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SEDIMENT TRANSPORT MODELS FOR ESTUARIES

G V Miles

Hydraulics Research Station
Wallingford
Oxon OX10 8BA
Telephone 0491 35381

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SUMMARY

This paper is a review of numerical models for studying sediment transport with special emphasis on the applicability of methods to estuarine conditions. The main features of estuaries are the wide range of sediments present, the absence of erodible material in some places, a combination of unsteady and nonuniform flow and lateral as well as longitudinal variations in the flow and suspended solids concentrations. Saline stratification can also be present. Not surprisingly no single model has so far been presented to simulate all the estuarine sediment processes but many models are described which can be used to study certain aspects. The models are separated into potential load models primarily geared to studying bed load transport, suspended sand models and suspended mud models.

The review covers the most significant papers published in recent years, supplemented by some case studies of projects carried out in the Hydraulics Research Station, Wallingford.

1 INTRODUCTION

An estuary is a partly enclosed body of tidal water where river water is mixed with and diluted by sea water. In a general sense the estuarine environment is defined by salinity boundaries rather than by geographical ones, but although the salinity has influence on the clay sediment fractions it is the currents generated by the tidal volume flowing in and out of the estuary which dominate the movement and distribution of sediments. The sediments themselves may have originated from natural erosion inland or from seawards. They consist of materials ranging from the finest clay particles to coarse sand and gravels. A convenient classification of sediments uses a geometric scale of sizes

	mm	phi units
Very coarse sand	1.0 - 2.0	- 1
Coarse sand	0.5 - 1.0	0
Medium sand	0.25 - 0.5	1
Fine sand	0.125 - 0.25	2
Very fine sand	0.064 - 0.125	3
Coarse silt	0.032 - 0.064	4
Medium silt	0.016 - 0.032	5
Fine silt	0.008 - 0.016	6
Very fine silt	0.004 - 0.008	7
Coarse clay	0.002 - 0.004	8
Medium clay	0.001 - 0.002	9

TABLE 1 SEDIMENT GRADINGS

A significant feature of estuaries is the wide range of sediment sizes found in them. These sediments are sifted and sorted by the tidal currents.

In the main channels bed stresses are usually too high to allow the finer materials to accumulate although they may settle temporarily at slack water. Only coarse sand and gravel can exist as permanent deposits in these high energy regions. Along the shallow margins of the estuary, and further upstream, the tidal currents are too weak to move the sand and either no sand is transported there or it is covered by silt or clay to produce characteristic mud flats. These mud flats are colonised by various forms of marine life and become the feeding grounds of birds. If conditions are suitable the level of the mud flats rises and eventually a salt marsh develops.

The study of sediment transport generally is a very difficult problem. The particular study of sediment transport in estuaries is especially complicated because

1. The water movements are continually changing with the rise and fall of the tide;
2. The wide range of sediments present in suspension and on the bed;
3. The absence of certain sediments in some parts of the estuary leading to unsaturated sediment loads in the water.

Although it is not possible to predict precisely how any single type of sediment will behave in the estuary, the recent advances in numerical modelling do enable some information to be obtained to give guidance to engineers and environmentalists for assessing the impact of engineering works.

In the following report various methods are described which have appeared in the engineering journals and proceedings of conferences over the last few years. The emphasis is on the formulation of models and applicability rather than on the details of particular numerical schemes used to obtain the solutions. We have tried to treat the methods in a systematic way starting with those appropriate for bed load transport and ending with models for studying fine silt and mud transport in suspension. There is inevitably some overlap between the modelling techniques when fine sand is concerned but it is hoped that the limits of applicability of any particular model will be clear. The author has attempted to fit all the relevant papers and reports known to him into this review, and apologises for any omissions which may have occurred through oversight.

2 POTENTIAL LOAD MODELS

The simplest type of sediment transport model is essentially a single equation representing conservation of bed material.

$$\frac{\partial m}{\partial t} + \frac{\partial}{\partial x}(T_s) = 0 \qquad (1)$$

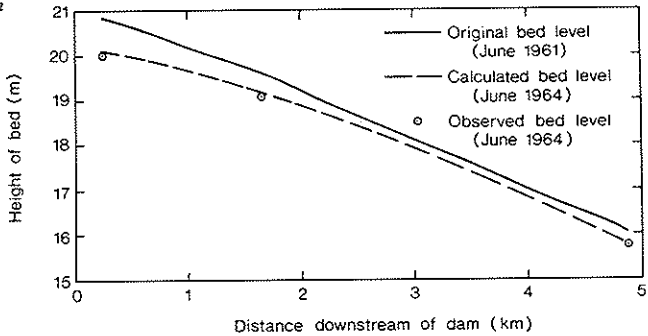
where m (kg/m²) is the quantity of material on the bed and T_s (kg/sec/m width) is the sand transport. The basic assumption for this type of model is that the flow is saturated with sediment, which means that the flow is carrying the maximum sand transport that can be maintained for the given hydraulic and sedimentary conditions. Under saturated conditions the transport can be calculated from one of the many sediment transport laws to be found in the literature. An appraisal of available methods is given by White et al (1973). The flow parameters (water depth, mean velocity and shear velocity) required for the transport calculation could be obtained from measurements in a physical model but it is usually quicker and cheaper to generate this data on a regular grid from a separate numerical model of water movements.

The sediment carrying capacity of flow increases significantly for high water velocities – typically in proportion to the fourth power. This means that the flow will tend to pick up material from the bed when it accelerates and to deposit excess material when it decelerates. If the flow is always saturated with sediment the differences in transporting capacity must define the quantity of material picked up or deposited on the bed. This is the basis for the potential load model. The computer is only used because it is many times faster than calculating by hand. In this way the sand transport calculation can be made for hundreds of points and repeated at intervals to define the variation of sand transport over the estuary and through the tidal cycle.

The potential load model is naturally most suited to situations where the bed material is narrowly graded and where there is an adequate supply of erodible material on the bed to maintain the saturated load. These conditions are more often met in rivers and it is in such situations that potential load models have been found most successful. See for example Cunge and Perdreau (1973), de Vries (1976), Thomas and Prasuhn (1977) and Bettess and White (1979). An example of the results from the last of these is given in Fig 1.

Lepetit and Haguel (1978) have extended the modelling approach used in river studies, to simulate 2-dimensional local scour round a jetty in a steady flow. The model is quasi-steady and uses a perturbation technique to feed the changes in depth back into the flow. Transport is calculated from a saturated bed load sediment law and bed changes calculated from the 2-dimensional form of equation 1, for conservation

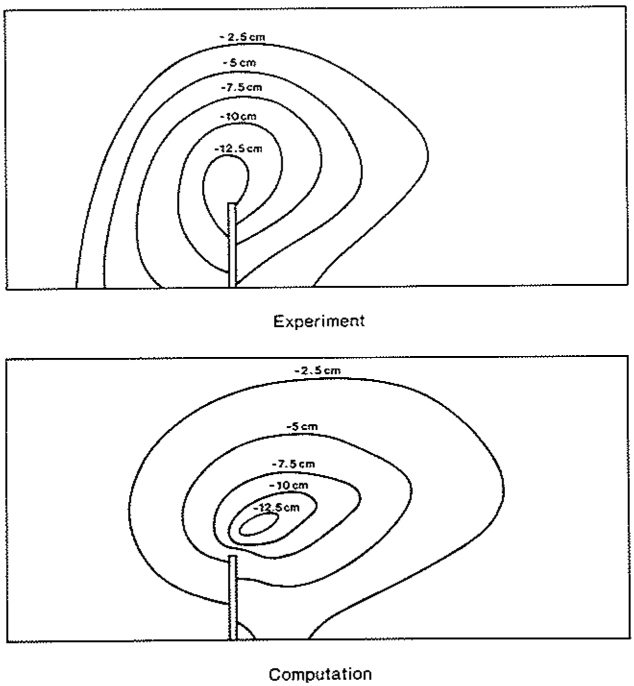
Fig 1. Degradation downstream of Milburn Dam



of sediment. The model results were shown to agree qualitatively with scour patterns measured in a mobile bed physical model (Fig 2).

