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Is steady sediment transport possible in graded sediments?

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IS STEADY SEDIMENT TRANSPORT POSSIBLE IN GRADED SEDIMENTS?

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Abstract

Measurements of bed load sediment in rivers frequently show pulsing. Such pulses have also been frequently noticed in laboratory experiments. This paper presents a one-layer model where the Exner equation was written for each sediment size to represent continuity and assumptions were made about hiding functions and their impact on the predictions. Results were compared with experimental tests which simulated very well the general behavior of the transport rate at the downstream end of the flume. However, by its nature, it cannot reproduce the unsteady behavior of bed-load pulses. In this paper it is suggested that bed load pulses arise from non-linear effects generated by the interaction of the different sizes forming the graded bed. Practical implications of the non-linearity are related to the measurement of bed-load in channels in which the duration of the sampling has to exceed the time-scale of the non-linear pulses and the bed composition may vary in each pulse.

1. Introduction

Measurements of bed load sediment transport in rivers frequently show pulsing of sediment transport rates over a range of spatial scales (Reid & Frostick 1987, Whiting et al 1988). This behaviour has been explained in the past as the product of different mechanisms such as the presence of bedforms, bar migration, the sampling device used or random inputs of transportable material into the channel upstream, (e.g. as a result of bank collapse). Unsteady behaviour is also observed in the more controlled context of laboratory experiments with graded sediments (Bettess 1983, Gomez 1983, Kunhle & Southard 1988, Veditti et al 2008) (Fig. 1).

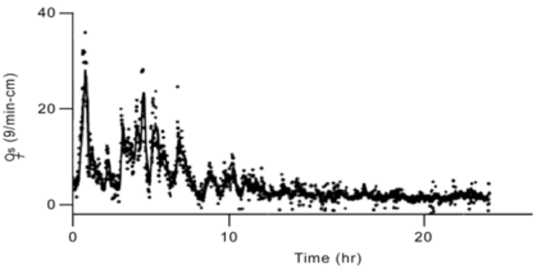

Figure 1. Bed load pulses in a degradational experiment (from Venditti et al 2008)

Degradational laboratory experiments with no-sediment feed show the same pulsing behaviour which suggests that pulses may be generated by other mechanisms (Cudden & Hoey 2003). The behaviour of the transport rate at the end of the flume in degradational experiments (Proffitt 1980, Lamberti & Paris 1992, Tait et al 1992) shows the following general behaviour:

- an initial short phase where a great amount of material is removed at a nearly steady transport rate; this phase is present for high values of shear stress
- a second phase of gradually reducing sediment transport as sediment is transported out of the flume and the surface of the sediment bed becomes armoured

Examination of the experimental results shows that during the second phase the reduction in the sediment transport rate is not monotonic in time but shows temporal fluctuations. This appears to be a feature of all such experiments carried out by a range of experimenters, despite using different flumes, different sediments and different flow conditions. The magnitude of the fluctuations does not appear to be related to the care with which the experiments have been carried out of the experimental techniques used. The theoretic modelling of such experiments provides a clue to the causes of the unsteady behavior.

Sediment transport rates in non-uniform sediments have been extensively studied using the continuity equation, Exner equation, the concept of mixing layers, where it was considered that changes in composition of bed material occurred, and the use of hiding functions to modify the threshold conditions of motion of graded sediments (Fig. 2).

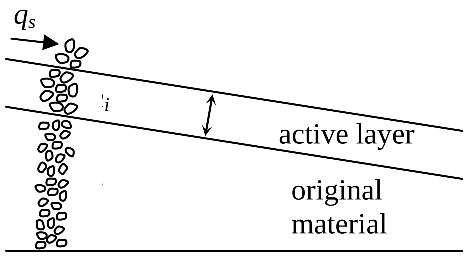

Figure 2. Sketch of the one-layer model

Based on these theories, this paper describes a wave model to simulate changes through time of the sediment transport rate. The model shows that bed sediment composition and transport during the second degradational phase of experiments can be described using non-linear wave theory. Though simple, the model simulates the development of self-armoured beds when compared with degradational experiments (Frangipane 1994).

The use of kinematic waves to describe aspects of sediment transport is not novel. Langbein and Leopold (1968) used kinematic wave theory to explain a number of aspects of sediment transport with uniform sediments. Lisle et al (2001) studied the translation and dispersion of bed material waves. The novelty of the present paper is to investigate the implications of the kinematic wave theory to describe the propagation of information relating to different sediment sizes within a graded sediment.

