



Hydraulics Research
Wallingford

SHORT CRESTED SEAS IN HARBOUR MODELLING

E C Bowers BSc PhD DIC

Report No SR 141
November 1987

**Registered Office: Hydraulics Research Limited,
Wallingford, Oxfordshire OX10 8BA.
Telephone: 0491 35381. Telex: 848552**

This report describes work funded by the Department of the Environment under Research Contract PECD 7/6/054 (PIF No HRL 3/85) for which the DoE nominated officer was Dr R P Thorogood. It is published on behalf of the Department of the Environment but any opinions expressed in this report are not necessarily those of the funding Department. The work was carried out in the Maritime Engineering Department of Hydraulics Research, Wallingford, under the management of Dr S W Huntington.

© Crown copyright 1987

Published by permission of the Controller of Her Majesty's Stationery Office.

Short Crested Seas in Harbour Modelling

E C Bowers BSc PhD DIC

Report No SR 141

November 1987

ABSTRACT

In the near future it is planned to construct a short crested wave-maker for use in physical models of harbours. This report is concerned with three aspects of that development.

Firstly, it highlights the differences in harbour and vessel response that are expected as a result of generating short crested waves instead of the long crested random waves used at present. This comparison makes use of analytical and computational results. For example, at ordinary wave periods, wave heights within a harbour are expected to be more evenly distributed and more vertical vessel movement in waves is expected in entrance channels. The response of the harbour at wave group periods is expected to be significantly smaller which, in turn, will tend to reduce the horizontal movements of model ships moored within the harbour. Overall, a significant effect on the outcome of model investigations is expected.

The second aspect concerns basic design requirements. A number of consequences follow from the need to keep the cost of the short crested wave-maker within acceptable limits while satisfying the requirements for a large range of mean wave directions and a sufficiently long wave front. It will be necessary to make the wave-maker mobile and to use separate individual paddles not more than 0.5m wide. Wave guides will be needed which, in turn, forces the use of symmetric functions to describe the directional spread of energy in the generated waves. An illustration of the use of wave guides indicates that the refraction and shoaling of waves as they propagate up to the harbour should be well represented.

Finally, it is shown that set-down compensation at the short crested wave-maker is necessary to limit the generation of spurious long waves. A technique for doing this is illustrated with a computer simulation. The results show that the technique is effective in minimising the amplitude of spurious long waves that can cause an unrealistic increase in harbour and moored ship response at wave group periods.

CONTENTS

	Page
1 INTRODUCTION	1
2 DIFFERENCES BETWEEN SHORT CRESTED AND LONG CRESTED WAVES	4
2.1 Wave pattern inside harbours	4
2.2 Ship movement at ordinary wave periods	5
2.3 Wave group responses	7
3 SHORT CRESTED WAVES FOR HARBOUR MODELS	9
3.1 Separate versus hinged paddles	10
3.2 Consequences of using wave guides	12
3.3 Proposed method	14
4 COMPENSATION FOR SET-DOWN BENEATH WAVE GROUPS	14
4.1 Theoretical model	15
4.2 Results of computer simulation	20
5 CONCLUSIONS	21
6 ACKNOWLEDGEMENTS	23
7 REFERENCES	24

FIGURES

1. Diagram of uniform, long crested random and short crested waves
2. Effect of short crestedness on wave diffraction
3. Vertical response of bulk carrier in head seas at 8 knots
4. Directional wave energy spectrum for 100 year return period for sector 240° in area E
5. Bow response of supertanker at 12 knots
6. Directional energy spectrum for bow movement in 100 year wave in sector 240° in area E
7. Set-down spectra for scaled Moskowitz wave spectrum ($H_s = 5\text{m}$, $T_p = 20\text{s}$) in 20 m water depth
8. Dover model layout
9. Schematic diagram of short crested wave-maker
10. Limiting angles for oblique wave generation
11. Effect of refraction on directional spread in coastal waters
12. Spread to be generated in deeper water to obtain best fit to predicted spread in shallower water
13. Effect of generating symmetric spread in deeper water with same mean spread as predicted (asymmetric) spread
14. Primary wave spectra
15. Spreading functions for primary wave spectra
16. Long period wave frequency spectra
17. Directional spread of long waves for frequency near long period spectrum peak
18. Long period spectrum for case (1) in depth equivalent to 20m
19. Long period spectrum for case (2) in depth equivalent to 30m

1 INTRODUCTION

Long crested random waves have been used in physical models at Hydraulics Research (HR) to study coastal harbours since the early seventies. Such waves are more realistic than the single period waves used previously. This is because the sea is known to consist of a superposition of wave components with a range of periods. The distribution of energy over the various components is displayed in the form of a wave spectrum. Since surface waves are dispersive, different components propagate at different speeds which tends to randomise the phasing between components. The result is that the sea surface has an irregular appearance; large waves form when components come into phase with one another, small waves occur when the components are largely out of phase with one another. This contrasts with single period waves where heights are uniform (Figs 1(a) and 1(b)).

One of the major improvements resulting from the use of random waves in harbour modelling was the more realistic wave pattern in a given harbour layout.

With single waves a small change in period produced a new pattern of wave height within the model harbour: high and low points, where reflected waves were in phase and out of phase with incident waves, altered position. As a result wave bridges were used to enable the largest wave height along a quay face to be identified. The position of this largest height varied with period: it also varied when the period was kept constant and the harbour layout altered. Thus, different schemes were judged by comparing maximum wave heights along a quay face over a range of single period tests. The varying position of this maximum height from period to period, and scheme to scheme, had to be ignored.

With random waves the various wave components representing local conditions are present in the one test. Apart from reducing the number of tests necessary to evaluate a scheme, it produces a more even distribution of wave height along a quay face even in the presence of reflections. As a result, wave bridges are no longer necessary and single point measurements become meaningful. This enables analyses to be performed of the time history of the wave pattern at various points of interest within the harbour. Such analyses lead to the definition of the significant wave height (the average of the highest one third of the waves) the zero crossing wave period (an average period of the wave record) the maximum wave height occurring over the duration of the test and, with spectral analysis, the distribution of

energy over the various wave components. All this information, from points of interest in the model harbour, enables a more meaningful comparison to be made between different layouts.

Greater realism in ship responses was another major benefit resulting from the use of random waves. Estimates of berth tenability can be obtained through the use of models of moored ships. Not infrequently, model vessels placed in single period harbour models displayed little of the behaviour known to occur within real harbours. While the models would pitch, roll and heave in the residual wave activity at a berth, they moved only slightly on their moorings when ranging (slow horizontal movements along and transverse to the quay face) was known to be a problem in the real port. All this altered when random waves were used. Models of vessels with 5,000 tonnes displacement and above were found to range at their natural periods of horizontal motion on their moorings, ie with periods from 20 seconds up to several minutes. Considerable research world-wide has been carried out in the past decade to explain this behaviour which is more in accord with observation. It has been shown that there are a number of mechanisms producing forces at wave group periods as well as forces at wave periods. Such forces were absent in single period models because wave heights were uniform. In random waves, groups develop naturally and this leads to models of moored vessels ranging on their moorings.

One of the most important sources of energy causing wave group forcing on vessels moored in harbours has been identified as set-down beneath groups. Due to non-linearities in wave motion a long period disturbance, travelling with groups of waves, develops naturally in the water. This causes a depression in the mean water level beneath groups of large waves with a rise between groups to compensate. While the phenomenon was described by Longuet-Higgins and Stewart in the sixties (Ref 1) its significance for coastal harbour design was only fully appreciated when random or irregular waves were used in harbour modelling. Set-down incident on a harbour was shown by Bowers (Ref 2) to cause long waves inside the harbour. The long waves, with periods of 20 seconds to several minutes, are able to excite natural period oscillations of vessels on their moorings. Due to their long wavelength character these disturbances penetrate more deeply into the harbour than ordinary waves with the result that ranging still occurs even in well sheltered berths.

Research carried out at HR (Ref 3) showed the importance of compensating for set-down at the wave-maker. Once a random wave system is formed, non-linearities within the wave motion ensure that set-down develops too. At the wave-maker, however, it is necessary to compensate for the water particle movements associated with set-down while generating the ordinary waves. If this is not done spurious long waves are produced. The earlier work on set-down compensation has led to a significant improvement in the accuracy of harbour modelling (Ref 4). Set-down compensation is now used on a regular basis in physical models of harbours employing long crested random waves, and it has led to a more realistic representation of vessel ranging at wave group periods.

The next step in making harbour models more realistic is to use short crested waves (see Fig 1(c)). Here, wave energy is distributed over a range of directions as well as a range of periods. Storm waves in deep water are known through measurement (Ref 5) to have a broad spread of energy in direction whereas swell from distant storms has a narrow spread, making it appear more long crested.

As waves propagate into coastal waters, wave refraction tends to line-up wave crests with the seabed contours, effectively reducing the directional spread of wave energy (Fig 11). This has been used in the past as an argument to justify the use of long crested waves in harbour models; it being thought that the spread in direction remaining at the harbour entrance after refraction was small enough not to cause a significant effect on results obtained with long crested waves. However, in Section 2 of the report it is shown that directional spread can have significant effects on harbour and ship motion responses. In particular using long crested wave results as a basis for comparison, set-down is some 50% smaller even for wave conditions with relatively narrow directional spreads. This implies that under storm conditions, even where spread may be relatively narrow after refraction, the long wave components directly excited within the harbour by set-down will be very much reduced. This in turn will reduce ranging of moored vessels at wave group periods. In the limit of minimal directional spread, which is probably the case with swell from very distant storms, the set-down associated with long crested waves will have more realistic magnitudes. Models using long crested waves are then realistic provided set-down compensation is used at the wave-maker.

In Section 3 of this report the proposed method of short crested wave generation is described. Set-down compensation for that method of wave generation is presented in Section 4 with results from a computer simulation to show its expected effect. Conclusions appear in Section 5.

2 DIFFERENCES BETWEEN SHORT CRESTED AND LONG CRESTED WAVES

2.1 Wave pattern inside harbours

Short crested waves produce a more even spread of wave height within a harbour. This quite logical result can be illustrated by representing short crested waves as a superposition of wave components from a range of directions for a single wave period. The spread of wave energy over those components is taken to be that measured in the Southern North Sea (Ref 5). When this wave system is assumed to approach a single breakwater arm, with its mean direction at right angles to the breakwater, the pattern of wave height contours produced behind the breakwater is shown by the dashed lines in Fig 2. This compares with the solid line produced by a long crested wave of the same period (8 seconds) approaching at right angles to the breakwater. It can be seen that a more gradual transition, to reduced wave height within the shelter of the breakwater, occurs with short crested waves than with long crested waves. This means short crestedness increases the residual wave activity well within the shelter of a breakwater.

The increase in wave height encountered above is balanced by a somewhat reduced wave height on the edge of the shadow cast by the breakwater (see area just to the left of the breakwater tip in Fig 2). This, in effect, means the breakwater casts a larger shadow when subjected to short crested waves. For the situation of a harbour entrance consisting of a gap between two breakwater arms, the larger shadows cast by the two breakwaters will result in smaller waves inside the harbour at positions opposite the entrance. There will, of course, be corresponding increases in wave height well within the shelter of each breakwater.

Another point to bear in mind is that the directional spread assumed in obtaining the result shown in Figure 2 is that expected in deep water. Generally, only the shorter period waves incident in harbours will possess such broad spreads. Longer waves are affected by wave

refraction which tends to reduce the directional spread (Fig 11). As a result, the differences in primary wave response to short crested and long crested waves can be less marked and will vary from site to site.

2.2 Ship movement at ordinary wave periods

The effect described in 2.1 will, in turn, affect the magnitude of vessel response at a berth: the broader the spread at the harbour entrance, the more the ship response will be affected. We can also consider whether directional spread can affect vessel response more directly.

Waves from a variety of directions can affect positions within a harbour simultaneously when reflections from internal harbour boundaries add to the main wave component entering directly through the entrance. Thus, internal wave reflections will tend to cause some directional spread in wave energy. Effects of such a spread on vessels moored inside the harbour will be represented with both short crested and long crested incident waves. In short crested waves the degree of directional spread introduced directly by the incident waves entering through the harbour entrance may be less significant than the spread caused by internal reflections, particularly where the harbour entrance consists of a gap between two breakwater arms. In this case, waves can only travel from a well defined harbour entrance to reach internal positions directly. This component will tend, therefore, to be almost uni-directional.

However, vessels in navigation channels outside harbours will experience the full directional spread present in the incident waves and we can consider its effect on vertical vessel motions in waves as this, in turn, can affect the dredged depth required in the navigation channel.

The multi-directional wave spectrum can be expressed in the form:

$$S_w(f, \theta) = S(f) G(\theta),$$

where $S(f)$ is the one dimensional wave spectrum for components of frequency f (inverse of wave period) and G is a normalised spreading function for components with angles θ to the mean direction. We can form the one dimensional ship's spectrum using the response $r(f, \theta)$ of the vessel to waves (r is the ratio of

vessel movement to wave amplitude at frequency f and direction θ).

$$\begin{aligned} S_s(f) &= \int r^2(f, \theta) S_w(f, \theta) d\theta \\ &= S(f) \int r^2(f, \theta) G(\theta) d\theta \\ &= S(f) R^2(f), \text{ say.} \end{aligned}$$

Here, $R(f)$ is the equivalent response function with directional spread taken into account. This response function can be compared with the response $r(f, \theta=0)$ to a unidirectional wave from the mean wave direction $\theta=0$.

The two responses are shown in Figure 3 for the case of a large bulk carrier underway in head seas at 8 knots with an underkeel clearance of 15% of its 20m draught. The results are from a mathematical model of vertical ship motion called UNDERKEEL which has recently been developed at HR and the response function is for the stern, which is the point on the ship with the largest vertical movement. Although the two curves appear similar, it can be seen that at frequencies between 0.05 and 0.08, ie in the wave period range 12 seconds to 20 seconds, the response of the vessel to a short crested sea is noticeably larger than the response to a long crested sea. This occurs because pitch increases as the waves are angled to the bow. The spreading function assumed in this case corresponds to short crested waves during a storm. For example, with the one dimensional wave spectrum $S(f)$ representing a significant wave height of 4m with a spectral peak at 15.8s, we find vertical movements at the stern are larger by 44% in short crested seas.

The effect of short crested seas on vertical motions of very large crude carriers (VLCCs) was one of the factors considered in a recent investigation that made use of the mathematical model UNDERKEEL (Ref 6). The study was carried out to establish a safe underkeel allowance for VLCCs passing through the Dover Strait. On one section of the route, in the western part of the Strait, the north east bound tankers carrying oil from the Middle East to Europoort at Rotterdam could experience large storm waves from a south westerly sector (mean wave direction 242°N). The significant wave height is predicted to be 7m for a 100 year return period and the degree of directional spread expected in that storm is shown in Figure 4. The vessel heading on that part of the route (18°N) means that such waves would approach the stern at an average angle of 44° . The full response function of the tanker obtained from UNDERKEEL is shown in Figure

5, the bow vertical movement being the largest in such conditions. The response is seen to be even larger at angles of approach to the stern of 50° to 60° than it is at 44°. Inspection of the full wave spectrum (Fig 4) shows that more energy is present in these waves (248°N to 258°N) than in waves from the mean direction (242°N). This is due to asymmetry in the spreading function. As a result, bow movement obtained with the expected amount of directional spread is considerably larger (see Fig 6 for the resulting multi-directional spectrum of vessel movement) than would be the case were uni-directional waves assumed to approach from the mean direction. The resulting effect on the underkeel allowance for vessel motions in waves is equally large. A figure of 5.6m is obtained for the multi-directional or short crested waves whereas an allowance of only 3.9m would result were uni-directional or long crested waves, assumed.

