Principle note and User Manual, LNH Report HE-42/95/34/B (In French)

ARTEMIS Release 2.0 - Validation Document, LNH Report HE-42/95/35/A (In English)

## MODEL DESCRIPTION : DIFFRAC

**Model Name** : DIFFRAC version 4.14

**Model Type** : Wave disturbance

**Author** : Delft Hydraulics

**Date** : 1992

**Users** : Posford Duvivier

**Application Areas :**

Harbour areas where refraction and shoaling are not significant and relection and diffraction are the dominant processes.

**Output :**

- Wave heights at individual locations in the harbour and contour plots of wave heights over the whole area being modelled.

**Physical Processes Modelled :**

- reflection
- diffraction

**Limitations of Use :**

DIFFRAC does not include the effects of refraction and shoaling, or energy dissipation processes such as sea bed friction and wave breaking.

**Input :**

- Constant depth of water over the whole model.
- Incident wave conditions specified in terms of wave period, wave direction and wave height.
- Locations of reflection boundaries, together with suitable reflection coefficients.

**Governing Assumptions :**

- Small amplitude waves.
- No wave breaking or bed friction.
- Any effects due to changes in the bed bathymetry can be neglected.

**Modelling Technique :**

DIFFRAC uses a boundary element method to solve the Helmholtz equation.

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + k^2 = 0$$

Where $\phi$ is the complex wave potential and $k$ is the wave number.

**Validation :**

The model has been compared with diffraction around a semi-infinite breakwater with good results.

**References :**

User documentation.

## MODEL DESCRIPTION : OUTDIF

**Model Name** : OUTDIF

**Model Type** : Wave disturbance

**Author** : HR Wallingford

**Date** : 1989

**Users** : HR Wallingford

**Application Areas :**

Coastal areas where refraction, shoaling and diffraction due to surface piercing structures are the dominant processes. OUTDIF can be used to transform offshore spectral waves to single inshore points, in areas where the effect of one semi-infinite breakwater is important.

**Output :**

- The significant wave height, wave period and mean wave direction at the inshore point.
- The wave energy spectrum, in terms of frequency and direction, at the specified inshore point.
- A plot of the wave ray paths.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction due to surface piercing structures
- directional and frequency spreading

**Limitations of Use :**

OUTDIF does not include reflection or energy dissipative processes. Since the model calculates the wave conditions at a specified point, OUTDIF is not particularly suitable for studies in which the wave conditions throughout an area need to be predicted.

**Input :**

- A rectangular grid of depth values which represent the bathymetry of the area being modelled.
- The location of a semi infinite breakwater which is representative of a coastal structure around which waves will diffract.
- The offshore wave conditions may be specified by wave height, period and direction, or in terms of a wave spectrum with frequency and/or direction spreading.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- A wave in water of local depth, d, will behave similarly to a wave in water of constant depth, d.
- Diffraction is modelled using the Sommerfeld solution for a semi infinite breakwater.
- Reflection and energy dissipation may be neglected.

**Modelling Technique :**

OUTDIF is an extension of the OUTRAY wave refraction model, which includes the effects of diffraction by a semi-infinite breakwater. OUTDIF is a back tracking ray model in which wave rays are tracked from an inshore point to the offshore boundary.

The computation in OUTDIF is split up into two stages. Firstly fans of rays, at small angular increments, are tracked from an inshore point of interest until they reach deep water. Wave refraction and shoaling are governed by Snell's Law and wave heights are calculated using the principle of conservation of energy between adjacent rays. By considering a large number of such ray paths, a set of matrices (transfer functions) are constructed. The transfer functions describe the transformation of wave energy between the offshore boundary and the inshore point of interest. The second stage of the OUTDIF model uses the transfer functions to modify the offshore spectrum. From this, the significant wave height, wave period and mean wave direction at the inshore point can be calculated.

Diffraction is represented in OUTDIF by calculating diffraction coefficients for a semi infinite breakwater using the Sommerfeld solution. The diffraction coefficients are then used to modify the transfer function and the offshore spectrum so as to include diffraction effects. The modified transfer function and offshore spectra are then used to calculate wave condtitions at the nearshore point of interest.

**Validation :**

OUTDIF has been validated against examples for which analytical wave data exists.

**References :**


HR Wallingford (1989) The OUTDIF Wave Refraction/Diffraction Model, Training and User Manual, HR Wallingford Report EX 1924,

Sommerfeld A. Mathematical theorie der diffraktion. Mathematische Annalen 47, p317-374. 1986 (In German)

## MODEL DESCRIPTION : PORTCGS

**Model Name** : PORTCGS

**Model Type** : Wave disturbance

**Author** : HR Wallingford

**Date** : 1994

**Users** : HR Wallingford

**Application Areas :**

PORTCGS may be used to predict wave conditions in the approaches to and within harbours where shoaling, refraction and diffraction due to the seabed are significant.

**Output :**

- Significant wave heights, wave phase and instantaneous surface elevation at each grid point throught the area being modelled.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction due to surface piercing structures and varying bathymetry
- reflection

**Limitations of Use :**

PORTCGS does not model energy dissipation effects such as wave breaking and seabed friction.

**Input :**

- A rectangular grid of depth values representing the bathymetry in the area.
- Boundary conditions including incident wave height, direction and period and reflection coefficients for solid boundaries.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theroy
- Slowly varying seabed

**Modelling Technique :**

PORTCGS was developed at HR Wallingford and is run in monochromatic mode. The model is based on the solution of the mild slope equation:

$$\nabla.(CC_g \nabla \phi) + \omega^2 \frac{C_g}{C} \phi = 0$$

where $\phi$ is the complex wave potential, **C** is wave celerity, $\mathbf{C_g}$ is the group velocity and $\omega$ is the wave angular frequency. Within PORTCGS this equation is solved over a finite difference grid using an iterative solution scheme based on the pre-conditioned conjugate gradient method.

**Validation :**

POTRCGS has been validated against analytic solutions to problems involving internal and external diffraction and against physical model results.

**References :**

Tozer N P and Lawson J (1994) Development of a wave disturbance model including seabed diffraction, HR Wallingford Report SR389.

## MODEL DESCRIPTION : PORTRAY

**Model Name** : PORTRAY

**Model Type** : Wave transformation and wave disturbance

**Author** : HR Wallingford

**Date** : 1988

**Users** : HR Wallingford

**Application Areas :**

PORTRAY may be used to predict wave conditions in the approaches to and within harbours where shoaling and refraction are important but diffraction due to the sea-bed is not significant.

**Output :**

- Significant wave heights, periods and directions defining the wave conditions at each grid point throughout the area being modelled.
- Information on rays used in the model run, including a plot of the ray paths.

**Physical Processes Modelled :**

- shoaling
- refraction
- diffraction due to surface piercing structures
- sea-bed friction
- wave breaking
- wave-current interaction
- reflection

**Limitations of Use :**

PORTRAY does not model diffraction due to changes in the bathymetry nor does it model bed friction for a varying sea-bed material. The model is monochromatic, but a spectral version is currently being developed at HR.

**Input :**

- A rectangular grid of depth values representing the bathymetry of the area.
- The incident wave condition defined in terms of wave height, period and direction.
- Appropriate values of the friction coefficient and the breaking coefficient.
- Reflecting boundaries can be specified, together with suitable reflection coefficients.
- If required, the current velocity at every node in the grid system.

**Governing Assumptions :**

- Weak non-linearity since the model is based on linear wave theory.
- A wave in water of local depth, d, will behave similarly to a wave in water of constant depth, d.
- Diffraction due to changes in the bathymetry may be neglected.

**Modelling Technique :**

PORTRAY is a monochromatic wave transformation model which uses a forward tracking ray method. The model may be run in one of two modes:

(1) Harbour mode is used to represent the wave disturbance within a port or a harbour.

