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NUMERICAL MODELLING OF SEDIMENT TRANSPORT IN COASTAL SITUATIONS

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CONTENTS

ABSTRACT

The problems of developing a numerical model to predict the movement of non-cohesive sediment in coastal situations are discussed. Existing wave and tidal models available at HR are considered and the way that they could be combined to model both waves and currents. The application of a theory of sediment transport under waves and currents to these situations is discussed.

1 INTRODUCTION

There is an almost continual movement of sediment in coastal areas and any engineering work which affects or interrupts such movement may have a major impact on the morphology of the area. Consequently there is an interest in the prediction of such movement. While there are accepted methods of proven accuracy for predicting the wave climate the problem of determining the resulting sediment movement is less amenable to solution. In most coastal situations the waves are the dominating feature but because the sediment transport rate is sensitive to the currents the presence of the currents may have a significant effect. Since spatial variations in the wave climate lead to wave-induced currents, currents are invariably a feature of coastal wave problems. It is, therefore, in general not sufficient to consider only the wave climate. At present, to make predictions about the impact of engineering works on the movement of coastal sediments resort has to be made to empirical methods or simple, one-dimensional numerical models. This report considers the problems of implementing a two dimensional, numerical model that will simulate the movement of non-cohesive sediments on a coast.

2 SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

Sediment transport in coastal situations may depend upon both the waves and the currents. To simulate the movement of non-cohesive sediments on a coast it is necessary to have a theory which will predict the local sediment transport rate. Many of the theories of sediment transport under waves and currents have been developed from theories of sediment transport under steady uni-directional currents. In the limit of zero waves these theories become identical to the equivalent, steady uni-directional flow equations which produce satisfactory predictions. The waves and currents theories can be regarded as extrapolations of the steady uni-directional flow theories. A comparison of such theories with field and laboratory data has indicated that, providing the effect of the current dominates that of the waves, they provide reasonable predictions but that when waves dominate the results are less satisfactory. The predictions in the case where waves predominate could be improved if a reliable predictor were available for the case of waves alone. The main findings of the comparison are summarised in Appendix 1.

The sensitivity of sediment transport rate to both waves and currents is demonstrated by Figures 1 and 2 which show the sediment transport rate calculated using Ackers and White-Swart equations. This sensitivity implies that to effectively include both waves and currents it is necessary to model both adequately.

Thus one is lead to the conclusion that to include the effects of sediment transport under waves and currents one should have a model which has both a flow component and a wave component.

When the waves and currents are not in the same direction consideration must be given to the direction of the resulting sediment transport. The net movement of the sediment may not necessarily be in the direction of the waves or the current. Since the direction of the net shear may be different at different levels throughout the depth it is possible that the bed load may move in one direction and the suspended load may move in another.

Careful attention must also be given to the methods used to simulate the wave conditions. Under many conditions it can be assumed that waves behave linearly so that wave solutions may be superimposed. Since sediment transport is, in general, a non-linear process, however, it is invalid to superimpose solutions including sediment movement. It follows that the wave-field must be determined completely before any sediment transport is calculated. If waves of different directions are present this produces a complex moving pattern of shear stress distribution whose effect on sediment transport is unknown.

In applications the influence of bed slope may also have to be considered. Laboratory experiments have normally been limited to flat beds or beds sloping gently in the direction of the waves but in real situations the bed slope may influence the direction and amount of sediment transport.

3 DETERMINATION OF WAVES AND CURRENTS

Numerical models have been developed at HR for the calculation of waves in coastal situations and the calculation of tidal movements. The only difficulty is in combining two such models and ensuring the appropriate interactions between the two.

Numerical modelling of short-period waves (2s to 15s) is normally performed using ray methods. These can allow for the effects of depth refraction, shoaling, reflections and diffraction (Southgate, 1981 and 1984). The area of interest is represented using a rectangular grid, typically each grid cell being of the order of 200m by 100m. Southgate (1984) described a method of using a ray method combined with a spatial averaging technique whereby account can be taken of trends associated with a bundle of rays rather than the behaviour of an individual ray. The results obtained are in the form of an array of spot wave heights and phases covering the whole studied area. The method has the advantage that the resultant of two or more interacting wave trains may be calculated. Such ray methods essentially assume steady state conditions for the wave climate.

A range of tidal flow models are available at HR to simulate the diverse conditions that may occur in tidal problems. A suitable model for determining the tidal currents in coastal situations would be 2-D in plan and depth averaged. The finite-difference scheme used may be implicit or explicit. In general explicit schemes involve fewer calculations per timestep but stability constraints limit the length of timestep that can be used. Implicit schemes can utilise larger timesteps but each timestep involves more computational effort. Explicit schemes can readily be adapted to take advantage of the DAP available at HR. A typical grid size would be in the range of 14m to 100m with timesteps varying from 54 sec for the largest grid size using an implicit finite-difference scheme down to 1 sec for the smallest grid sizes with an explicit scheme.

In combining the wave and current models there would be advantages if they could use the same numerical grid or if one was a subset of the other.

