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HRPP 441

Idealised model for flow towards a dam breach

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Reproduced from a paper presented at:
1st European IAHR Congress
Edinburgh
4 – 6 May 2010

IDEALISED MODEL FOR FLOW TOWARDS A DAM BREACH

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ABSTRACT

Most models of the flow over an embankment or through a breach treat the flow as that over a broad-crested weir. This may be appropriate when the flow width is large compared with the depth, and the streamlines of the flow in the reservoir towards the embankment or breach nearly normal to the crest. However, evidence from laboratory and large scale field tests (such as in the EC IMPACT project) show strong curvature of the flow towards a narrow breach with approximately semi-elliptical or semi-circular shaped water profiles in plan for the reservoir upstream of the breach. This paper presents a new analytic model for the two dimensional flow towards a slot in an otherwise solid flow boundary. The surface profiles predicted from the model are semi-elliptical in plan, becoming semi-circular in the limit of an infinitesimally thin slot. Analysis of this latter idealized case shows that critical flow occurs at a radial distance inside the reservoir of r_c given by:

$$r_c^2 = \frac{27Q^2}{8g\pi^2 E^3}$$

where Q is the discharge through the breach and E is the total energy level in the reservoir measured above the base of the breach. Non-dimensionalising the horizontal distance along the streamlines leads to a new universal solution for the water level profile for this idealised case. This solution provides insight into both the erosion mechanisms that control breach growth and the validity of assuming flow is similar to a broad-crested weir.

INTRODUCTION

The origin of this paper comes from discussions at the final meeting of the EC research project IMPACT in Zaragoza (see http://www.impact-project.net/). The IMPACT project included large scale field tests in Norway where temporary dams were constructed and then failed in controlled circumstances (Morris *et al*, 2007). The photographs and video taken during the failure showed a sequence of flow in the impoundment conditions towards breach. Figure 1 shows three characteristic types of flow which we shall call here: weir flow, converging flow, breach flow; these photographs and more appear in the FLOODsite project analysis of the IMPACT project data (Morris, 2009).

The purpose of the analysis in this paper is to illustrate the difference between flow over a broad-crested weir as shown in Figure 1(a) and the two dimensional converging flow towards an opening in an otherwise solid flow boundary, typified by Figure 1(b), since water surface profiles evident in the photographs show the character of the flow changes as the breach develops. Note in Figure 1(b) the steeply curved water surface in the vertical plane as the flow approaches the breach and the approximately circular or elliptical shape in plan of contours of equal surface level as the flow converges towards the breach. Many models of breach formation in an embankment assume that the flow at the crest obeys a broad-crested weir equation, whereas the photographs above call this assumption into question. The analysis presented below concentrates on the two-dimensional converging flow and shows that the effective crest-length in this case exceeds the width of the breach assumed in the broad-crested weir approximation of many models.

Flow over a broad crested weir

For one-dimensional flow towards a broad-crested weir, in the upstream reservoir the energy level E and water surface level h_0 are equal; we take the vertical datum as the weir crest level and can set:

$$E = h_0 \qquad (1)$$

At the weir crest the flow is critical and we apply Bernoulli's equation using the average velocity U_c and local depth over the crest h_c:

$$E = h_c + \frac{1}{2g}U_c^2 \qquad (2)$$

$$U_c = [gh_c]^{1/2} \qquad (3)$$

which has the solution:

$$h_c = {}^2\!/_3\, h_0 \qquad (4)$$

The discharge Q for a weir of crest width B is then given by:

$$Q = (2/3)^{3/2} B\sqrt{g}\,h_0^{3/2} \qquad (5)$$

This leads to the idealised coefficient of discharge for the broad-crested weir as

$$C_d = \left({}^2\!/_3\right)^{3/2} = 0.5443 \qquad (6)$$

The development of these equations can be found in standard textbooks e.g. Section 2.7 of Kay and Nedderman (1974).