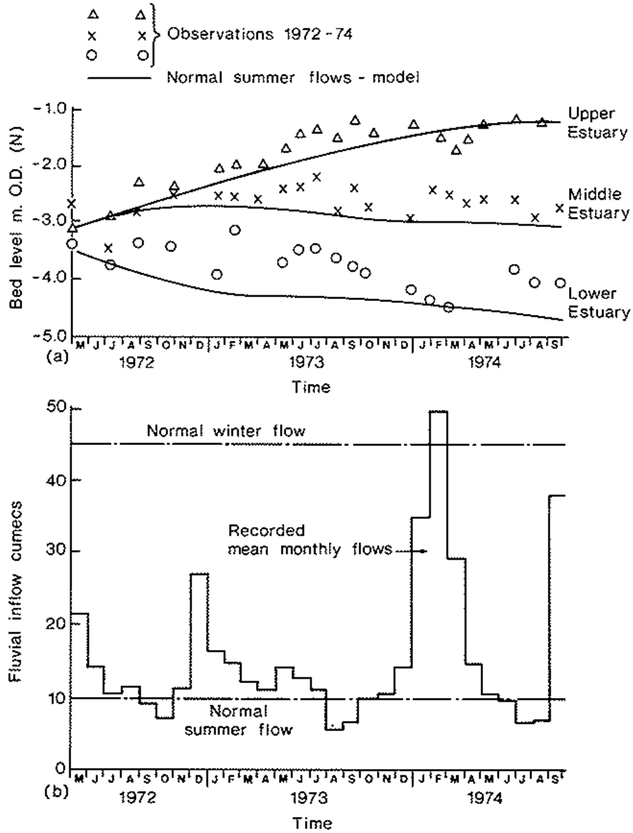
The previously mentioned models have the common feature that the water flow is either steady or varying very slowly. Under those circumstances the potential load model can be applied to total (bed plus suspended) loads. In estuaries, where lag effects are more important, the potential load modelling approach is only appropriate, if at all, to medium and coarse sands which move mainly as a bed load and respond relatively quickly to the changing flow conditions. Odd, et al (1976) describe two instances where potential load models have been applied to tidal conditions. The first application to the River Great

Fig 2. Comparison between measured and computed erosions. (From Fig 3 in Lepetit and Haguel (1978); courtesy American Society of Civil Engineers)



Ouse proved most successful because the bed contained an adequate supply of erodible sand and inertial lag effects were small. Using a model of the river it was possible to reproduce the evolution of bed level profiles over a period of two years (Fig 3) and predict the changes which would occur following the construction of a tidal barrage and/or extracting fresh water. A second application to a 2-dimensional area in the south east corner of the Wash was less satisfactory. The complicated nature of outfall channels prevented the collection of enough data to calibrate the model properly. Nevertheless it was still possible to use the model to predict the changes in sediment transport patterns which could occur following the construction of reservoir schemes.

Fig 3. Bed levels and fluvial flows in the Great Ouse



Recently Katoh et al (1979) have presented a 2-dimensional potential load model which can take into account several distinct sand sizes. According to the English synopsis of their paper they have been able to forecast long term bed changes for a site in the Ariake Sea. The predicted bed changes were apparently fed back into a complementary flow model without causing stability problems.

These studies showed that potential load models are valuable tools for the engineer if there is an adequate supply of erodible material on the bed. The experiences of the Hydraulics Research Station in a study of the Conwy Estuary (Ref 15) demonstrate clearly the shortcomings of this type of model in situations where the bed contains areas of inerodible material. Under these circumstances the actual transport, T, may be less than the saturation transport T_S. The consequences of this are very serious as regards predictions because

1. erosion may not in fact occur in a region of potential erosion identified by the model if there is no erodible material on the bed;
2. deposition may not in fact occur in a region of potential deposition identified by the model if the actual sediment load of the approaching water is insufficient to saturate even the slower flow;
3. erosion may in fact occur in an area of potential deposition if the sediment load of the approaching flow is very low for example after flowing over an area of rock bed.

In order to overcome these limitations it is necessary to reformulate the sediment transport in terms of an equation representing conservation of suspended sediment.

3 SUSPENDED SAND TRANSPORT MODELS

The potential load model is not suitable for studying sand transport in estuaries where there is not a continuous supply of erodible material on the bed. The reason is that that sort of model cannot take into account where the water carrying the sediment has been nor how much sediment is actually being carried by the flow. To do these requires a different sort of model based on conservation principles which simulates the sediment transport in terms of a suspended solids concentration. The erosion or deposition of material on the bed can then be assumed in the model depending on whether the actual load is less or greater than the saturated load which would obtain under steady, uniform flow conditions at the same values as the instantaneous flow. This assumption effectively ignores the differences in turbulence which occur in accelerating or decelerating flow. Under these circumstances the suspended solids concentration, c, (kg/m^3) satisfies, (eg Graf (1971))

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + (w - w_s)\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}(D_x\frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(D_y\frac{\partial c}{\partial y}) + \frac{\partial}{\partial z}(D_z\frac{\partial c}{\partial z}) + S \qquad (2)$$

where

u, v, w are the velocity components

x, y, z are space co-ordinates, with z vertically upwards

D_x, D_y, D_z are the (turbulent) diffusion coefficients

w_s is the settling velocity

S is the sink or source term representing erosion or deposition of material on the bed

t is time.

Most solutions to be found in the literature are for special cases of this equation. The earlier solutions by Schmidt (1925) and Lane et al (1941), and later Hunt (1965) are essentially geared to providing insight into the vertical structure of the suspended solids profile. These assume one-dimensional, uniform, steady flow conditions for which equation 2 reduces to

$$w_s\frac{\partial c}{\partial z} + \frac{\partial}{\partial z}(D_z\frac{\partial c}{\partial z}) = 0 \qquad (3)$$

This equation represents the equilibrium profile obtained as a balance between settling and vertical diffusion due to the turbulence. It is important to appreciate that equilibrium defined in this way does not mean saturation. Indeed the sediment load can be in equilibrium if the bed is not mobile, even when the flow is under saturated with sediment. Integration with respect to z and the application of a boundary condition of zero flux of sediment at the free surface (and implicitly also at the bed) yields the governing equation

$$w_s c + D_z \frac{\partial c}{\partial z} = 0 \tag{4}$$

This can be integrated further if the vertical structure of the diffusivity is prescribed. These profiles have proved valuable in the understanding of sediment transport but they are not relevant to the unsteady, unsaturated flow conditions which are the main concern here, so these special solutions are not considered further. Graf (1971) is a good source of additional information on these solutions.

A class of solutions which have more relevance to estuaries have been presented by Kalinske (1940), Dobbins (1943), Mei (1969) and Lean (1980), for unsteady and uniform or steady and non uniform conditions governed respectively by the equations

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z}\right) + w_s \frac{\partial c}{\partial z} \tag{5}$$

$$u_o \frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z}\right) + w_s \frac{\partial c}{\partial z} \tag{6}$$

These equations are mathematically the same when u_o is constant. The first represents a concentration changing with time following a change in the magnitude of a uniform flow, while the second represents the concentration changing as a function of position as might occur for example when clear water flows from an area with an inerodible bed into an area where erosion can commence. Solutions of these equations provide information about the time or distance of travel required for the sediment concentration to adapt to changes in the flow conditions.

The assumption of constant eddy diffusivity permits analytic solution of these equations. The diffusivity normally used is

$$D_z = \frac{1}{6} \kappa u_* d \tag{7}$$

which is the depth averaged value of the parabolic eddy viscosity

$$\nu_T = \kappa u_* z(1 - z/d) \tag{8}$$

consistent with the logarithmic velocity profile. κ is the Von Karman constant. Apmann and Rumer (1970) present experimental evidence that supports this assumption. The exact solutions, which involve the use of Laplace Transforms, or similar, may be expressed in the form of infinite series. Mei (1969) recognised that an approximate solution, valid for small times or distances of travel, could be obtained from the expansion of the Laplace Transform for large values of the transform parameter. Under most conditions this expansion is valid for distances of the order of twenty water depths or the equivalent in a time dependent situation. Lean (1980) proposed an alternative bed boundary condition and the present author has reworked Mei's solution for this case. This solution is used in the sediment transport model proposed in the next section.

The solution of the unsteady equation 5 requires an initial condition at say t = 0 and boundary condition at z = 0 (the bed) and z = d (the free surface). The surface boundary condition is clearly zero vertical flux of sediment viz

$$w_s c + D_z \frac{\partial c}{\partial z} = 0 \text{ at } z = d \quad (9)$$

There are two possible conditions at the bed which admit analytical solution. firstly one could assume (Mei 1969) that the concentration $c(o, t)$ at the bed responds instantaneously to the changing flow conditions.

That is

$$c(o,t) = c_s(o,t) = \beta_s \bar{c}_s(t) \quad (10)$$

where

$c_s(o,t)$ is the concentration of the equilibrium profile at the bed when the flow is saturated with sediment

$\bar{c}_s(t)$ is the depth averaged value of this profile

and

$$\beta_s = c_s(o,t)/\bar{c}_s(t) \text{ is a profile factor} \quad (11)$$

This is a much more realistic condition than that implicit in a potential load model which assumes that the full load responds instantaneously. However, the condition still implies an infinite rate of exchange of material at the bed at $t = 0$ (or at $x = 0$ in the non uniform version).