(2) Coastal mode is used to model the propagation of waves from further offshore towards and into a harbour.

Only the coastal mode is described here.

Parallel wave rays are sent out at equal intervals from a line offshore and are tracked, in the direction of propagation, inshore. Each ray is tracked until either the grid boundary is reached, the ray runs ashore or the energy associated with the ray falls below a specified minimum. Sufficient rays need to be tracked to ensure that the inshore region of interest is adequately covered. Wave refraction and shoaling are governed by Snell's Law and wave heights are calculated from the principal of conservation of energy between adjacent wave rays. Each ray is tracked through a triangular grid and an energy ratio is calculated for each triangle. This ratio records the proportion of the offshore energy retained by that ray after modifications for shoaling, friction, breaking and reflections have been taken into account. Energy loss due to sea-bed friction and wave breaking is included within PORTRAY. The model uses the non-linear formula of Bretschneider and Reid (1954) to calculate the energy loss due to sea-bed friction. The energy dissipated by wave breaking, assuming a Rayleigh distribution of wave height, is calculated whenever the wave height exceeds the breaking wave height (taken as $0.78 \cdot h$, where $h$ is the total water depth).

The reflection of waves can also be modelled within PORTRAY. The user must specify a reflection coefficient for each reflecting boundary. When a wave ray intersects one of these boundaries, the wave energy is decreased appropriately and the angle of reflection is set equal to the angle of incidence.

An additional model, PORTURAY, has been developed as an extension to PORTRAY. PORTURAY includes the effects of current refraction on waves. As before, wave refraction and shoaling are governed by Snell's law, but are modified by current effects.

**Validation :**

PORTRAY has been validated against both physical model data and measured field data at HR Wallingford.

**References :**

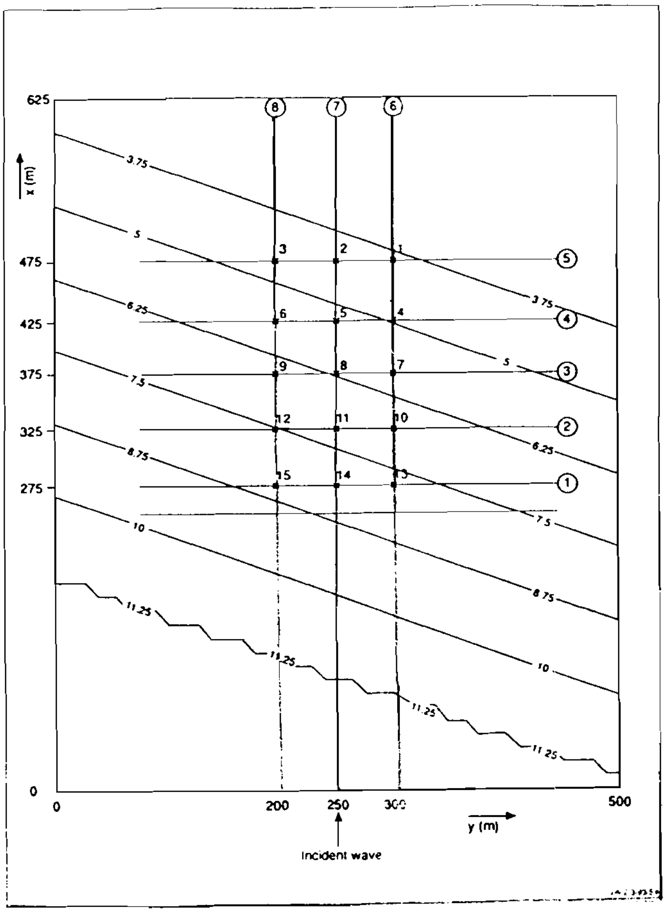
HR Wallingford (1988) The PORTRAY Harbour Wave Disturbance Model Training and User Manual, HR Wallingford Report EX 1774.

# Appendix 3

Bemnchmark test descriptions

TEST A

| | | |
|---|---|---|
| **Title** | : | Linear Beach. |
| **Physical Processes** | : | Refraction and shoaling due to the sloping seabed. |
| **Data Available** | : | The solution to this test case is given by Snell's law which can be computed at any required point in the area being modelled. |
| **Description** | : | The bathymetry used in this test case is shown in the diagram below. |




The plane slope (1:50) rises from a region of constant depth, $h$ = 11.25 m, and is at an angle of 20° to the normal. The slope is given by

$$h = \begin{cases} 11.25 & x' < -145.5 \\ 11.25 - 0.02(145.5 + x') & x' \geq -145.5 \end{cases}$$

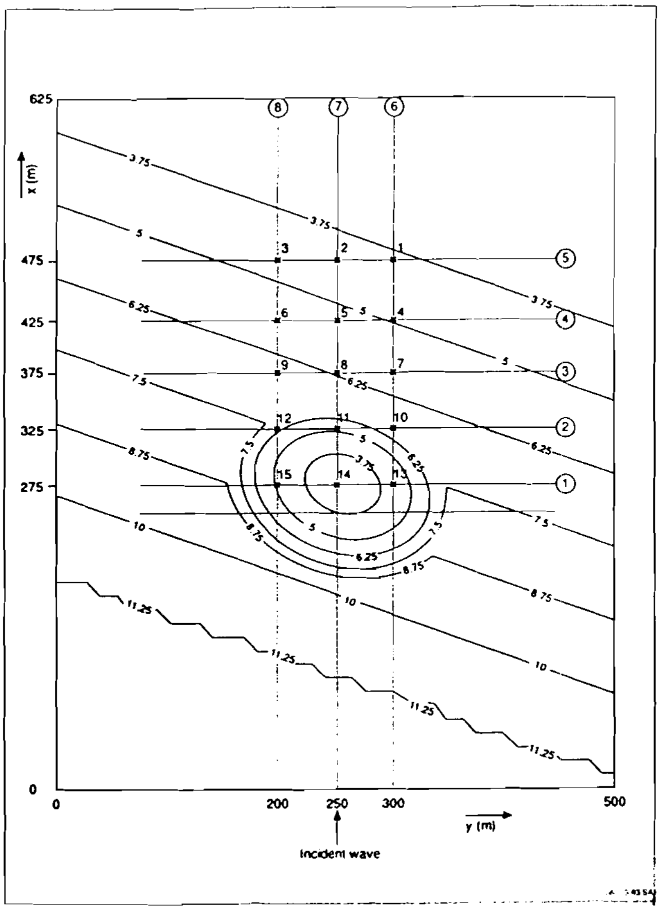
where $(x', y')$ are slope-oriented coordinates related to $(x, y)$ by

$$x' = (x - 262.5)\cos 20° - (250 - y)\sin 20°$$
$$y' = (x - 262.5)\sin 20° + (250 - y)\cos 20°$$

| | | |
|---|---|---|
| **Offshore Boundary** | : | The incident wave conditions are specified along $x = 0$. |
| **Bed Friction** | : | None. |
| **Wave Breaking** | : | Should be applied if included in the model. |
| **Input** | : | A monochromatic wave which has an amplitude of 0.58m and a period of 3.7 seconds. The incident direction is 0° to the x axis. |
| **Results Required** | : | The wave height coefficients at the fifteen analysis points and along each of the eight transects shown on the figure. |

## TEST B

| | | |
|---|---|---|
| **Title** | : | Elliptic Shoal. |
| **Physical Processes** | : | Refraction, shoaling and diffraction due to the varying bathymetry. |
| **Data Available** | : | Wave height coefficients along the transects measured during a laboratory experiment. |
| **Description** | : | The bathymetry used in this test case is shown in the diagram below. |




