Hydraulics Research
Wallingford

A numerical sand transport model with time-dependent bed exchange

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ABSTRACT

The study of sediment transport generally is very difficult but more so in the case of estuaries because:

- the water movements are continually changing with the rise and fall of the tide
- a wide range of sediment exists on the bed and in suspension
- certain sediments are not found in some parts, leading to unsaturated loads in the water.

In recent years sediment transport models have been developed and used for making engineering assessments of the impact of works on the sediment regime. At present the full potential of the models cannot be realised because of the lack of calibration and verification data, and gaps in our understanding of the fundamental sedimentation processes.

The basic aim of this research was to improve the representation of sand transporting processes in computer models. It is relevant to the sand transport consequences of civil engineering works on the operation of ports and harbours and on environment aspects, and will ultimately benefit the industry by helping to minimise maintenance dredging of ports and navigation channels.

The first phase of the project, covered in report SR 75(1), concentrated on finding the best numerical model representation of the exchange of sand between the bed and the flow, based on assessments of the available data and theoretical analyses. The report also contained descriptions of the fundamental physical processes affecting sand transport in estuaries and a brief review of existing numerical models of sand transport. A sand transport model, based on theoretical and empirical relationships, was tested to verify that it simulates the relevant physical processes. It was found that the most appropriate formulation for entrainment of sand at the bed was in terms of an entrainment rate. The connection between this entrainment rate and the associated sand transport law was considered and it was concluded that strictly only one of these should be specified. The model was compared with some flume data to test its response to a change in the sediment load. It was shown that the model simulation could be calibrated by adjusting the settling velocity and vertical diffusivity parameters.

The second stage of the project, covered in this report, concentrates on the implications on the suspended load due to the unsteadiness in accelerating and decelerating flow and the numerical simulation of these effects. The efficacy of the model is demonstrated by comparison against some sand flux observations from the Thames Estuary. The work involved use of standard sand transport relations and brief descriptions are included of three particular solution procedures.

CONTENTS

1 INTRODUCTION

1.1 Research objectives

The study of sediment transport generally is very difficult but more so in the case of estuaries because

- the water movements are continually changing with the rise of the tide

- a wide range of sediments exist on the bed and in suspension

- certain sediments are not found in some parts leading to unsaturated loads in the water

In recent years sediment transport models have been developed and used for making engineering assessments of the impact of works on the sediment regime. At present the full potential of the models cannot be realised because of the lack of calibration and verification data, and gaps in our understanding of the fundamental sedimentation processes.

The basic aim of this research was to improve the representation of sand transporting processes in computer models which involved some work to gain a better understanding of the underlying physics.

The first phase of the project, covered in report SR75 (1), concentrated on finding the best numerical model representation of the exchange of sand between the bed and the flow, based on assessments of the available data and theoretical analyses. The report also contained descriptions of the fundamental physical processes affecting sand transport in estuaries and a brief review of existing numerical models of sand transport.

The second phase of the project, covered in this report, concentrates on the implications on the suspended load due to the unsteadiness in accelerating and decelerating flow and the numerical simulation of these effects. The efficacy of the model is demonstrated by comparison against some sand flux observations from the Thames Estuary. The work involved use of standard sand transport relations and brief descriptions are included of three particular methods of solution.

2 SAND FLUX OBSERVATIONS AT FOULNESS

During work on proposed navigation developments at Foulness, large amounts of sand flux data were collected in the Thames estuary in 1972 (2). Simultaneous concentration and velocity profiles were obtained using the equipment described in Appendix 1.

Programs have been written to store the Foulness data in a database. Information can be extracted from the database according to date or measuring position; new processing programs can be used to plot profiles for assessing the quality of the data, to summarise the depth-mean quantities (velocity, suspended sediment concentration, flux) through the tide, and to relate the flux to the velocity. These analyses can be carried out for single measurement positions or results from several positions can be combined.

Examples of the graphical output from the analyses programs are shown in Figures 1 to 4. (The processed results are also tabulated and available for further analysis if required). The hysteresis which is characteristic of sand transport in varying currents is apparent in Figures 3 and 4; the sediment flux (and concentration) at a given magnitude of the current is smaller when the current is increasing than when it is decreasing.

This is due to the finite time required for the sediment load to reach equilibrium following a change in current. There are also differences between the flood and ebb stages of the tide. These arise from variation in the availability of material in the flood and ebb directions and from variation of the dependence of current on time between rising and falling tides.

The Foulness sand flux database and the set of processing programs comprise a versatile system for the testing of numerical sand transport models.

3 THE SAND TRANSPORT MODEL

The sand transport model was set up according to the analysis described by Miles (1986) (1). This was a solution of a simplified time dependent equation for the concentration in a water column as it approaches the saturation profile from some initial profile. (The initial concentration is assumed to decrease exponentially with height and represents an equilibrium profile for some other saturation concentration). As the model can deal with departures of the concentration from saturation it is suitable for use with unsteady flows or non-uniform bed composition. This contrasts with other sediment transport models in which saturation is assumed and entrainment or deposit occur instantaneously to maintain saturation in different parts of the flow.

At each cell and timestep the model determines the saturation concentration. If this is less than the predicted concentration then sediment is deposited, the quantity being proportional to the excess of concentration above saturation. If the predicted concentration is less than saturation and there is erodible material present on the bed, sediment is entrained. If the flow is unsaturated but no erodible material is present no exchange with the bed occurs. The amount deposited or entrained is used to calculate the new concentration which is used with advection and diffusion subroutines to determine the concentrations at the next timestep. The model stores the net deposition or erosion accumulated at each cell; this permits realistic treatment of the situation in which material is deposited on a rocky surface at some states of the tide and removed later.

The constant of proportionality in the model connecting super or subsaturation to exchange is related, as described in reference 1, to water depth, settlement velocity, vertical diffusion coefficient (and hence friction velocity) and timestep by the equation

$$S(\tau)/(\bar{c}_o - \bar{c}_s) = \beta_s D_z w_s^{-1}\left(4\tau^2(1 + \tau^2)\mathrm{erfc}(\tau)\right.$$

$$\left. + \mathrm{erf}(\tau) - (2\tau/\sqrt{\pi})(1 + 2\tau^2)\exp(-\tau^2)\right)$$

where

S = vertical flux per unit area integrated over the timestep Δt

$\bar{c}_o$ = initial depth-averaged concentration

$\bar{c}_s$ = saturated depth-averaged concentration

β_s = proportionality factor $c_{z=o}/\bar{c}$
$= (w_s d/D_z)/\left(1 - \exp(-w_s d/D_z)\right)$

D_z = depth-mean vertical diffusion coefficient
$= ku_* d/6$

w_s = settlement velocity

τ = dimensionless time = $w_s \sqrt{(\Delta t/4D_z)}$

$\mathrm{erf}(\tau)$ = error function = $(2/\sqrt{\pi})\int_0^\tau \exp(-x^2)\, dx$

$\mathrm{erfc}(\tau) = 1 - \mathrm{erf}(\tau) = (2/\sqrt{\pi})\int_\tau^\infty \exp(-x^2) dx$

d = total depth

u_* = friction velocity

k = von Karman's constant

The equation is valid when $\tau < 1$. An implicit formulation is used to give the exchange for the middle of the timestep. The condition that $\tau < 1$ for validity of the solution is less demanding than the normal condition for numerical stability of the finite difference scheme, $\Delta t < \Delta x/u$, where u is the depth-averaged velocity.

The expression for the bed exchange requires a number of parameters which may be supplied explicitly as part of the control data for the program or calculated within the program: depths and currents are supplied from output from the 2D flow models; w_s is related to particle size by Stokes' law and, in its current version, the model is supplied with a Darcy Weisbach friction factor (f) to calculate the friction velocity from

$$u_* = u\sqrt{f/8}$$

and hence the vertical diffusion coefficient D_z, and bed friction if required.

The calculation of saturated concentration is of great importance. At present the model allows a choice of three methods; a modification of that of Odd (3), those of Ackers and White (4), and van Rijn (5). These are all semi-empirical relationships between concentrations and the flow parameters, based on different assumptions and with varying applicability.

4 SAND TRANSPORT LAWS

4.1 Power law

The approach used by Odd (3) was based on Bagnold's hypothesis (6) that the rate of work done in keeping sediment in suspension is related to the turbulent kinetic energy of the flow and hence to the bed stress. This work is assumed to be partitioned between the sediment fractions according to the relative area, i, of the bed occupied by the fraction. An equation is derived which implies

$$\bar{c}_s = \frac{Ei\tau_b us}{g(s-1)w_s d} = \frac{Ei\rho sf}{8g(s-1)w_s}\frac{u^3}{d}$$

where s is the specific gravity of the sand particles, τ_b the bed stress, and E is a parameter which is weakly dependent on flow conditions and which is tabulated in reference 3. As the bed stress depends on the square of the current, it follows that the concentration essentially depends on the cube and the sediment flux on the fourth power of the current. The settling velocity can be obtained from Stokes' law and the bed stress from (eg) $\tau_b = \rho f u^2/8$ to obtain a closed set of equations.

