



Hydraulics Research
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USE OF HARMONIC CONSTITUENTS IN
NUMERICAL MODELLING

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CONTRACT

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SUMMARY

The tide observed in nature is made up of constituents with periods equal to the periods of the relative astronomical motions of the Earth, Sun and Moon, but with amplitudes and phases which are unique to each location. The tidal predictions printed in tide tables, are derived from a large number of constituents, and the future astronomical positions of the Earth, Sun and Moon. The numerical modeller often requires tides idealised in some manner, and his use differs from that of the hydrographer. This report is a practical guide to the numerical modeller in his specialised use of these constituents.

At the time of privatisation in April 1982 Hydraulics Research Limited had the basis of a comprehensive set of computer models for simulating and predicting the effect of engineering works on conditions in tidal waters, which has subsequently become known as the TIDEWAY system.

The most important component of any TIDEWAY application is a TIDEFLOW model which generates the basic water movements for transporting the other quantities. The models are based on well established principles of conservation of mass and Newtons Laws of motion, including friction due to bed roughness, pressure forces due to density variations and wind stress.

Features of the models are the facility to vary the grid size, to give high resolution in areas of interest, and local distortion of boundary elements to improve the representation of the natural geometry of the study area.

The models can be used to predict flooding due to tides, fluvial floods, storm surges, cyclones and tropical storms. They are also commonly used to predict the effect of engineering works on tidal currents and hence navigation. Flow results from the models are stored in the computer and subsequently used to transport sediment, salt, heat and pollutants.

An important aspect of any TIDEFLOW-2D model application is the boundary conditions used to generate the tidal flow. Ideally the boundary conditions would be prescribed from observations of tidal currents or tidal levels for specific types of tides or tidal sequences. However, in practice, tidal observations may not be available at the required position or for the required period. Under these circumstances it is often necessary to predict the tide using tidal constituents obtained from cotidal charts or analysis of recording tide gauges.

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1 INTRODUCTION

At the time of privatisation in April 1982 Hydraulics Research Limited had the basis of a comprehensive set of computer models for simulating and predicting the effect of engineering works on conditions in tidal waters, which has subsequently become known as the TIDEWAY system.

The most important component of any TIDEWAY application is a TIDEFLOW model which generates the basic water movements for transporting other quantities (Refs 1 and 2). The models are based on well established principles of conservation of mass and Newtons Laws of motion, including friction due to bed roughness, pressure forces due to density variations and wind stress.

Features of the models are the facility to patch areas of finer meshes dynamically into the basic model, and local distortion of boundary elements to improve the representation of the natural geometry of the study area.

The models can be used to predict flooding due to tides, fluvial floods, storm surges, cyclones and tropical storms. They are also commonly used to predict the effect of engineering works on tidal currents and hence navigation. Results from the models are stored in the computer and subsequently used to transport sediment, salt, heat and pollutants.

An important aspect of any TIDEFLOW-2D model application is the boundary conditions used to generate the tidal flow. Ideally the boundary conditions would be prescribed from observations of tidal currents or tidal levels for specific types of tides or tidal sequences. However, in practice, tidal observations may not be available at the required position or for the required period. Under these circumstances it is often necessary to predict the tide using tidal constituents obtained from cotidal charts or analysis of recording tide gauges.

The standard method of tidal analysis and prediction is the harmonic method, in which an observed tidal record is analysed for its constituents, and these constituents may then be recombined to produce tidal predictions during periods of no observations. This report does not set out to give a description of the harmonic method which can readily be found elsewhere (Ref 3). Rather it sets out to illustrate the mechanics of using harmonic constituents for numerical modelling applications.

