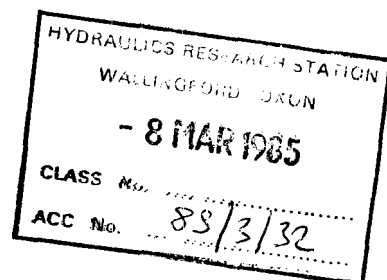




Hydraulics Research
Wallingford

NUMERICAL SIMULATION OF THE
EROSION OF MARINE MUD

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SUMMARY

For many years HR have been applying their suite of mud transport computer models called MUDFLOW to study the impact of engineering schemes on siltation and erosion of mud in estuaries. These models are part of the HR TIDEWAY computer system for modelling tidal flows and the transport of sediment, salt, heat and pollution. It has not been possible, however, to collect field data of sufficient quality to thoroughly check that all aspects of the models are working properly. The difficulty is that erosion and deposition are normally too slow and the supply of sediment too variable to enable any bed level changes to be quantified in the field.

This report deals with testing the representation of erosion processes in the existing MUDFLOW-2D model using experimental results obtained from the HR erosion flume. In the past this flume was used primarily to determine erosion thresholds and erosion rates of mud beds for use in model studies and other engineering assessments. However the experimental results can also be analysed to hindcast the original structure of the eroded bed and this provides an ideal opportunity for testing the model under well defined conditions.

Brief descriptions of the flume and experimental procedures are included together with details about the formulation of the model and the underlying physics. A new graphical method is presented in Fig 2 for determining a single representative value of the erosion constant in Eq 3 from concentrations and bed stresses measured in the erosion flume. It is shown how erosion in the flume can be accurately simulated by the model from a knowledge of the shear strength structure of the bed. The simulation of erosion using density-mass and erosion strength-density relations derived from the experimental results were less satisfactory. This is attributed to the quality of the relations rather than to a shortcoming of the model.

The study concludes that the main sources of inaccuracy in simulating mud transport in real estuaries would be in the determination of the bed density profile and the relationship between the critical erosion stress and the density of the bed. Further basic research is needed to gain a better understanding of consolidation processes before a fully dynamic model of the bed can be produced. Nevertheless the existing models can still be applied to a wide range of engineering problems involving deposition and erosion in ports, harbours, rivers and estuaries.

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1 INTRODUCTION

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Before testing the model in this way, we first describe the underlying physics and formulation of the model, and give a few details about the experimental procedures.

2 FORMULATION OF MODEL

The sediments under consideration have high proportions of clay and silt particles of size less than 0.06mm which give marked cohesive properties. Deposits of this material are usually referred to as mud, and this term is often also used loosely to describe similar mixes of sediments in suspension, even though the various size fractions present in the suspension do not move as an entity.

The main factors controlling the transport and erosion or accumulation of bed deposits are the physical processes in the fluid

- transport by currents
- settlement under gravity
- turbulent diffusion in all directions (but only the vertical component is of significance under most circumstances)

and the physical processes at the water-bed interface

- deposition near slack water
- consolidation of deposits under the weight of subsequent deposits and as a function of time
- resuspension of deposits (or erosion of original bed) when currents increase.

Fortunately, it is not always necessary to model the complete structure of the flow. Advection effects usually predominate, and, provided the mud is fairly well mixed throughout the water depth, it is sufficient to consider depth-averaged equations. This approximation is particularly appropriate for moderate suspended concentration of less than a few thousand mg/l of fine cohesive particles with low settling velocities. Under these circumstances, the depth-averaged concentration, $c(x,y,t)$, satisfies the conservation of mass equation.

$$\frac{\partial}{\partial t}(dc) + \frac{\partial}{\partial x}(duc) + \frac{\partial}{\partial y}(dvc) = \frac{\partial}{\partial s}(dD_s \frac{\partial c}{\partial s}) + \frac{\partial}{\partial n}(dD_n \frac{\partial c}{\partial n}) + \frac{dm}{dt} \quad (1)$$

where (u, v) = depth-averaged components of velocity (m/s)

D_s = longitudinal (shear flow) dispersion coefficient (m^2/s)

D_n = lateral (turbulent) diffusivity (m^2/s)

(x, y) = cartesian co-ordinates in horizontal plane (m)

(s, n) = natural co-ordinates (parallel with and normal to mean flow) (m)

t = time (sec)

d = water depth (m)

$\frac{dm}{dt}$ = erosion from or deposition on the bed ($kg/m^2/s$)

Deposition

Net deposition from a dilute suspension is usually assumed to occur when the bed stress falls below a critical value τ_d and the deposition rate for stresses lower than this is usually represented as

$$\frac{dm}{dt} = -cw_s (\tau_d - \tau) / \tau_d \quad (2)$$

where $w_s(c)$ = settling velocity

τ_d = critical deposition stress

This can be thought of as a deposition proportional to τ_d and a vertical diffusive flux proportional to τ . When $\tau = \tau_d$, the rate of settling is balanced by the vertical diffusive flux due to turbulence and there is no deposition. As the current slows down, its associated bed stress and turbulent intensity reduces, thereby weakening the vertical diffusive flux, resulting in a net deposition. This is a simplification of what, in nature, is a very complicated process. For example, there will not be an abrupt cut off τ_d for deposition but rather a gradual change over from a non-depositional to a fully depositional state. In between, there is likely to be some interchange of suspended particles with bed particles. However, for the purpose of making engineering judgements it is sufficient to represent the deposition in the above manner, and accordingly to simulate deposition, values are required for $w_s(c)$ and τ_d . However, since τ exceeded τ_d in all of the erosion flume runs the accuracy of expression 2 was not tested.

Erosion

A similar approach is used to represent erosion. Intuitively, one would expect erosion to start when the stress exerted by the flow exceeded the shear strength of the exposed bed and the erosion rate to depend on the excess shear. If the erosive power of the stream is low we would not expect much erosion to take place. As in the deposition situation, there will be times when a burst of turbulence slightly higher than average hits a slightly weaker part of the bed causing untypical erosion, but for practical purposes we can ignore these and assume that we have a similar cut off for erosion as assumed for deposition. This critical erosion stress is normally higher than the critical stress for deposition because deposits undergo some degree of consolidation. The most common representation of erosion is

$$\frac{dm}{dt} = Am_e (\tau - \tau_e) \quad (3)$$

where m_e = erosion constant (kg/N/s)

τ_e = erosion strength of the bed (N/m^2)

A = area of erosion (m^2)

Given the strength profile of the bed $\tau_e(z)$ and an erosion rate then we have a procedure for calculating erosion.

