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WAVE ACTION AT A SERIES OF OFFSHORE BREAKWATERS: A Mathematical Model of Wave Diffraction and Overtopping

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ABSTRACT

This report describes the development of a mathematical model of wave diffraction and overtopping by a series of offshore breakwaters. The method used combines the solutions of a number of insular breakwater and breakwater gap problems. For the case of a single insular breakwater or breakwater gap the results from this model are compared with published results. For a number of offshore breakwaters a preliminary comparison is made with the results from a physical model. The model is subsequently used to test the sensitivity of wave height in the lee of the breakwaters to changes in incident period and direction, and adjustments in breakwater length: gap width ratio.

NOTATION

a	Half gap width or half breakwater length, see Figure 2
$\hat{a}$	Incident wave amplitude
g	Gravitational acceleration
$H_o^{(1)}$	Hankel function of the first kind, zeroth order
h	Water depth
i	Complex unit, $\sqrt{-1}$
k	Wave number, $2\pi/L$
K_o	Overtopping coefficient
L	Wave length
$\underline{q}$	Velocity field
t	Time
T	Wave period
x	Cartesian co-ordinate
y	Cartesian co-ordinate
z	Cartesian co-ordinate, measured vertically upwards
β	Incident wave angle, see Figure 2
η	Fluid elevation, Equation 2
η_b	Fluid elevation in the lee of a series of breakwaters, Equation 14
η_g	Fluid elevation in the lee of a series of breakwater gaps, Equation 16
π	3.1415 ...
σ	Angular frequency, $2\pi/T$
ϕ	Velocity potential, Equation 2
ϕ_d	Diffracted wave potential
ϕ_i	Incident wave potential, Equation 3
ϕ_r	Reflected wave potential
ϕ_d^j	Diffracted potential due to jth breakwater, Equation 12
ψ_d^j	Diffracted potential due to jth gap, Equation 16

CONTENTS

1 INTRODUCTION

In recent years there has been an increase in interest in the use of offshore breakwaters in coast protection. A recent review of shore protection breakwaters (Ref 1) has indicated that whilst mathematical models are available to predict the shoreline response due to changes in wave climate there are few models which determine the effect of offshore structures on the nearshore wave climate. This report describes the development of a mathematical model of wave diffraction and overtopping by a series of offshore breakwaters. The model is intended to give a first estimate of the performance of such structures and does not include refraction effects.

The technique used for the model is based on the idea of combining the solutions from a series of individual insular breakwater and breakwater gap problems. A detailed description of the mathematical model is given in section 2. In section 3 we give results from the individual breakwater and gap calculations using the method described in section 2, and compare these with results from existing models. In addition a number of results are presented for combinations of a series of breakwaters with different overtopping characteristics. In the final section we give our conclusions and suggestions for further research.

2 DESCRIPTION OF THE MATHEMATICAL MODEL

2.1 Outline of method

In constructing the mathematical model we require two different mechanisms to be included, namely, wave diffraction by the breakwaters and the effect of waves overtopping the structures.

To model wave diffraction around a number (say n) of insular breakwaters the fairly straightforward approach of combining the solutions of n single breakwater problems was used. This can be expected to provide a reasonable approximation to the diffracted field for most practical purposes, although, it does not allow for interaction between the diffracted waves emanating from adjacent breakwater tips. Clearly this will lead to an approximation which becomes less accurate as the spacing between the breakwaters decreases. However provided the breakwaters are of the order of two wavelengths apart this method should not produce significant errors.

In order to include overtopping of the structures a less obvious approach was employed. As overtopping waves by definition can only enter the sheltered side of the breakwaters over the structures themselves the complementary problem was considered. This consisted of a single infinite length breakwater with n gaps in the same positions as the insular breakwaters. Overtopping was modelled by diffraction of waves through the breakwater gaps, the wave heights on the sheltered side of the breakwaters being reduced by a factor to account for the proportion of waves overtopping the structure. The overtopping coefficient is dependant on the incident wave height and the depth of submergence/emergence of the breakwater crest relative to the mean water level. In the examples presented here a range of values for the overtopping coefficient were determined from physical model tests, but the model may be used for any overtopping coefficient. The wave field due to the breakwater with n gaps was calculated by combining the solutions for n individual breakwater gap problems. Similar comments to those made for the insular breakwater apply to the accuracy of the combination of breakwater gap solutions. The method used in the mathematical model is shown schematically in figure 1. In the next section we outline the technique used to calculate the diffracted field for a single insular breakwater and a breakwater gap. The method used to combine solutions for the n breakwater case is described in section 2.3.

2.2 Mathematical considerations for the individual breakwater and gap problems

2.2.1 General requirements

To simplify the construction of a mathematical model of wave diffraction by an insular breakwater or a breakwater gap certain assumptions about the flow field and breakwaters are made. These are as follows:

1. The water is assumed to be of uniform undisturbed depth, h, and in irrotational motion to which linear theory may be applied.

2. The fluid motion is induced by a train of small amplitude mono-frequency plane waves. (The case of random waves may be obtained from the single frequency calculations using linear superposition.)

3. The breakwaters are taken to be infinitesimally thin with vertical faces which are perfectly reflecting. This assumption is made to simplify the equations and boundary conditions. Its consequences will be discussed later in section 3.2.

These hypotheses allow the velocity field to be expressed in terms of a potential function $\Phi(x,y,z,t)$ by $\underline{q} = -\underline{\nabla}\Phi$. Here (x,y,z) are Cartesian co-ordinates with z measured vertically upwards, the plane z=o coincides with the undisturbed free surface. It is assumed that Φ is periodic in time with angular frequency σ. Since the breakwaters are supposed vertical sided and the water depth,h, is constant the vertical and time dependence of the fluid motion can be anticipated and subsequent calculations simplified by setting.

$$\Phi = \text{Re}\left[\frac{g}{i\sigma}\frac{\cosh k(z+h)}{\cosh kh}\phi(x,y)e^{-i\sigma t}\right] .$$

The wave number k is given by the dispersion relation

$$\sigma^2 = gk \tanh(kh) \quad ,$$

where g is the gravitational acceleration. The vertical displacement of the free surface from its equilibrium position is

$$\eta(x,y,t) = \text{Re}\left[\phi(x,y)e^{-i\sigma t}\right] \quad , \qquad (1)$$

where $\phi(x,y)$ is a reduced potential satisfying the Helmholtz equation

$$\phi_{xx} + \phi_{yy} + k^2\phi \equiv (\nabla^2 + k^2)\phi = 0 \quad , \qquad (2)$$

at all points (x,y) corresponding to the fluid domain.

It should be noted that the equations to be solved are in two dimensions only. This means that the breakwaters are represented in two dimensions by their intersection with the plane z=0. An insular breakwater is taken to lie along the x axis and is located between x=-a and x=+a. An infinite breakwater will be represented by the x axis with a gap located between x=-a and x=+a. A plan view of both breakwater configurations as given in Figure 2. In both cases it is assumed that waves are incident on the structure from $y=-\infty$ on a line making an angle β with the y axis. The incident wave potential is given by,

$$\phi_I(x,y) = \hat{a}\exp\left(ik(x\sin\beta + y\cos\beta)\right) \quad , \qquad (3)$$

where $\hat{a}$ is a prescribed wave amplitude. In the subsequent analysis we will also have a diffracted wave potential $\phi_d(x,y)$ which is required to satisfy the Sommerfeld radiation condition,

$$\lim_{r \to 0} r^{\frac{1}{2}} \left(\frac{\partial}{\partial r} - ik\right) \phi_d = 0 \quad , \qquad (4)$$

where $r = (x^2 + y^2)^{\frac{1}{2}}$

We now proceed to outline the equations which are to be solved in order to determine the flow field around an insular breakwater and breakwater gap. In both cases the method used is due to Gilbert and Brampton and a more detailed derivation of the equations is given in Ref 2.