4 INTERACTIONS OF WAVES AND CURRENTS

Under certain circumstances the mutual interaction of waves and currents may be significant (Noda et al 1974). In these cases it is not sufficient to calculate the currents and waves independently. Even in the absence of tidal currents local variations in wave height result in wave-induced Reynolds' stresses which drive currents (Longuet-Higgins, 1970 and Noda 1974). Thus in these situations the wave field must be determined first, the Reynolds' stresses calculated and their effect included when determining the current field.

Variations in the spatial velocity distribution cause wave refraction. Thus where this is significant the current field must be known first and then the wave field may be calculated.

The magnitude of these current-wave interactions will vary with the situation and conditions and may frequently be insignificant. Initially some effort should be expended on characterising those conditions where such interactions may be ignored and those where it is important that they be included.

To include all these interactions it is necessary that the part of the model that calculates the wave refraction should be capable of including the effect of currents and that the flow part should be capable of including the Reynolds' stresses induced by variations in wave height. If the waves and currents elements are separate in the model then some iterative technique is required in which the wave and current fields are solved for successively. A suitable structure for such an iteration is given in Figure 3.

If the flow element is solved using a time-stepping procedure consideration should be given to the frequency with which the wave-field needs to be calculated. In many tidal situations, provided the waves are only weakly dependent upon the currents, the timescale for variations in the current field will be very much smaller than that for variations in the wave field. Consideration may then be given to whether different timesteps could be used for the calculation of the wave and current fields so that the wave field is up-dated less frequently than the currents, see Figure 3.

5 EFFECT OF WAVES AND CURRENTS ON BED FEATURES

The presence of waves affects the bed features that are developed on a mobile bed and hence influences the hydraulic roughness; a knowledge of which is important in the determination of the flow field. Swart (1976) relates the hydraulic roughness to the bed form steepness though as he says this 'does not solve the problem of the determination of the bed roughness, it just shifts it'. Swart presents equations to predict bed form steepness and hence hydraulic roughness under wave dominated conditions but it is unclear how reliable such equations are.

6 CONCLUSIONS

At present the various theories considered for sediment transport under waves and currents based on extensions of steady uni-directional sediment transport theories provide satisfactory predictions only when the effect of currents predominate. It seems likely that a simple, reliable method of predicting sediment transport under waves and currents may be developed utilising theories for sediment transport under waves alone and under steady uni-directional flow.

A model to predict the movement of sediment under waves and currents in coastal situations must include both an appropriate wave model from which may be calculated Reynolds' stresses generated by variations in wave height and an appropriate flow model.

It is suggested that more work is directed at:

(a) the development of a reliable theory for sediment transport under waves alone so that improvements can be made to predictions in the wave dominated area

(b) the direction of sediment transport when waves and currents are not in the same direction

(c) the prediction of the hydraulic roughness of the bed under waves and currents.

APPENDIX

SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

This was a summary of the work described in Sediment transport under waves and currents, HR Report SR 22, 1985

(a) Shear stress under waves and currents

Bijker (1967) assumed that the velocities under waves and currents could be assumed to be the sum of a uniform flow logarithmic velocity profile,

$$v(y) = \frac{v_*}{\kappa} \ln \frac{y}{yo}, \tag{1}$$

and the wave velocity given by the first-order wave theory

$$u_o = \frac{\omega H}{2 \sinh kd} \tag{2}$$

where H is the wave height, d is the depth and k is the wave number. Bijker then calcualted the mean component of the resultant bed shear in the direction of the current to give

$$\frac{\tau_{wc}}{\tau_c} = \frac{2}{T} \int_{T/4}^{T/4} \left[(1+\xi \frac{u_o}{V} \sin\omega t \sin \phi) \sqrt{(1+\xi^2 \frac{u_o^2}{V^2} \sin^2 \omega t + 2\xi \frac{u_o}{V} \sin\omega t \sin\phi)}\right] dt \tag{3}$$

where

$$\xi = \frac{p\kappa C}{g^{\frac{1}{2}}}, \qquad T \text{ is the wave period,} \tag{4}$$

p = 0.45, C is the Chezy roughness coefficient given by $C = 18 \log_{10} \frac{12d}{r}$, r is the bed roughness and ϕ is the angle between the wave crests and the normal to the current. Bijker evaluated the elliptic integral in (3) for a range of values of ϕ, u_o and V and fitted an approximation to (3) of the form

$$\frac{\tau_{wc}}{\tau_c} = a + b\left(\xi \frac{u_o}{V}\right)^c \tag{5}$$

for various values of ϕ, see Bijker (1967) for details. These approximations are only valid in particular parameter ranges and should only be used in those ranges. In the case where the component of the shear in the direction of the current is always positive then (3) reduces to

$$\frac{\tau_{wc}}{\tau_c} = 1 + \tfrac{1}{2}\left(\xi \frac{u_o}{V}\right)^2 \tag{6}$$

but this should only be used if $u_o < V$.

