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DEVELOPMENT AND DEMONSTRATION OF SYSTEMS BASED ESTUARY SIMULATOR: ESTSIM

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Key Words

Estuary Morphology; Behavioural Modelling; Systems Approach.

Abstract

Managing estuaries is an essential part of Flood and Coastal Erosion Management. Since 1998, Defra and the Environment Agency have run the UK Estuaries Research Programme (ERP) as part of their joint R&D programme. Within Phase 2 of the programme, tools have been further developed to enhance our ability to assess morphological change in estuaries. Running in parallel to this development, the EstSim project (FD2117: Development and Demonstration of System Based Estuary Simulators) investigated an alternative, yet complementary, methodology to the more conventional approaches pursued within other strands of ERP2. This involved application of a systems-based approach to the large-scale geomorphological behaviour of estuaries.

This paper describes the systems-based approach to simulating estuary geomorphological behaviour (EstSim) that complements quantitative morphological modelling. A structured qualitative framework has been developed through which it is possible to assess the behaviour of geomorphological features present in estuaries, the links between them, and how they respond to changes brought about through natural processes, engineering works and estuary management activities. This behavioural model is based on a working typology for UK estuaries (i.e. seven standard behavioural types) that has been formalised through an expert assessment via a mathematical framework to allow computational simulations to be made. A prototype simulator has been developed for evaluating the gross properties and behaviour of estuaries and this can be accessed through the world wide web. Development and pilot testing on the Ribble Estuary and Southampton Water, along with the Thames and Teign Estuaries, has demonstrated that the simulator can be applied both to specific and user-defined estuaries as well as the standard estuary configurations. The limitations of applicability have been documented.

Although systems-based thinking (as applied within this research) is a well-established conceptual framework in geomorphology, methodologies for converting system diagrams into

practical simulation tools have not previously been investigated. In investigating this approach, EstSim has provided a qualitative framework to understand and explain the behaviour of geomorphological features within estuaries, the linkages that exist between them and hence their response to change. The work has been successful in providing exploratory level research that has been used to demonstrate the potential of such a systems-based approach.

In conclusion, the knowledge gained on estuary geomorphological properties and behaviour through the study, combined with the results from applying the research level simulator, has led to an enhancement in the qualitative information available to support sustainable estuary management.

Introduction

There is a requirement to facilitate strategic and sustainable decisions regarding flood and coastal defence in estuaries with an understanding of estuarine morphological change. The Environment Agency / Defra funded UK Estuaries Research Programme (ERP) has sought to address this need through the development of tools to predict morphological change in estuaries over the medium to long term. Numerical modelling approaches provide quantitative outputs that can inform on trends and directionality of an estuarine system response. However, a need to capture knowledge of estuary response with other complementary, morphological tools and expert knowledge within a qualitative framework has been identified. This has been explored using a systemsbased approach, as an alternative, yet complementary, method to those research lines undertaken in the other strands of research within ERP (e.g. morphological concepts, bottom-up, top-down and hybrid methods). The "Development and Demonstration of Systems Based Estuary Simulators" (FD2117, EstSim) has been completed as part of Phase 2 of ERP to specifically address this need.

The overall aim of the research completed within EstSim was to extend our ability to simulate estuarine response to change. This has involved the provision of a qualitative framework to assist in understanding the:

- Presence and behaviour of geomorphological features in an estuary;
- Linkages that exist between them; and
- Their response to change.

The Rationale for Applying a Systems-Based Approach

A systems-based approach provides a structured framework for the incorporation of qualitative knowledge of the behaviour of estuarine systems. The highly interactive nature of estuary systems, with a variety of positive and negative feedback loops, means that they are well suited to this approach. For example, a change or action in one compartment or location of the system, such as the introduction of a flood defence, can have much wider-scale impacts.

The rationale for applying a more formalised systems-based approach derives from the inherent complexity of interactions between physical processes, sediment transport and geomorphological form. With incomplete knowledge of these relationships, there is a benefit in developing and applying qualitative models and descriptions that do not solely rely on having precise knowledge of the physical laws governing 'bottom-up' approaches. Such methods do not preclude the use of more quantitative approaches, but are more likely to use the knowledge gained from these methods in the development of the behavioural knowledge of systems. The requirement for estuarine research in these fields has been discussed by Townend (2002).