TWO-DIMENSIONAL FLOW IN PLAN

Stream function, velocity potential and complex potential

To analyse this case we assume that the flow is incompressible, inviscid and irrotational. These assumptions are the same as made in the theory underlying the flow over a broad-crested weir above. We also assume that the flow can be treated as steady (either the inflow equals the outflow to the reservoir or the reservoir area is very large compared with the magnitude of the outflow). Thus we have the continuity equation for the flow:

$$\nabla \cdot \mathbf{q} = -\frac{\partial h}{\partial t} \approx 0 \qquad (7)$$

Here **q** is the two-dimensional unit flow vector defined as the integral over depth of the 3-d velocity **u**

$$\mathbf{q} = \int_{z_b}^{h} \mathbf{u}\,dz \qquad (8)$$

(a) Weir flow (b) Converging flow (c) Breach flow

Figure 1 Three types of flow from the IMPACT project field tests

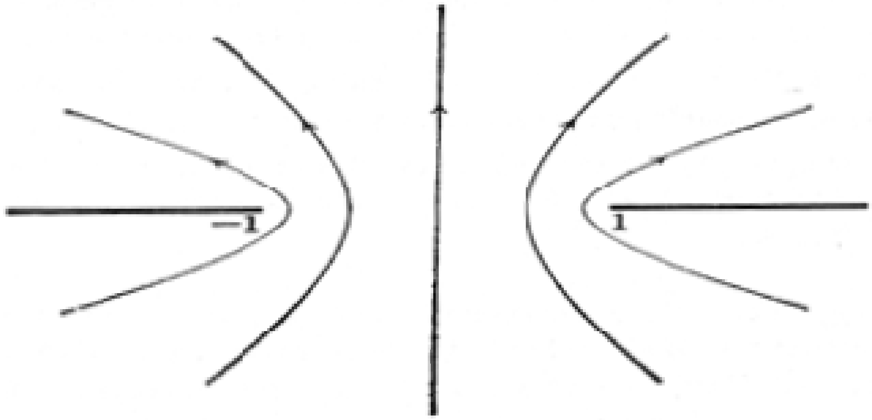

Figure 2 Streamlines for potential flow through a unit aperture (after Kreyszig, 1999)

This form of the continuity equation for steady flow allows the unit flow to be represented by a stream function Ψ as

$$\mathbf{q} = \nabla \times (0,0,\Psi) \qquad (9)$$

The assumption of irrotational flow leads to the representation of the unit flow vector by a velocity potential Φ thus

$$\mathbf{q} = \nabla\Phi \qquad (10)$$

The stream function and velocity potential are conjugate functions each satisfying Laplace's Equation

$$\begin{aligned} \nabla^2\Phi &= 0 \\ \nabla^2\Psi &= 0 \end{aligned} \qquad (11)$$

These can be combined to form a complex potential F

$$F = \Phi + i\Psi \qquad (12)$$

Standard texts on fluid dynamics and potential flow theory allow the complex potential (and thus the stream function and velocity potential) to be identified for a variety of boundary conditions (see for example Acheson, 1990, Rutherford, 1959). Kreyszig (1999) gives the solution for potential flow through a unit aperture (-1<x<1; y = 0) as:

$$F(z) = \cosh^{-1}(z) = \log_e\left(z + \sqrt{z^2 - 1}\right) \qquad (13)$$

in which z is now the complex variable in two spatial dimensions

$$z = x + iy \qquad (14)$$

The streamlines are hyperbolae with foci at ±1 and the equipotential lines are ellipses, also with foci at ±1.

We scale this solution to obtain the complex potential for flow through the general width aperture (-a<x<a) and introduce a scaling constant C to allow for an arbitrary total discharge thus:

$$F(z) = C\log_e\left(\frac{z}{a} + \sqrt{\left(\frac{z}{a}\right)^2 - 1}\right) \qquad (15)$$

It is interesting to note that this potential solution for the unit flow vector does not depend explicitly on the depth of the flow or on the bottom topography, thus it could be used (if the physical assumption of 2-D flow still applies) on the sloping upstream face of an embankment dam.

FAR-FIELD CASE – RADIAL FLOW

In the subsequent analysis we accept that the idealised model of purely radial flow will break down in real fluids at some finite distance from the origin. For large distances from the origin, i.e. |z| >> a, the complex potential of Equation (15) can be approximated by:

$$F(z) = C \log_e \left(\frac{2z}{a} \right) \tag{16}$$

Equation 16 is the complex potential for flow towards a sink or from a source, depending upon the sign of C, the streamlines are radii from the origin and the equipotential lines are concentric circles centred on the origin. This limiting case therefore facilitates the identification of the constant C without recourse to the use of elliptic integrals to calculate the circumference of the semi-elliptical equipotential lines of Equation (15). We take Ψ=0 on one side of the small aperture (θ = 0) and Ψ= -Q/π on the other side (θ = π), where Q is the discharge through the aperture. We have in polar coordinates (r, θ)

