

Hydraulics Research Wallingford

SLUDGE DISPOSAL IN COASTAL WATERS

Mathematical modelling of the transport of heavy metals in Liverpool Bay

James G. Rodger B.Sc. Ph.D. Nicholas V. M. Odd B.Sc. (Eng) FICE

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Registered Office: Hydraulics Research Limited, Wallingford, Oxfordshire OX10 8BA. Telephone: 0491 35381. Telex: 848552

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ABSTRACT

A new two-layer, two-dimensional in-plan mathematical model was developed to simulate tidal flow, circulations and heavy metal transport in coastal waters.

In its present state of development (April 1985) the model is designed and coded in a fairly general form to have the following features:

- 1. The geography of a coastal area may be defined by a number of variably sized, patched and locally distorted grids each containing a maximum of about 4000 elements.
- 2. The ability to calculate tidal motion, gravitational circulations based on a prescribed salinity field and steady wind driven flows in two layers.
- 3. The ability to calculate the erosion, transport and deposition of marine mud and sewage sludge particles.
- 4. The ability to calculate the movement of dissolved metal adsorbed onto marine mud or sewage sludge.

The model was set up to simulate conditions in the Eastern Irish Sea, Liverpool Bay and the Mersey Estuary using three interactively nested grids with element sizes of 2700m, 900m and 300m, respectively. The model was used to simulate the physical dispersal of recently discharged zinc, irreversibly adsorbed onto marine mud in suspension and on the surface layers of the bed of Liverpool Bay and its adjacent estuaries, during a repeating mean tide cycle.

The results from the demonstration tests show that the model is an effective means of predicting the physical dispersal of recently discharged metal from a number of different sources, including sewage, sewage sludge, muddy dredged spoil and industrial waste. For the Liverpool Bay model to reach its full potential as a predictive planning tool it needs to be calibrated more exactly, in terms of tidal and residual flows and mud transport.



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In November 1982, Hydraulics Research Limited (HR) published Report DE 59 on the feasibility of mathematically modelling transport of heavy metals in Liverpool Bay (Fig 1). Although the report paid special attention to Liverpool Bay, the authors are of the opinion that the proposed model could in principle be applied to any coastal region in the British Isles.

In March 1983, HR was commissioned by the Department of the Environment to start development and preliminary testing of the model, which is the subject of this report. The funding for this part of the work ceased in March 1985. The most critical part of the work, namely using the model to make preliminary simulations, was done in the first quarter of 1985.

The objectives of the research were fairly ambitious considering the timescale, available funding and the nature of the modelling problem. The authors considered it necessary that the first application of the method should be to simulate conditions in Liverpool Bay, rather than some idealized or simpler situation. This was because the method will only be of practical use in helping solve the problem of the disposal of sludge in Liverpool Bay (Fig 2) by the North West Water Authority (NWWA) if it can be applied to the real situation in terms of scale and detail of the geometry, tidal flows and metal transport patterns in Liverpool Bay and the Mersey Estuary (Fig 3). As presented, the model attempts to simulate not only the whole of the Eastern Irish Sea at a grid size of 2700 m, and Liverpool Bay at a grid size of 900 m but also the whole of the Mersey Estuary at a grid size of The model also simulated the tidal basins and 300 m. canals in the Mersey Estuary on a grid size of less than 300 m.

This meant that HR had to:

1

- (a) develop a new type of two-dimensional two-layer model, which could handle three dynamically patched grids with water movement (TIDEFLOW-2D2L), mud (MUDFLOW-2D2L), particulate BOD (sewage), and metal transport (METALFLOW-2D2L) on HR's DAP computer;
- (b) solve the logistic problem of schematising the detailed bathymetry, flows and mud transport patterns in the Eastern Irish Sea, Liverpool Bay and the Mersey Estuary and fit it into the active core of the DAP computer;

(c) ensure all parts of the model interact correctly then to use it to simulate conditions in Liverpool Bay.

The preliminary test runs were unusually long and cumbersome to analyse. The inclusion of the detailed model of the Mersey Estuary pushed the DAP computer to its limits both in terms of core, storage and realistic run times. The authors considered that the results of the preliminary tests illustrated in the report are quite promising considering no detailed attempt was made to calibrate the model. The model will require considerable refinement before it can be used to make firm predictions on the dispersal of heavy metals in Liverpool Bay and this report should be read in conjunction with an earlier report on the same subject (Ref 1).

GEOMETRY OF THE MODEL

2

Originally, the authors had planned to use four model grids (Ref 1). However, because there were so few elements in the 8,100 m grid of the Eastern Irish Sea and because each slice of the DAP could accommodate up to 3,844 elements (62×62) without extra computer time, it was decided to extend the 2,700m grid beyond the limit of Liverpool Bay out to the Isle of Man. The revised limits of the 2,700m, 900m and 300m grids, which cover the Eastern Irish Sea (EIS), Liverpool Bay (LB) and the Mersey Estuary (ME), are shown in Fig 4.

The bed level was defined at the centre of each side of each cell. The data was compiled from Admiralty Charts of the EIS and LB and detailed surveys of the ME. All the bed levels were reduced to ODN (Ordnance Datum Newlyn - Mean Sea Level).

The contoured bathymetry of the EIS, LB and ME as represented in the model are shown in Figs 5a-c. In order to help resolve the vertical variations in the water column, the water body contained in the model was divided into a bed and surface layer by a horizontal interface set at a level 6.5 m below OD(N), except where the lower layer would be less than 1.5 m deep; in which case the interface was dropped to the bed level. This schemetization meant that the lower layer penetrated into the Mersey Narrows but not into the shallow regions upstream. The lower layer did not penetrate a significant distance into either the Dee or Ribble estuaries, or Morecambe Bay. The horizontal interface at -6.5 m OD(N) divided the water column in Liverpool Bay and the Mersey Narrows (bed level at about -17m OD(N)) into two layers of approximately equal thickness (10m) at high water mean spring tide (+4m OD(N)). At low water mean spring tide

(-4m OD(N)), the upper layer was only about 2.5m thick.

The method of mapping the variables describing the flow, water volume, mud and metal concentrations in each layer within each grid in the active store of the DAP is illustrated in Fig 5d. This shows that the fine grid required to resolve the spatial variations in the Mersey Estuary, especially in the narrows, took up half of the active core store in the DAP.

The orientation of the grid was set to line up with the axis of the Mersey Narrows in order to get the best representation of tidal flow in the Mersey.