The maximum bed shear stress is given by

$$\left(\frac{\tau_{wc}}{\tau_c}\right)_{max} = 1 + \left(\xi \frac{u_o}{V}\right)^2 + 2\ \xi \frac{u_o}{V} \sin\ \phi \tag{7}$$

Swart (1976) suggested replacing equation (4) by the equation

$$\xi = C\left(\frac{f_w}{2g}\right)^{\frac{1}{2}} \tag{8}$$

where f_w is the Jonsson wave friction factor which can be approximated by

$$f_w = \begin{cases} \exp\left(-5.98 + 5.21\left(\frac{a_o}{r}\right)^{-0.19}\right) & \frac{a_o}{r} > 1.57 \\ 0.3 & \frac{a_o}{r} < 1.57 \end{cases} \tag{9}$$

where a_o is the orbital amphitude at the bed, that is

$$a_o = \frac{Tu_o}{2\pi} \tag{10}$$

Bijker's model, using a logarithmic velocity profile for the current and a first order wave theory, is relatively crude and must be seen as a major weakness in any theory of sediment transport that uses it.

There are more sophisticated models but these frequently involve solving a differential equation through the depth to obtain the flow structure and hence obtaining the shear stress (Bakker, 1974 and Fredsoe, 1983). Such models would be computationally expensive to implement. The possibility arises, however, of developing appropriate 'look-up' tables which might lead to savings in computation.

(b) Sediment transport

The following theories have been compared with field and flume observations:

1. Frijlink-Bijker (Bijker, 1967)
2. Ackers and White-Swart (Swart, 1976)
3. Ackers and White-Willis (Willis, 1978)
4. Ackers and White-van de Graaff and van Overeem (1979)
5. Engelund and Hansen-Swart (Swart, 1976)

The above sediment transport equations have been derived from equations for uni-directional sediment transport. As an illustration of the methods involved we describe the Ackers and White-Swart method.

Ackers and White-Swart

The Ackers and White equations for sediment transport utilise four parameters, n,A,m and C which depend upon the dimensionless grain size Dgr of the sediment. Swart keeps these unchanged when applying the equations to sediment transport under waves and currents. The mobility in the case of currents above is defined by

$$F_{gr} = \frac{v_*}{\sqrt{(gD(s-1))}} \left[\frac{V}{\sqrt{(32)}\log(10d/D)}\right]^{1-n} \qquad (11)$$

and Swart defines the corresponding variable for waves and currents, F_{gr}^{wc}

$$F_{gr}^{wc} = F_{gr} \times \left(\frac{v_{*wc}}{v_{*c}}\right)^{n}, \qquad (12)$$

where $v_{*wc} = v_{*c} \left(1+\left(\xi \frac{u_o}{V}\right)^2\right)^{\frac{1}{2}}$

The equation for sediment concentration in the case of currents alone is

$$X = C_{gr} \frac{sD}{d} \left(\frac{V}{v_*}\right)^n \tag{13}$$

In the case of currents and waves Swart replaces this by

$$X = C_{gr} \frac{sD}{d} \left(\frac{V}{v_{*wc}}\right)^n \tag{14}$$

Thus, denoting v_{*wc}/v_{*c} by β the equation for the sediment concentration becomes

$$X = C \left(\frac{F_{gr}}{A} \beta^n - 1\right)^m \frac{sD}{d} \left(\frac{V}{v_{*c}}\right)^n \frac{1}{\beta^n} \tag{15}$$

The comparisons with observations indicated that, in general, the methods performed satisfactorily when the effects of the currents dominated those of the waves but that they uniformly over-predicted the transport rate when waves dominated, Fig A1. This was in part due to the expression adopted by the various methods for the shear stress under waves and currents. When equation (6) was replaced by equation (3) then improved predictions resulted though transport rates were still overpredicted, Fig A2. An ad-hoc correction to the shear stress was employed which improved the predictions.

In general the methods based on Ackers and White sediment transport theory performed better than the others. Within the group there were variations in the behaviour of the different adaptions but, taking into account the inadequacies of the data upon which these methods were tested, it would be difficult to argue that these variations were significant.

It is unlikely that predictions in the wave donimated region can be significantly improved until more reliable equations are available for sediment transport under waves alone.

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NOTATION

A		Value of sediment mobility at initiation of motion (Ackers and White)
a_o	m	Amplitude of orbital excursion at the bed
C		Coefficient in sediment transport function (Ackers and White)
C_h	$m^{\frac{1}{2}}s^{-1}$	Chezy coefficient
D	m	Sediment diameter
D_{gr}		Dimensionless grain 'size'
d	m	depth
F_{gr}		Sediment mobility (Ackers and White)
F_{gr}^{wc}		Sediment mobility under waves and currents (Ackers and White)
f_w		Jonsson wave friction factor
G_{gr}		Dimensionless sediment transport rate (Ackers and White)
g	m/s^{-2}	Acceleration due to gravity
H	m	Wave height
k	m^{-1}	Wave number
m		Exponent in sediment transport function (Ackers and White)
n		Transition exponent in sediment transport function (Ackers and White)

p		Constant
r	m	Bed roughness
s		Specific gravity of sediment
T	s	Wave period
t	s	Time
u_o	m/s	Wave orbital velocity at bed
V	m/s	Current velocity
v_*	m/s	Shear velocity
v_{*c}	m/s	Shear velocity due to current
v_{*wc}	m/s	Shear velocity due to waves and currents
X		Sediment concentration
y	m	Distance above bed level
y_o	m	Depth
β		V_{*wc}/V_{*c}
ϕ		angle between the wave crests and the normal to the current
κ		Van Karman's constant
ξ		parameter, depending on the bed roughness and water depth
τ_c	kg/m/s ^{2}	Shear stress under currents
τ_{wc}	kg/m/s ^{2}	Shear stress under waves and currents
ω	s $^{-1}$	Wave frequency