$$q_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta} = -\frac{Q}{r\pi} \tag{17}$$

where the negative sign denotes flow radially inwards. We now let D(r) denote the local depth and we have for the depth average velocity:

$$U(r) = \frac{q_r}{D(r)} = -\frac{Q}{r\pi D(r)} \tag{18}$$

RADIAL FLOW OVER A HORIZONTAL BED

This formula for the depth average velocity can be used in Bernoulli's equation to calculate the water surface profile as flow accelerates towards the aperture. We now make the assumption that the bed resistance can be ignored and the geometric simplification that the bed is horizontal. In this case the depth D becomes the water surface level, on taking the constant bed level as the vertical datum. Bernoulli's equation on a streamline (choosing the water surface) is

$$E = h(r) + \frac{U(r)^2}{2g} = Const. \tag{19}$$

On substituting for U(r) we obtain

$$E = h(r) + \frac{1}{2g}\left(\frac{Q}{r\pi h(r)} \right)^2 \tag{20}$$

Thus the depth, h(r), satisfies the following cubic equation:

$$h^3 - Eh^2 + \frac{1}{2g}\left(\frac{Q}{r\pi} \right)^2 = 0 \tag{21}$$

This equation is nearly the same as that for rectilinear flow, the difference (apart from the scaling constant) is the radial distance in the zero order term. For very large values of r this equation has the solutions approximately of h = E and a double root h = 0. The first of these is physically the correct root for imperceptible flow velocity far away from the embankment.

In view of the 3-D nature of the problem, we may consider the total energy flux crossing a semi-cylindrical surface centred on the sink at the origin. The total energy flux, T, across any semi-cylindrical surface is

$$T = QE \tag{22}$$

We define the flow as critical when T is a minimum, i.e. when E is a minimum, or

$$\frac{dE}{dh} = 1 - \frac{1}{g}\left(\frac{Q}{r\pi} \right)^2 h^{-3} = 0 \tag{23}$$

This may be inverted to give the critical radius, r_c as

$$r_c^2 = \frac{Q/\pi}{\left(gh_c^3\right)^{1/2}} \tag{24}$$

At this point we have:

$$h_c = {}^{2E}\!/_{3} \tag{25}$$

$$r_c^2 = \frac{27Q^2}{8g\pi^2 E^3} \tag{26}$$

Thus we note that the location of the control point occurs at some radial distance r_c upstream of the embankment and is not controlled in this simplified case by the shape or level of the breach. Equation (26) may be rearranged as

$$Q = (2/3)^{3/2} r_c \pi \sqrt{g} h_0^{3/2} \tag{27}$$

This is of identical form to the broad crested weir flow Equation (5) above with the weir crest width, B, being replaced by the circumference, πr_c, of the semicircle at the critical radius.

We now manipulate the Equation (20) to examine the behaviour of h(r) as the flow approaches the sink at the origin. by rescaling the distance according to this value of r_c, that is we set:

$$X = \frac{r}{r_c} \tag{28}$$

$$h^3 - Eh^2 + \frac{4E^3}{27X^2} = 0 \tag{29}$$

Differentiating this equation with respect to X we obtain the equation for the surface slope:

$$\left(3h^2 - 2Eh\right)\frac{dh}{dX} = \frac{8E^3}{27X^3} \tag{30}$$

When h = 2E/3, i.e. for critical flow we deduce that the surface slope becomes infinite since E and X are both positive and finite, unlike the case for rectilinear flow where the transition occurs with finite surface slope.

On an examination of the roots of the cubic equation for h, we find that upstream of the critical point there are three real roots for h for any given value X. The two which give positive values of depth are physically meaningful and are the sub and supercritical depth. The roots to the cubic can be easily established in inverse form by deriving an expression for X(h) – that is the non-dimensionalised, radial distance at which a given value of water level h is achieved. We have:

$$X = \left[\frac{4E^3}{27\left(h^3 - Eh^2\right)}\right]^{1/2} \tag{31}$$

$$X = \frac{2}{3\sqrt{3}}\left(\frac{1}{Y}\right)\frac{1}{(1-Y)^{1/2}} \tag{32}$$

Here Y is the non-dimensionalised water level, Y = h/E and this equation admits solutions for all 0<Y<1, see the Figure 3 below. Alternatively Cardan's solution for a general cubic equation can be employed to give the value of h at any desired value of X (See for example Chapter XXI of Ferrar (1948)).]

