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ABSTRACT: Flood defence maintenance strategies require understanding of the time-dependent behaviour of flood defences. Quantitative risk and reliability methods provide a rational decision-making basis for flood defence management. Failure mechanisms influencing the flood defence reliability are expressed by a limit state equation and organised in a fault tree. One or more random variables in the limit state equation may be time-dependent. Quantitative information about and understanding of time-dependency of flood defences is often limited. A modelling methodology identifying the relevant variables and uncertainties involved with time-dependent processes, the character of the process and appropriate statistical models is therefore introduced. The modelling methodology is demonstrated on anchored sheet pile walls in a flood defence system in the Thames Estuary.

1 INTRODUCTION

Any kind of flood defence maintenance strategy requires understanding of the time-dependent flood defence behaviour. Quantitative risk and reliability methods form a rational decision-making basis for flood defence management, Vrijling (2003), Faber (1997), Sayers et al. (2002), Vrouwenvelder et al. (2001), CUR 141 (1990), Dawson & Hall (2006). Failure mechanisms influencing the flood defence reliability are expressed by limit state equations and organised in a fault tree. One or more flood defence characteristics in the limit state equation may be time-dependent. A modelling methodology identifying the relevant variables and uncertainties associated with the time-dependent processes, the character of the process and the appropriate statistical models is introduced to deal with the often limited information availability on time-dependency. The time-dependent process is subsequently embedded within the time-dependent flood defence reliability analysis.

Section 2 addresses time-dependent reliability analysis of flood defences. Section 3 introduces the modelling methodology for time-dependent processes. Section 4 demonstrates the methods on a flood defence system in a case study. Section 5 presents conclusions.

2 TIME-DEPENDENT RELIABILITY ANALYSIS OF FLOOD DEFENCES

2.1 *Time-dependent reliability analysis and stochastic processes*

As part of the quantitative reliability analysis the main flood defence types in a flood defence system are identified. Subsequently, the failure mechanisms, i.e. the chains of events leading to structural failure of the flood defence types, are established. The mutual relations between failure mechanisms are represented in a fault tree. The failure mechanisms are expressed by one or more limit state equations. Time-dependent reliability analysis defines the lifetime distribution based on the time-dependent limit state equation $Z(X(t), t) \leq 0$, where $X(t)$ is a vector of processes: $X_1(t), \ldots, X_n(t)$. The lifetime probability distribution $F_L(T)$ is defined as a function of time t as follows:

$$
\begin{aligned}
F_L(T) &= P(L \leq T) \\
&= 1 - P\left[Z(X(t),t) > 0; \forall t \in [0,T] \right]
\end{aligned} \quad (1)
$$

in which $P(L \leq T)$ is the probability of the lifetime is smaller or equal than a period T. The vector $X(t)$ contains the flood defence characteristics in the limit

state equation. These characteristics can be hydraulic loading conditions, i.e. water level and wave conditions, as well as geometry, vegetation, revetment or soil properties. A variety of time-dependent processes, mainly directed at modelling hydraulic loading conditions, have been discussed in literature, e.g. Melchers (1999). This paper intends to model time-dependent flood defence characteristics which are not hydraulic loading conditions or are not broader system-scale processes that influence multiple flood defence assets simultaneously, e.g. a morphological model or a hydrodynamic model for river water levels and currents.

Traditionally, time-dependent processes are modelled without considering the variations in time in the process, such as the deterministic process in the 1st panel in figure 1. In this paper statistical models for time-dependent processes are developed consisting of components based on three different types of representation of time-dependency. Firstly, a *stochastic process* where the time-dependent variable of interest $X_i(t)$ is modelled by a stochastic process (3rd panel figure 1). Secondly, a *hierarchical process* consisting of random variables and which has one or more stochastic processes embedded in it, so for example $X_i(t) = f(D_1, .., D_i(t), .., D_n)$, where $X_i(t)$ is a function of random variables D_1 to D_n, among which $D_i(t)$ is a stochastic process (3rd panel figure 1). Thirdly, a *parametric process* where $X_i(t) = f(D_1, .., D_n, t)$ so $X_i(t)$ is a deterministic function of random variables $D_1, ..., D_n$ and time t (2nd panel figure 1). Strictly according to the definition of a stochastic process, all three processes mentioned above are stochastic processes. Equation (2.2) can generally not be solved analytically. Engelund et al. (1995) suggest to approach the lifetime probability distribution $F_L(T)$ with a Poisson distribution based on the assumption of independent outcrossings:

$$F_L(T) \approx 1 - \exp\{-E[N^+(T)]\} \qquad (2)$$

in which $E[N^+(T)]$ is the mean number of crossings of $X(t)$ into the failure domain during $[0,T]$. In

the stationary case $E[N^+(T)] = v^+T$, where v^+ is the outcrossing intensity. In this paper the lifetime period of interest $[0,T]$ is subdivided into N time intervals Δt. The mean number of crossings in a time interval is approximated by $P_f(\Delta t_i)$, the time-dependent probability of failure during a period Δt_i. The numerical implementation to calculate $P_f(\Delta t_i)$ is addressed in the following section.