This assumes that erosion is gradual which is not necessarily the case for certain types of newly formed slack water deposits but although there is no physical reason for assuming erosion rate to be directly proportional to the excess shear, Puls (Ref 1) shows that this is a better variable for describing mud erosion than others. In any event it is not too important in tidal conditions to know the erosion rate precisely because the erosion process is self correcting in the sense that if the erosion constant is too high then too much erosion occurs in the early stages, but this exposes stronger bed material and erosion slows down accordingly. The opposite happens if m_e is under-valued. The ultimate result in any case would be erosion down to the bed level, where the strength of the exposed material corresponds to the maximum bed stress of the tidal cycle or of the spring-neap cycle, if longer periods are being considered. This does not mean that there is no need to do further work on these processes - we would always prefer to use the correct formulation if it were known - but rather that we can still get useful information even if we do not know all of the erosion properties of the bed material. This point will be considered again in the mud erosion flume simulation.

Consolidation

The erosion model just presented has a severe limitation because it assumes that the bed strength profile is known. There are two problems in extending this model to real sites. Firstly, in a pure erosional situation there is the difficult task of prescribing the initial bed strength profile in nature. Fortunately, one can usually identify the most likely erosion areas associated with a particular engineering project and measure the density profile using cores or radio-active density probes. The more difficult problem in tidal situations is when both erosion and deposition occur at different stages of the tide. The strength profile of the new deposits depends on the weight of overlying sediment and on the time which elapses from original deposition until erosion. In addition, the strength of the bed can be weakened by the

flow of pore water escaping from deeper deposits and increased by the action of flowing water. It is thought that this latter occurs as a result of turbulent fluctuations which re-arrange the bed matrix. On average the weaker bonds break up and form new bonds. Some of these will be weaker and these would presumably be broken again. But some would slip into stronger configurations and be maintained and the final result from a period of working would be a stronger bed than at the outset. These four consolidation processes will modify the strength of the bed. In some cases this will not be enough to resist the next stage of the tide and all material will be resuspended. In other cases some deposits will remain, particularly during neap tides, and these residues could then lie undisturbed for several tides after which they may or may not have gained enough strength to resist the following spring tide currents.

From this it is clear that a fully dynamic model would require a theoretical description of the bed consolidation so that the density and shear strength profiles could be predicted as functions of time.

3 HR EROSION FLUME

The flume used for the erosion tests consists of an entry section (8.5m long), a working section (7.3m long) and an exit section (1.6m long). It is made of fibre-glass; the cross section is rectangular with a width of 30cm and a depth of between 14cm and 20cm. The flume is covered by a metal roof. The flume is part of a closed loop: the water flows from the exit section back to the entry section through a 20cm diameter return pipe. Precautions are taken that there is no air in the flow system. The flow is driven by an axial pump, which is located just downstream of the exit section.

The trough for the mud bed has slopes with a gradient of 1:20 at both ends in order to minimise end effects. The mud beds have a maximum thickness of 60mm; the minimum bed thickness during the tests presented in this paper was 38mm. For preparing the bed for an erosion test, the roof of the working section is removed and replaced by an open-bottomed tank (2m high, 0.3m wide, 7.3m long). The flume's working section is thus converted into a 2.2m deep settling tank.

The flow discharge is measured by orifice plates in the return pipe (recently replaced by an acoustic flow meter). The shear stress being applied to the walls of the flume's working section by the flowing water is determined by measuring the head loss along the length of the working section. The head loss is evaluated from the static pressures that are measured at seven positions in one side wall of the working section. At each position there are three pressure tapings mounted vertically in line. The pressure tapings are connected to a central bank of stilling wells via a manifold and a silt trap. The water levels in the stilling wells are measured by micrometer screw point gauges.

The suspension concentration in the flume is determined from 20ml samples that are withdrawn from the flow. The sampler is situated just downstream of the axial pump where the flow is well-mixed. The concentration is immediately determined by using a photo-absorptiometer. While running a test, it is thus always known whether the concentration is changing or whether it remains constant.

Bed preparation in the erosion flume

The mud bed in the flume's working section is created under quiescent conditions. For creating the bed, the roof of the working section is removed and replaced by the settling tank. The tank is filled with water of the same salinity that will be used for the following erosion test. A slurry of mud of the same salinity (slurry prepared one day before) is then pumped into the settling tank. The slurry is injected through 6 valves that are evenly spaced along the top of the tank. The mean suspension concentration in the settling tank is in the order of 4000mg/l.

After a short time, the mud begins to deposit on the floor of the working section. When most of the mud has reached the bed (after about 3 hours), the water in the settling tank is drained off. Finally, the tank is removed and the flume's roof is bolted on to the working section.

The bed is left to consolidate until the erosion test begins. Through the glass windows in the side walls of the working section it can be

seen that the surface of the mud bed is very smooth, even above the slopes at the two ends of the working section.

The thickness, H , of the mud bed is measured when the erosion test starts.

Erosion test procedure

At first the axial flow pump is switched on at its lowest speed for about an hour, in order to pick up spurious solids in the flume circuit. At this minimum discharge the mud bed in the working section is not eroded. The suspension concentration measured at this time is the so-called background concentration (about 40 mg/l).

In the sequel, the flume discharge is increased in discrete steps by increasing the speed of the pump. The time between the discharge-increases varies between about 20 minutes and 300 minutes. Readings (at least one every ten minutes) are taken of the pressures, the flow discharge and the suspension concentration. Because the flume is part of a recirculating system, the rate of erosion of the mud bed is proportional to the rate of increase in the suspension concentration of the flowing water. When erosion ceases, the concentration remains constant (assuming that there is no deposition of mud, which is true for a bed shear stress that is greater than about 0.05N/m^2 , Ref 2).

The flow discharge is increased to a new value soon after a constant concentration has been observed. The flow discharge is also increased to a new value, if a constant concentration is not attained during an appropriate time. Output from the experiments consists of tables and graphs of bed stress and concentrations plotted against time. Figure 1. After every increase in discharge the concentration increases at first rapidly (strong erosion), then more smoothly (modest erosion), and finally the concentration might remain constant (no erosion), until the next increase in discharge causes strong erosion again. The situation with no erosion is called an equilibrium. The maximum acceptable erosion depth, z (measured downwards from the top of the initial bed) is about $2/3$ of the initial bed thickness.

At the end of a run the flume is run at maximum discharge so that the rest of the bed is rapidly eroded and all the mud is put into

suspension. From the measured final suspension concentration, the total amount of mud in the initial bed is determined.

Evaluation of the erosion constant, m_e

In the past the erosion constant was evaluated from erosion rates determined at different times from the start of each test. This appears to show that the erosion constant is a function of duration of erosion (Ref 3). A more useful analysis can be made by assuming that the increase in the shear strength of the bed during any discharge run is proportional to the eroded mass. The constant of proportionality for a discharge (Q) run is given by

$$\alpha = \frac{\tau_b - \tau_o}{c_e - c_o} \quad (4)$$

where τ_b is applied bed stress for the discharge run
 τ_o is unit strength of bed at start = equilibrium from previous Q run
 c_e is equilibrium concentration at end of Q run
 c_o is initial concentration = equilibrium concentration at end of previous Q run

This does not assume that there is a linear relation between strength of bed and overlying weight for the complete bed. This overall structure is fixed by the equilibrium conditions at the end of each discharge run. We have merely assumed a linear variation from one equilibrium state to the next and α can vary for each run.

