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WAVE GROUPS

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This report describes work carried out under contract DGR 465/33, "Wave progagation with application to port design" funded by the Department of Transport to 31 March 1984 and thereafter by the Department of the Environment. The departmental nominated officer at report date was Dr R P Thorogood, CIRU. The study was carried out by Mr R M Shuttler in the Maritime Engineering Department of Hydraulics Research. Dr S W Huntington, Head, Maritime Engineering Department, was the Company's nominated project officer. This report is published on behalf of the Department of the Environment but any opinions expressed are not necessarily those of the Department.

Abstract

The existence of wave groups incident upon a floating structure, or within a harbour can give rise to low frequency resonance effects. In many harbours this has significant impact upon berth tenability due to unacceptable ship motions. It is essential therefore that the wave groups are realistically represented in both physical and numerical modelling studies.

Questions have been raised as to whether the conventional specification of a sea state as a gaussian surface elevation determined solely by the spectrum is adequate to define the groups correctly. This report examines this question to examine whether existing modelling techniques based on the conventional assumption can be shown to be in error. One way to show that the conventional understanding is sound would be to demonstrate that the time domain characteristics of the wave groups can be predicted from a knowledge of the spectrum alone.

Wave records from field measurements, the laboratory and numerical simulation have been analysed with particular reference to wave groupiness.

A prominent feature is the scatter in all the parameters chosen for describing the groupiness. It is of equal extent for all three sources of data considered.

A theoretical determination of the distribution of the wave groups from the mean length of the groups is confirmed by field and laboratory data. The determination of the mean length of the wave groups from the spectrum is yet to be achieved.

Nevertheless, the characteristics exhibited by all sets of data are similar, indicating that there are no significant differences between the field data sets and the laboratory and numerical simulations. The primary reason for not being able to find a correlation between mean group length and spectral shape is the very large scatter in the groupiness parameters arising from finite length records of waves.

The overall conclusion of this work is that there is no reason to believe that the conventional specification of a sea state as a gaussian surface elevation determined solely by the spectrum will lead to errors in the representation of groups.

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1 Introduction

Waves on the sea surface are always more or less random giving the surface an appearance varying irregularly in space and time. One aspect of this irregularity is that at a fixed position the waves arrive in runs or groups of varying numbers of high waves. It is generally accepted that many floating structures have certain responses dominated by the excitation caused by groups. Inshore, it is the group structure of waves that can lead to harbour resonances, with the associated large slow drift motions of moored ships.

Analysis of field wave records and numerical simulation studies by various researchers suggest that the occurrence and length of wave groups is related to the wave spectrum. There is, however, considerable scatter in the data.

Questions have also been raised as to whether the conventional specification of a sea state as a gaussian surface elevation determined solely by the spectrum is adequate to define the groups correctly. This question reflects upon the adequacy of wave generation and modelling techniques which rely on the above specification.

This report describes the main groupiness parameters used and presents results from field and laboratory measured waves and computer simulated waves.

2 Wave group parameters and analysis procedures

Previous researchers have identified a number of parameters which characterise wave groupiness and spectral shape and these may be divided into four categories.

Energy spectrum shape parameters 2.1

Cartwright and Longuet-Higgins (1956) introduced the spectral width parameter

$$\epsilon = (1 - m_2^2/m_0/m_4)^{1/2}$$

where

m_0, m_2 and m_4 are the zeroth, second and fourth moments of the energy spectrum.

Goda (1970) introduced the spectral peakedness factor

$$Q_p = \frac{2}{m_0^2} \int_0^\infty f[E(f)]^2 df$$

where

$E(f)$ is the energy density with frequency, and

m_0 is the zeroth moment of the energy spectrum.

Nolte and Hsu (1972) introduced the parameter

$$K = ((m_0^2)/(m_2 m_0 - m_1^2))^{1/2}$$

where

m_0, m_1 and m_2 are the zeroth, first and second moments of the energy spectrum. The inverse of K is the RMS width of the energy spectrum about its centroid.

Wave correlation factor 2.2

Sequential individual wave heights (H_i) along the wave record are identified using a zero down crossing counting analysis based on the mean of the record. The wave correlation factor is then calculated from

$$\phi_H(n) = \frac{1}{\phi_H(o)} \cdot \frac{1}{N_z - n} \sum_{i=1}^{N_z - n} (H_i - \overline{H})(H_{i+n} - \overline{H})$$

where

$\overline{H}$ is the mean wave height

N_z is the number of zero crossing waves in the record

$$\phi_H(o) = \frac{1}{N_z} \sum_{i=1}^{N_z} (H_i - \overline{H})^2$$

when n = 1 the correlation coefficient between adjacent waves is found. For North Sea storms, Rye (1974) and Arhan and Ezraty (1978) found this value to lie between 0.2 and 0.3. For values of n greater than 1 the coefficient is usually small and may be positive or negative. Similar coefficients have been used for the wave by wave period and for the wave by wave joint height and period.

Individual wave groups 2.3

From the sequential individual wave heights (H_i) of a wave record the length of each group of successive waves exceeding a given height can be measured. Generally the significant wave height ($\overline{H}_3$) is taken as the reference height and for this report is always calculated from the energy spectrum using

$$\overline{H}_3 = 4\, m_0^{1/2}$$

By definition a single wave greater than or equal to $\overline{H}_3$ is counted as a group of 1 wave.

The data thus calculated is usually plotted to show the probability of occurrence against group length and is compared with theoretical curves.

The mean encounter period of the wave groups, TGBAR, is obtained by dividing the record length by the total number of groups.

Gross groupiness parameters 2.4

These give an overall measure of the groupiness of the record in a single value.

The first, (NGBAR), is the mean length of all the wave groups as identified in (2.3) above.

The second is the Groupiness Factor (GF) proposed by Funke and Mansard (1979). This is obtained by computing the Smoothed Instantaneous Wave Energy History (SIWEH) and transforming it to obtain the SIWEH spectrum. The Groupiness Factor is then found from

$$GF = \sqrt{m_{\epsilon 0}}/m_0$$

where

$m_{\epsilon 0}$ and m_0 are the zeroth moments of the SIWEH and energy spectra respectively.

The SIWEH is defined by

$$E(t) = \frac{1}{T_p} \int_{-\infty}^{+\infty} \eta^2 (t + \tau) \cdot Q(\tau) \cdot d\tau$$

where $Q(\tau)$ is a smoothing or window function

$$Q(\tau) = 1 - \tau/T_p \text{ for } -T_p \leq \tau \leq T_p$$
$$= 0 \text{ elsewhere}$$

Where T_p is the period of the peak of the spectrum. The mean group period is obtained from the zeroth and second moments of the SIWEH spectrum by

$$\overline{T}_{SIWEH} = (m_{\epsilon 0}/m_{\epsilon 2})^{1/2}$$

3 Results

Field waves 3.1

Field waves have been recorded by HR at Perranporth off the north coast of Cornwall since 1975. Measurements were made using a Datawell Waverider buoy anchored in 27m of water. 2048 data points of water surface elevation are recorded on magnetic tape at a sampling rate of $2H_z$ at 3 hourly intervals. A sample of these records, from June to December 1982, have been analysed to yield the wave groupiness parameters defined in the previous section.

Plots of GF and NGBAR against Q_p and NGBAR against GF suggest a direct correlation between the parameters, albeit very weak in some cases (Figures 1, 2 and 3). However, the main feature of the plots is the large scatter in the data; a fact reported by other researchers (eg Rye 1981). Plots of the other wave groupiness parameters show the same feature but are not presented in this report.

The field wave data consists of a wide variety of conditions: storm generation and decay, breaking and non-breaking waves, deep water and non-deep water spectra, swell and locally generated waves, etc. Selection of wave records having smooth single peaked deepwater spectra and with non-breaking waves did not reduce the scatter of the data.

Laboratory waves 3.2

Long crested deep water non-breaking random wave trains were generated in the Offshore Sea Facility at HR. The principle of generating the waves in this facility is based on the digital filtering of a white noise source. This method gives rise to waves which are according to the conventional understanding of random waves (Fryer, Gilbert and Wilkie, 1973). Four spectra were used: bimodal, Moskowitz, Jonswap (γ = 3.3) and narrow (Figure 4). A sample from each wave train is given on Figure 5. For each spectrum 192k consecutive water surface level values were taken giving at least 7 hours laboratory time.

The data was analysed, using the procedures described above, in blocks of 2k data points thus giving 96 estimates of each parameter for each spectrum.

Laboratory/field comparisons 3.2.1

Plots of GF and NGBAR versus Q_p and NGBAR versus GF (Figures 6, 7 and 8) corresponding to Figures 1, 2 and 3 show the same correlations between the parameters as for the field data. Superposed on each plot is a dashed line which outlines

the spread of the field wave data (cf Figures 1, 2 and 3).

Three points arise from these results:

1) The scatter of the laboratory data is not significantly different to that of the field data despite the stationarity of the input laboratory wave spectra. The data for the Jonswap spectrum was re-analysed for 192/1k blocks, 48/4k blocks and 24/8k blocks. Plotting the standard deviation of Q_p, GF and NGBAR against block length (Figure 9) shows that some reduction in the scatter will result from using longer block lengths, particularly for the parameter Q_p, but that little further reduction would be gained with block lengths greater than 8k. Large scatter in the wave groupiness parameters appears to be an inherent feature even under controlled laboratory conditions. It may be noted that an 8k block of field data corresponds to about one hour in which time large variations of $\overline{H}_3$ can occur which will further increase the scatter in the parameters.

2) The Groupiness Factor (GF) does not appear to be sufficiently sensitive. The four spectra were chosen to give a wide range of groupiness. The mean and standard deviation of GF for each spectrum measured over 2k sample lengths is:

Bimodal 0.781 ± 0.079
Moskowitz 0.748 ± 0.061
Jonswap 0.814 ± 0.066
Narrow 1.055 ± 0.113

These values and the data plotted on Figure 6 highlight the problems with GF. Given a value of GF the spread of the data is such that it would be impossible to identify the spectrum particularly between the bimodal, the Moskowitz and the Jonswap spectra. The difference in GF between the Jonswap and the narrow spectra is also relatively small bearing in mind the scatter of the data. The parameter, NGBAR, shows a better distinction between the Jonswap and the narrow spectra.

3) The spectral peakedness factor Q_p appears to be insensitive at the lower values with little distinction between the Moskowitz and the bimodal spectra (Figures 6 and 7). Plotting NGBAR against the parameter K normalised by $\overline{T}$, the mean zero crossing wave period, Figure 10, shows a slightly better distinction between the bimodal and the Moskowitz spectrum but at the expense of a smaller distinction between the Moskowitz and the Jonswap spectrum. The K parameter has less scatter than Q_p but neither appear totally adequate. The spectral width parameter, ϵ, was also calculated but was obviously inadequate. This parameter is highly dependent on the high frequency cut off (Rye 1981). The data for K and Q_p show some dependance upon the high frequency cut off. To minimise this effect all the laboratory data has been evaluated for the same cut off frequency. The field data has also been evaluated to a constant (but different) cut off frequency. Goda (1983) shows the parameter Q_p to be dependent upon the degree of freedom. For consistency all spectra in this present report have the same degrees of freedom.

Other groupiness parameters 3.2.2

The remaining groupiness parameters considered in this report are the wave by wave correlation factors , the individual wave groups and the mean occurrence period of the wave groups.

(i) The correlation factor, $\phi_H(n)$, defined in Section 2.2, for values of n is plotted on Figure 11. The value plotted is the mean of the 96/2k samples. For clarity the standard deviation is noted beside each point. As expected the coefficient increases with increasing groupiness and for each spectrum decreases with increasing values of n. An indication of the scatter in the parameter under stationary conditions is shown in the following values from two blocks of 2k data taken from the Jonswap wave data set. For n = 1, 2 and 3 $\phi_H(n)$ was 0.487, 0.279 and 0.230 for one block and 0.349, −0.042 and −0.058 for the other block. Those latter values are similar to the mean values for the Moskowitz data set (Figure 11). A comparison with the field data was made using only those data with smooth single peaked spectra. The data shows reasonable agreement with the laboratory generated waves (Figure 11).