An additional feature of the model is the distortion of the cells at the coastal boundaries of the model which improves the resolution of coastal features, tidal basins and the lower reaches of the ship canal.

3 THEORY FOR TWO-LAYER TIDAL FLOW MODEL

The model, TIDEFLOW-2D2L, was based upon the differential equations describing the conservation of mass and momentum averaged over the depth of each layer or over the total depth in the shallow regions. Apart from the interaction between the two layers, the equations are similar to and are derived in a similar manner to the depth averaged equations given in Ref 2. These layer averaged equations can be written as follows:

Conservation of water mass

$$\frac{\partial uz_B}{\partial t} + \frac{\partial vz_B}{\partial t} + w = 0$$
(1)
$$\frac{\partial h}{\partial t} + \frac{\partial ud}{\partial x} + \frac{\partial vd}{\partial y} - w = 0$$
(2)

3.1 Conservation of

momentum

$$\frac{\text{Bed layer}}{\partial t} + \frac{u \partial u}{\partial x} + \frac{v \partial u}{\partial y} + \frac{g \partial h}{\partial x} + \frac{P}{x} + \frac{f u (u^{2} + v^{2})^{\frac{1}{2}}}{z_{B}}$$
$$- D \left(\frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} u}{\partial y^{2}}\right) - \Omega v = \frac{21 \frac{2}{m}}{z_{B}} \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \frac{\partial u}{\partial z} \quad (3)$$

$$\frac{\partial v}{\partial t} + \frac{u}{\partial x} + \frac{v}{\partial y} + \frac{g}{\partial y} + \frac{g}{\partial y} + \frac{g}{\partial y} + \frac{fv(u^{2}+v^{2})^{2}}{z_{B}}$$

$$- D \left(\frac{\partial^{2}v}{\partial x^{2}} + \frac{\partial^{2}v}{\partial y^{2}}\right) + \Omega u = \frac{21\frac{2}{m}}{z_{B}} \frac{\partial(u^{2}+v^{2})^{2}}{\partial z} \cdot \frac{\partial v}{\partial z} \quad (4)$$

where:

$$P_{\mathbf{x}} = \frac{gd}{\rho} \cdot \frac{\partial\rho}{\partial \mathbf{x}} + \frac{gz_B}{2\rho} \quad \frac{\partial\rho}{\partial \mathbf{x}}$$
(3a)
$$P_{\mathbf{y}} = \frac{gd}{\rho} \cdot \frac{\partial\rho}{\partial \mathbf{y}} + \frac{gz_B}{2\rho} \quad \frac{\partial\rho}{\partial \mathbf{y}}$$
(4b)

Surface layer

$$\frac{\partial u}{\partial t} + \frac{u}{\partial x} \frac{\partial u}{\partial x} + \frac{v}{\partial y} \frac{\partial u}{\partial x} + \frac{g}{2\rho} \frac{\partial h}{\partial x} - D(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$

$$-\Omega v = -\frac{21\frac{2}{m}}{d} \frac{\partial (u^2 + v^2)^{\frac{1}{2}}}{\partial z} \cdot \frac{\partial u}{\partial z} + \tau_{xw}$$
(5)

$$\frac{\partial \mathbf{v}}{\partial \mathbf{t}} + \frac{\mathbf{u} \, \partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\mathbf{v} \, \partial \mathbf{v}}{\partial \mathbf{y}} + \frac{\mathbf{g} \, \partial \mathbf{h}}{\partial \mathbf{y}} + \frac{\mathbf{gd}}{2\rho} \cdot \frac{\partial \rho}{\partial \mathbf{y}} - \mathbf{D}(\frac{\partial^2 \mathbf{v}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{v}}{\partial \mathbf{y}^2})$$

$$+\Omega v = -\frac{21\frac{2}{m}}{d}\frac{\partial(v^{2}+v^{2})^{\frac{1}{2}}}{\partial z}\cdot\frac{\partial v}{\partial z} + \tau_{yw}$$
(6)

where

- - w = vertical velocity component between layers
 (m/s)
 - h = surface level relative to datum (m)
 - $z_B = bed layer depth (m)$
 - d = surface layer depth (m)
- $d_T = total depth (m)$
- P_x , P_y^{\perp} = density component of pressure gradient (m/s^2)
 - f = friction parameter
 - D = coefficient of horizontal eddy viscosity (m²/s)
 - Ω = Coriolis parameter

 τ_{xw} , τ_{yw} = surface wind stress components (N/m²)

l_m = momentum mixing length (m)

The density gradients $\frac{\partial \rho}{\partial x}$ and $\frac{\partial \rho}{\partial y}$ were prescribed for observed salinity distributions in the EIS, LB (Ref 7) and the Mersey Estuary (Ref 8) as shown in Figures 6a-c.

3.2 Interfacial mixing

The turbulent exchange of momentum between the two layers was represented in terms of a momentum mixing length (l_m) in equations (3) to (6). The mixing length is a function of the total depth and the bed layer thickness and the degree of stratification between the layers represented in terms of a bulk Richardson number defined as:

$$R_{i} = \frac{z_{B}g \Delta \rho}{\rho [(u_{1}-u_{2})^{2} + (v_{1}-v_{2})^{2}]}$$

Subscripts 1 and 2 refer to the upper and lower layers respectively.

(7)

For Liverpool Bay, it was assumed that there was a fixed salinity difference of 2 kg/m^3 between the bed and surface layers. The form of the mixing length function is described in Ref 3.

The model was set up with the intention of using the ICL DAP (Distributed Array Processor) at HR. This computer can carry out 4,096 operations in parallel and was essential for the two-layer model envisaged. Two-dimensional, depth-averaged models (Ref 4) and a three-dimensional model (Ref 5) have been implemented on the DAP and the two-layer model of Liverpool Bay was a development of both models. The model equations 1-6 were solved using an explicit finite-difference scheme for horizontal derivatives and an implicit scheme for the vertical derivatives and, in this respect, the model followed methods employed in the depth averaged and three-dimensional models (Refs 4 and 5). In order to model Liverpool Bay and the Mersey Estuary with an adequate resolution, three patched finite-difference grids were used. At each patched line the dimension of the finite difference grids reduces by a factor of three. Each grid also featured distorted grid cells along the coastal boundaries to improve the resolution of coastal features and small harbours.

3.3 Solution procedure

3.4

Boundary conditions

The model was driven along the open sea boundary using prescribed tidal levels generated by linear interpolation of published Admiralty data for the M_2 and S_2 tidal components at four stations along the open sea boundary.