If we further assume that the erosion rate for exposed mud surface area is given by equation (3) as recommended in Ref 1, we obtain, using (4) and replacing m by cV

$$\frac{dc}{dt} = Am_e \alpha (c_e - c_o) / V \quad (5)$$

where V is the volume of water in the flume ($2.30m^3$) and $A = 1.95 m^2$. This can be integrated as

$$(c_e - c) = (c_e - c_o) \exp(- Am_e \alpha t / V) \quad (6)$$

This solution exhibits the expected behaviour of concentrations tending to equilibrium values for large times.

Based on the analytic form of this theoretical solution, the erosion flume results for tests on Mersey mud (Fig 1) have been normalised and plotted in Fig 2 using linear (αt) and logarithmic $((c_e - c_o)/(c_e - c))$ axes to give a representative erosion constant for the tests.

The best straight line eye fit to the normalised data gives the following exponential relationship for all test runs

$$c_e - c = (c_e - c_o)\exp(-0.0018\ln(10)\alpha t) \quad (7)$$

and the corresponding erosion constant is

$$m_e = 0.0018\ln(10)V/A \sim 0.005 \text{ kg/N/s} \quad (8)$$

which is in good agreement with the erosion constant for Mersey mud found in Ref 1 based on instantaneous rates of erosion. The unusual behaviour in discharge run 4 was probably caused by a patch of stronger than average bed. But even in this run the erosion rate given by the slope of the line eventually settled to a similar value to the other cases.

An advantage of the method used in the current work is that the density structure of the bed in the flume is not required to evaluate m_e .

4 NUMERICAL SIMULATION

Simulation with prescribed bed strength profile

Using the measured equilibrium shear stress (τ_e) and concentration (c_e) values from the erosion flume experiment we can construct the shear strength profile in terms of the eroded mass of mud. In the case of Mersey mud (Fig 3), a linear representation

$$\tau_e = 0.355 + 0.37c \quad (9)$$

was found appropriate where c is the excess concentration (kg/m^3) above the background concentration $c_b = 0.039 \text{ kg/m}^3$ at the start of the

test. Since the excess concentration is proportional to the eroded mud equation (9) can be recast as

$$\tau_e = 0.355 + 0.31 w_o \quad (10)$$

in terms of the overlying mass w_o (kg/m^2) making use of the flume volume to erosion area ratio $V/A = 1.185$

A bed was set up in the computer model with erosion strength given by equation (10). The model updated the strength of the exposed surface according to this equation as erosion proceeded.

Using this bed structure we ran the model for a series of discharges (stresses) corresponding as closely as possible to the effective bed stresses of the experiment. Settling velocities were obtained from the empirical relation given in Fig 4, however, this did not affect the results because only erosive cases were simulated. The average erosion constant (0.005) found in Section 4 was used. A comparison of the computed and measured concentrations in the flume is shown in Fig 5. In each discharge run the model concentration increased rapidly at first and tended towards the correct equilibrium value. The model did not quite reach equilibrium in individual runs, (a consequence of the exponential nature of the theoretical solution and a phenomenon observed by Krone (Ref 4)) and the shapes of the concentration-time curves did not exactly match the observed curves. Nevertheless, the final concentration in the model agreed very closely with that at the corresponding stage in the experiment. Therefore the model must have eroded the bed down to the same depth as in the experiment. (We did not compare depth of erosion explicitly because this is difficult to measure and was not recorded in the experiments). This emphasises the point that weak deposits left over at one stage are quickly eroded away at the next stage, leading to the satisfactory result just described.

Simulation using the density structure measured in the settling column

The first simulation demonstrates that the model can reproduce the main features of erosion when the erosion strength structure of the bed is prescribed. For practical applications it is more likely that data on the properties of the bed would be supplied in the form of its density structure. Under these circumstances, the only extra information

required is a relationship between bed shear strength and density. In a fully dynamic simulation the density would be estimated from a consolidation relation $\rho_b(w_o, t)$ as described earlier and the bed shear may also need to take account of increasing bed strength due to history of bed stresses working on the bed. The simulation of the dynamic consolidation is beyond the scope of the present work. In the following, we describe a simulation based on a known density structure together with a stress-density relation.

As part of the routine tests on mud samples, the density profiles of deposits are studied in a settling and consolidation column, using a transmission gamma ray probe. The measurements are presented as a non-dimensional density profile (Fig 6) and integrated to give the cumulative mass profile, $w_o(z)$ (Fig 7). In these figures, H is the total bed thickness (m), h is the height above the bottom of the bed (m), ρ and $\bar{\rho}$ are the local and mean bed densities (dry weight per unit wet volume) and $z = H - h$ is distance below the surface of the bed. The variables are non-dimensionalised because, during processing, the tube relations are referred to the flume bed for different values of H and $\bar{\rho}$. However, for the model simulation, it is convenient to combine the two relations, (Fig 8) and express the absolute dry density in the settling column as a function of the overlying weight of mud

$$\rho_b = 126.7 + 19.2 \log_{10}(w_o(z)) \quad (11)$$

The nature of this expression is not significant. Its purpose is to convey information about the density structure to the model. A stress-density relation (Fig 9) was constructed from the results of the flume and settling column experiments as follows. The erosion strength of the bed at each equilibrium condition is assumed equal to the steady bed stress at the end of each discharge run, and the corresponding bed density was obtained from the cumulative mass profile (Fig 7) using the equilibrium excess concentration to define the weight of eroded mass.

The procedure in the model simulation was to obtain the strength of the bed from the density-stress relation using the eroded mass to define the bed density. The result from the simulation shown in Fig 5 does not agree very well with the experiment. The reason seems to be (Ref 1) that the density structure of the flume does not exactly match the density structure derived from the settling column measurements.

Simulation using a derived density structure for the flume

An approximate relationship for the density profile in the flume

$$\rho_b = 130.0 + 19.2 \log_{10}(w_o(z)) \quad (12)$$

was derived from the flume equilibrium stresses and concentrations, together with the stress-density relation of Fig 9. The results of a simulation using this density relation (Eq 11) are more satisfactory (Fig 5) than when the settling column density was used directly.

5 DISCUSSION

The study has shown that the existing computer modelling techniques can simulate the erosion of a muddy bed when the erosion strength of the bed is known directly (ie. as in Equation 10) or indirectly from the bed density structure and a corresponding stress-density relation.

The quality of the results reflects the quality of the relations and parameters used in the simulations rather than casting doubts on the validity of the model. Even under the controlled conditions in the laboratory, it was not possible to quantify the erosion rate and structure of the bed exactly, and it would be still more difficult to do so in the field. It is not too important to know the erosion rate precisely because the erosion process is self-correcting in the sense that too much (or little) erosion in the early stages exposes stronger deposits (or leaves weaker deposits exposed) which inhibit (or allow faster) erosion later on. On the other hand, it is essential to have a good definition of the bed erosion strength if there is to be any chance of predicting the correct depth of scour or deposition in practical applications.