Background

The Systems Approach Defined

The systems approach involves separating out sub-systems and their interactions in order to understand the system organisation and define its behaviour. Thus it combines both the physical elements and the dynamics of the interactions between those elements in order to explain how the different elements that make up the system interact and respond to change (Cowell & Thom, 1994; Capobianco *et al.* 1999).

The systems approach has been applied and reviewed by various workers (Chorley & Kennedy, 1971; White *et al.*, 1984; Cowell & Thom, 1994; Capobianco *et al.*, 1999; Townend, 2003) and some of the key issues identified from these studies are summarised here.

A system-based approach is necessarily an abstraction of the real system, focussing on the scale and aspects of the system that are of interest to the user. To this end, systems diagrams provide a means of capturing the key attributes of a system by identifying the system elements and their interactions. A systems diagram is a flowchart representation and its ability to capture the behaviour of the systems will depend upon the fundamental knowledge of estuary processes and the ways in which these are expressed. The objective of defining systems diagrams is to represent the interactions between system components. Ideally this should capture the behavioural attributes of the system and inform abstraction and aggregation to different system levels.

Behavioural / Qualitative Modelling

The systems approach and the use of systems diagrams can be used to map the system components (elements and interactions) at a specified level of interest. However, attempting to model this detailed system is limited by current mechanistic understanding of the underlying processes. This is one of the reasons why the behavioural systems approach was being investigated as an alternative to detailed process modelling. The limitations of the systems diagram approach are however fully recognised (Townend, 2003). It is noted that that whilst systems diagrams make clear the nature of flows of energy and matter (e.g. sediment), and the interactions and feedbacks between elements, they say little about the relationship between components and the character of any response.

This is where behavioural or qualitative modelling (Capobianco et al., 1999) can be thought of as extending the basic systems approach. The concept of behavioural modelling is to develop an understanding of the behaviour of the system by capturing the nature of relationships between system components and mapping it onto a simple model, which exhibits the same behaviour, but which does not need to have any specified relationship to the underlying physical processes. Whereas systems diagrams highlight the presence of interactions, the behavioural approach places emphasis on developing the interaction as a relationship (response). In the context of an estuarine system the identification of a behavioural system is an attempt to integrate geomorphological units that are spatially contiguous into a unified entity that reflects how one or more of these units are likely to change.

Formal system definition

In order to apply the systems approach, a formal definition of UK estuary systems provides a useful rationale for identifying the key components. This was developed by defining a typology to classify all UK estuaries, according to behavioural type, and so identify the range of geomorphological elements within each behavioural type (ABPmer, 2007a). Formal definition was provided through the mapping out of the geomorphological sub-systems and exploration of systems diagrams and their ability to encapsulate different types of behavioural response (ABPmer, 2007a). The formal definition of UK estuary systems in turn provided the basis for the development of a behavioural model through mathematical formalisation of the defined systems.



Estuary Typology

The estuary classification adopted in the EstSim project builds on work undertaken from a number of previous studies (Hume & Herdendorf, 1988; Pritchard, 1989; Davidson *et al.* 1991; Townend *et al.* 2000) and more

recently as part of Futurecoast (Defra, 2002). The Futurecoast scheme has been amended and simplified to provide a working typology with which to progress the study for UK estuaries (Table 1).

Table 1Estuary Typology

Type	Origin	Behavioural Type	Spits ¹	Barrier Beach	Dune	Delta	Linear Banks ²	Channels ³	Rock Platform	Sand Flats	Mud Flats	Salt Marsh	Cliff	Flood Plain ⁴	Drainage Basin
1	Glacial valley	Fjord	Х					Х	Х	Х			Х		Х
2		Fjard	0/1/2					Х	Х	Х	Х	Х		Х	Х
3	Drowned	Ria	0/1/2					Х	Х		Х	Х	Х		Х
4	river valley	Spit-enclosed	1 / 2		Х	E/F		X/N		Х	Х	Х	Х	Х	Х
5		Funnel- shaped	Х		Х	E/F		Х		Х	Х	Х		Х	Х
6	Marine/fluvia l	Embayment			Х		X	Х		Х	Х	X		Х	
7	Drowned coastal plain	Tidal inlet	1 / 2	X	Х	E/F		X		X	X	X		X	

Notes:

¹ Spits: 0/1/2 refers to number of spits; E/F refers to ebb/flood deltas; N refers to no low water channel; X indicates a significant presence.