2. Review of kinematic wave theory

We now briefly review the features of kinematic wave theory that are relevant to the following discussion. For a fuller account of kinematic wave theory the reader is directed to any standard account of non-linear wave theory, for example, Whitham (1974).

Kinematic waves occur where there is a continuous distribution of either material or some state of the medium in which the flux $q(x,t)$ per unit time depends upon some function Q of the density $\rho(x,t)$ per unit length, that is, $q = Q(\rho)$.

The wave equation describing the propagation of ρ is then

$$\rho_t + c(\rho)\rho_x = 0,$$

where

$$c(\rho) = Q'(\rho).$$

The mathematical system can be interpreted as describing a system in which information is propagated in the downstream direction in such a way that different values of ρ propagate with velocity $c(\rho)$. The solution at time t can be constructed by moving each point on the initial curve $\rho = f(x)$ a distance $c(\rho)t$ to the right, the distance moved being different for different values of ρ. The dependence of c on ρ introduces non-linear behaviour and results in a distortion of the initial wave as it propagates. When $c'(\rho) > 0$, higher values of ρ propagate faster than lower ones.

Kinematic wave theory can be applied to rivers in which the 'density' ρ becomes the cross-sectional area of the flow. In this case the $c'(A) > 0$. The consequence is that during a flood the higher part of the flood wave propagates faster than the lower parts. The result is that the flood wave deforms and steepens as it propagates downstream. Similar theory predicts the development of roll waves in steep channels.

3. Wave model

A one-layer model to simulate the development of static armour during a degradational experiment first developed in Frangipane (1994) is presented. The Exner equation was written to represent sediment continuity for each sediment size. With the assumptions of constant bed porosity (λ_0) and constant width of the control volume, the continuity equation becomes:

$$\frac{\partial(am_i)}{\partial t} + p_i \frac{\partial(z-a)}{\partial t} + \frac{1}{1-\lambda_0}\frac{\partial(t_i q_s)}{\partial x} = 0\,, \quad (1)$$

where p_i, m_i and t_i are the fractions of the i-th sediment class with average representative dimension D_i, in the original material, in the exchange layer and in motion, respectively (Fig. 2); z is the bed elevation, a the thickness of the mixing layer and q_s the volume rate of sediment transport for unit width.

To simplify the equation some assumptions are made:

- The sediment transport rate of each sediment class, $t_i q_s$, is considered related to the proportion of that sediment size present on the bed,

$$t_i q_i = m_i q_{si} \quad (2)$$

and the volume rate of transport for the i-th sediment class, q_{si}, varies in space and time
- A constant thickness of the mixing layer is imposed, $a=a_0$
- For small amounts of bed degradation the bed level can be assumed unchanged hence, the second term of the equation is negligible in comparison with the third one.

With these assumptions equation (1) takes the form:

$$\frac{\partial(m_i)}{\partial t} + c\frac{\partial(m_i)}{\partial x} = 0 \quad (3)$$

Equation (3) describes a kinematic wave. Each value of m_i propagates at a speed, c, that depends on the bed load transport rate associated with that sediment size, and is equal to

$$c = \frac{q_{si}}{a_0(1-\lambda_0)} \quad (4)$$

The model simulates the movement of sediment down the flume as a series of waves progressing downstream.

The model is used to simulate the general behavior of the sediment transport rate at the downstream end of a flume in experiments from Tait et al. (1992) and Tait (1992). Laboratory data was obtained from four experiments on static armour formation during which the change in time of bedload composition was recorded.

The fluctuations observed in the experiments (Fig. 3) show the typical non-linear wave behaviour where any initial variability on the bed would lead to the creation of sediment waves down the flume with different wave speeds. Cudden & Hoey 2003 suggested that bed load pulses may be generated by changes in the size and structure of the bed material.

Equation (3) is solved in Frangipane (1994) considering two sediment transport formula, Meyer-Peter & Muller and Ackers & White, and different hiding functions to modify the shear stress applied to each grain size. It is also considered that q_{si} is constant so the wave speed c is assumed to be constant in space and time. With the appropriate boundary conditions equation (3) can be solved analytically.

The model results considering Meyer-Peter & Muller simulate very well the general behaviour of the sediment transport rate in self-armouring experiments predicting a monotonic reduction in transport rates (Fig. 3). The model predicts the initially uniform transport rate followed after a period of time by a steadily reducing transport rate. The model indicates that this arises due to the different speeds with which information on the upstream boundary condition propagates down the flume.