4 THEORY FOR TWO-LAYER TRANSPORT MODEL

The transport model used the stored results from the two-layer flow model and was based on the equations describing the conservation of mass. As with the flow model, these equations are similar to those used in the HR MUDFLOW two-dimensional depth averaged models and can be written as follows:

4.1 Conservation of mud

Surface layer

$$\frac{\partial dc}{\partial t} + \frac{\partial u dc}{\partial x} + \frac{\partial v dc}{\partial y} - (w - w_s) c - \frac{\partial}{\partial x} (Dd \frac{\partial c}{\partial x})$$
$$- \partial (\frac{Dd}{\partial y}) + \frac{1}{c} \frac{1}{m} \left| \frac{\partial (u^2 + v^2)^{\frac{1}{2}}}{\partial z} \right| \frac{\partial c}{\partial z} = 0$$
(8)

Bed layer

$$\frac{\partial z_{B}c}{\partial t} + \frac{\partial uz_{B}c}{\partial x} + \frac{\partial vZ_{B}c}{\partial y} + (w \not(w_{S}) c - \frac{\partial}{\partial x} (Dz_{B} \frac{\partial c}{\partial x})$$

(9)

$$-\frac{\partial}{\partial y}\left(Dz_{B}\frac{\partial c}{\partial y}\right) - \frac{1}{c} \frac{1}{m} \left|\frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z}\right| \frac{\partial c}{\partial z} = \frac{dm}{dt} + L_{mud}$$

where

с	-	suspended mud concentration (kg/m^3)			
		(in upper or lower layer)			
1 _m	=	momentum mixing length (m)			
l_c^{m} = solute mixing length (m)					
Ď	=	coefficient of lateral dispersion (m^2/s)			
w _s	=	settling velocity (m/s)			
dm/dt	=	bed exchange (kg/m ² /s)			

(erosion or deposition) $L_{mud} = loading (kg/m³/s)$

 \bar{c} = suspended solids concentration in upper layer if (w-w_s) < 0 (kg/m³)

or

ē

= suspended solids concentration in lower
layer if (w-w_s) > 0 (kg/m³///)

4.2 Conservation of adsorbed metal

Surface layer

$$\frac{\partial dc_{c}}{\partial t} + \frac{\partial u dc_{c}}{\partial x} + \frac{\partial v dc_{c}}{\partial y} - (w - w_{s}) \overline{c_{c}}_{am}$$

$$- \frac{\partial}{\partial x} \left(Dd \frac{\partial c_{am}}{\partial x} \right) - \frac{\partial}{\partial y} \left(Dd \frac{\partial c_{am}}{\partial y} \right)$$

$$+ 1_{c} 1_{m} \left| \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right| \frac{\partial c_{am}}{\partial z} = - \frac{-ddc_{d}}{dt}$$
(10)

Bed layer

$$\frac{\partial z_{B}c_{am}^{c}}{\partial t} + \frac{\partial uz_{B}c_{am}^{c}}{\partial x} + \frac{\partial vz_{B}c_{am}^{c}}{\partial y} + (w-w_{s})\overline{c_{am}^{c}}$$

$$= \frac{\partial}{\partial x} \left(Dz_{B}\frac{\partial c_{am}}{\partial x}\right) - \frac{\partial}{\partial y} \left(Dz_{B}\frac{\partial c_{am}}{\partial y}\right) + 1c_{am}^{1} \left|\frac{\partial \left(u^{2}+v^{2}\right)^{\frac{1}{2}}}{\partial z}\right|^{\frac{\partial c}{am}} \frac{\partial c_{am}^{c}}{\partial z}$$

$$= -\frac{dz_{B}c_{d}}{dt} + c_{amdt}^{dm} + L_{am}$$
(11)

where

С	=	the concentration of suspended mud (kg/m^3)
Cam	=	the concentration of adsorbed metal
		(kg/kg) (in the upper or lower layer)
cc _{am}	=	cc _{am} in upper layer if (w-w _s) < 0
ccam	=	cc_{am} in lower layer if $(w-w_s) > 0$
$\bar{1}_{c}$	=	solute mixing length (m)
Lam	=	loading of adsorbed metal per unit area of
		bed $(kg/m^2//s)$

4.3 Conservation of sewage sludge

Similar equations define the movement of sewage sludge particles incorporated into the mud flocs except for the addition of an extra term representing the oxidation (decay) of the biodegradable matter.

4.4 Conservation of dissolved metal

Surface layer

$$\frac{\partial dc_{d}}{\partial t} + \frac{\partial u dc_{d}}{\partial x} c_{d} + \frac{\partial v dc_{d}}{\partial y} - \frac{\partial c_{d}}{\partial x} (Dd \frac{\partial c_{d}}{\partial x})$$
$$- \frac{\partial}{\partial x} (Dd \frac{\partial c_{d}}{\partial y}) + 1_{c} 1_{m} \left| \frac{\partial (u^{2} + v^{2})^{\frac{1}{2}}}{\partial z} \right| = R + L_{dm}$$
(12)

Bed layer

$$\frac{\partial z_{B}c_{d}}{\partial t} + \frac{\partial uz_{B}c_{d}}{\partial x} + \frac{\partial vz_{B}c_{d}}{\partial y} - wc_{d} \partial x \quad (Dz_{B} \frac{\partial c_{d}}{\partial x})$$
$$- \frac{\partial}{\partial y} (Dz_{B} \frac{\partial c_{d}}{\partial y}) - 1_{c}1_{m} \left| \frac{\partial (u^{2}+v^{2})^{\frac{1}{2}}}{\partial z} \right| \frac{\partial c_{d}}{\partial z} \neq (13)$$

where

- c_d = the concentration of dissolved metal (kg/m³) (in upper or lower layer)
- $L_{dm} = R =$
- loading of dissolved metal (kg/m²/s)
 rate of loss or gain of metal due to adsorption or desorption of metal on
 - adsorption or desorption of metal on suspended mud (kg/m²/s)
- 4.5 Interfacial mixing

The turbulent exchange of suspended mud and adsorbed metal between the two layers was represented by a mixing length technique, where the solute mixing length (1_c) and the momentum mixing length (1_m) were obtained using the functional form described in Ref 3. These functions depend on the relative depth of the lower layer, the difference in the salinity between the two layers - assumed to be $2kg/m^3$ - and whether or not the flows in each layer were in the same or opposing directions.