2.2 *Numerical implementation*

This section describes generic aspects about the numerical implementation of stochastic processes and the incorporation in time-dependent reliability analysis. Since many time-dependent processes require a specific approach, some time-dependent processes are specifically addressed in the case study site application in section 4. The numerical implementation in this paper is based on Monte Carlo computations.

A hierarchical process (section 2.1) contains both stochastic process and parametric process components. Therefore, with understanding of the numerical implementation of the hierarchical process the stochastic process and the parametric process are explained as well. A hierarchical process is defined as a function $X_i(t) = f(D_1, .., D_i(t), .., D_n)$ in which $D_1, ..., D_{i-1}, D_{i+1}, ..., D_n$ are constant variables in time, like in a parametric process, and $D_i(t)$ is a stochastic process. The first simulation in the time series sample requires sampling of $D_1, ..., D_{i-1}, D_{i+1}, ..., D_n$ and a sample of the increment of $D_i(t)$ in the interval $t = 0$ and $t = 1$. In subsequent time steps the sample of $D_1, ..., D_{i-1}, D_{i+1}, ..., D_n$ remains equal to the first and constant throughout the time series. The increment of $D_i(t)$ is sampled and accumulated for each subsequent time interval. Based on $D_1, ..., D_{i-1}, D_{i+1}, ..., D_n$ and the accumulated $D_i(t)$ the overall quantity $X_i(t)$ is calculated for each time step. The appearance of time series samples is similar to those illustrated for the stochastic process in figure 1 (right panel). Following the previous explanation a parametric process samples variables D_1 to D_n once for a time series and varies deterministic time t. A stochastic process D_i is sampled and accumulated for each time step in the time series.
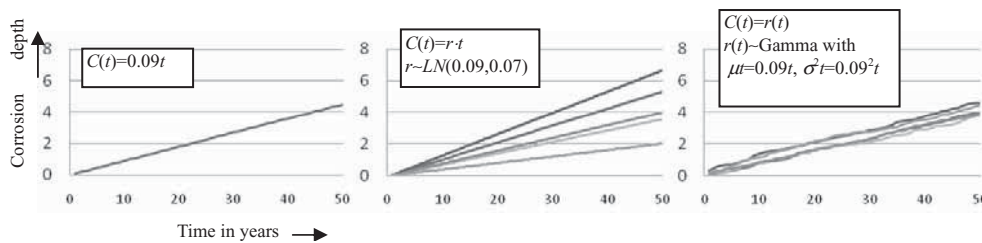

Figure 1. Examples of corrosion depth time series samples for a deterministic model (left), a parametric process (middle) and a stochastic process (right).

The incorporation of time-dependent processes in flood defence fragility and reliability starts by subdividing the flood defence system into a number of flood defence sections. Each of these sections is characterised by one cross section and can fail in multiple ways, the failure mechanisms. A limit state equation is used to define a failure mechanism. The logical relations between the failure mechanisms are represented in a fault tree. The flood defence properties in the limit state equations form a vector of random variables $X_1, .., X_{i-1}, X_i(t), X_{i+1}, ..., X_n$. One or more of these variables is a time-dependent process, $X_i(t)$, according to the definitions in section 2.1.

The first simulation in the time series, requires sampling of all random variables $X_1, .., X_{i-1}, X_i(t), X_{i+1}, ..., X_n$. If $X_i(t)$ is a time-dependent function $X_i(t) = f(D_1, .., D_i(t), .., D_n)$, the vector $D_1, .., D_i(t), .., D_n$ is also sampled. Based on the sample of the random variables the limit state equations corresponding with the different failure mechanisms are computed and evaluated whether $Z(X(t) \leq 0)$. According to the fault tree an evaluation is made of whether the cross section fails or not. For the subsequent time steps only the time-dependent quantities $X_i(t)$ are sampled and accumulated as explained above. For the subsequent time steps and the newly sampled values of $X_i(t)$ the limit state equations are evaluated. After completing the evaluation of the limit state equations for one time series all the time-dependent and time independent variables are sampled in a second time series simulation. This simulation procedure is repeated a large number of times to calculate the overall probability of failure of the cross section as a function of time, $P_f(t)$:

$$P_f(t) = \frac{N_{tot \leq 0}}{N_{tot}} \tag{3}$$

$N_{top \leq 0}$ is the number of simulations which entails failure of the cross section and N_{tot} is the total number of simulations. During the simulations the time period $[0,T]$ is discretised into time intervals Δt_i for which the time-dependent processes are sampled and the probability of failure is calculated. The probability of failure $P_f(\Delta t_i)$ is representative for the time interval Δt_i and is implemented in equation (2.3).

A further result of interest is time-dependent fragility, or the probability of failure conditional upon the water level. The procedure is similar to that described above except that the water level is not random. The water level is instead subdivided into a number of intervals. The time interval $[0,T]$ is discretised into time intervals Δt_i and for each moment in time the probability of failure given a water level h is calculated as $P_f(t|h)$:

$$P_f(t|h) = \frac{N_{tot \leq 0}}{N_{tot}} \tag{4}$$

$N_{top \leq 0}$ is the number of simulations for which the cross section fails given a water level h. N_{tot} is the total number of simulations given a water level h.