As with harbour response to ordinary wave motion (see 2.1) the differences in vessel response to short crested waves and long crested waves gradually diminish as the directional spread decreases. This means the importance of short crestedness in these ordinary wave responses will vary from site to site. In the case of moored ship responses, however, the situation is different because even after refraction, storm waves will have sufficient spread to produce a significant effect on disturbances at wave group periods. This aspect is considered in the next sub-section.

2.3 Wave group responses

As explained in the introduction, one of the most important sources of energy causing vessels moored inside harbours to respond at wave group periods is set-down. Here we consider the effect of short crestedness on set-down.

To demonstrate this, it is convenient to plot the expected spectrum of set-down associated with various primary wave conditions. Examples are given in Figure 7 for primary waves in a water depth of 20m with a significant height of 5m and a spectral peak at 20s. The spreading function takes the form:

$$G(\theta) = \frac{1}{\sigma \sqrt{\pi}} e^{-(\theta/\sigma)^2}$$

This allows the amount of spread to be varied simply by varying σ . The significant height of set-down is

defined in the same way as the primary wave height, ie 4 times the square root of the area under the spectrum of set-down which is equivalent to 4 times the standard deviation of set-down.

The result for long crested waves ($\sigma=0$) shows a very large set-down with an exponentially shaped spectrum falling off rapidly as frequency increases.

The spectrum of set-down alters completely with even a limited amount of spread ($\sigma=15^\circ$ - see case (1) in Fig 15). There is far less energy (its height is approximately halved from 1.64m to 0.86m) and a peak develops at a frequency of 0.015 ie at a period of about 70s, with energy falling off rapidly at longer periods (lower frequencies).

The situation with a broader spread ($\sigma=30^\circ$ - see case (2) in Fig 15) is that set-down retains its peak at about 70s but its height drops further to 0.63m.

These results clearly show that in coastal waters set-down is considerably smaller in short crested waves than it is in long crested waves. In particular, even with the relatively narrow spreads expected at sites where wave refraction effects are strong, the resulting height of set-down is likely to be roughly half the height obtained with long crested waves. This implies that in models where long crested waves are used to represent short crested storm waves, harbour and moored ship responses at wave group periods could be up to double their true value. The case of a site subject to swell, however, should be better represented in a random sea model using long crested waves.

In addition, whether the appropriate waves for a given site are long crested or short crested, care needs to be taken to use set-down compensation at the wave-maker to avoid the generation of spurious waves at group periods. This has already been demonstrated for long crested waves (Ref 4) and it is shown in Section 4 to be equally necessary for short crested waves.

Also, for wave group responses, care is needed in representing the effects of surf beats in harbour models. This requires careful attention to be paid to the effect of the coastline if the reflection of the incoming set-down is to be well represented; it being thought that surf beats are associated with reflections of set-down from the coastline.

3 SHORT CRESTED
 WAVES FOR
 HARBOUR MODELS

Apart from the need to generate short crested seas, there are two important requirements for generating waves in harbour models.

The first requirement is for a large range of mean wave directions. Figure 8 shows wave-maker positions used in studying Dover Harbour. The waves were, of course, long crested but the mean directions are different by more than 90° for south westerly storms (position 1) and easterly storms (position 3). The nearshore wave directions took account of any changes in direction due to depth refraction as the waves travel from deep water to the depths represented at the wave-maker.

The large range in mean direction is typical of harbour studies for the following reason. It is usually necessary to provide shelter from a dominant wave direction (SW waves in the case of Dover) and so breakwaters are designed to maximize shelter for that direction. However, this will leave the harbour more open to secondary wave directions at large angles to the dominant direction. For example, the Western Docks at Dover are less open to the dominant SW direction than they are to smaller easterly wave running in through the Western Entrance. The end result is that schemes have to be tested with waves from both dominant and secondary directions to ensure that they perform adequately.

This requirement has been satisfied up to now by making the long crested wave-makers mobile. With the ability to generate short crested seas, ie multi-directional waves, it might be thought that one could use a fixed wave-maker capable of generating different mean directions as well as a spread in direction about the mean. But Figure 8 shows that such a wave-maker would need to extend around almost 3 sides of the wave basin to be capable of providing the required mean directions. This option would be very costly. Therefore, we limit consideration to a mobile wave-maker in order to satisfy the requirement for a large range of mean directions without having an extremely long wave front.

The second requirement is that sufficient wave front be generated to obtain a realistic harbour response. For example, overtopping of breakwaters can sometimes add to the disturbance within a harbour and so the wave front must cover the relevant length of breakwater as well as the harbour entrance. Sometimes

the harbour has two entrances. This may mean sufficient wave front is needed to cover both entrances. For example, in Dover (Fig 8), it was found that under SW wave attack the Eastern Docks were affected by the disturbance coming through the Eastern Entrance as well as a larger wave component coming through the Western Entrance. Thus, the wave front had to be sufficiently long to cover both entrances. It was also necessary for the wave front to cover the Admiralty Pier breakwater (the western arm of the Western Entrance). This was because the vertical walled breakwater produced a strong reflection under SW wave attack, and this reflection contributed to the disturbance diffracted into the harbour through the Western Entrance.

This sort of requirement on the length of wave front is quite typical in harbour studies. It has been satisfied up to now, without having to generate a very long wave front, by using wave guides (see dashed lines at the ends of the various wave generator positions in Fig 8). These guides are simply walls that are positioned to guide the wave front up to the model harbour taking account of any depth refraction. They prevent a loss of wave height that would otherwise occur due to diffraction of energy off to the sides of the wave front. It is clear that a short crested wave-maker, supplying a similar length of wave front as the present long crested generators, will also need to employ wave guides. The alternative of having a long enough generator to avoid the need for guides is too costly. With guides present, component directions in the short crested sea will reflect off the guides as wave fronts propagate towards the harbour. Thus, reflection from the guides will become an essential part of the multi-directional sea and the control signal driving the multi-element paddles must take this into account. Fortunately, such a control signal has already been developed at HR for driving a multi-element wave-maker in a deep wave basin used for testing offshore structures.

In summary, two requirements of harbour modelling lead to a mobile short crested wave-maker that makes use of wave guides to produce a multi-directional sea over a sufficient length of wave front. Such a generator keeps the cost of producing adequate short crested waves for harbour modelling, within reasonable limits.

3.1 Separate versus hinged paddles

A schematic picture of a short crested wave-maker in a water depth d is given in Figure 9. A wave

propagating at an angle Θ to the paddle normal can be generated by programming individual paddles of width ℓ . Although each paddle must have its own drive, neighbouring paddles are correlated in such a way as to give the correct resolved wavelength along the face of the wave-maker. When operating, the face of the wave-maker appears to wriggle like a snake and such generators were called serpent wave-makers in the past when single period waves were used.

If the individual paddles are separate, so that they can slide relative to one another, they will produce a "stepped" approximation to the required variation along the face of the generator. The theory for this case has been described by Gilbert (Ref 7). It might be thought that by linking individual paddles with vertical hinges, it would be possible to obtain a smoother approximation to the required variation along the generator. A prototype generator employing this idea was tested at the Danish Hydraulic Institute. If effective, it would mean fewer hinged paddles could be used to give as good a match as a larger number of separate paddles. However, in practice, the benefits of using vertically hinged paddles appear small.

An example is shown in Figure 10. The water depth at the wave-maker is taken to be 0.3m, a typical depth for harbour modelling. The limiting angle for oblique wave generation (Θ in Fig 9) is plotted as a function of the ratio of wavelength (λ) to individual paddle width (ℓ in Fig 9). The two lower scales show the wave periods corresponding to particular ratio values when $\ell=0.3\text{m}$ and when $\ell=0.5\text{m}$. These periods are given at full scale assuming a harbour model scale of 1 to 100; a typical choice. Figure 10 shows that, provided there are at least 2 individual paddles per wavelength, a full range of directions can be generated up to 90° to the wave-maker normal. If individual paddles are 0.3m wide, this means waves with periods longer than 6s can be generated at any angle. The limiting period becomes 8s for a paddle width of 0.5m. The situation with fewer than 2 paddles per wavelength is that a kind of aliasing can take place if waves at too great an angle are generated. If the main wave angle is greater than a limiting value (Θ_c) defined by,

$$\sin \Theta_c = \frac{\lambda}{\ell} - 1,$$

additional waves would also be generated with very variable amplitude and directions. They cannot, therefore, be used as part of a required directional wave spectrum. For this reason they must be either

excluded or kept at low amplitude relative to the main wave.

Figure 10 shows that a small benefit is obtained with hinged paddles for wavelength to paddle width ratios of less than 1; waves up to 10° to the normal could be generated if spurious wave amplitudes up to 1/10th of the main wave were accepted. There is no benefit from using hinged paddles for ratio values between 1 and 2; spurious waves of large amplitude would be generated for main wave angles only slightly larger than the limiting angle. The full range of directions can be generated for ratio values above 2 whether hinged or separate paddles are used.

Overall, hinged paddles appear to be of little benefit.

3.2 Consequences of using wave guides

It was explained at the beginning of this section that wave guides are necessary if the overall length of the wave generator is to be kept within reasonable limits and yet sufficient wave front is to be maintained between the generator and the model harbour. Guides prevent wave diffraction off to the sides and an unrealistic loss of wave height. In this situation, wave components in a multi-directional sea will both reflect off the guides and refract over the depth contours before they reach the harbour entrance. These reflections will clearly contribute to the directional spread of wave energy. With long crested or uni-directional waves, reflections do not occur because the guides follow lines of refraction.

To judge the effect of reflections off the guides we can first consider just the refraction of 10s waves, with a $\cos^2\theta$ directional spread in deep water, approaching straight parallel contours with a mean direction at an angle of 60° to the contours (mean crest at 30° to the contours). This spread is shown in Figure 11 as the deep water value. When the waves propagate into shallower water, the spread tends to become asymmetric about the direction with maximum energy, which itself changes due to refraction. The estimated spreads on the 30m, 20m and 10m contours are shown in Figure 11. The latter spreads are not normalised in order to show how the overall wave energy is reduced by refraction. Thus, with 75% of the deep water energy present on the 10m contour, the wave height will be 0.87 of the deep water value. If the waves were long crested instead of short crested, the corresponding wave height reduction would be less

with a height on the 10m contour of 0.94 times the deep water value.

Figure 11 demonstrates the narrowing in the spreading function that will occur due to depth refraction as wave crests tend to line up with the contours. It also shows that asymmetric spreads should be generated in order to precisely represent the effects of refraction (see spreads on 20m and 30m which are typical of depths represented at the wave-maker).

With guides present, reflections will alter the spread. If we assume initially that an attempt is made to produce the predicted spread on the 10m contour, then we can estimate the sort of spread needed on the 30m contour and compare that with the spread actually predicted ie that shown for the 30m contour in Figure 11. The results appear in Figure 12. In making these estimates, the length of the generator is taken to be 15m, the wave guides follow the changes in the maximum energy direction due to refraction, the point of interest on the 10m contour is in the middle of the working area, and the change in depth from 30m to 10m occurs in a 15m length measured perpendicular to the contours. The 10m depth can be considered typical for a harbour entrance.

Figure 12 demonstrates the difficulties involved in attempting to reproduce asymmetric spreads. Some of the energy required on the 10m contour, that for angles greater than about 34° , cannot even be generated due to the combination of reflection and refraction. In addition, the final asymmetric spread to be generated on the 30m contour is quite different from the predicted spread for that depth (dashed line).

An alternative approach is to accept that symmetric spreads must be generated if wave guides are used. The result of this is shown in Figure 13. The symmetric spread chosen for generation is assumed exponential with the same root mean square spread and energy level as the predicted spread on the 30m contour. The mean directions on that contour are also assumed the same and, as the predicted spread is asymmetric, its direction of maximum wave energy lies to one side of the mean. Refracting and reflecting the symmetric spreading function up to the 10m contour leads to the estimated spread in Figure 13. In spite of reflections the estimated spread is slightly asymmetric whereas, in practice, the guides may well force complete symmetry. These additional effects of the guides are difficult to take into account but they are not expected to significantly alter the estimated

spread shown in Figure 13. In general, the agreement between the estimated spread and the directional spread predicted for the 10m contour is very encouraging. In addition, the reduction in wave height due to refraction is also well represented; a value of 0.88 of the deep water height is obtained on the 10m contour. This compares with a reduction to 0.87 of the deep water height when just refraction is considered (Fig 11).

In summary, wave guides force the use of symmetric directional spreading functions when generating short crested seas, whereas asymmetry can occur in the real sea due to depth refraction. However, this is not expected to lead to significant errors in harbour response; the combination of refraction and reflection from the guides that occurs as the waves travel from the generator to the model harbour appears to result in an adequate representation of the effect of refraction on the directional spread in the real sea.

For waves approaching a harbour with a mean direction perpendicular to the contours, a case which not infrequently leads to the largest waves because depth refraction has the least effect in reducing wave height, the spreading function will tend to remain symmetric. In this case the depth refraction effects will be represented exactly through the use of wave guides.

3.3 Proposed method

In view of the above results, the method of short crested wave generation proposed for harbour models makes use of separate individual paddles, a mobile generator, wave guides and symmetric functions about the mean wave direction to describe the directional spread expected in the real sea in the water depth represented at the wave-maker.

4 COMPENSATION FOR SET-DOWN BENEATH WAVE GROUPS

It was explained in the introduction how set-down occurs in the real sea and that it will occur naturally in model random seas but that care is needed in wave generation to avoid spurious long waves. It was mentioned that a technique has already been developed, and implemented at HR, for minimising the spurious long waves associated with long crested random wave-makers (Ref 4). This involves compensating for set-down at the wave generator. In application, set-down compensation has been found to have a significant effect on the response of harbours and moored ships at wave group periods. Here, a

technique is described for set-down compensation when short crested waves are generated. Such compensation is equally necessary for short crested seas since the spurious long wave amplitudes are again comparable with set-down amplitudes.

4.1 Theoretical model

The basic theoretical model assumes irrotational motion which allows the water particle velocity vector \underline{q} to be described in terms of a potential ϕ . To represent short crested waves, the velocity \underline{q} is taken to have two horizontal components u and v and a vertical component w in a right handed orthogonal co-ordinate system (x, y, z) . The x and y axes lie in the horizontal plane and the vertical axis points upwards with the origin at the mean water level (Fig 9).

$$\underline{q} = -\nabla\phi = (u, v, w).$$

Incompressibility results in the basic equation for the velocity potential,

$$\nabla^2\phi = 0. \tag{1}$$

The boundary conditions to be satisfied by surface waves with amplitude η are as follows.