6 CONCLUSIONS

1. A new theoretical method has been devised for determining a single representative value of the erosion constant m_e from concentrations and bed stresses measured in the HR mud erosion flume, which does not require direct observations of the density profile of the initial bed.

2. It has been shown that the rate of erosion in the mud flume can be accurately simulated in terms of the excess shear stress (Eq 3) from a knowledge of shear strength structure of the bed.
3. The main sources of inaccuracy in simulating mud transport in real estuaries would be in the determination of the bed density profile and the relationship between the critical erosion stress and the density of the bed. Further basic research is needed to gain a better understanding of consolidation processes before a fully dynamic model of the bed can be produced. Nevertheless the models can still be applied to a wide range of engineering problems involving deposition and erosion in ports, harbours, rivers and estuaries.

7 REFERENCES

1. Puls W. Erosion characteristics of Estuarine muds. Hydraulics Research Report IT 265, Feb 1984.
2. Hydraulics Research. Properties of Belawan mud. Hydraulics Research Report EX 880, April 1979.
3. Thorn M F C and Parsons J G. Erosion of cohesive sediments in estuaries : an engineering guide. Proc. 3rd Int. Symp on Dredging Technology. Paper F1 BHRA Bordeaux 1980, pp 349 - 358.
4. Krone R B. Flume studies of the transport of sediment in estuarial shoaling processes. Univ of California Hyd Eng Lab and Sanit Eng Res Lab. 1962.