2.2.2 Insular breakwater problem

For the insular breakwater problem the potential $\phi(x,y)$ satisfying equation (2) will have the form

$$\phi(x,y) = \phi_i(x,y) + \phi_d(x,y) \qquad (5)$$

at all points in the fluid domain. The incident potential ϕ_i is given by (3) and the diffracted potential ϕ_d must satisfy (4). The diffracted potential $\phi_d(x,y)$ is given by the following expressions,

$$\phi_d(x,y) = \frac{i}{2} \int_{-a}^{a} g(x_o) \left[\frac{\partial H_o(1)}{\partial y_o} (kR)\right]_{y_o=0} dx_o \quad , \; y \geqslant 0$$

$$\phi_d(x,y) = -\phi_d(x,-y) \quad , \quad y < 0 \qquad (6)$$

where

$$R^2 = (x-x_o)^2 + (y-y_o)^2$$

and $H_o^{(1)}$ is the Hankel function of the first kind, zeroth order. The function $g(x_o)$, $-a \leqslant x_o \leqslant a$, is a solution of the integral equation,

$$\int_{-a}^{a} g(x_o) \, H_o^{(1)} (k|x-x_o|) \, dx_o$$

$$= G(x) + Ae^{-ikx} + Be^{ikx}, \; -a \leqslant x \leqslant a \quad , \qquad (7)$$

where G(x) is a particular integral of the differential equation

$$\frac{\partial^2 G}{\partial x^2} + k^2 G = 2k\hat{a}\cos\beta \exp(ikx\sin\beta)$$

and A and B are chosen to satisfy the boundary condition

$$g(x_o) = 0 \quad \text{at} \quad x_o = \pm a \quad .$$

Once (7) has been solved for $g(x_o)$, $-a \leqslant x_o \leqslant a$ the diffracted potential in the flow field may be calculated using (6) and the total potential recovered from (5) and (3). The method used to find a numerical solution to equation (7) is given in Ref 2. Further discussion of the results obtained from this method of solution and a comparison with existing results is given in section 3.1.

2.2.3 Breakwater gap problem

In the breakwater gap case the potential may be decomposed as

$$\phi(x,y) = \begin{cases} \phi_i(x,y) + \phi_r(x,y) + \phi_d(x,y), & y < 0 \\ \phi_d(x,y) & , y \geqslant 0 \end{cases} \tag{8}$$

where ϕ_i and ϕ_d, are the incident and diffracted potentials. The potential of the waves reflected from the barrier as given by,

$$\phi_r = \hat{a} \exp\left(ik(x\sin\beta - y\cos\beta)\right) \quad . \tag{9}$$

Gilbert and Brampton (Ref 2) have shown that ϕ_d is given by the following expressions,

$$\phi_d(x,y) = -\frac{i}{2}\int_{-a}^{a} f(x_o)\left[H_o^{(1)}(kR)\right]_{y_o=0} dx_o \quad , y \geqslant 0,$$

$$\phi_d(x,y) = -\phi_d(x,-y) \quad , \quad y < 0 \; . \tag{10}$$

The unknown function, $f(x_o)$, $-a < x_o < a$, is a solution of the integral equation

$$\int_{-a}^{a} f(x_o)\, H_o^{(1)}\left(k\left|x-x_o\right|\right) dx_o = 2i\hat{a}\exp(ikx\sin\beta), \quad -a \leqslant x \leqslant a \quad . \tag{11}$$

The method used to calculate $f(x_o)$ is similar to that used to calculate $g(x_o)$ for the insular breakwater case, and is given in detail in Ref 2. Once $f(x_o)$ has been calculated the diffracted field may be evaluated from (10) and thence the total wave field from (8), (9) and (3).

2.3 Combination of solutions for insular breakwaters and breakwaters gaps

The mathematical model of wave diffraction and overtopping by a series of insular breakwaters may be simplified by making the following assumptions.

i) The breakwaters lie along a single straight line (in this case the x-axis).

ii) The breakwaters are all of the same length and are separated by gaps of uniform length.

As indicated in section 2.1 modelling of diffraction and overtopping by a series of insular breakwater may be broken down into two separate parts. For diffraction we combine the solutions of a number of individual insular breakwater problems and for overtopping the same number of breakwater gap solutions are combined. To complete the model the results of the overtopping and diffraction calculations are put together with due account being taken of an overtopping coefficient. The technique used to combine individual solutions for the diffraction and overtopping problems are similar. Therefore we will give details of the method used for the diffraction problem and simply state the results for the overtopping problem.

To this end we consider n insular breakwaters which satisfy both the assumptions given above and those in section 2.2.1. A typical breakwater layout is illustrated in Figure 3. For each breakwater the diffracted potential due a specified incident wave may be calculated using the method given in section 2.2.1. We denote by $\phi_d(x_j,y_j)$, $j = 1,..n$, the diffracted potential of waves associated with the j th breakwater at a position (x_j,y_j) relative to an origin at the centre of the j th breakwater. If we introduce the global co-ordinate system (X,Y), such that the breakwaters lie along the X axis with their centres at $(X_j,0)$, $j = 1,..n$, the diffracted potential at a point (X,Y) due the j th breakwater will be $\phi_d^j(X,Y)$ where

$$\phi_d^j(X,Y) \equiv \phi_d^j\,(X_j+x_j,y_j) \equiv \phi_d(x_j,y_j)$$

Some allowances must also be made for the diffracted field being due to incident waves arriving at the structures at different times. Clearly, for waves which are not normally incident there will be a delay between the arrival time at the first and subsequent breakwaters. If the time of arrival of the incident wave at the j th breakwater is t_j, then at time t the fluid elevation, from (2), at any point in the field will be given by,

$$\eta_b(X,Y,t) = \text{Re}\ \{\phi_i(X,Y)e^{-i\sigma t} + \sum_{j=1}^{n} \phi_d^j(X,Y)e^{-i\sigma(t-t_j)}\}. \tag{12}$$

Here σ is the radian frequency ($\sigma = 2\pi/T$) and the first term represents the global incident wave which in the notation of figure 3 is,

$$\phi_i(X,Y) = \hat{a}\ \exp\ \left(i(X\sin\beta + Y\cos\beta)\right) \tag{13}$$

where $\hat{a}$ is a prescribed amplitude.

Expression (12) may be further simplified by considering the field at time t=0. This leads to the fluid elevation being given by,

$$\eta_b(X,Y) = \text{Re}\ \{\phi_i(X,Y) + \sum_{j=1}^{n} \phi_d^j(X,Y)\ \exp\ (i\sigma t_j)\} \tag{14}$$

The times of arrival at the incident waves may be shown to be,

$$t_j = \frac{X_j}{c\ \sin\ \beta}\quad,\quad j = 1,\dots,n\ . \tag{15}$$

where c is the wave speed. Expressions (13) and (14) allow the fluid elevation at all points in the field due to a series of insular breakwaters to be calculated. A similar method may be used to find the fluid elevation due to a number of gaps in an infinite breakwater. The fluid elevation in the lee of such an arrangement is,

$$\eta_g(X,Y) = \text{Re}\ \{\sum_{j=1}^{n} \phi_d^j\ (X,Y)\ \exp\ (i\sigma t_j)\},\ Y > 0\ , \tag{16}$$

where ϕ_d^j (X,Y) is the diffracted potential at a point (X,Y) due to the j th gap j=1,...,n. The centre of the j th gap will be at $(X_j,0)$, j=1,....n and t_j will be given by (15).

The total fluid elevation in the lee of the breakwaters, $\eta(X,Y)$ where Y> 0, due to both diffraction and overtopping may be calculated from (14) and (16) by using the expression,

$$\eta(X,Y) = [(\eta_b(X,Y))^2 + (K_o\eta_g(X,Y))^2]^{\frac{1}{2}},\ Y > 0, \tag{17}$$

where K_o is the overtopping coefficient, $0 \leqslant K_o \leqslant 1$. The value of K_o will be a function of the crest height of the breakwater relative to mean water depth and the incident wave height. A range of values for K_o, determined from physical model tests, will be discussed in the results section.

Therefore, from individual insular breakwater and breakwater gap solutions, a model of wave diffraction and overtopping by a series of insular breakwaters, has been constructed.

3 RESULTS

3.1 Results for individual insular breakwater and breakwater gap calculations

Before considering diffraction and overtopping by a series of offshore breakwaters we first present some results for wave diffraction by a single insular breakwater and a breakwater gap. The numerical method used to find a solution to the integral equations describing diffraction for both of the breakwater arrangements is given in section 2.2.

For both of the problems discussed in section 2.2 an analytic section of the governing equations may be derived, the resulting expression for the diffracted potential for both problems is in terms of an infinite series of Mathieu functions (see, for example, Abramowitz and Stegun (Ref 3)). There are certain difficulties associated with the evaluation of Mathieu functions and an accurate computation of analytic solutions is not easily achieved. Therefore, from a practical point of view, calculation of the approximate numerical section to both problems as given here is preferable to attempting to evaluate the exact solution involving an infinite series. However, we can compare the results from the present method with published results from a method based on the analytic solutions to both diffraction problems.

Details of the derivation of the analytic solution for both the insular breakwater and breakwater gap diffraction problems are given by Montefusco (Refs 4 and 5). Having obtained an expression for the exact solution he derives a numerical approximation for the diffracted potential. The approximation for both problems uses the idea of relative contribution to the total energy flux as is measure of the importance of the terms in the infinite series. For both problems the approximation gives a fairly accurate representation of the diffracted field although in the case of the breakwater gap the approximation seems to become less accurate as the gap width increases relative to the incident wave length (see Ref 5).

The results from the present method were compared with those of Montefusco for both the insular breakwater and the breakwater gap for several different cases. We present here only a selection of the results in order to demonstrate the accuracy of the calculation method.