(ii) The number of groups containing J successive waves greater than or equal to $\overline{H}_3$ (Section 2.3) are listed below for the laboratory data and plotted on Figures 12-15 as the probability of occurrence of a group of J waves, P(J) versus J.

J	Bimodal	Moskowitz	Jonswap	Narrow
1	2089	1813	1350	497
2	397	488	510	275
3	55	71	159	130
4	8	8	40	153
5	2		11	114
6			2	75
7			1	55
8				49
9				36
10				31
11				16
12				11
13				7
14				4
15				2

Superposed on each figure is the theoretical curve based on the assumption that adjacent waves are correlated and by considering the problem as a two state Markov process. The theory is given in Appendix 1 and shows the probability of a given wave group length to be a function of the mean group length. Given the mean group length the theory predicts the group distribution. Comparison of the measured laboratory values with this is good. Similar plots of the field wave data are shown on Figure 16. One set of points shows the results using all the data measured over the six month period. The other set of points, giving higher probabilities of occurrence, is for selected smooth single peaked spectra having a value of

$Q_p > 2.5$. Superposed on the figure are the theoretical curves for the laboratory Moskowitz and Jonswap spectra. The wave grouping of the laboratory generated spectra compares well with the field data.

(iii) The mean occurrence period of the wave groups is shown plotted against K on Figure 16 using both the individual wave group definition (Section 2.3) and the SIWEH definition (Section 2.4). The mean and standard deviation of 24/8k data blocks for each spectrum are plotted and the period is normalised by the mean zero crossing period of the waves. The data show an increasing trend of occurrence period with increasing K. The near 3 fold difference in the two sets of results reflects the fact that the individual wave group method only identifies groups of higher waves in the record ($\overline{H}_3$ in this report) whereas the SIWEH method identifies all groupiness.

Wave directionality 3.2.3

Laboratory wave records were also made of short crested seas characterized by the Moskowitz and Jonswap spectra already described (Section 3.2) but with a $\cos^2\theta$ spreading function applied. There is no evidence of a statistically meaningful difference in any of the calculated groupiness parameters between the long crested and short crested records.

Computer simulated waves 3.3

The possibility that wave dispersion effects might contribute to the wide scatter in the groupiness parameters was checked using computationally derived wave data.

The wave signal was derived simulating the generation technique used in the HR spectrum synthesizer namely a digitally filtered white noise source generated by shift register techniques. The programme generated a time series wave train for a given wave spectrum producing a file of 204 800 sequential water surface elevations (ie 100 x 2048). Wave trains for the following 5 spectra were generated:

1. Moskowitz
2. Jonswap $\gamma = 3.3$
3. Jonswap $\gamma = 7.0$
4. Narrow
5. Narrow

These data were analysed to yield the spectral and groupiness parameters as for the field and laboratory measured data. Figure 18 shows the results plotted NGBAR versus Q_p. Comparison with Figure 7, the laboratory measured data, shows no significant difference due to wave dispersion effects.

The parameter NGBAR 3.3.1

The parameter NGBAR, the mean length of all wave groups in a record, is based upon waves equal to or greater than $\overline{H}_3$ and hence only uses about 10-15% of the waves. The wave data was further analysed for groups of waves exceeding $\overline{H}$ to see if the scatter in the data reduced. Figure 19 shows the results, NGBAR versus Q_p, plotted as the mean and standard deviation of the 100 records for both the $\overline{H}_3$ and $\overline{H}$ definition of groups. There is about the same scatter for both sets of results.

4 Conclusions

There is an inherent large scatter in the wave groupiness parameters even under carefully controlled and stationary laboratory conditions.

This scatter can be reduced by increasing the sample length up to about 8k. Above this value little further reduction occurs.

Wave groupiness increases with increasing peakedness of the spectrum.

The parameter, K, appears to give a better definition of spectral peakedness at the lower values.

The Groupiness Factor parameter (GF) is less sensitive than the mean group length parameter (NGBAR) in defining groupiness in relationship to the scatter in the data.

Wave directionality does not affect the groupiness.

The distribution of wave groups is well defined from the mean group length but the mean group length cannot yet be determined from the spectrum.

Overall, the characteristics exhibited by all sets of data are similar, indicating that there are no significant differences between the field data sets, and the laboratory and numerical simulations. The primary reason for not being able to find a correlation between mean group length and spectral shape is the very large scatter in the groupiness parameters arising from finite length records of waves.

However, despite this, there is no reason to believe that the conventional specification of a sea state as a gaussian surface elevation determined solely by the spectrum will lead to errors in the representation of groups.

5 Acknowledgements

This report was written by Mr R Shuttler. The theory in the Appendix was derived by Dr S W Huntington and Mr G Gilbert.

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Appendix

Appendix

Theory for wave group distribution

Let the probability that j waves larger than $H_{1/3}$ succeed each other in one single run be denoted by P(j). Provided that only waves larger than $H_{1/3}$ are considered then

$$\sum_{j=1}^{\infty} P(j) = 1 \qquad (1)$$

And, provided the wave heights follow a Rayleigh distribution

$$\text{Prob}\,(H > H_{1/3}) = P = \exp\left(-\frac{H_{1/3}^{\,2}}{8m_0}\right) = \exp(-2) = 0.135 \qquad (2)$$

Uncorrelated waves

After a wave of height greater than $H_{1/3}$ the chance of a second being found is P, and the chance of a second not being found is $1-P$, so

$$P(1) = 1-P$$

$$P(2) = P(1-P)$$

$$P(j) = P^{j-1}(1-P) \qquad (3)$$

note that $\sum_{j=1}^{n} P(j) = (1-P)\,(1 + P + P^2 \ldots\ldots P^n)$

$$= (1-P)\left(\frac{1-P^n}{1-P}\right)$$

$= 1$ for large n since $P < 1$ as equation (1).

The mean length of a run is given by

$$m = \sum_{j=1}^{\infty} j\,P(j) \qquad (4)$$

Now $\sum_{j=1}^{n} j\,P(j) = 1(1-P) + 2(P(1-P)) + 3(P^2(1-P)) + \ldots\ldots\ldots$

$$+ n(P^{n-1}(1-P))$$

$$= 1 + P + P^2 + P^3 \ldots\ldots + P^{n-1} - nP^n$$

$$= \frac{1-P^n}{1-P} - nP^n$$

as $n \to \infty$ then $m = \dfrac{1}{1-P}$ (5)

It is convenient to plot LogP(j) against j.

Now $\text{Log}P(j) = (j-1)\log P + \log(1-P)$

$$= j \log P + \log \frac{1-P}{P}$$

So we should get a straight line of slope logP and intercept $\log (1-P)/P$.

The slope and intercept can be fixed with a knowledge of the mean run length only.

$$\text{Slope} = \log \frac{m-1}{m} \tag{6}$$

$$\text{Intercept} = \log \frac{1}{m-1} \tag{7}$$

Correlated waves

Allow for correlation between waves by considering the problem as a two state Markov process. After having found a wave height greater than $H_{1/3}$ let the probability of finding a second be p, and hence the probability of not finding one is $1-p$. However, if a wave height smaller than $H_{1/3}$ is found let the probability of the next wave also being less than $H_{1/3}$ be q, and hence the probability of returning to the state of exceeding $H_{1/3}$ is $1-q$. The overall probability of any wave exceeding $H_{1/3}$ remains as P. P, p and q are related by

$$P = pP + (1-q)(1-P) \tag{8}$$

Now

$$P(j) = p^{j-1}(1-p) \tag{9}$$

and the mean length of runs of waves greater than $H_{1/3}$

$$m = \frac{1}{1-p} \tag{10}$$

Now, define n as the mean length of runs of waves less than $H_{1/3}$

$$n = \frac{1}{1-q} \tag{11}$$

and define a mean total run length M where a total run is the number of waves between the first exceedance of $H_{1/3}$ by a group of waves and the first exceedance by the succeeding group of waves.

Hence

$$M = m + n \tag{12}$$

now, using equation (8)