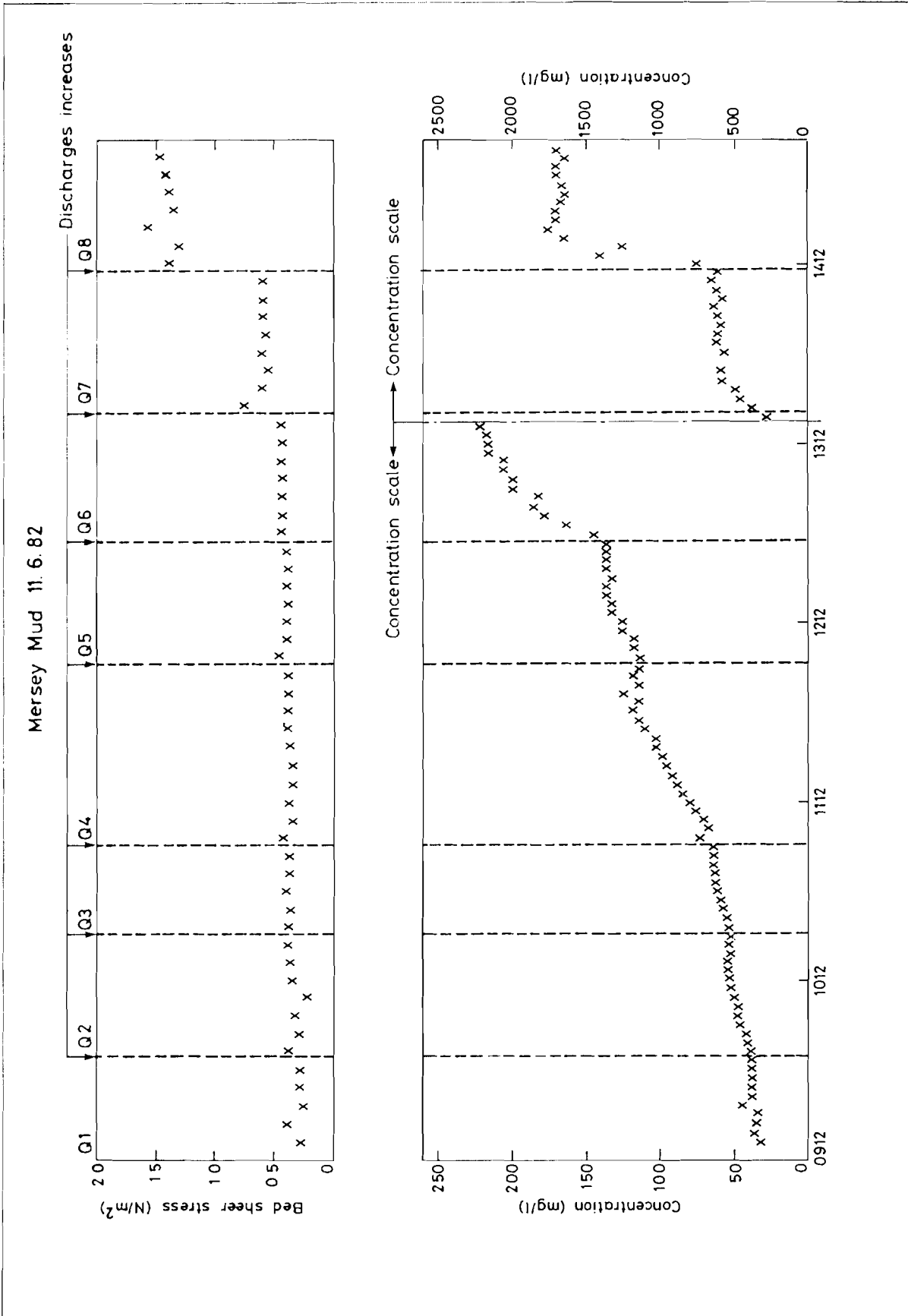


Fig 1 Variation of concentration and bed shear stress with time

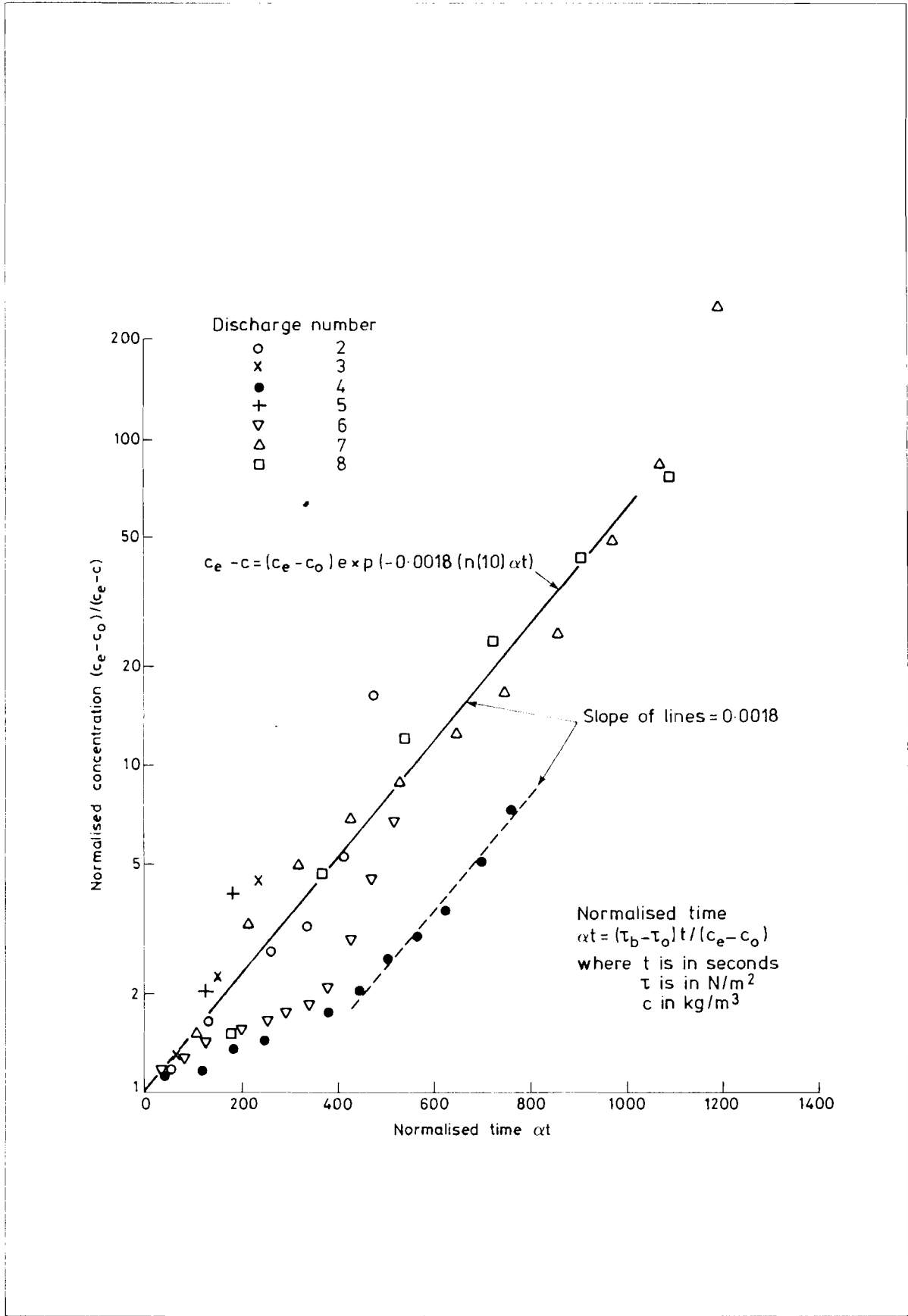


Fig 2 Analysis for erosion constant