The conditions given for applicability and relevant to use in the present model are: no cohesive material; Stokes' law applies (so particle size is between 60μ and 300μ); sorting coefficient of material $\sqrt{(D_{25}/D_{15})}$

$\geqslant$ 0.80; flow is well mixed and deep compared to grain size.

The parameter E incorporates an efficiency of the turbulence in keeping sand in suspension and a threshold velocity for initiation of suspension. The model approximates the threshold and variation of efficiency by using the equation

$$\bar{c}_s = \text{const}\ (1 - \frac{u_t^2}{u^2})\ \frac{u^3}{d}$$

The constant of proportionality and threshold velocity can be evaluated by plotting $E i \rho s f / [g(s-1)\ w_s]$, as calculated according to reference 3, against $1/u^2$ and taking the intercept and gradient.

Although in ref 3 the friction factor f is given as a variable, dependent on current as well as grain size, it was found in this case that better results were obtained using a constant f appropriate to a mid-range value of current. This is because the roughness elements are sand waves which do not vary with current over the tidal cycle in the same way as they would with a slowly varying current.

4.2 Ackers-White method

The Ackers-White method (4) divides the transport into (i) a bed process (coarse sediment) in which the effective stress on the grains is related to the mean flow velocity, depth and a roughness length (= grain size); and (ii) a suspended process (fine sediment) in which the suspended concentrations are related to the total turbulence and hence the total energy degradation as determined from the hydraulic gradient. This distinguishes between the total stress acting on the bed (which includes a contribution from pressure on bed forms) and shear stress acting on bed grains. A sediment transport parameter is derived which incorporates an efficiency of transport, either in terms of the ratio of (work done transporting coarse grains against bed friction):(total grain shear) or, in the case of fine sediment, (work keeping the sediment in suspension):(total turbulence work).

A set of relationships, involving parameters whose form depends on whether the sediment is coarse ($\gtrsim$ 2.5mm) or "transistional", are given to calculate sediment mixing ratio in terms of depth-mean current, depth, friction, velocity, particle size, particle specific gravity and viscosity. These relationships are shown in Appendix 2.

In the case of mixtures of sediments the authors say that D_{35} can be used as a representative particle size. The method is not appropriate for individual sand fractions in a mixture, nor for sediments finer than about 0.15mm (150μ).

4.3 van Rijn method

The van Rijn method (5) also uses separate grain shear and total shear stresses. This is used to distinguish between initiation of suspension and the maintenance of an equilibrium profile. The solution allows for the existence of locally turbulent bursts of suspension, for reduction of settlement velocity at high concentration, damping of turbulence by sediment and differences between sediment and fluid diffusion. Approximations to numerical solutions for the equilibrium profile are given together with an expression for concentration at a reference height near to the bed. This allows calculations of the average concentration and sediment transport as shown in Appendix 2.

The method is valid for sediment grain sizes in the range 0.1 mm - 0.5 mm (100μ-500μ) and allows for mixtures of sediments by the use of a representative grain size calculated from D_{16}, D_{50} and D_{85}. It is not suitable for use with individual fractions in a mixture. It is computationally more complex and therefore more costly than the Ackers-White or power law methods described in Sections 4.1 and 4.2.

4.4 Availability of methods

The Ackers-White and van Rijn methods are both available in subroutines with standardised input and output parameters. The subroutines can be called by the sandflux modelling program or by a simple program which allows the user to tabulate fluxes for a range of currents at specified particle sizes and depths. Sample output from the routines, for a range of water depths and particle sizes are shown in Figures 5 and 6. Tne predictions from both methods are comparable for grain sizes down to about 0.2mm (200μ). At diameter 0.15mm (150μ) the Ackers-White method predicts considerably greater fluxes and its runaway behaviour for fine sediments is apparent at diameter 0.1mm (100μ).

Other methods for calculating the saturation concentration are summarised in reference 7. It is intended to make some of these available to the sand transport program as part of the future development work.

5 SIMULATION OF CONDITIONS AT FOULNESS

5.1 Conditions simulated

The numerical model was run with a simple flume geometry for comparison with the Foulness data. The observed currents and depths were applied uniformly across and along the channel. Periodic boundary conditions were used at the ends of the channel (ie material carried off one end was reintroduced at the upstream end). This allowed the basic physics of the model to be checked against real field data at a low cost per run of the model. The effects of two dimensional flow and of variations in bed characteristics, present in the data, could not be modelled in this way. The costs of running a fully two dimensional model would, however, have been excessive at this developmental stage.

There was a wide spectrum of particle sizes in the data (Fig 7) and the model was used to simulate individual fractions.

5.2 Results

Figures 8 and 9 show a comparison of the model and observed results. On the corresponding model run the transport of the 75-100μ sand fraction was simulated. The saturation concentration was calculated using the power law (section 4.1). The results are presented as plots of observed concentrations, in parts per million by weight, against u^3/d where u is the depth-averaged current and d the total depth. (On this site and at this location the average depth was approximately 16 m and the maximum current was 1 m/s). The results are plotted at half-hourly intervals through the tide with a sequence number for each point showing the progression through the tide. Ebb and flood stages are shown separately. The initial concentration in the model was obtained from the observations but subsequently values were determined entirely by the model because of the use of periodic boundary conditions. In any case for practical purposes, apart from the first three points, the model was self-initiating because the suspended concentration dropped to effectively zero at slack water.

It can be seen that the model results exhibit a similar hysteresis effect to the observations, both on the flood and ebb tides. The width of the loop is in good agreement with the observations on the ebb tide. The observations on the flood tide show an anomalously low concentration at sequence number 22 which gives rise to a narrower loop than in the model results; this is probably due to some variation in the upstream conditions not represented in the simple model geometry.

The model systematically underestimated concentration on the ebb tide, while on the flood there was no systematic difference from the observations. The variation of observed concentration is smoother on the ebb than the flood; this suggests that the conditions upstream in the ebb direction are more uniform and, therefore, better approximated by the simple model. It follows that the underestimate in the model is genuine and that a larger saturation concentration should be used in the model calculations.

6 CONCLUSIONS AND RECOMMENDATIONS

It is clear that, as is the case with potential load models, the correct calculation of saturation concentration is of great importance. Accurate reproduction of the field data from a real estuary would also require representation of the two dimensional flow patterns and variable bed characteristics. Nevertheless, the demonstration of realistic hysteresis behaviour, even in the simplified model geometry used here, is very encouraging as it confirms the applicability of the innovatory part of this sand transport model (the non instantaneous bed exchange calculation) in a natural situation. Furthermore, since the non equilibrium concentration in a time varying flow is mathematically similar to that in flow over a non uniform bed, the use of the bed exchange functions in areas of variable sediment supply is also supported by these results.

In order to improve the usefulness of this new model in engineering applications a number of developments, some of which are the subject of continuing research, are required:

1) A realistic treatment of mixtures of sands of different sizes for simulation of estuarine sediments. This should incorporate different saturation concentrations for the different fractions; different bed exchange rates resulting from variable settling velocities; variable bed composition arising from differential erosion and deposition; the possibility of "armouring" of finer by coarser particles. (To be considered within the scope of Research Contract PECD 7/6/112)

2) Efficient and accurate methods of calculating saturation concentrations. This has to be done at every timestep for all active cells. The power law of section 4.1 is not very demanding of time but other methods such as the Ackers-White or van Rijn formulae use subroutines of several lines

to provide a value. Some form of look-up table would be desirable; it would have to give the required accuracy over a large range of values. (To be considered within the scope of Research Contract PECD 7/6/112)

3) A method of initialising the bed composition in a situation of variable supply of erodible material.

4) These developments should be applied to previously studied estuaries so that comparisons can be made with field data and with earlier models in terms of cost and accuracy.

7 REFERENCES

1 Hydraulics Research Ltd. Report SR 75 (1986)
Numerical model simulation of entrainment of sand bed material

2 Hydraulics Research Station. Report EX 619 (1972)
Foulness sea port. Navigation channel siltation.

3 Hydraulics Research Station. Report DE9 (1973).
The Wash water storage scheme. Numerical model studies of the Great Ouse estuary: A transport function for fine sand in the estuary.

4 Ackers P, White WR (1973). Sediment transport: New approach and analysis. Proc. ASCE, 99, HY 11 pp 2041-2060.

5 van Rijn L C (1984). Sediment transport, part II: suspended load transport. Journal of Hydraulic Engineering, 110, 11, pp 1613-1641.

6 Bagnold R A 1966. An approach to the sediment transport problem from general physics. US Geol survey, Prc paper 4221

7 Hydraulics Research Ltd. Technical note OD/TN 27 (1987).
A compilation of methods for predicting friction and sediment transport in alluvial channel.

FIGURES.