The elliptic shoal is situated on a plane slope (1:50), rising from a region of constant depth, $h$ = 11.25 m, and which is at an angle of 20° to the wave paddle. The slope is given by

$$h = \begin{cases} 11.25 & x' < -145.5 \\ 11.25 - 0.02(145.5 + x') & x' \geq -145.5 \end{cases}$$

where $\{x',y'\}$ are slope-oriented coordinates related to $\{x,y\}$ by

$$x' = (x - 262.5)\cos 20° - (250 - y)\sin 20°$$
$$y' = (x - 262.5)\sin 20° + (250 - y)\cos 20°$$

and the origin $\{x',y'\} = (0,0)$ corresponds to the centre of the shoal. The boundary of the elliptic shoal is given by

$$\left(\frac{x'}{75}\right)^2 + \left(\frac{y'}{100}\right)^2 = 1$$

and the depths in the region of the shoal are defined by

$$h = h_{slope} - 12.5\left[1 - \left(\frac{x'}{93.75}\right)^2 - \left(\frac{y'}{125}\right)^2\right]^{1/2} + 7.5$$

| | | |
|---|---|---|
| **Offshore Boundary** | : | The incident wave conditions are specified along $x = 0$. |
| **Bed Friction** | : | None. |
| **Wave Breaking** | : | Should be applied if included in the model. |
| **Input** | : | A monochromatic wave which has an amplitude of 0.58m and a period of 5 seconds. The incident direction is 0° to the x axis. |
| **Results Required** | : | The wave height coefficients at the fifteen analysis points and along each of the eight transects shown on the figure. |

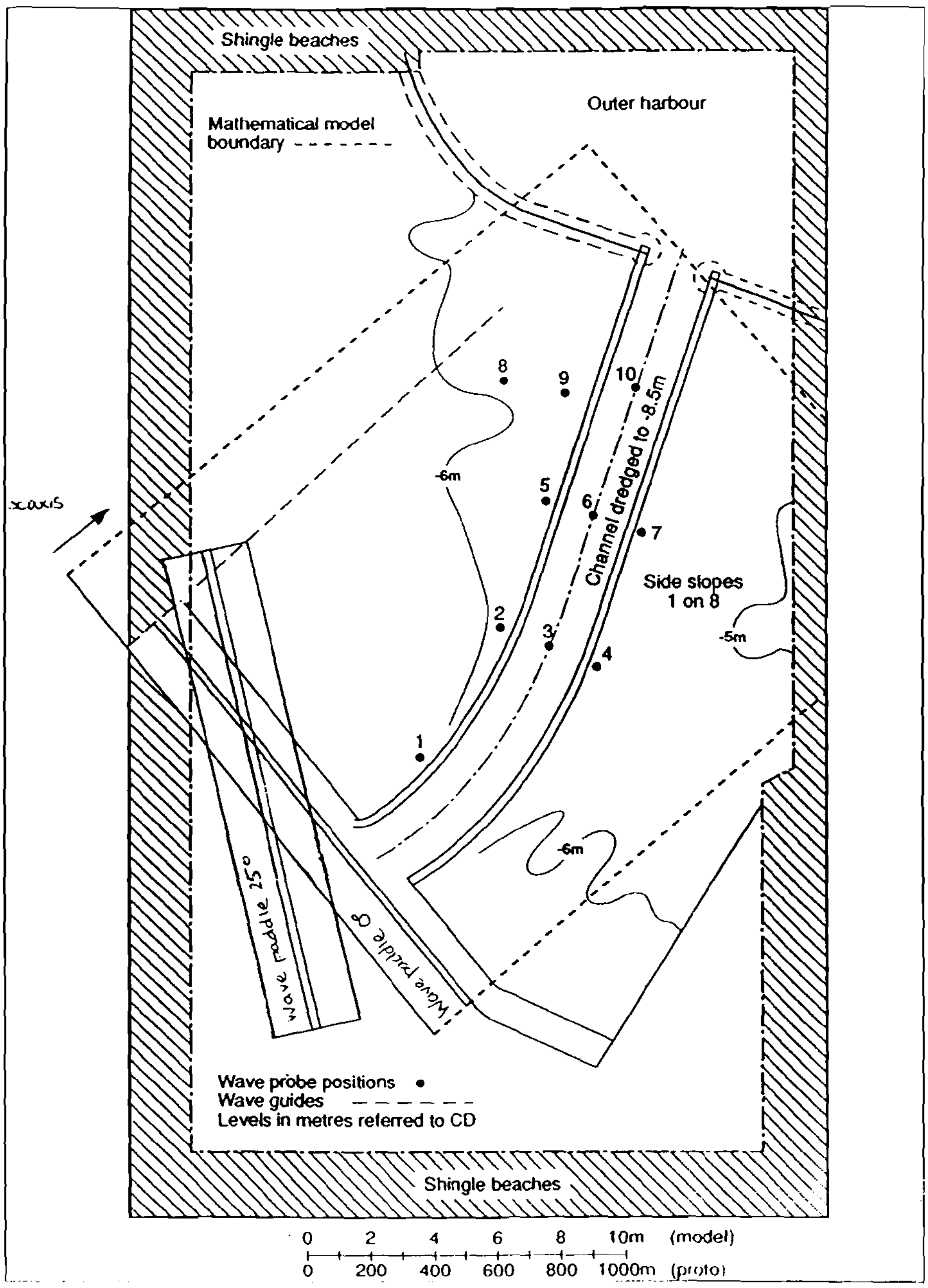
**TEST C**

**Title** : Harbour Approach Bathymetry.

**Physical Processes** : Refraction, shoaling and diffraction due to the varying bathymetry.

**Data Available** : Wave height measurements at ten positions (marked on the figure) recorded during a physical model study.

**Description** : The bathymetry used in this test case is shown in the diagram below.



This bathymetry is typical of a dredged harbour approach channel and so will be a good test of the models' capabilities.

**Offshore Boundary** : The incident wave conditions are specified along $x = 0$.

**Bed Friction** : Should be applied if included in the model. The seabed is relatively smooth with no significant bed formations such as rock outcrops etc.

**Wave Breaking** : Should be applied if included in the model.

**Tidal Levels** : The test should be run with a tidal level of +1.9m CD.

**Input** : The wave conditions to be specified along the boundary are given in Table 1. A JONSWAP spectrum should be used if the model can be run spectrally.