$$P = \frac{m-1}{m} P + (1-(\frac{n-1}{n}))(1-P)$$

$$= \frac{m}{M} \text{ as expected.} \tag{13}$$

Figures

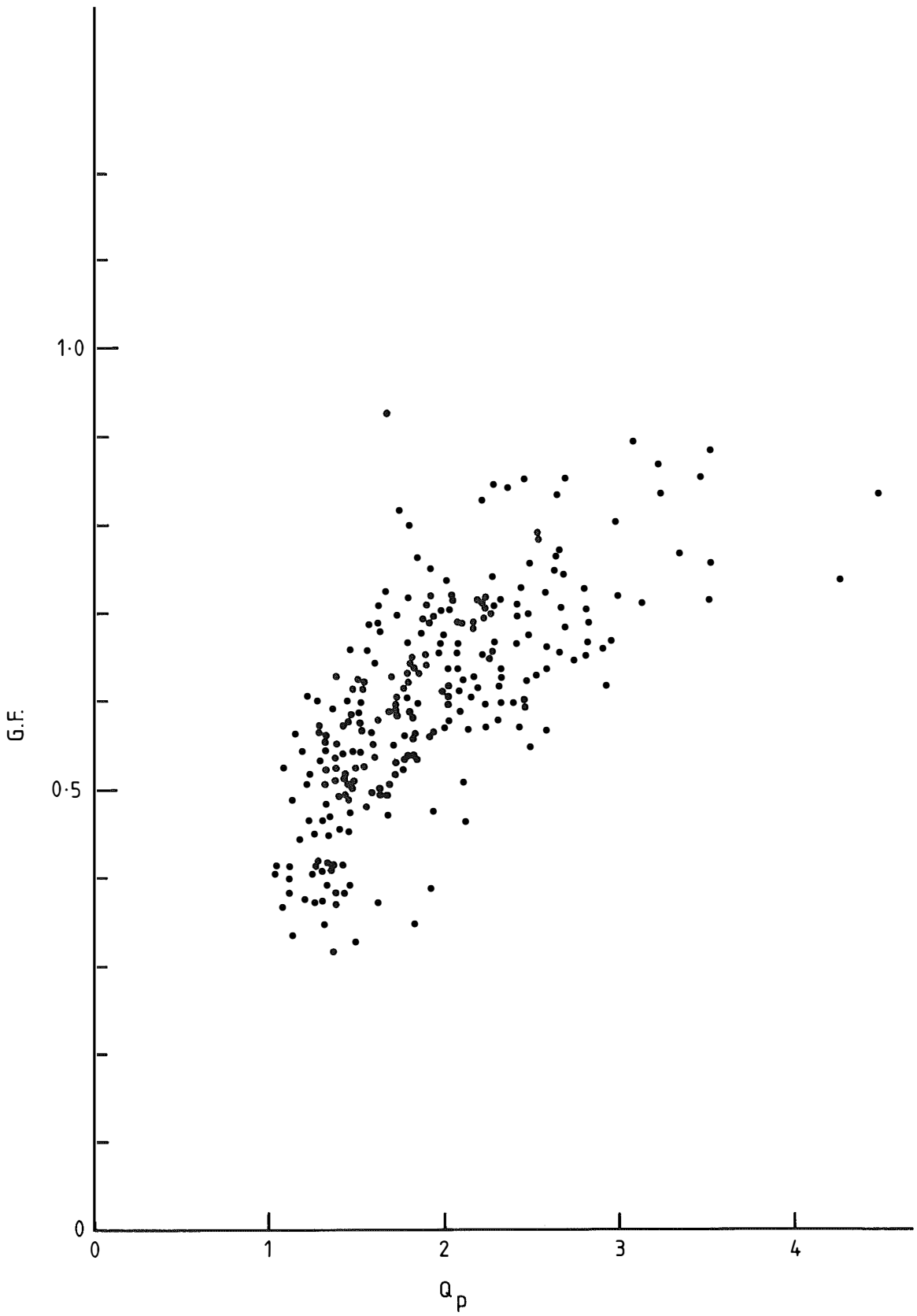

Fig. 1 Field wave data - Groupiness Factor vs Spectral Peakedness Parameter

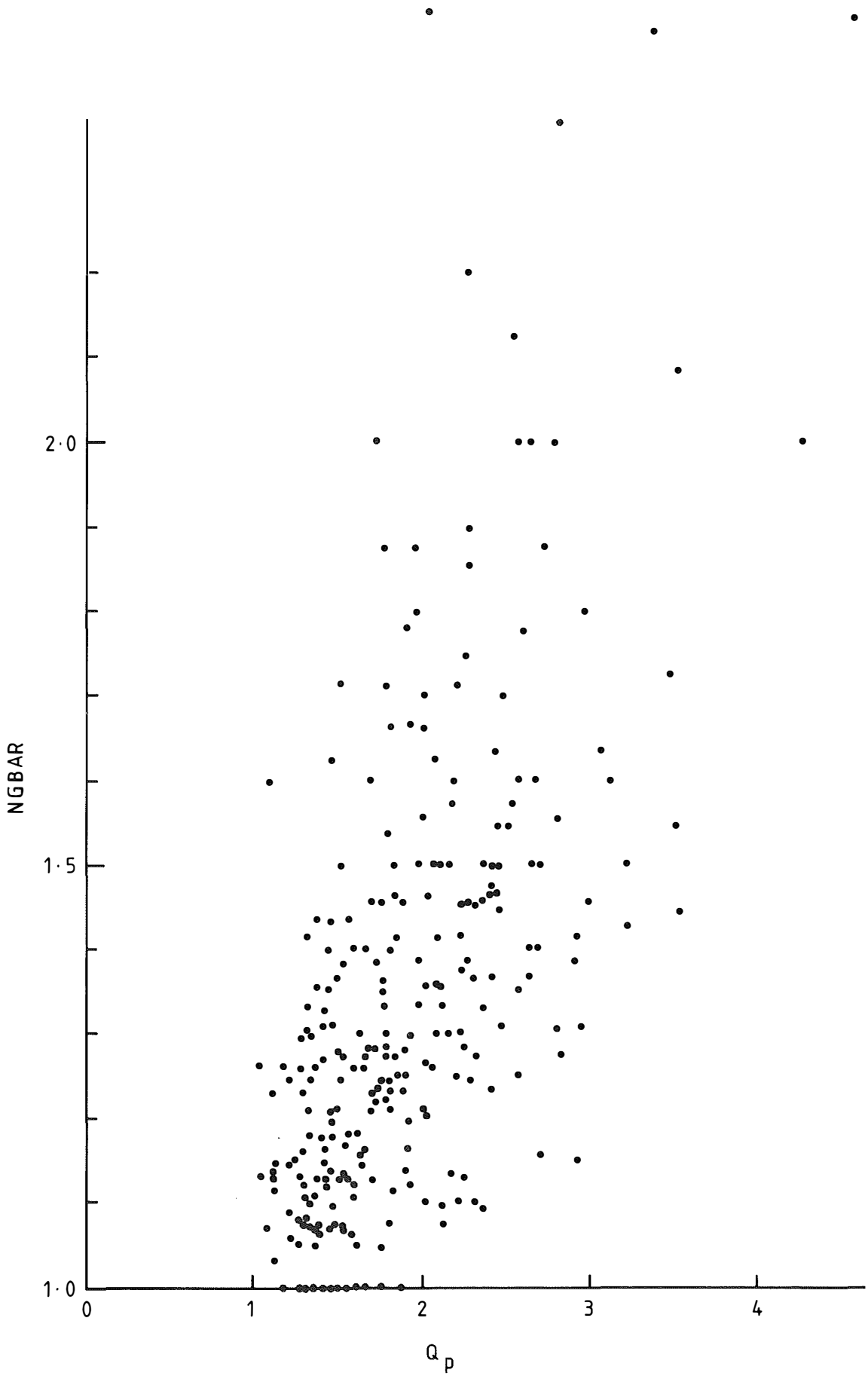

Fig. 2 Field wave data - Mean Group Length vs Spectral Peakedness Parameter

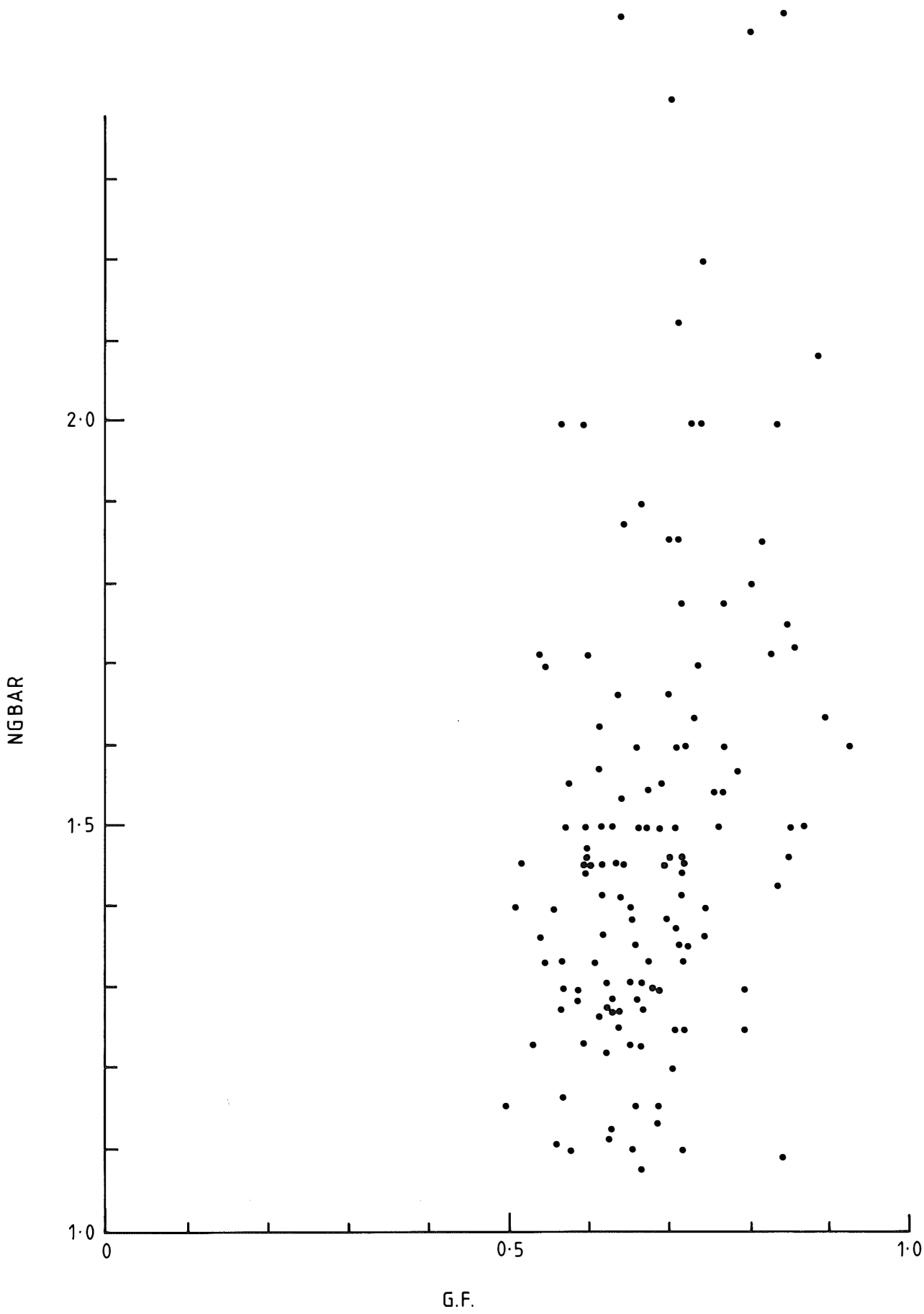