3 MODELLING METHODOLOGY FOR TIME-DEPENDENT PROCESSES OF FLOOD DEFENCES

3.1 Introduction

The availability of historical observations on time-dependent processes of flood defences is usually limited. The possibilities to test the quantitative predictions of the statistical models are therefore also constrained. This section introduces a modelling methodology that structures the way in which a time-dependent statistical model is formulated, based on both theoretical and practical considerations. The modelling methodology in this paper is aimed at time-dependent processes that can decrease as well as improve the flood defence performance. The processes are therefore not strictly deterioration processes, which affect the flood defence performance only negatively.

3.2 Steps in the modelling methodology

The methodology begins with an analysis of (i) the failure mechanisms in the flood defence system and (ii) the processes that may lead to time-dependent behaviour in those failure mechanisms. As equation (2.2) implies, time-dependent behaviour may act through change in the distribution of basic variables, or through change in the limit state function, though in practice the latter of these is parameterised by introducing new basic variables. Figure 2 presents the steps in the modelling methodology. These steps are explained in more detail in the following.

3.2.1 Identify existing knowledge
The starting point is the identification of existing scientific understanding about the time-dependent process of the flood defence asset. This analysis uses site specific information as well as more generic reviews of flood defence processes, e.g. Floodsite (2007) or HR Wallingford (2005).

3.2.2 Identify relevant flood defence properties
The second step in the modelling methodology identifies the flood defence properties contributing to the time-dependent process. The uncertainties associated with the flood defence properties provide insight in how time variability is introduced in the time-dependent process. *Excitation features* are the flood defence properties that initiate and drive the time-dependent process of interest. Without those features no
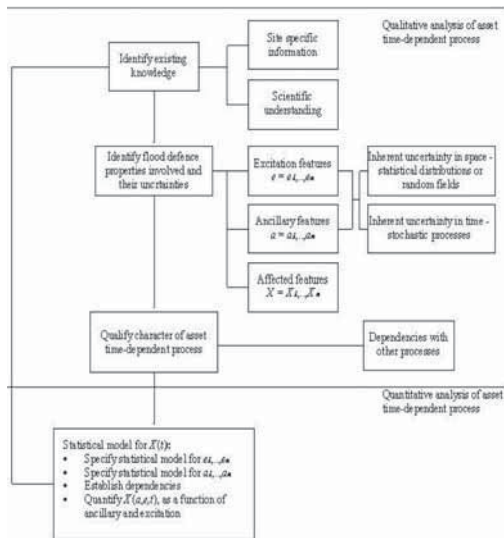
Figure 2. Steps in modelling methodology for time-dependent processes of flood defences.

Table 1. Some examples of excitation features.

Excitation feature	Characteristics time-dependent behaviour
Wave climate: significant wave height, wave period and wave direction	Recurrent/storm sequence
River current velocity	Reversing tides/recurrent high water events
Pore pressure distribution in flood defence structure or foundation	Rainfall and drought cycles/seasons/event sequence/accumulation
Water head difference over flood defence	Tides/recurrent/storm sequence/rainfall events/ rainfall sequence
Third party loading, e.g. traffic (vehicle weight, tyre acceleration, profile)	Recurrent/event sequence
Third party loading, e.g. animal burrowing	Three-dimensional random walk/entrance shifting with river water levels
Superimposed loading	Constant in time
Corrosion: the presence of oxygen and moisture	Seasonal/climate

Table 2. Stochastic processes and the type of time variability that they represent.

Stochastic process	Type of time-dependent behaviour
Rectangular wave process (e.g. Borges Castanheta)	Seasonality in hydraulic loading variables such as: high river discharges/ floodplain water levels/ pore pressures
Pulse/Poisson process	Arrival of storm events/arrival of trafficking events/arrival of pit corrosion
Gamma process	Strictly increasing excitation features, ancillary features or overall quantity $X_i(t)$
Compound renewal process (e.g. superposed, alternating, cumulative)	Arrival of trafficking events causing cumulative damages
Gaussian/Brownian	Continuous process

asset time-dependency takes place. Table 1 tabulates a number of excitation features and their time-dependent and spatial variation. *Ancillary features* are the flood defence properties that additionally influence the time-dependent process by transforming the excitation features into the time-dependent process. For example, the damage of the revetment caused by wave impact is a function of the structure slope, shape and weight of the revetment, as well as the excitation features of wave height and period.

3.2.3 *Qualify the character of the time-dependent process*

The character of the time-dependent process depends on how the variability in time is introduced and transformed. Table 2 shows examples of suitable stochastic processes for different types of time-dependent behaviour. These stochastic process models are suitable for both excitation and ancillary features introducing time variability. The character of the overall process is then quantified as a function of ancillary features and the excitation of the asset time-dependent process.

3.2.4 *Dependencies between time-dependent processes*

The fourth step in the modelling methodology in figure 2 is to represent the dependencies among asset time-dependent processes. Examples of such dependencies are: processes sharing the same excitation, one process forming (partly) the excitation of the other process, processes sharing similar ancillary features. Modelling such dependencies is fairly straightforward if process-based models are available for both time-dependent processes. The common excitation and/or ancillary features then appear in both process models. Dependency models in the form of threshold values and fault tree analysis provide a structured approach as well. Alternatively dependencies may be expressed in terms of correlation structure.