Lean (1980) assumes that the rate at which material is entrained into the flow is the quantity which responds most readily to changes in flow. In this case the boundary condition at the bed would be

$$\left(D_z \frac{\partial c}{\partial z}\right)_{z=0} = \left(D_z \frac{\partial c_s}{\partial z}\right)_{z=0} \quad (12)$$

or, from equation 4, the net vertical flux at the bed is prescribed as

$$w_s(c_s - c)_{z=0} = w_s \beta_s (\bar{c}_s - \bar{c}) \quad (13)$$

This provides an alternative boundary condition to condition 8. The asymptotic form of solutions to equation 5 for initial concentration $\bar{c}_o$ are

$$c(t,z) = \beta_s \bar{c}_o e^{-R\zeta} + \frac{1}{2}\beta_s(\bar{c}_s - \bar{c}_o)(\text{erfc}(\sigma\zeta + \tau) + e^{-R\zeta}\text{erfc}(\sigma\zeta - \tau)) \quad (14)$$

and

$$c(t,z) = \beta_s \bar{c}_o e^{-R\zeta} + \frac{1}{2}\beta_s(\bar{c}_s - \bar{c}_o)(\text{erfc}(\sigma\zeta + \tau) + e^{-R\zeta}\text{erfc}(\sigma\zeta - \tau)) - \beta_s(\bar{c}_s - \bar{c}_o)((1 + \frac{1}{2}R\zeta + 2\tau^2)\text{erfc}(\sigma\zeta + \tau) - \frac{2}{\sqrt{\pi}}\exp(-(\sigma\zeta + \tau)^2)) \quad (15)$$

where

$$R = w_s d/D_z \quad (16)$$

$$\tau = (w_s^2/4D_z)^{1/2} t^{1/2} \quad (17)$$

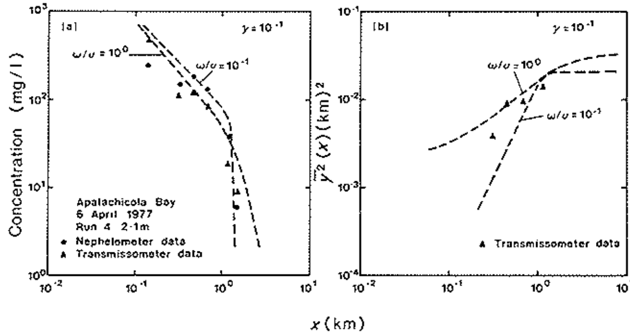
$$\sigma = (d^2/4D_z)^{1/2} t^{1/2} \quad (18)$$

$$\zeta = z/d \quad (19)$$

for the bed concentration boundary condition (Mei) and the bed entrainment boundary condition (Lean) respectively. These solutions are valid when $\tau \leqslant 1$, ie for times while water flows over distances equal to 10 to 100 water depths.

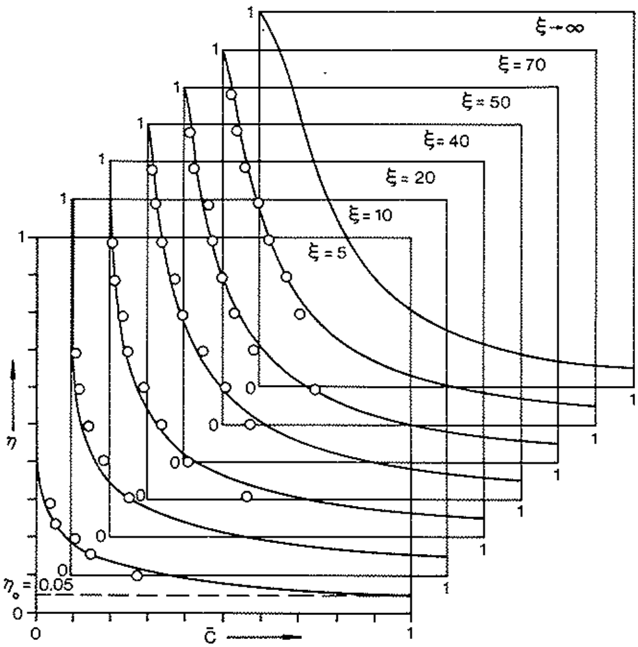
There is another class of special analytic solutions which are related to the plume models used for thermal or sewage pollution studies. These models are usually depth integrated and are designed for situations

Fig 4. *Comparison of observed plume structure in Apalachicola Bay.*
(From Fig 6 in Wilson (1979); courtesy Academic Press Inc. (London) Ltd)



when the lateral spread of the plume is considered to be more important than the vertical profile. Wilson (1979) describes a model of this sort with an integral solution representing the balance between advection with the ambient current, lateral and longitudinal diffusion and settlement of particles of various sizes. The model is presented as a tool for studying sediment plumes produced from dredging losses. A typical comparison between results from the model and observations is reproduced in Fig 4. Another integral solution for studying plumes from continuous discharges is presented by Christodoulou et al (1974), and Chen et al (1978) present a special model which computes bed changes in a reservoir due to settlement of suspended material carried in by the incoming water. The water flow in the latter case is obtained from the classical structure of a turbulent jet, and the suspended solids concentrations are computed using a finite difference scheme. Chen et al (1978) also present some experimental results which could be of value in testing models of this type.

Fig 5. *Comparison of observed and calculated concentration profiles.*
(From Fig 3 Yalin and Finlayson (1973); courtesy International Association for Hydraulic Research)



3.1 Numerical sand transport models

Although the special solutions described above provide insight into the sediment transport processes they lack many of the factors which are important in estuaries, namely

1. the combination of non uniformity with unsteadiness
2. variable supply of erodible material
3. lateral as well as longitudinal variation.

The inclusion of these factors completely precludes any possibility of analytic solution and leads to the need for numerical models.

The simplest form of numerical sediment transport model takes the form of finite difference or finite element solutions of the approximate equations 5 and 6. Apmann and Rumer (1970) and Yalin and Finlayson (1973) present models of this type. The advantage of seeking numerical solutions is that more realistic eddy diffusivities and velocity profiles can be incorporated. Yalin employs an eddy diffusivity equal to the parabolic eddy viscosity (eq 8) consistent with the logarithmic flow profile. The model was tested against experimental measurements (Fig 5). Though the model is not immediately relevant to estuaries there is no inherent difficulty in extending the numerical techniques to non uniform, unsteady conditions.

Kerrsens et al (1979) have developed a multi-layer, 1-D model of this type which allows non uniform cross-sections to be considered but it has apparently only been applied under steady flow conditions. The basic flow equation

$$\bar{u}\frac{d\bar{u}}{dx} + g\frac{dd}{dx} + g\frac{dz_b}{dx} = -g\frac{\bar{u}|\bar{u}|}{C^2 R_h} \qquad (20)$$

where

- d is water depth
- z_b is elevation of the bed
- C is Chezy coefficient
- R_h is the hydraulic radius

is used to calculate the magnitude of the depth averaged current $\bar{u}(x)$. A logarithmic profile is assumed for the vertical structure of the flow. The suspended solids concentration is computed from the 1-dimensional form of the sediment concentration equation 2 using non uniform vertical grid to give greater accuracy near the bed. Kerrsens et al (1979) assumed the turbulent diffusivity, D_z to equal the parabolic eddy viscosity (eq 8) appropriate for logarithmic flow. The boundary condition at the bed is taken to be the equilibrium concentration, equation 10, which would occur at the instantaneous flow conditions. The solution simulates the transient evolution of the equilibrium profile from a non equilibrium condition. The model was tested against infill rates measured in a gas pipeline trench in the Western Scheldt. Some results from the paper are reproduced in Fig 6.

Koutitas and O'Connor (1980) present a similar model, but use the alternative bed condition (12) instead of prescribing an equilibrium concentration at the bed. Their paper is mainly concerned with the mathematics of finite element versus finite difference solutions. They show that the qualitative structure of the results are not sensitive to the computational method used but the full finite element model proved to be more expensive than the preferred hybrid finite element/

finite difference model. The validity of the model was verified for classical diffusion solutions.

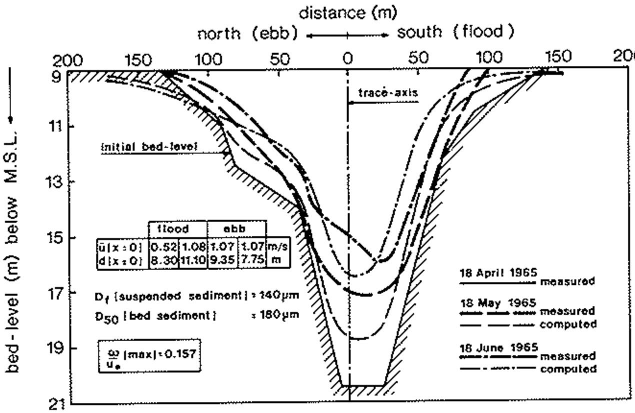


Fig 6. Measured and computed bed levels.
(From Fig 11 in Kerssens et al (1979); courtesy American Society of Civil Engineers)

Farmer and Waldrop (1977) have presented a similar model which has been applied to study a true three-dimensional situation. This model uses finite difference techniques to solve the steady state version of equation 2 ie approaching the full estuary conditions. In the model trajectories of particles from a river flow are computed under the influence of water currents, settlement and vertical diffusion to simulate the delta formation in the sea. The model was designed to study the Mississippi Delta. The model incorporates a bottom boundary condition that includes a probability relation that a particle is reflected back into the flow and there is a scouring rate coefficient.

Another model for studying three dimensional problems has been reported by Sengupta et al (1978). This is formulated as a full 3-dimensional model including time variations. The proposed boundary condition at the bed contains both concentration and diffusion flux at the bed and two parameters for the bed deposition rate and for the stickiness of the sediment. The model has been applied to Biscayne Bay, Florida. However Sengupta, et al recognised the lack of an adequate data base for verifying the model and consequently it was tested by studying the influence of the settling velocity and bed deposition parameter on the concentration profiles arising from an initial line source of suspended sediment. The results, reproduced in Fig 7, show that the surface concentration is more sensitive to settling velocity than to the bed deposition parameter. The reason for this is that although changes to the settling velocity and the bed deposition parameter are expected to have the same effect at the bed, the settling velocity also effects the feed of sediment through the water column.