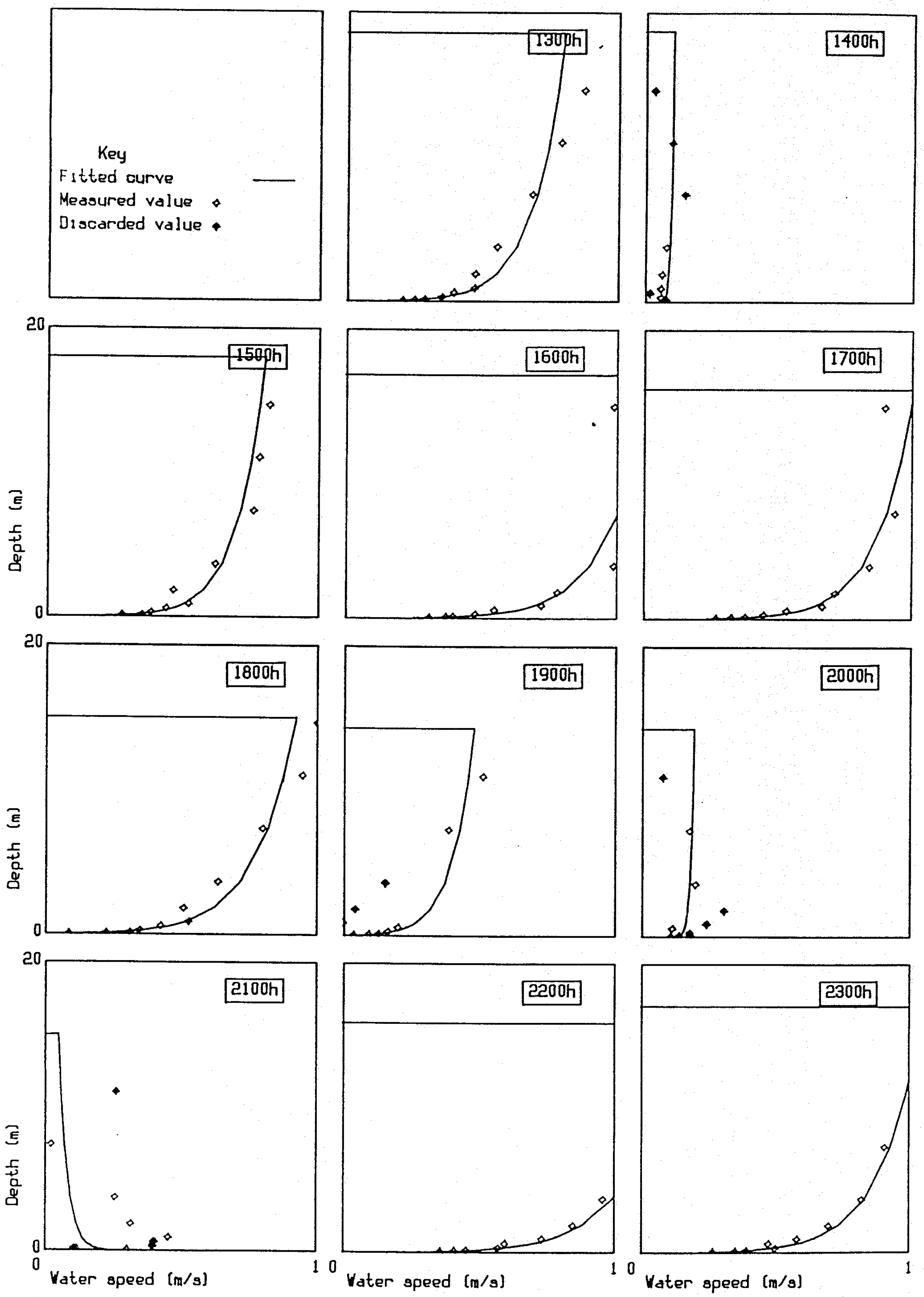


Fig 1a Sand flux processing program output: current profiles

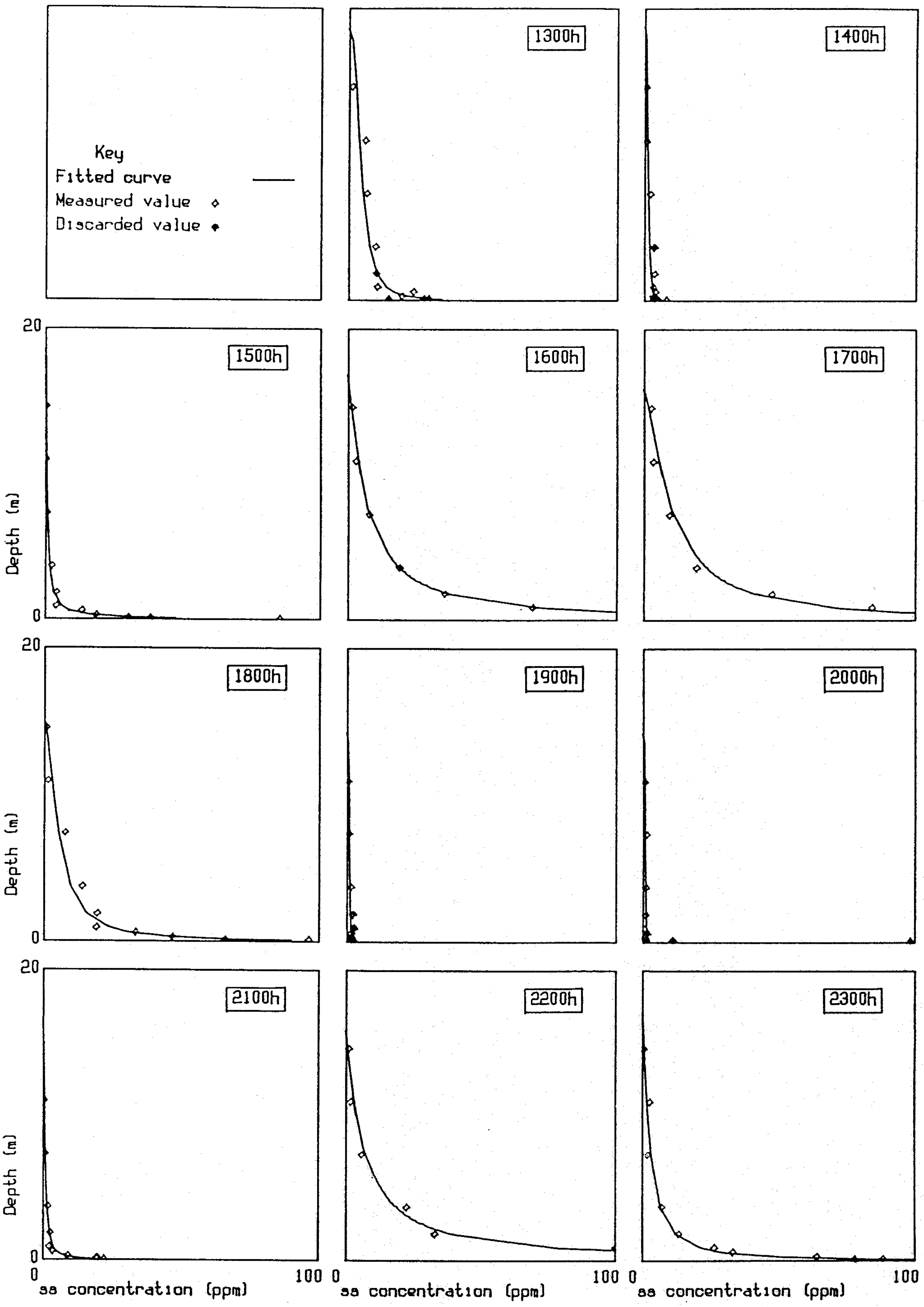


Fig 1b Sand flux processing program output: concentration profiles

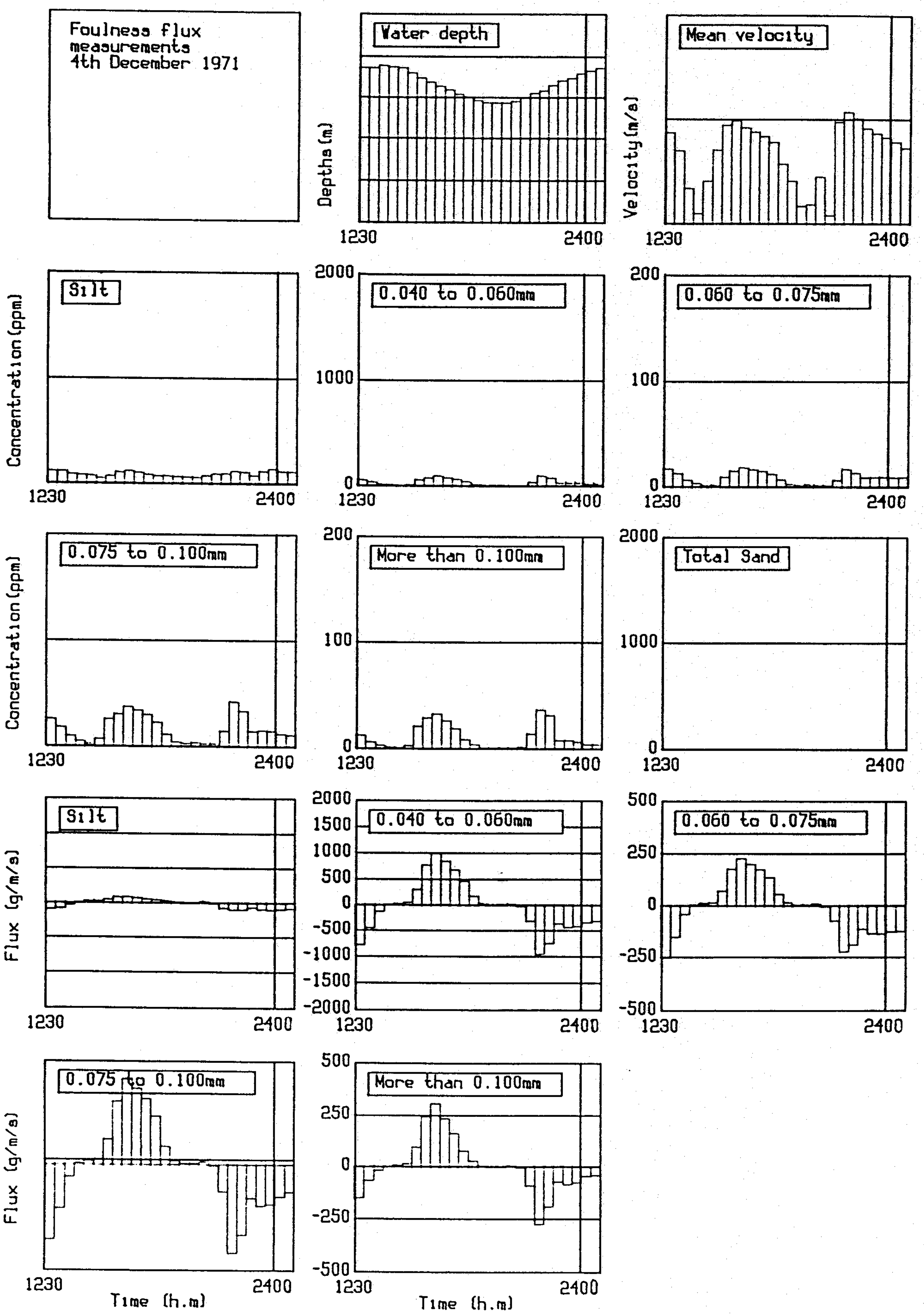


Fig 2 Sand flux processing program output: depth-averaged quantities