On the bottom ($z = -d$)

$$w = 0. \tag{2}$$

On the free surface ($z = \eta$)

$$\frac{\partial\eta}{\partial t} + u\frac{\partial\eta}{\partial x} + v\frac{\partial\eta}{\partial y} - w = 0, \tag{3}$$

$$\frac{1}{2} \underline{q}^2 + g\eta - \frac{\partial\phi}{\partial t} = 0. \tag{4}$$

Equation (2) expresses the condition that the vertical velocity vanishes on the bottom, (3) is the equation of motion of the free surface and (4) arises from Bernoulli's equation after allowing for constant air pressure at the free surface.

The above set of equations is non-linear but an effective method of solution is to expand the basic wave quantities \underline{q} , ϕ and η in powers of the wave amplitude. This assumes the product terms in (3) and (4) are small in comparison with the other terms in the equations. This procedure, known as a Stokes' expansion, has been found to lead to a good description of second order effects like set-down in long crested random waves: measurements in a wave

flume are explained by predictions from the theoretical model.

Denoting first order quantities with a superfix one and second order (in the wave amplitude) quantities with a superfix two we find (1) to (4) become in lowest order:

$$\nabla^2 \phi^{(1)} = 0, \quad (5)$$

$$w^{(1)} = 0 \quad \text{on } z = -d, \quad (6)$$

$$\frac{\partial \eta^{(1)}}{\partial t} - w^{(1)} = 0 \quad \text{on } z = 0, \quad (7)$$

$$g\eta^{(1)} - \frac{\partial \phi^{(1)}}{\partial t} \quad \text{on } z = 0. \quad (8)$$

The surface elevation $\eta^{(1)}$ is assumed to be a superposition of wave components with a range of frequencies ω_j and directions θ_n^j .

$$\eta^{(1)} = \sum_j \sum_n a_n^j \cos(\omega_j t - k_j \sin \theta_n^j y - k_j \cos \theta_n^j x + \epsilon_n^j).$$

The j, n component has frequency ω_j , direction θ_n^j (see Fig 9) relative to the mean wave direction or wave-maker normal, amplitude a_n^j , wave number k_j and random phase ϵ_n^j . Each frequency will have a range of wave directions, including positive and negative values of θ_n^j . The amplitudes are defined by the wave spectrum:

$$(a_n^j)^2 / 2 = S(f_j) df G(f_j, \theta_n^j) d\theta$$

Here G describes the angular spread in wave energy and it is normalised, ie,

$$\sum_n G(f_j, \theta_n^j) d\theta = 1.$$

$S(f_j)$ is the usual one dimensional wave frequency spectrum where,

$$2\pi f_j = \omega_j.$$

Solving equations (5) to (8) enables the first order water particle velocity $u^{(1)}$ to be defined and the usual dispersion relation for surface waves to be derived:

$$\omega_j^2 = k_j g \tanh k_j d.$$

So far, the results apply equally to physical model and real sea conditions. But an additional boundary condition has to be satisfied in the physical model because the waves are generated by a paddle. Taking a piston paddle with a horizontal displacement ζ which is independent of the depth:

$$\frac{\partial \zeta}{\partial t} = u \quad \text{on } x = \zeta. \quad (10)$$

To lowest order this is simply:

$$\frac{\partial \zeta(1)}{\partial t} = u(1) \quad \text{on } x = 0,$$

and the movements of the individual paddle elements (see Fig 9) required to generate any particular component of the elevation (9) can be derived using the method given by Gilbert (Ref 7). Let the superposition of component paddle movement be $\zeta(1)$.

In second order (1) gives:

$$\nabla^2 \phi(2) = 0 \quad (11)$$

Expanding (4) about $z = 0$ gives:

$$\eta(2) = \frac{1}{g} \left(\frac{\partial \phi(2)}{\partial t} + \eta(1) \frac{\partial^2 \phi(1)}{\partial z^2 \partial t} - \frac{1}{2} (q(1))^2 \right) \quad (12)$$

Expanding (3) about $z = 0$ gives:

$$\frac{\partial \eta(2)}{\partial t} + u(1) \frac{\partial \eta(1)}{\partial x} + v(1) \frac{\partial \eta(1)}{\partial y} + \frac{\partial \phi(2)}{\partial z} + \eta(1) \frac{\partial^2 \phi(1)}{\partial z^2} = 0$$

Substituting for $\eta(2)$ from (12) into the above equation gives the boundary condition to be satisfied by the second order potential on the surface $z=0$:

$$\frac{\partial^2 \phi(2)}{\partial t^2} + g \frac{\partial \phi(2)}{\partial t} = \eta(1) \left(\frac{\partial^2 w(1)}{\partial t^2} + g \frac{\partial w(1)}{\partial z} \right) + 2q(1) \frac{\partial q(1)}{\partial t} \quad (13)$$

In obtaining (13) use has been made of (7) and (8) to rearrange terms on the right hand side. These product terms give rise to summations involving differences and sums of wave component frequencies. It is the terms at difference frequencies that lead to set-down.

Equation (13) shows the origin of set-down beneath wave groups. Surface effects, like the reduction in water pressure due to the $\frac{1}{2}(q(1))^2$ term in Bernoulli's equation (12), drive a wave-like flow beneath the surface with velocity potential $\phi(2)$. These surface

effects appear on the right hand side of (13) and they can be evaluated after substituting for the first order elevation $\eta^{(1)}$ and velocity $u^{(1)}$. When this is done, the solution for $\phi^{(2)}$ can be found by solving Laplace's equation (11) subject to boundary condition (13) on the surface and the requirement that the vertical velocity vanish on the bottom $z = -d$, ie

$$w^{(2)} = - \frac{\partial \phi^{(2)}}{\partial z} = 0 \quad (14)$$

The final expression for set-down is then obtained from (12) after substituting for $\phi^{(2)}$ and retaining the difference frequency terms. It is clear from (12) that, due to its sign, the third term on the right hand side will tend to cause a depression beneath groups of large waves where the orbital wave velocity $u^{(1)}$ is large. After solving for $\phi^{(2)}$ it is found that the phasing of the first term on the right hand side of (12) is the same as the third term. This means the wave-like flow represented by $\phi^{(2)}$ tends to reinforce the depression beneath groups of large waves making set-down larger. In coastal waters the contribution from $\phi^{(2)}$ dominates over the other terms on the right side of (12). It is this term that increases rapidly as the waves propagate into shallow water.

So far, the discussion of second order quantities applies equally well to model and real sea conditions. Set-down will arise naturally through non-linearities in the model waves once the first order wave system described by (9) has been generated. The only difficulty in the model occurs at the wave-maker.

The boundary condition to be satisfied at the generator is obtained from (10) to second order by expanding about the mean paddle position $x=0$, ie

$$\frac{\partial \zeta^{(2)}}{\partial t} = - \left[\frac{\partial \phi}{\partial x} \right]_{x=\zeta}^{(2)} = - \left[\frac{\partial \phi^{(2)}}{\partial x} + \zeta^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial x^2} \right]_{x=0} \quad (15)$$

Equation (15) expresses the fact that a second order (in wave amplitude) paddle movement $\zeta^{(2)}$ is required to balance water particle velocities associated with set-down (the $\phi^{(2)}$ term) and a term introduced by the first order movement of the paddle. As the set-down contribution dominates in coastal models, the additional paddle signal $\zeta^{(2)}$ can be called the set-down compensation signal.

If the wave-maker is only programmed to generate the first order, or primary wave system, then the left

hand side of (15) is zero and the only way the equation can be satisfied is through introducing a free long wave. Denoting its velocity potential by $\phi_L^{(2)}$ equation (15) gives:

$$\frac{\partial \phi_L^{(2)}}{\partial x} = - \frac{\partial \phi^{(2)}}{\partial x} - \zeta^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial x^2} \quad \text{on } x = 0 \quad (16)$$

The form of this free wave is obtained by solving Laplace's equation (11) subject to boundary condition (13) with the right hand side set to zero and boundary condition (14) on $z=-d$. The amplitude of the free wave is then defined by (16). It differs from set-down in that it propagates at the usual phase velocity for surface waves. This is faster than the group velocity which is the speed of propagation of set-down. The result is that the free wave, which is 180° out of phase with set-down at the generator (equation (16)), can enhance the overall long period energy at some distance from the generator as its phasing alters relative to set-down. In the case of long crested random waves this unrealistic enhancement can result in increases in moored ship response of some 20 to 30% in a typical harbour model (Ref 4).

The approach taken at HR to counteract this problem is to approximate to the required second order paddle movement (defined by (15)) in "real time". Working in real time means that an approximate long period movement of the wave-maker is calculated instantaneously according to the particular wave grouping pattern being generated at the time. This imposes no limits on the length of test that can be employed which is a great advantage where responses are non-linear and long tests are necessary to establish the statistics of extreme movements and loads. As the expressions on the right hand side of (15) are complex, they involve four summations (two over frequency components and two over component directions) it would be very difficult to calculate exactly the instantaneous second order paddle movement necessary to satisfy (15). Instead, a simpler form for the second order paddle movement is adopted in such a way as to limit the amplitude of the spurious long waves. In this case (15) becomes:

$$\frac{\partial \phi_L^{(2)}}{\partial x} = - \frac{\partial \phi^{(2)}}{\partial x} - \zeta^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial x^2} - \frac{\partial \zeta^{(2)}}{\partial t} \quad (17)$$

where the real time second order paddle movement $\zeta^{(2)}$ minimises $\phi_L^{(2)}$. The results of this approach are described in the following sub-section.

4.2 Results of computer simulation

To illustrate the technique described above for real time set-down compensation, a multi-directional wave-maker is assumed that makes use of wave guides in the manner described in 3.2. The overall length of the wave-maker, or distance between the guides is taken to be 15m. Separate paddles, each 0.5m wide, are assumed to be generating short crested seas.

Two multi-directional wave spectra are considered. Each is taken to be of the form:

$$S(f) \frac{1}{\sigma \sqrt{\pi}} e^{-(\theta/\sigma)^2}$$

where $S(f)$ is the usual one dimensional wave frequency spectrum and the other (normalised) term describes an exponential spread of wave energy in direction.

The first example represents a long period (spectral peak at 20s) sea with a significant wave height of 5m in a depth equivalent to 20m (case (1) in Fig 14). These parameters are given at full scale assuming a model scale of 1 to 100 which is typical for a harbour study. After refracting into a 20m depth the directional spread will be much reduced due to the long wave period and so a mean spread of only 15° is assumed (case (1) in Fig 15).

The second example represents a more typical storm condition with a significant height of 4.8m and a spectral peak at 14.7s (case (2) in Fig 14). The water depth is equivalent to 30m and so less refraction will have taken place: the average spread is taken to be 30° (case (2) in Fig 15).

The period scale shown on Figure 10 for 0.5m individual paddle widths in a model depth of 0.3m (30m full scale) indicates a limiting spread of 30° at a period of about 7s ie at a wave frequency of $0.14c/s$ at full scale. This allows the full directional spreads in cases (1) and (2) to be well represented (Fig 15) as there is little energy in either spectrum at higher frequencies (Fig 14) where the spread that can be generated is more limited.

The one dimensional long wave frequency spectra associated with sea conditions (1) and (2) are presented in Figure 16. All of these spectra range over periods of 30 seconds ($0.03c/s$) to 3 minutes ($0.006c/s$) at full scale. There are two main

components ie set-down itself and the spurious long waves. The latter are presented with and without real time compensation at the wave-maker. It can be seen that the compensation technique is more effective for the broader spread, deeper water wave condition (2). Without compensation the spurious long waves have amplitudes comparable with set-down. Taken together the results with real time set-down compensation are very encouraging with the remaining spurious long wave energy expected to enhance the overall long period response of harbours and moored ships by 4 to 8%.

To gain an idea of how the spurious long wave energy is distributed in direction we can consider the spread in a wave frequency component close to the peak in long wave energy ie 0.015 (67s) in case (1) and 0.021 (48s) in case (2). The spread in energy of the spurious waves without set-down compensation is not dissimilar from the directional spread in the primary waves in the two cases: compare Figure 17 with Figure 15. Figure 17 also shows that real time set-down compensation is particularly effective for the mean wave direction component (0°).

The latter point mentioned in the above paragraph is well illustrated in Figures 18 and 19. Spectra are shown for cases (1) and (2), respectively, with energy in the mean wave direction shown alongside the rest of the energy contained in all the components propagating at an angle to the mean.

5 CONCLUSIONS

1. The effect of short crested seas on harbour and vessel response has been considered using computational and analytical work. Taking long crested waves as a basis for comparison the following differences are expected.
 - (a) The distribution of wave energy within a harbour is likely to alter with larger wave heights inside the shelter of breakwaters and smaller wave heights at positions opposite the entrance.
 - (b) For vessels entering and leaving harbours, the vertical motions (generally dominated by pitch and heave) are expected to be higher in short crested waves. The exception is for a vessel experiencing beam seas where roll becomes the dominant movement. In this case short crested waves should cause less vertical movement.

- (c) Wave refraction, which tends to line up wave crests with seabed contours, will tend to reduce the directional spread in wave energy in coastal waters which in turn will reduce the differences in primary wave response mentioned above.
 - (d) Even after allowing for strong refraction at a coastal site, the remaining directional spread is expected to significantly reduce the long period energy associated with wave grouping. The largest source of energy in storm conditions, set-down beneath wave groups, is expected to be some 50% smaller in short crested seas implying that set-downs represented in the present long crested random wave physical models are too large by a factor of two. On this basis alone, the use of short crested seas in physical models can be expected to have a significant effect on the outcome of harbour investigations because wave grouping responses have been found to be so important in past studies.
2. Certain aspects in harbour modelling, ie the large range of mean wave directions needed in testing and the requirement for sufficient wave front to cover harbour breakwater arms as well as the entrance, can only be accommodated at reasonable cost by having a mobile wave-maker that makes use of wave guides to maintain the wave front up to the harbour area.
 3. The use of wave guides for a multi-directional sea means reflections off the guides become an integral part of the final wave spectrum and that symmetric spreading functions must be generated at the wave-maker. Although wave refraction can cause asymmetries in the directional spread, approximating with a symmetric spread in depths typically represented at the wave-maker is not expected to cause significant errors in harbour response.
 4. There appears to be no real advantage in using vertically hinged paddles instead of separate paddles to approximate to the wave variations across the face of the generator.
 5. Spurious long waves will again be a feature in short crested wave generation as they are for long crested wave generation. A technique for real time set-down compensation at the wave-maker

has been simulated computationally and found to be effective in minimising the spurious long waves. The long wave energy remaining after compensation should increase the overall response of harbours and moored ships at wave group periods only slightly, ie by less than 10%.

6. The proposed short crested wave-maker is expected to improve the accuracy of physical models of harbours to a significant degree and have a profound effect on the final outcome of investigation.