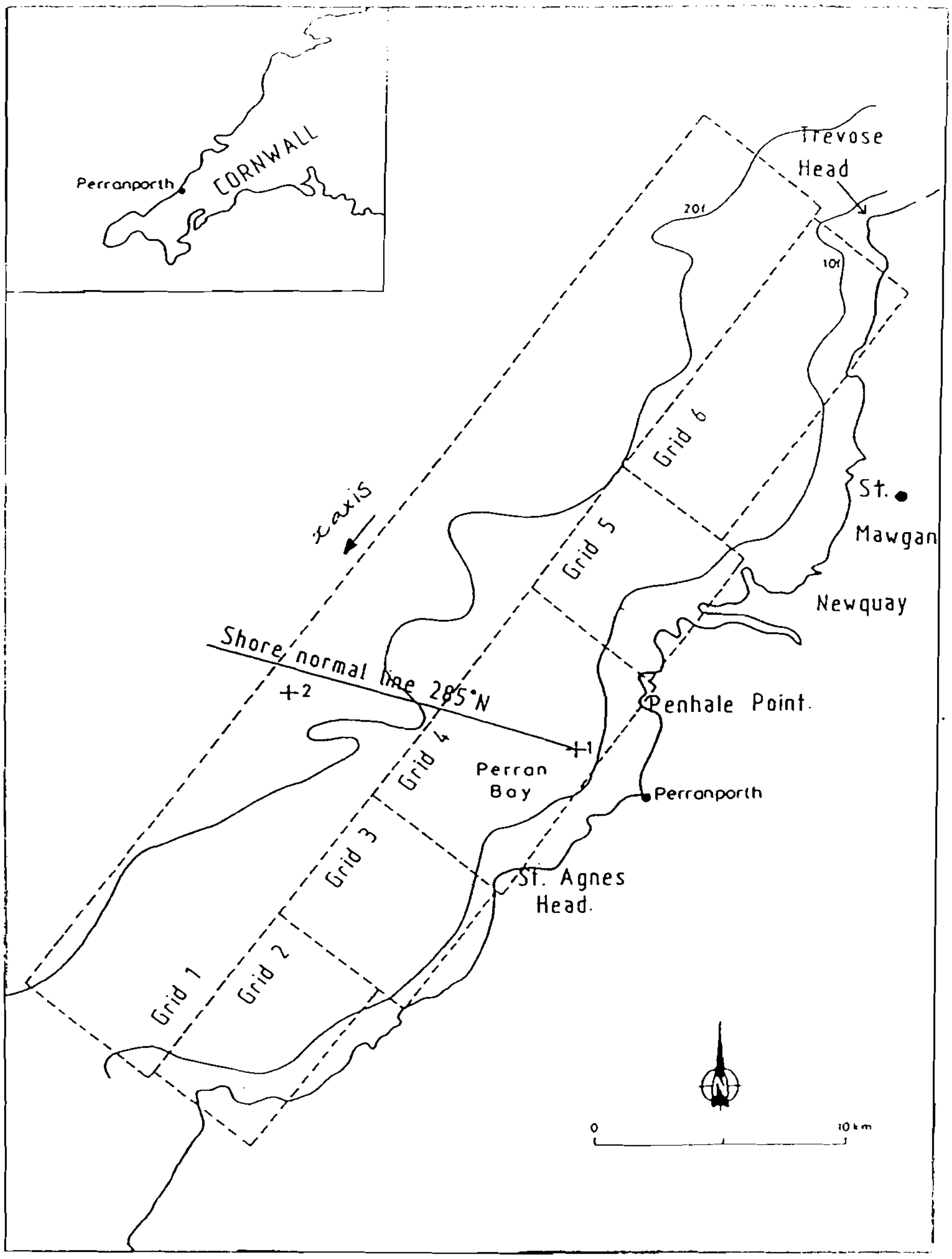
**Results Required** : The wave height coefficients at the ten analysis points shown on the figure for each of the input conditions.

**TABLE 1** Input wave conditions

| Case number | Case name | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Direction (°) |
|---|---|---|---|---|
| 1 | storm 0° | 4.3 | 8.6 | 0 |
| 2 | typical 0° | 1.9 | 6.0 | 0 |
| 3 | storm 25° | 6.0 | 10.0 | 25 |
| 4 | typical 25° | 3.2 | 7.5 | 25 |

**TEST D**

| | | |
|---|---|---|
| **Title** | : | Perranporth. |
| **Physical Processes** | : | Refraction and shoaling as the waves travel inshore. |
| **Data Available** | : | Wave recordings at two waverider buoys (shown in the figure below) for a 10 month period over the winter of 1978/79. |
| **Description** | : | The area to be modelled is shown in the figure below. |



This area extends 20km either side of Perranporth, which ensures that all important incoming waves are modelled. The x axis of the grid system used makes an angle of 36.5° with true North.The inshore waverider buoy (site 1) was situated in 24m of water approximately 2km offshore. The offshore waverider buoy (site 2) was approximately 11km from the coast in 48m of water.

The coastline in the area being modelled is relatively straight with approximately parallel depth contours. The seabed is sandy, which means that friction effects should be negligable.

Ten storms were chosen from the recorded data for this validation exercise. The frequency spectrum for each storm was obtained from a spectral analysis of the field wave data. Since the waveriders did not record wave direction, the measurements were augmented with directional spreading derived using mathematical modelling.

| | | |
|---|---|---|
| **Offshore Boundary** | : | This is shown in the figure. It lies in a water depth of approximately 50m below ODN. |
| **Bed Friction** | : | None. |
| **Wave Breaking** | : | Should be applied if included in the model. |
| **Tidal Levels** | : | A previous study at Perranporth showed that the effect of the tidal range is small. So, in this study, one tidal level should be used corresponding to mean sea level (0.25m above ODN). |
| **Input** | : | The wave conditions to be specified along the offshore boundary are given in Table 1. These correspond to recordings at the offshore waverider buoy (site 2) during the selected storms. Directional spectra at the boundary for each of the storms are provided using a $\cos^2\theta$ directional spreading function. |
| **Results Required** | : | The wave height coefficients at the inshore waverider buoy (site 1) for each input condition. |

**TABLE 1** Storm conditions recorded by offshore waverider buoy at Perranporth

| Storm number | Offshore wave conditions (site 2) | | |
|---|---|---|---|
| | Significant wave height, $H_s$ (m) | Zero crossing period, $T_m$ (s) | Predicted mean wave direction (°N) |
| 1 | 4.0 | 7.0 | 259 |
| 2 | 3.2 | 7.3 | 253 |
| 3 | 7.1 | 10.5 | 253 |
| 4 | 3.2 | 6.1 | 11 |
| 5 | 3.4 | 8.0 | 255 |
| 6 | 4.2 | 7.3 | 271 |
| 7 | 4.5 | 7.2 | 273 |
| 8 | 5.8 | 7.7 | 343 |
| 9 | 3.6 | 6.7 | 276 |
| 10 | 3.4 | 6.2 | 279 |