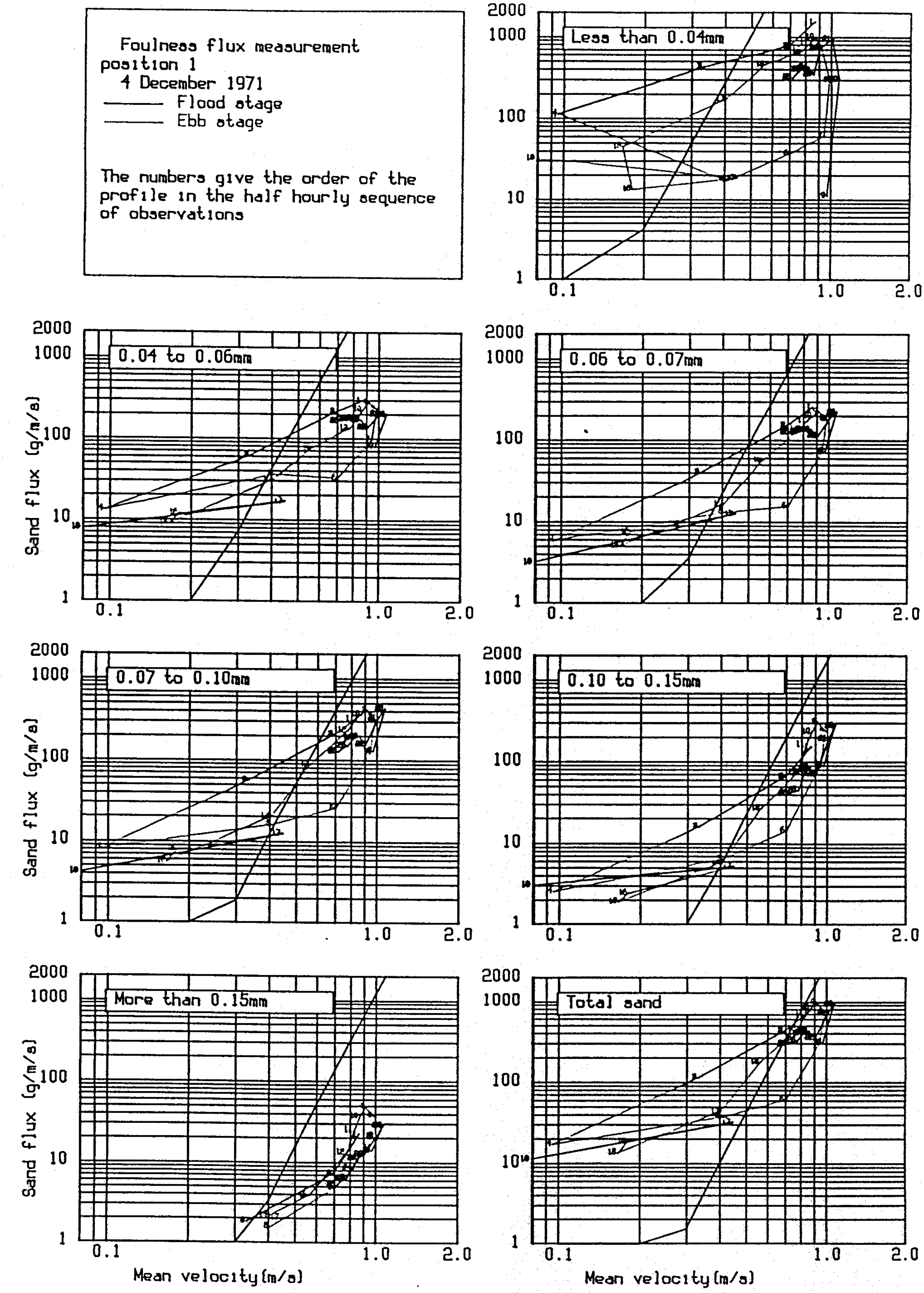


Fig 3 Sand flux processing program output:
flux dependence on current & grain size

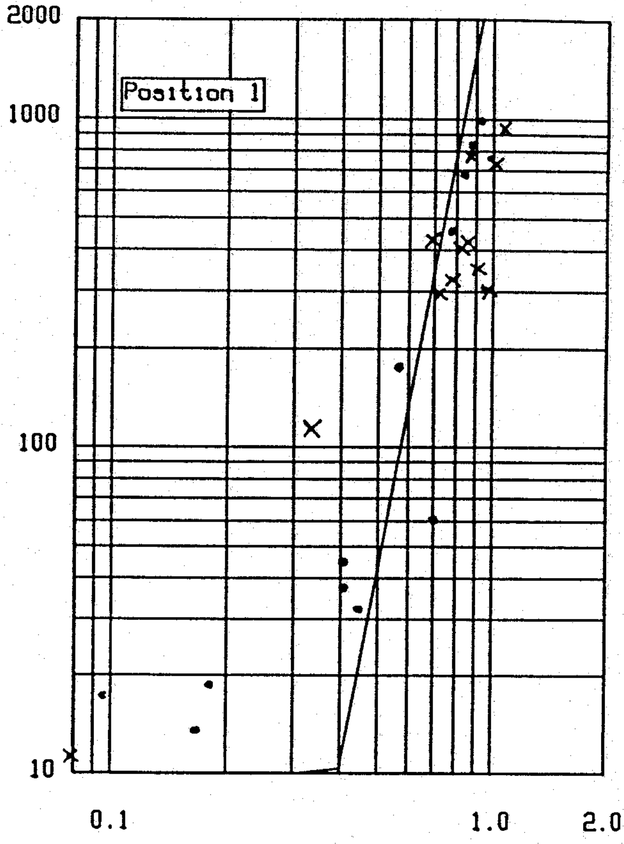
Transport relation for

Total sand

X x Flood stage

• • Ebb stage

These graphs show the results for total sand at specified site(s) together with the combined results from all sites