4.6 Exchange between the bed and the flow

The erosional deposition of mud at the bed was prescribed by relationships used by Odd and Owen (Ref 6) which can be summarized as follows:

Deposition
$$\frac{dm}{dt} = w_s c (1 - \frac{\tau_b}{\tau_d})$$
 when $\tau_b < \tau_d$ (14)

Erosion
$$\frac{dm}{dt} = M(\tau - \tau)$$
 when $\tau > \tau$ (15)

where

 $\begin{aligned} \tau_b &= \text{bed stress } (N/m^2) \\ \tau_d &= \text{critical stress for deposition } (N/m^2) \\ \tau_e &= \text{critical stress for erosion } (N/m^2) \\ w_s &= \text{settling velocity } (m/s) \\ M &= \text{empirical erosion constant } (kg/s/N) \end{aligned}$

For this study, the settling velocity was assumed to depend on the suspended mud concentration through the empirical relationship:

 $w_{s} = 0.002c$

(16)

The critical stress for deposition τ_d was taken to be $0.1N/m^2$ and the critical stress for erosion (τ_e) was set to be $0.4N/m^2$. The constant M was assigned the value of 0.003 kg/s/N.

Exchange of adsorbed metal between the lower layer and the bed was calculated from the rate of erosion or deposition of mud on the assumption that the concentration of adsorbed metals on the suspended mud or on the bed were known. A basic assumption was that the adsorbed metal settled on the bed would become uniformally distributed throughout the surface layer of settled mud.

4.7 Adsorption and desorption

The adsorption and desorption of a metal to and from a suspended mud load is defined as a function of the difference between the actual and equilibrium concentration of dissolved metals, c_{d_p} , for the

prevailing concentration of adsorbed metal, c_{am} (Ref 1).

$$c_{d_e} = \frac{\beta}{\alpha} c_{am}$$

where

 β = an empirical constant (m⁻¹) α = the specific surface area of the mud (m²/kg)

If, locally in a model element, c_d and c_{am} change and $c_d > \frac{\beta}{\alpha} c_{am}$ then

$$\frac{dc}{dt} = -\frac{dc}{dt} = -k_a \left(c_d - \frac{\beta c_{am}}{\alpha}\right)$$
(18)

where

 k_a = the rate constant (t^{-1}) for the process of adsorption of the metal onto the mud

If, locally, c_d falls below the equilibrium value and $c_d < \frac{\beta}{\alpha} c_a$ there will be a tendency for desorption.

$$\frac{dc_{am}}{dt} = -\frac{dc_{d}}{dt} = k_{d} \left(\frac{\beta}{\alpha}c_{am} - c_{d}\right)$$
(19)

where

k_d = the rate constant for the process of desorption of the metal from the mud

 k_a is probably very high and k_d is almost certainly very small.

5 DEMONSTRATION TEST CONDITIONS

> In order to test the basic model, TIDEFLOW-2D2L was set up to simulate conditions in Liverpool Bay and in the adjacent waters during a repeating mean tide with no wind.

5.1 Seaward boundary conditions

The tidal levels were specified along the open boundaries of the outermost grid (Fig 4) between near Whitehaven on the Cumbrian Coast (5.44m range) to near Ramsey (4.96m) on the Isle of Man and between near Port St Mary (3.78m) southwards to Anglesey (3.90m). The tidal levels were synthesised from published

admiralty harmonic constants for the M2 and S2 constituents (Table 1). The tide was adjusted to repeat every 12.5 hours by raising the height of the

second high water by approximately 0.2m (6% of the tidal range). An allowance was made to account for the variation in the mean tidal level along the open boundaries of the model due to density effects. In the absence of field observations of mean tide levels, HR used values computed by Heaps and Jones (Ref 7) using a salinity distribution observed in September 1972 (Fig 6a). At that stage it was not practical to take into account variations in the mean tidal level along the seaward boundaries of the model due to non-linear effects of tidal motions in the Irish Sea.

Ideally, HR would have preferred to have extended the outer grid of the model to two more clearly defined sections at Port Patrick in the North Channel and opposite Wicklow Head in the St George's Channel (Fig 4). But the active core of the DAP was filled to capacity with the data defining the 300m grid within the Mersey estuary. However, it was considered that errors arising from minor inconsistencies in the boundary conditions would not have a significant influence on flows within Liverpool Bay. The model was not intended to predict conditions accurately within about 10-15km off the seaward boundaries, an area which must be considered to be a buffer zone. The discharge of the River Mersey was said to remain constant at a steady 50m³/s.

5.2 Salinity density effects

The pressure terms P_x and P_y (Eqs 3a and 4b) require a knowledge of the horizontal density variations within the model area. For the purpose of the present study it was considered to be impractical to calculate the salinity (or temperature) distribution within the model because of the very long reaction time of the system to changes in river flow. Instead, salinity observations were used to prescribe an unchanging density field. Plots of the prescribed salinity distributions for EIS, LB and ME are shown in Figures 6a-c respectively. The data was derived from several sources (Refs 7-12). The effect of variations in water temperature on the horizontal density gradients was ignored.

5.3 Bed roughness

The effective roughness of the bed (k_s) was estimated to be 40mm in EIS, LB and ME. This value was considered to be representative of a rippled sandy bed.

6 TIDAL PROPAGATION

The model test was started with an arbitrary initial condition with a nearly flat water surface and the water at rest. The model was then run for several

tidal cycles until the tidal flows reached a state of dynamic equilibrium repeating almost exactly every 12.5 hours. The results were not submitted to a rigorous analysis in terms of detailed comparisons with observations. Instead, they were compared qualitatively with readily available standard admiralty data on tidal streams in the EIS and LB.

6.1 Tidal levels

Comparisons between the tidal levels simulated by the model at sites along the coasts of the EIS and LB (Figs 1-4) and synthesised levels based on local admiralty tidal constants are shown in Figure 7a. The model simulated the correct degree of amplification of the tidal range of about 20% between the seaward boundary and the coast and the correct phase. The deviation in the water levels on the second high water is partly due to the small diurnal inequality in the local synthesised tide. The model was least accurate at Princes's Pier within the Mersey Estuary (Fig 7a). There was no immediately available data to compare with the simulated tidal levels higher up the Mersey Estuary (Fig 7b).

6.2 Tidal streams

The main feature of the pattern of the flood and ebb tidal streams in the bed layer below the level of -6.5m OD(N) agreed with information from the admiralty tidal atlas (Ref 13) with most of the flow passing between Anglesey and the Isle of Man (Figs 8a-b), with peak flood and ebb velocities of about 0.75m/s in an easterly and westerly direction. The flood and ebb tidal streams in the bed layer in Liverpool Bay were in a SW and NE direction with speeds in the lower layer of about 0.5m/s on a mean tide (Fig 9a-b).