2 THE HARMONIC CONSTITUENTS

Over 400 constituents have been identified, but many of these are small and for most modelling purposes only a limited number need to be used. These break down into four groups: Long Period Constituents; Diurnal Constituents; Semidiurnal Constituents and Higher Order (Shallow Water) Constituents. The most important are tabulated below:

| Name of Constituent | Symbol | Coefficient Ratio ($M_2=100$) | Speed ($^{\circ}$ /solar hour) | Period (solar hours) |
|----------------------------|--------|---------------------------------------|------------------------------------|-------------------------|
| Lunar fortnightly | M_f | 17.2 | 1.09803 | 327.86 |
| Principal lunar diurnal | O_1 | 41.5 | 13.94304 | 25.82 |
| Principal solar diurnal | P_1 | 19.4 | 14.95893 | 24.07 |
| Luni-solar diurnal | K_1 | 58.4 | 15.04107 | 23.93 |
| Longer lunar elliptic | N_2 | 19.2 | 28.43973 | 12.66 |
| Principal lunar | M_2 | 100.0 | 28.98410 | 12.42 |
| Principal solar | S_2 | 46.6 | 30.00000 | 12.00 |
| Luni-solar semidiurnal | K_2 | 12.7 | 30.08214 | 11.97 |
| Lunar quarter diurnal | M_4 | - | 57.96821 | 6.21 |
| Luni-solar quarter diurnal | MS_4 | - | 58.98410 | 6.10 |

The coefficient ratios are theoretical values from the tractive forces, and are subject to modification by dynamic effects. However, it is usual for constituents within a group (semidiurnal or diurnal) to have relative magnitudes similar to the theoretical values.

The long period, M_f constituent arises from variations in the declination of the moon. Its main effect is to alter the mean level of a particular tide.

The diurnal constituents tend to be smaller than the theoretical coefficient ratios in European waters, and only produce a small daily inequality in the two daily tides. In other parts of the world, such as parts of South East Asia, the diurnal constituents predominate and the tide may be almost fully diurnal. In areas of diurnal tides the relative movement of the O_1 and the K_1 constituents give the spring-neap cycle. In areas where the tide is predominantly semidiurnal the spring-neap cycle is given by the constituents M_2 and S_2 .

The N_2 constituent arises from the elliptical orbit of the moon around the earth, and effects the range of successive sets of spring or neap tides. This causes high and low springs in some months, and high and low neaps in others. These sets of high and low springs or neaps occur in different months each year, repeating approximately every 4 years, though more years are needed for a complete cycle.

The effect of the other major semidiurnal constituent K_2 has a more definite seasonal effect. This constituent has nearly the same period as S_2 and so in some months reinforces and other months reduces the effect of S_2 . Reinforcement occurs at the spring and autumn equinoxes giving rise to larger spring and smaller neap ranges during March/April and September/October. These constituents are in opposition during mid-winter and mid-summer resulting in smaller spring and larger neap ranges during December/January and June/July. The effect of this seasonal variation will often be masked during individual spring or neap tide periods by the rather larger effects of N_2 . The average from each pair of spring or neap tides will, however, reflect the seasonal trend.

Shallow water and bottom friction distort the shape of the tide in the inner portions of most estuaries. This gives rise to the higher order constituents which can be higher order harmonics of the primary constituent (eg for M_2 ; M_4 , M_6 , M_8 etc, or for S_2 ; S_4 , S_6 , S_8 etc) or compounds arising from the interaction of the constituents such as MS_4 , $2MS_6$, $2SM_2$, MK_3 etc. The quarter diurnal constituents M_4 and MS_4 are usually the most important. As with the primary constituents the suffix indicates the approximate number of tides per day. A number of prefixing the constituent is a scalar on the first term, eg $MS_4 = M_2 + S_2$
 $2MS_6 = 2 \times M_2 + S_2$
 $2MS_2 = 2 \times M_2 - S_2$

From the convention it is possible to derive the astronomical argument and the speed of the constituent from the arguments and speeds of the primary constituents.