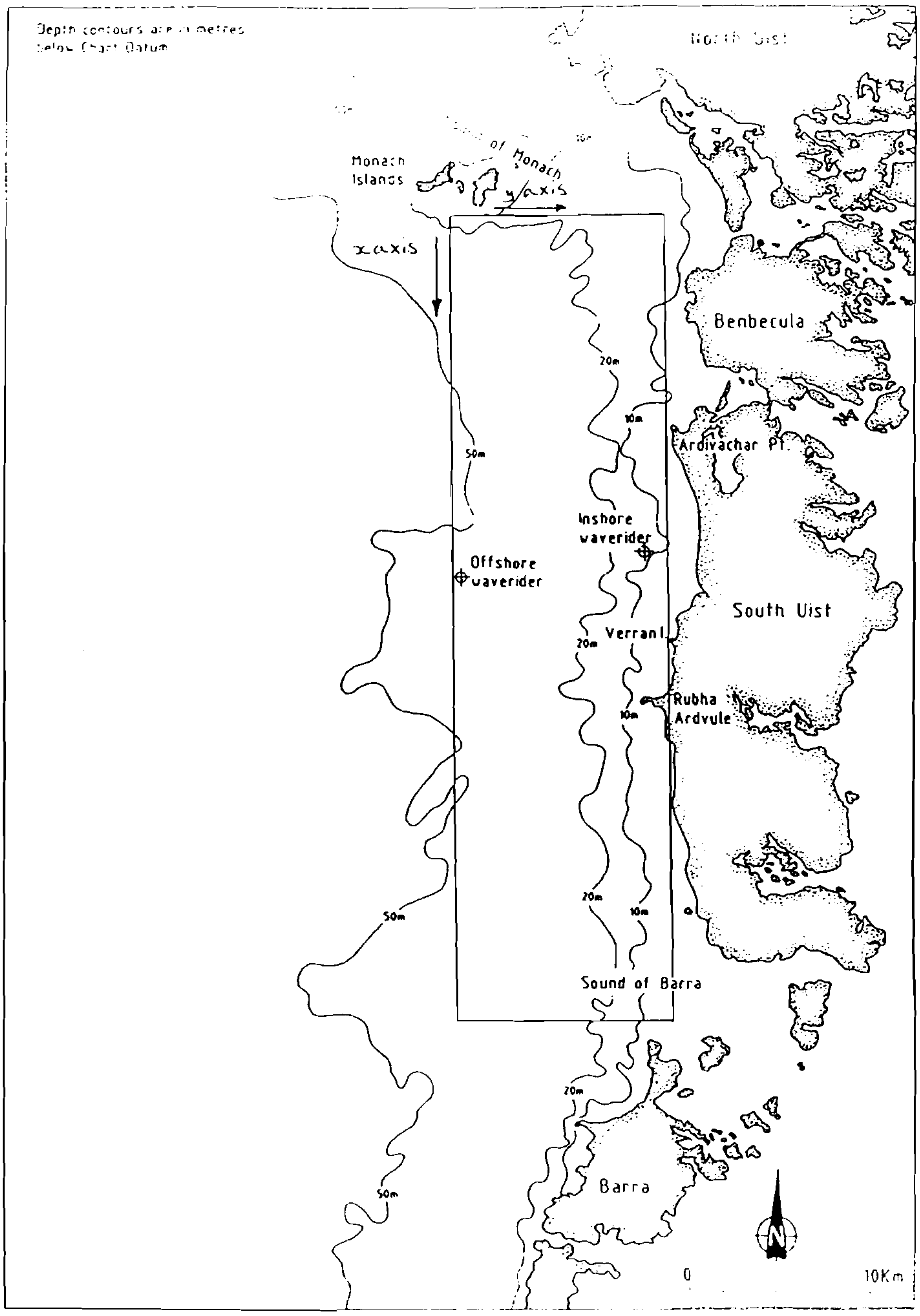
## TEST E

**Title** : South Uist.

**Physical Processes** : Refraction and shoaling as the waves travel inshore at a site where seabed friction is significant.

**Data Available** : Simultaneous wave recordings at two waverider buoys over a 12 month period from August 1978 to August 1979

**Description** : The area to be modelled is shown in the figure below.



The area being modelled covers an approximately 45km long section of the Outer Hebrides, between North Uist and Barra. The positive x axis of the computational grid runs north to south. The inshore waverider buoy was situated in 18m of water approximately 4km offshore and the offshore waverider buoy was approximately 14km from the coast in 44m of water. Both buoys are shown in the figure.

The coastline is relatively straight and open to the sea. Since the seabed is rough and uneven friction is likely to be significant.

Ten storms were chosen from the recorded data for this study. The frequency spectrum for each storm was obtained from a spectral analysis of the field wave data. Since the waveriders did not record wave direction, the measurements were augmented with directional spreading derived by mathematical modelling. The offshore waverider was exposed to swell waves as well as wind generated waves and the frequency spectra for the thirty five storms show that most were dominated by swell. Therefore, the swell direction has been used for the whole energy spectrum of each storm.

**Offshore Boundary** : This is shown on the figure. It lies in a water depth of approximately 40m.

**Bed Friction** : Should be applied if included in the model. The seabed to the west of South Uist is rocky with extensive kelp forests.

**Wave Breaking** : Should be applied if included in the model.

**Tidal Levels** : A previous study at South Uist showed that the tidal level had little effect on wave refraction and shoaling. However, since the loss of energy due to seabed friction is dependent on the water depth separate tidal levels should be used for each storm. Table 1 gives the tidal level for each storm.

**Input** : The wave conditions to be specified at the offshore boundary are given in Table 2. These correspond to recordings at the offshore waverider buoy during the selected storms. Frequency spectra at the offshore boundary for each of the storms are provided. If the model uses directional spectra, it is suggested that a $\cos^2\theta$ directional spreading function is applied.

**Results Required** : The wave height coefficients at the inshore waverider buoy for each input condition.

**Table 1** Tidal levels

| Storm number | Tidal level (m above CD) |
|---|---|
| 1 | 3.6 |
| 2 | 2.8 |
| 3 | 3.2 |
| 4 | 1.9 |
| 5 | 2.9 |
| 6 | 2.3 |
| 7 | 1.9 |
| 8 | 3.6 |
| 9 | 1.9 |
| 10 | 2.5 |

**TABLE 2** Storm conditions recorded by offshore waverider buoy to the West of South Uist.

| Storm number | Offshore wave conditions | | |
|---|---|---|---|
| | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Predicted mean wave direction (°N) |
| 1 | 3.5 | 9.0 | 240 |
| 2 | 6.0 | 10.0 | 255 |
| 3 | 4.0 | 10.0 | 260 |
| 4 | 7.5 | 14.1 | 280 |
| 5 | 5.7 | 11.7 | 265 |
| 6 | 3.7 | 14.5 | 225 |
| 7 | 4.8 | 9.5 | 350 |
| 8 | 4.8 | 13.6 | 265 |
| 9 | 5.3 | 11.7 | 265 |
| 10 | 3.5 | 8.3 | 330 |

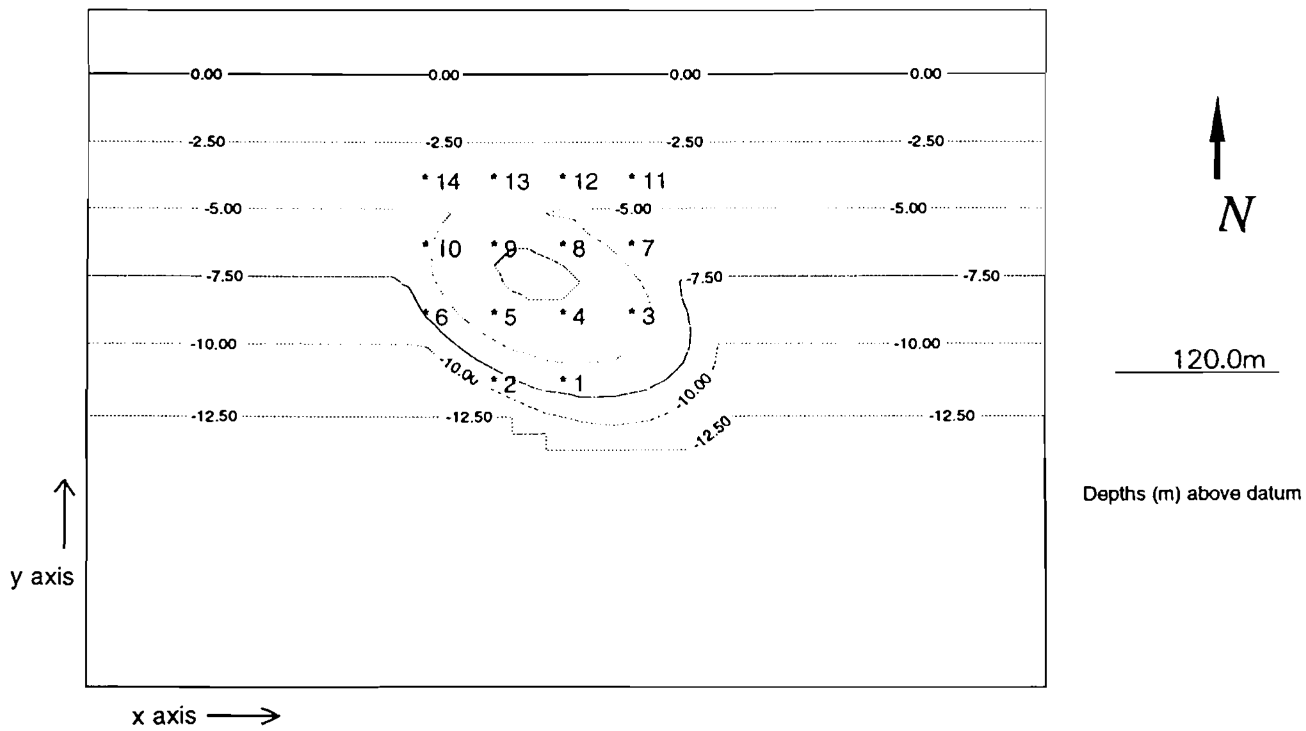
# Test F

**Title** : Elliptic Shoal With Currents

**Physical Processes** : Refraction, shoaling and diffraction due to varying bathymetry, and refraction due to the effects of currents.