Although none of the models described so far have been applied to real estuary conditions there is no technical reason why this should not be done. The main problems preventing this at present seem to be the high expense of running 3-dimensional models and deficiencies in our knowledge of sediment transport processes in estuaries. In an attempt to gain an understanding of the consequences of unsaturated flow in estuaries the present author has proposed a 2-dimensional, depth integrated model. This type of model requires special provision to take into account the vertical profile effects of the sediment concentration.

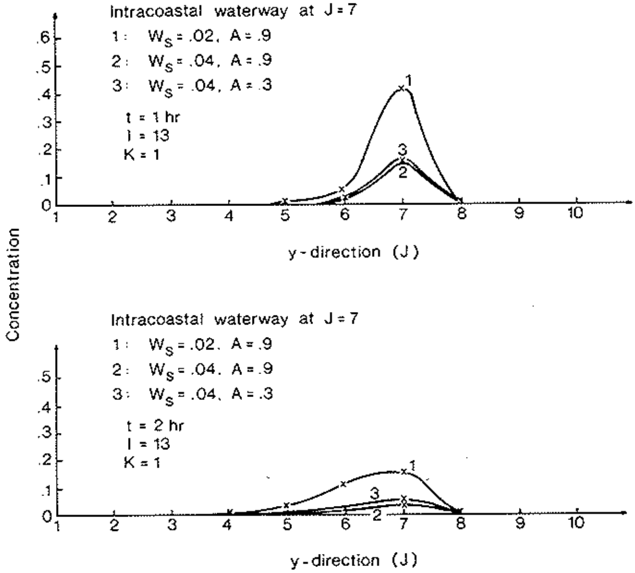


Fig 7. Sediment particle concentrations versus lateral distance at the surface (From Fig 6-1 in Sengupta et al (1979); courtesy University of Miami)

The depth averaged concentration $\bar{c}(x,y,t)$ satisfies the depth integrated form of equation 2 which may be written

$$\frac{\partial}{\partial t}(\bar{c}d) + \alpha\left(\frac{\partial}{\partial x}(d\bar{c}\bar{u}) + \frac{\partial}{\partial y}(d\bar{v}\bar{c})\right) = \frac{\partial}{\partial s}\left(dD_s\frac{\partial \bar{c}}{\partial s}\right) + \frac{\partial}{\partial n}\left(dD_n\frac{\partial \bar{c}}{\partial n}\right) + \beta_s w_s(\bar{c}_s - \bar{c}) \qquad (21)$$

where

D_s is a longitudinal dispersion coefficient due to the vertical profile

D_n is the lateral (turbulent) diffusion coefficient

(s,n) are natural co-ordinates in the direction and normal to the flow

Nihoul and Adam (1975) proposed a model similar to this. The parameters α and β_s are introduced to account for the vertical concentration and velocity profiles. For example

$$\alpha = \frac{1}{\bar{q}\bar{c}d}\int_0^d qc\,dz \qquad (22)$$

where

$q = (\bar{u}^2 + \bar{v}^2)^{1/2}$ is the horizontal water speed

represents the factor required to recover the true transport of sediment from the product of depth averaged quantities. Since high concentrations occur near the bed it follows that $\alpha < 1$, and $\beta_s > 1$.

Miles et al (1980) have analysed sediment transport measurements from the Conwy estuary in terms of α and β_s. The results for α in Table 2 are the most consistent on both a station basis and overall for the estuary and the mean is in good agreement with theoretical values from Sumer (1977) for conditions typical of flow in the Conwy estuary. The scatter in β_s, illustrated by the standard deviation, is greatest for stations 1 and 10 where no observations were made lower than 0.5m above the bed, and the omission of stations where less information was collected gave a mean β_s value of 5.0 with a standard deviation of only 2.4. This value is in good agreement with theoretical values

$$\beta_s = 2R(1 - \exp(-2R))^{-1} \tag{21}$$

from the exponential equilibrium profile of Lane et al (1941) for conditions typical of the flow in the Conwy Estuary.

Station	Date	Number of useful profiles	α Mean	α Standard deviation	β_s Mean	β_s Standard deviation
1	20 July 78	6	0.82	0.21	8.8*	7.3
3	20 July 78	3	0.98	0.15	4.1	0.9
4	20 July 78	2	0.96	–	1.8*	–
6	24 July 78	12	0.77	0.19	6.3	2.7
6	14 June 79	8	0.97	0.11	3.1	1.5
6	15 June 79	11	0.82	0.10	5.0	2.0
7	21 July 78	7	0.82	0.10	6.4*	2.3
7	24 July 78	1	0.80	–	6.3	–
8	21 July 78	1	0.70	–	8.1*	–
10	23 July 78	9	0.80	0.16	9.6*	5.3
All	–	60	0.84	0.16	6.2	4.2
All except*	–	35	0.84	0.17	5.0	2.4

* Profile calculated from measurements at minimum depths 0.5m above the bed

TABLE 2 PROFILE FACTORS FOR THE CONWY ESTUARY

The results show that it is possible to obtain sensible profile factors from field measurements and, although, the relations found are valid only for the Conwy, the techniques involved could be applied to any site. With this empirical method of prescribing α and β_s this model has the basic features for studying sediment transport in estuaries. It has advection by currents, dispersion in the direction of flow due to the vertical profile and lateral diffusion by turbulence. Deposition or erosion takes place depending as to whether the instantaneous sediment load exceeds or falls short of the saturated load, and, if required, erosion may be prevented if there is no sediment of the appropriate size available on the bed. A shortage of material on the bed would be reflected in a low concentration of suspended solids being advected away by the flow.

The most unsatisfactory aspect of this model is the use of the settling velocity as the main scaling factor for the exchange rate of material between the flow and the bed. A better approximation for this exchange rate can be determined from the analytical solutions (14) or (15) for transient conditions. The rates of exchange of sediment at the bed

$$F_z(\tau) = -(D_z\frac{\partial c}{\partial z} + w_s c)_{z=0} \tag{24}$$

are

$$F_z(\tau) = \frac{1}{2}\beta_s w_s(\bar{c}_s - \bar{c}_o)(\frac{1}{\tau\sqrt{\pi}}e^{-\tau^2} - \mathrm{erfc}(\tau)) \tag{25}$$

and

$$F_z(\tau) = \beta_s w_s(\bar{c}_s - \bar{c}_o)((1 + 2\tau^2)\mathrm{erfc}\ (\tau) - \frac{2\tau}{\sqrt{\pi}}e^{-\tau^2}) \tag{26}$$

for the boundary conditions of Mei and Lean respectively. The nature of these relations is shown in Fig 8 for a typical flow condition. Since relation 25 implies an infinite flux of sediment at t = 0 the second formulation is favoured in the following. If preferred, the analysis could be repeated for the other case.

Fig 8. Rates of exchange of sediment at the bed

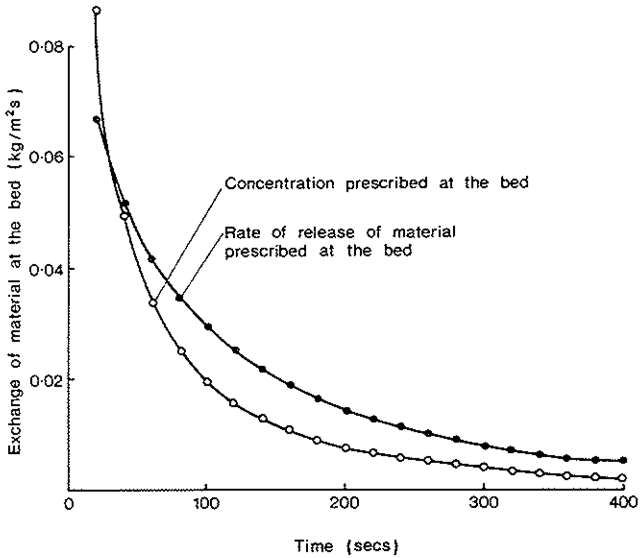


Fig 8 shows that the rate of exchange of sediment at the bed varies considerably over times of the order of timesteps normally used in numerical models. Accordingly it is advisable to integrate equation 26 over a model timestep. That is the vertical flux of sediment, $S_s(\tau)$, during the interval (o, t) is

$$S_s(\tau) = w_s\beta(\tau)(\overline{c}_s - \overline{c}_o) \qquad (27)$$

where

$$\beta(\tau) = \beta_s D_z w_s^{-2}(4\tau^2(1 + \tau^2)\text{erfc}(\tau) + \text{erf}(\tau) - \frac{2\tau}{\sqrt{\pi}}(1 + 2\tau^2)e^{-\tau^2}) \qquad (28)$$

is the bed exchange scaling factor that incorporates the effects of the vertical structure and the lag time for the concentration profile to adjust to the changing flow conditions. The nature of $\beta(s)$ is shown in Fig 9 for a typical sediment fraction and timestep.

A new sand transport model has been developed by the author at the Hydraulics Research Station using $S_s(\tau)$ given in equation 27 to replace the source/sink term on the right hand side of equation 21. This is equivalent to assuming that the sediment load is in equilibrium with the flow, which is reasonable provided the flow is slowly varying in space and time. However it follows that the new method cannot be absolutely precise because the theoretical solutions 14 and 15 have a gradual transition while the approximate computation is calculated from the assumed equilibrium profile at the start of each timestep. However, results under uniform flow conditions shown in Fig 10 indicate that the errors involved are less than the differences which result from applying the two alternative boundary conditions 10 and 12.