For an insular breakwater of length 0.9 wavelengths with waves incident at 0^0 (normal) and 45° both sets of results are displayed in Figures 4 and 5. The lines shown in these figures are contours of equal wave height coefficient (wave height at a particular point/incident wave height). It can be seen from Figures 4 and 5 that the agreement between the results from the present method and that of Montefusco is very good.

In the case of the breakwater gap agreement between the two sets of results was found to be fairly good. However, as the gap width increased discrepancies between the results became apparent. These differences were thought to be due in part to the accuracy of the Montefusco approximation which was discussed above. A comparison of the results for the widest gap width (1.75 wavelengths) tested by Montefusco, and therefore in some sense the worst case, for 0^0 and 45^0 incidence is given in Figures 6 and 7. It can be shown from these results that the maximum discrepancy between the two methods is of the order of 15%, but in most cases the discrepancy is less than 10%.

In summary, the results for the insular breakwater and the breakwater gap problems, obtained using an integral equation method, have been shown to be in good agreement with existing results at points near the breakwater gap. For positions further away from the breakwater or gap the accuracy of the method has been demonstrated by Gilbert and Brampton, see Ref 2. Having established that the method provides a good approximation to the diffracted field for an insular breakwater or a breakwater gap we proceed to give some results from the model of diffraction and overtopping by a series of offshore breakwaters.

3.2 Results for the model of wave diffraction and overtopping by a series of offshore breakwaters

Before discussing the results from the mathematical model it is worth commenting on the simplifying assumptions which have been made, and the ways in which these assumptions will effect the interpretation of the results. One of the assumptions that is made is that the breakwaters are vertical sided structures which are infinitesimally thin and perfectly reflecting. In this idealised situation the breakwaters act to redistribute the incident wave energy through the processes of reflection and diffraction. In practice the breakwaters will probably be either rubble mound or surrounded by rock armour. The structures themselves will then dissipate wave energy due to frictional effects, and by causing the waves to break. In addition the prototype breakwaters may have rounded ends which will scatter and dissipate the incoming waves and thus reduce the total amount of energy travelling landward of the line of the breakwaters. Therefore, it may be expected that the mathematical model will provide a conservative approximation of wave heights on the sheltered side of the breakwaters.

The results from the mathematical model have so far only been compared with theoretical results for a single insular breakwater and a breakwater gap. Before proceeding further some evidence of the validity of the model when compared with the physical situation is required. The mathematical model described here was first used in a study to examine the possibility of siting a series of offshore breakwaters on the South Coast of England (See Ref 6) as part of a coast protection scheme. The mathematical model was further used in that study to optimise the layout of offshore breakwaters so that the amount of testing required in a mobile bed physical model could be minimised. From the mathematical and physical model tests it was possible to make an initial comparison of results as a check on the accuracy of the mathematical model. This comparison between the results showed that the mathematical model of diffraction and overtopping provided a good first estimate of the behaviour of waves in the physical model.

Several tests were run in the mathematical model to investigate the effect of factors, such as wave height, wave period and breakwater spacings on wave heights in the lee of the breakwaters. All of the tests included here were for a series of five offshore breakwater lying parallel to the shoreline situated in water of depth 8m. Although any number of breakwaters can be used in the model, it was thought that five breakwaters would allow the effects well into the lee of the structures to be properly examined. Results from the model were in the form of wave height coefficients which were output on a line 200m landward of the breakwater at 10m intervals.

The first series of tests that were run in the mathematical model concentrated on investigating the effects of diffraction by a series of breakwaters. At this stage the overtopping coefficient (K_0) was set to zero, results showing the effects of overtopping will be examined later in this section.

The effect of change in incident direction on wave height coefficients are illustrated in Figure 8 for breakwaters of length 200m with 200m gaps. (For 8s waves in 8m of water a wavelength is approximately 65m). For normally incident waves the graph of wave height coefficient is symmetric about a line through the centre of the third breakwater and there is also local symmetry about centre lines of the second, third and fourth breakwaters. It can also be seen that at normal incidence the minimum wave height coefficient occurs at two positions in the lee of each breakwater with a slight increase in wave height between the two minimum values. This is characteristic of the 'lobe' patterns which are observed in the insular breakwater results, see Figure 4. As the incident wave angle increases the minimum value of the wave height coefficient remains directly in the lee of the breakwaters but the profile is, as expected, no longer symmetric. Similar behaviour is observed for the maximum wave height coefficients in the breakwater gaps. The range covered by the wave height coefficients increases slightly as the incident angle increases.

Next we examine the variations in wave height coefficient which are due to changes in incident wave period. For a series of breakwaters of length 200m with 200m gaps runs of the mathematical model were made with incident waves periods 8s, 9s and 10s, the results are shown in Figure 9. It should be noted that as these waves are normally incident, the graph of wave height coefficients is symmetric about the centre of the middle breakwater and therefore only half of the field needs to be displayed. The effect of changing the wave period in constant depth is equivalent to changing the length of the breakwater relative to the incoming wavelength. The breakwater lengths relative to the incident wavelength for each period tested are also shown in Figure 9. For all of the periods tested the minimum wave height coefficient occurs as expected in the lee of the breakwater and the maximum in the breakwater gap. A fairly clear trend emerges for the maximum wave height coefficients, ie. as the period increases these increase in value. For the maximum period tested there is evidence of subsidiary 'peaks' in the graph near the breakwater ends. There is no such clear trend for the minimum values wave height coefficient.

For both 8s and 10s the curves show a similar shape behind the breakwaters there are two identical minimum values with a peak in between, whereas for 9s there is some sign of similar behaviour but the 'peak' is not as marked.

For fixed incident wave conditions it is possible to investigate the effects of breakwater spacing and breakwater length on the diffracted field. The maximum, minimum and average wave height coefficients for breakwaters of fixed length with different gap spacings are given in Table 1. Here the average wave height coefficient is an average of the values between the centres of the first and fifth gaps, and this will give an indication of the wave height, well into the shelter of the line of breakwaters. From Table 1 it can be seen that there is no strong trend in either the minimum or maximum wave height coefficients. However, the average wave height coefficient increases steadily as the ratio of breakwater length: gap width decreases. This is a result of the increased proportion of gap width to total length through which wave energy permeates.

The wave height coefficients for a fixed breakwater length: gap width ratio are shown in Table 2. It can be seen that for this fixed ratio the average wave height coefficient remains the same for all breakwater lengths tested. This bears out the comments made above concerning the amount of energy transmitted through the gaps.

Finally we consider the effect of including overtopping on the wave height coefficients. In Figure 10 we illustrate the wave height coefficients for a breakwater system with zero overtopping and with an overtopping coefficient of 0.5. It can be seen that including overtopping leads to a general increase in wave height coefficient at all points along the curve. This is as expected because the amount of energy transmitted into the lee of the breakwater has increased. Also as expected the increase in wave height coefficients is greatest behind the breakwater gaps. On including overtopping in this model the general shape of the curves, compared with that for zero overtopping, is maintained.

Figure 11 shows the average wave height coefficients as a function of the overtopping coefficients for four offshore breakwater layouts. In practical situations such a graph would be used to compare the performance of a number of different schemes. In practise the overtopping coefficient can be determined from flume tests for different breakwater crest heights. This would allow not only the layout to be varied but also the crest elevation of the breakwaters, making the mathematical model described here a very useful tool in preliminary studies of offshore breakwater schemes.

4 CONCLUSIONS AND RECOMMENDATIONS

The conclusions and recommendations arising from this report are as follows:

1. Using the integral equation method to find a solution for wave diffraction by an insular breakwater, or a gap in an infinite breakwater, has been found to give results which are in good agreement with those from the analytic solutions to the same problems.

2. A mathematical model of wave diffraction and overtopping has been developed to allow a preliminary assessment of the performance of a series of offshore breakwaters to be made.

3. The model has been found to give reasonable quantitative agreement with results available from physical model tests. Clearly a more complete comparison of results would be desirable for any future development of the model.

4. A number of aspects of the behaviour of a series of offshore breakwaters have been examined. In particular, the effects of change in wave direction and period and also variation of breakwater length and gap width on wave diffraction have been assessed. Detailed results for all of these cases are given in Chapter 3.

5. On including wave overtopping in the model a general increase in wave height coefficient over the case with zero overtopping was noted. Although, the shape of the curve of wave height coefficient against distance was similar with and without overtopping.

6. Figures illustrating the average wave height coefficient for several breakwater arrangements and a range of overtopping coefficients were presented. Such illustrations were shown to be useful in selecting the most effective breakwater configuration taking into account breakwater length, gap width and crest height.

7. Clearly this model is capable of further development, for example including the effects of retraction would be desirable. In addition some modification could also be made to allow breakwaters which were not shore parallel or co-linear in plan to be represented. Once a more

complete model of the effects of the breakwaters and water depth on the wave field has been achieved attention could be focussed on modelling changes in beach plan shape due to the introduction of offshore breakwaters. Whilst these developments are not straightforward it seems probable that in future more realistic modelling of the effects of offshore breakwaters will be achieved.