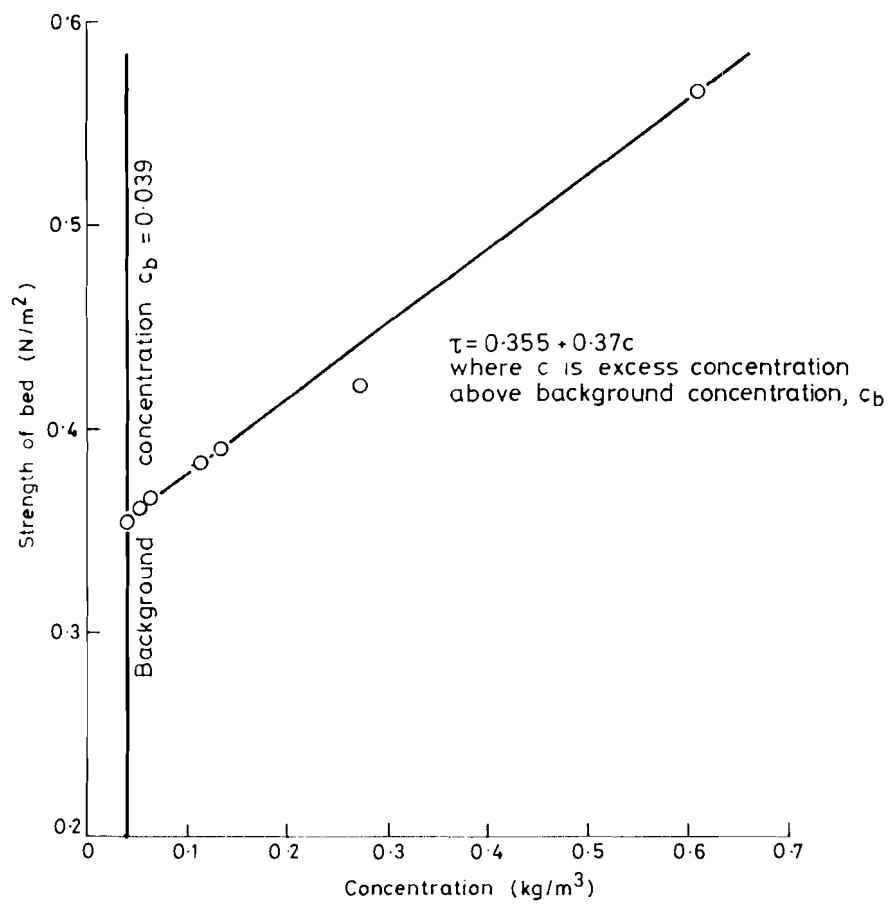


Fig 3 Shear strength relation for Mersey Mud

W_{50} Mersey mud

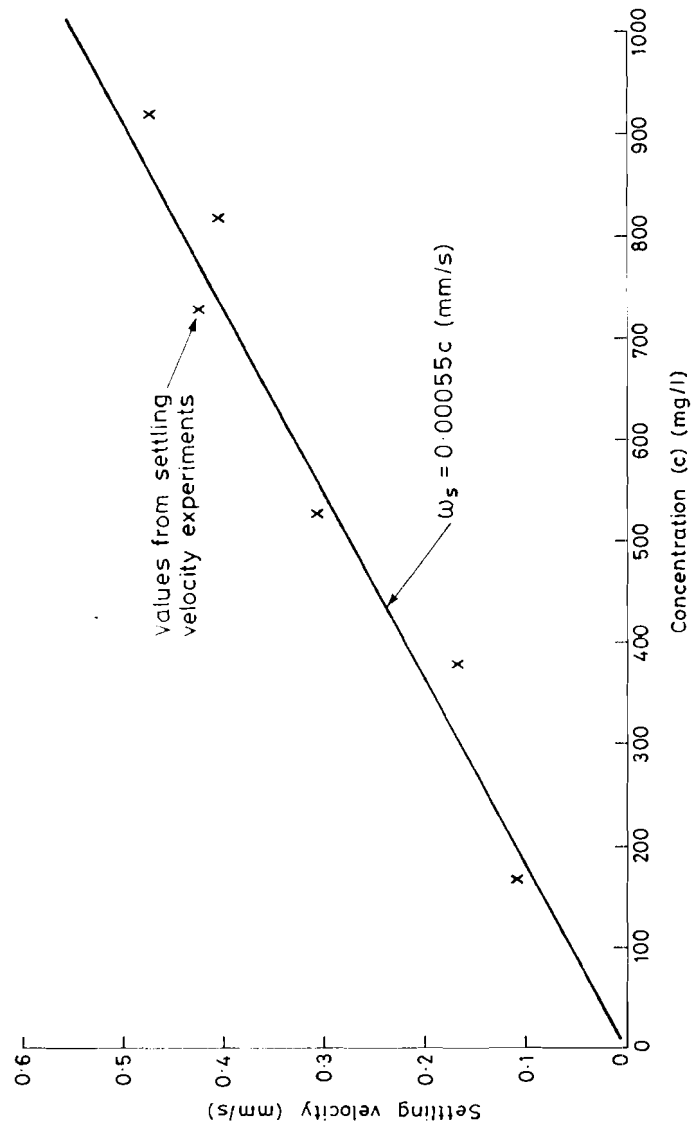


Fig 4 Variation of settling velocity with concentration