FIGURES

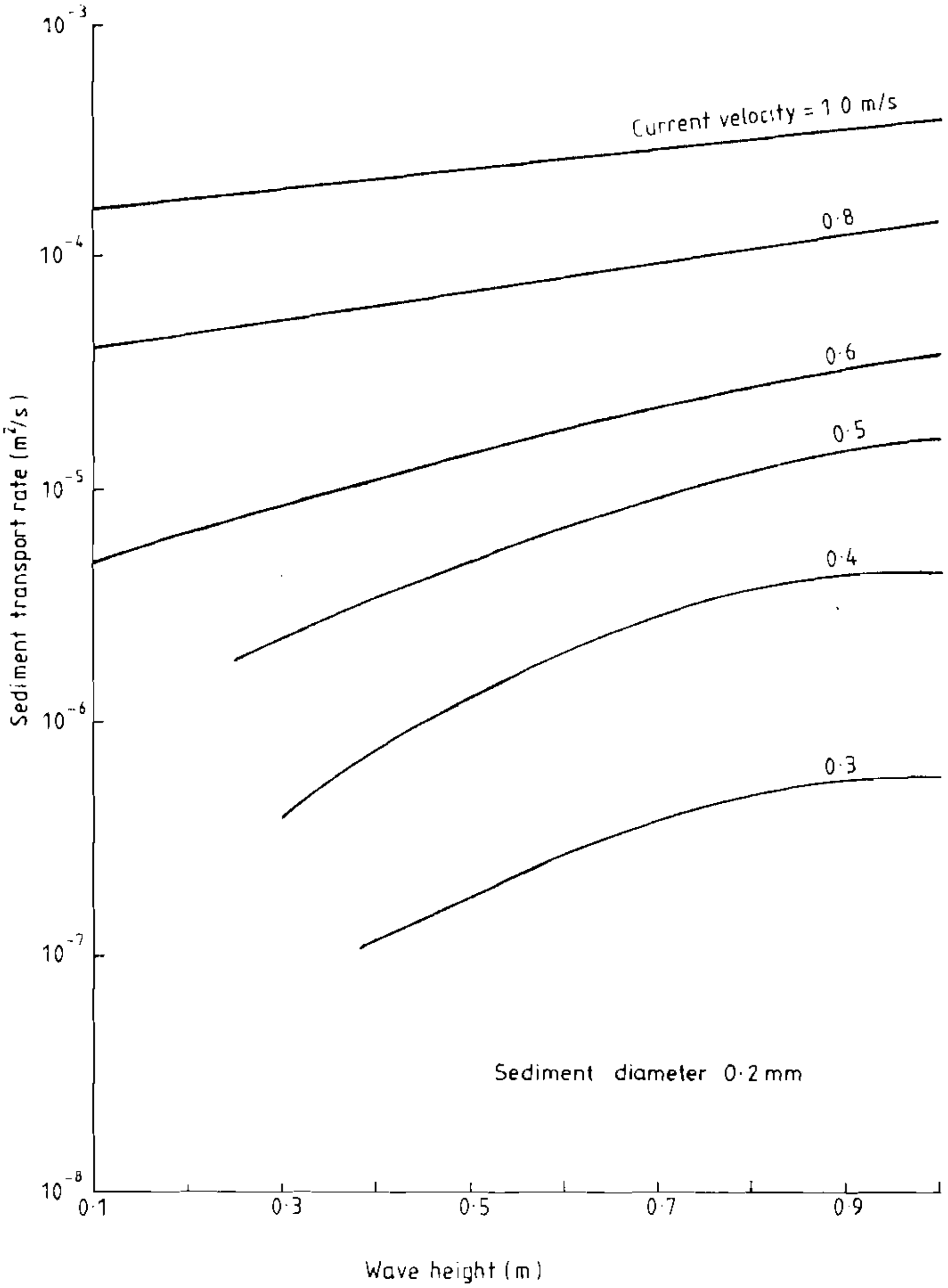


Fig 1 Sediment transport rate against wave height

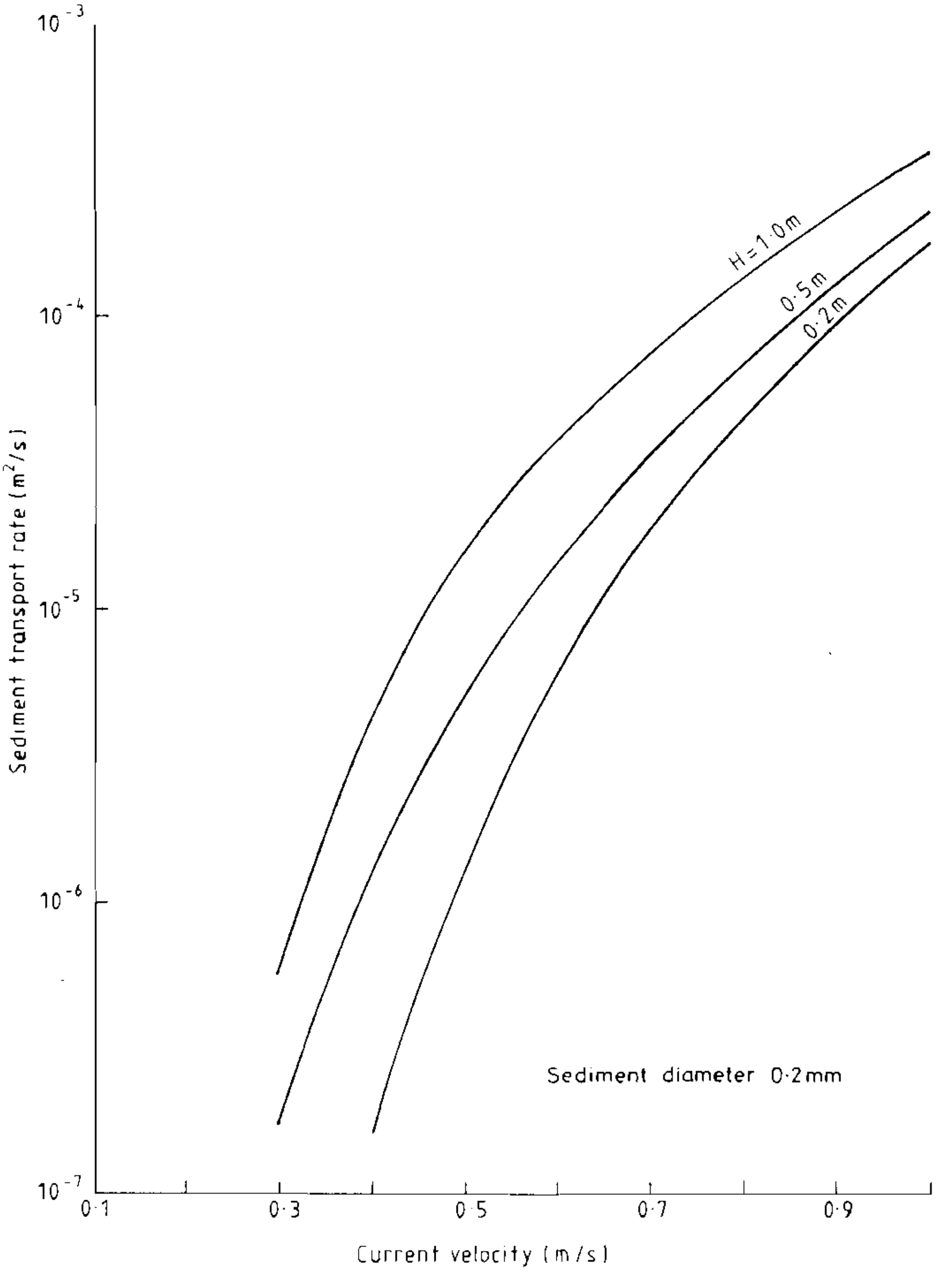