5 ACKNOWLEDGEMENTS

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Tables

TABLE 1 **Wave height coefficients for fixed length breakwater, variable gap width**

Breakwater length (m)	Gap width (m)	Ratio breakwater:gap	Wave height coefficients Min	Max	Average
120	120	1 : 1	0.24	1.22	0.68
120	180	2 : 3	0.45	1.26	0.74
120	280	3 : 7	0.38	1.18	0.81

TABLE 2 **Wave height coefficients for fixed (breakwater length : gap width) ratio**

Breakwater length (m)	Gap width (m)	Ratio breakwater:gap	Wave height coefficients Min	Max	Average
90	210	3 : 7	0.36	1.18	0.81
120	280	3 : 7	0.38	1.18	0.81
150	350	3 : 7	0.26	1.24	0.81

Figures

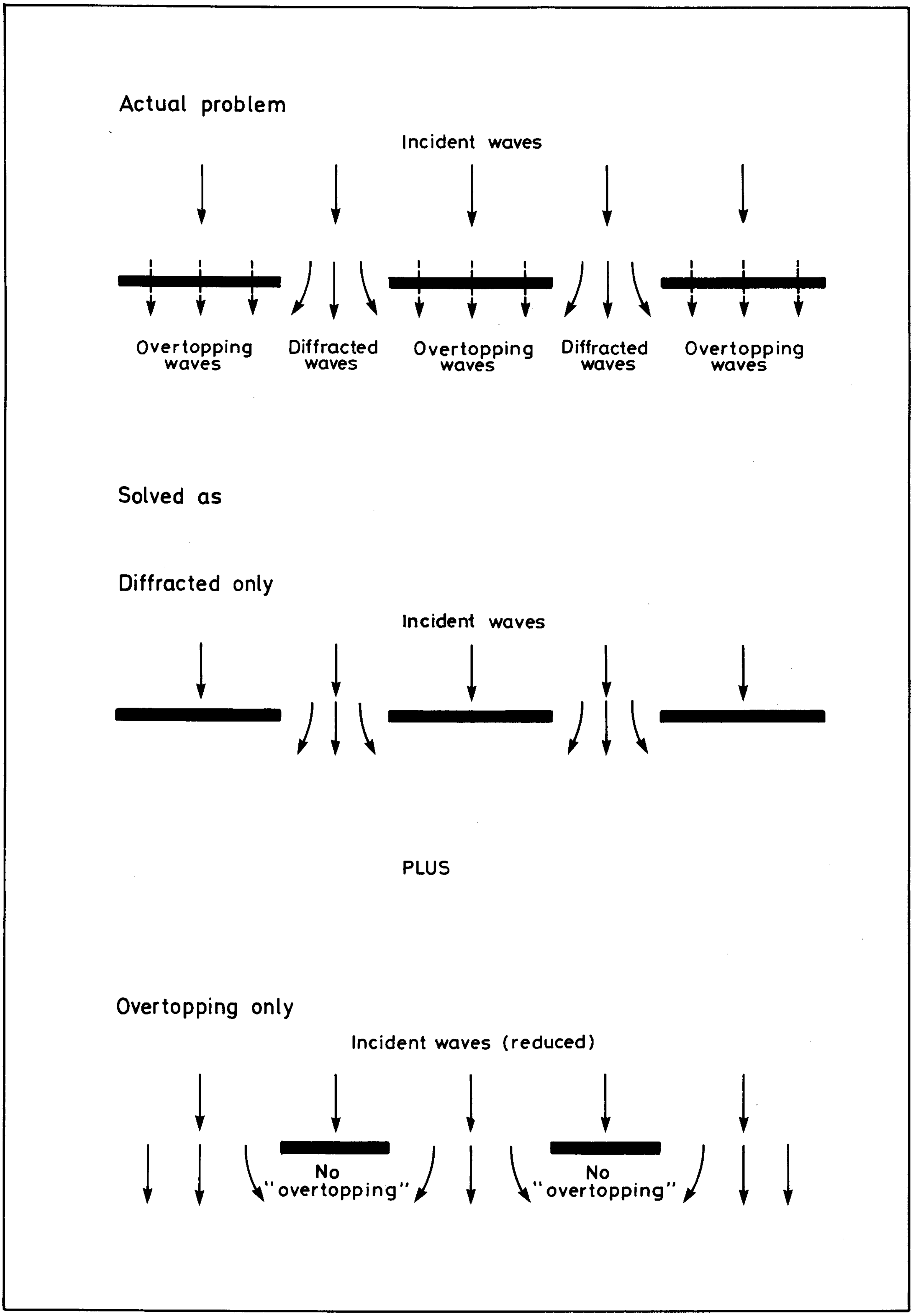


Fig 1 Schematic representation of mathematical model

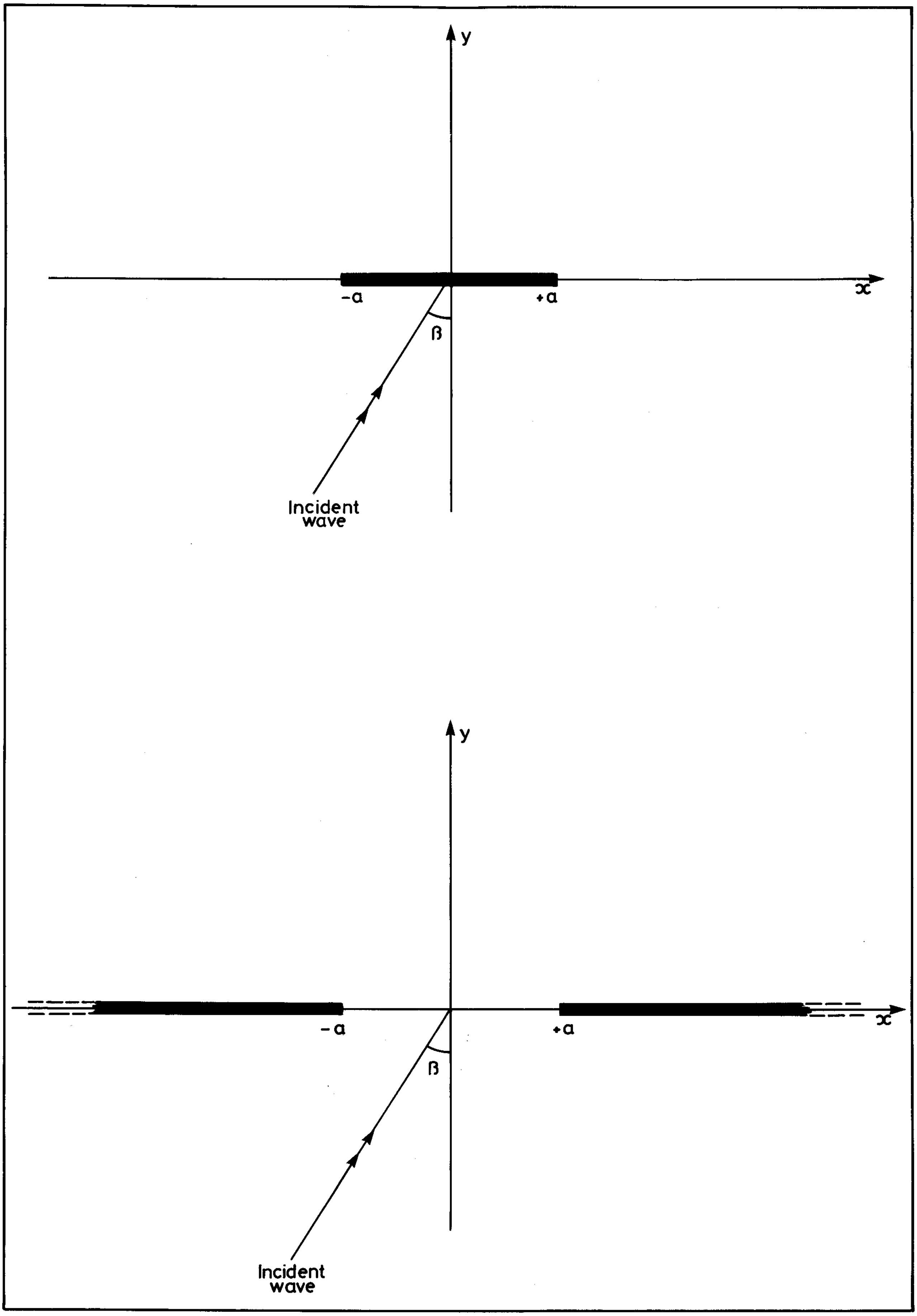


Fig 2 Plan of insular breakwater and breakwater gap

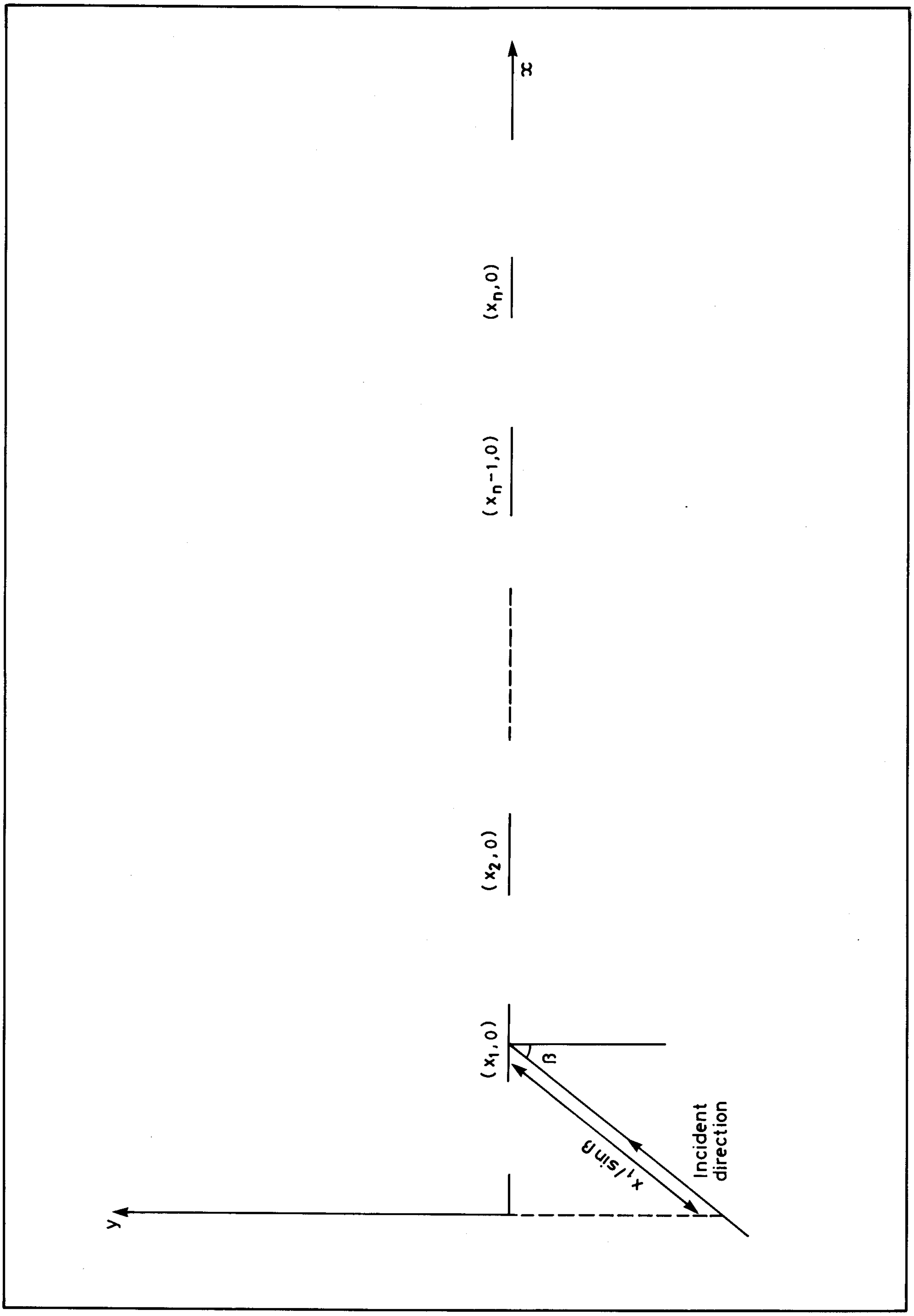


Fig 3 Typical breakwater layout

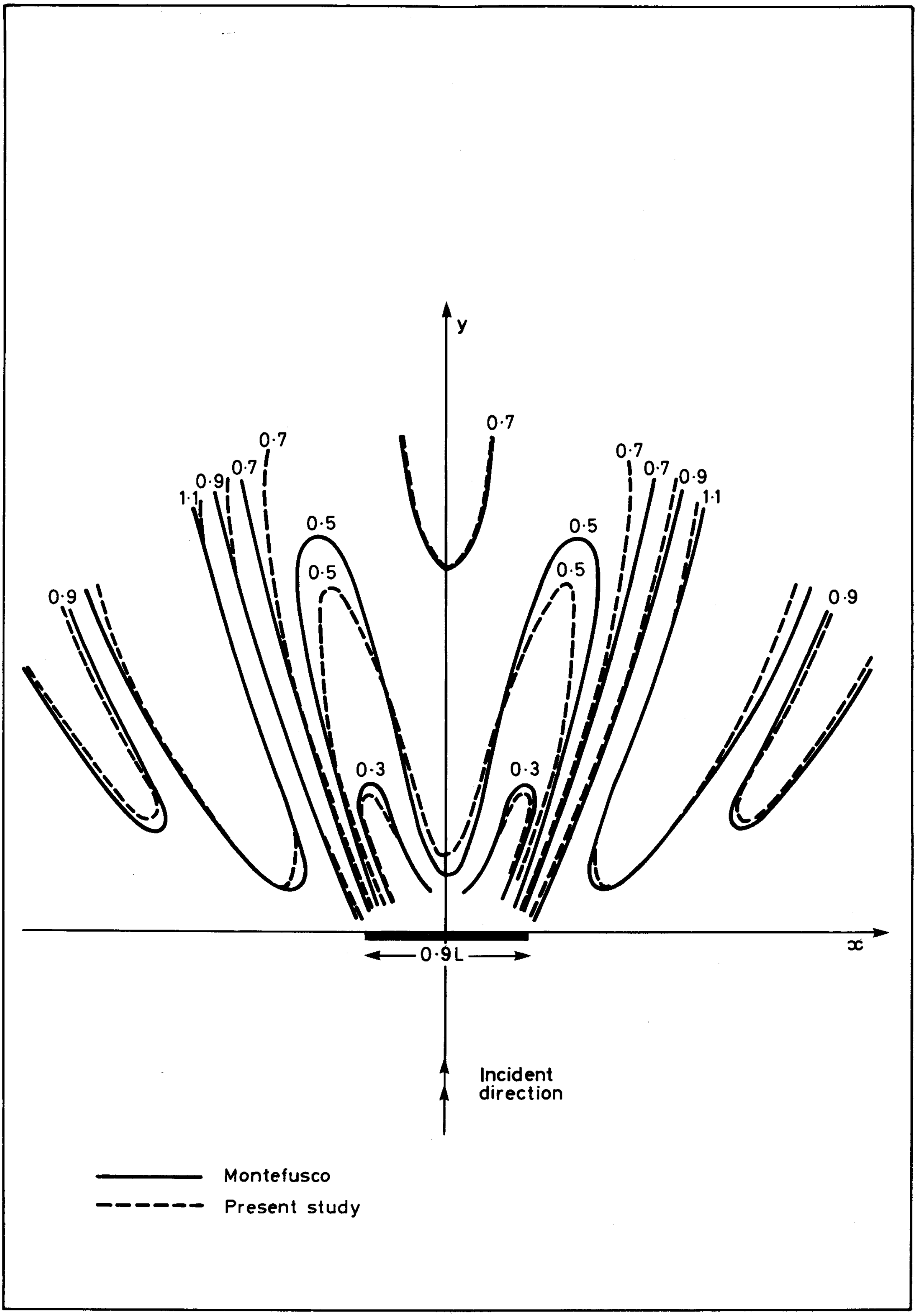


Fig 4 Wave height coefficients for an insular breakwater, length 0·9 wavelengths (L), normal incidence