For example:

$$\begin{aligned} \text{arg. of } MS_4 &= \text{arg. of } M_2 + \text{arg. of } S_2 \\ \text{and } \omega_{MS_4} &= \omega_{M_2} + \omega_{S_2} \\ &= 28.98410 + 30.0000 \\ &= 58.98410 \text{ }^\circ/\text{hour.} \end{aligned}$$

For the numerical models there is no need for a detailed knowledge of the astronomical arguments as daily values of phases can be found in the Admiralty tide tables.

3 TIDAL CONSTITUENT DATA

For each constituent at a site two values are given, g and H . g is the local phase lag of the constituent compared with the phase of the same constituent of the theoretical equilibrium tide (derived by Newton) at Greenwich. The value of g is associated with a time zone. Consequently if the time zone is changed, phase g is changed. H is the amplitude of the constituent. Both of these values are essentially fixed for a site, although there may be a small seasonal variation.

The data is normally available in one of two basic forms; sets of values at a location, or a chart/computer printout showing the values of H and g for a constituent over an area. Commonly, chart/computer printout for an area is restricted to four constituents, M_2 , S_2 , K_1 and O_1 . These four values are published in the Admiralty Tide Tables for a large number of locations. Fuller sets of constituents may be obtained from the Institute of Oceanographic Sciences or the International Hydrographic Bureau.

4 TYPICAL APPLICATIONS

The use of harmonic constituents for numerical modelling falls into two main categories:

1. Reproduction of a typical tide;
2. Reproduction of a specific tide.

4.1 Reproduction of a typical tide

The example illustrated here is a typical mean spring tide for European waters. As we are dealing with only a typical tide, the long period constituents can be ignored. The diurnal components can also be ignored as they are small. For a mean tide it is usually only necessary to consider the M_2 and S_2 semidiurnal constituents. the phase lag g is ignored and a good representation of a mean spring tide can be obtained by assuming that the two constituents M_2 and S_2 are in phase. Similarly a mean neap tide can be computed assuming M_2 and S_2 are 180° out of phase, and therefore acting in opposition. The tide produced by these constituents is approximately a sine curve, and does not reproduce the strong asymmetry found in areas where higher order (shallow water) constituents are significant. In these areas higher order constituents need to be included.

The phase of the higher order constituents cannot be chosen arbitrarily; once the phases of M_2 and S_2 have been chosen then so implicitly have M_4 , S_4 , MS_4 , M_6 etc.

This can be seen as follows. Suppose it is desired that the M_2 tide shall commence at high water, ie

$$(g_{M_2} t_0 - \omega_{M_2}) = 0 \text{ (modulo } 360^\circ)$$

where

$$\omega_{M_2} = \text{speed of } M_2 \text{ (}^\circ/\text{hr)}$$

$$g_{M_2} = \text{phase of } M_2 \text{ (}^\circ)$$

t_0 = starting time of tide curve

then the M_4 tide will commence at a phase given by

$$(g_{M_4} - \omega_{M_4} t_0) = (g_{M_4} - 2\omega_{M_2} t_0) = (g_{M_4} - 2g_{M_2})$$

ie, the state of M_4 tide is determined.

In general the relationship takes the form:

$$\theta_c = \sum_{i=1}^n \phi_i \alpha + g_c - \sum_{i=1}^n \pm \phi_i g_i$$

where:

n is the number of primary constituents used to produce the higher order constituent. For tidal predication this normally does not exceed 3, and for modelling applications 2 is likely to be the limit.

θ_c is the phase of the constituent

ϕ_i etc are scalars to be applied

α etc are the phases of the primary constituent

g_i etc are the phases of the primary constituent

g_c is the phase g of the constituent

This is best illustrated by examples:

if the M_2 tide is to commence at phase θ^0 , and the S_2 tide at phase ϕ^0 , say, then:

$$\begin{array}{l} M_4 \text{ will have phase } 2\theta + g_{M_4} - 2g_{M_2} \\ S_4 \quad \quad \quad 2\phi + g_{S_4} - 2g_{S_2} \\ MS_4 \quad \quad \quad \theta + \phi + g_{MS_2} - g_{M_2} - g_{S_2} \end{array}$$

and so on.