The advantages of the new model become apparent from the following results for the special case of a steady flow Q per m width along

Fig 9. Bed change factor

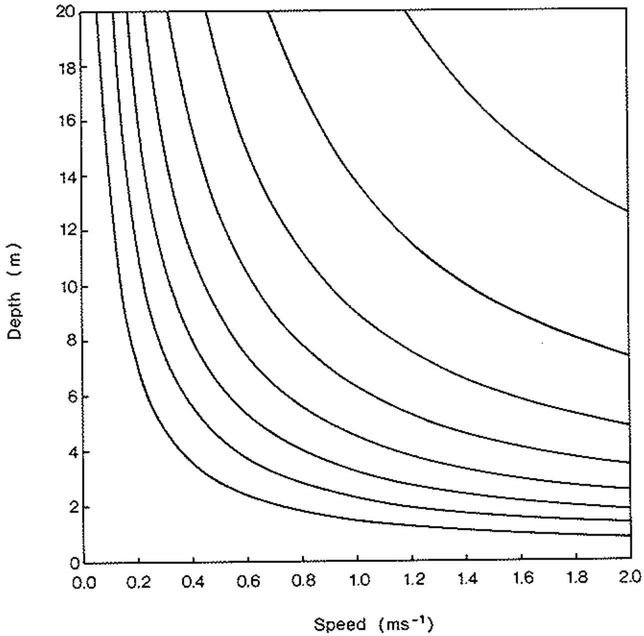
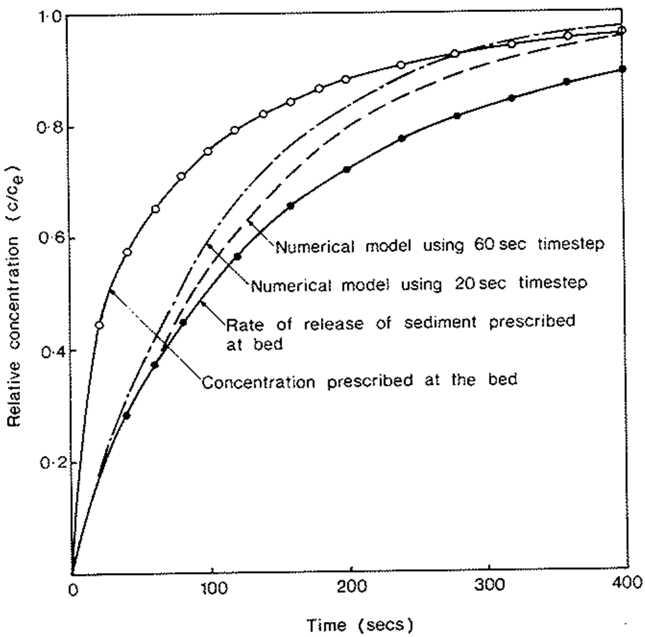


Fig 10. Theoretical and calculated adaption rates



the idealised channel shown in Fig 11 which has depth variations d(x). The bed of the channel is assumed to contain only three patches of mobile sediment. Clear water is fed into the left hand end of the model and the concentration along the channel was calculated using a source/sink term given by equations 27 and 28.

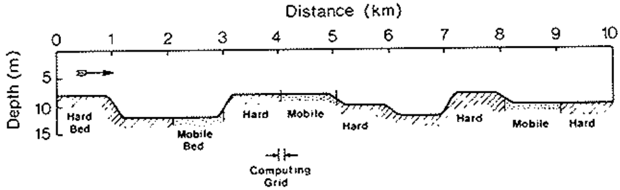


Fig 11. Geometry of test channel

Figure 12 shows the calculated concentration, $\overline{c}$, compared against the saturation concentration, $\overline{c}_s$, obtained from the simple transport relation

$$T_s = \alpha Q\overline{c}_s = E\overline{u}^2(\overline{u}^2 - \overline{u}^2_{crit}) \text{ kg/sec/m width} \qquad (29)$$

where u_{crit} is the threshold velocity for initiation of motion and E is a coefficient depending on the sediment size. The calculated concentrations in Fig 12 tend towards the saturated concentration from above (A) or below (B) depending whether the flow is over or under saturated. If the flow is over saturated, sand is deposited as the actual concentration drops. However, in the other case erosion is only permitted if there is mobile material on the bed. If the bed is hard the concentration in the model remains constant even if it is unsaturated (C).

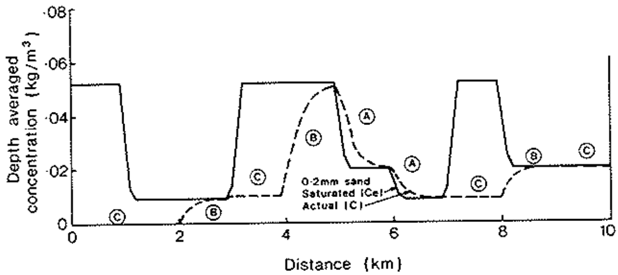


Fig 12. Calculated concentrations along test channel

Figure 13 shows an alternative method of viewing the problem using the saturation ratio $c_r = \overline{c}/\overline{c}_s$. Areas with $c_r > 1$ will suffer deposition.

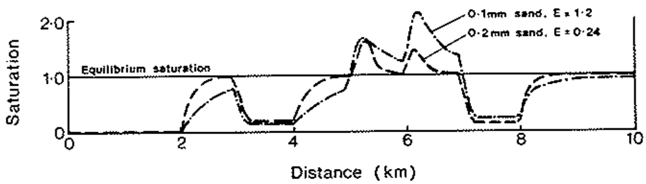


Fig 13. Calculated saturation ratio along test channel

Areas with $c_r < 1$ are potential areas of erosion but erosion only occurs if there is mobile material on the bed. Note how the finer sand takes longer to adapt to the new flow conditions.

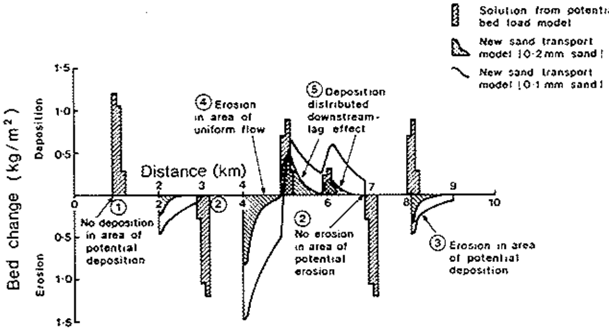
Figure 14 shows the bed changes predicted in the model for two representative sediment sizes, together with the corresponding changes from a potential load model. The idealised channel was designed to emphasise the limitations of potential load models in estuarine conditions, namely

1. Erosion may not in fact occur in an area of potential erosion if there is no erodible material on the bed – see (1) in Fig 14.
2. Deposition may not in fact occur in a region of potential deposition if the sediment load of the approaching water is insufficient to saturate even the slower flow – see (3) in Fig 14.
3. Erosion can even occur in an area of potential deposition if the sediment load of the approaching flow is very low – see (3) in Fig 14.

The results also show that

4. Erosion in the new model can occur in an area of uniform flow – see (4) in Fig 14.
5. The new model distributes deposits downstream and the finer sediment is deposited over a greater distance due to its lower settling velocity – see (5) in Fig 14.

Fig 14. Calculated bed changes in test channel



The fact that the new model can simulate these properties of sand transport makes it a much more powerful tool than potential load models. The next step will be to calibrate and verify the model in a real situation. It is hoped that this will be possible in the near future using the Conwy Estuary to enable a direct comparison to be made against the results of the potential load model previously used on that estuary. In addition there are plans to extend the model to deal simultaneously with several sand fractions.

4 MUD TRANSPORT MODELS

The relatively high density of sand grains produces a vertical structure in the sediment load which has to be taken into account in the sediment modelling. The most pronounced structure arises when the largest sand fractions are involved. At the other extreme one might expect the very fine silt and clay materials to form even distributions through the water depth and be more suitable for studying with the aid of depth integrated models. This is certainly possible in some situations, but the smallest particles ($D < 0.002$mm) cannot be considered independently because they are subjected to mutual gravity and electrochemical forces. In fresh water the particles tend to disperse but salinity cancels out part of the electrochemical forces leaving a net attractive force. Particles which come in contact or pass close to each other are drawn together and form permanent bonds in saline water. Clusters of particles are produced which have higher settling velocities than their individual constituent particles. This process is called flocculation. The rate at which flocculation occurs depends on the concentration because more collisions or near collisions are likely to take place when more particles are present in the water. At very high concentrations layers of mobile or static suspensions are formed, such as those shown in Fig 15 from Parker and Kirby (1977). It is obvious that under these circumstances special techniques will be required to describe the vertical profile. The vertical distribution is often also important in tidal rivers especially when the fresh water flow is strong enough to restrict the seawater to a salt wedge. In these situations there is a net landward drift of seawater

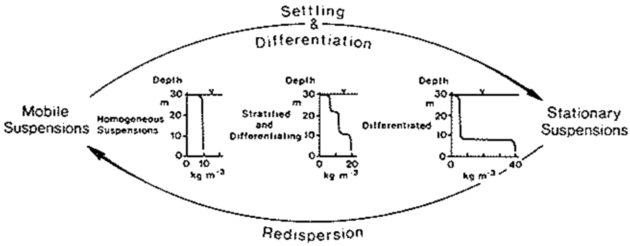


Fig 15. Development of suspension structures over one tidal cycle (From Fig 2 of Parker and Kirby (1977); courtesy BHRA Fluid Engineering)

near the bed to balance the seaward drift entrained by the surface water. Special techniques will again be required to study the transport of mud in these situations.