A more detailed comparison of the tidal streams in the EIS at stations 1-4, whose location is shown in Figure 5a, indicated that the velocity in the bed layer below -6.5m OD(N) was similar to velocities measured at the admiralty stations. The velocities in the thin surface layer were between 50-100% higher than the depth averaged values for the bed layer. The model was least accurate at stations 1 and 2 in the upper zone close to the northern boundary of the model (Fig 5a) and most accurate at station 4 in the middle of the EIS (Fig 10a). In Liverpool Bay (Figs 5b and 10b) the simulated tidal currents in the bed layer agreed most closely with admiralty observations. The tidal stream in the surface layer tended to be 50-100% greater than the lower layer and to rotate in a clockwise direction, whereas the tidal stream in the lower layer tended to rotate in an anti-clockwise direction. The observed depth-mean admiralty tidal currents tended to fall between the bed and surface values simulated in the model.

The simulated tidal velocities in the bed layer below -6.5m OD(N) at stations 10-14 in the Mersey Estuary (Fig 5c) are illustrated in Figure 10c. The velocity in the surface layer was about 100% higher than in the bed layer, which was similar to variations observed in the Mersey Narrows in September 1983 (Ref 14). The simulated patterns of peak tidal currents in the bed and surface layers in the Mersey Estuary are shown in Figures 11a-b. The bed layer did not extend landward of the Narrows. The representation of the flow in and out of the Manchester Ship Canal was incorrect, because at present the model does not simulate the tidal gate. However, this could be added relatively easily at a later stage of development of the model.

6.3 Bed stress

The stress exerted by the flow on the sea-bed determines the conditions for erosion and deposition of muddy sediments and sewage sludge. The patterns of peak bed stress simulated in the model in EIS, LB and ME are shown in Figures 12a-c respectively. There is a zone of high bed stress between the Isle of Man and Anglesey in the EIS. The other main zones of high bed stress are within Morecambe Bay, the Dee Estuary and the Mersey Estuary. Cohesive mud or sludge is unlikely to form a permanent deposit on the bed where the peak bed stress on a mean tide exceeds about $0.5N/m^2$. The bed stress downstream of the Manchester Ship Canal is probably unrealistically high because the model did not include the effect of the lock gates.

6.4 Residual flows

The pattern of residual discharges per unit width in the lower layer below -6.5m OD(N) in EIS, LB and ME are shown in Figures 13a-c. The vectors adjacent to the coast are unreliable because the method of analysis assumes that the velocities are zero along the coast- line. Any circulation between the Isle of Man and Anglesey should be ignored because it is the buffer zone close to the seaward boundary of the model. The model predicted a residual current in the bed layer leaving Liverpool Bay in a north-westerly direction, and the general pattern was some seaward residuals in the Mersey Narrows. The magnitude of the residuals are realistic but the directions were incompatible with the prescribed density field and do not agree with the reported observations (Ref 1).

The model reproduced a correct landward flow when it was run without a tide. The main cause of the unexpected seaward residuals appears to be a combination of the choice of the position of the fixed interface, the method of calculating the shear stress between the two layers and to a lesser extent the effect of using a constant density field. In the

present series of tests the upper layer accommodates the whole tidal range. This means that the surface layer is thinner at low water and thick at high water. Observations by the Water Research Centre (WRC) (Ref 17) show that there is a three layer flow (landward at the bed and surface and seaward at mid-depth) in the Mersey Narrows if one analyses the results in terms absolute levels relative to OD(N). The landward flow in the surface layer is caused by the fact that these layers are only full of water when the tide is high and still flowing landward (immediately before and after high water). The landward flow in the bed layer is generated by the longitudinal density gradient along the estuary. There is a compensating seaward flow in the mid-depth layers. This three-layer flow effect becomes more pronounced in estuaries with a large tidal range-to-depth ratio. As set-up, the model appears to exaggerate the effect to such an extent as to reverse the direction of the residual flow in the bed and surface layers. A two-layer model (Ref 6) can usually simulate the correct residual flow in the bed layer even if the surface layer contains an opposing residual flow in the surface and mid-depth regions. Observations by WRC in the Mersey Narrows appear to indicate that the changeover from landward to seaward drift is several metres below the level of the interface set in the model at -6.5m OD(N). In the authors' opinion the model would work realistically in terms of residual flows if the interface was dropped below the point of reversal of drift currents as measured by WRC (Ref 17).

The strength of the landward gravitational circulation in the bed layer is also sensitive to the prescribed mixing coefficients which determine the shear stress between the bed and surface layers. Strong tidal currents and high interfacial stresses tend to reduce the strength of the gravitational circulation. In the present demonstration test series, HR made no attempt to optimise the value of this coefficient which depends on the degree of stratification. Finally, in nature, the horizontal salinity-density field and velocities continually adjust themselves according to prevailing tidal currents, tidal mixing, coriolis forces and average antecedent fluvial flows. The problem in defining a fixed salinity field from sparse and non-simultaneous observations is that the position of the iso-halines (contours of equal salinity) may not match the computed tidal flows. This can distort the pattern of residual flows. In the authors' opinion, it should be possible to adjust the model parameters so that it reproduces the main features of residual flows.

It should be noted, that unlike dissolved matter, particulate matter is only influenced by the residual flows when it is in suspension on the main run of the tide. The net direction of transport of particulate matter is also strongly influenced by the strength of the tidal currents on the flood and ebb phases of the tide. The authors considered that the flow simulation was adequate for the purposes of a demonstration test.

7 MUD TRANSPORT

7.1 Muddy zones

The muddy zones in the three different model grids representing the EIS, LB and ME were prescribed using data from admiralty charts, and special surveys (Refs 15-16). In these zones, the mud was assumed to be uniformly distributed in the surface layer at a density of 100kg/m^2 . The areas of the muddy zones are shown in Figures 14a-c.

7.2 Wave action

Wave action was assumed to prevent the deposition of mud on the bed in the exposed coastal zone of the EIS and LB, where the bed level was less than 5m below ODN, as shown in Figure 15. Due to an oversight, this condition was not applied to the outer zone of the fine grid of the Mersey Estuary (solid black zone, Fig 15) in the demonstration test. Wave action was assumed to be ineffective at preventing deposition in the relatively sheltered reaches of the Mersey Estuary and the upper regions of the Dee Estuary.