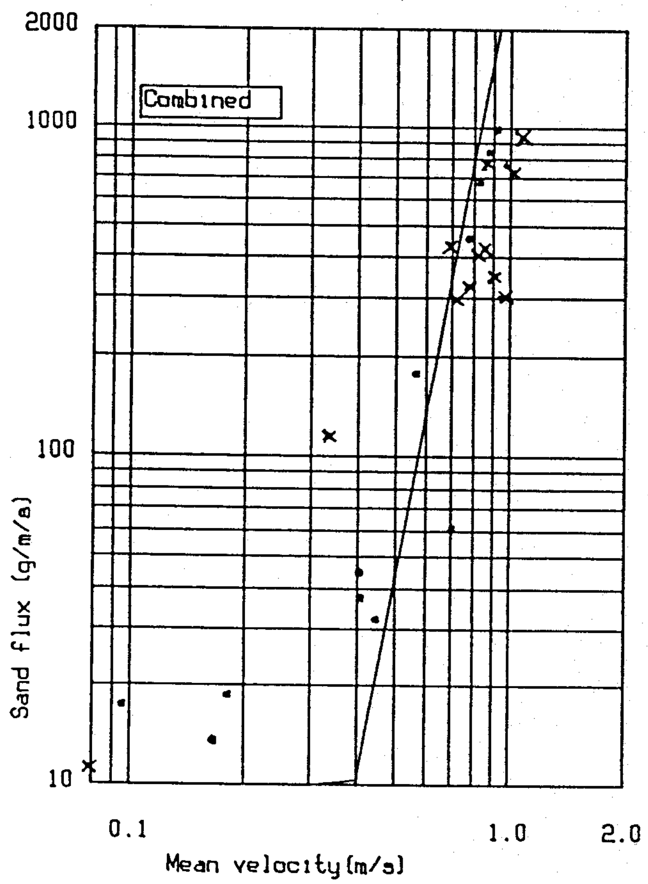


Fig 4 Sand flux processing program output: total sand flux dependence on current

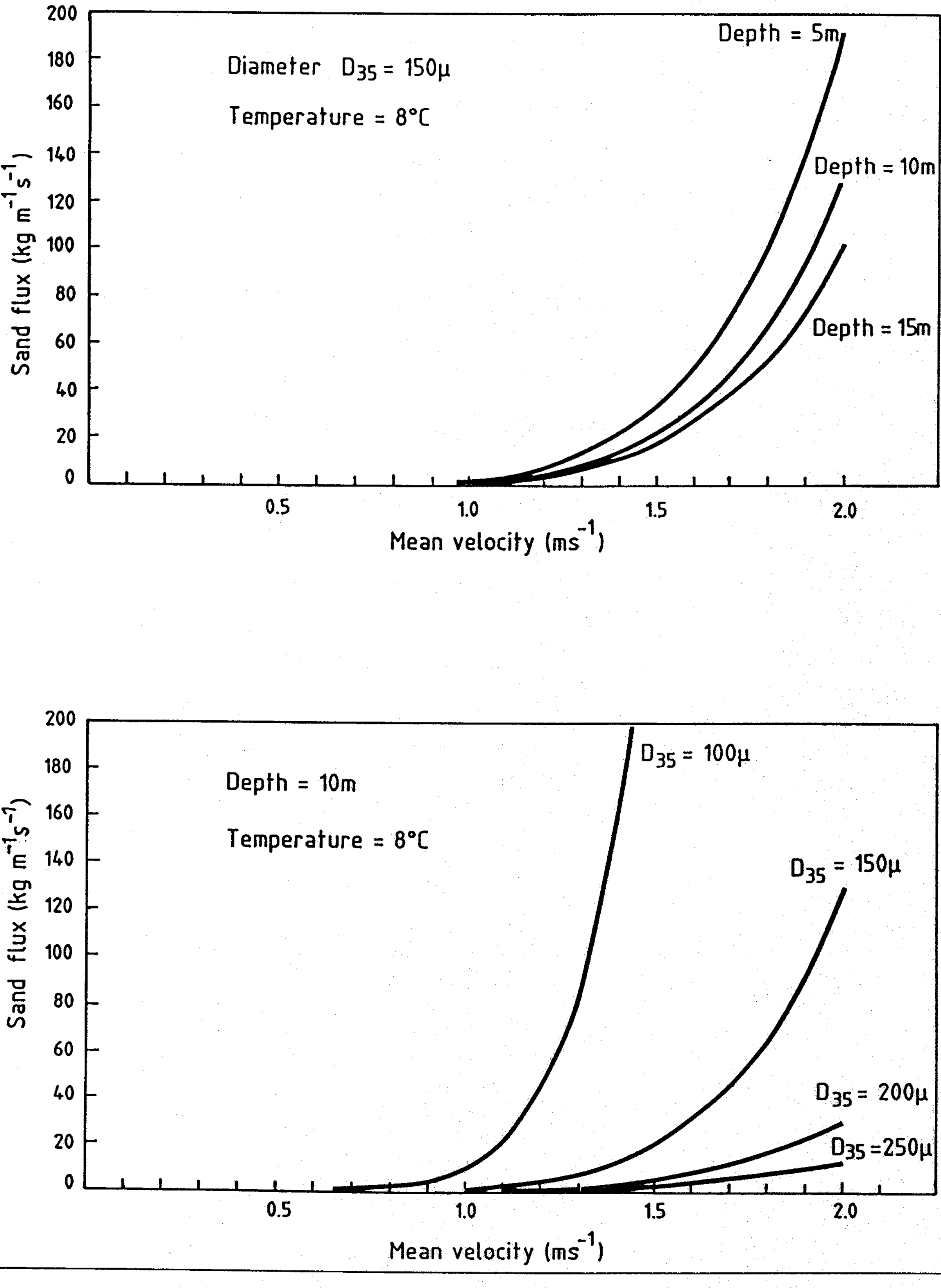


Fig 5 Predicted sand flux using Ackers - White method

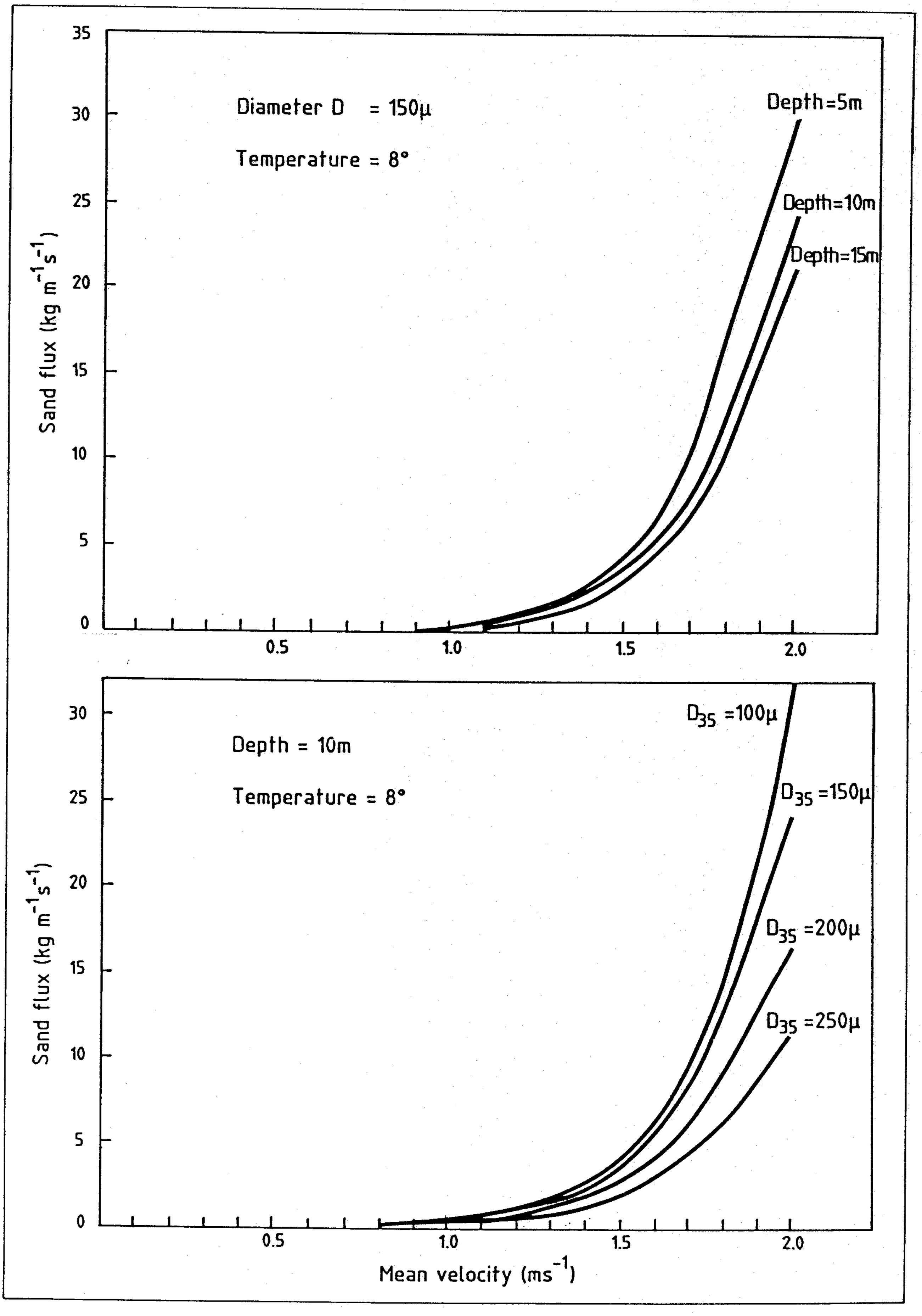


Fig 6 Predicted sand flux using van Rijn method

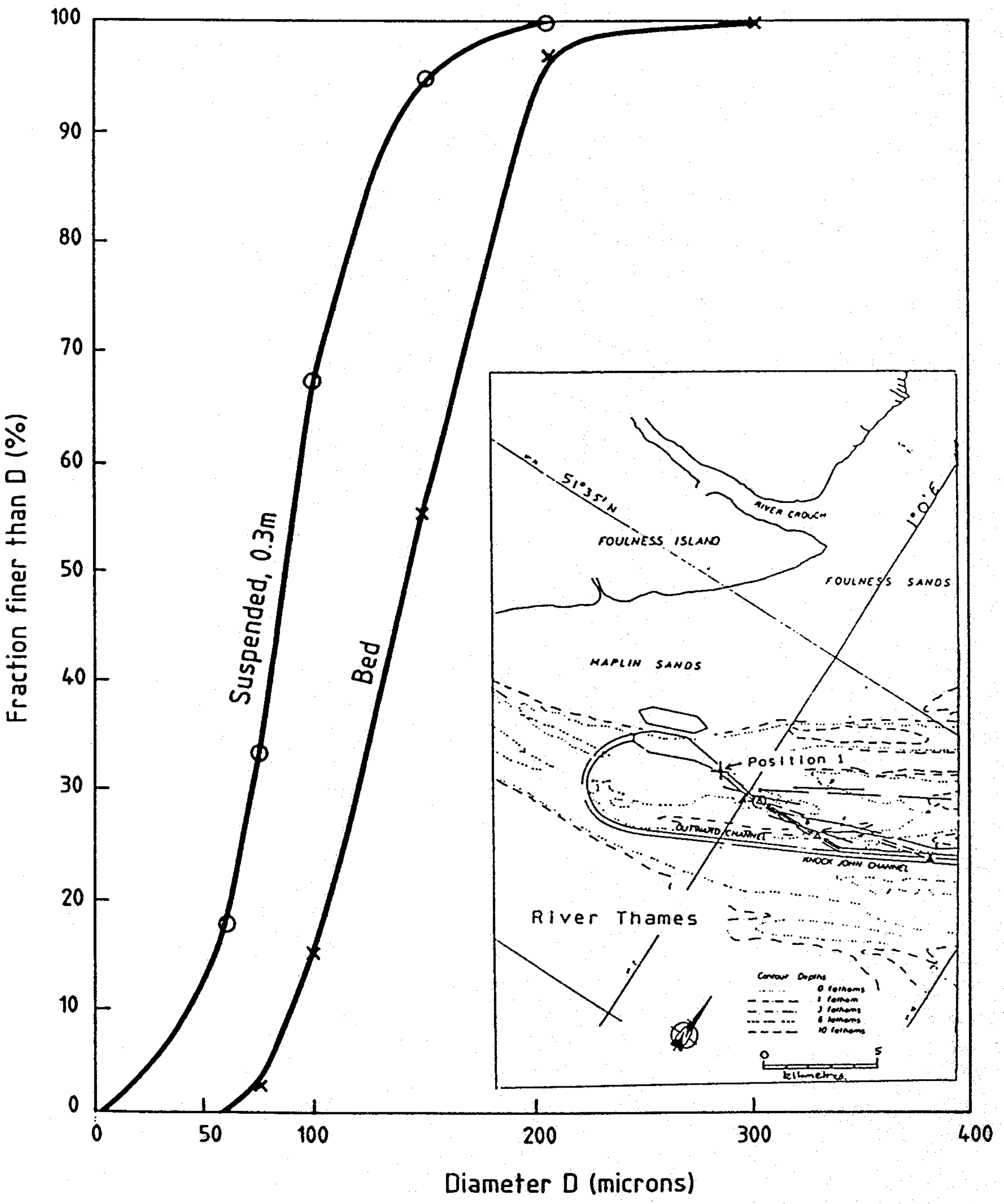


Fig 7 Size spectrum observed at Foulness position 1

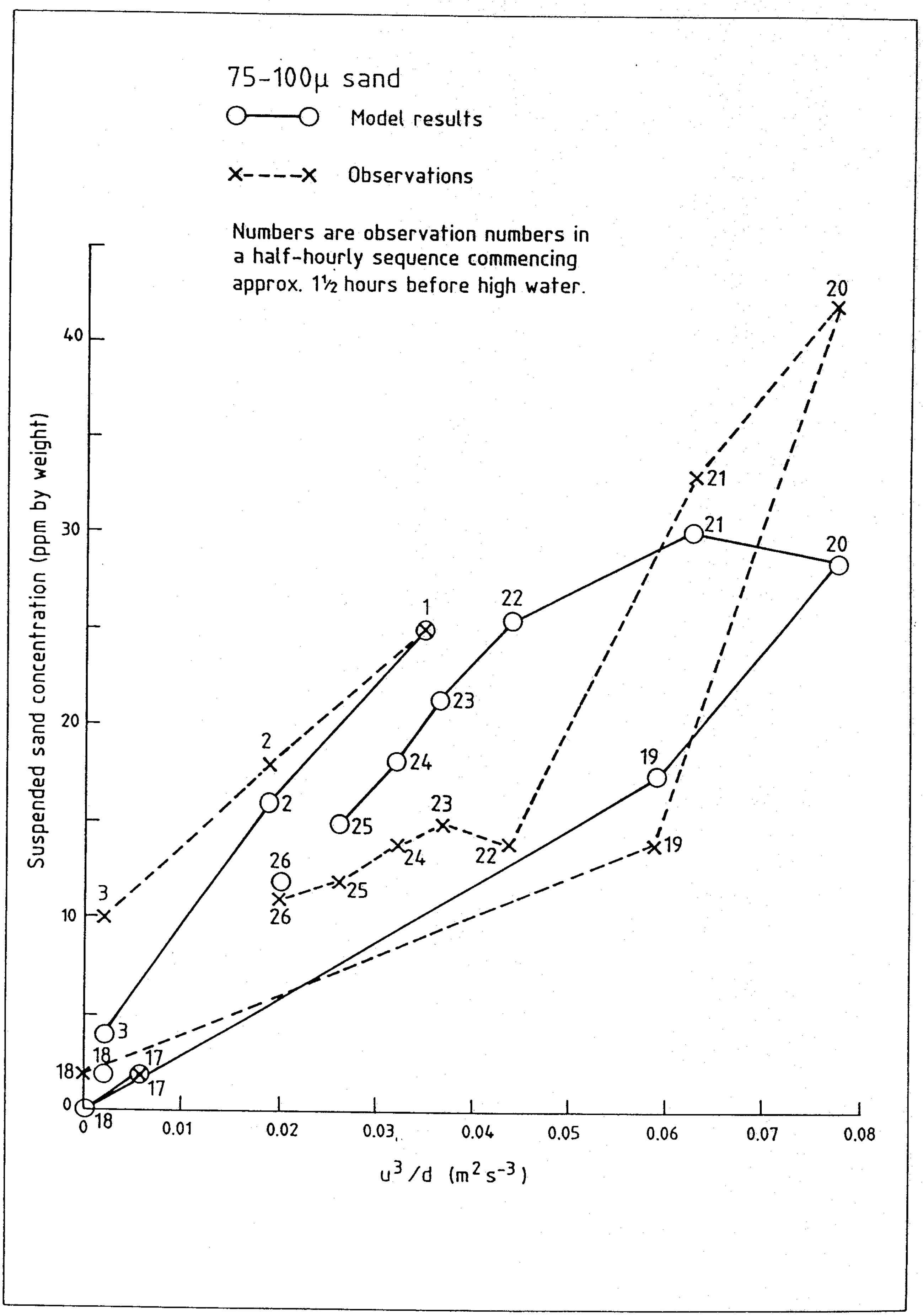


Fig 8 Simulation of Foulness position 1: flood tide

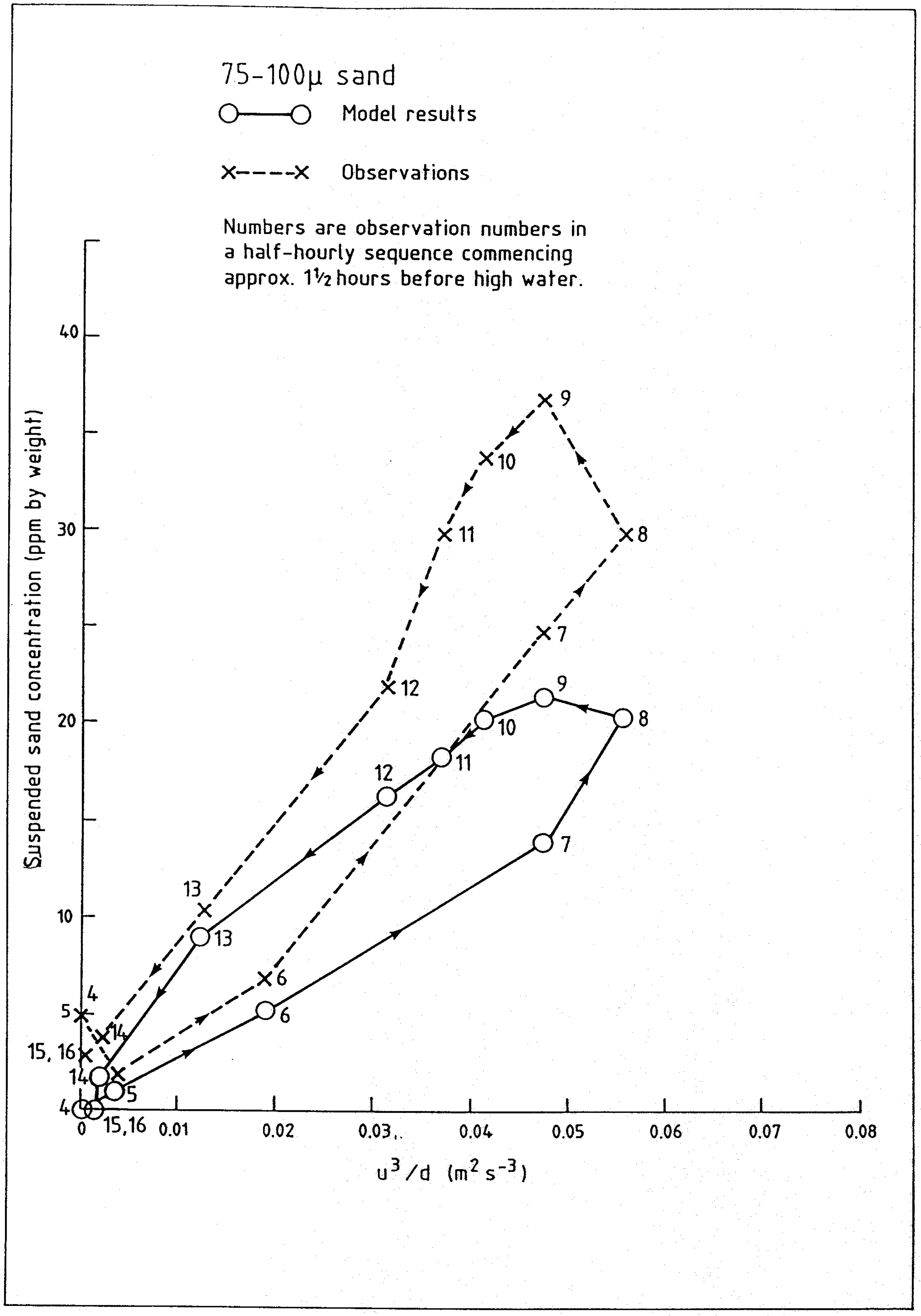


Fig 9 Simulation of Foulness position 1: ebb tide

APPENDICES.

APPENDIX 1

Hydraulics Research
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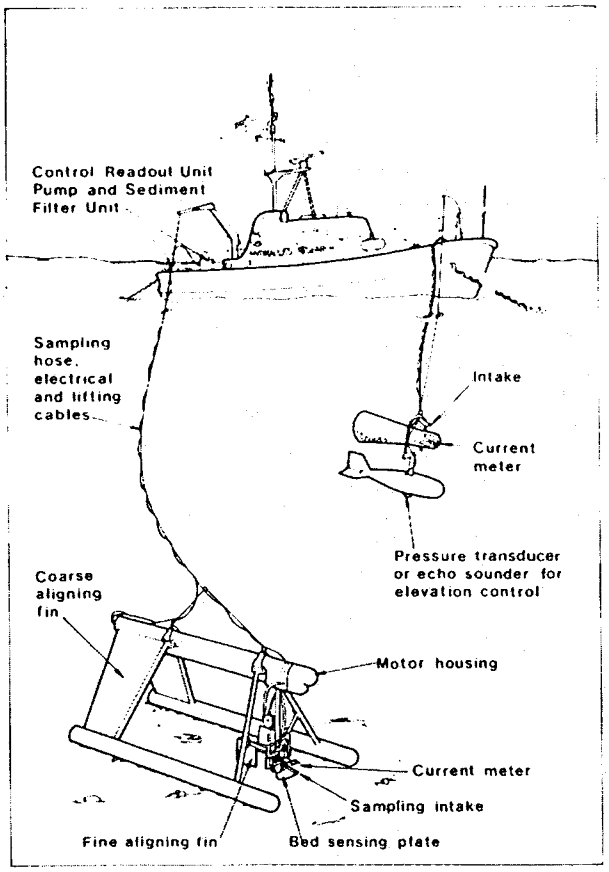
Equipment notes 11

Sand fluxmeter (standard version)

Before valid predictions can be made of the effects that proposed engineering works are likely to have on the accretion–erosion processes in an estuary or a river channel, it is important to be able to measure existing sediment transport rates. It is particularly important to be able to measure these rates close to the bed where sediment movement is greatest.