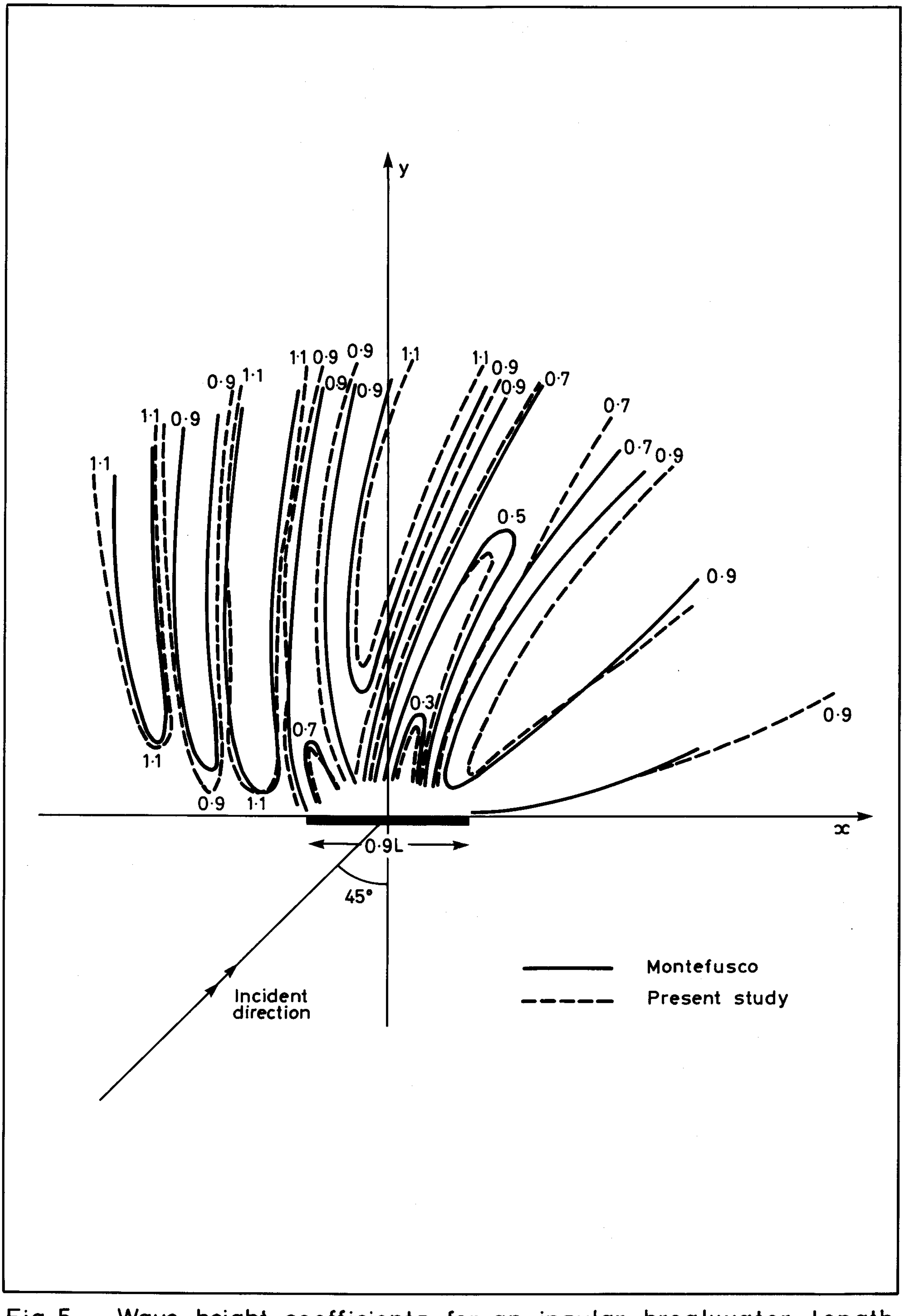


Fig 5 Wave height coefficients for an insular breakwater, length 0·9 wavelengths (L), incident angle 45°

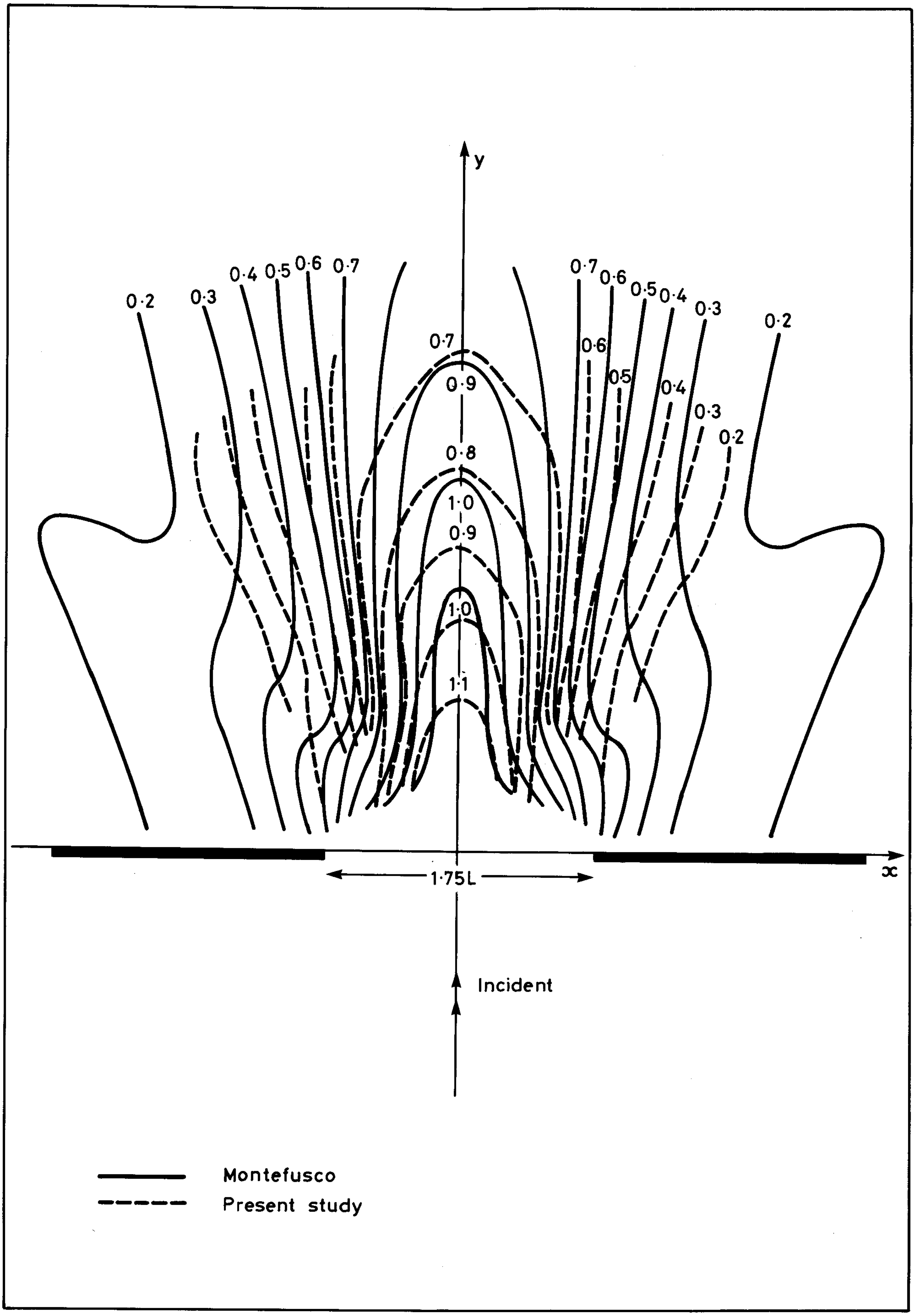


Fig 6 Wave height coefficients for a breakwater gap, width 1·75 wavelengths (L), normal incidence