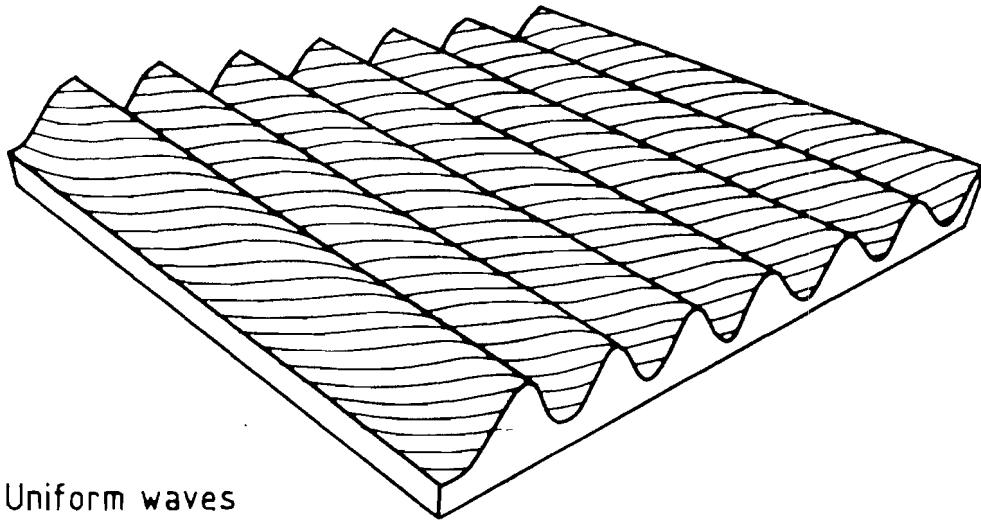
6 ACKNOWLEDGEMENTS

I am grateful to Mr J M A Spencer for his invaluable help with the numerical computations and to Mr G Gilbert for many helpful discussions.

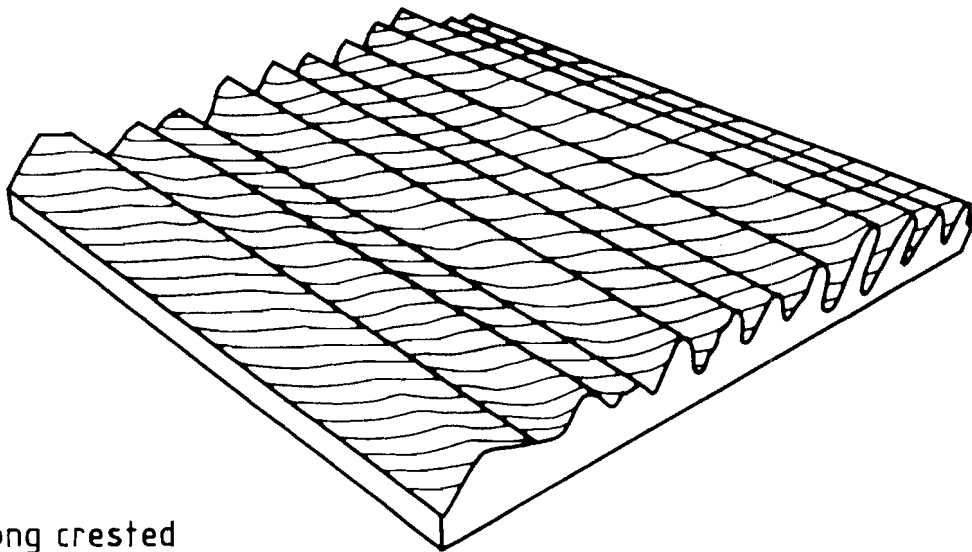
7 REFERENCES

1. Longuet-Higgins M S and Stewart R W. Radiation stresses in water waves: a physical discussion with applications. Deep Sea Research Vol II p 529, 1964.
2. Bowers E C. Harbour resonance due to set-down beneath wave groups. J Fluid Mech. Vol 79 p 71, 1977.
3. Bowers E C. Long period disturbances due to wave groups. 17th Int Conf Coastal Eng. Sydney, Vol I p 610, 1980.
4. The correct representation of set-down beneath wave groups in random wave harbour modelling. HR Report SR 101, 1986.
5. Hasselmann D E et al. Directional wave spectra observed during JONSWAP 1973. J Phys Oceanogr. Vol 10 p 1264, 1980.
6. Underkeel allowance for deep draughted vessels in the Dover Strait. HR Reports EX 1309, 1985 and EX 1432, 1986.
7. Gilbert G. Generation of oblique waves. HRS Notes No 18, May 1976.

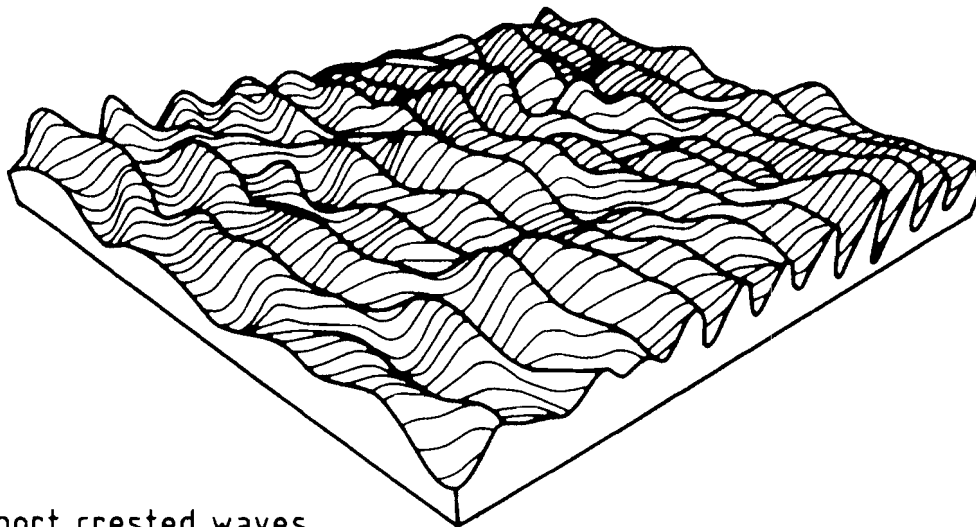
FIGURES.



(a) Uniform waves



(b) Long crested
random waves



(c) Short crested waves

Fig 1 Various wave representations

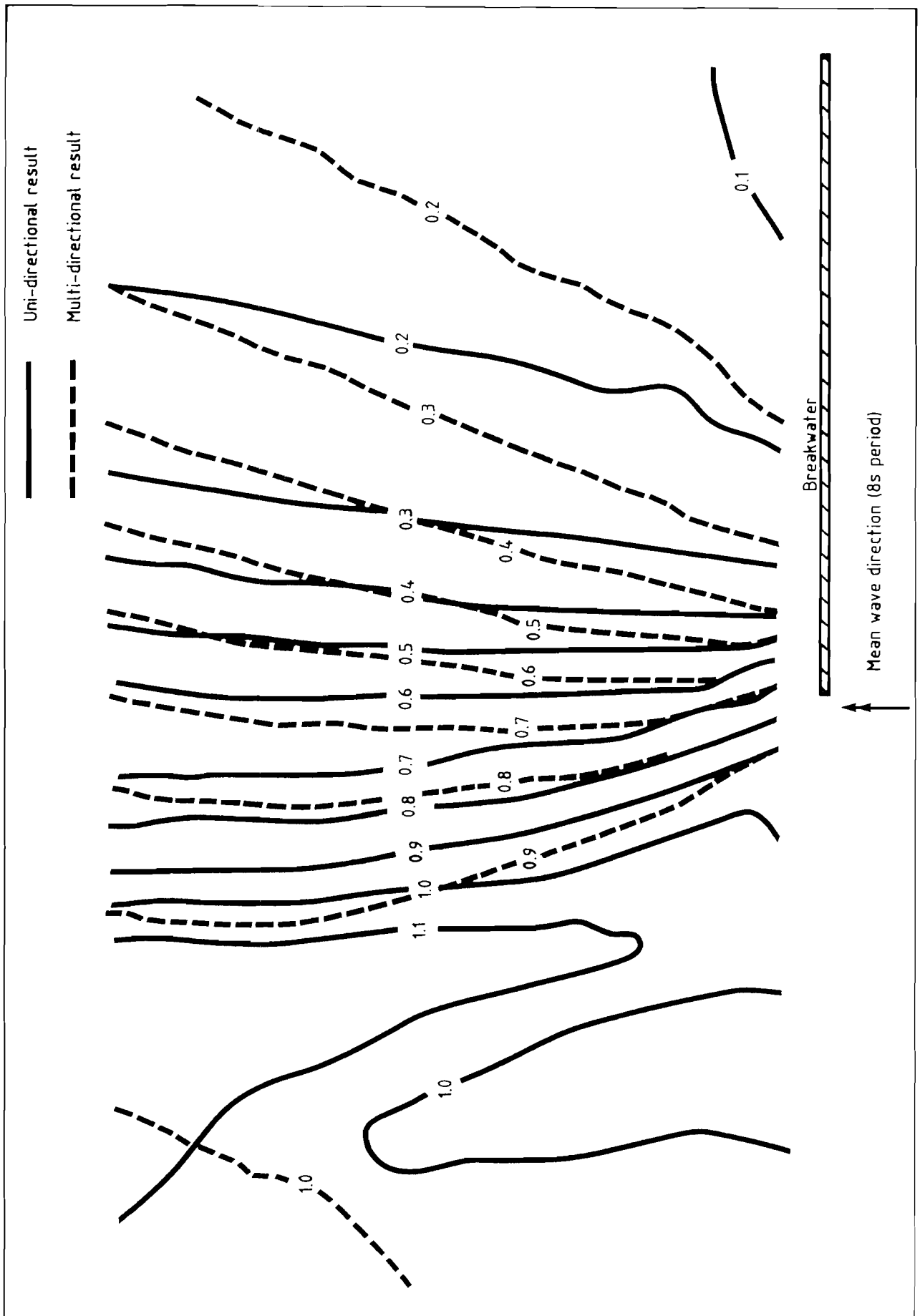


Fig 2 Effect of short crestedness on wave diffraction

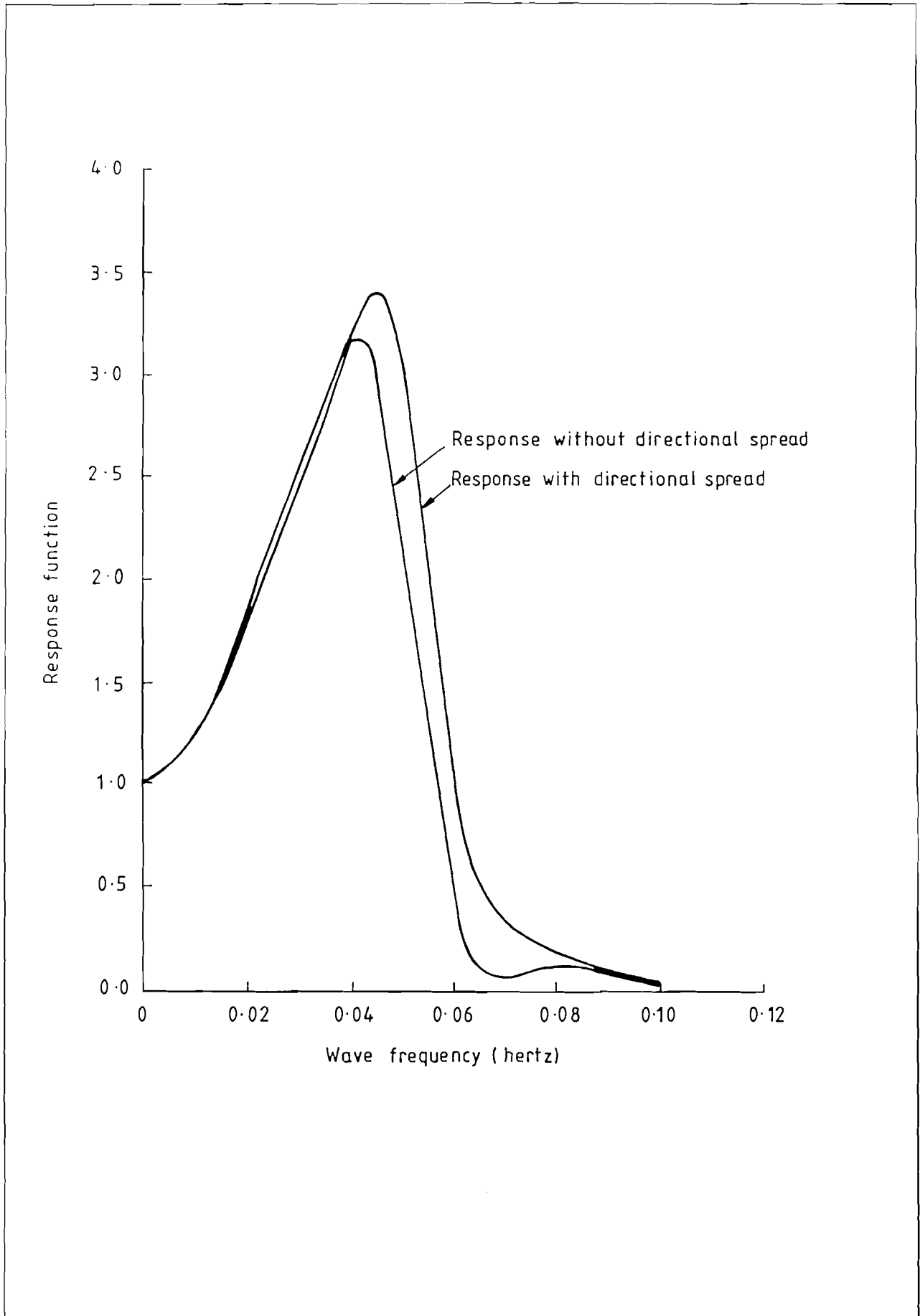
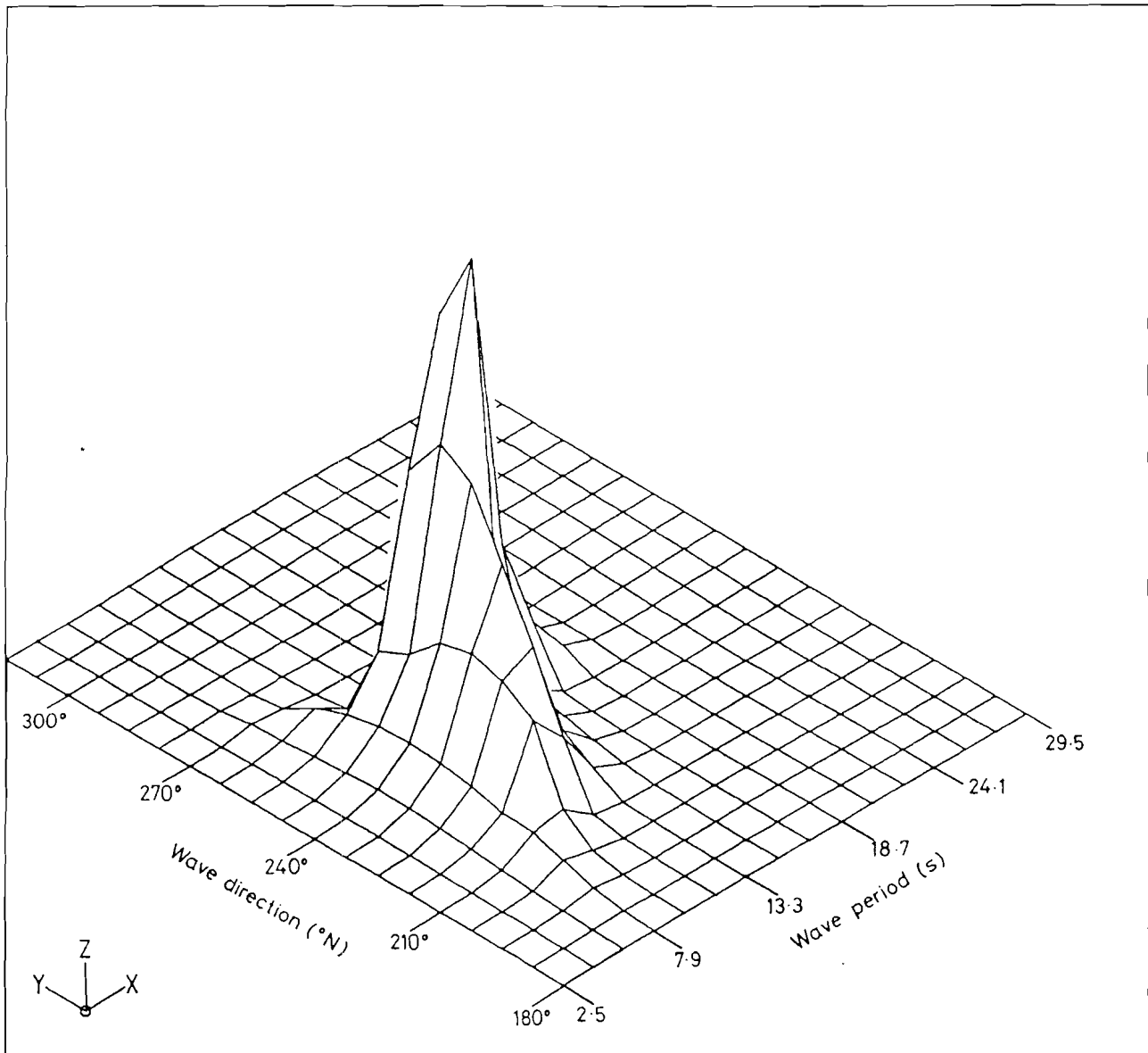