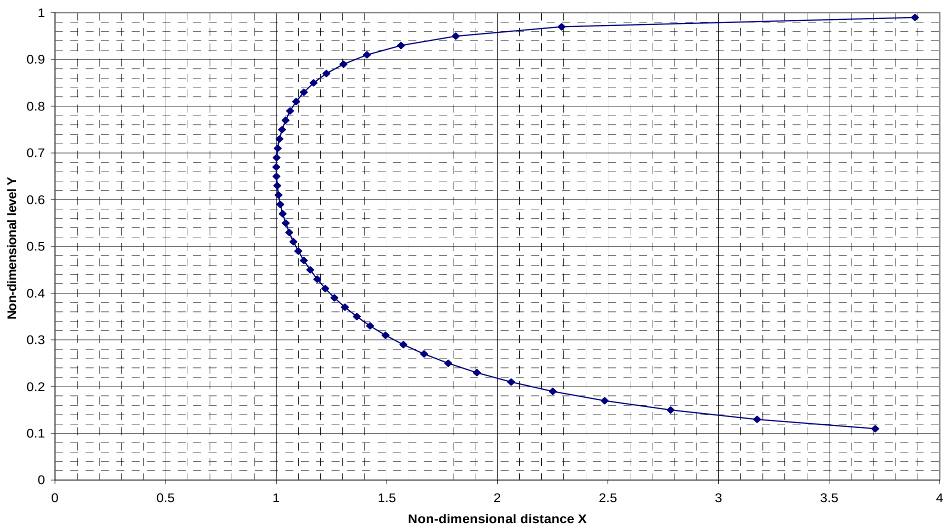

Figure 3 Idealised water surface profile for radial flow towards a breach

DISCUSSION

Limitations of the analytic model of radial flow

As X tends to the value 1 from "above" ($X>1$), the two sequent depths become progressively closer in value until at the point $X=1$ they are a double root of the cubic equation. At $X = 1$ there is a "catastrophe" for the water level. For $X<1$ the only root is the physically meaningless negative depth. This is in contrast to the rectilinear case of flow over a broad crested weir when there is a smooth transition from subcritical conditions upstream to critical conditions over the crest and potentially supercritical conditions downstream. We conclude that at $X=1$ the model of assuming shallow water flow (negligible vertical acceleration) has broken down and that a 3-D analysis is needed. Moreover, for an infinitesimally small aperture width the flow velocity would increase without limit for a fixed discharge rate close to the aperture. Hence the radial flow approximation should only be used for the far-field, with certainly a limit X_R, which obeys the restriction $X > X_R > 1$.

Finite sized aperture

To apply the Bernoulli analysis for a finite sized aperture we need to consider the velocity along the streamline hyperbolae and this will depend (via continuity) on the differential arc length on the orthogonal ellipses. It is well known that the arc length on the circumference of an ellipse involves an incomplete elliptic integral of the second kind, for which there is no closed form analytic relationship, and so progress on this must be made by numerical means; this paper does not pursue this approach further.

Comparison of qualitative features of the flow

We note the photographs of flow towards a dam breach, such as Figure 1(b) above, indicate that water level is constant on circular or elliptical arc upstream, with very steep water surface slopes developed at the "critical" point some distance upstream of the dam crest as suggested in this simplified analysis in Equation (30). This converging flow case operates once initial overtopping has led to an erosion of the embankment crest level, but ceases before the final breach dimensions are achieved and the reservoir in substantially emptied, see Figure 1(c).

Limitation of 2-D steady flow

The stream function of Equation (9) only is defined for the case of 2-D steady flow. Thus the analysis will only be an approximation to the physical case for a finite sized reservoir where the inflow does not match the outflow through the breach. Two physically important time scales can be deduced – one is the ratio of the reservoir volume to the net outflow rate and the other is for the time taken for the water to exit the breach, say the ratio of r_c to U_c of Equations (18) and (26). The first of these timescales dominates and so the analysis is likely to be typical of the situation during the initial phase of the enlargement of a breach. The reasonable correspondence with observation of the inferred properties of the flow from this solution also gives encouragement that the analysis has captured the essence of the physics of the converging flow case (Figure 1b).