Equipment capable of doing this has been developed by Hydraulics Research, Wallingford. The equipment is unique in that it can be used in water depths down to 30 m and will measure flow velocities and sand concentrations along any vertical profile starting as close as 0.05 m above the bed. A lightweight version for shallow water is described in Equipment Note 12.

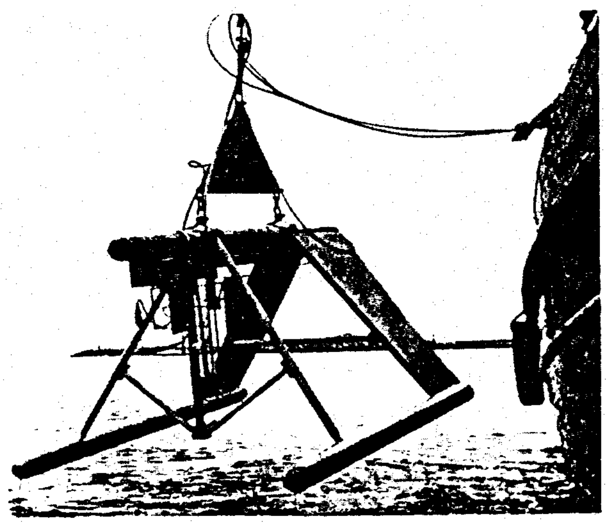
Figure 1 The sand fluxmeter system



Bed frame unit

The equipment consists of a tubular frame which is lowered to rest on the seabed and supports a sampling intake and current meter whose height above the bed is controlled from the attendant survey vessel. The weight of the frame is adjustable between 135 and 400 kg to suit the strength of the ambient currents.

Figure 2 The bed frame unit

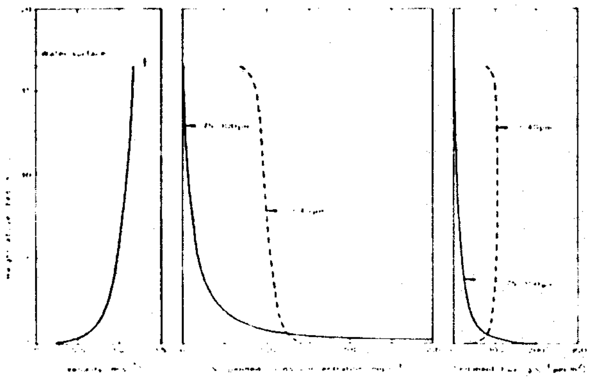


Sediment in suspension is drawn from the sampling intake to the deck of the vessel by a 24 V pump either onboard, or submerged if the required hose length exceeds 50 m. Pump flow is metered and sediment greater than 40 μm particle diameter is recovered by on-line filtration. Flow volumes up to 40 l are pumped to provide representative time-averaged samples of the suspended sediment. They also yield enough sediment for size-grading by simple sieving without creating the storage and transport inconveniences normally associated with large water samples.

Readouts for either an impeller or an electromagnetic current meter are fitted to the onboard control unit.

On the underwater unit the intake and current meter are hauled up and down on an endless chain driven by an integral 12 V motor. For setting up, the measuring head is driven downwards until a plate makes contact with the bed and operates a switch to interrupt the power supply. This position is taken as the bed datum and the measuring head is raised to the desired height, as indicated by a counter on the control unit. The bed unit is also instrumented to show its pitch and roll angles when resting on the bed.