Buijs et al. (2005) is an illustration of correlated gamma processes for deterioration.

3.2.5 Definition of the statistical model

This paper demonstrates three main types of stochastic process models on the case study site: a hierarchical process, a parametric process and a gamma process model. The motivation to apply these statistical models is explained in more detail in section 4.2 in the application to the case study.

3.2.6 Parameterisation of the statistical model

Unfortunately, historical time series samples of the time-dependent quantity $X_i(t)$ are usually scarcely populated. Calibration is then supported by checking whether the time series samples of the asset time-dependent quantity $X_i(t)$ are in a sensible order of magnitude and display a sensible variation. Corroboration of the model is supported by comparing the qualitative behaviour of the process following from the modelling methodology with the behaviour of the time series samples. The increasing availability of future observations can be used for further calibration and corroboration or for Bayesian updating of the prior distributions. The necessity of further calibration and corroboration depends on the complexity of the time-dependent process and the sensitivity of the reliability and life cycle costing model to the time-dependent process.

4 APPLICATION TO CASE STUDY SITE

4.1 Site description

The time-dependent reliability methods are demonstrated on the Dartford Creek to Swanscombe Marshes flood defence system along the Thames Estuary in the UK (Figure 3). Flooding in 1953 was the trigger for a major flood defence improvement scheme which was carried out in the 1970s and 1980s. As part of this improvement scheme new flood defences were designed and existing defences improved. Considerable sums are invested annually in inspection and maintenance, and the strategy for maintaining and/or upgrading the defences during the coming decades is now being reviewed. An impression of the anchored sheet pile wall in the case study site and its main failure mechanisms are shown in figure 4. The time-dependent processes that are further analysed are: corrosion of the sheet pile cross section (section 4.3), corrosion of the sheet pile anchors (section 4.4) and changes in the river bed levels in front of the anchored sheet pile wall (section 4.5). Section 4.6 presents the incorporation of the time-dependent processes in the fragility and lifetime reliability of the anchored sheet pile wall. Before giving details of the time-dependent
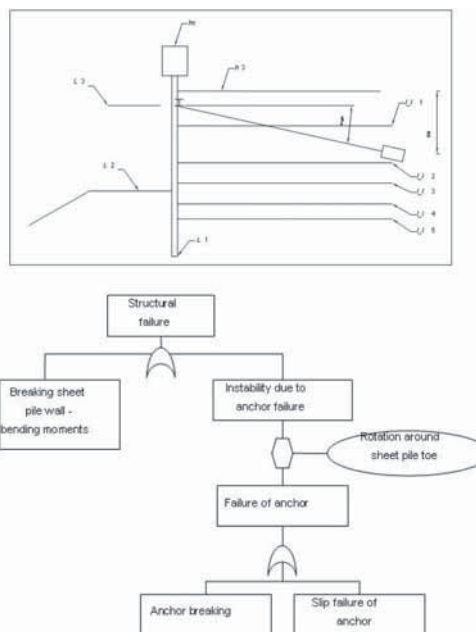

Figure 3. Anchored sheet pile wall and simplified fault tree.

Figure 4. An indication of the flood contours along the Thames.

processes the motivation to apply three main types of statistical models is explained in section 4.2.

4.2 Motivation for three main types of statistical models

Three types of statistical models are demonstrated on the Dartford Creek to Swanscombe Marshes case study site. The first type of statistical model is *a hierarchical process* developed according to the modelling methodology described in figure 2. The hierarchical process model allows the decomposition of the time-dependent process into contributions by different flood defence properties. Under some circumstances it may be preferable to instead choose for an aggregate representation of the time-dependent process, e.g.: lack of scientific understanding; the availability of field data on the deteriorating quantity only rather

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than on the contributing flood defence properties; financial and time constraints (broad risk assessments). *The gamma process model* is demonstrated in the form of an aggregate model as the second type of statistical model. This model provides a strictly increasing time-dependent stochastic process and is therefore deterioration. The gamma process either takes the behaviour according to an existing process model into account or allows expert elicitation on the average deterioration rate μ and its standard deviation σ, Noortwijk & Van Gelder (1996). The deterioration increments x are then a gamma process according to:

$$P(x) \sim Ga(x|a,b)$$
$$= \left[b^a / \Gamma(a) \right] x^{a-1} \exp\{-bx\} I_{\{0,x\}}(x) \quad (5)$$

in which

$$a = \frac{\mu^2 t}{\sigma^2} \quad \text{and} \quad b = \frac{\mu}{\sigma^2}$$

The third type of statistical model that is demonstrated on the case study is a conventional engineering approach based on a *parametric process*. An existing process-based model or, in the absence of such a model, a random deterioration rate as a function of deterministic time serves this purpose.

4.3 *Corrosion of the sheet pile cross section*

This section applies the modelling methodology to corrosion of the sheet pile cross section. The statistical models mentioned in the previous section are applied to the corrosion process.