The simulation of mud transport requires a more comprehensive knowledge of the physical properties of the sediment than was required for studying sand transport. The properties of mud, such as settling velocity, erosion stress, deposition stress and erosion rate have been the subjects of many experimental studies over the years (Parthenaides (1965), Krone (1962), Owen (1971), and Thorn and Parson (1980)).

It appears that the settling velocity of cohesive material in suspension for concentrations less than 5000mg/l is proportional to the concentration with a proportionality factor of between 0.001 and 0.002mm/s per mg/l. This is because there is a greater likelihood of collisions when there are more particles present. Above 5000mg/l the settling velocity does not increase linearly because of hindered settling.

If the local bed stress is too high mud particles or flocs which drop onto the bed cannot settle. Krone (1962) postulated that the probability, P, that flocs stick to the bed increases linearly from zero as the bed stress, τ_b, falls below the critical depositon stress, τ_d, thus

$$P = (1 - \tau_b/\tau_d) \qquad \tau_b < \tau_d \tag{30}$$

The net deposition rate can then be expressed as

$$S_d = P w_s \bar{c} = w_s \bar{c}(1 - \tau_b/\tau_d) \tag{31}$$

with the value of τ_d determined from tests carried out on mud from the estuary under investigation.

It is well known (Ariathurai and Krone (1976) and Thorn and Parsons (1980)) that erosion does not commence until the bed shear exceeds a certain critical value, τ_e, and that the erosion rate seems to be proportional to the excess shear. However this critical stress varies with the density of the exposed mud surface, which in turn varies with time. As mud is eroded a more consolidated (and hence denser) mud is exposed which requires a corresponding higher bed stress to put it into suspension. Parthenaides (1965) proposed representing erosion as

$$S_e = M(\tau_b/\tau_e - 1) \qquad \tau_b > \tau_e \tag{32}$$

in which the erosion rate M has to be prescribed from experiments in addition to the critical erosion stress.

The fundamental equation governing the transport of mud in suspension is identical to equation 2 with the source/sink term given by equations 31 or 32. The special nature of these boundary conditions at the bed precludes the formulation of analytic solutions thus all the models described below are numerical. The models reported in the literature are often designed to meet the particular requirements of certain problems. In the following the models are separated into those which involve just one horizontal dimension ie when lateral effects are neglected, and those which take into account lateral as well as longitudinal variations. Either sort of model can include a description of vertical effects.

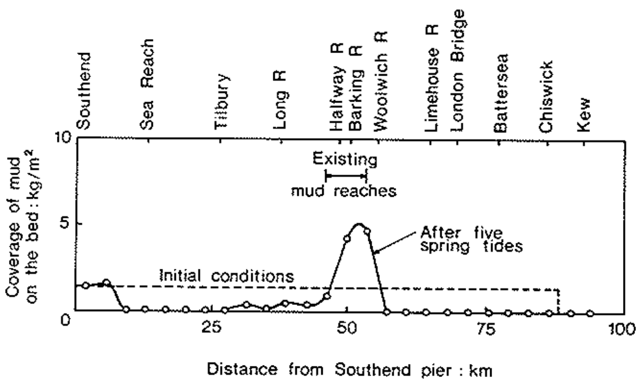
4.1 One-dimensional mud models

One-dimensional models are designed for estuaries or tidal channels where lateral variations are not important. Unfortunately these situations are likely to be more affected by salinity effects insofar that secondary currents can be created in the vertical plane. Even if the mud appears to be well mixed through the water column the presence of non-logarithmic velocity profiles necessitates the use of 2 or more layers in the vertical to simulate the mud transport.

The earliest model of this type presented by Odd and Owen (1972) used 2 layers to simulate the mud transport processes in the Thames Estuary. The lower layer was given a fixed thickness and the flow and concentration calculated in each layer using a finite difference method. The exchange of material between the lower layer and the bed was based on relations of the type given in equations 31 and 32 in which the critical stresses were estimated from laboratory experiments. Exchange of material between the upper and lower layer was computed from the net flux of mud due to settling, upwelling and vertical mixing by turbulence. The model was verified by comparing predicted and observed concentrations. The results showed very good agreement between areas of siltation predicted by the model and the existing muddy reaches of the river (Fig 16). The application of the model to study the effects of a tidal barrage clearly demonstrated its value for the engineer.

This model was subsequently improved to allow the interface between the layers to adjust dynamically to the flow and salinity variations. It has been applied to study tidal flows, saline intrusion and sediment transport in the Rotterdam Waterways. A series of sensitivity tests was carried out to demonstrate the effect of altering mixing and entrainment coefficients on the pattern of saline intrusion. Comparison with two independent sets of field observations showed that the model simulated all the gross features of the pattern of saline intrusion in the waterways for average and low fluvial flows with the same set of model parameters. The model also simulated correctly all the important periodic variations in the concentrations of suspended mud in both layers during a period of low fluvial flows.

Fig 16. Effect of repeating spring tides on the distribution of mud on the bed (From Fig 6 of Odd and Owen (1972); courtesy Institute of Civil Engineers)



Bonnefille (1976) has applied a very similar 2-layer model with a constant lower layer to the Gironde.

Experience in the use of these 2-layer models at the Hydraulics Research Station identified the need for and led to the development of numerical techniques for describing the vertical structure of the flow with many layers. A model of this type has been applied to study the siltation in the extensions to the Port of Brisbane. The essentials of the model are described by Rodger (1980). The model uses relations of the form in equations 31 and 32 for exchanges of mud between the bed and the lower layer, and includes a layered description of the bed to keep account of the age of deposits. Exchange of material between the layers is simulated as a balance between settling, upwelling and turbulent mixing as for the earlier 2-layer model but in addition the rate of turbulent mixing is adjusted dynamically to allow for statifications in the flow as described by Odd and Rodger (1979).

Watanabe et al (1978) propose a different method of representing the vertical profile using a finite element approach. At present this model is limited to uniform flow but the paper contains some flume data which could serve as a useful check on model response to changes in flow.

A model to study a special fluid mud problem in the River Avon was developed at the Hydraulics Research Station (HRS Ex 703 (1975)). Mud concentrations in this river exhibit the same type of behaviour described by Parker and Kirby (1979) for the Severn Estuary into which the river in fact flows. The model was designed to simulate three distinct phases of suspension – a static bed layer with a uniform concentration of 150 000ppm, a fluid mud layer with concentrations varying between 75 000 and 10 000ppm and a suspended mud layer with a variable concentration distributed uniformly through the layer. The flow and transport were calculated dynamically using a finite difference technique. The exchange relations between the layers were similar to those used by Odd and Owen in their 2-layer model. The erosion rate was chosen during calibration to produce the correct thickness of static layer observed in the river. The model was used to study the influence of a proposed barrage on the mud regime of the River Avon.

4.2 2-dimensional mud transport models

The extension of the numerical techniques used in the 1-dimensional models to study mud transport in 2-horizontal dimensions have not yet reached the stage for presentation in the literature. One reason is undoubted because of the high cost of 2-dimensional layered models on the computer but there is also the extra difficulty in defining the properties of the bed over an area. To date the only known models for two-dimensional areas are in fact solutions of the depth integrated equation for the mud concentration, $\bar{c}(x,y,t)$

$$\frac{\partial(\bar{c}d)}{\partial t} + \frac{\partial}{\partial x}(\overline{cu}d) + \frac{\partial}{\partial y}(\overline{cv}d) = \frac{\partial}{\partial s}(dD_s\frac{\partial\bar{c}}{\partial s}) + \frac{\partial}{\partial n}(dD_n\frac{\partial\bar{c}}{\partial n}) + S \quad (33)$$

where S is defined by relations of the form given in equations 31 and 32. The main difference between this equation and equation 21 is that no profile parameters are considered necessary when very fine sediments are involved. Models based on equation 33 are therefore mainly valid in situations where the suspended material is well mixed in the vertical (concentrations < 5000mg/l) and when lateral effects are more important.

The only published model of this type (Ariathurai and Krone (1976)) makes use of a finite element technique. The rate of exchange between the flow and the bed was simulated by relations like those given in equations 31 and 32. The model was tested by comparing predicted siltation against siltation measured in a flume experiment (Fig 17). The experimental set up consisted of a permeable barrier to create non uniformities in a steady flow. Siltation occurred in the shelter of the barrier. The potential of the model was demonstrated by considering various alternatives for a hypothetical harbour problem.