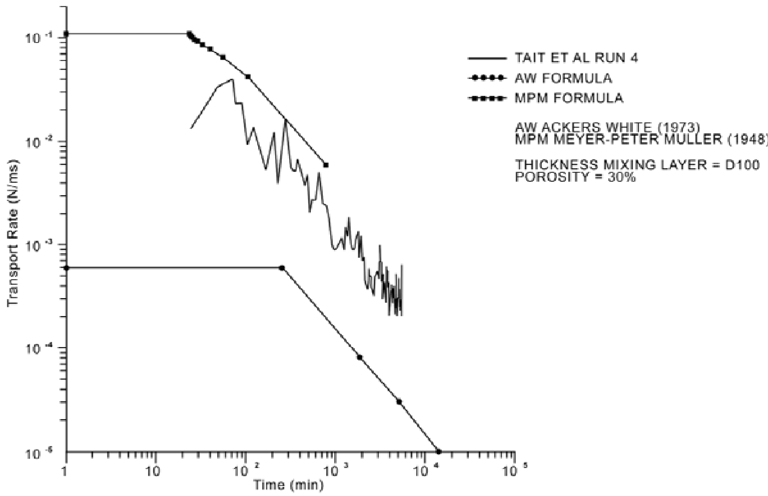

Figure 3. Total sediment transport against time

In reality the wave speed depends upon local values of the variables and so varies in both space and time. In this situation equation (3) is a non-linear kinematic wave equation. This means that as the waves progress down the flume they change in shape, as described above.

When the kinematic wave theory is applied to the case sediment movement, it can be shown that the gradient of the wave speed associated with individual sediment sizes is greater than zero. This implies that if there are initial perturbations in the initial bed grading that these are modified and exaggerated as the kinematic wave propagates down the flume. Any perturbations in the initial conditions will deform as the waves pass down the flume. Due to the nature of the expression for the wave speed, ‘shocks’ are likely to develop in the bed composition as the waves progress down the flume.

Changes in the proportion of any one sediment size present in the bed results in changes in the sediment transport rate. Thus the observed fluctuations in sediment transport rate at the downstream end of a flume reflect the propagation and deformation of ‘waves’ associated with different sediment sizes.

It is suggested then, that the bed pulses arise from the non-linear effects generated by the interactions of the different sediment sizes forming the graded bed.

In the Frangipane model solution the movement of individual sediment sizes is described by separate equations, that is, it is assumed that the different sediment sizes do not interact. At a particular location, however, the proportion of the bed that is formed by one sediment size cannot change independently of the other sediment sizes. This emphasizes the need to take account of interactions between different sediment sizes. This implies that in reality equation (2) should be of the form

$$t_i q_i = m_i q_{si} + f(m_j, q_{sj}) \qquad (5)$$

for j not equal to i for some function f.

It should be pointed out that this phenomenon will only occur where there are a range of sediment sizes present on the bed. In uniform sediments both the wavespeed and the value of m_i are constant. Thus one would not expect this type of behavior to occur in uniform sediments. In such experiments, however, it is possible to have time fluctuating sediment transport rates resulting from the migration of bed forms.

Observations made in flumes suggest that in the later stages of degradation experiments the magnitude of the bed pulses appears to reduce. The model presented would suggest that during the experiment the range of sediment sizes moving on the bed should reduce as the finer sediments pass through the system and the bed coarsens. As this process continues the composition of the sediment in motion should gradually become increasingly more uniform. In this case one would expect that a phenomenon associated with the movement of graded sediments would gradually reduce in magnitude.

If the movement of graded sediments down channels is non-linear and inherently unstable then this would imply that the theoretic construct of steady sediment transport rarely occurs in natural systems.

4. Implications

If the transport of graded sediments is inherently subject to pulsing, even in steady flow conditions then this has a number of implications. In the context of laboratory experiments, measurements of sediment transport have to be made over a period which exceeds the time-scale of the non-linear pulses of bed load. Kuhnle & Southard (1988) demonstrate the problems of taking transport sample for too short based on experimental results. They predict the probability of obtaining accurate results for a sample taken over a given time interval in the system. They found sample lengths necessary as a function of the most prevalent periods of fluctuation.

Another practical implication to consider is that bed sediment sampling should take into account the possibility of variations in bed composition associated with such bed load pulses.