Linear Banks: considered as alternative form of delta.

³ Channels: refers to presence of ebb/flood channels associated with deltas or an estuary subtidal channel.

Flood Plain: refers to presence of accommodation space on estuary hinterland.

Application of the typology to classify all UK estuaries allowed for the development of behavioural statements and systems diagrams that are applicable to the range of estuarine types found in the UK. In doing this, application of the classification ensured the behavioural relationships explored, developed and applied throughout the research could be inter-compared within a consistent framework.

Behavioural Statements and Systems Diagrams

The development of behavioural statements has facilitated (i) our understanding of relationships between different elements and the key forcing factors, and (ii) the translation of these relationships into a model domain capable of simulation.

High-level descriptions of each of the seven types estuary behavioural including identification of the component geomorphic elements, using a systems diagram, were developed. In addition, textural descriptions were generated for each of the eleven geomorphic elements identified from the typology as being present in UK estuaries Each geomorphic element is (Table 1). represented by a systems diagram, which covers the short to medium-term and medium to long-term.



These behavioural statements and system diagrams capture, in a qualitative sense, the components and linkages at different levels within generic estuary systems in order that these definitions could be formalised to develop a behavioural model.

Behavioural model development

The formal system definition has been developed into a mathematical framework. The approach adopted for this mathematical formalisation involved the use and development of a Boolean network approach.

The Boolean Network Approach

The Boolean network approach was described by Nicolis (1982) in a pioneering application of the technique to climate dynamics; and has since been developed with applications to different fields including seismology, climatology and meteorology (e.g. Ghil et al., 1987, Wohlleben & Weaver 1995; Saunders & Ghil 2001; Zaliapin et al., 2003). It provides a modelling framework that is particularly suited to the mathematical formulation of conceptual models of systems that exhibit threshold behaviour, feedbacks and time delays.

Within the EstSim project, a Boolean approach has been used to develop a mathematical formalisation of long-term morphodynamic evolution of complex estuary systems (Karunarathna and Reeve, 2007). This has involved development of Boolean networks combining geomorphological elements within the estuary system with external forcing driving the morphological evolution, and derivation of Boolean expressions that define the interactions between the (network) elements. The method provides a formal mathematical language that allows qualitative geomorphological 'rules' (i.e. as provided by the formal system definition) to be encapsulated and manipulated in a rigorous manner.

Boolean networks were initially constructed for each behavioural estuary type (Table 1). Each element in a Boolean network has two

states, 'high' or 'low' (also called 'on' or 'off', 'true' or 'false'). To indicate its state, each element has an associated value 1 for 'high' and 0 for 'low'. The future state of one element in the network depends on the states of the other elements in the network, which are designated as that element's inputs. The element may feedback its own state as a selfinput. The state of an element in a Boolean network at a future time is governed by a logical rule or Boolean function, which operates on the element's inputs. Each geomorphological element and the external forcing parameters that drive morphological changes in the estuary are represented by an individual element in the network.

Once the elements of the Boolean network have been defined based on the estuary system diagram, feedback loops between geomorphological elements and external forcing that drive the morphological evolution of the estuary are developed from the behavioural description and the system diagram of the geomorphological elements. The effects of change in environmental forcing parameters on the morphological evolution of the estuary are incorporated through waves and tides. Human interference is modelled through feedback from control structures (e.g. training wall, jetties) and The feedbacks from the subdredging. systems are represented by the sediment flow.

Once the network is completed, a Boolean variable is assigned to each element in the network. A Boolean function for each variable is derived by combining Boolean variables within a logical framework. The logical framework operates on the feedback from designated 'input' elements in the network. A truth table is computed by solving the logical expressions for Boolean functions. The truth table gives Boolean states corresponding to various combinations of Boolean variables and resulting Boolean functions. The concept is illustrated for a simplified-representation of a generic tidal inlet in Figure 1. The geomorphological rules for that network are defined by the statements contained in Equation (1).