Fig. 3 Field wave data - Mean Group Length vs Groupiness Factor

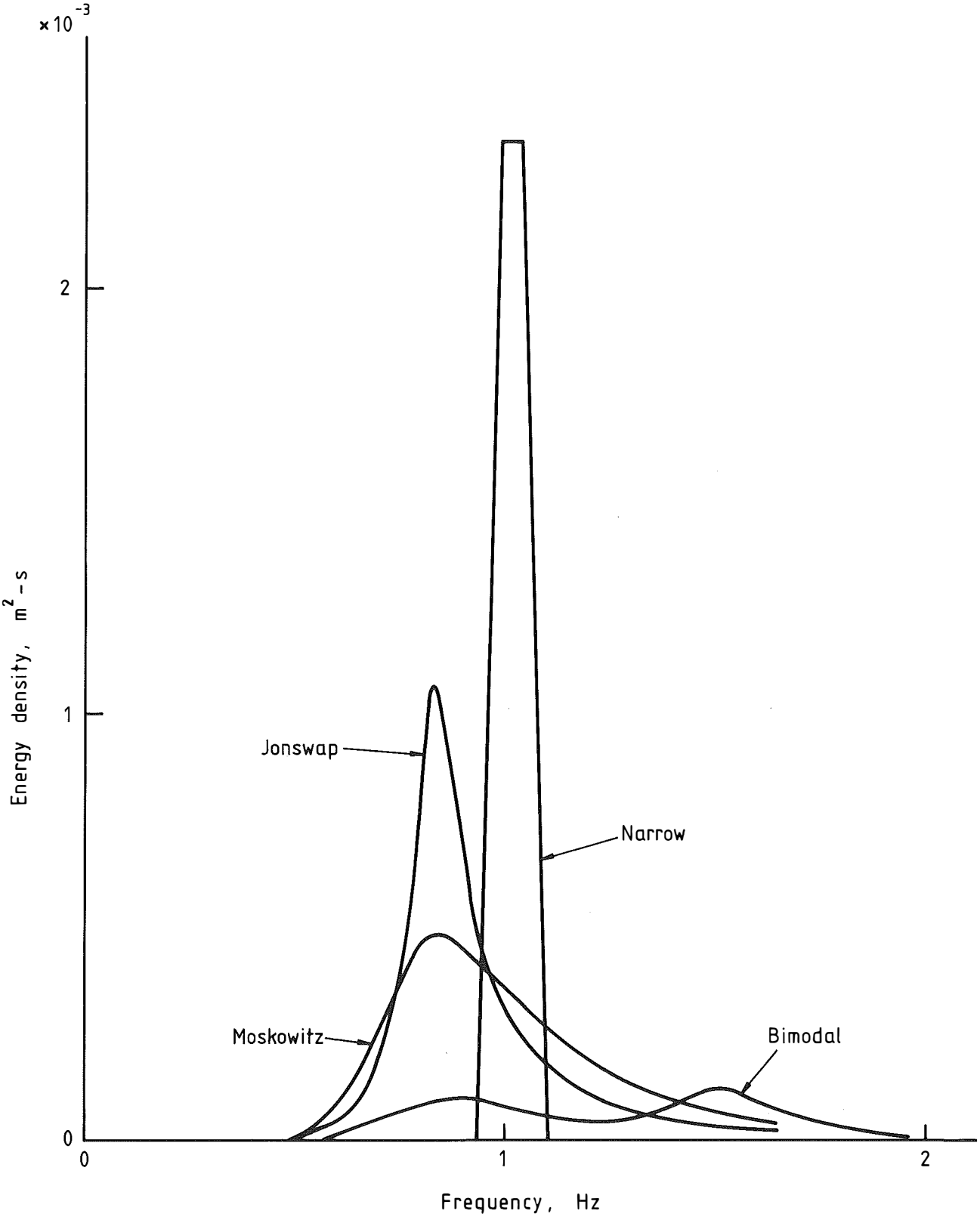

Fig. 4 Laboratory wave spectra

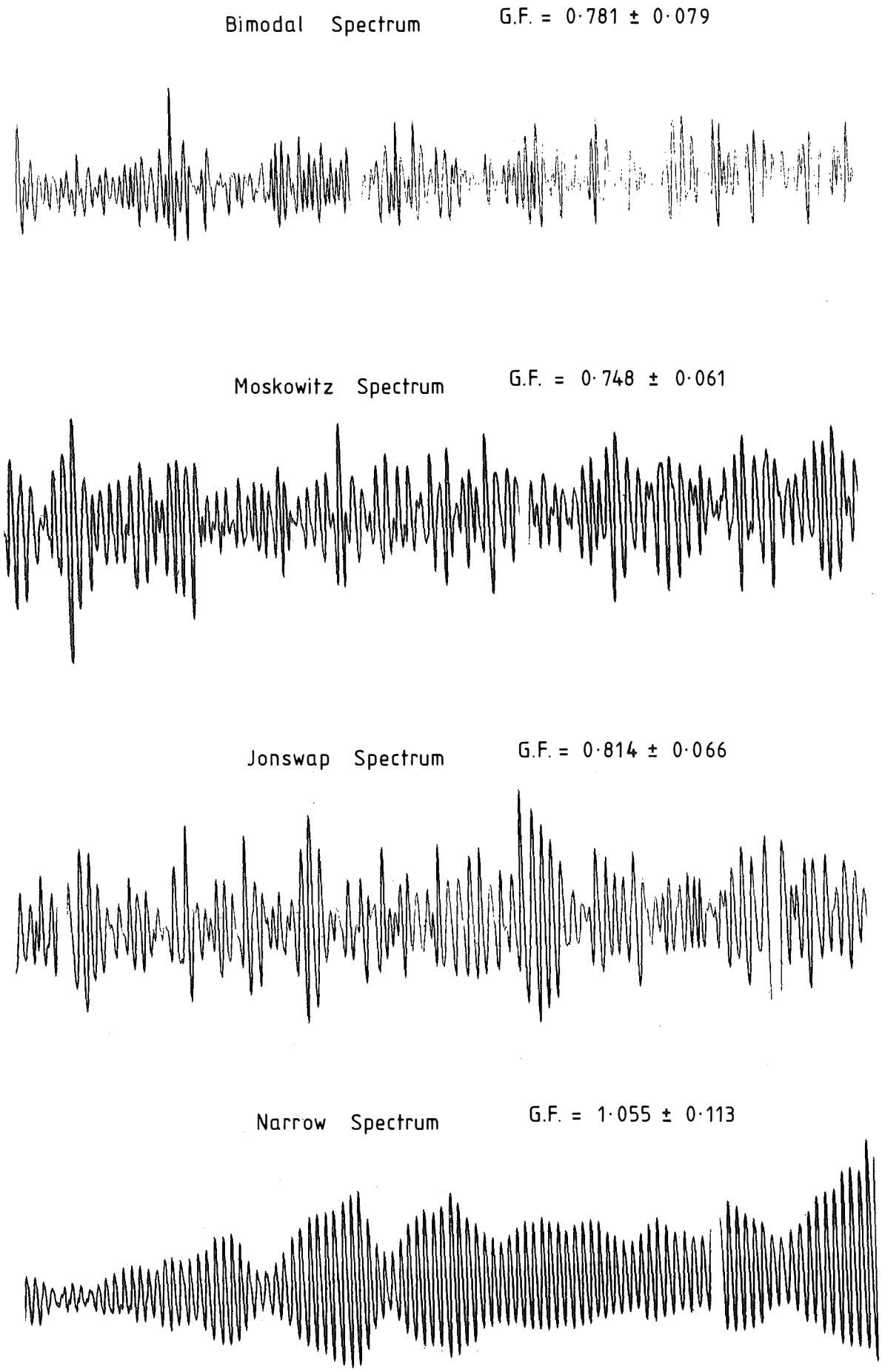

Fig. 5 Sample wave traces from four laboratory wave spectra

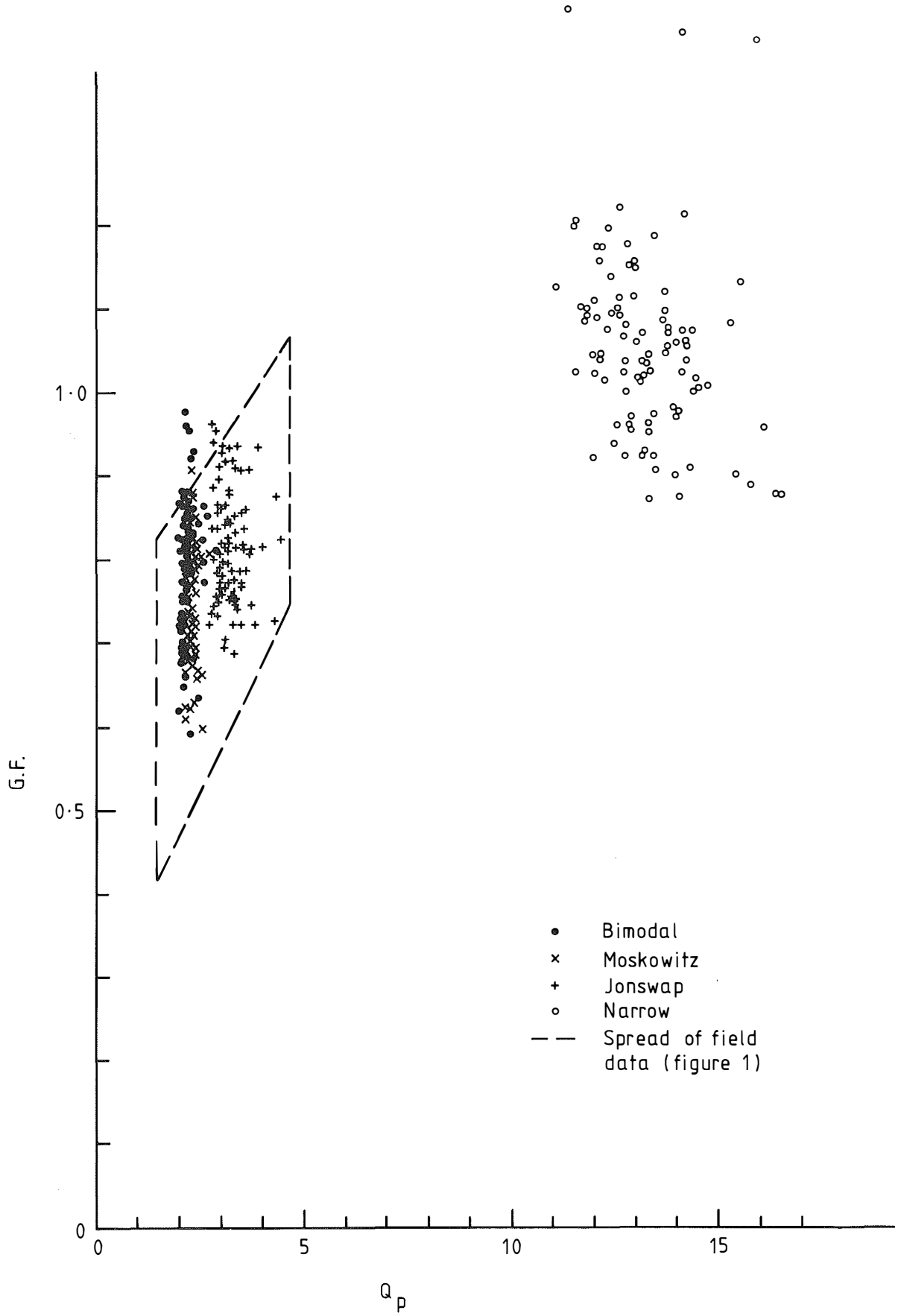

Fig. 6 Laboratory wave data - Groupiness Factor vs Spectral Peakedness Factor

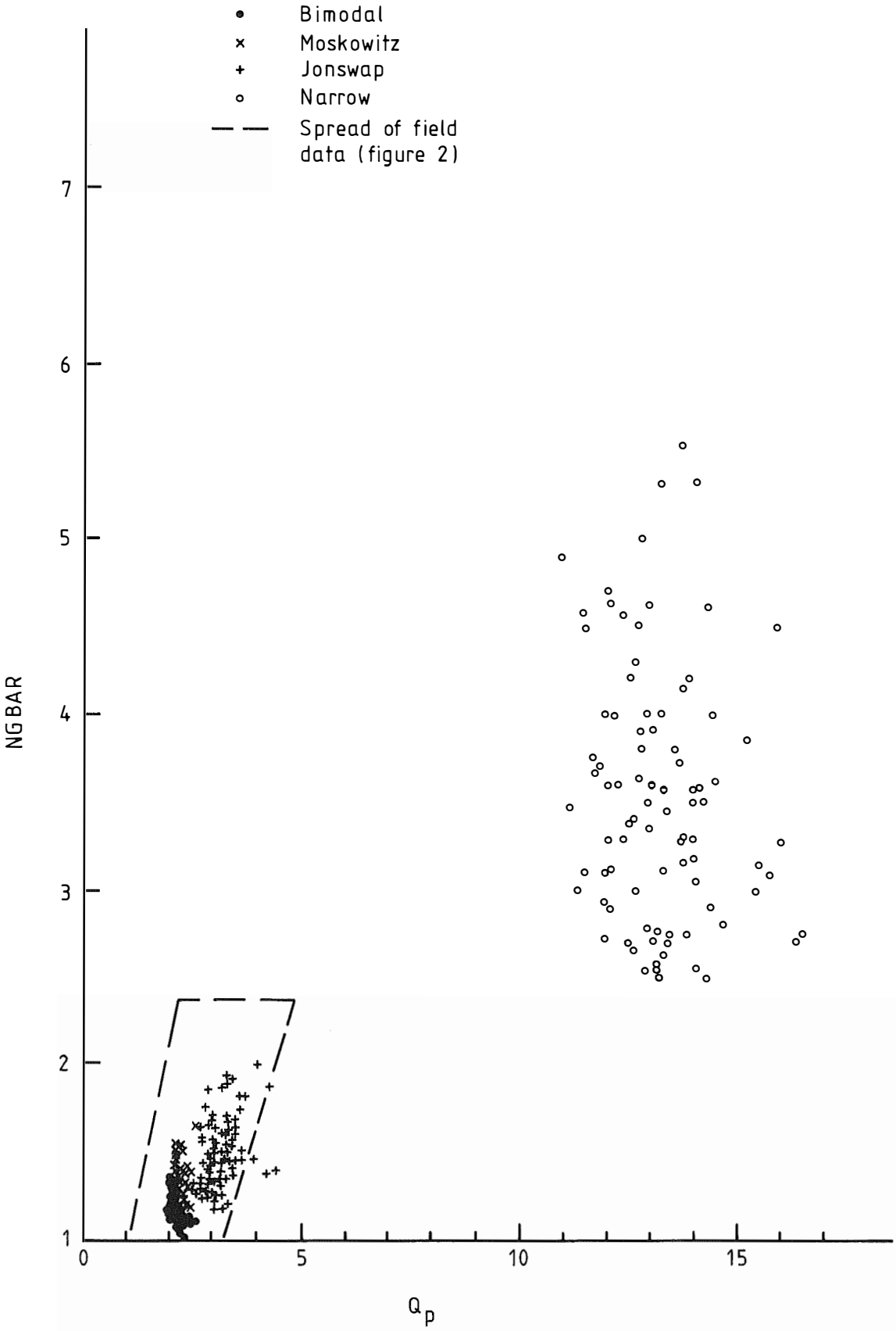