Fig 2 Sediment transport rate against current velocity

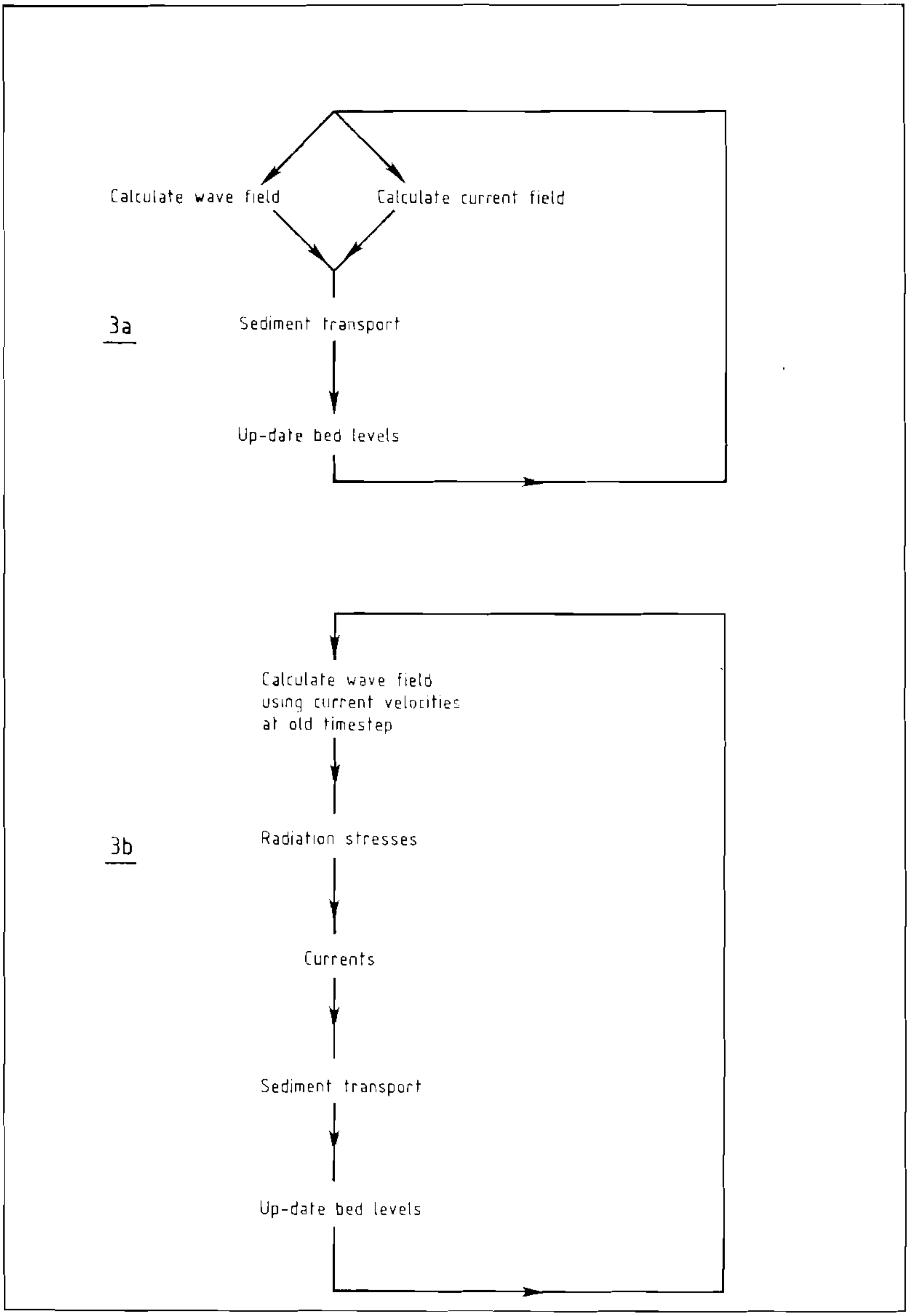