**Data Available** : Wave heights measured during a random wave physical model test, at the points shown in the diagram below.

**Description** : The bathymetry used in the test is shown in the diagram below.




**Reflections** : None.

**Offshore Boundary** : The incident wave conditions are specified along $y=0$.

**Bed Friction** : Assume a smooth bed.

**Wave Breaking** : Should be applied if included in the model.

| | | |
|---|---|---|
| **Input** | : | The wave conditions to be specified at the offshore boundary are given in Table 1. Both tests should be run at a still water level of 0.0m, and with a parabolic current profile, details of which are provided. A JONSWAP spectrum with the values of $H_s$ and $T_p$ specified in Table 1, should be used for models which can be run in spectral mode. |
| **Results Required** | : | The significant wave height coefficients at the 14 points indicated on the diagram for both incident wave conditions. |

**Table 1 Input wave conditions**

| Case number | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Direction (°N) |
|---|---|---|---|
| 1 | 2.5 | 5.0 | 180 |
| 2 | 3.1 | 6.0 | 180 |

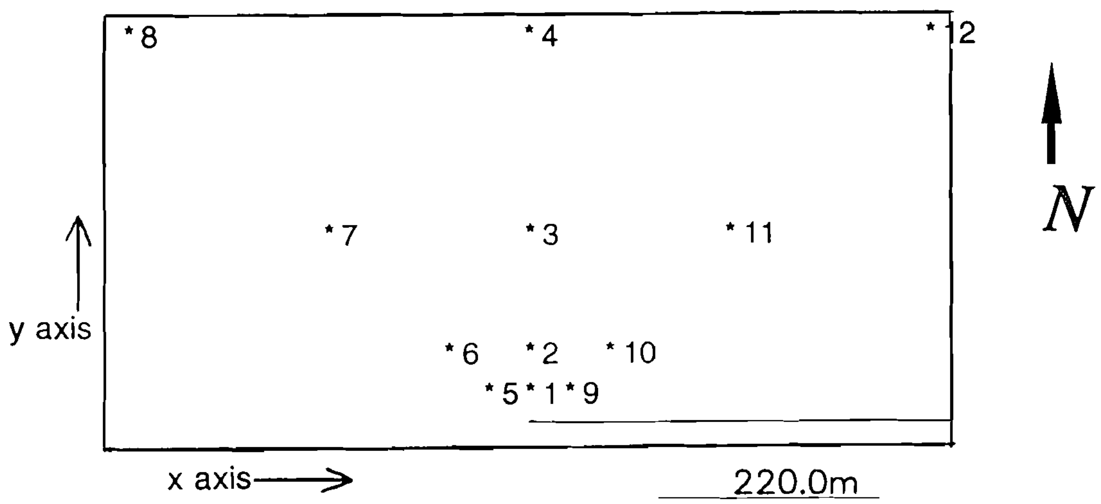
## Test G

**Title** : Semi-Infinite Breakwater

**Physical Processes** : Diffraction around a surface piercing structure.

**Data Available** : The solution to this test is given by the Sommerfeld solution which can be computed at any point in the area being modelled.

**Description** : The bathymetry used in this test is shown in the diagram below.




Constant depth of 6m.

**Reflections** : Assume the breakwater is fully reflective.

**Offshore Boundary** : The incident wave conditions given should be applied along an infinite line just south of the breakwater.

**Bed Friction** : Assume a smooth bed.

**Wave Breaking** : Apply if present in the model.

| | | |
|---|---|---|
| **Input** | : | A monochromatic wave which has an amplitude of 1.0m, and a period of 5.9s. There are five incident directions 120°N, 150°N, 180°N, 210°N and 240°N. |
| **Results Required** | : | The wave height coefficients at the 12 analysis points indicated on the diagram for the five incident wave directions. |

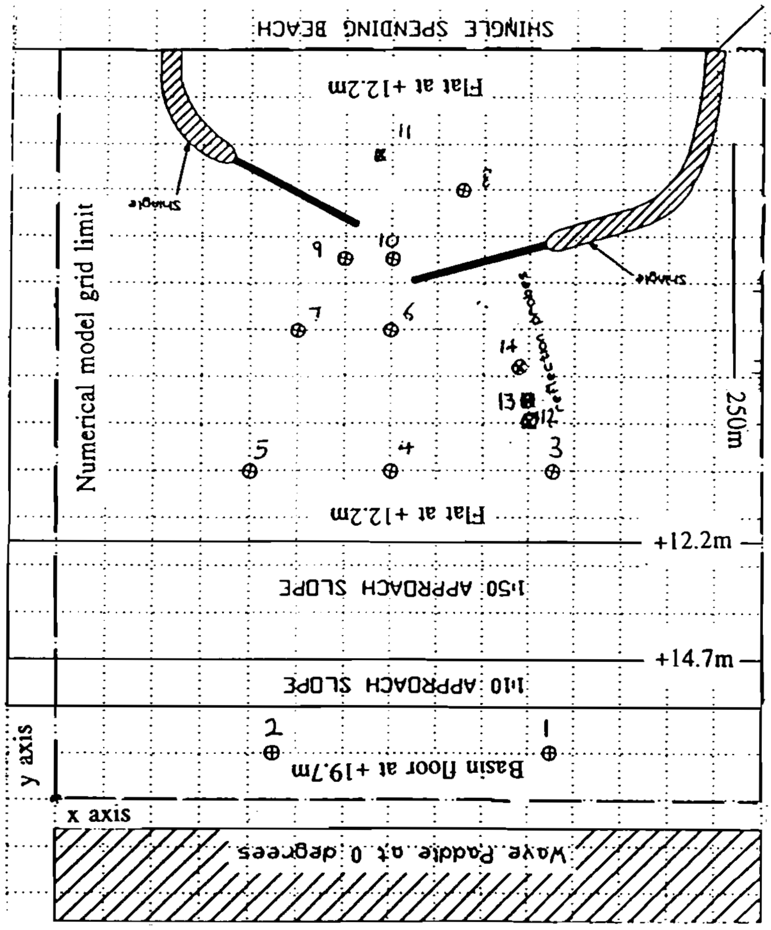
# Test H

**Title** : Idealised Harbour Entrance

**Physical Processes** : Refraction and shoaling due to varying bathymetry, and diffraction about surface piercing structures.

**Data Available** : Details of a random wave physical model test which was recently carried out at HR Wallingford.

**Description** : The bathymetry used in the test is shown in the diagram below.




Water depth at structure = 12.2m

Water depth at paddle = 19.7m

**Reflections** : Assume both breakwaters are fully reflective up to the part labeled shingle, which is a 1 : 2 shingle breakwater.