Fig 3 Vertical response of bulk carrier in head seas at 8 knots



DOVER STRAIT - POINT E HS=7 M TPEAK=12.55

DIRECTIONAL WAVE ENERGY SPECTRUM

WIND FROM 239.0 DEG. NORTH

AXES X - PERIODS
 Y - DIRECTION DEG N
 Z - WAVE ENERGY

Fig 4 Directional wave energy spectrum for 100 year return period wave for sector 240° in area E

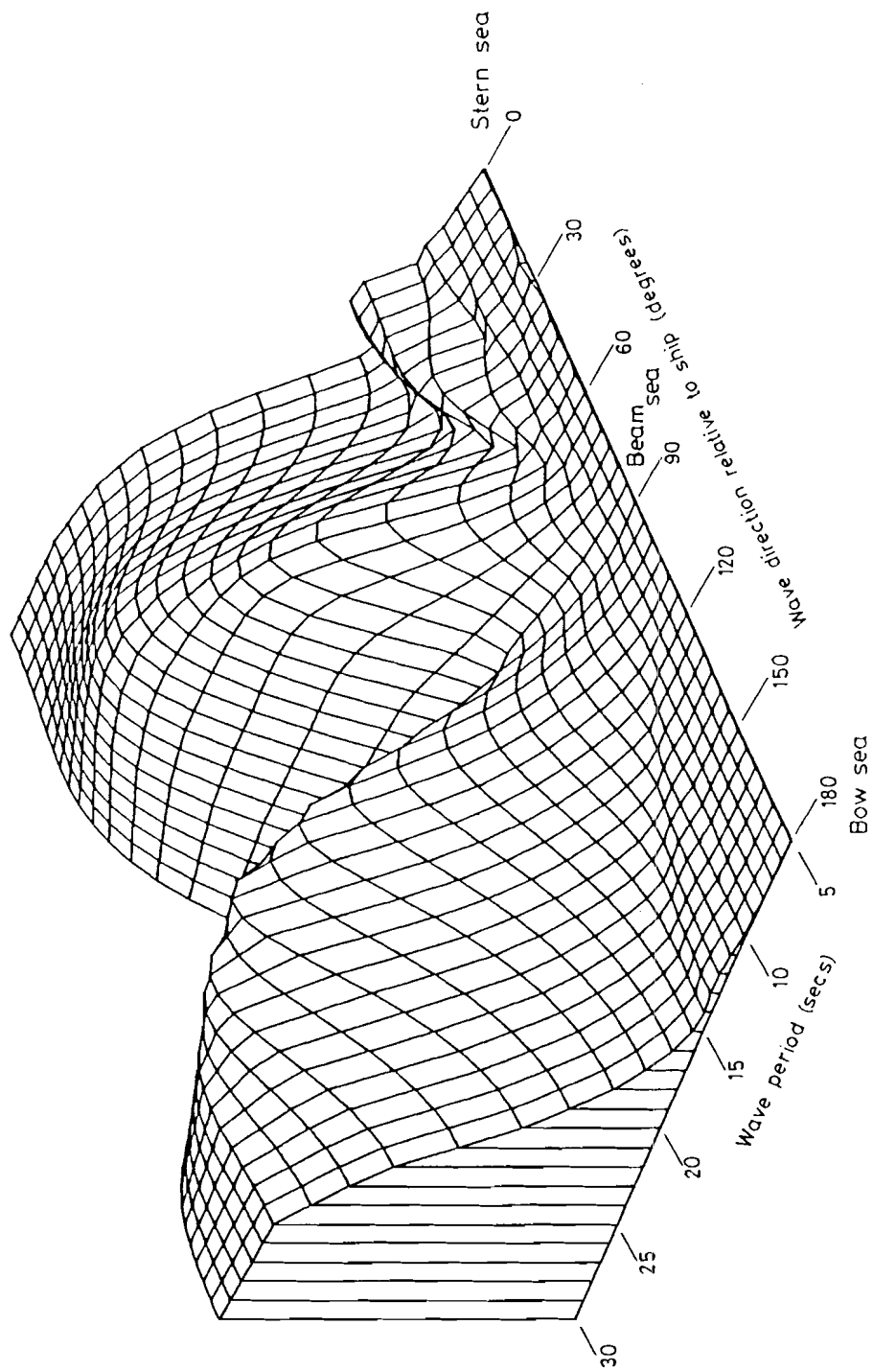


Fig 5 Bow response of supertanker at 12 kts

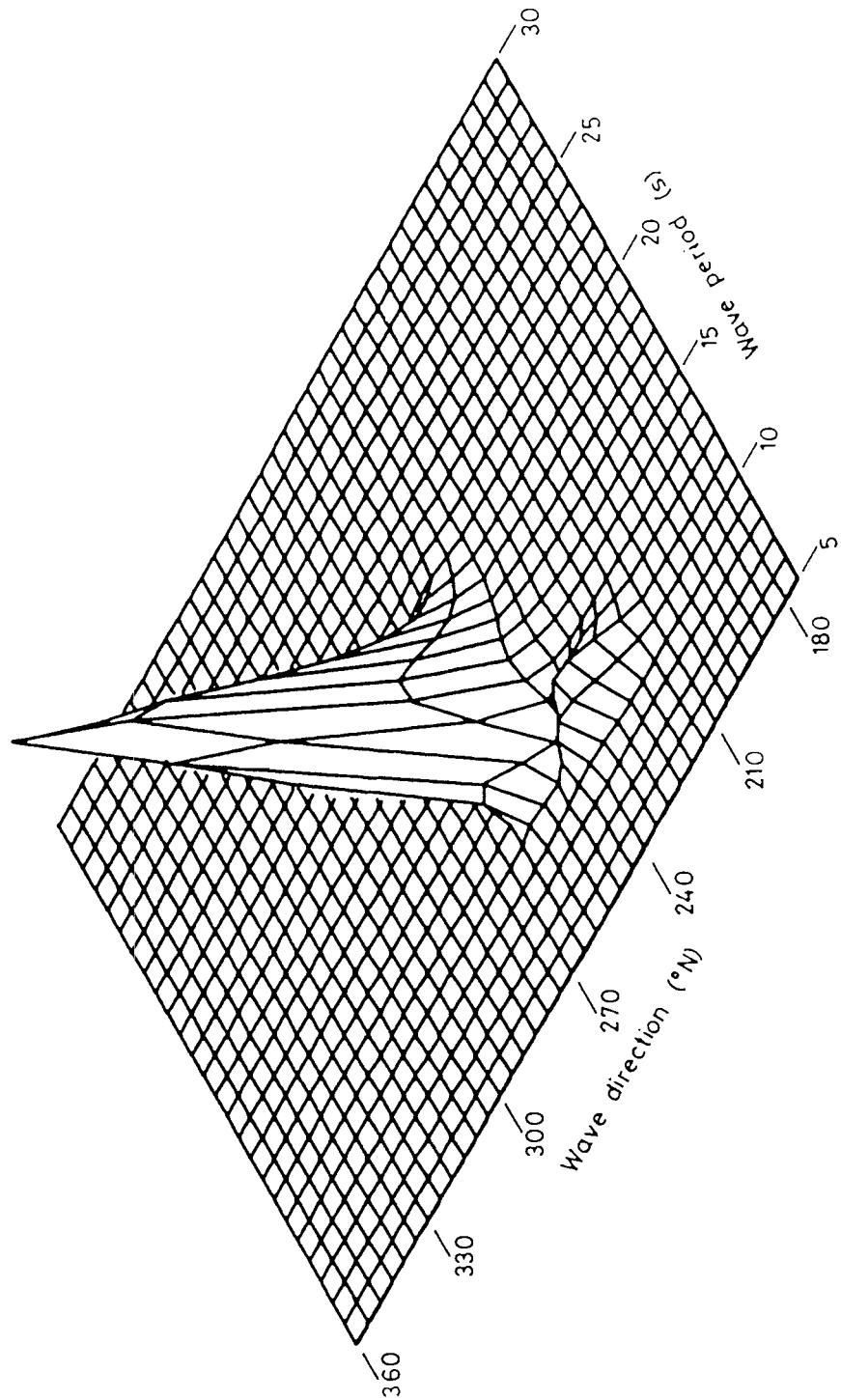


Fig 6 Directional energy spectrum for bow movement in 100 year wave in sector 240° in area E

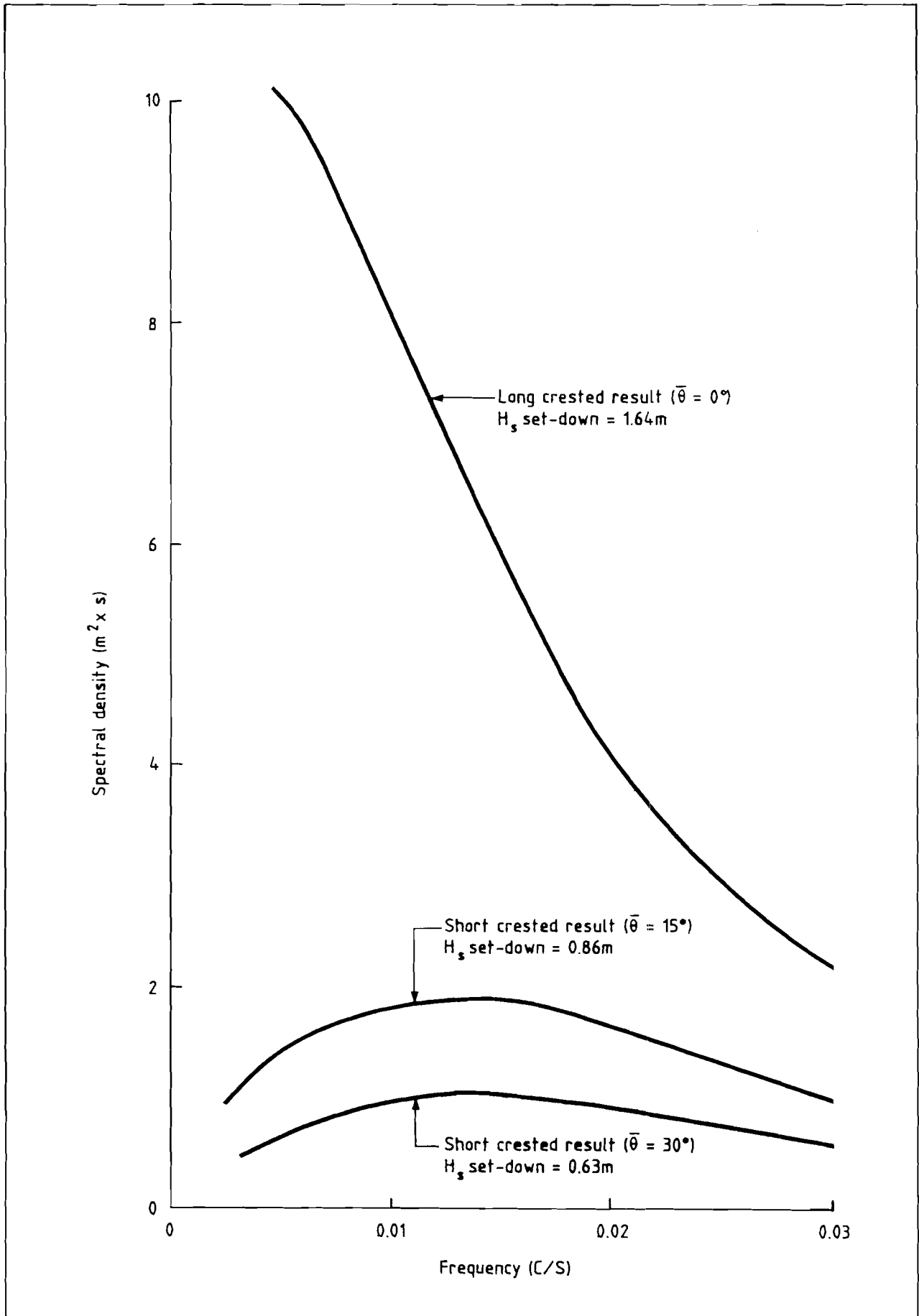


Fig 7 Set-down spectra for scaled Moskowitz wave spectrum ($H_s = 5m, T_p = 20s$) in 20m water depth