Fig 3a & 3b Flow diagrams for numerical model

3c

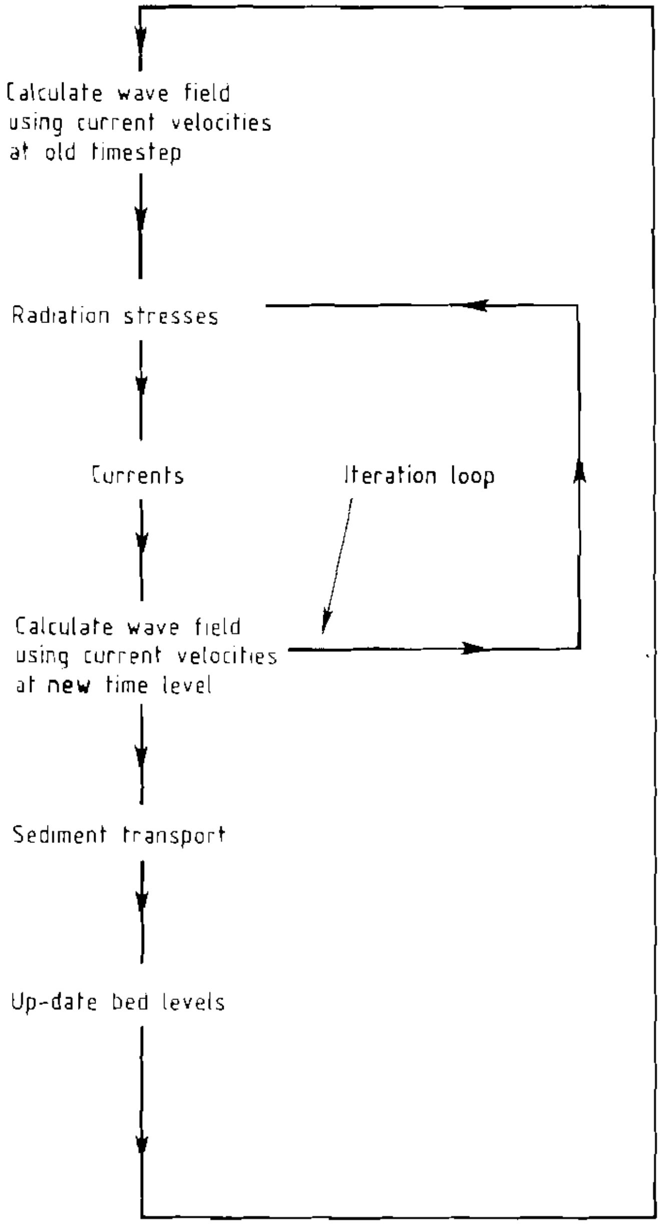
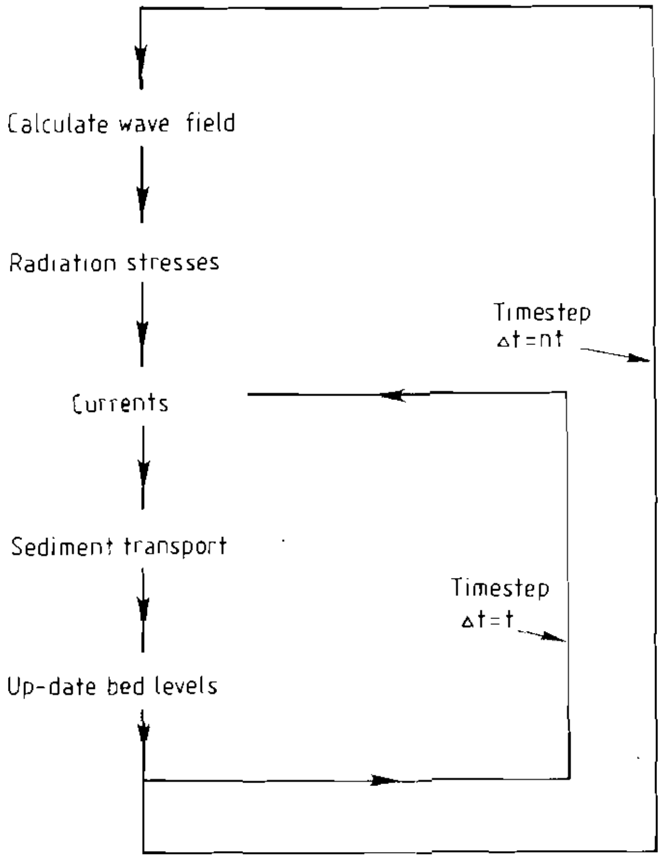