MS_4 although a long period tide is essentially a shallow water constituent which sometimes can be significant, and has the argument of S_2 - argument of M_2 . Hence the relationship for the higher order constituents can be applied.

MSf will have phase $\phi + \theta + g_{MSf} - g_{S_2} + g_{M_2}$

It is common practice to adjust the speeds of the constituents to give a repeating tide. For example, taking $M_2 = 29^\circ/\text{hr}$, $S_2 = 30^\circ/\text{hr}$, $O_1 = 14^\circ/\text{hr}$ and $K_1 = 15^\circ/\text{hr}$ would give a repeating 15 day spring-neap tide cycle. This approach also has the advantage that it can be seen when transients in a model solution, arising from poor starting conditions, have disappeared.

4.2 Reproduction of a particular tide

To obtain a tide for a particular day the constituents need to be combined at the correct phasing for that particular day. The constituents listed in the table (Section 2) should be sufficient to give a good reproduction of the tide, but further constituents will increase accuracy. Often only M_2 , S_2 , K_1 and O_1 are available, and an alternative approach (Admiralty Method), which modifies the phase and amplitude of these constituents to allow for the lesser constituents, may be better. The modified phase is known as the tide angle 'A' and the factor F is used to modify the amplitude. These values are based on the assumption that constituents close to each other in speed will have approximately equal phase lags (g), and their amplitude (H) will be approximately proportional to their theoretical magnitudes. How good these assumptions are will vary from site to site, but the Admiralty have found (Ref 4) that the differences compared with a full prediction using 60 constituents were no greater than those found using the 25 constituents which can be obtained by one month's analysis.

The two approaches using the modified or true values, are basically similar and can be described together. Reference needs to be made to the Admiralty Tide tables Table VII or Table VIII. Table VIII gives the true value of phase ($E_0 + u$) and monthly values of f for 13 constituents. Where: E_0 is the phase of the equilibrium constituent at Greenwich and u is a correction to allow for the effect of the 18.6 year variation in the declination of the moon on the phase of the equilibrium constituent. f is the nodal factor and is a scalar applied to amplitude H to allow for the effect of the 18.6 year cycle on the amplitude of a constituent. (Alternatively, these values may be computed directly (Ref 3)). Table VII gives the modified values A and F for M_2 , S_2 , K_1 and O_1 as previously described.

The nodal function f or the Tide function F can be taken straight from the tables. The value of phase is calculated as follows:

Table VII

$$\text{Phase} = g - [\omega t + 360 - A]$$

where

ω is the speed of the constituent

t is the time in hrs past 0000h

or Table VIII

$$\text{Phase} = g - [\omega t + (E_0 + u)]$$

[For certain modelling applications subsequent rounding of speeds may be needed to give a repeating tide. However, exact values of speed should be used to the above relationships when obtaining the initial phase.] Where simplification has taken place (ie reducing the number of harmonics, rounding off speeds etc) it is advisable to examine the boundary tide to make sure it gives a reasonable tide curve. The decision whether to use Table VII or Table VIII values will depend largely on what is available. It may be possible to calculate the boundary tide using both methods and examine which gives the best synthesis.

5 TIME ZONES

As has been stated phase g is only correct for the stated time zone. If a model crosses a time zone, it is necessary to reduce phase g values to a common time zone. This is accomplished in the following manner:

$$g_c = (T_o - T_c)\omega_c + g_o$$

where

T_o is the time zone of phase g ;

T_c is the common time zone to be used;

ω_c is the angular speed of the constituent ($^\circ$ /hour);

g_o is phase g at time zone T_o ;

g_c is phase g at common time zone.

6 ACKNOWLEDGEMENTS

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