7.3 Initial mud concentrations

Initial concentrations of mud in suspension in the model was set to be uniform in both layers at 50ppm, 100ppm and 1000ppm in the 2700m, 900m and 300m grids representing the EIS, LB (including the Dee Estuary) and the ME, respectively.

7.4 Boundary conditions

The concentration of suspended mud in the incoming water on the seaward boundaries was set to be a uniform 10ppm.

The pattern of mud transport in an estuary system usually reacts rapidly to a change in flow conditions because the suspended load is sensitive to the bed stress. This means that the transient effects arising from poor initial conditions are soon lost from the solution. The distribution of mud on the bed reacts more slowly to the flow because the deposits usually contain large quantities of sediment.

The periodic pattern of suspended mud transport and the associated concentration fields were approaching a state of dynamic equilibrium after about ten tidal cycles.

7.5 Simulated mud concentration fields

Contoured distributions of the mud concentrations in the bed and surface layers in the EIS, LB and ME on the peak flood and peak ebb phases of a mean tide are shown in Figures 16a-f and 17a-f, respectively. Coloured diagrams showing the distribution of suspended mud concentrations in the whole of the model area at the time of peak flood tidal velocities in the surface and bed layers are shown in Figures 18a and 18b, respectively.

Time histories of the variation in the suspended mud concentrations in the bed layer at stations 1-5 in EIS (Fig 5a), stations 6-10 in LB (Fig 5b) and stations 11-14 in ME (Fig 5c) are shown in Figures 19a-c, respectively. The model results show three well-defined zones of high turbidity. The main one obviously being in the Mersey Estuary. The other two peaks in turbidity are associated with patches of mud in the EIS (Fig 14a). The one in the upper zone is probably artificially created by the strong tidal currents and high bed stresses simulated by the model along the southwest shore of the Isle of Man. This zone is probably entirely spurious and would disappear as soon as all the excess mud had been scoured from off the bed surface. Compare the size of zone A in Figures 14a and 18c at the beginning and end of 10 repeating tidal cycles.

The second zone of turbidity is generated by the erosion of mud at the extreme western end of the prescribed muddy zone (B) on the outer boundaries of Liverpool Bay (Fig 18c). This turbidity peak is also probably wholly spurious because the extent or density of the prescribed muddy zone is too great. The concentration of suspended mud in the two aforementioned zones in EIS is higher and more extensive in the lower as compared to the upper layer (Figs 17a-b).

7.6 Liverpool Bay and Mersey Estuary

> The pattern of mud transport in LB and ME was approaching a state of dynamic equilibrium above the 10th repeating mean tide. For example, the suspended mud concentration in the bed layer at station 8 close to the entrance of the training walls (Fig 5b) almost repeats at the end of each tidal cycle (Fig 19b). Normally one would expect the suspended mud concentrations to peak twice in the tidal cycle as the mud is re-suspended after each slack water period. A

single peak, as shown in Figure 19b, usually occurs on the edge of a turbidity front where there is a sudden change in concentrations. The results are probably still slightly affected by the initial conditions which included a sudden change in concentration from 1000ppm to 100ppm between the Mersey Estuary and Liverpool Bay. The predicted mud concentrations at station 12 in the Mersey Narrows (Fig 5c) almost repeated at the end of each tidal cycle (Fig 19b and 19c) peaking before low water on the ebb tide. A more pronounced double peak was evident with a peak concentration of about 750ppm occurring in the bed layer at station 14 close to the landward end of the Narrows (Fig 5c, and 19c). The simulated range of suspended mud concentrations in the bed layer in the Mersey Narrows (Fig 19b) was in the range 350ppm to 750ppm which is somewhat higher than the observed values near the bed on a mean tide (100-500ppm) given in Reference 8.

The model predicted that the suspended mud concentrations would be in the range 200-700ppm in the surface layer in the wide regions of the upper Mersey as far as Runcorn bridge (Figs 16e and 17e), which are similar to those observed on spring tides in February 1983 (Ref 8) (Figures 6-10). There was evidence that the weak seaward residual velocities in the bed layer were steadily reducing the mud concentrations in the Mersey Estuary (station 13 and 14 in Fig 19c). However, the model did reproduce the main features in terms of the spatial and temporal distribution of the suspended mud concentrations (Figs 16a-f, 17a-f and 18a-b).

The model predicted a continuous zone of muddy water with concentrations in excess of 50ppm covering the southeast corner of Liverpool Bay (Fig 16c-d and 17c-d) including the Dee Estuary with concentrations rising rapidly towards the entrance to the Mersey Estuary. The mud transport aspect of the model needs to be calibrated by detailed comparison with field observations. However, the preliminary simulation was considered to be adequate to demonstrate the capabilities of the modelling method.

8 HEAVY METAL TRANSPORT

> The capability of the model to simulate the transport of heavy metals was demonstrated by calculating the transport and dispersal of new adsorbed zinc discharged simultaneously at several offshore and shoreline sites. The calculation was restricted to simulating the dispersal of newly discharged metal because it was considered that the initial background values (Refs 18 and 19) would swamp the small increases predicted by the model over a period of ten

tidal cycles. Zinc was chosen as a representative metal which is readily adsorbed onto the mud. The metal was assumed to be firmly fixed to the mud with no transfer between the dissolved and adsorbed states, although the model was designed so that it could in the future take this additional complication into account.

The position and magnitude of the zinc loads are shown in Fig 20. The offshore loads include two almost equal ones of 670 km/day at the sludge disposal ground and the dredge spoil disposal ground (site Z) in Liverpool Bay. The metals were assumed to be discharged at a constant rate and to be mixed simultaneously with the muddy sediment in the top 150 mm of the bed. In the case of the dredged spoil disposal ground, the model took into account the fact that 1,400 tons of cohesive mud on average would be added to the bed each day from dredger hoppers.

The shoreline discharges included a large industrial effluent of 3,750 kg/day in the Dee and 230 kg/day in the Mersey estuary from the rivers and sewage outfalls. An additional 130 kg/day were discharged into the Ribble estuary. The aforementioned distribution of loads approximates to conditions in about 1980.

8.1 Results

Colour images showing the spatial distribution of new adsorbed zinc on suspended mud at the time of peak flood tide velocities after about 10 repeating mean tidal cycles for the surface and bed layers are illustrated in Figs 21a-b, respectively. The corresponding distribution of new zinc concentrations on the mud deposits on the bed surface after 10 tidal cycles is shown in Fig 22. More detailed contoured patterns of concentrations of new zinc adsorbed on suspended mud in the bed and surface layers on the main run of the flood and ebb phase of the tide in Liverpool Bay and the Mersey Estuary are shown in Figs 21c-j. Time histories of variations of the concentrations of adsorbed metals on suspended mud in the bed layer at stations 5-10 in Liverpool Bay are shown in Fig 23a and at stations 11-14 in the Mersey Estuary in Fig 23b.