Fig 3c Flow diagram for numerical model

3d



3a. Shows flow diagram if waves and currents are assumed independent.

3b To enable wave and current interactions to be included the wave field is calculated using information on current velocities calculated at the previous timestep. This can be regarded as the zeroeth order iteration in a full iteration procedure for both waves and currents illustrated in 3c. If the timescale for the variations in the wave field is very much larger than the timestep required for the flow calculations then advantage can be taken of this by calculating the wave field at less frequent intervals than those used for the flow, see 3d.

Fig 3d Flow diagram for numerical model

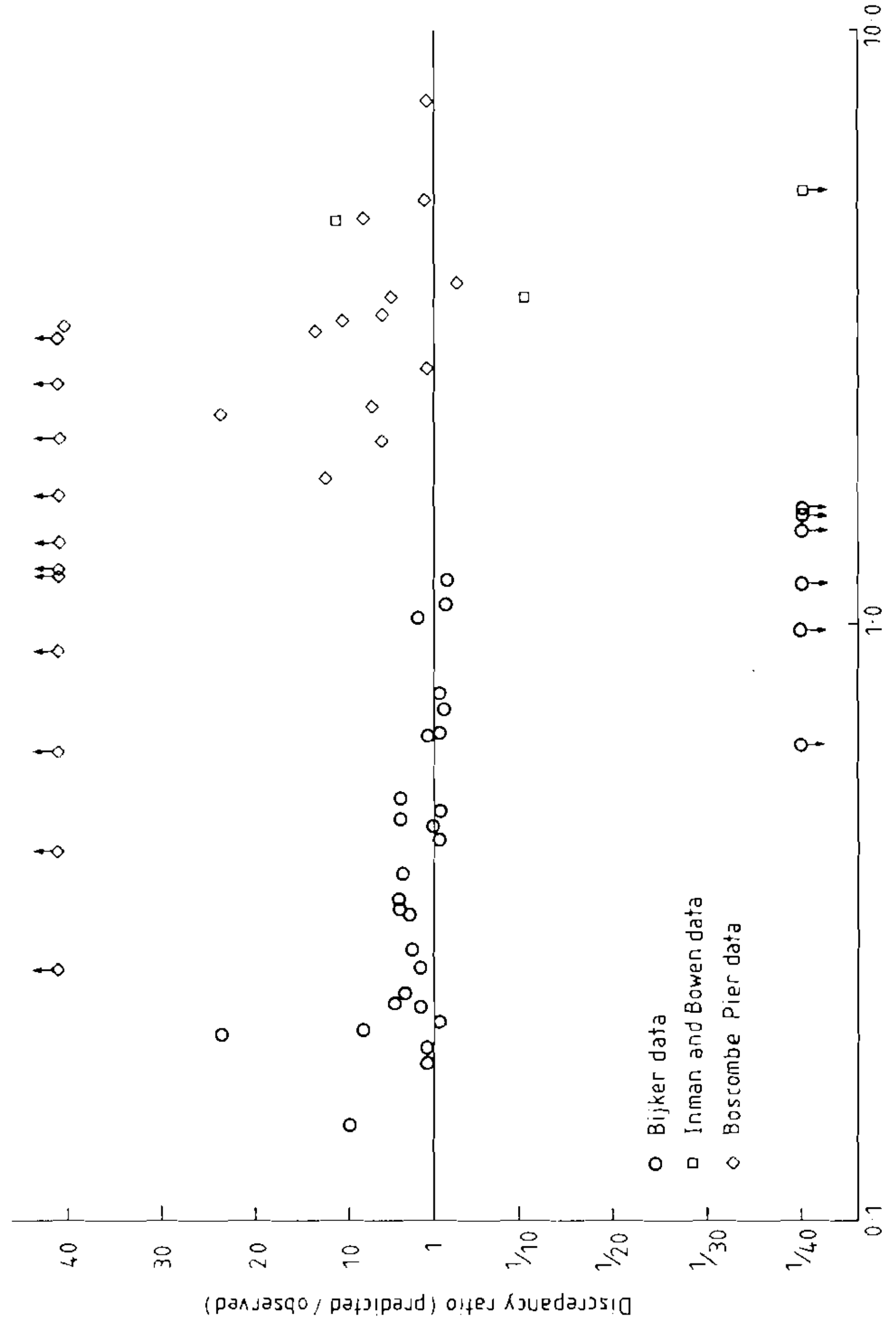


Fig A1 Discrepancy ratio against u_0/V, Ackers and White-Swart

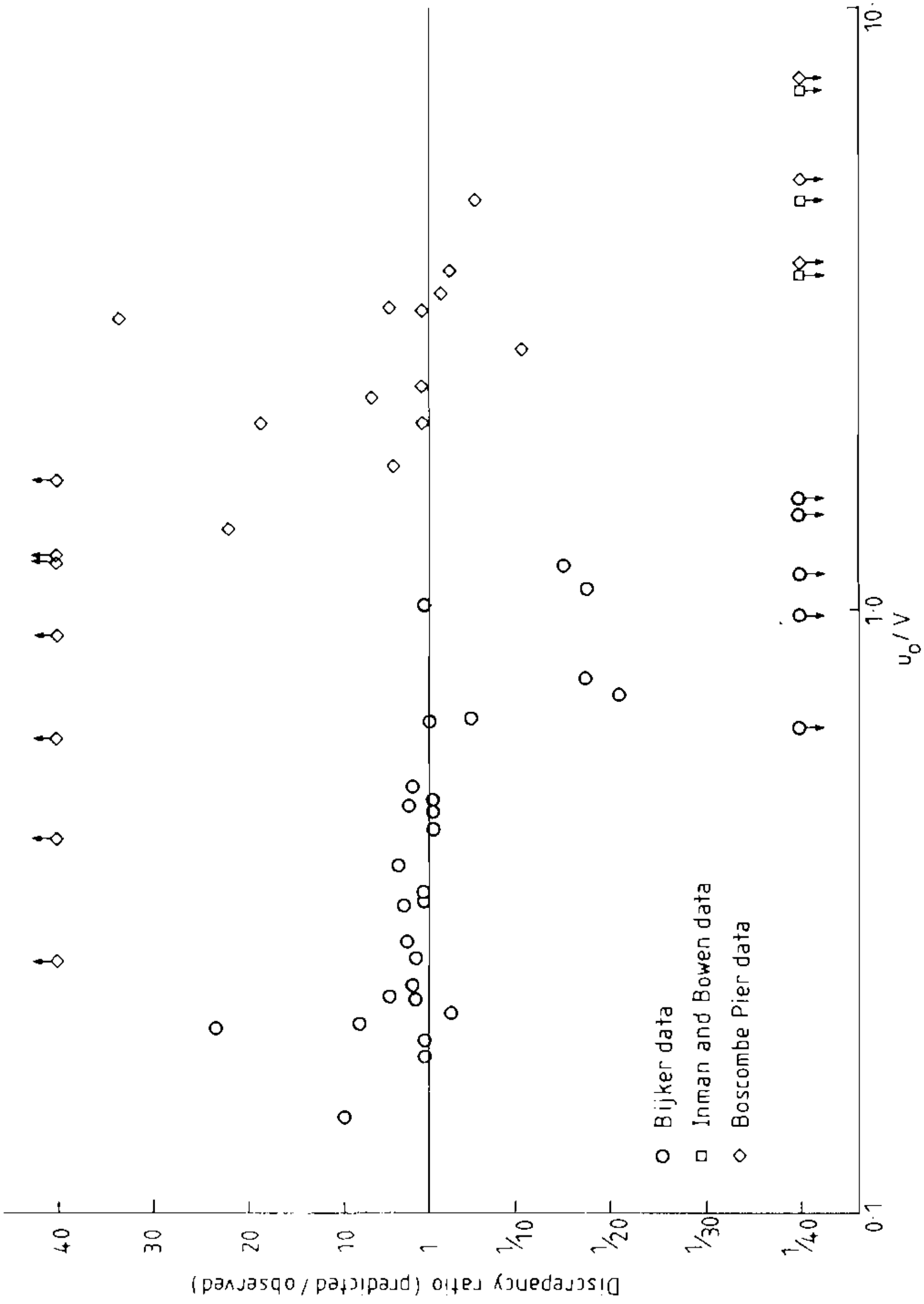


Fig A2 Discrepancy ratio against u_0/V, Ackers and White-Swart with Bijker shear stress integration