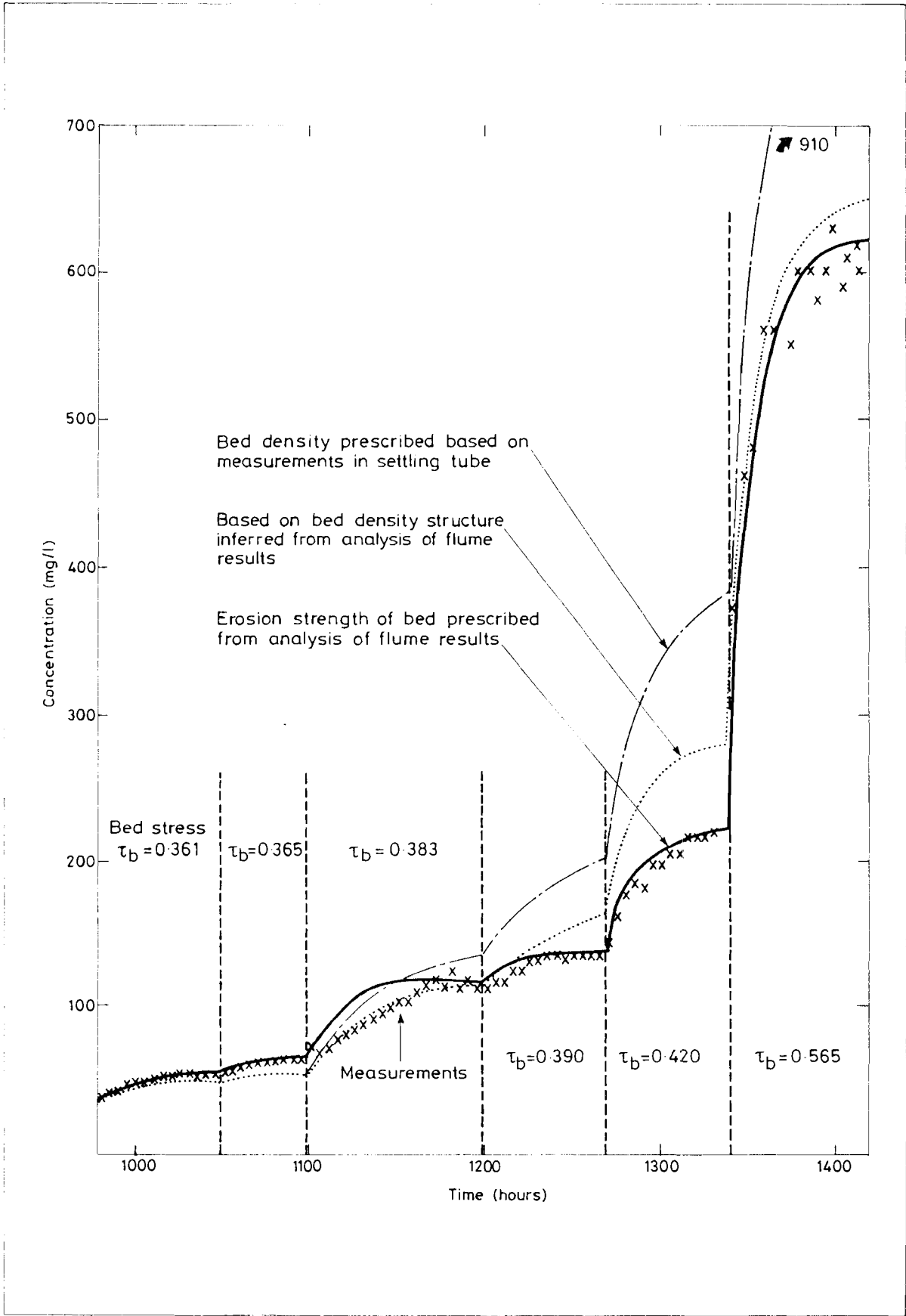


Fig 5 Comparison of computed and measured concentrations

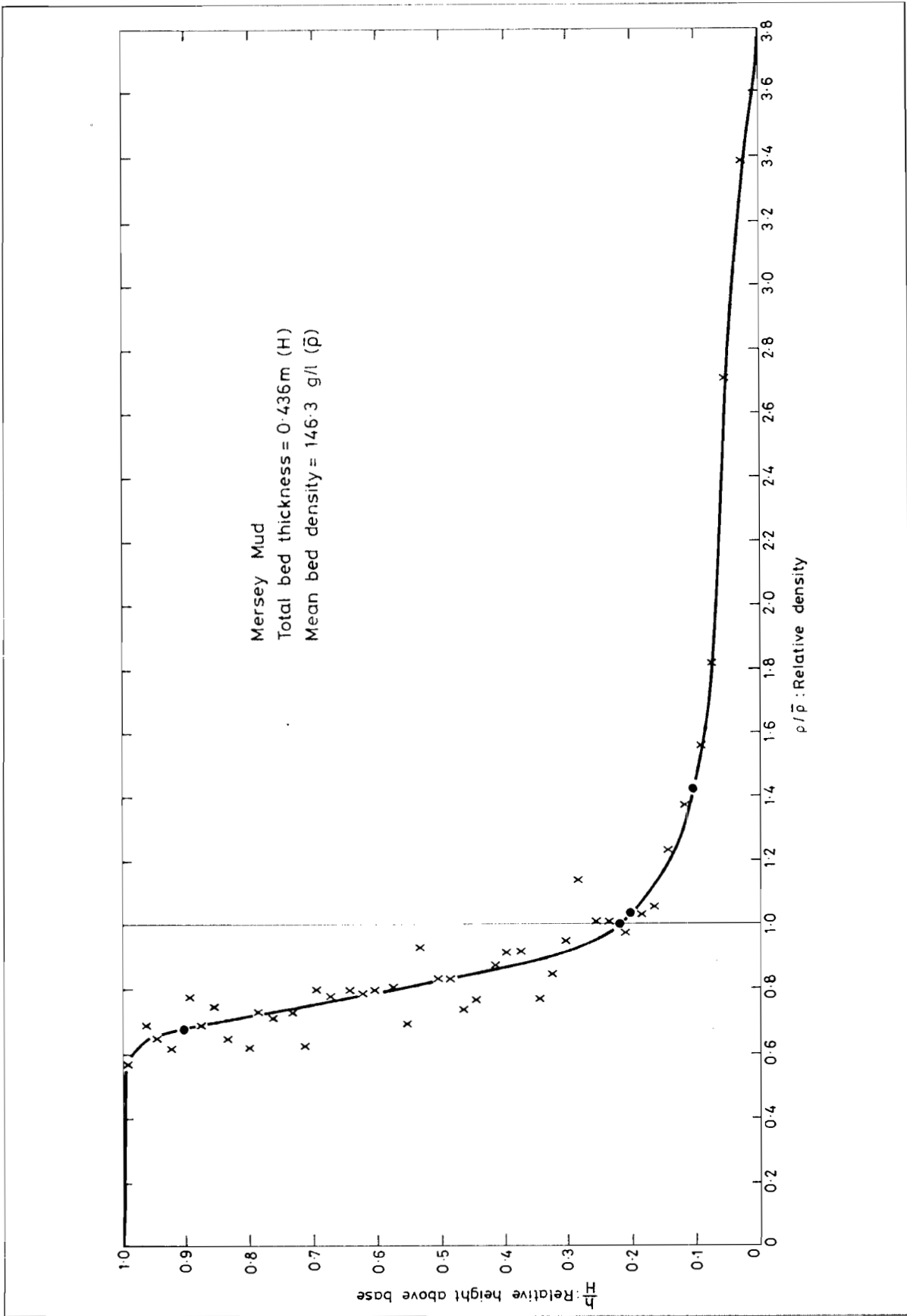
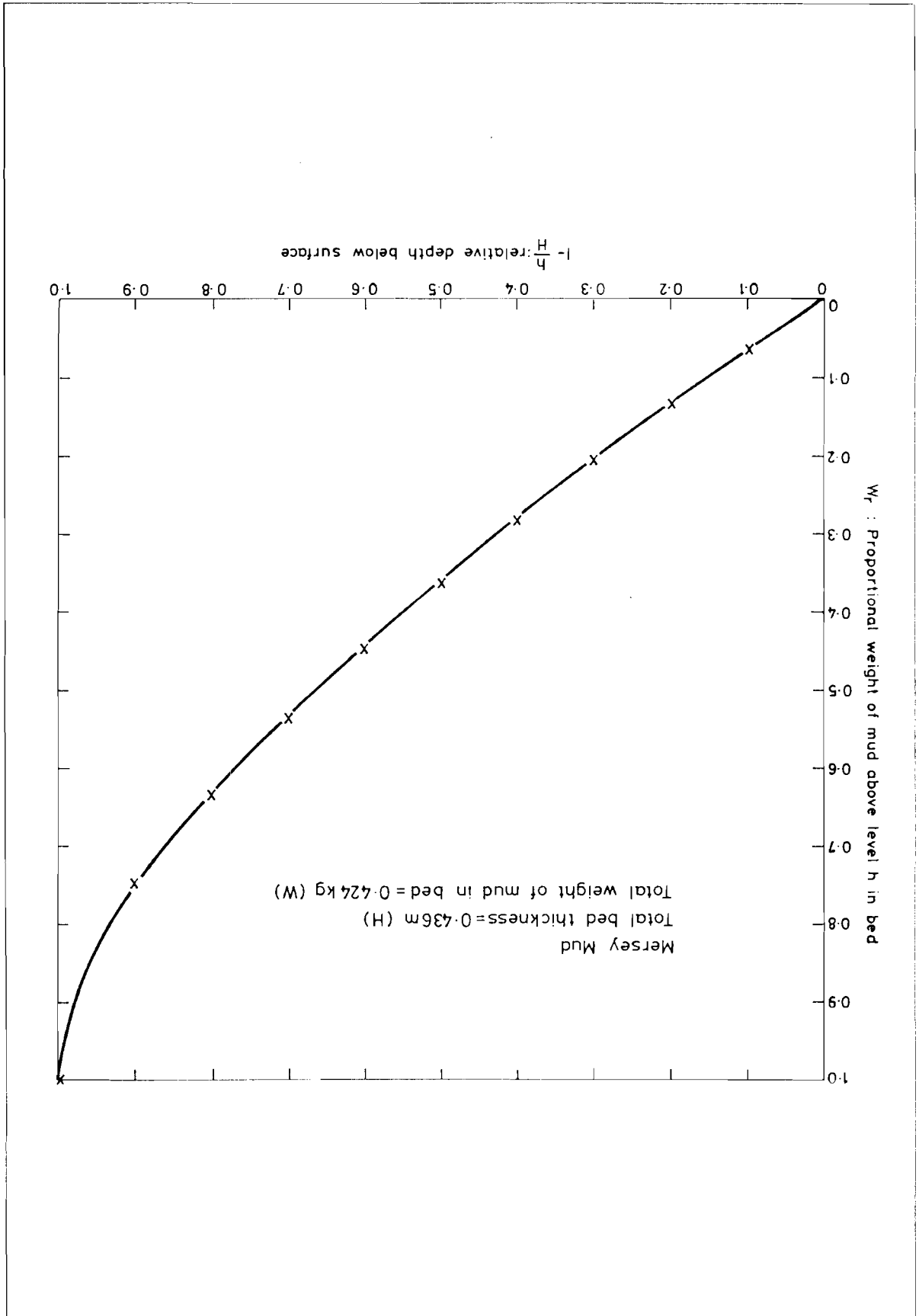