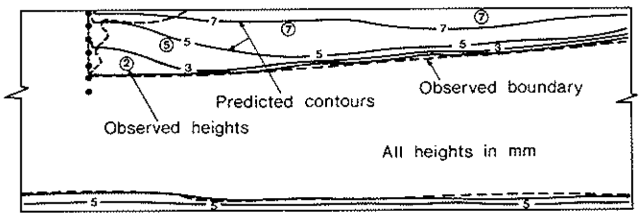


Fig 17. Observed and predicted deposition in flume (From Fig 4 in Ariathurai and Krone (1976); courtesy American Society of Civil Engineers)

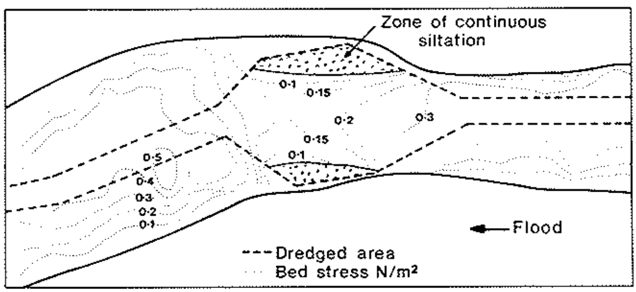
Doubtless other models of this type have been developed and are in use on particular engineering studies. The reason why accounts of such studies have not yet filtered through to the literature is probably the reluctance of the modeller to present results of the models that have not been thoroughly verified. The results of such models require considerable interpretation but with care they do provide useful information for the design engineer.

Such models are used in this way at the Hydraulics Research Station. In recent years the Station has carried out siltation studies for projects at Esmeraldas, Ecuador, the Port of Brisbane, Australia, and in the Conwy Estuary, UK. All studies were based on finite difference solutions of equation 33 and incorporated a deposition relation but erosion was not modelled explicitly. The reason for this was the lack of knowledge about the erosive properties of the existing bed. Nevertheless it was still necessary to take erosion into account otherwise it was not possible to distinguish areas where deposition could accumulate from areas which suffer only from temporary deposits at slack water. The methods of achieving this are described in connection with the particular applications.

The first application of the HRS transport model to a 2-dimensional mud problem was made for advising on siltation problems in the Port of Esmeraldas, Ecuador (Hydraulics Research Station (1977)). Siltation in the Port is apparently caused by sediment brought down the fast flowing Esmeraldas River in the wet season. Some of this settles into the sluggish underlying salt wedge in the lower estuary and eventually reaches the bed. A special 2-layer, 2-dimensional flow model was constructed to represent the river flow and the salt wedge, and a mud transport model was constructed for the surface layer; Miles (1978). It was not possible to collect enough data to calibrate the model properly and as a consequence it was necessary to make sensitivity tests on the relative significances of turbulent mixing, settling and entrainment parameters which controlled the amount of material dropping into the lower layer. The model was used to predict changes in suspended solids concentration and the settlement into the salt wedge and to the bed which would occur if a new harbour was built.

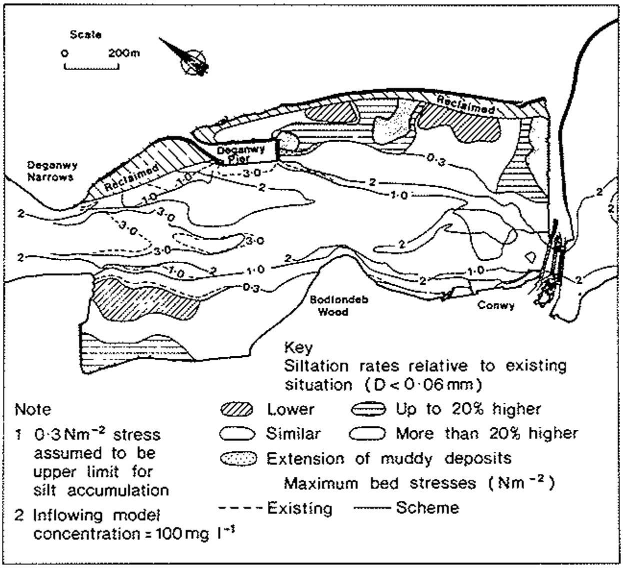
The next application by the Hydraulics Research Station was made to provide information about the lateral distribution of siltation in the Brisbane River to complement the longitudinal distribution of siltation predicted by the HRS multi-layer model. This work was commissioned by the Port of Brisbane Authority. An outline of the study was presented by Odd and Baxter (1980) and the particular details of the 2-dimensional model are described in Ref 14. The 2-dimensional model was run under steady peak flood and ebb flows on the principle that deposition under these conditions would identify areas where mud could accumulate. The model was verified against cores collected from areas identified by the model to be muddy or not muddy (Fig 18). The siltation rate was calibrated by reference to known dredging rates under existing conditions and the model used to predict areas and rates of siltation in the new docks.

Fig 18. Bed stress distribution in the Fisherman Islands Swing Basin



The most recent application was to study siltation aspects of proposed engineering works in the Conwy Estuary (Ref 15). The 2-dimensional models were in this case run over spring tidal cycles and again deposition was governed by relation 31. There was no information available on the erosion properties of the Conwy mud and consequently the effect of erosion was taken into account by choosing a critical stress for the accumulation of deposits. This critical stress was determined by comparing the maximum bed stress contours (predicted by the model) against the existing muddy areas. The same critical stress was subsequently used to identify the possible advance or retreat of the muddy areas following the construction of works. There were no siltation rates or dredging records available from the estuary to calibrate the model. Accordingly siltation rates in the developed state were presented in the form shown in Fig 19 as ratios relative to the existing (but unknown) siltation rate at the same point. This information was used by environmentalists to assess the impact of the works on the ecology as well as by the engineers to assist in the design of works.

Fig 19. Influence of permanent works on fine sediments



5 STATE OF SEDIMENT TRANSPORT MODELLING

From this review of published information it is clear that the expertise already exists to formulate and develop the appropriate numerical techniques. Furthermore computers are becoming more powerful and machines exist which can cope with the escalating number of calculations involved as the models become more sophisticated. The main factor limiting the advance of numerical sediment transport models is probably the uncertainties that exist about the physical relations which have to be fed into the models. Table 3 shows a list of the main relations that influence sediment transport in estuaries. The quality of the relation is graded from very good to poor merely to give a relative guide, but hopefully it will also inspire some researchers to improve the situation.

6 CONCLUSIONS

Many numerical models for studying sediment transport have been presented in recent years but very few are suitable for application to estuaries. If the author's experience is typical, it is likely that many engineering studies have been made that have not been publicised because of pressures of other work or because the models were not fully calibrated or verified. From a scientific point of view an unverified model is of very little value. However this is not necessarily true in an engineering sense provided the modeller or engineer recognises the shortcomings of the model and takes care in interpreting the model results. Results should not be taken on their face value but used to supplement the experience of the engineer, and to pinpoint the areas that the engineer should concentrate his attention on. Used in this way even an unverified model is a valuable tool. For this reason there is a need for modellers and engineers to publish more readily their sediment transport case studies. Other workers in the field will appreciate the reason for limitations in such studies and the ensuing discussion can only serve to improve sediment models generally in the long-term.

Relation		Model required by	Method of studying	Quality of relation
Longitudinal dispersion	D_s	SM	Theory/L/F	Good
Lateral diffusion	D_n	SM	Lab/Field	Fair
Vertical diffusion	D_z	SM		Fair
Sand settling velocity	$w_s(D)$	S (sand)	Lab	V good
Advection deficit	α	S	Field	Fair
Profile factor	β_s	S	Theory	Good
Saturation S/T law	T_s, c_s	S	Field	Fair
Mud settling velocity	$w_s(c)$	M (mud)	Lab/Field	Good
Deposition stress	τ_d	M	Lab	Fair
Erosion stress	$\tau_e(\rho_b)$	M	Lab	Good
Erosion rate	M	M	Lab?	Fair
Consolidation rate	$\rho_b(t)$	M	Lab?	Poor

TABLE 3 STATE OF KNOWLEDGE ABOUT PHYSICAL RELATIONS

7 ACKNOWLEDGEMENTS

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8 REFERENCES

1 APMANN R P and RUMER R R (1970). Diffusion of sediment in developing flow. Proc. ASCE 96 HY1.

2 ARIATHURAI R and KRONE R B (1976). Finite element model for cohesive sediment transport. Proc. ASCE 102 HY3, pp 323-338.

3 BETTESS R and WHITE W R (1979). A one-dimensional morphological river model. Hydraulics Research Station, Wallingford. Report IT 194.

4 BONNEFILLE R (1976). Suspended sediment transports in the Gironde Estuary. Paper presented to 15th Coastal Engineering Conference, Honolulu (Abstract).

5 CHEN Y H, LOPEZ J L and RICHARDSON E V (1978). Mathematical modelling of sediment deposition in reservoirs. proc. ASCE 104 HY12 pp 1605-1616.

6 CHRISTODOULOU G C, LEIMKUHLER W F and IPPEN A T (1974). Mathematical models of Massachusets Bay. III A mathematical model for the dispersion of suspended sediments in coastal waters. Mass Ins of Technology Report 179.

7 CUNGE J A and PERDREAU N (1973). Mobile bed fluvial mathematical models. La Houille Blanche 7 pp 561-581.

8 DE VRIES M (1976). Morphological computations, Delft University of Technology.

9 DOBBINS W E (1943). Effect of turbulence on sedimentation Trans Am Soc Civil Engineers. 109.

10 FARMER R C and WALDROP W R (1977). A model for sediment transport and delta formation. Coastal sediments 77. Proc 5th Symp. of Waterways, Ports and Coastal Div. ASCE New York 1977 pp 102 - 115.

11 GRAF W H (1971). Hydraulics of sediment transport. McGraw Hill Book Company, New York.

12 HYDRAULICS RESEARCH STATION. Report Ex 703 (1975). River Avon, Bristol. Tidal Barrage at Aston, Part II of sediment study.