**Offshore Boundary** : Incident wave conditions should be specified along *y=0*.

**Bed Friction** : Assume a smooth bed.

**Wave Breaking** : Should be applied if included in the model.

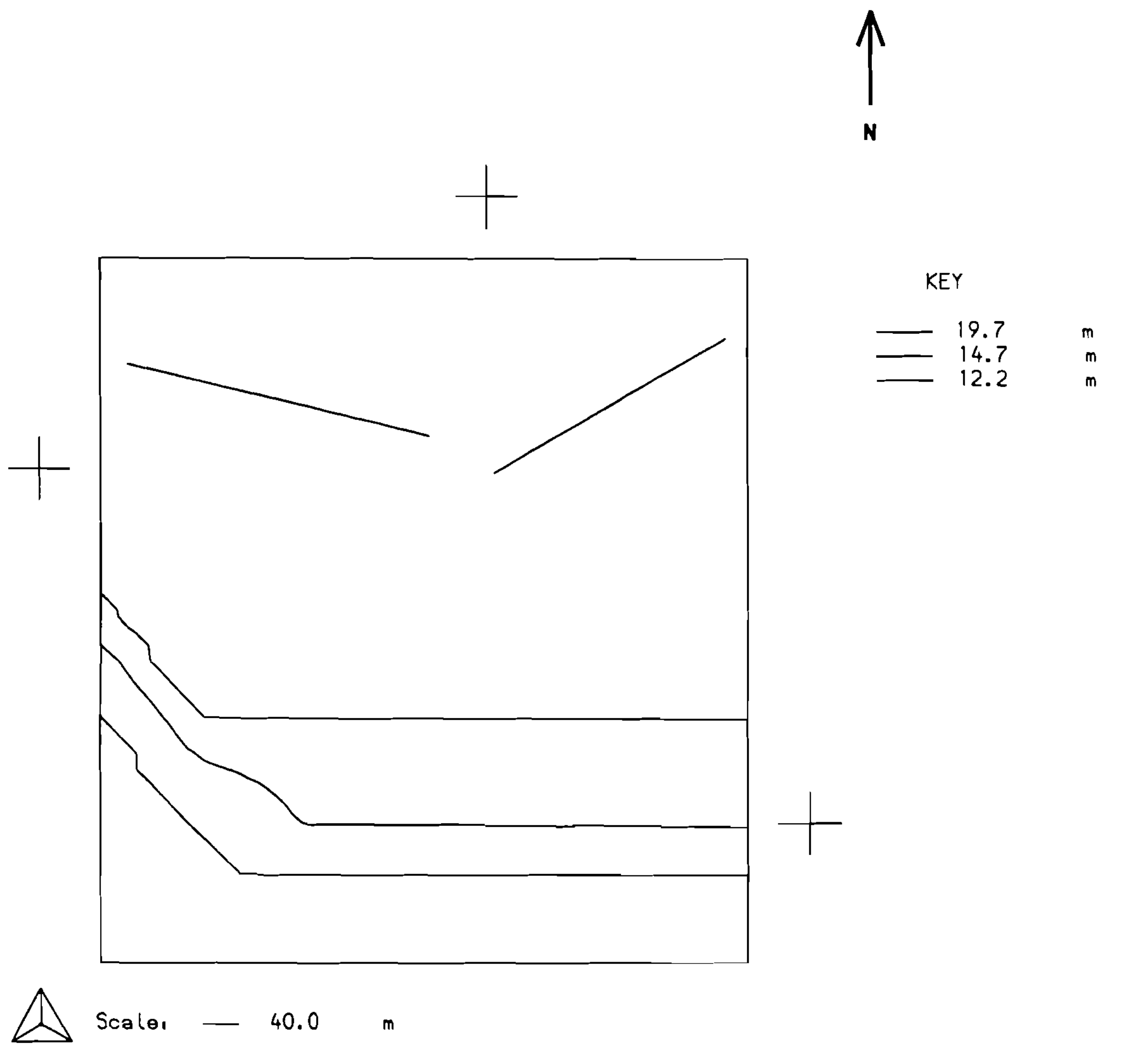
**Input** : The wave conditions to be specified at the offshore boundary are given in Table 1. Both tests should be run at a water level of 0.0m . A JONSWAP spectrum with values of $H_s$ and $T_p$ specified in Table 1 should be used for models which can be run in spectral mode.

**Results Required** : The wave height coefficients at the 14 points specified in the diagram for both incident wave conditions.

**Table 1 Input wave conditions**

| Case number | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Direction (°N) |
|---|---|---|---|
| 1 | 2.5 | 6.6 | 180 |
| 2 | 4.2 | 7.3 | 180 |

Idealised harbour en HR Wallingford
11:27 28-02-95 from file ideal.mrg



# Test I

**Title** : Pittenweem Harbour.

**Physical Processes** : Refraction, shoaling and diffraction due to varying bathymetry. Reflection from and diffraction around surface piercing structures.

**Data Available** : Wave height measurements at nine positions (marked on figure) recorded during a physical model study.

**Description** : The bathymetry used in this test case is shown in Figure 1.

**Reflections** : All harbour walls at Pittenween are smooth vertical concrete walls, and reflection coefficients should be assigned to represent this.

**Bed Friction** : Should be applied if included in the model. The seabed offshore of Pittenweem consists mainly of rocky outcrops.

**Wave Breaking** : Should be applied if included in the model.

**Tidal Levels** : The test should be run at a tidal level of +2.1m OD for case1 incident wave conditions, and +1.2m OD for case 2 incident wave conditions.

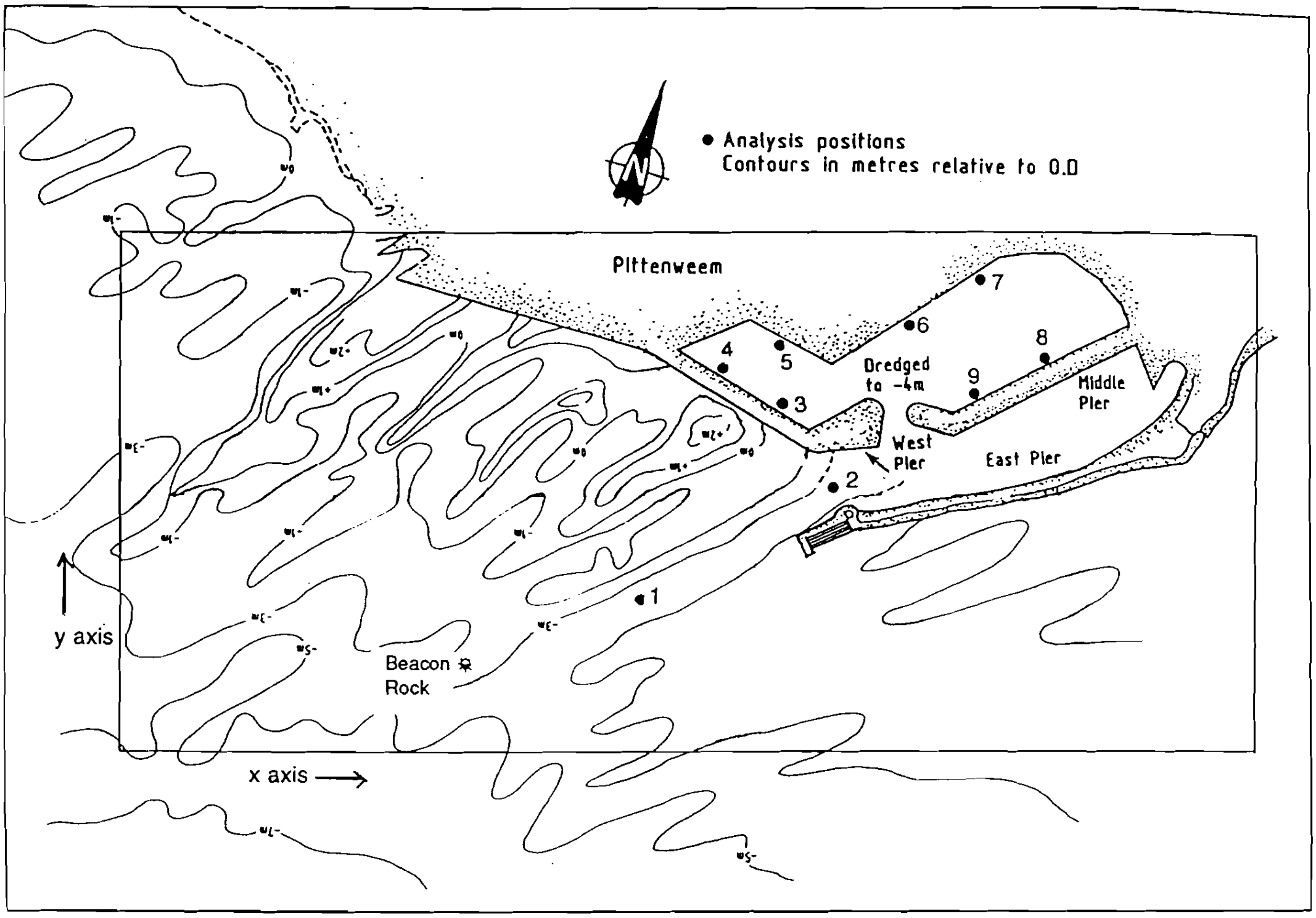
**Input** : The model should be run using the two sets of wave conditions from Beacon Rock, which are shown in Table 1. The Spectra supplied should be used if the model can be run in spectral mode.