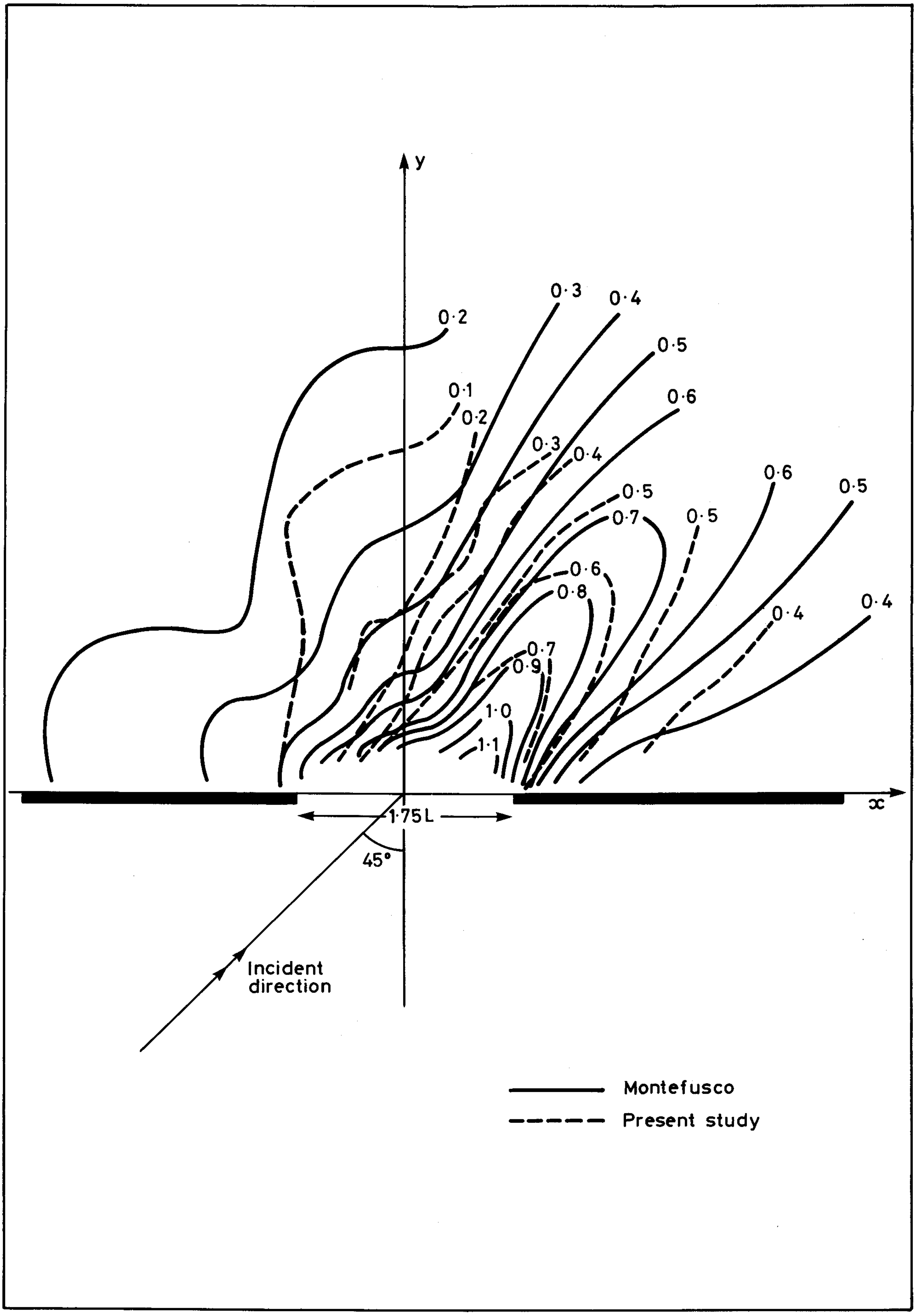
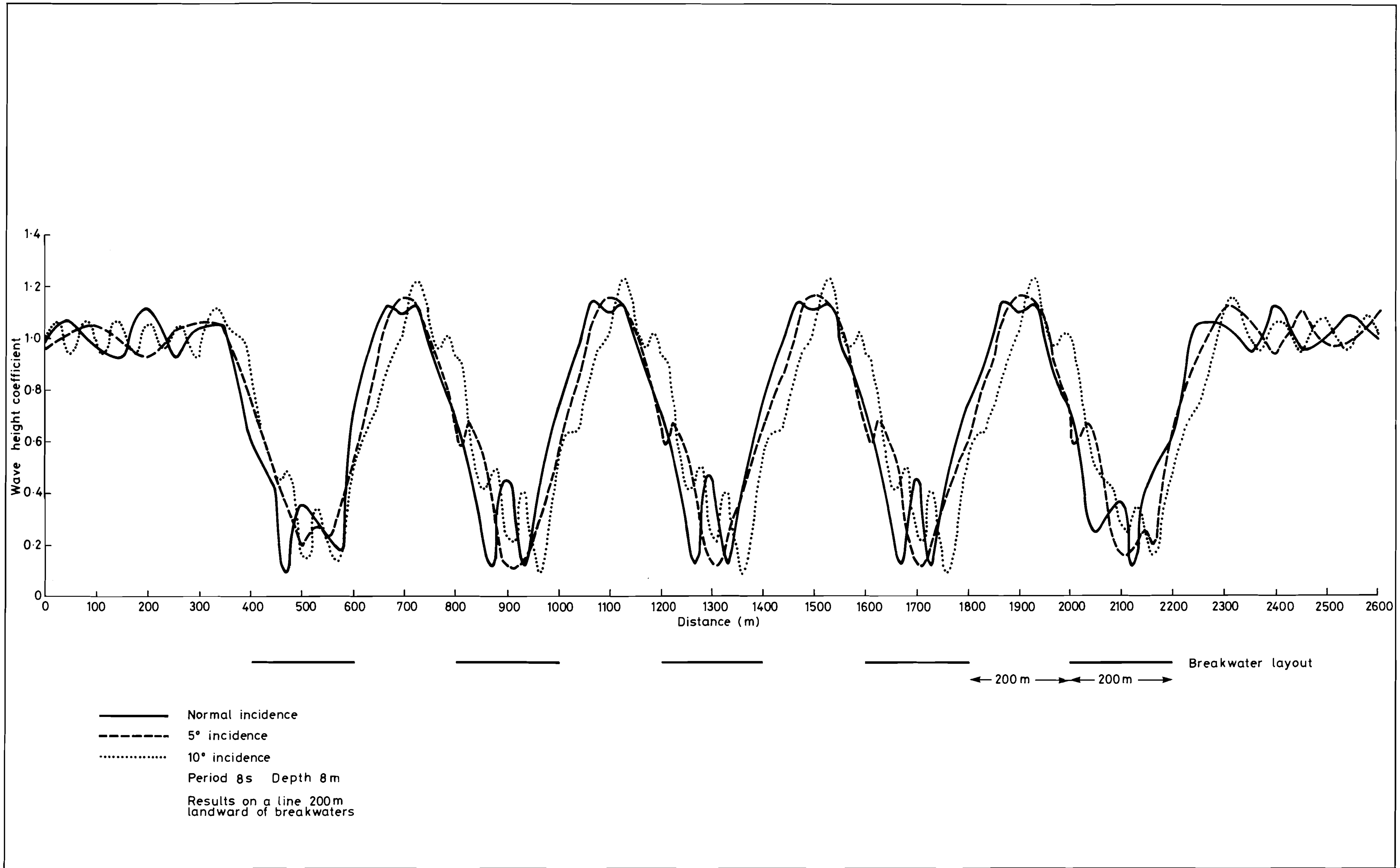


Fig 7 Wave height coefficients for a breakwater gap, width 1·75 wavelengths (L), incident angle 45°



Wave height coefficent for variable incident direction (no overtopping)

Fig 8

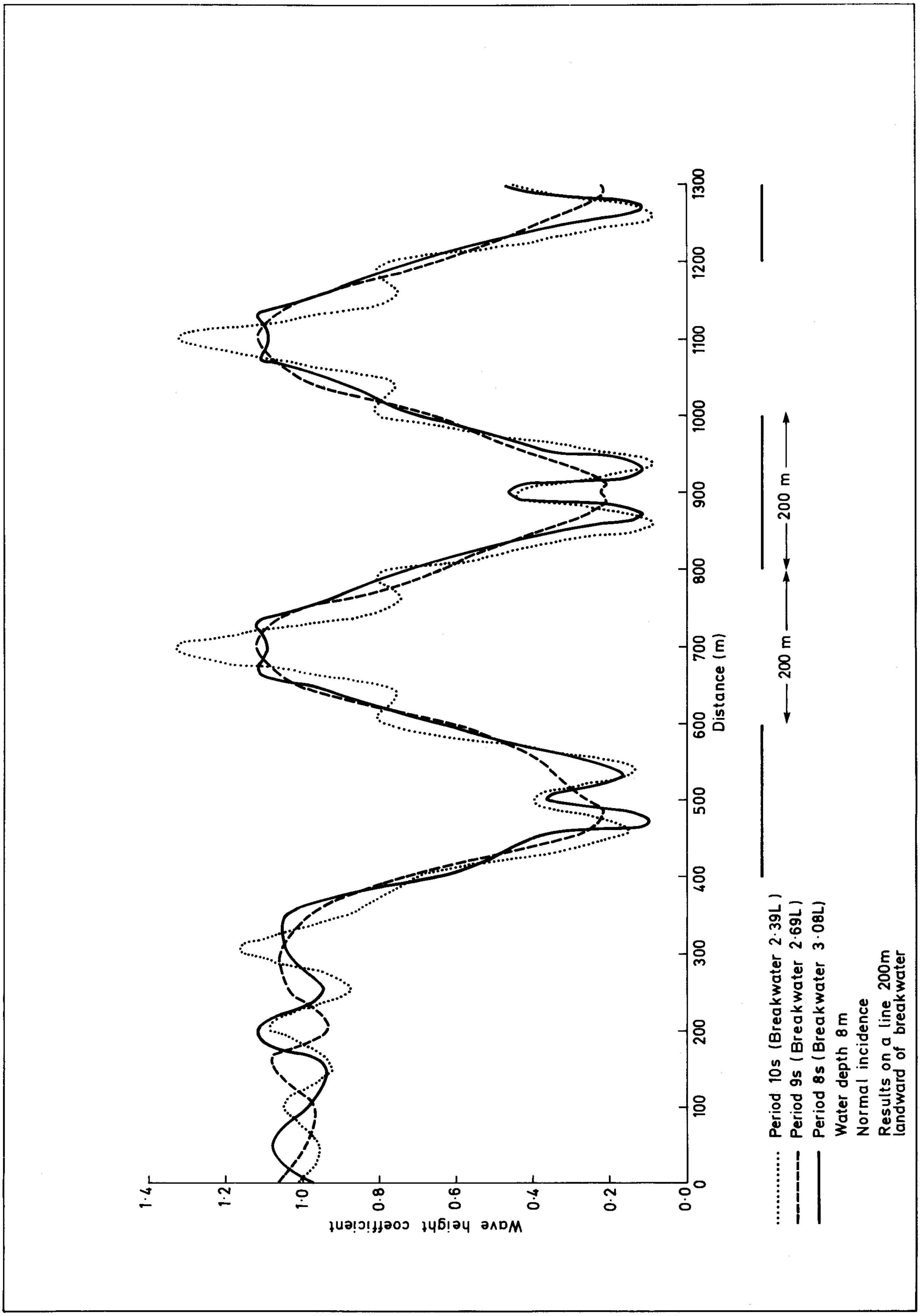


Fig 9 Wave height coefficients for variable incident period (No overtopping)