Fig. 7 Laboratory wave data - Mean Group Length vs Spectral Peakedness Factor

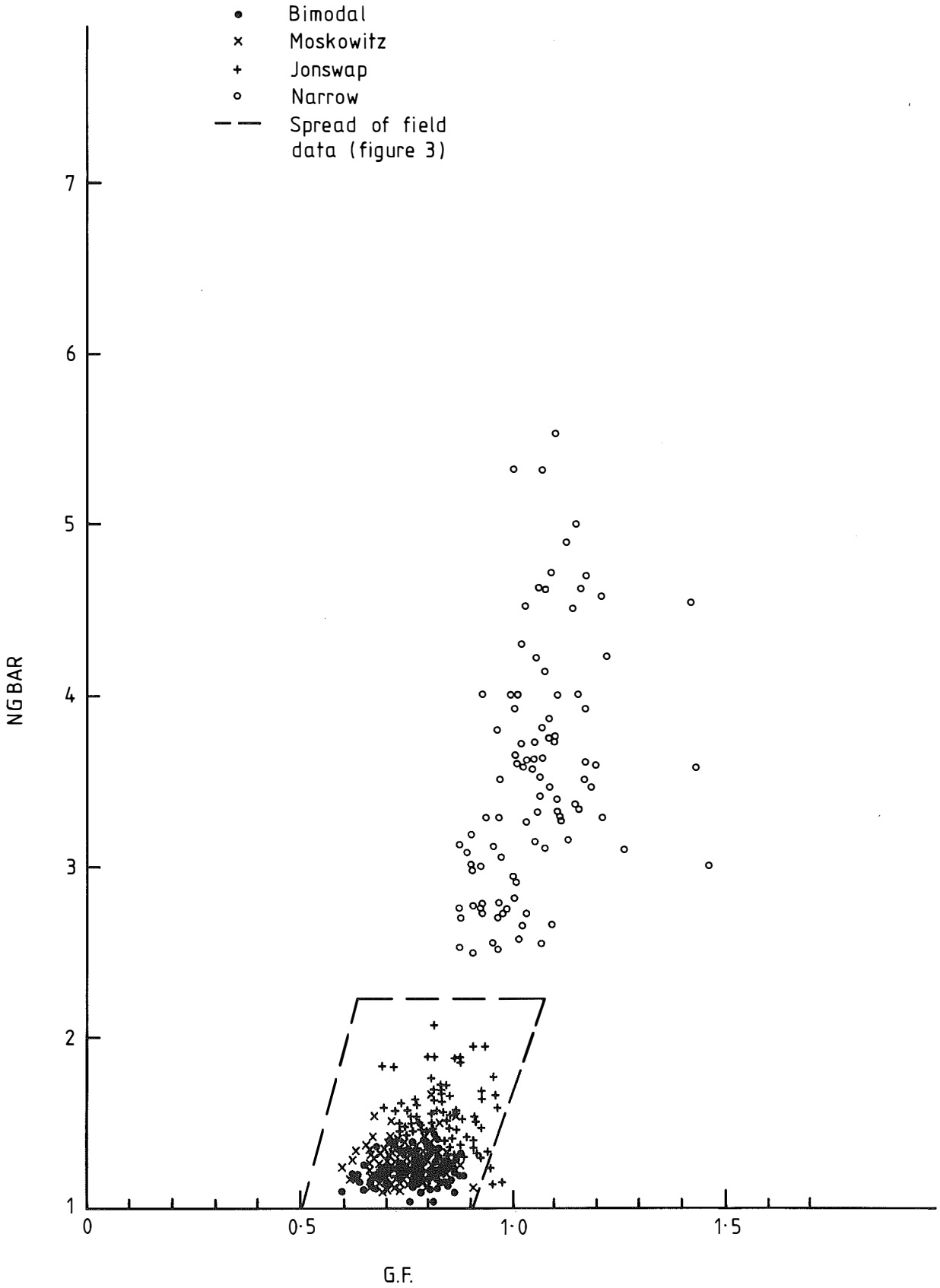

Fig. 8 Laboratory wave data - Mean Group Length vs Groupiness Factor

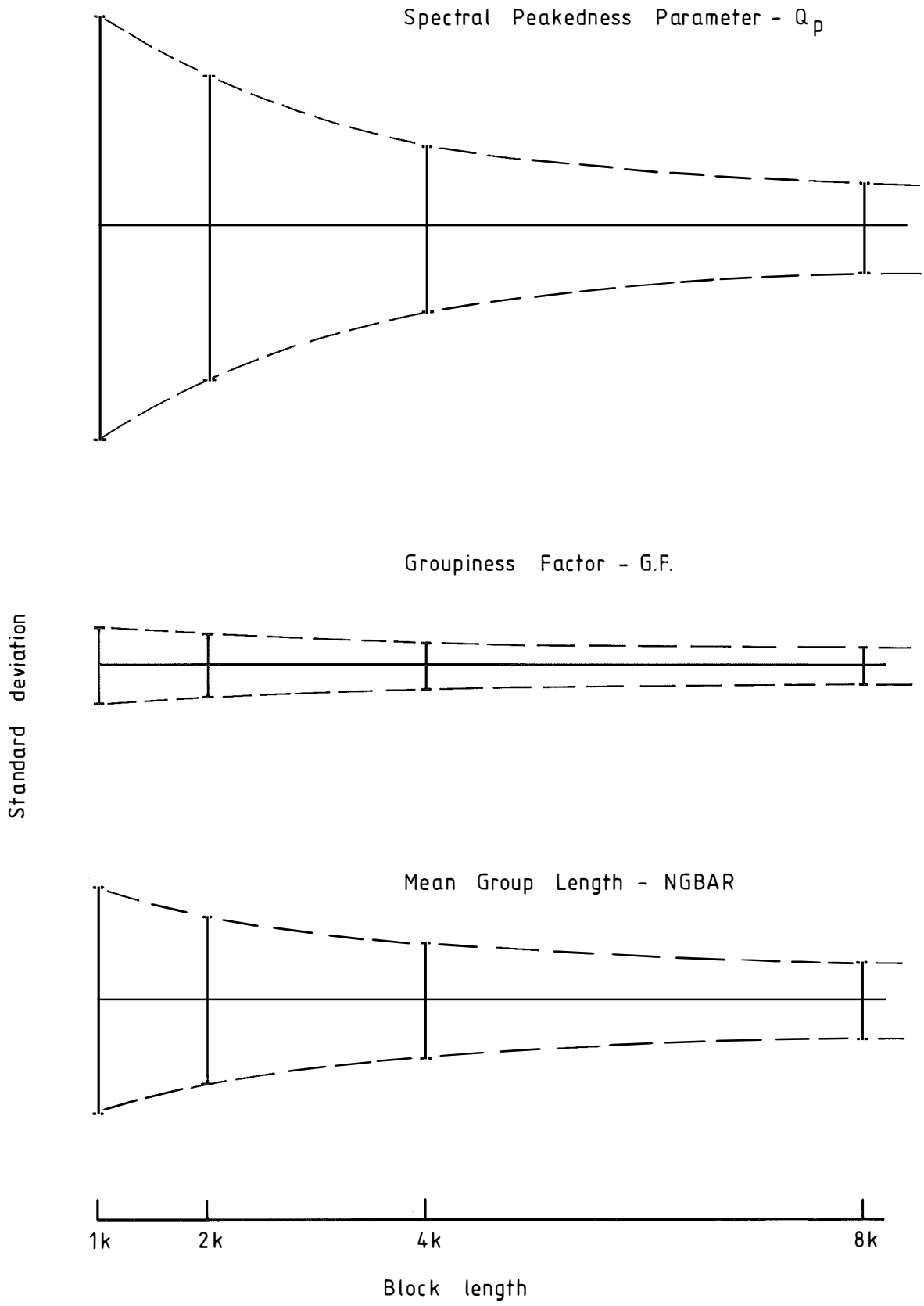

Fig. 9 Reduction of scatter with increasing block length

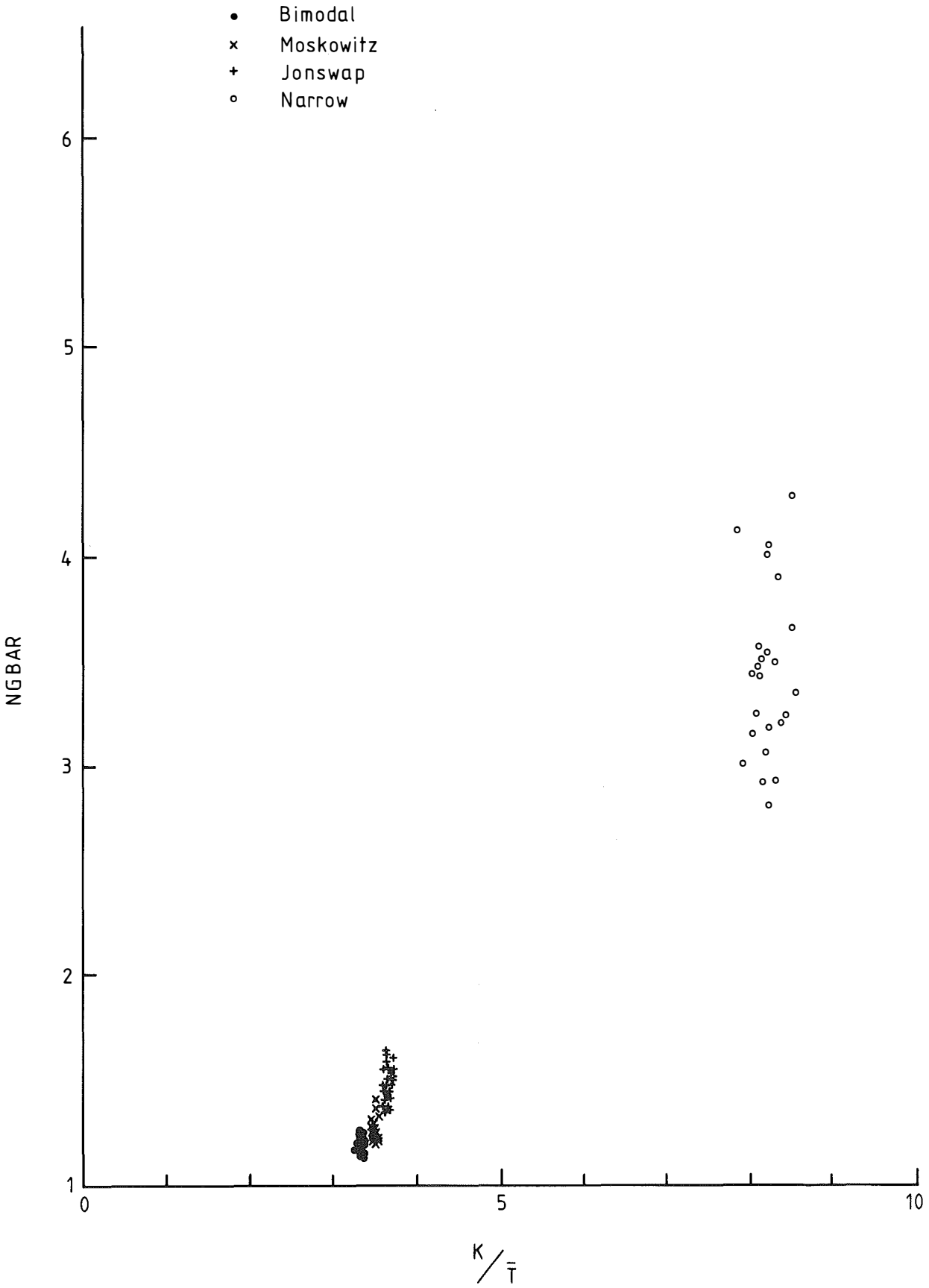

Fig. 10 Laboratory wave data - Mean Group Length vs $K/\bar{T}$