Dark arrows and broken arrows in the network represent positive and negative feedback respectively

Figure 1 Boolean network for a generic tidal inlet (little or no sediment flow from outside)

$W = sm' \lor tf' \lor cc \lor dd' \lor ss'$	(a)	
$T = sm' \lor tf' \lor cc \lor dd' \lor ss'$	(b)	
$SM = (w' \wedge t \wedge tf)$	(c)	
$TF = ((sm \lor cc) \land t) \lor (tf' \land w')$	(d) (1)	I)
$CC = (w \lor t) \land (tf \lor dd) \lor cc'$	(e)	
$DD = (w' \lor t') \land ss' \lor (t \land cc)$	(f)	
$SS = (w' \lor t') \land dd$	(g)	
The following notations stand for the variables u	used in Equation 1:	

Network Element	Boolean Variable	Boolean Function
Waves	W	W
Tides	t	Т
Saltmarsh	sm	SM
Tidal flats	tf	TF
Channels	сс	CC
Delta	dd	DD
Sand spit	SS	SS

The following convention is used to form the logical expressions:

Convention	Description
<i>a</i> '	not a
a v b	<i>a</i> or <i>b</i>
$a \wedge b$	$a ext{ and } b$

A Boolean state, in which all the Boolean variables and corresponding functions take the same value indicates a stable state where the system is bound to no further changes (Nicolis, 1987). When a stable state is reached, neither the Boolean variables nor the Boolean functions will change.

Development of the Prototype Simulator

The initial Boolean network model outlined above provided a proof of concept exercise. The Boolean network model was extended to incorporate a broader set of morphological and process components and a more realistic representation of estuary behaviour at a whole system level (French and Burningham, 2007).

The formal system definition was extended to incorporate a basic spatial division into an outer estuary sub-system (which interacts with elements of the adjacent coastal system, and potentially contains beach, spit, dune, and tidal delta or linear bank features, as well as sand or mud flat and saltmarsh) and an inner estuary sub-system (which contains an assemblage of sub-tidal channel and intertidal flat and saltmarsh features). Such a scheme resembles the estuary sedimentary facies model of Dalrymple et al. (1992) in that outer and inner estuary zones are likely to be dominated by marine and marineinfluenced fluvial processes respectively.

The generic scheme is visualised in Figure 2, which sets out the spatial arrangement of the major morphological components and various external and internal influences on their evolutionary behaviour. Three broad types of system component are distinguished:

- External (imposed) forcing and interventions;
- Process state variables; and
- Morphological components (similar to the geomorphological elements derived from the estuary typology with a number of minor modifications).

Boolean functions were developed to define, within the network model, the linkages between each of the above variables for the coast-estuary interface, outer-estuary and inner-estuary sub-systems. In applying what is essentially a 'rule-base' approach at this level of detail, it is recognised that the translation of a qualitative understanding of geomorphological elements, and their interaction with processes, into Boolean functions represents a significant challenge.

To provide an illustration of the extension of the initial Boolean network model, as described above, an example Boolean variable and function within the Outer Estuary sub-system is presented below in Table 2, together with the rationale for the specific function. The extended Boolean network described above is referred to as the Prototype Simulator.

Table 2	Example of a Boolean Variable and associated Boolean, to illustrate 'Rule Base'
	approach of Boolean network model

Sub-System	Outer Estuary					
Boolean	OUTER_MARSH_LOW					
Variable						
Boolean	(~outer_marsh_low & (outer_mudflat outer_sandflat) &					
Function	(outer_flood_defence outer_marsh_high outer_cliff) &					
	(marine mud (cliff mud & coastal cliff erosion)) &					
	~(outer_estuary_swell & outer_estuarywaves)) (outer_marsh_low					
	& (outer_mudflat outer_sandflat) & ~(outer_estuary_swell) &					
	~(((outer_flood_defence outer_marsh_high bedrock) & slr) & (slr					
	<pre> ~(marine_mud (cliff_mud & coastal_cliff_erosion)))))</pre>					
Descriptive	Forms under conditions of mud supply (from marine or coastal cliff					
Rationale	sources) and negligible wave action. Can be lost through coastal					
	squeeze if backed by higher marsh, cliff, or flood defence and					
	subjected to SLR.					