Fig 6 Fitted 2-day dimensionless density profile of mud bed

Fig 7 Cumulative weight profile of mud bed after 2 days



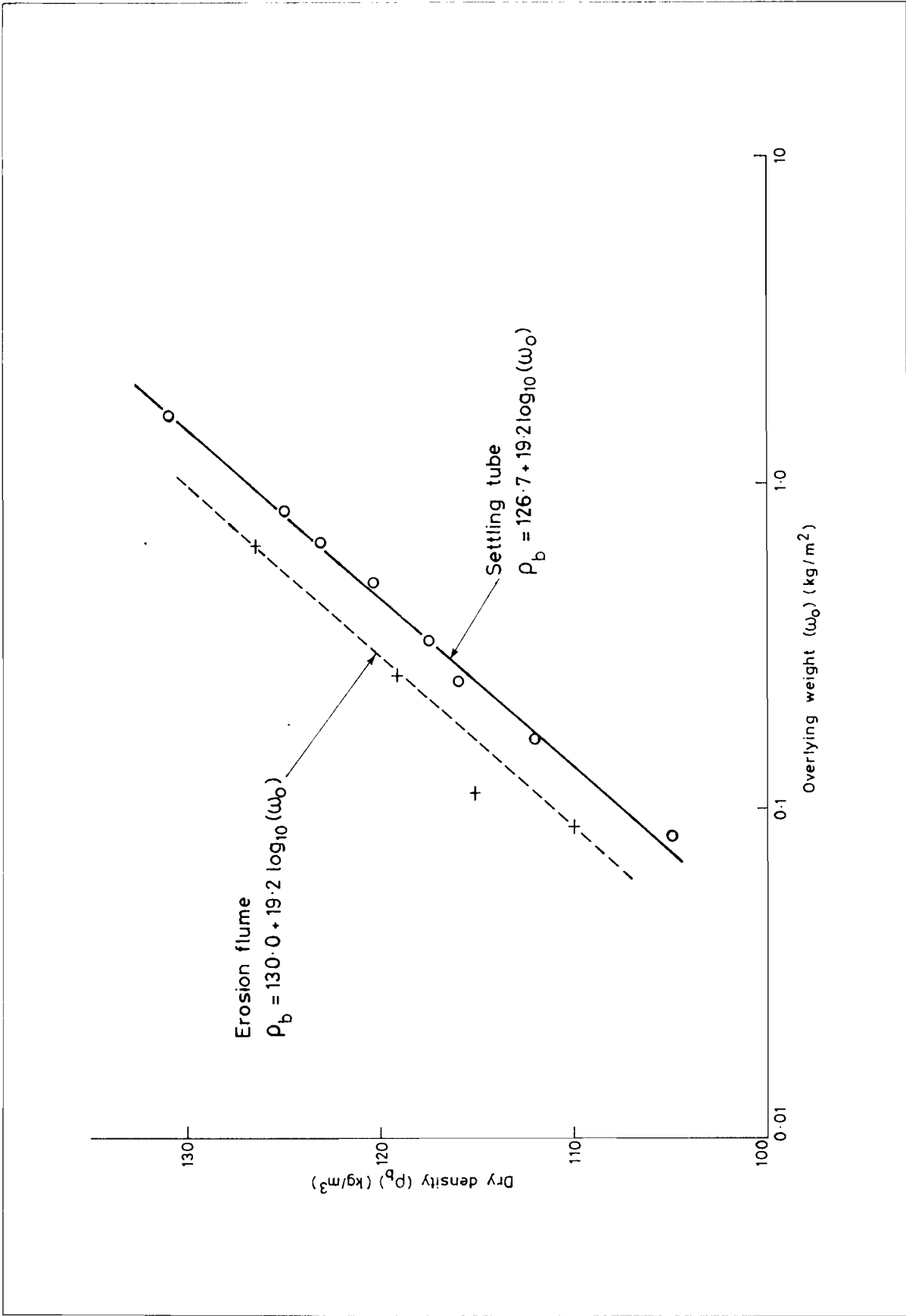


Fig 8 Structure of bed

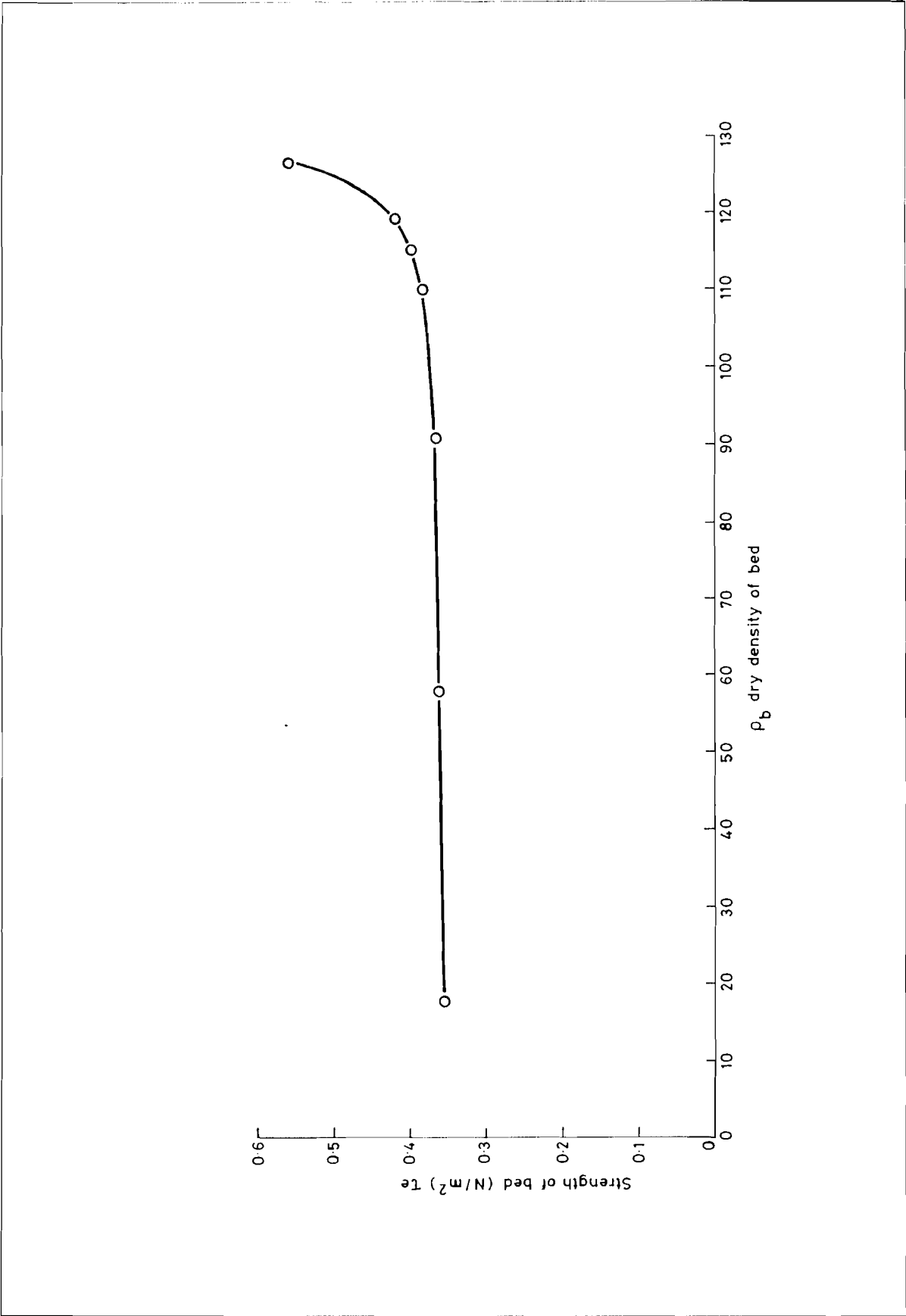


Fig 9 Variation of bed erosion strength with bed density