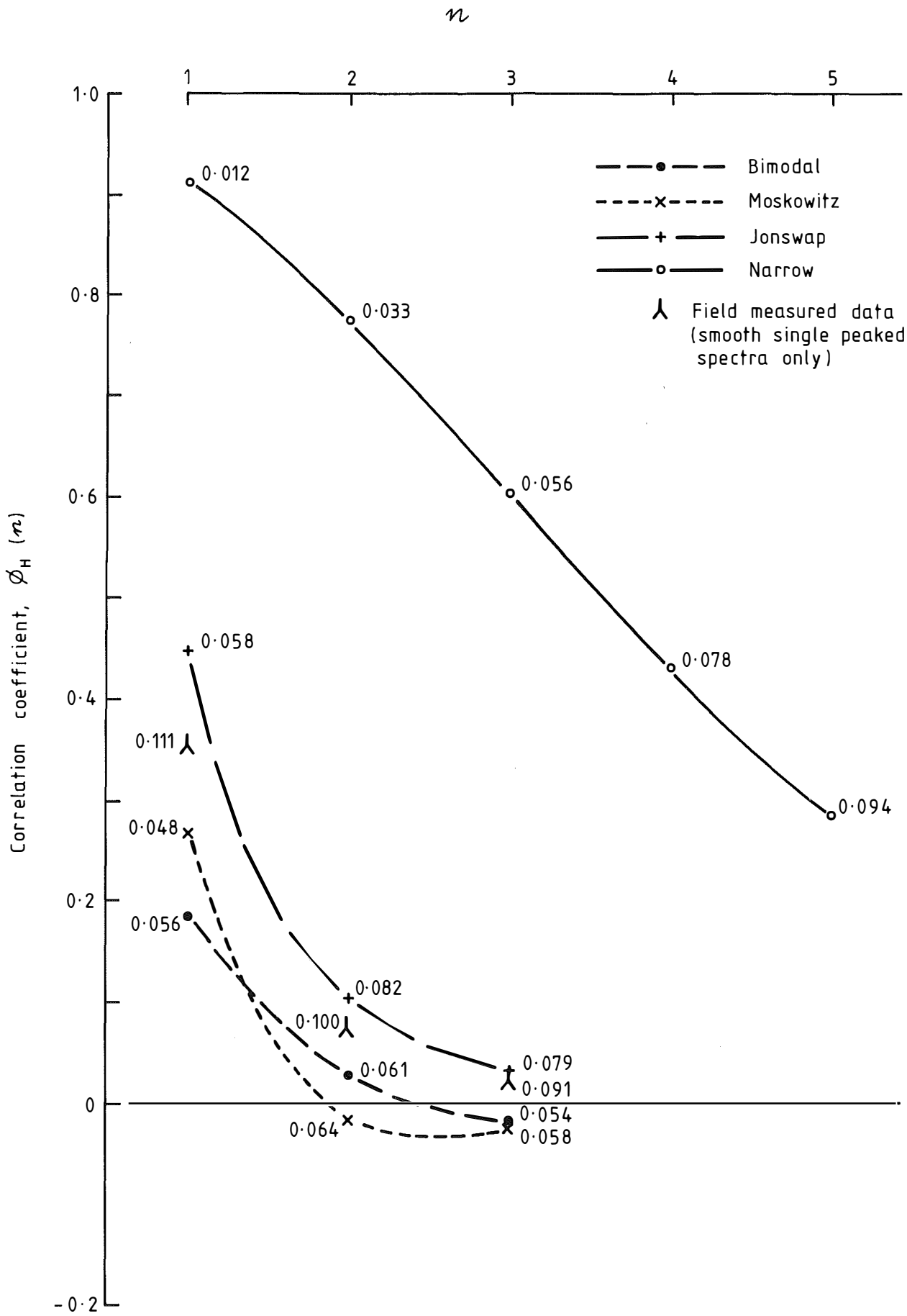

Fig. 11 Laboratory wave data - Correlation coefficients

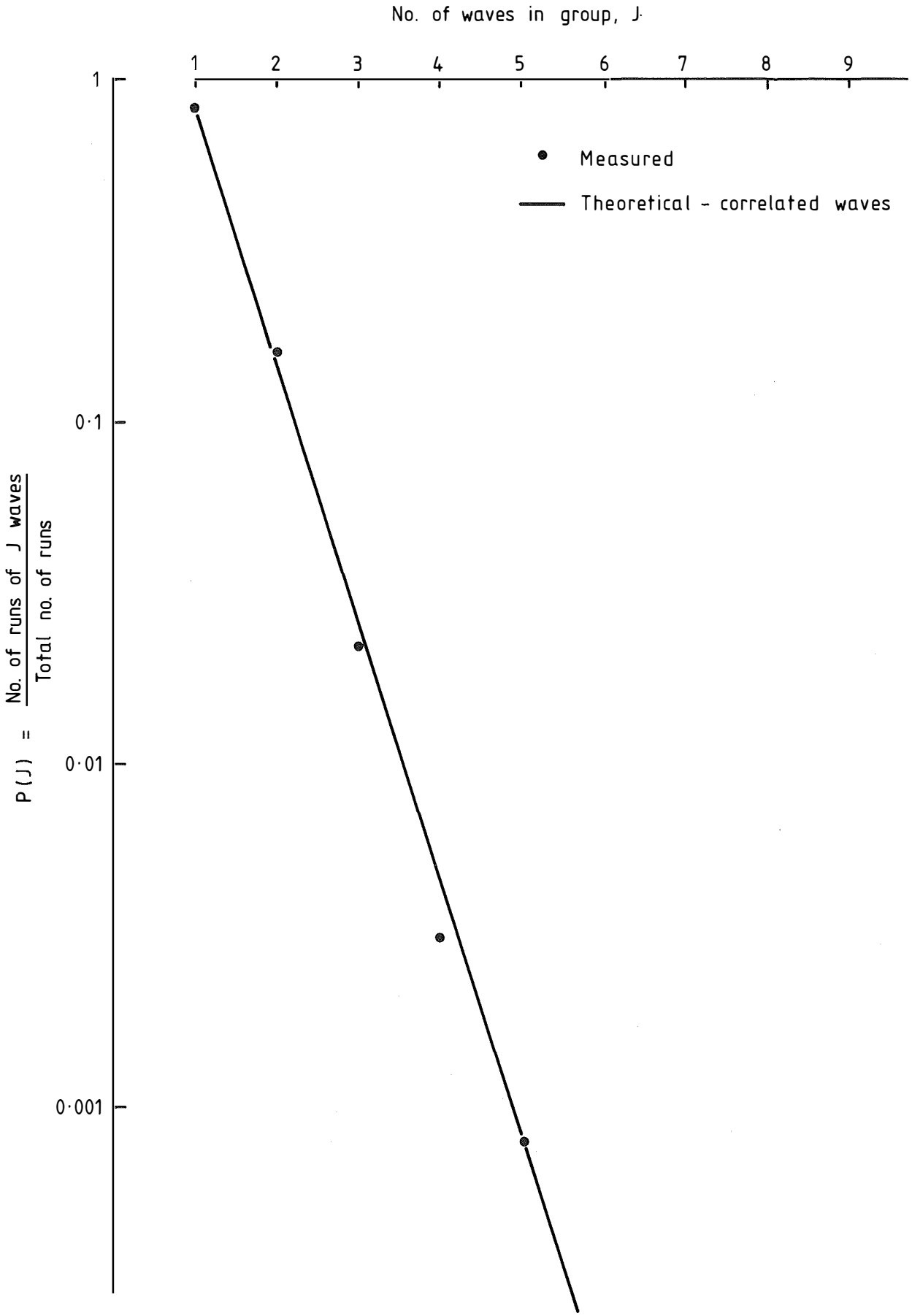

Fig. 12 Laboratory wave data - Bimodal Spectrum.
Runs of waves greater than or equal to $\bar{H}_3$

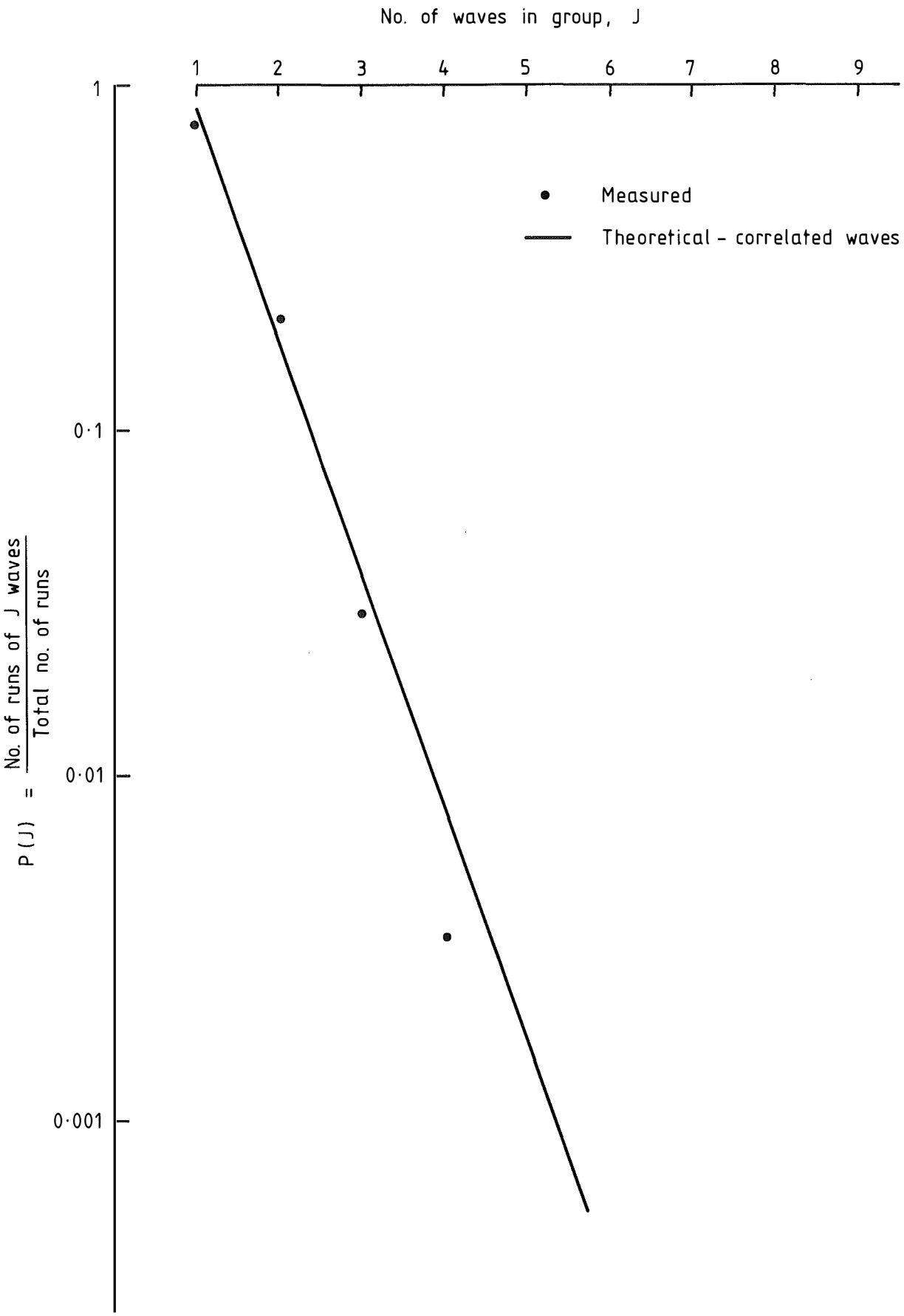

Fig. 13 Laboratory wave data - Moskowitz Spectrum. Runs of waves greater than or equal to $\bar{H}_3$