Corrosion of the sheet pile cross section reduces the thickness of the sheet pile cross section (figure 4). The probability of breaking of the sheet pile under bending moments increases. The probability of holes developing through the piles also increases. Holes in the pile may lead to wash out of fines and the instability of the retained ground behind the anchored sheet pile wall.

Existing knowledge about corrosion is available in e.g. British Steel (1997), on traditional marine corrosion, and CIRIA (2005), on Accelerated Low Water Corrosion (ALWC). Southern Water (1989) and Halcrow (2006) provide sheet pile thickness measurements specifically at several locations along the Greenhithe anchored sheet pile frontage in the Dartford Creek to Swanscombe Marshes flood defence system. There is no evidence for ALWC in these measurements. The corrosion of the sheet pile cross section is therefore modelled as traditional marine corrosion. Reports and construction drawings (figure 4) indicate that the sheet pile walls along the case study flood defence line were originally constructed around 1950. They

were refurbished in 1979. In the period from 1979, two sets of sheet pile thickness measurements for the Greenhithe (figure 3) anchored sheet piles are available: in 1989 and 2006. Each of those measurements was followed by remediation activities. The precise effect of these activities is not clear. The second set, dating from 2006, targets the top, middle and bottom zones of the sheet pile walls and is applied as the basis for the statistical model, see table 3. The influence of measurement errors is not further investigated. British Steel (1997) relates different corrosion rates to six environmental exposure zones of the steel sheet pile. The measurements in table 3 subdivide the sheet pile wall length only into three zones and form the basis for the statistical model.

Excitation features of traditional marine corrosion are the presence of moisture and oxygen. These are subject to environmental variations such as daily or seasonal changes as well as spatial variations. The moisture and oxygen therefore introduce inherent uncertainties in time and space. Ancillary features are e.g. the steel composition or spatial variations in the exposure or surface deposits.

The probability of corrosion holes compares the sheet pile thickness with the corrosion depth at a moment in time. As mentioned above, the probability of corrosion holes does not entail the probability of failure of the steel sheet pile wall. The second failure mechanism of the sheet pile wall that is affected by corrosion of the steel sheet pile is breaking of the sheet pile cross section. The section modulus is reduced according to the corrosion depth in the three different exposure zones shown in table 3. Subsequently the proportion between the bending moment, the corrosion reduced section modulus and the vertical pressure exerted by the anchor is evaluated for each of the exposure zones.

Corrosion processes are traditionally modelled based on one or more corrosion rates either as a linear or polynomial function of deterministic time. The corrosion models are therefore not based on the chemical reaction between moisture and oxygen or the influence

Table 3. Sheet pile thickness measurements in 2006. 'Traditional' corrosion rates are based on the assumption that full restoration was achieved in 1989.

Site investigation 2006, Assuming lifetime 17 years

	Corrosion rate bottom (mm/year)	Corrosion rate middle (mm/year)	Corrosion rate top (mm/year)
mean	0.029	0.087	0.039
stdv	0.018	0.072	0.030
V=	0.65	0.84	0.78

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of the steel quality or other corrosion accelerating factors previously mentioned. In this paper, the hierarchical process model for the corrosion depth $d(t)$ is as follows (similar to the 3rd panel in figure 1):

$$d(t) = \left(m_{c;1} + m_{c;2} + m_{c;3}\right) \cdot c_d \cdot t \tag{6}$$

in which $m_{c;1}$ is a lognormally distributed constant model uncertainty in time. $m_{c;2}$ is a wave renewal model uncertainty covering seasonal variations in the exposure to moisture and oxygen. The year is subdivided into four seasons, for each season a new value for the normally distributed $m_{c;2}$ is sampled. $m_{c;3}$ is a model uncertainty represented by a Brownian Motion capturing the daily environmental variations in the exposure to moisture and oxygen. c_d is the lognormally distributed corrosion rate according to table 3. t is deterministic time. In a second variation on the hierarchical model, the seasonal variations are captured by a wave renewal model for the corrosion rate. The year is subdivided into four seasons and each season a new corrosion rate is sampled from a lognormal distribution based on table 3. The hierarchical process model does not include the influence of ancillary features or corrosion accelerating factors.

The second statistical model is the gamma process model for the corrosion depth (similar to the 3rd panel in figure 1), based on equation (2.6). Whereby the average corrosion rate μ is 0.09 mm/year and the standard deviation is taken higher than the value stated in table 3, σ is 0.09 mm.

Often stochastic process theory is not acknowledged in time-dependent statistical models. The corrosion rate is then considered as function of a constant random variable in time, c_d, and a deterministic time t:

$$d(t) = c_d t \tag{7}$$

Equation (2.6) corresponds with the third statistical model mentioned in section 4.2, the parametric process model (2nd panel in figure 1).