Notes: Boolean Functions are defined here using logical AND (& in MATLAB), OR (|) and NOT (~) operators.



Figure 2 Morphological components of estuary simulator and various external and internal influences on their evolutionary behaviour



Pilot Testing

Two estuaries were used to provide independent pilot tests to evaluate the capabilities and limitations of the prototype simulator. In order to allow extensive and rapid testing the Thames and Teign were chosen due to availability of good data sets and extensive previous study. In addition, during development phases, the prototype simulator was tested on the Ribble Estuary and Southampton Water (Rossington, et al., 2007). Research was completed to identify an up to date and relevant list of management questions applicable to UK estuaries, mapped against relevant legislation (ABPmer, 2007b).

Discussion: validation and applicability

Through the various development testing and pilot testing a number of key observations were made regarding the validation of the approach and the overall applicability of the prototype simulator at its present level of development.

Testing indicates that the simulator can obtain a largely correct depiction of gross estuary properties with the generic estuary types and rule base. This conclusion is made in terms of the qualitative model output when compared with observed estuarine features and responses; in reality these are value judgements rather than quantifiable results. There are subtle estuary-specific aspects of inherited morphology, sediment transport, hydrodynamics, and intervention history that would require customisation of the model functions. The ability to customise the prototype simulator was investigated to a limited extent by Rossington et al. (2007) who concluded that EstSim was able to reproduce the observed features of the Thames and Teign. However, further validation studies are required to obtain more confidence in the results, i.e. by verifying the rule-base and examining the response to particular effects in specific documented cases.

In its generic form, the prototype simulator can be applied to any one of the seven UK

estuary types, as well as user-defined estuaries. The model requires expert knowledge of estuary morphology to set up the model for specific estuaries and in order to interpret results. In addition, minor modifications may be required to capture particular estuary specific aspects of morphology, processes and which additionally requires a good understanding of processes and morphology. The present implementation of the model does not allow for the magnitude of an effect to be determined, or for the scale of the presence of a morphological variable, e.g. saltmarsh; it cannot distinguish between a few square metres of marsh or a hectare of marsh. The approach makes use of system-based abstractions (idealised simplifications) of the estuary as a whole and its component geomorphological features. The model can be used to determine the directions of change but, in its present form, is not able to determine sensitivities of the estuary system to change due to its discrete (all or nothing) approach.

Some of the limitations noted mean the prototype simulator is not a suitable tool, in isolation, to address estuary management options. However, the approach provides a useful means of formalising some of the more qualitative geomorphological knowledge and capturing characteristic behaviour. It is recommended that detailed consideration be given to the capabilities and limitations of the approach prior to any application.

Conclusions

The developments within the EstSim project have been successful in providing exploratory level research into the systemsbased approach for the simulation of estuary change. The research has been formalised and the resulting prototype simulator is beginning to reveal potential in this field, although it must be emphasised that at this stage this is still primarily a research tool. In addition, the research has provided a valuable qualitative framework for the application of the systems based approach to estuaries.

The study has provided formal definition of UK estuaries, in systems terms, and included in this is a database of UK estuary behavioural types. Behavioural descriptions have been produced at the generic level providing a reference source and also providing a framework for specific estuary behavioural statements.

The definition of UK estuaries has been mathematically formalised to develop a behavioural or qualitative model in the form of the prototype simulator. This predictive systems-based tool is capable of capturing characteristic morphological behaviour and provides a framework for formalising qualitative geomorphological knowledge.

Accessibility

Further details of the research completed within EstSim can be found at a web-based interface that provides a visualisation tool for the prototype simulator and additionally hosts a number of other key outputs from the project

(http://www.discoverysoftware.co.uk/EstSim /EstSim.html). The interface provides a means to disseminate the research and promote knowledge and understanding of the systems-based approach. In addition a 'research code web-page' provides access to the MATLAB code developed to implement the prototype simulator. This enables future evaluations and developments by other researchers

(http://www.geog.ucl.ac.uk/ceru/EstSim).

Summary details of the EstSim project can be found on the Estuary Guide website (http://www.estuary-guide.net/) in the context of other methods and models available to assess morphological change in estuaries developed within ERP and other R&D.

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