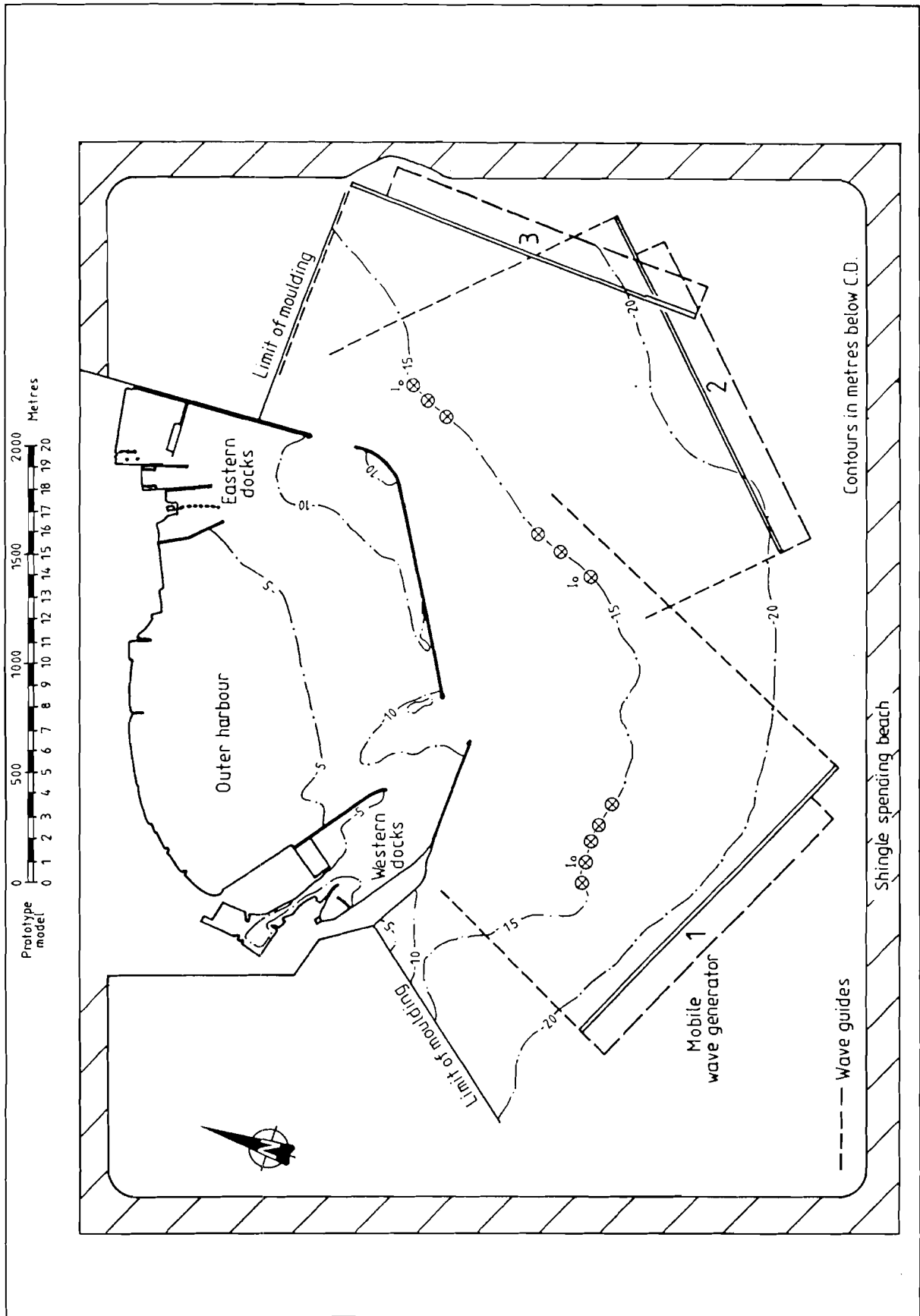


Fig 8 Dover model

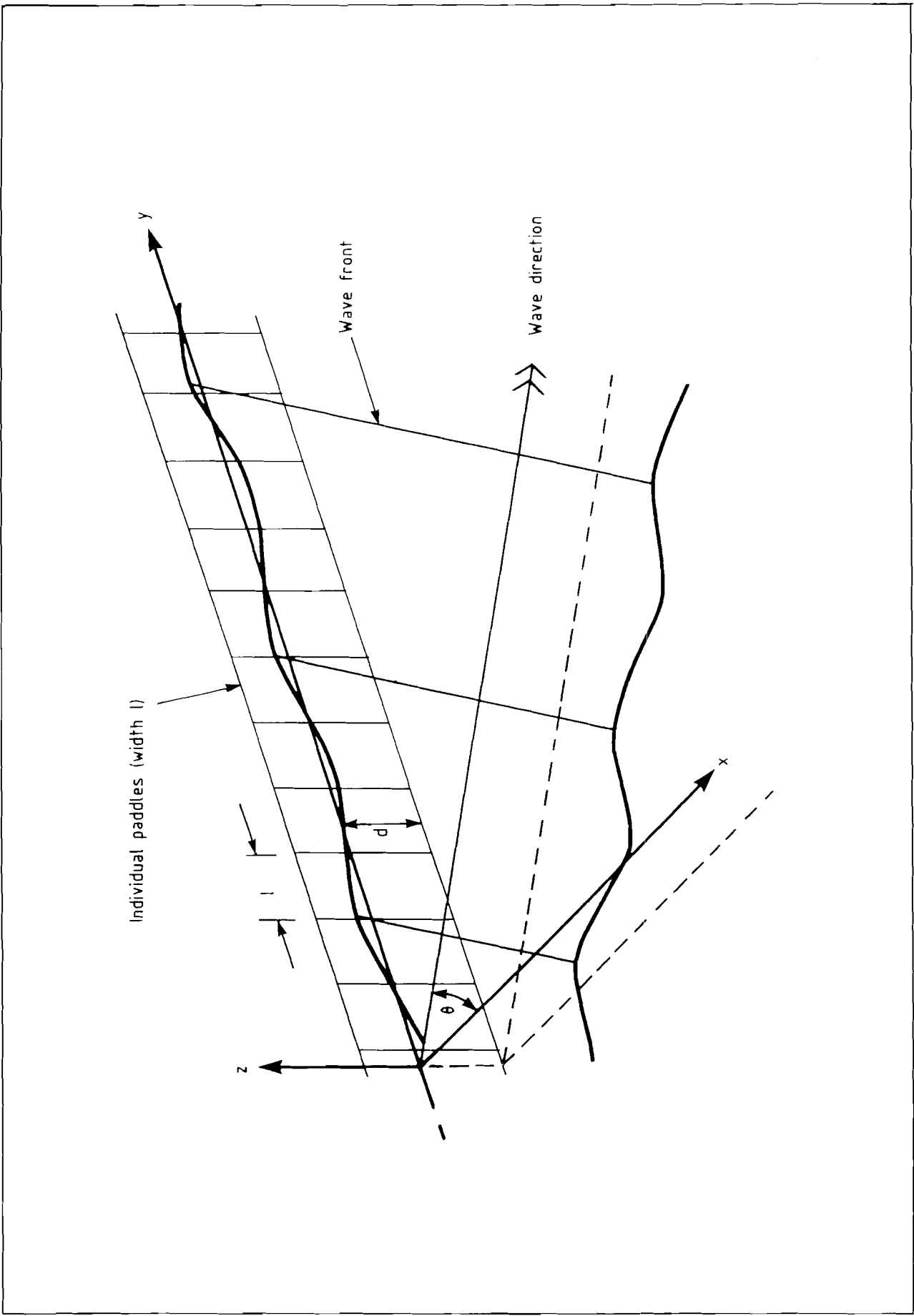


Fig 9 Schematic diagram of short crested wave-maker

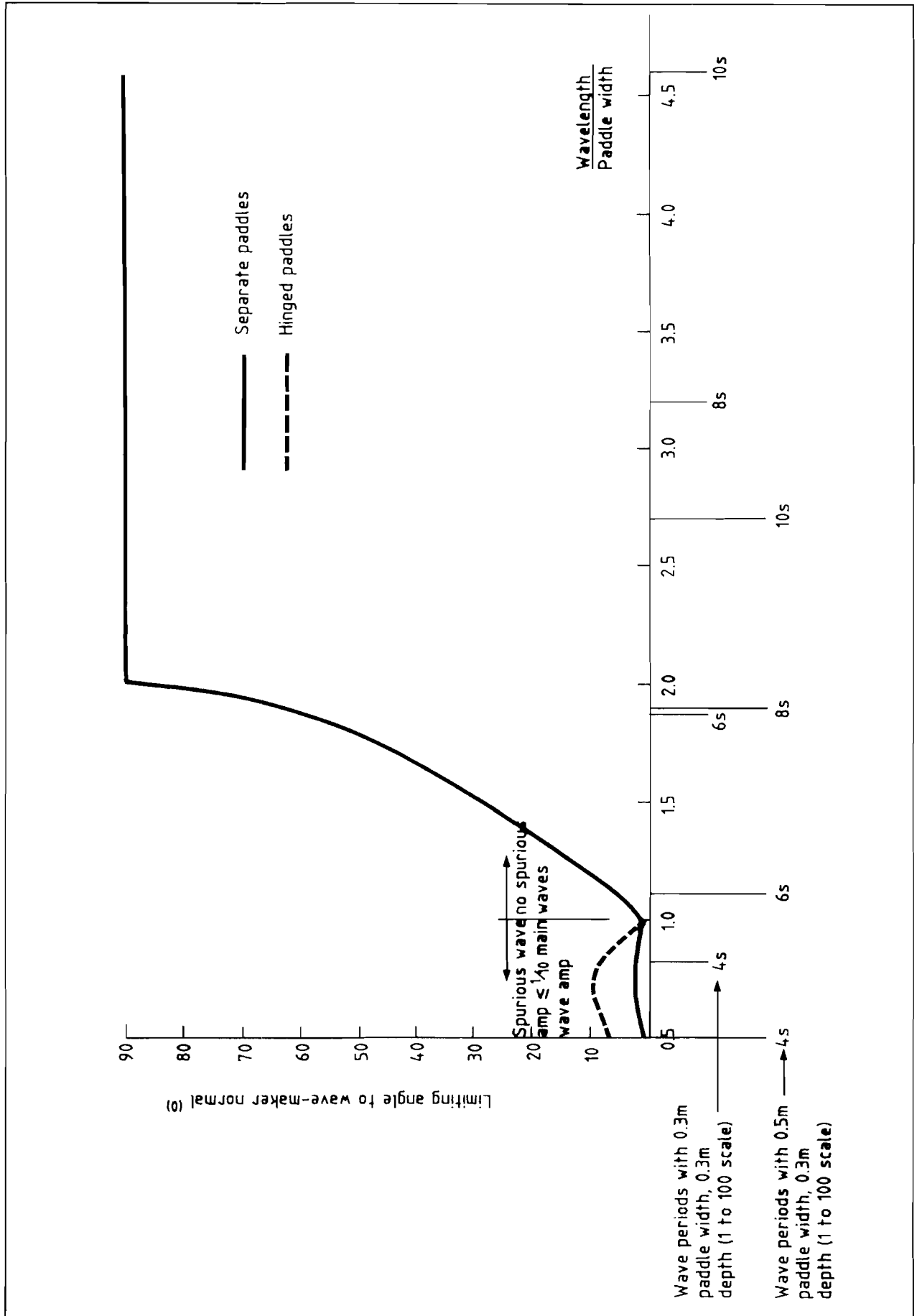


Fig 10 Limiting angles for oblique wave generation

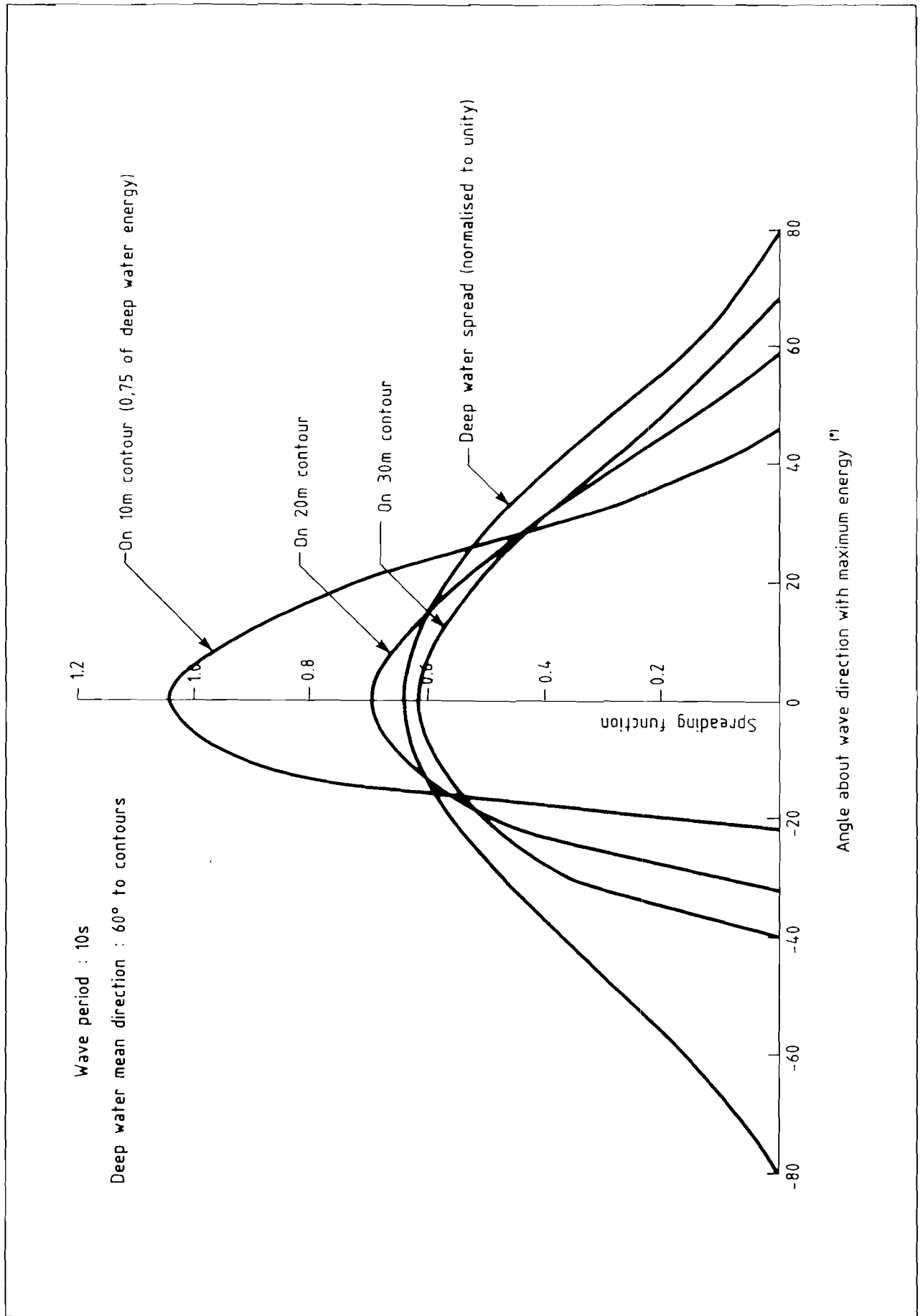


Fig 11 Effect of refraction on directional spread in coastal waters