Figure 5 shows the expected values and standard deviations of the sheet pile corrosion statistical models. The time series samples of the parametric process model consist of straight lines (such as panel 2, figure 1). The difference between the seasonality emphasis on the model uncertainty and on the corrosion rate shows that the first has more variation than the latter time series. The two different models demonstrate a difference in emphasis on spatial uncertainty or variations in time. The gamma process does not distinguish between seasonality or daily environmental fluctuations and does not enable a shift of emphasis on spatial uncertainties or uncertainties in time.
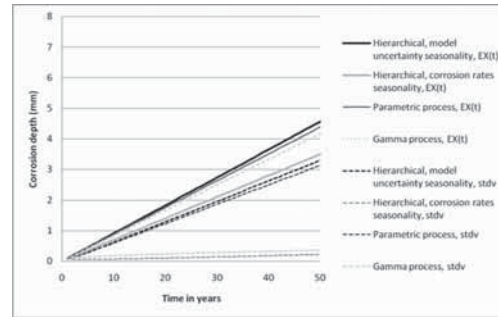

Figure 5. Expected values and standard deviations for the statistical models of sheet pile corrosion.

4.4 Corrosion of the sheet pile anchor

Corrosion of the sheet pile anchor reduces the area of the anchor. The reduction in the sheet pile anchor area reduces the strength capacity of the anchor, increasing the probability of anchor failure. After anchor failure, rotational failure of the anchored sheet pile wall is required for full failure of the anchored sheet pile wall.

British Steel (1997), British Standard (1989) and British Standard (2000) give qualitative background information about anchor corrosion. Southern Water (1989) reports about anchor excavations, corrosion depth measurements and corrosion pitting at the Greenhithe site (figure 3 and 4). As mentioned in section 4.3, the anchored sheet pile frontage along Greenhithe was originally constructed in 1950/1951. In 1979 improvement and maintenance of this frontage was carried out, there is no mention of work undertaken on the anchorages as part of this remediation scheme. The site investigation in 1989 measured the corrosion propagation at three places along the anchor: at the wall, midway and at the connection with the anchor block.

Three types of anchor corrosion appear to be relevant from these measurements: generalized corrosion, corrosion pitting and bimetallic corrosion. The statistical models for these corrosion types are addressed below. During the time series samples all three corrosion processes are computed. It is assumed that the force in the anchor is equally large along the length of the anchor and is not influenced by the shape and development of the corrosion area. The smallest anchor cross section at each moment in time then represents the smallest capacity and is the weakest link.

Generalised corrosion occurs uniformly over the tie rods, with the highest rates close to the wall. Excitation of this type of corrosion is the exposure to oxygen and moisture, subject to spatial variations and time variability. It is assumed that the moisture in the soil is influenced by seasonal variations and daily
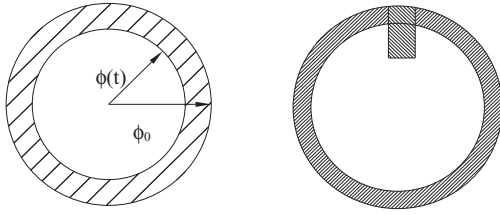
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Figure 6. Left: generalised corrosion uniformly affecting the anchor cross section. Right: a combination of a uniformly by corrosion affected anchor cross section and a corrosion pit.

fluctuations. Ancillary features are e.g. the type and properties of metal, the presence of corrosion acceleration factors and the presence of a protective layer around the anchor. The hierarchical process model is given by:

$$\varphi(t) = \varphi_0 - \left(m_{g;1} + m_{g;2} + m_{g;3}\right)c_g t \qquad (8)$$

$$A(t) = \pi \cdot \left(\varphi(t)\right)^2 \qquad (9)$$

in which $\varphi(t)$ is the radius of the anchor cross section in time, see figure 6, φ_0 is the anchor radius at the outset of the corrosion process, $m_{g;1}$ is the constant model uncertainty on the uniform corrosion rate. $m_{g;2}$ is the seasonal model uncertainty on the uniform corrosion rate and $m_{g;3}$ covers the daily environmental fluctuations.

Localised corrosion/corrosion pitting occurs at most of the observed locations. It is assumed that the main excitation feature of this type of corrosion is bacterial attack. This type of corrosion is very local and the influence of environmental exposure is negligible. Ancillary features are e.g. the presence of a protective layer around the anchors, the type and properties of the metal or accelerating circumstances for bacterial activity. The hierarchical process model is based on the assumption that, in addition to the previously described uniform corrosion, corrosion pits arrive according to a Poisson process, see figure 6. Upon arrival of the corrosion pit, the corrosion depth $d_p(t)$ increases linearly with a random rate c_p and a deterministic time t:

$$d_p(t) = c_p \cdot t \qquad (10)$$

The width of the corrosion pit is assumed to grow with approximately half the corrosion depth: $\frac{1}{2}d(t)$.

Accelerated *bimetallic corrosion* occurs midway of the anchorages due to the presence of turnbuckles. The excitation features of this type of corrosion are the presence of moisture and oxygen as well as two different types of metal. The location of these turnbuckles is midway of the anchors where the environmental influence is considered to be constant. Ancillary features are e.g. the area of the anchor, the types and properties of the metal or the presence of a protective layer around the anchor. The hierarchical process model for turnbuckle corrosion c_{tb} is defined as follows as a function of time t:

$$\varphi(t) = \varphi_0 - \left(m_{tb;1} + m_{tb;2}\right) \cdot c_{tb} t \qquad (11)$$

$m_{tb;1}$ is a constant model uncertainty and $m_{tb;2}$ is a model uncertainty covering the seasonality in the moisture and oxygen in the ground.