13 HYDRAULICS RESEARCH STATION. Report Ex 788 (1977). Port of Esmeraldas, Ecuador. Siltation study.

14 HYDRAULICS RESEARCH STATION. Report Ex 883 (1979). Saline intrusion and sediment transport in Rotterdam Waterways. Two-layer numerical model simulation studies.

15 HYDRAULICS RESEARCH STATION. Report Ex 899 (1980). Port of Brisbane Siltation Study 4th Report. Mathematical model studies of peak tidal flow and mud deposition patterns at Fisherman Island.

16 HYDRAULICS RESEARCH STATION. Report Ex 902 (1980). A55 North Wales Coast Road. Conwy Estuary Crossing Numerical model studies.

17 HUNT J N (1954). The turbulent transport of suspended sediment in open channels. Proc. Roy Soc. A. 224. No 1158 pp 322-335.

18 KALINSKE A A (1940). Suspended material transportation under non-equilibrium conditions. Trans Am Geophy Union Vol 21.

19 KATOH K, TANAKA N, NADAOKA K (1979). Tidal simulation on tidal marsh and numerical forecasting of its topographic deformation. Report of the Port and Harbour Research Institute, Vol 18 (4), pp 4-76 (In Japanese with English synopsis).

20 KERRSENS P M J, AD PRINS and VAN RIJN L C (1979). Model for suspended sediment transport. Proc ASCE 105 HY5 pp 461-476.

21 KOUTITAS C and O'CONNOR B (1980). Numerical modelling of suspended sediment. Advances in Wat Res. 3 pp 51-57.

22 KRONE B R (1962). Flume studies of the transport of sediment in estuarine shoaling processes. Technical report Hydraulics Engineering Laboratory. Univ. of California, Berkeley, California.

23 LANE E W and KALINSKE A A (1941). Engineering calculations of suspended sediment, Trans Am Geophys Union Vol 20.

24 LEAN G H (1980). Estimation of maintenance dredging for navigation channels. Hydraulics Research Station, Wallingford, UK.

25 LEPETIT J P and HAGUEL A (1978). A numerical model for sediment transport. Paper presented to 16th Coastal Engineering Conference, Hamburg pp 1715-1723.

26 MEI C C (1969). Non-uniform diffusion of suspended sediment, Proc. ASCE HY1 pp 581-584.

27 MILES G V (1978). Two-dimensional, two-layer salt and mud transport model. Report IT 182, Hydraulics Research Station, Wallingford, UK.

28 MILES G V, OZASA H and WEBB D G (1980). Sediment transport relations for estuary models. In preparation.

29 NIHOUL J C J and ADAM Y (1975). Dispersion and settling around a waste disposal point in a shallow sea. J Mar Res 13 (2) pp 171-186.

30 ODD N V M and OWEN M W (1972). A two-layer model of mud transport in the Thames Estuary. Proc Ins Civil Engineers, London Supplement ix paper 7517S pp 175-205.

31 ODD N V M, MILES G V and MANN K (1976). Mathematical and physical model studies for the Wash Water Storage Scheme. Paper presented at joint ICE/CWPU Symposium on Wash Water Storage Scheme, 9 Nov. 1976. Hydraulics Research Station Reprint, Wallingford UK.

32 ODD N V M and RODGER J G (1978). Vertical mixing in stratified tidal flow. Proc ASCE HY3 pp

33 ODD N V M and BAXTER T (1980). Port of Brisbane siltation study. Paper presented to 17th Coastal Engineering Conference, Sydney.

34 OWEN M W (1971). The effect of turbulence on the settling velocity of silt flocs IAHR Congress, Paris.

35 PADMAVALLY K (1977). Speed of inertial bed load disturbances. Proc. ASCE (103) HY7 pp 777-795.

36 PARKER W R and KIRBY R (1977). Fine sediment studies relevant to dredging practice and control. Proc 2nd Int. Symp on Dredging Technology. Vol 1. Paper B2 pp 15-26, Published by BHRA, UK.

37 PARTHENAIDES E (1965). Erosion and deposition of cohesive soils. Proc ASCE 91 HY1 pp 105-139.

38 RODGER J G (1980). Simulation of stratified flows in estuaries. Paper presented to 2nd Int. Symp. Stratified Flow, Trondheim, Norway. June 1980.

39 SCHMIDT W (1925). Der Massenaustausch in freier Luft und verwandte Erscheinungen, in "Probleme der kosmischen Physik", Bd 7 Hamburg.

40 SENGUPTA S, LEE S S and MILLER H P (1978). Three-dimensional numerical investigations of tide and wind induced transport processes in Biscayne Bay. Sea Grant Ball No. 39. Miami University. Dept Mech Engineering. Coral Gables. FL.

41 SUMER B M (1977). Dispersion of suspended particles in open channel flow. A review. DCAMM Report 130. Institute of Hydro and Hydraulic Eng. Tech Univ. of Denmark, pp 1-23.

42 THORN M F C and PARSONS J G (1980). Erosion of cohesive materials in estuaries. An engineering guide. Paper presented to 3rd Int. Sym. on Dredging Technology. Mar 1980.

43 THOMAS W A and PRASUHN A L (1977). Mathematical modelling of scour and deposition. Proc. ASCE (103) HY8 pp 851-863.

44 WATANABE A, THIMAKORN P and DAS GUPTA A (1978). Concentration of suspended clay in periodic flow. Proc. 16th Coastal Engineering Conference, Hamburg, Aug 27 - Sept 3 1978. Pub. ASCE New York 1979, Vol 2 pp 1819-1931.

45 WHITE W R, MILLI H and CRABBE A D (1973). Sediment transport: an appraisal of available methods. Vol 1: Summary of existing theories, Vol 2: Performance of theoretical methods when applied to flume and field data. HRS Report INT 119.

46 WILSON R E (1979). A model for the estimation of the concentration and spatial extent of suspended sediment plumes. Estuary and Mar. Coastal Sci. 9 pp 65-78.

47 YALIN M S and FINLAYSON G D (1973). On the development of the distribution of sediment load. Paper presented to 15th IAHR Congress, Istanbul pp 287-295.

9 LIST OF SYMBOLS

$c\ (x,y,z,t)$	concentration of suspended solids (kg m^{-3})
$\bar{c}\ (x,y,t)$	depth-integrated concentration (kg m^{-3})
$\bar{c}_o$	initial concentration (kg m^{-3})
$c_r\ (x,y,t)$	saturation ratio, $\bar{c}/\bar{c}_s$
c_s	concentration under saturated conditions
$\bar{c}_s$	depth-integrated, saturated concentration
C	Chezy friction factor
d	water depth (m)
D	sediment grain size (mm)
D_n	lateral (turbulent) diffusion coefficient (m^2 s^{-1})
D_s	longitudinal (shear) dispersion coefficient (m^2 s^{-1})
D_x, D_y, D_z	diffusion coefficients in 3-D axes (m^2 s^{-1})
E	sand transport coefficient (kg s^3 m^{-5})
F_z	net vertical flux of sand (kg m^{-2} s^{-1})
g	acceleration of gravity (m s^{-2})
m	quantity of mobile bed material (kg m^{-2})
M	erosion rate (kg m^{-2} s^{-1})
n	natural co-ordinate normal to flow
P	probability of deposition

q	speed of current, $(\bar{u}^2 + \bar{v}^2)^{1/2}$ (ms^{-1})
Q	discharge per unit width ($m^2\ s^{-1}$)
R	settling velocity Reynolds Number, $w_s d/2D_z$
R_h	hydraulic radius
s	natural co-ordinate in direction of flow
S	source/sink term at the bed ($kg\ m^{-2}\ s^{-1}$)
S_d	deposition rate ($kg\ m^{-2}\ s^{-1}$)
S_e	erosion rate ($kg\ m^{-2}\ s^{-1}$)
S_s	vertical flux of sand ($kg\ m^{-2}\ s^{-1}$)
t	time
T_s	sand transport ($kg\ m^{-1}\ s^{-1}$)
(u,v,w)	components of velocity vector (ms^{-1})
$(\bar{u}, \bar{v})$	depth integrated velocity components (ms^{-1})
$\bar{u}_{crit}$	threshold velocity for sand transport (ms^{-1})
u_o	uniform velocity (ms^{-1})
$w_s(D)$	settling velocity (ms^{-1})
(x,y,z)	cartesian co-ordinates
z_b	elevation of bed (m)
α	profile parameter, $\int ucdz/d\bar{u}\bar{c}$
$\beta(\tau)$	bed exchange factor
β_s	profile parameter, $c(x,y,o,t)/\bar{c}(x,y,t)$
κ	Von Karman constant
ν_T	eddy viscosity ($m^2\ s^{-1}$)
ρ_b	consolidation ratio of deposited mud ($kg\ m^{-3}$)
σ	dimensionless variable, $(w_s^2/4D_z)^{1/2}\ t^{1/2}$
τ	dimensionless variable, $(d^2/4D_z)^{1/2}\ t^{1/2}$
τ_b	bed stress (Nm^{-2})
τ_d	critical stress for deposition (Nm^{-2})
τ_e	critical stress for erosion (Nm^{-2})
ζ	dimensionless vertical co-ordinate, z/d