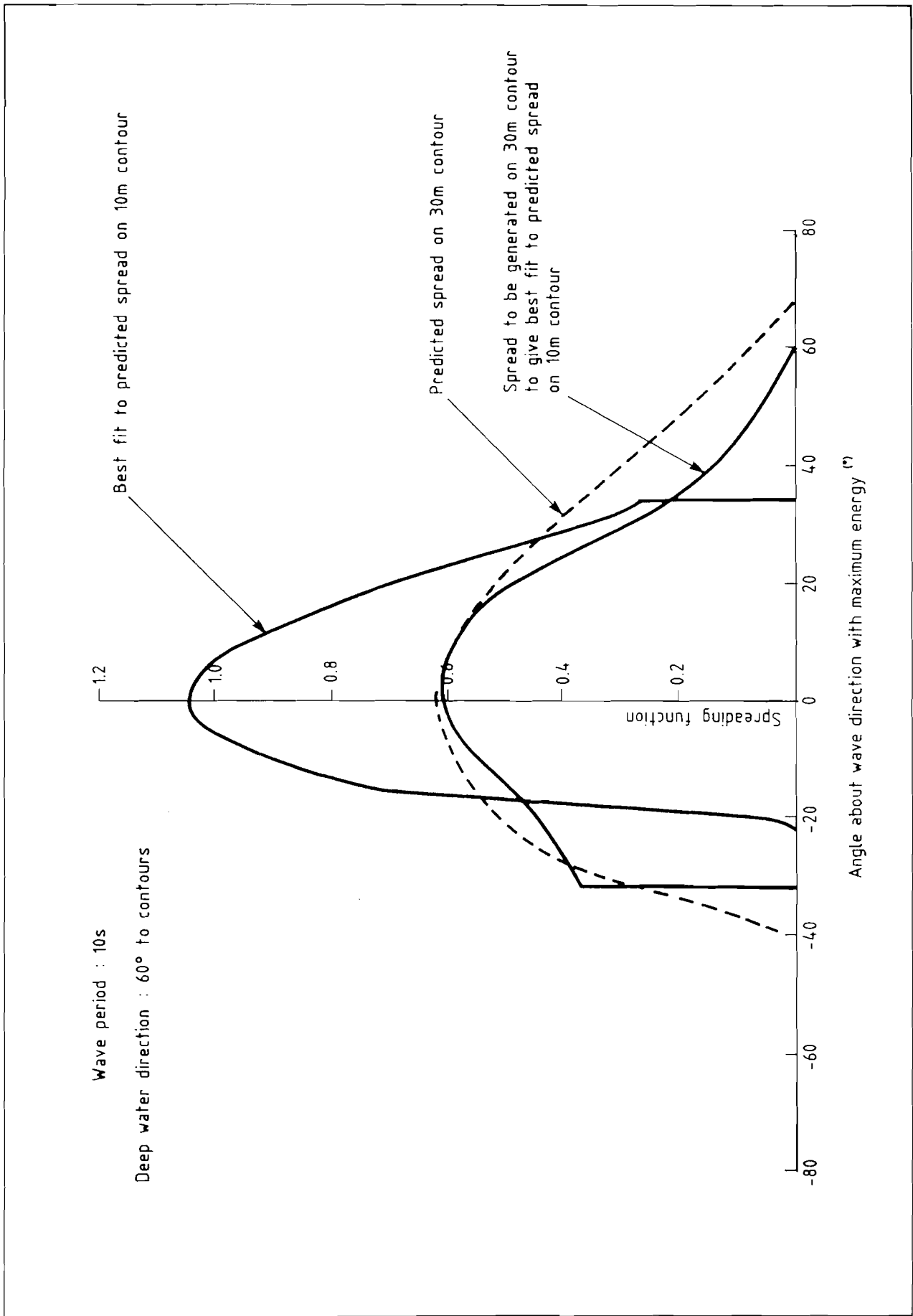


Fig 12 Spread to be generated in deeper water to obtain best fit to predicted spread in shallower water

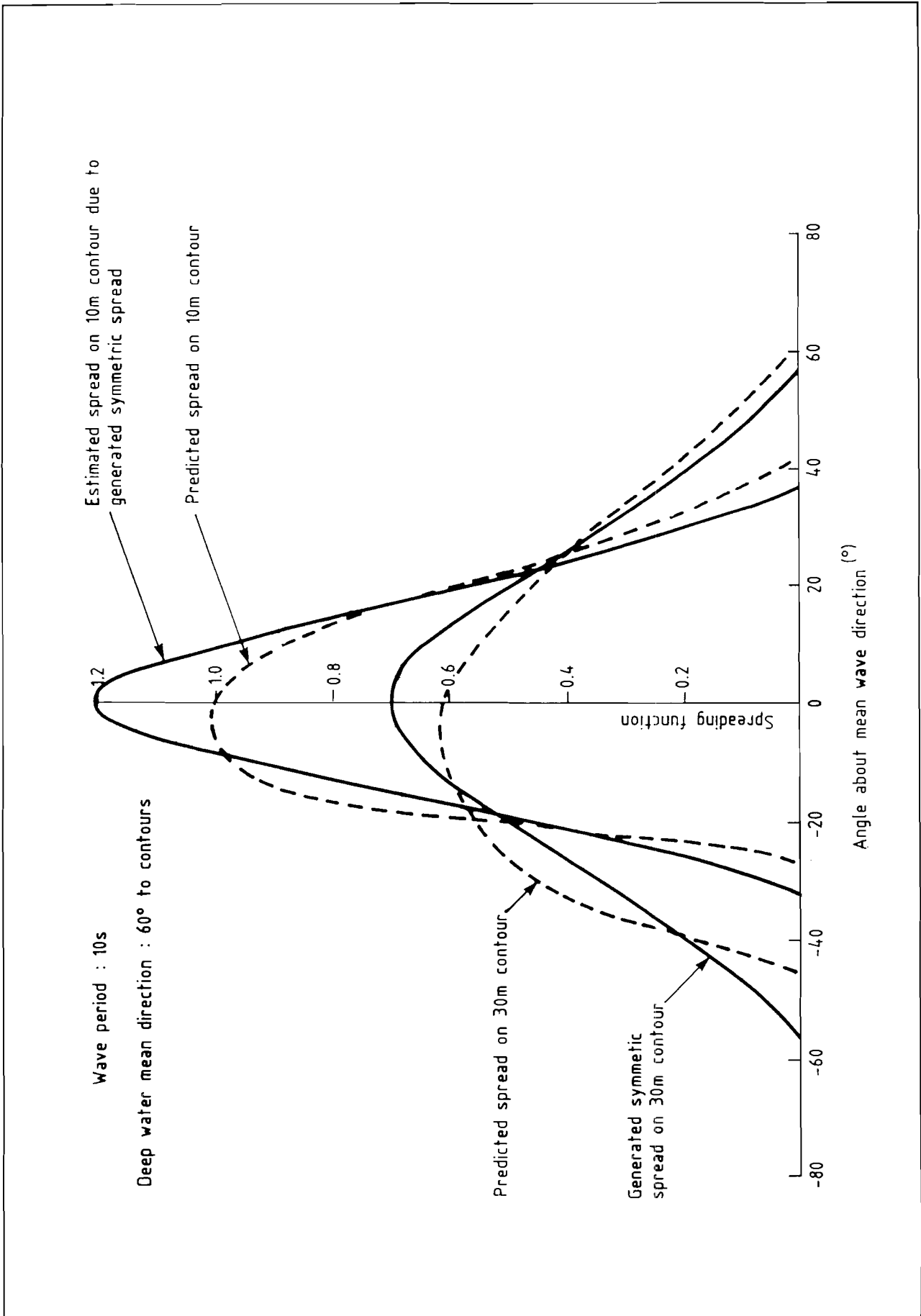


Fig 13 Effect of generating symmetric spread in deeper water with same mean spread as predicted (asymmetric) spread