The gamma process model covers all uncertainty types with one overall average corrosion rate μ and a standard deviation σ, forming a stochastic process according to equation (2.6). The parametric process model covers all uncertainty types with one overall random corrosion rate and increases with a deterministic time t.

The corrosion rates for uniform corrosion and turnbuckle corrosion are derived from site measurements described in Southern Water (1989). The corrosion and arrival rate of the pitting corrosion are based on assumptions. The top graph in figure 7 contains the time series samples for the hierarchical process model and the parametric process model. Figure 8 shows the expected values and standard deviations of the anchor corrosion statistical models. Two time series samples of the hierarchical process model show the influence of corrosion pitting. In the instances that corrosion pitting occurs, according to these simulations, the life of the anchor is nearly halved. The parametric process model on average overestimates the anchor strength in long term predictions and underestimates the strength in short term predictions with respect to the hierarchical process model. The time series samples of the gamma process model in the bottom graph of figure 7 are clearly a stochastic process. However, comparable to the parametric process model, the gamma process model does not consider the arrival of corrosion pitting. Corrosion pitting nearly halves the lifetime of the anchor according to this model.

4.5 Toe level changes

The toe level in front of the anchored sheet pile wall is subject to e.g. morphological changes, the local wave climate, naval activity or dredging activities. The toe level occurs in all the relevant failure mechanisms of the anchored sheet pile wall, determining the amount

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Figure 9. Sheet pile fragility given low water level for anchor breaking and rotational failure where $t = 0$ is present.
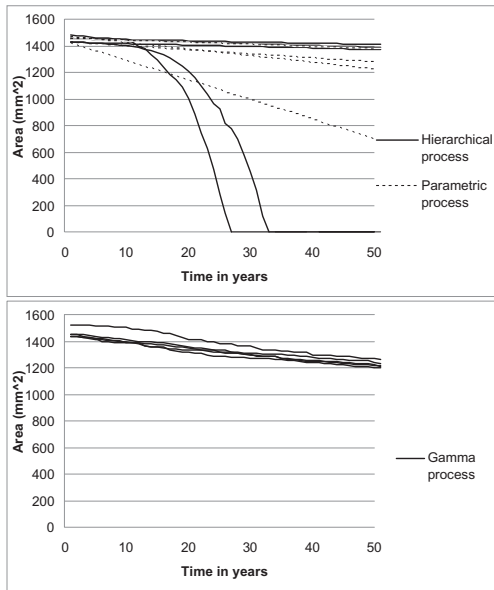
Figure 7. Top: Time series samples for the hierarchical process model and the parametric process model. Bottom: Time series samples for the gamma process model.
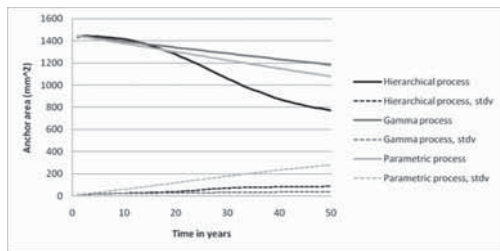

Figure 8. Expected values and standard deviations for the statistical models of anchor corrosion.

of passive ground pressure on the riverside. The river bathymetry has been monitored regularly over the past century. Comparisons between river bed levels in the 1910s and the 1990s have been made, pointing out that along the Greenhithe frontage (figure 3) the toe level has increased with 1.5 to 2 meter over the last century.

There are several excitation features for the toe level changes: morphological accretion, under influence of e.g. the river discharge or tidal currents, local wave climate, dredging activities or naval activity. Ancillary features are for example the river bathymetry or the shape of the frontage alignment.

It is not possible to model all these influences. It is unclear when dredging activities will take place in
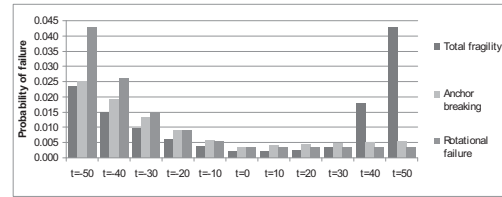
the future, and most naval activities have ceased. The change in the sheet pile toe level is modelled as an increasing accretion process, regularly reduced by the local wave climate:

$$dh_t = dh_a - dh_w \tag{12}$$

in which dh_a is the accretion due to morphological changes, modelled with a Brownian Motion. dh_w is the lognormally distributed change in toe level due to the erosion caused by the local wave climate. The arrival of the storms associated with the wave climate is assumed to be Poisson distributed. The time series samples therefore allow a positive as well as a negative development of the toe level changes.

The gamma process model applies equation (2.6) with an average accretion rate and a standard deviation. The parametric process model applied on overall random accretion rate r_a as a function of deterministic time t to obtain the total amount of accretion $h_{acc}(t)$:

$$h_{acc}(t) = r_a t \tag{13}$$

Figure 9 contains the time series samples of the statistical models for toe level changes. The hierarchical process model allows time series samples with accretion as well as erosion. The parametric process model only considers the accretion rate at the start as a random variable. Once the accretion rate has been sampled it remains constant throughout the time series. The gamma process model in the bottom plot is clearly a stochastic process with increments varying for different time intervals. However, a gamma process model only allows positive increments. Both the parametric process and the gamma process model do not reflect the possibility of a negative accretion rate.