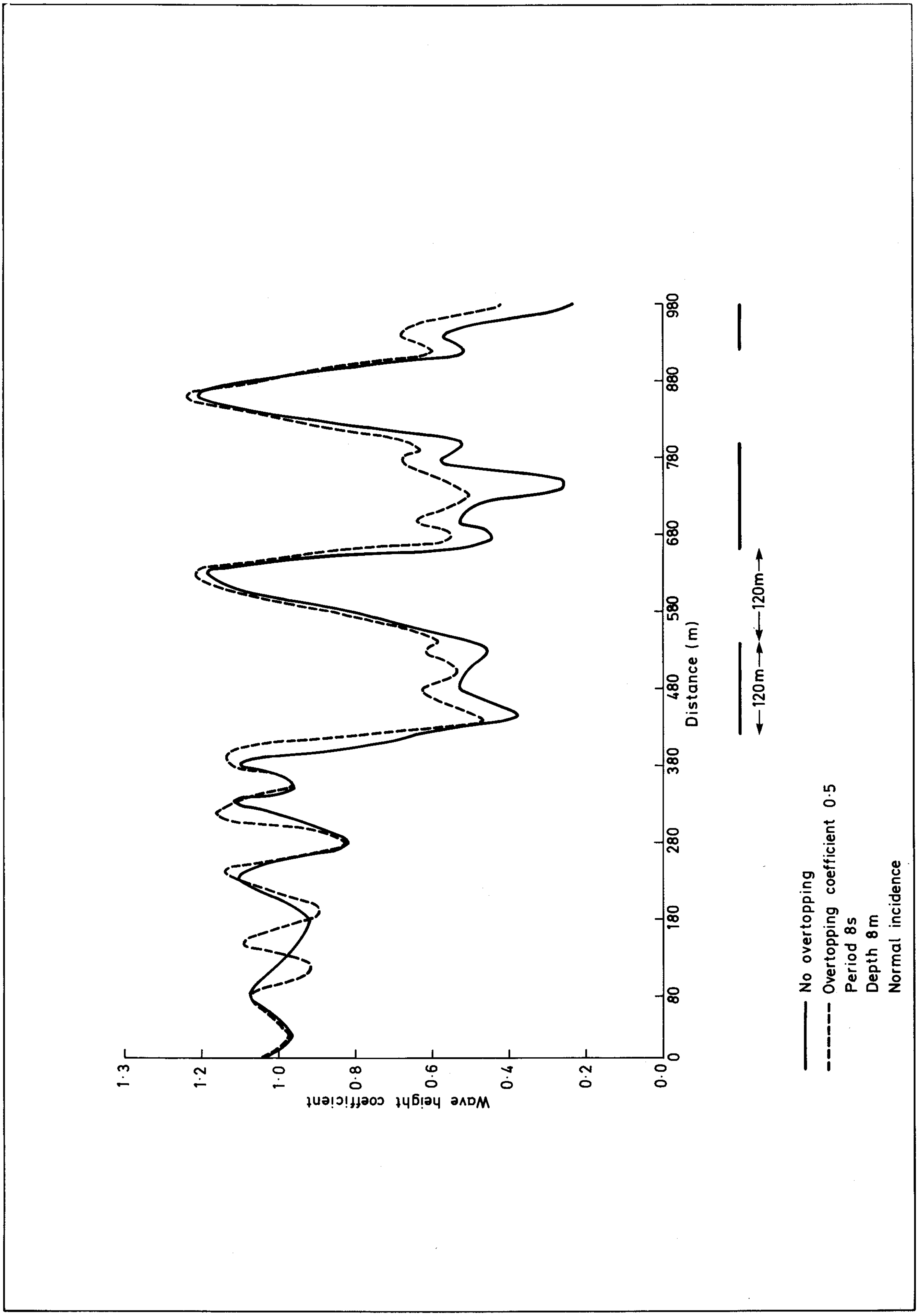


Fig 10 Wave height coefficient with and without overtopping

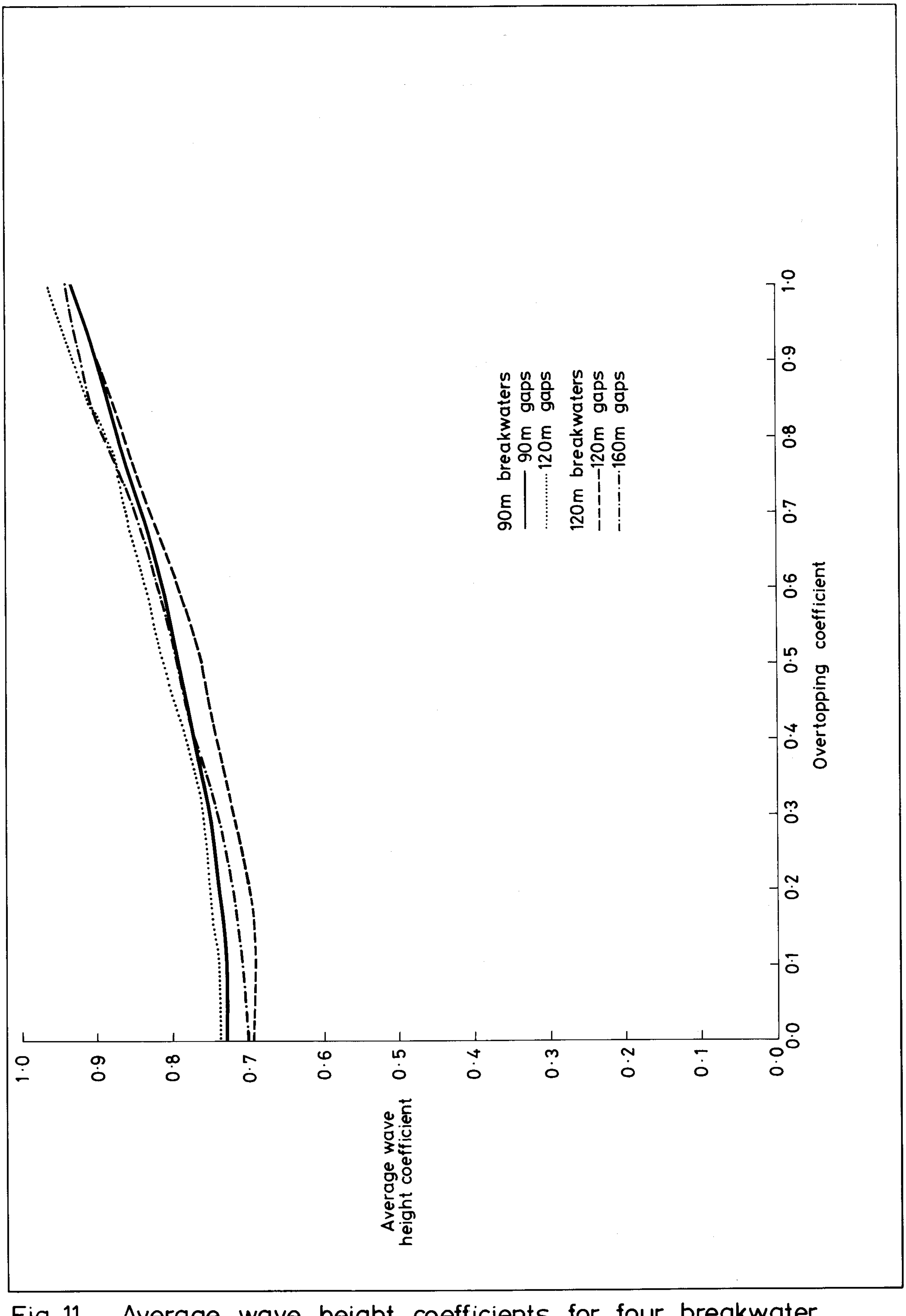


Fig 11 Average wave height coefficients for four breakwater arrangements, results on a line 200m landward of the breakwaters