**Results Required** : Significant wave height coefficients at the nine analysis points shown on the figure for each of the input conditions.

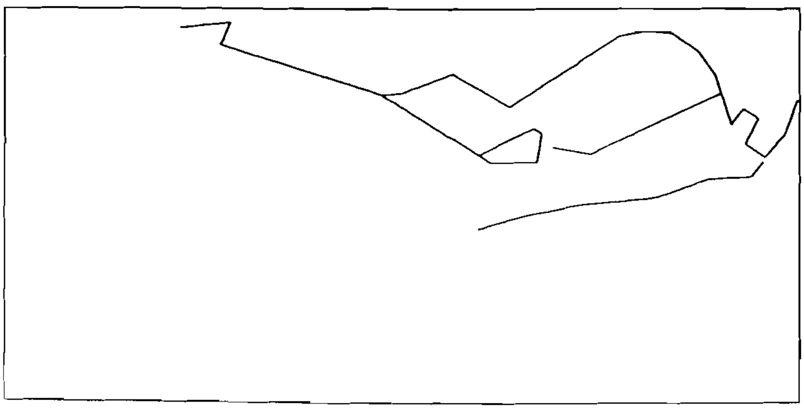
**Table 1 Input wave conditions**

| Case number | Water level (m above OD) | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Mean period, $T_m$ (s) | Direction (°N) |
|---|---|---|---|---|---|
| 1 | +2.1 | 2.8 | 10.4 | 6.5 | 139 |
| 2 | +1.2 | 1.5 | 5.3 | 3.8 | 186 |

## Figure 1




## Supplied boundary information



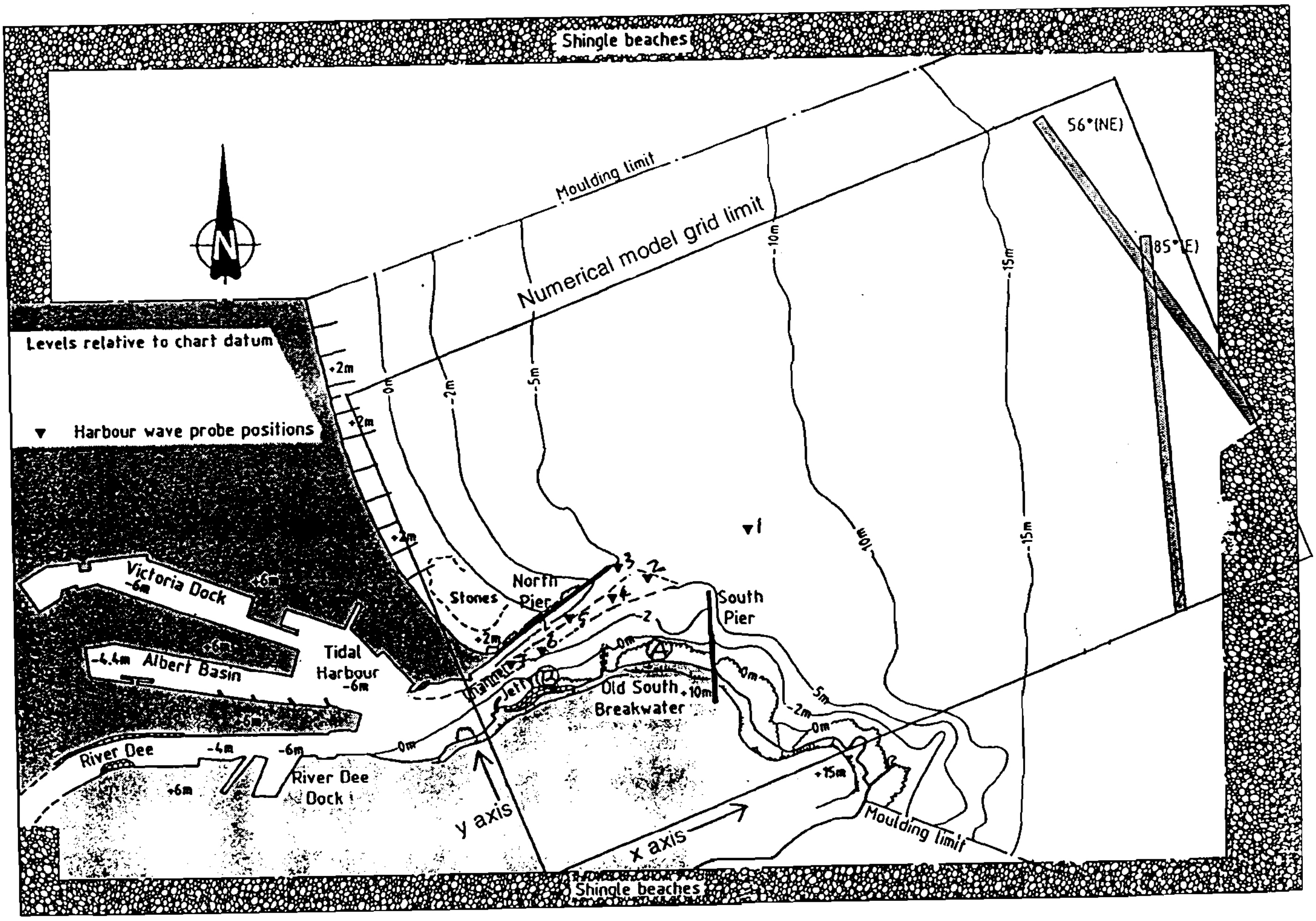
## Test J

| | | |
|---|---|---|
| **Title** | : | Aberdeen Harbour Entrance. |
| **Physical Processes** | : | Refraction, shoaling and diffraction due to varying bathymetry. Reflection from and diffraction around surface piercing structures. |
| **Data Available** | : | Wave height data at seven positions (marked on figure) recorded during a physical model study. |
| **Description** | : | The bathymetry used in this test case is shown in Figure 1. |
| **Reflections** | : | The North Pier, South Pier, Jetty and Old South Breakwater all have smooth vertical walls. Partial reflection should be represented on the south side of the harbour. Area A, between the South Pier and the Old South Breakwater is a rocky shore with a shallow slope. Area B, between the jetty and the Old South Breakwater is a smooth shallow slope. |
| **Bed Friction** | : | Should be applied if included in the model. The seabed around Aberdeen Harbour is sandy. |
| **Wave Breaking** | : | Should be applied if included in the model. |
| **Tidal Levels** | : | The model should be run at the levels given in Table 1 for the incident wave conditions given. |
| **Input** | : | The wave conditions given in Table 1 were produced in the physical model at the paddles indicated in the diagram. The spectra supplied should be used if the model can be run in spectral mode. |
| **Results Required** | : | Significant wave height coefficients at the seven analysis points shown in Figure 1 for each of the input conditions. |

**Table 1 Input wave conditions**

| Case number | Water level (m above CD) | Significant wave height, $H_s$ (m) | Peak period, $T_p$ (s) | Mean period, $T_m$ (s) | Direction (°N) |
|---|---|---|---|---|---|
| 1 | +3.4 | 4.5 | 9.2 | 7.0 | 85 |
| 2 | +3.9 | 5.4 | 10.1 | 7.6 | 85 |
| 3 | +3.5 | 2.6 | 8.4 | 6.0 | 56 |
| 4 | +3.8 | 4.2 | 11.2 | 7.6 | 56 |

**Figure 1**



## Supplied boundary information