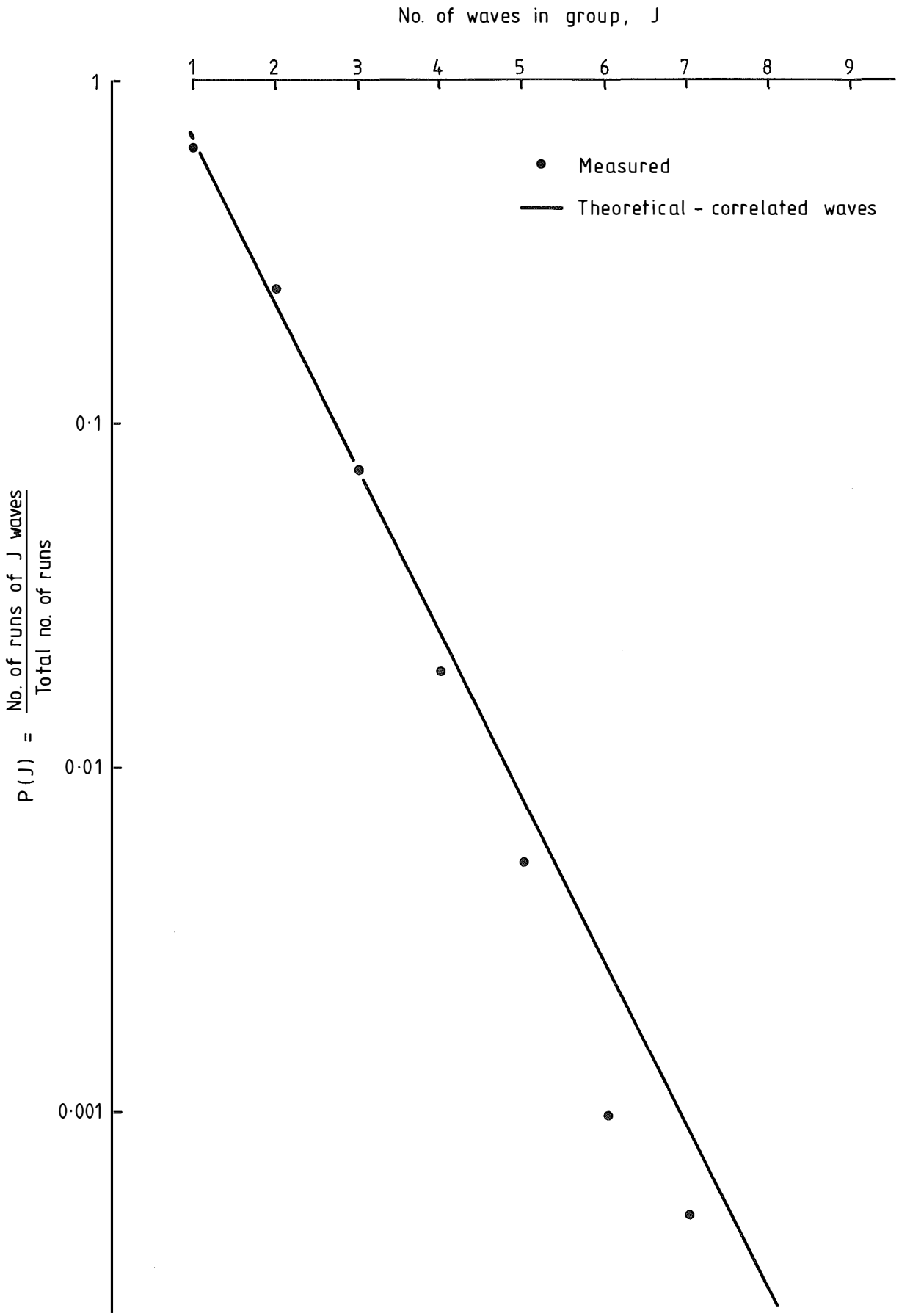

Fig. 14 Laboratory wave data - Jonswap Spectrum. Runs of waves greater than or equal to $\bar{H}_3$

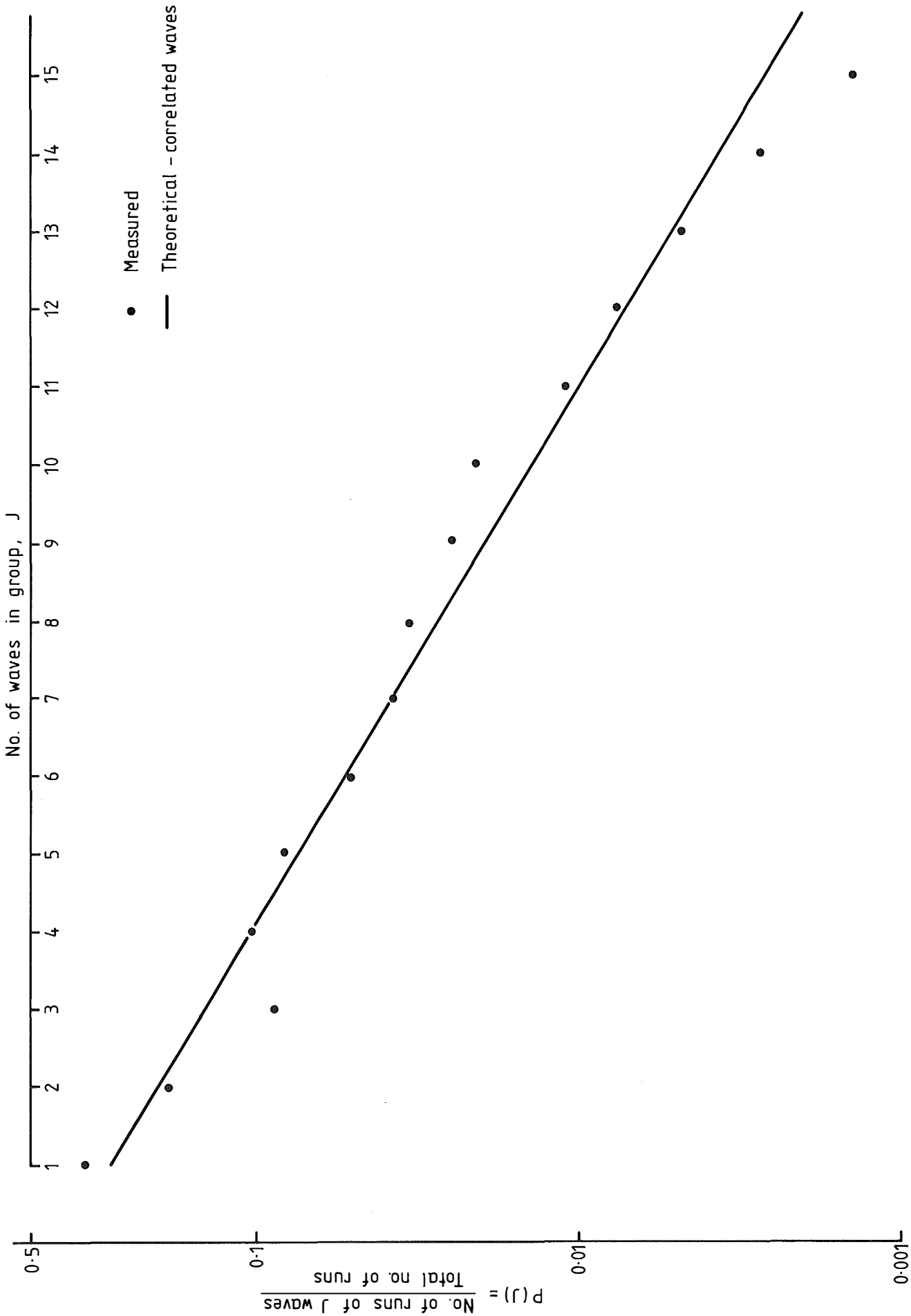

Fig. 15 Laboratory wave data - Narrow Spectrum. Runs of waves greater than or equal to $\bar{H}_3$