4.6 Anchored sheet pile wall: time-dependent fragility and reliability

With the time series samples of the time-dependent processes discussed in sections 4.3 to 4.5 the

anchored sheet pile wall fragility and time-dependent probability of failure is computed according to section 2.2. The results are displayed in figure 10 and 11. Fragility is the probability of flood defence failure given a deterministic water level. The water levels in fragility represent the high water level during one storm event, rather than the sequence of rising and subsiding water levels during a storm event. The probability of failure of the anchored sheet pile wall is at its highest for the lowest water level during a storm event. The fragility is therefore represented by one probability of failure. In figure 10 the time-dependent behaviour of the fragility is given. Figure 4 shows how the failure mechanisms are combined in a fault tree. The increasing river bed levels are simulated over the first fifty years until $t = 0$. It is unlikely that the river bed levels will rise another two meters in the next 50 years, depending on e.g. dredging activities. Anchor corrosion and sheet pile corrosion are simulated over the whole period. The probability of rotational failure and anchor breaking over the first 50 years decreases while the rising toe levels and hence increasing horizontal pressures stabilise the anchored sheet pile wall. After present time the probability of anchor breaking increases due to anchor corrosion while the probability of rotational failure remains constant. The influence of anchor corrosion is not that large as the sheet pile wall has been stabilised due to the rising river bed levels. The total fragility increases later due to the increasing probability of sheet pile holing; this probability is not the probability of the sheet pile wall failing (section 4.3). The probability of sheet pile breaking is negligible. Figure 11 contains the time-dependent probability of failure of the anchored sheet pile wall, with random local water levels. The probability of anchor breaking first stabilises due to the rising river bed levels and subsequently increases. The influence on the total probability of failure is flattened out by the low probability of rotational failure (figure 4). The probability of sheet pile holing increases steeply (see section 4.3).
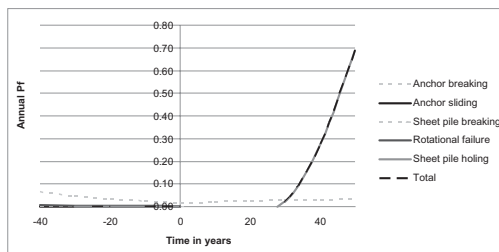

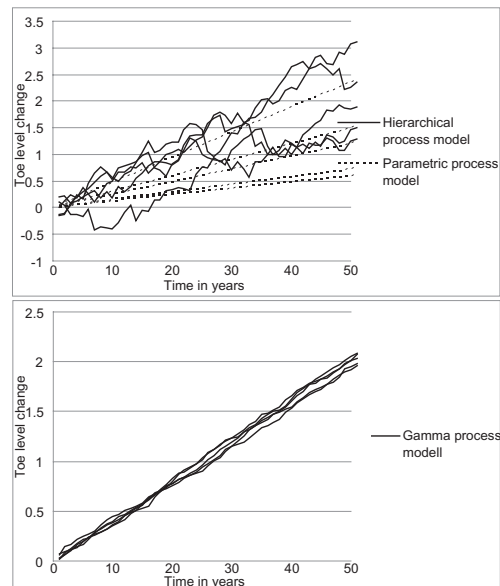Figure 10. Time-dependent probability of failure of the anchored sheet pile wall.

Figure 11. Top: Time series samples of the hierarchical and parametric process model of the toe level changes. Bottom: Time series samples of the gamma process model.

5 CONCLUSIONS

A variety of stochastic process models have been explored to demonstrate deterioration processes and associated uncertainties for examples of flood defence systems. Unlike the commonplace assumption (in the absence of further information) of steady deterioration of flood defences for the remainder of their residual life, the models developed here demonstrated how deterioration is often unsteady and non-linear. The time series samples show that the hierarchical process models better capture the character of the time-dependent process than aggregate representations, i.e. a parametric process or gamma process. The hierarchical process enables the decomposition of the time-dependent process into different stochastic process contributions, as opposed to an aggregate model. The modelling methodology allows the recognition of different flood defence characteristics contributing to the variations in time and space. The excitation features often have a large influence on the time series samples. The excitation formed by the presence of moisture and oxygen for sheet pile corrosion is captured in the corrosion rate. The arrival of corrosion pitting can halve the lifetime of the anchor in case of anchor corrosion. Overall accretion over a long period of time can show significant variations in between due to erosion by the local wave climate or other influences. The availability of quantitative

observations about the time-dependent process is often limited. The predictions of the time-dependent processes can therefore often not be quantitatively tested. The modelling methodology provides a way to check whether the predictions appropriately reflect the qualitative aspects of the time-dependent behaviour which have been identified. The necessity to further quantitatively test the time series samples depends on the complexity of the time-dependent process and the sensitivity of the flood defence reliability and life cycle costing model. The time-dependent fragility and probability of failure calculations demonstrate how time-dependent processes can be implemented in the reliability model.

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