Figure 3 Typical vertical profiles for velocity, concentration and sand flux

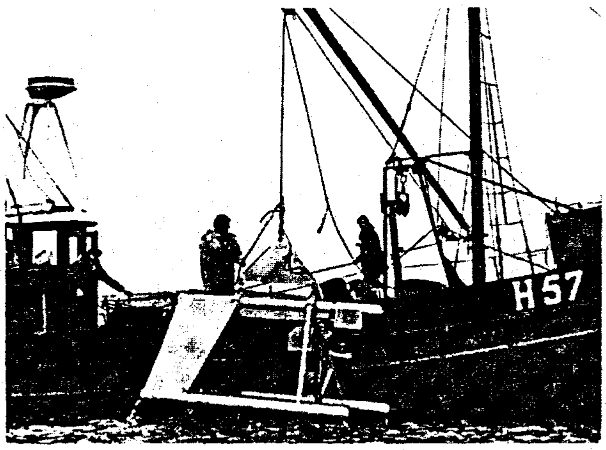


Profile measurements and analysis

Typically, pumped samples are taken and velocities measured at 0.05, 0.10, 0.15, 0.30 and 0.60 m above the bed by means of the bed frame. The flow from one metre above the bed to the surface is observed at the same time with an independent wire-suspended unit. Velocity and concentration gradients are normally less steep at these upper levels and consequently close elevation control is no longer so critical. In most tidal applications observations are made at ten points over the depth every 30–45 minutes.

Hydraulics Research has computer programs for deriving the best-fit theoretical profiles to the measured velocities and concentrations of individual size fractions and thereafter the instantaneous sediment flux.

Figure 4 Sand fluxmeter in use



Applications

Operation of the equipment calls for a stable floating platform capable of holding close station over the bed frame. Together with an earlier version functioning on the same general principle the sand fluxmeter has now been used on various applications on most of the major UK estuaries.

References

1. Crickmore M J and Aked R F. Pump samplers for measuring sand transport in tidal waters. IERE Conference on Instrumentation in Oceanography, Bangor, N Wales, 1975.
2. Crickmore M J and Teal C J. Recent developments in pump samplers for the measurement of sand transport. IAHS Symposium on erosion and sediment transport measurement. Florence, 1981.

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APPENDIX 2

PREDICTION OF TOTAL BED MATERIAL LOAD

(a) USING THE ACKERS AND WHITE (1973) FORMULAE

The sediment concentration in parts per million by weight is calculated from:

d	depth (m)
S	surface slope
$\bar{u}$	mean velocity (m/s)
D_{35}	grain size (mm)
S_g	specific gravity of particles (can be taken as 2.65 for sand)
and ν	kinematic viscosity

1. Calculate D_{gr}, the dimensionless particle size

$$D_{gr} = \frac{D_{35}}{1000} \left\{\frac{g(S_g - 1)}{\nu^2}\right\}^{1/3}$$

2. If $D_{gr} \geqslant 60$, then

$n = 0.0$, $A = 0.17$, $m = 1.5$, $c = 0.025$

and if $1 \leqslant D_{gr} \leqslant 60$ then

$$n = 1 - 0.56 \log_{10} D_{gr}$$

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14$$

$$m = \frac{9.66}{D_{gr}} + 1.34$$

$$\log_{10} c = 2.86 \log_{10} D_{gr} - (\log_{10} D_{gr})^2 - 3.53$$

3. $u_* = \sqrt{gdS}$

4. Calculate mobility number, F_{gr}, from the equation

$$F_{gr} = \frac{u_*^{\,n}}{\sqrt{0.001\ g\ D_{35}(S_g - 1)}} \left\{ \frac{\bar{u}}{\sqrt{32}\ \log_{10}\left(\frac{10^4 d}{D_{35}}\right)} \right\}^{1-n}$$

5. Calculate G_{gr} from equation:

$$G_{gr} = c\left\{\frac{F_{gr}}{A} - 1\right\}^{m}$$

(Note: if G_{gr} is negative then there is no sediment movement and $X_{AW} = 0$)

6. And finally calculate X_{AW}, the sediment concentration, from

$$X_{AW} = 1000\ \frac{G_{gr}\ S_g\ D_{35}}{d} \left\{\frac{\bar{u}}{u_*}\right\}^{n} \qquad \text{ppm}$$

PREDICTION OF TOTAL BED MATERIAL LOAD

(b) USING THE VAN RIJN (1984) FORMULAE

The sediment concentration in parts per million by weight is calculated from:

d	depth (m)
S	surface slope
r	hydraulic radius (m), the relation $r = \frac{bd}{b+2d}$ can be used where b = mean width (m)
$\bar{u}$	mean velocity (m/s)
D_{50}, D_{90}, D_{16}, D_{84}	grain sizes (mm)
S_g	specific gravity of particles (can be taken as 2.65 for sand)
and ν	kinematic viscosity

1. Compute dimensionless particle parameter

$$D_{gr} = \frac{D_{50}}{1000} \left[\frac{(S_g - 1)g}{\nu^2}\right]^{1/3}$$

2. Compute Chezy coefficient related to grains:

$$C' = 18 \log_{10} \left\{\frac{4r}{D_{90}}\right\} + 54$$

3. Compute effective bed-shear velocity:

$$u_*' = \frac{\sqrt{g}\ \bar{u}}{C'}$$

4. Compute critical bed-shear velocity: u_{*Cr}

$$\Theta_{Cr} = 0.24(D_{gr})^{-1.00} \quad \text{for } D_{gr} \leqslant 4$$

$$\Theta_{Cr} = 0.14(D_{gr})^{-0.64} \quad \text{for } 4 \leqslant D_{gr} \leqslant 10$$

$$\Theta_{Cr} = 0.04(D_{gr})^{-0.10} \quad \text{for } \leqslant D_{gr} \leqslant 20$$

$$\Theta_{Cr} = 0.013\ (D_{gr})^{0.29} \quad \text{for } 20 \leqslant D_{gr} \leqslant 150$$

$$\Theta_{Cr} = 0.055 \quad \text{for } D_{gr} \geqslant 150$$

$$u_{*Cr} = \{0.001\ \Theta_{Cr}\ (S_g-1)\ g\ D_{50}\}^{1/2}$$

5. Compute transport stage paramter

$$T_S = \frac{(u'_*)^2 - (u_{*Cr})^2}{(u_{*Cr})^2}$$

If $T_S \leqslant 0$ then there is no sediment movement

and $X_{VR} = 0$ ppm

6. Compute bed load transport:

$$X_b = \frac{5.25}{\bar{u}\ d\ 10^6} \frac{T_S^{2.1}}{(D_{gr})\ 0.3} \sqrt{S_g - 1}\ D_{50}^{1.5}$$

7. Calculate reference level, a :

a = maximum of

$$0.00692\ d\ [\frac{D_{50}}{d}]^{0.3} [1 - e^{-0.5T_S}]\ [25 - T_S]$$

and 0.01 d

8. Compute reference concentration, C_a from:

$$C_a = 1.5 \times 10^{-5} \frac{D_{50}}{a} \frac{T_S^{\ 1.5}}{D_{gr}^{\ 0.3}}$$

9. Calculate geometric standard deviation of particle size from:

$$\sigma_S = \sqrt{D_{84}/D_{16}}$$

10. Compute particle size of suspended sediment:

$$D_S = \left(1 + 0.011(\sigma_S - 1)(T_S - 25\right) D_{50} \qquad \text{(mm)}$$

11. Obtain fall velocity of representative sediment size D_S. Let w be fall velocity in m/s.

12. Compute overall bed shear stress velocity, u_*:

$$u_* = \sqrt{grS}$$

13. Compute β factor:

for $\frac{w}{u_*} \leqslant 1$

$$\beta = 1 + 2 \left[\frac{w}{u_*}\right]^2$$

for $\frac{w}{u_*} \geqslant 1$ take

$$\beta = 3$$

14. Compute ϕ factor from the following equation:

$$\phi = 2.5 \left[\frac{w}{u_*}\right]^{0.8} \left[\frac{C_a}{0.65}\right]^{0.4}$$

15. Compute suspension parameter of suspended load transport:

$$Z_y = \frac{w}{0.4\beta u_*}$$

16. Obtain modified suspension number:

$$Z'_y = Z_y + \phi$$

17. Compute F factor:

$$F = \frac{\left[\frac{a}{d}\right]^{Z'_y} - \left[\frac{a}{d}\right]^{1.2}}{\left[1 - \frac{a}{d}\right]^{Z'_y} \left[1.2 - Z'_y\right]}$$

18. Compute total suspended load concentration (by volume), X_S, from the following relation:

$$X_S = F\, C_a$$

19. Adding the bed load concentration, X_b, and suspended concentration, X_S, we get the total load concentration (by weight):

$$X_{VR} = (X_b + X_S)\, S_g\, 10^6 \qquad \text{ppm}$$