The concentration of adsorbed metal per unit mass of suspended mud varies directly with the magnitude of the loads and inversely with the amount of mud available in the water column and in the bed surface layer.

The tidal action is obviously very effective at flushing adsorbed metal on mud suspended in the surface layer in the Dee and Ribble estuaries, which have large tidal volumes. Very little water remains in these estuaries at low tide. New metal discharged into the surface layer at the head of the Mersey Estuary is obviously not flushed into Liverpool Bay at anything like the same rate as metal discharged into the Dee and Ribble estuaries. In reality, the two-layer gravitational circulation in the narrows and the imbalance of flood and ebb tidal currents would help to contain the suspended mud in the upper Mersey Estuary.

There are two small but distinct peaks in the concentration of new zinc adsorbed onto suspended mud in the lower layer in the vicinity of the sewage sludge disposal zone and the dredged spoil disposal zones in the Liverpool Bay (Fig 21b). Concentrations of new metal adsorbed onto mud in the lower layers of Liverpool Bay were increasing each tidal cycle as shown in Fig 23a. The model would have to be run for a large number of tidal cycles before the metal concentration reached values which were in equilibrium with the metal loadings. The model also predicted a gradual build-up in the metal concentrations in the mud deposits on the bed of the bay as shown in Fig 22. The predicted concentrations of adsorbed new metal in the muddy zones (Fig 14a-c) are relatively low compared to concentrations of metals on the suspended mud because the model assumed that the surface layers of mud in each square metre of the bed in these muddy zones would be continually perturbed and vertically well mixed to a depth of about 150 mm. As a result there are no significant concentrations of new zinc shown in Fig 22 in the prescribed muddy zones. The areas of concentration of new zinc on the bed represent new slack water deposits. The concentration of metals in these very thin mud deposits, on an otherwise sandy bed, react fairly rapidly to a change in metal loadings. Due to an oversight, the model allowed mud to deposit on the north west shoreline of the Wirral Peninsula and the shallow region within the area of the 300 m grid covering the mouth of the Mersey Estuary. In reality, wave action prevents the accumulation of mud deposits above the 5 metre contour on the coastline of Liverpool Bay.

9 DISCUSSION AND CONCLUSIONS

The main objectives of the research project were achieved, namely, to develop a new type of mathematical model called METALFLOW-2D2L and to use it to simulate the transport of heavy metals disposed of in Liverpool Bay and the adjacent estuaries.

The original objectives of the project, which included modelling the transport of up to six different metals both in solution and adsorbed onto inert mud mixed with fast and slow decaying biodegradable sewage particles, was fairly ambitious considering the complexity of the physical and chemical interactions which control the processes.

In its present state of development (April 1985) the model is designed and coded in a fairly general form to have the following features:

- (i) the geography of a coastal area may be defined by a number of variably sized, patched and locally distorted grids, each containing a maximum of about 4000 elements.
- (ii) the ability to calculate tidal motion, gravitational circulations and wind driven flows in two layers.
- (iii) the ability to calculate the erosion, transport and deposition of marine mud and sewage sludge.
- (iv) the ability to calculate the movement of dissolved metal and metal adsorbed onto marine mud or sewage sludge.

The number of variables (metals, fast and slow decaying sludge, etc) which can be handled by the model is limited by the available size of the active core of the HR DAP computer.

The demonstration test was restricted to simulating the physical dispersal of recently discharged zinc, irreversibly adsorbed onto marine mud in suspenion and on the surface layers of the bed of Liverpool Bay and its adjacent estuaries, during a repeating mean tide. At present, the model could only handle more variables if the total number of elements were reduced. However, the program coding could be modified to add extra virtual storage to the DAP computer. This would allow the machine to deal with more variables but it would considerably increase the elapse time of the calculation, because the additional data would have to be shifted in and out of the core of the machine every few time steps.

The results from the demonstration test showed that the Liverpool Bay model is an effective means of predicting the physical dispersal of recently discharged metal from a number of different sources, including sewage, sewage sludge, muddy dredged spoil and industrial waste.

For the Liverpool Bay model to reach its full potential as a predictive planning tool it needs to be calibrated more exactly in terms of tidal and residual flows and mud transport. The model could then be used to make useful predictions of the dispersal of a newly discharged metal irreversibly adsorbed onto marine mud. If the model were to be used to simulate the interaction between the adsorbed and dissolved state it would be necessary to work in terms of the total rather than the new metal concentrations.

There is obviously plenty of scope for improving the theoretical description of the desorption process, which may be important in the case of some of the more soluble heavy metals.

The same stored results from the flow, mud or sewage transport sub-models could be used repeatedly to calculate the transport of one metal for a long period or several metals for a shorter period. However, there is probably no advantage in attempting to calculate the transport of several metals simultaneously. The optimum mode of using the model will depend on the problem to be solved.

A successful application of the model will need a considerable amount of computing time, but it should be fairly easy to accommodate this demand by running the DAP computer overnight.

In the authors' opinion, this type of mathematical model is the only means of forecasting the effects of changes in policy of disposing of conservative pollutants in coastal waters in terms of their physical dispersal, a necessary pre-requisite to understanding their effect on the marine ecosystem.

The North West Water Authority are presently considering how a calibrated version of the model, in terms of flow and mud transport, could be used to predict the rate and pattern of physical dispersal of heavy metals associated with sewage sludge disposal in Liverpool Bay.