In natural rivers in which variations in the discharge hydrograph occur over a similar time period to that of the bed load pulses then it may not be possible to average out the impact of the sediment transport pulses by measuring over long time periods. This means that it may be impossible to use measurements to establish a sediment rating curve and that the concept of a sediment rating curve may be meaningless in this case.

The development of time-dependent sediment transport at the downstream end of flumes has implications for the use of re-circulating sediment flumes in which sediment at the downstream end of the flume is returned to the upstream end of the flume. If the sediment transport rate at the downstream end of the flume is unsteady then the sediment input at the upstream end will also be unsteady. This may have disadvantages if a steady input is required for the experiments. It may result, however, in the flume behaving like a section of a much longer channel, which may more closely reflect a natural river.

5. Conclusions

Bed load pulses are observed in both the field and the laboratory. Such pulses have been particularly noticed in degradational experiments. Simple, linear models of the development of static armour based on linear wave theory reproduce the observed general sediment behavior very successfully but, by their nature, they cannot reproduce the bed-load pulses. It is suggested that these bed load pulses arise from the non-linear effects generated by the interactions of sediment sizes forming the graded bed. This would suggest that in some situations, the movement of graded sediments down channels may be inherently unstable and hence always unsteady.

This implies that the bed load pulses observed at the downstream end of flumes in experiments with graded sediments are not related to deficiencies in the experimental technique. Such pulses cannot be reduced in magnitude by more careful control of the initial conditions and the flow conditions. They are intrinsic to the movement of graded sediments.

The presence of bed load pulses has implications for the measurement of sediment transport in both laboratories and the field and the development and application of sediment rating curves.

References

Reid , I. & Frostik, L.E. 1987. Towards a better understanding of bed load transport. In F.G. Ethridge, R.M.Flores & M.D.Harvey (ed), *Recent Development in Fluvial Sedimentology*. Society of Economic Paleontologists and Mineralogists. Special Publication (39): 13-10.

Whiting, P.J., Dietrich, W.E., Leopold, L.B. Drake, T.G. & Shreve, R.L (1988) Bedload sheets in heterogeneous sediment. *Geology*. 16(2): 105-108.

Cudden, J.R. & Hoey, T.B. 2003. The causes of bedload pulses in a gravel channel: the implications of bed-load grain-size distributions. *Earth Surface processes and landforms* 28:1411-1428.

Bettess, R. 1983. Degradation of river beds and associated changes in the composition of the sediments. Mechanics of Sediments Transport: 237-242. Rotterdam:Balkema

Gomez, B. 1983. Temporal variations in bedload transport rates: the effect of progressive bed armouring. *Earth surface processes and landforms*. 8:41-54

Kunhle, R.A. & Southard, J.B. 1988. Bed load transport fluctuations in a gravel bed laboratory channel. *Water Resources Research* 24(2):247-260

Veditti, J.G., nelson, P.A. & Dietrich, W.E. 2008. The domain of bedload sheets. *Marine and River Dune Dynamics*. Leeds, UK 1-3 April

Proffitt, G.T. 1980. *Selective transport and armouring of non-uniform alluvial sediments*. Rep. No.80/22 Dept. of Civ. Eng., Univ. of Canterbury. Cristchurch, New Zeland

Lamberti, A. & Paris, E. 1992. Analysis of armouring process trough laboratory experiments. In P. Billi, R.D. Hey, C.R. Thorne & P. Tacconi (eds) Dynamics of gravel-bed rivers: 227-250. John Wiley & Sons

Tait, S.J., Willetts, B.B. & Maizels, J.K. 1992. Laboratory observations of bed armouring and changes in bed load composition. In P. Billi, R.D. Hey, C.R. Thorne & P. Tacconi (eds) Dynamics of gravel-bed rivers: 205-225. John Wiley & Sons

Frangipane, A. 1994. *A one-layer model to predict development in time of static armour*. HR Wallingford Report SR382

Tait, S.J. 1992. Personal communication

Whitham, G.B. 1974. *Linear and nonlinear waves*. Wiley-Interscience publication

Langhein, W.B. & Leopold, L.B. 1968. River channel bars and dunes. Theory of kinematic waves. US Geological Survey Professional Paper 422-L

Lisle, T.E., Cui, Y., Parker, G., Pizzuto, J.E., Docc, A.M. 2001. *Earth surface processes and landforms*. 26:1409-1420

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