Implications for the rapidly varied, converging flow case

Comparison of the two discharge Equations (5) and (27), shows that in the case where the head over the crest is small, the broad crested weir flow should be used as the critical radius becomes small compared with the crest width. However as the embankment crest erodes and the discharge increases, the critical radius increases and the control point moves upstream into the reservoir and the flow is no longer controlled by the width of the breach in the crest. This is in accord with analysis of the videos taken during the IMPACT field tests (Morris, 2009). The implication under these circumstances is that breach discharge calculations based upon broad crested weir flow through the breach width may underestimate the discharge.

Figure 4 Evidence of flow separation and vorticity generation from field test

A consequence of the control point moving upstream is that at this point there is strong curvature of the water surface in the vertical plane as well as curvature of the streamlines in plan. These conditions imply there is acceleration of the mean flow velocity both in plan and in the vertical plane, leading to non-hydrostatic pressure distribution in the vertical and potentially to intense generation of vorticity and to flow separation. This is illustrated in the images from field tests in Figure 4, taken from the downstream side of the embankment and looking overhead.

The strong generation of vorticity implied by the curvature of the streamlines leads to concentrated erosion at the margins of the breach. Some of the images from the IMPACT tests show intense concentrations of sediment coming in a helical fashion from the corners of the breach implying possible hot-spots of erosion and undercutting of the embankment sides. Any undercutting of the embankment sides will give rise to mass failure of blocks of embankment material as observed in the IMPACT field tests.

It is evident that the flow conditions through the breach both from the approximate analysis in this paper and from the field observations in IMPACT field tests are far removed from the one-dimensional steady, uniform conditions used to derive general sediment transport formulae. Thus this analysis calls into question the use of such equilibrium sediment transport formulae in any physically-based model of breach formation and so reinforces the need to use erosion-based approaches for breach modelling (Morris *et al*, 2008a).

The next steps for deterministic modelling of breaching of embankments should therefore consider the use of fully three-dimensional non-hydrostatic computational fluid dynamics (CFD) software closely coupled with an erosion-based breach growth model. The breach growth model should also capture other observed soil mechanics processes of the failure of an embankment, such as mass failure of blocks of material (Morris, 2009). The analytic approach in this paper could enable a hybrid CFD-analytical model to be developed with reduced computational resources over a full CFD model of the reservoir, with the CFD providing the detail close to the breach and the analytical model providing the far-field. This approach should assist understanding of the complex fluid flow and soil mechanics interactions in the critical phase of formation and enlargement of the breach after the initiation stage leading up to the final, relatively stable, breach dimensions (see Morris *et al* (2008b) for a discussion of the phases of breach formation).

CONCLUSIONS

This paper has presented a new analytic solution for the free-surface flow towards the breach in an embankment, based upon potential flow theory and the use of Bernoulli's Equation.

The analytic model predicts features of the flow observed in the IMPACT project field tests during the growth of the breach in an embankment. In particular:

- The movement of the flow control upstream
- The elliptical shape of equipotential (water level) surfaces
- Strong curvature of the water surface in the vertical plane and of streamlines in plan.

The implication of this analysis is that models based on assuming the discharge through the breach to be calculated from the broad-crested weir equation will potentially underestimate the flow through the breach for the converging flow case. This analysis indicates that traditional approaches (weir flow based on the breach width and equilibrium sediment transport) used in some deterministic models of breach flow are not consistent with the hydrodynamics of the flow during the phase of breach growth characterised by "converging" flow discussed in this paper.

The analytic model in this paper has potential for use in the next generation of deterministic breach models through coupling with CFD for the near-field flow and soil mechanics processes for the failure of the embankment.

ACKNOWLEDGEMENTS

We would like thank Dr John Fenton of *Technische Universität Wien* for his comments on an early version of this text and for suggesting the analysis in terms of total energy flux crossing a surface (Equations 22 to 24 above).

The work described in this publication was supported in part by the European Community in the Fifth Framework Programme through the IMPACT Project, Contract EVG1-CT-2001-00037 and in the Sixth Framework Programme through the grant to the budget of the Integrated Project FLOODsite, Contract GOCE-CT-2004-505420. This paper reflects the authors' views and not those of the European Community. Neither the European Community nor any member of the IMPACT or the FLOODsite Consortia is liable for any use of the information in this paper.

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