10 ACKNOWLEDGEMENTS

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TABLES



TABLE 1 Tidal Constants

Location	M	S ₂	s ₂		
	'g' phase (⁰)	amplitude (m)	'g' phase (⁰)	amplitude (m)	
A	301	2.00	339	0.67	
В	325	1.95	3	0.63	
C	328	2.43	13	0.96	
D	331	2.78	11	0.88	

Locations shown in Fig 4.
FIGURES

















FIG 5a MODEL BATHYMETRY - EASTERN IRISH SEA



FIG 56 MODEL BATHYMETRY - LIVERPOOL BAY



FIG 5c MODEL BATHYMETRY - MERSEY ESTUARY





FIG 6a PRESCRIBED DENSITY (SALINITY) FIELD: EASTERN IRISH SEA



FIG 66 PRESCRIBED DENSITY (SALINITY) FIELD: LIVERPOOL BAY



FIG 6 c PRESCRIBED DENSITY (SALINITY) FIELD: MERSEY ESTUARY



FIG 7a SIMULATED TIDAL LEVELS - LIVERPOOL BAY

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SIMULATED TIDAL LEVELS (MODEL ONLY) MERSEY ESTUARY FIG 7b



FIG. 8a

a SIMULATED PEAK-FLOOD TIDAL VELOCITIES (BED LAYER) EASTERN IRISH SEA



FIG 8b SIMULATED PEAK-EBB TIDAL VELOCITIES (BED LAYER) EASTERN IRISH SEA



FIG 9a SIMULATED PEAK-FLOOD TIDAL VELOCITIES (BED LAYER) LIVERPOOL BAY



FIG 9b SIMULATED PEAK-EBB TIDAL VELOCITIES (BED LAYER) LIVERPOOL BAY



FIG 10a

TIDAL VELOCITIES IN BED LAYER - EASTERN IRISH SEA

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FIG 10b TIDAL VELOCITIES IN BED LAYER - LIVERPOOL BAY



FIG 10 CTIDAL VELOCITIES IN BED LAYER - MERSEY ESTUARY NARROWS

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FIG 11a SIMULATED PEAK-FLOOD TIDAL VELOCITIES (SURFACE LAYER) MERSEY ESTUARY



FIG 11b SIMULATED PEAK-FLOOD TIDAL VELOCITIES BED LAYER) MERSEY ESTUARY



FIG 12a PEAK BED STRESS IN EASTERN IRISH SEA



FIG 12b PEAK BED STRESS IN LIVERPOOL BAY



FIG 12c PEAK BED STRESS IN MERSEY ESTUARY



FIG 13a RESIDUAL DISCHARGES (BED LAYER) EASTERN IRISH SEA



Fig 13b RESIDUAL DISCHARGES (BED LAYER) LIVERPOOL BAY



FIG 13c RESIDUAL DISCHARGES (BED LAYER) MERSEY ESTUARY



ig 14a Prescribed muddy zones – Eastern Irish Sea





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FIG 14c MUDDY ZONES IN MERSEY ESTUARY



Fig 15 Wave action zones – Eastern Irish Sea



FIG 16a MUD CONCENTRATIONS AT PEAK-FLOOD. SURFACE LAYER - EASTERN IRISH SEA



FIG 16b MUD CONCENTRATIONS AT PEAK-FLOOD. BED LAYER - EASTERN IRISH SEA


Fig 16c MUD CONCENTRATION AT PEAK-FLOOD -SURFACE LAYER - LIVERPOOL BAY



FIG 16d MUD CONCENTRATION AT PEAK-FLOOD -BED LAYER - LIVERPOOL BAY



FIG 16e MUD CONCENTRATIONS AT PEAK-FLOOD -SURFACE LAYER - MERSEY ESTUARY



FIG 16f MUD CONCENTRATIONS AT PEAK-FLOOD -BED LAYER - MERSEY ESTUARY



FIG 17a MUD CONCENTRATIONS AT PEAK-EBB. SURFACE LAYER - EASTERN IRISH SEA



FIG 17^b MUD CONCENTRATIONS AT PEAK-EBB. BED LAYER - EASTERN IRISH SEA



FIG 17c MUD CONCENTRATION AT PEAK-EBB SURFACE LAYER LIVERPOOL BAY



FIG 17d MUD CONCENTRATION AT PEAK-EBB (BED LAYER) LIVERPOOL BAY



FIG 17e MUD CONCENTRATIONS AT PEAK-EBB -SURFACE LAYER - MERSEY ESTUARY



FIG 17f MUD CONCENTRATIONS AT PEAK-EBB - BED LAYER - MERSEY ESTUARY



less than 50 ppm 50-250 ppm 250-450 ppm greater than 450 ppm

land

Note: Part of the blue areas in the estuaries are dry intertidal flats

Fig 18a Suspended mud concentrations - surface layer - at peak flood velocities





land

Note: The lower layer does not extend inland from the -6.5 m contour (Fig 5a-c)



FIG 19a MUD CONCENTRATION TIME HISTORY - BED LAYER - EASTERN IRISH SEA



19b MUD CONCENTRATION LAYER - LIVERPOOL TIME BAY FIG HISTORY - BED



FIG 19c MUD CONCENTRATION TIME HISTORY - BED LAYER - MERSEY ESTUARY NARROWS







- less than 1 ppm mud
 - 1-5 ppm mud
 - 5-50 ppm mud
- greater than 50 ppm mud

land

Note: Part of the blue areas in the estuaries are dry intertidal flats

Fig 21a New adsorbed zinc concentrations on suspended mud - surface layer - at peak



- less than 1 ppm mud
 - 1-5 ppm mud
 - 5-50 ppm mud

greater than 50 ppm mud

land

Note: The lower layer does not extend inland from the -6.5 m contour (Fig 5a-c)

New adsorbed zinc concentrations on suspended mud - bed layer - at peak flood velocities - Liverpool Bay



FIG 21c ADSORBED ZINC CONCENTRATIONS. PEAK-FLOOD SURFACE LAYER - LIVERPOOL BAY



FIG 21d ADSORBED ZINC CONCENTRATIONS. PEAK-FLOOD BED LAYER - LIVERPOOL BAY



FIG 21e ADSORBED ZING CONGENTRATIONS. PEAK-FLOOD SURFACE LAYER - MERSEY ESTUARY



FIG 21f ADSORBED ZINC CONCENTRATIONS. PEAK-FLOOD BED LAYER - MERSEY ESTUARY



FIG 21g ADSORBED ZINC CONCENTRATIONS. PEAK-EBB SURFACE LAYER - LIVERPOOL BAY



FIG 21h ADSORBED ZINC CONCENTRATIONS. PEAK-EBB BED LAYER - LIVERPOOL BAY



FIG 21 ADSORBED ZING CONGENTRATIONS. PEAK-EBB SURFACE LAYER - MERSEY ESTUARY



FIG 21 ADSORBED ZINC CONCENTRATIONS. PEAK-EBB BED LAYER - MERSEY ESTUARY





land

Fig 22 New adsorbed zinc concentrations on mud deposits - at peak flood velocities -



FIG 23a ADSORBED ZINC CONCENTRATIONS - BED LAYER - LIVERPOOL BAY

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FIG 23b ADSORBED ZINC CONCENTRATIONS - BED LAYER - MERSEY ESTUARY

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