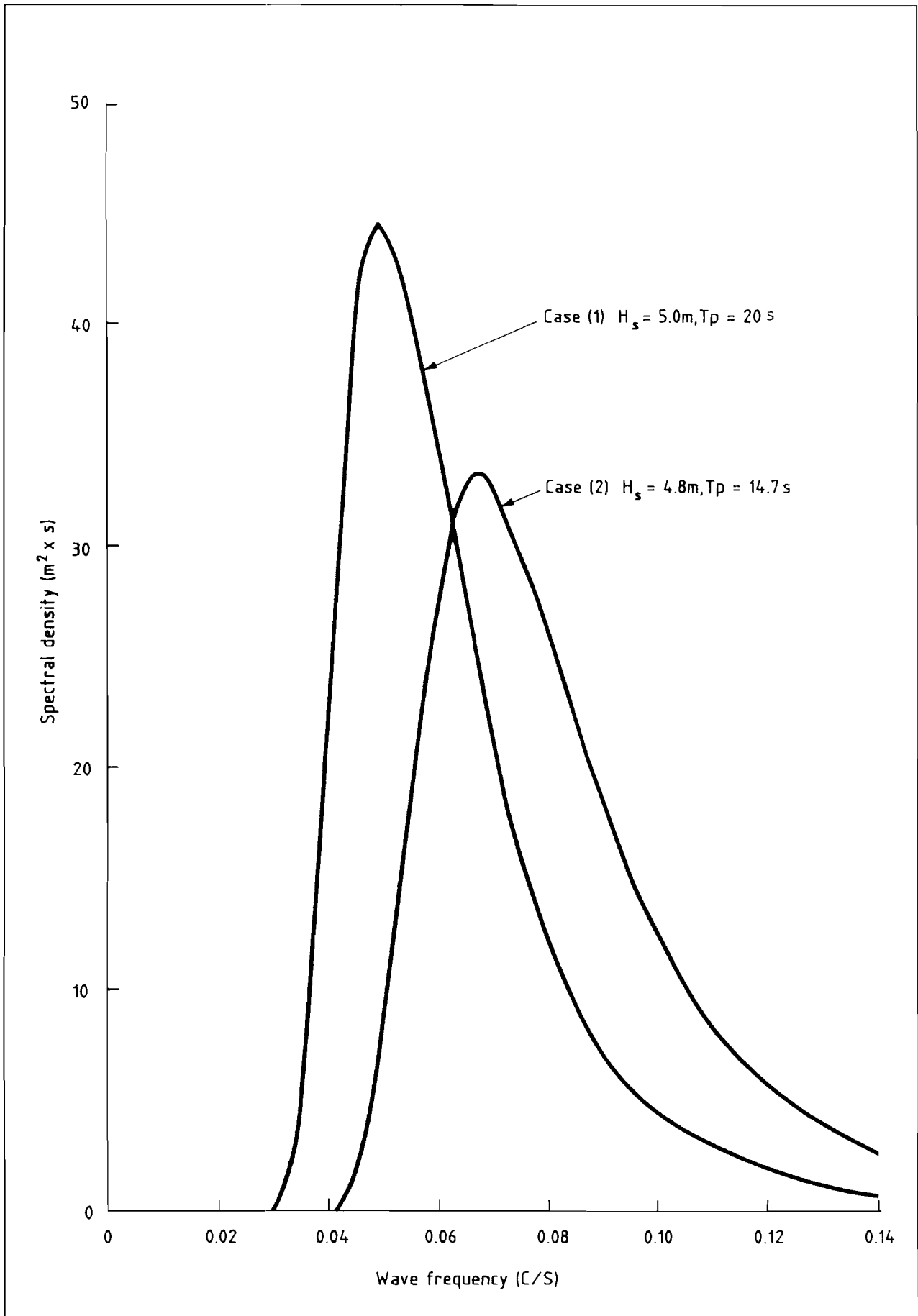


Fig 14 Primary wave spectra

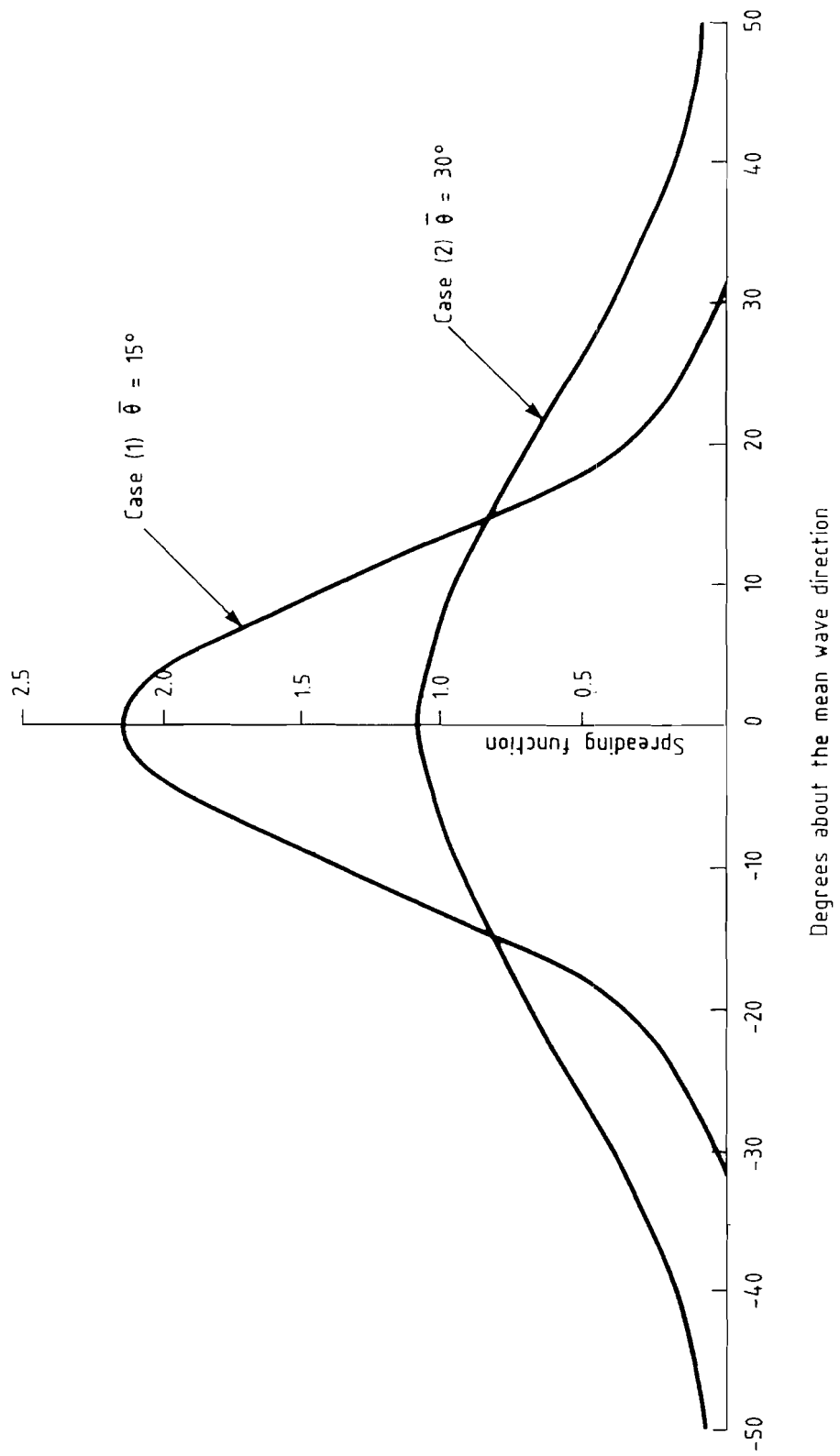


Fig 15 Spreading functions for primary wave spectra

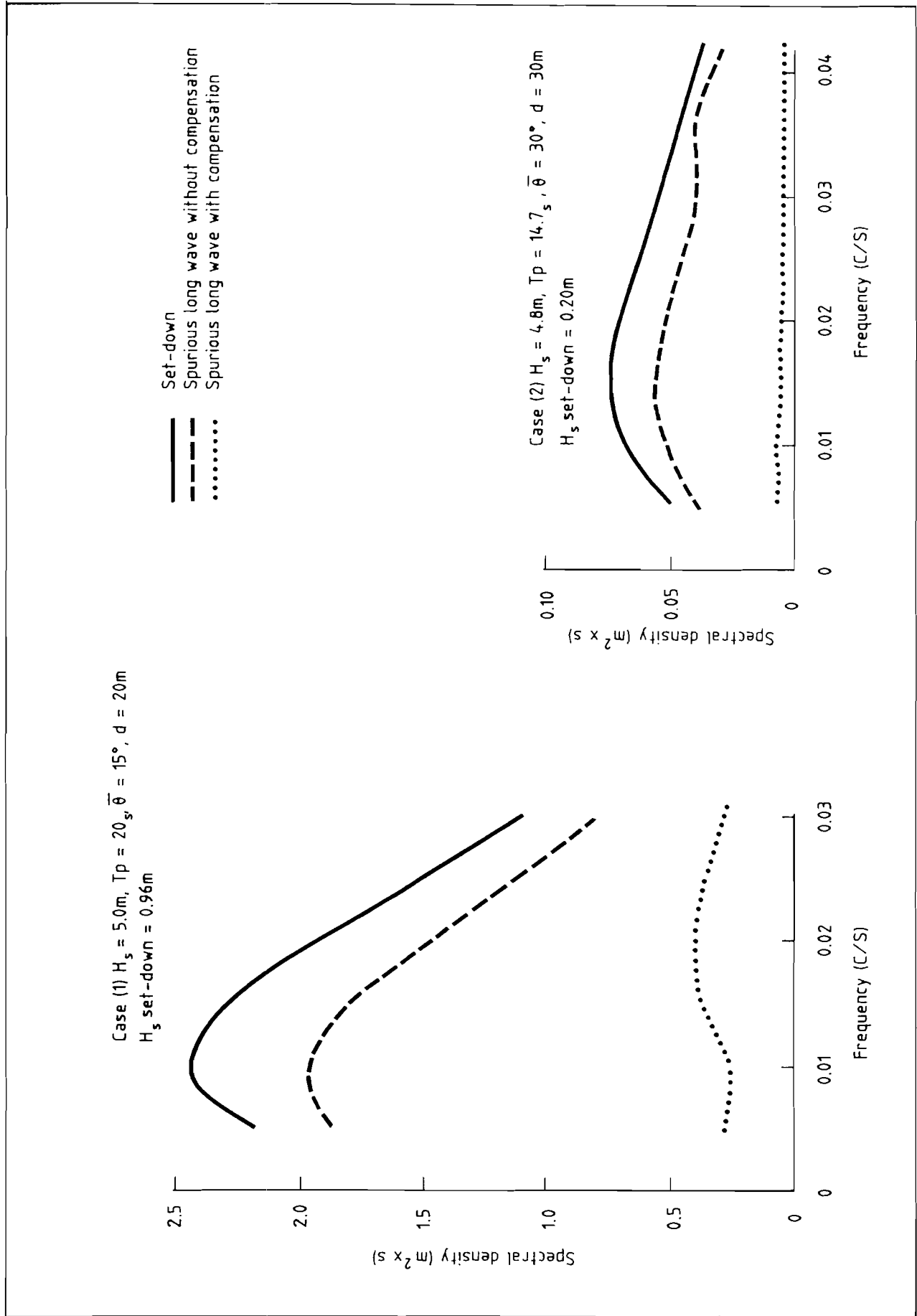


Fig 16 Long period wave frequency spectra

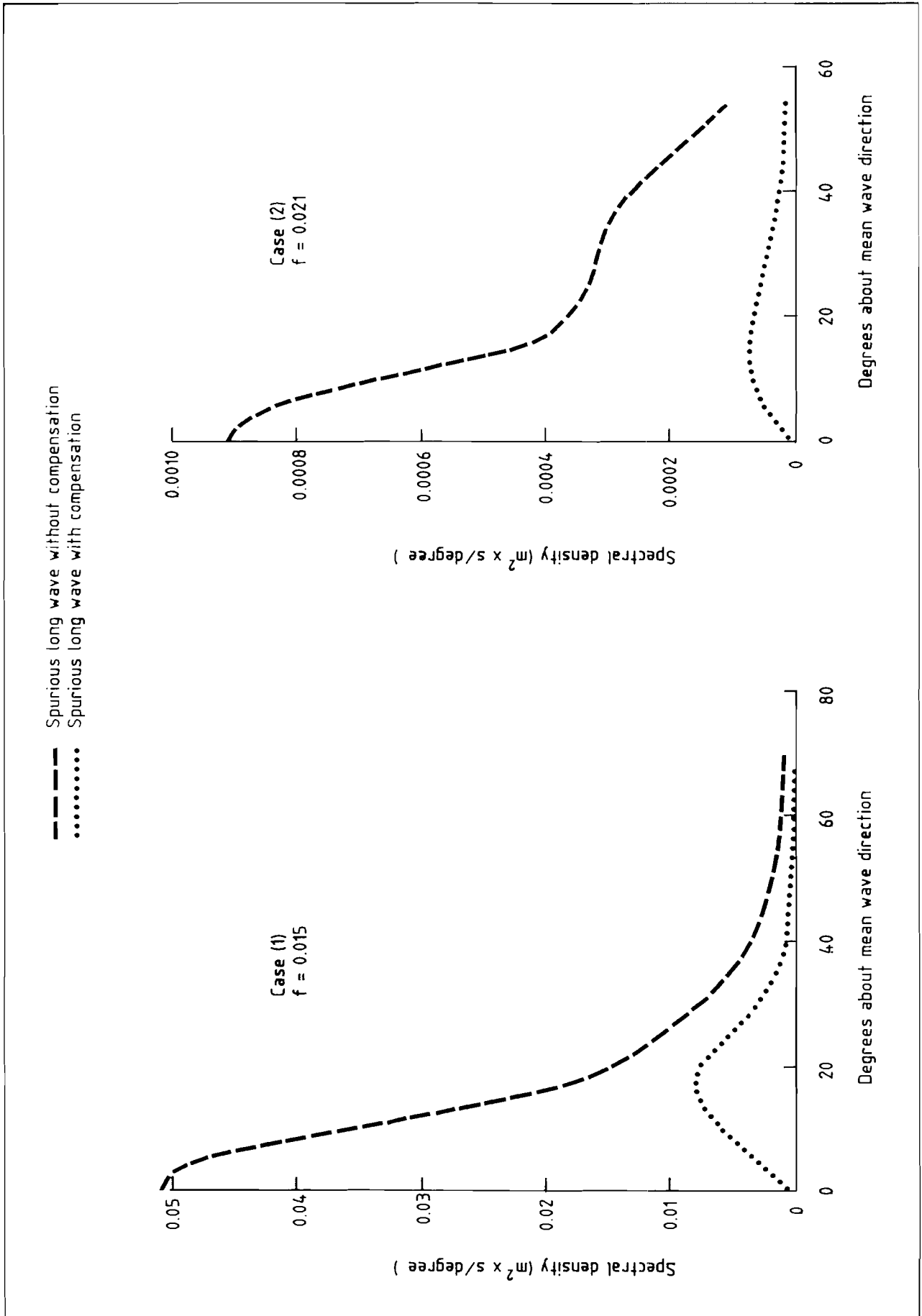


Fig 17 Directional spread of long waves for frequency near long period spectrum peak

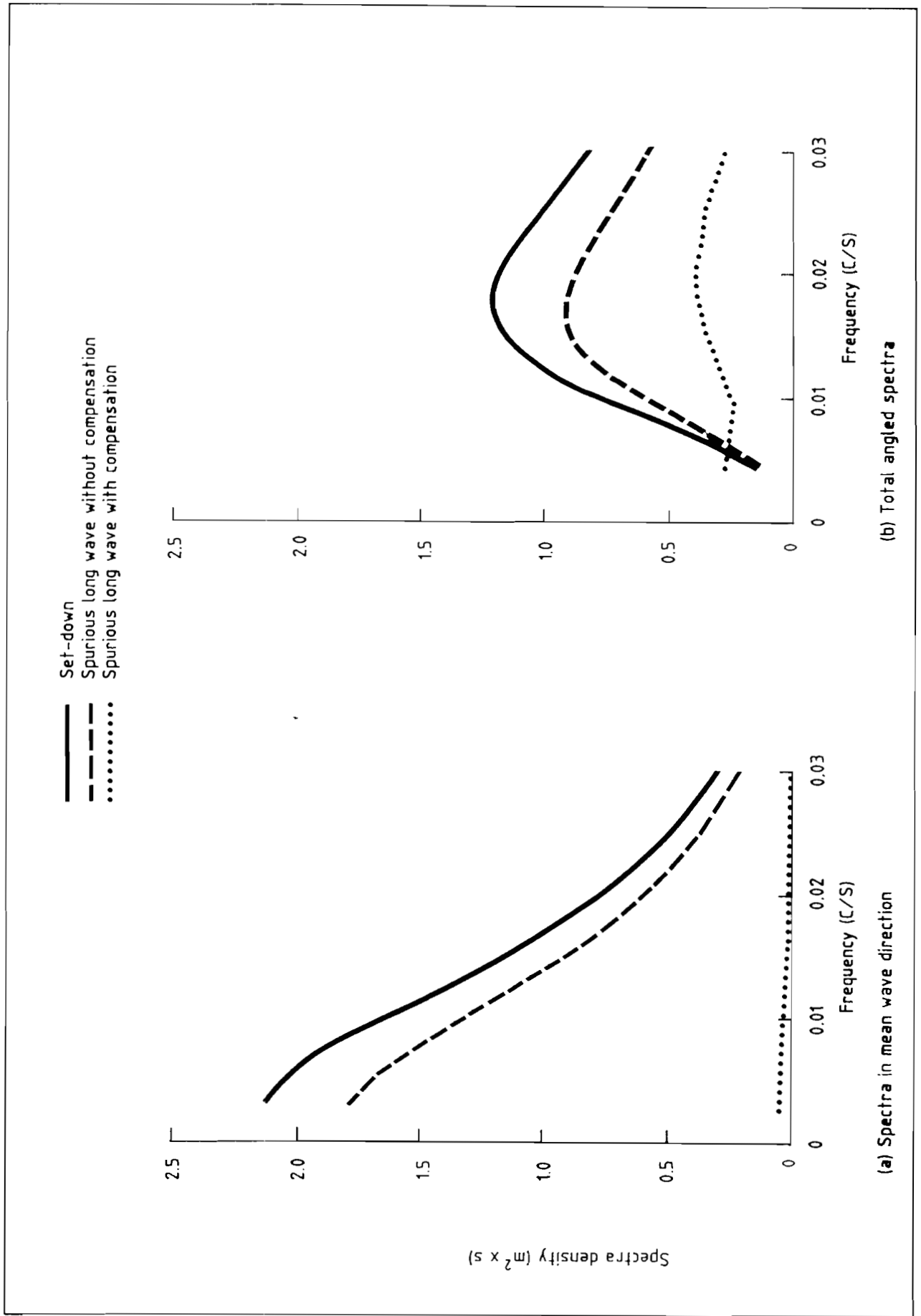


Fig 18 Long period spectra for case (1) in depth equivalent to 20m

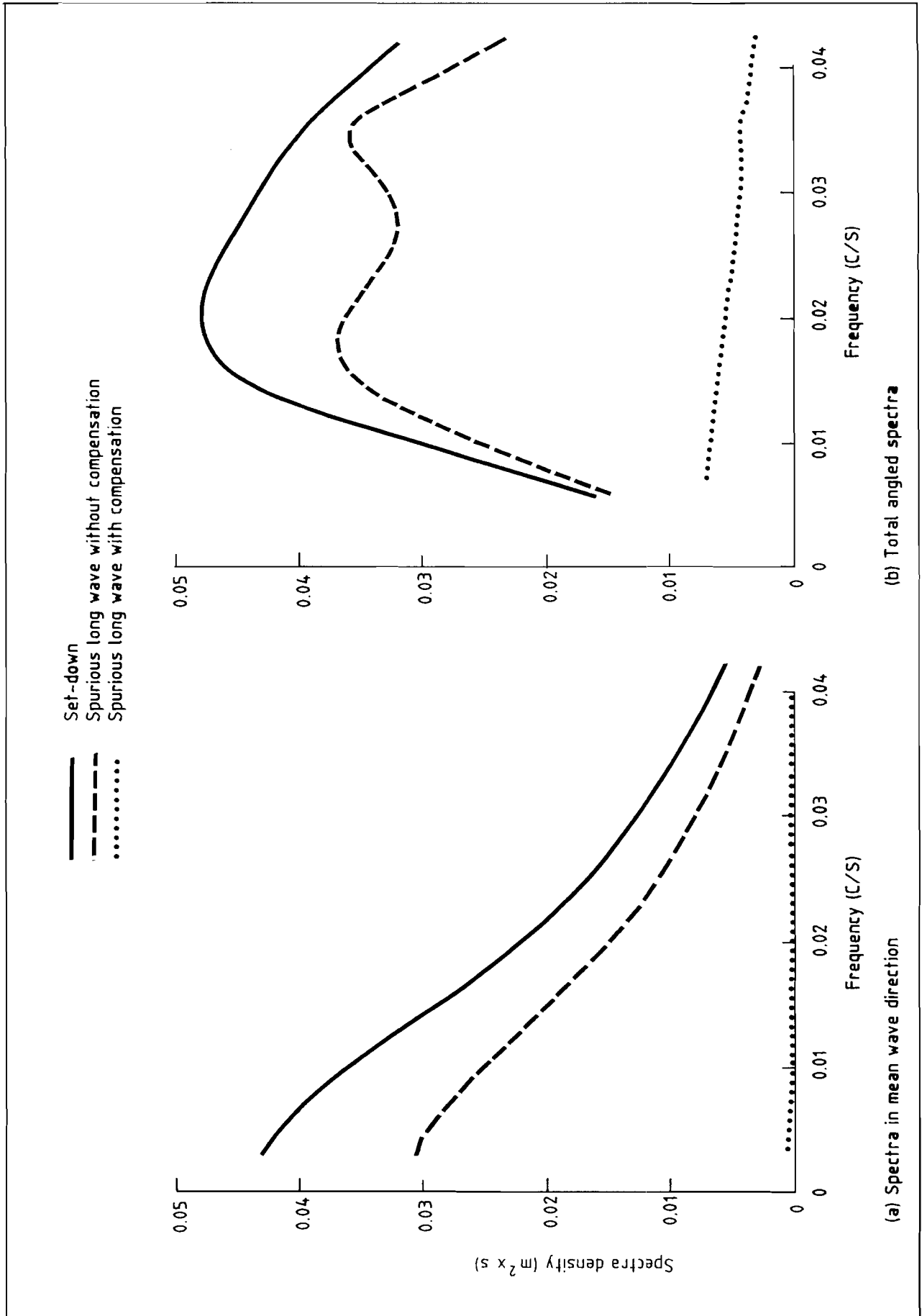


Fig 19 Long period spectra for case (2) in depth equivalent to 30m