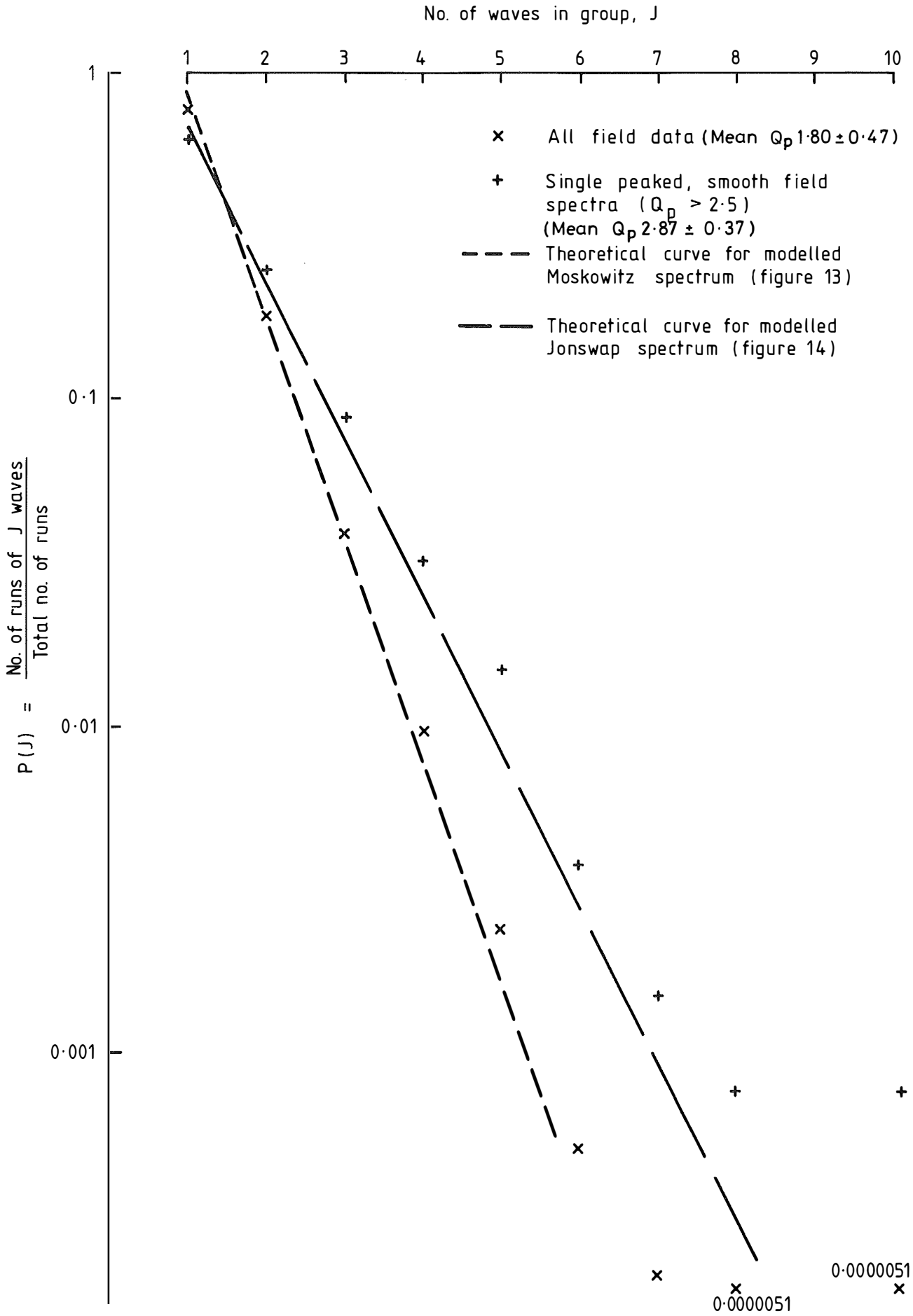

Fig. 16 Field wave data -
Runs of waves greater than or equal to $\bar{H}_3$

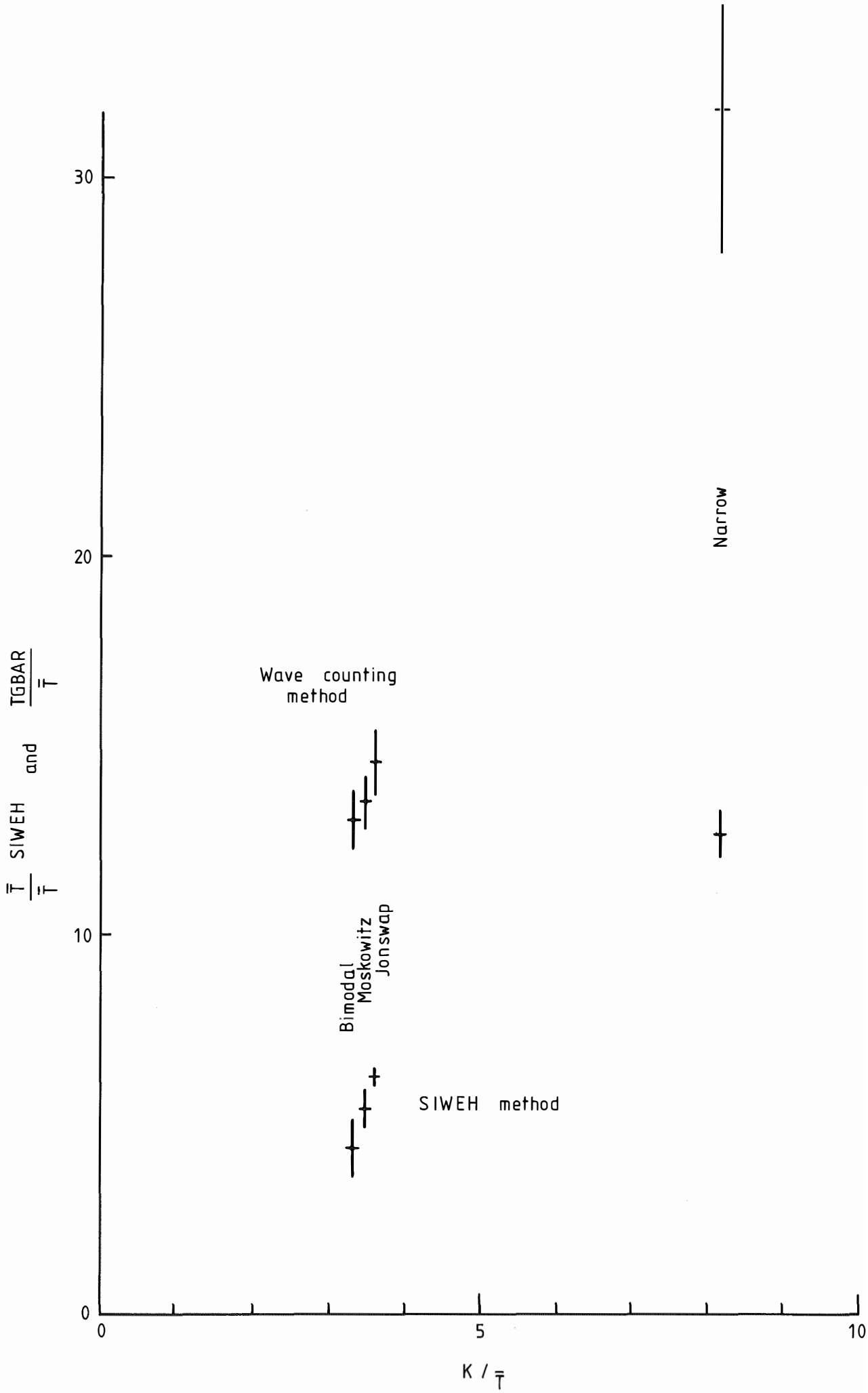

Fig. 17 Laboratory wave data - Mean Group Occurrence Period vs K / $\bar{T}$

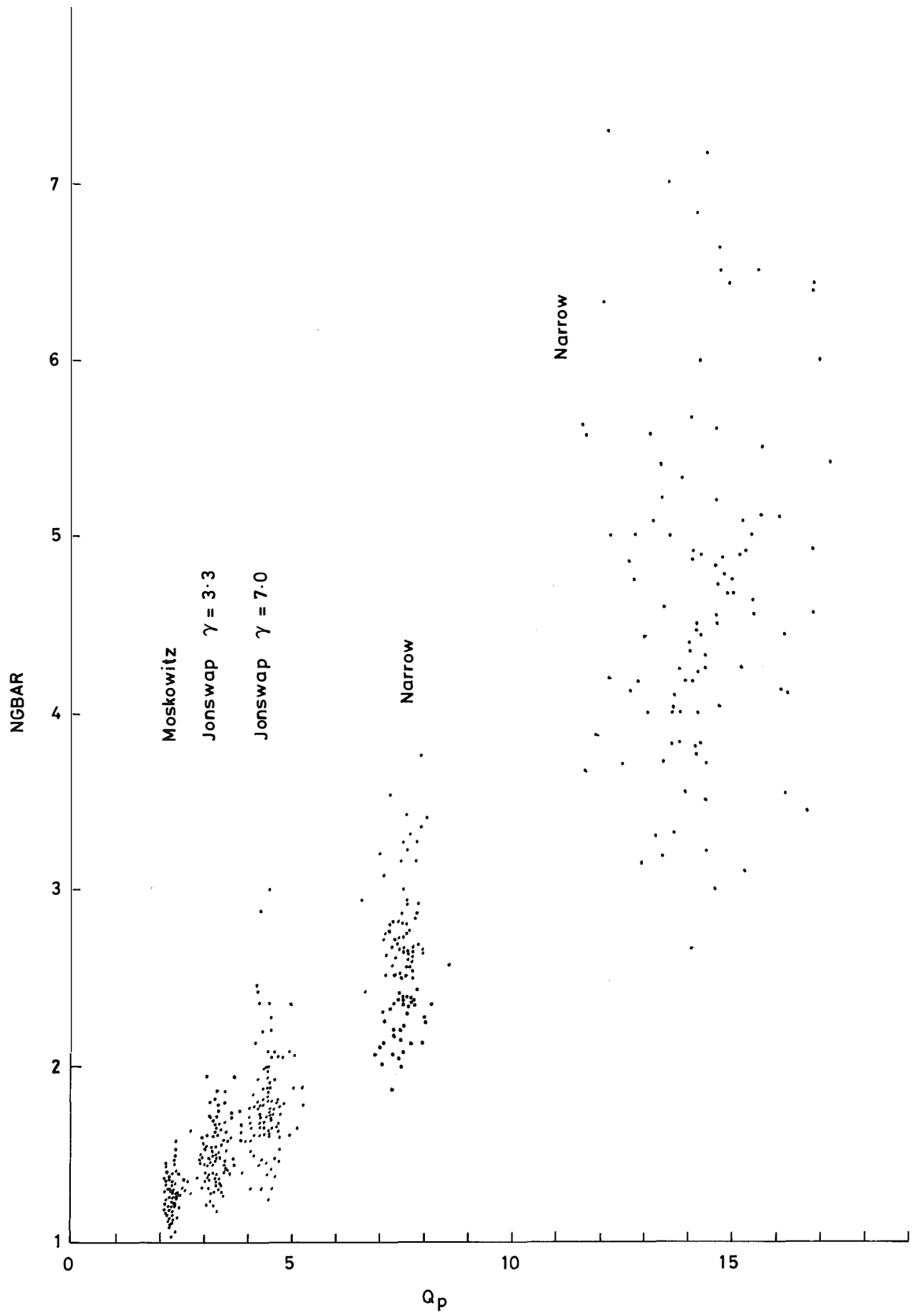

Fig 18 Computer simulated wave data - Mean Group Length vs Spectral Peakedness Factor

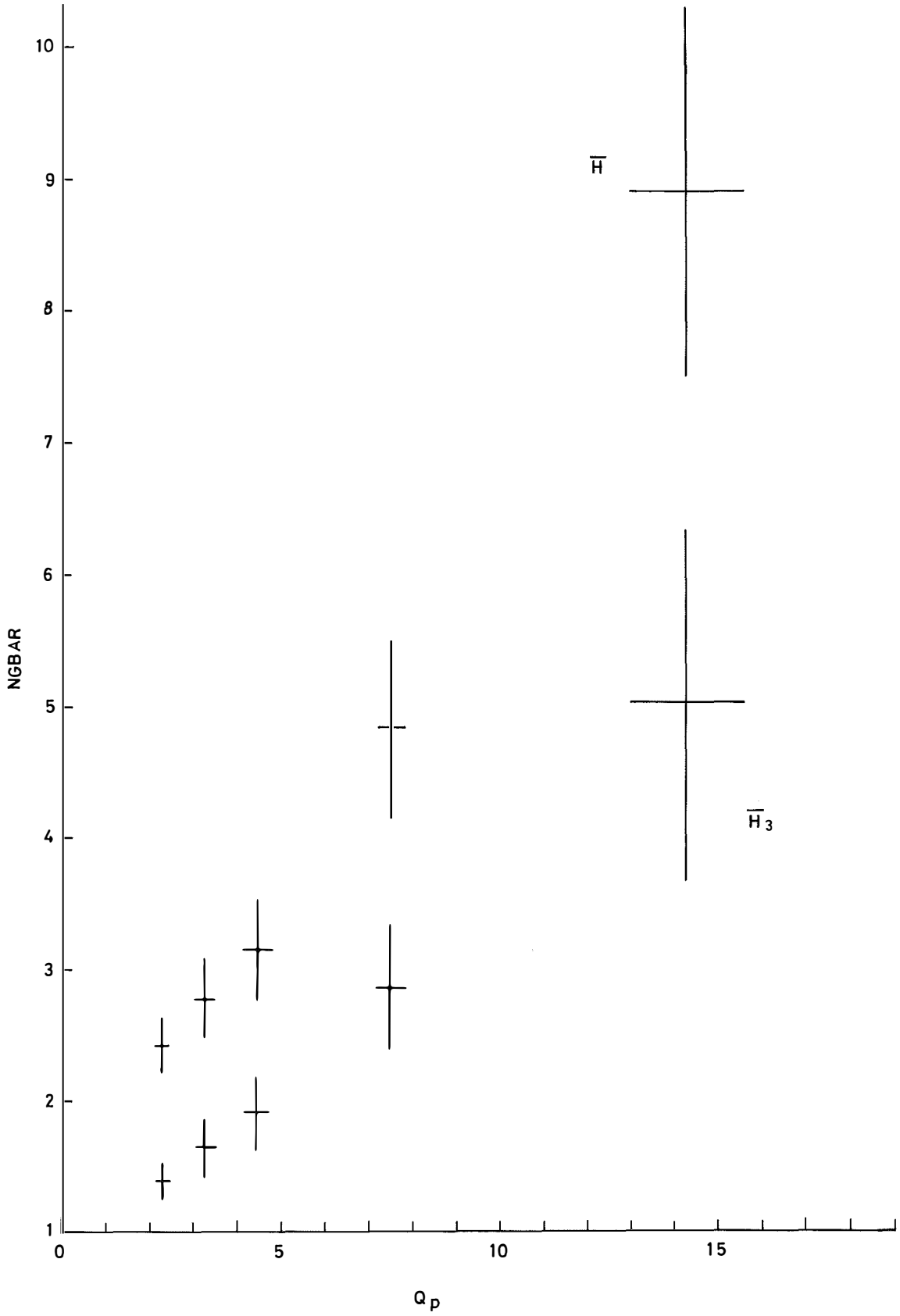

Fig 19 Computer simulated wave data - The effect of redefining NGBAR upon the scatter of the groupiness data