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**A MATHEMATICAL MODEL OF DIFFRACTION BY  
BREAKWATERS IN RANDOM WAVES**

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# **A Mathematical Model of Diffraction by Breakwaters in Random Waves**

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## **ABSTRACT**

This report gives details of a method to predict diffracted wave heights in the lee of a breakwater for random waves. It uses a technique whereby the diffraction coefficients for unidirectional monofrequency wave components are combined according to a specified incident frequency spectrum and directional spread. An example of its application, and effect, is given for a breakwater gap. Some illustrations of the use of the method for the production of diffraction diagrams are provided for island breakwaters and breakwater gaps.



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## 1 INTRODUCTION

There are several publications which present diagrams which may be used to calculate wave heights in the vicinity of a breakwater (see, for example, Refs 1 and 2). In general the diagrams which are available are applicable only to unidirectional monofrequency waves. In this report details are given of a method for predicting diffracted wave heights for random incident waves, which uses results from numerical models already developed for monofrequency incident waves (Ref 3).

The technique which is used to calculate diffracted wave heights in random wave is based on the idea of combining spectral components taking into account the diffraction coefficient for each of them. Details of the method used is given in Chapter 2, together with a discussion of the effect on diffraction coefficients of including directional and frequency spreading in the calculations. In Chapter 3 examples of diagrams are presented which allow diffracted wave heights for random incident waves to be calculated for island breakwaters and breakwater gaps. The conclusions and recommendations arising from this study are given in the final chapter.

## 2 DIFFRACTION COEFFICIENTS FOR RANDOM WAVES

### 2.1 Theoretical considerations

The diffraction coefficients for a given breakwater configuration with fixed incident wave conditions is defined as the ratio of the wave height in the area affected by diffraction to the incident wave height. It is usually denoted by  $K_d$  where,

$$K_d = \frac{H_d}{H_i}, \quad (1)$$

and  $H_d$  and  $H_i$  are the wave heights in the area affected by diffraction, and the incident wave height respectively. In (1) it is assumed that  $H$  is the monochromatic wave height, and as a consequence of that  $K_d$  is a function of both the frequency and direction of the monofrequency incident wave.

In random waves the sea state is usually characterised in terms of the significant wave height ( $H_s$ ) where,

$$H_s = 4 \left[ \int_0^\infty \int_0^{2\pi} S(f, \theta) d\theta df \right]^{\frac{1}{2}}, \quad (2)$$

and  $S(f, \theta)$  is the spectral density which is a function of frequency ( $f$ ) and direction ( $\theta$ ). Extending the definition given in (1) and (2) the diffracted wave height at a given location in random waves is given by,

$$(H)_{sd} = 4 \left[ \int_0^\infty \int_0^{2\pi} K_d^2(f, \theta) S(f, \theta) d\theta df \right]^{\frac{1}{2}}, \quad (3)$$

where  $K_d(f, \theta)$  is the diffraction coefficient at that location for a monofrequency wave with frequency  $f$  and direction  $\theta$ . Thus, for random waves the diffraction coefficient is given by

$$(K_d)_{\text{ran}} = \frac{(H)_{sd}}{(H_s)_i} = \frac{\left[ \int_0^\infty \int_0^{2\pi} K_d^2(f, \theta) S(f, \theta) d\theta df \right]^{\frac{1}{2}}}{\left[ \int_0^\infty \int_0^{2\pi} S(f, \theta) d\theta df \right]^{\frac{1}{2}}}. \quad (4)$$

This definition is similar to that given in Goda (Ref 4).

In practise the values of the functions  $K_d$  and  $S$  will be known for certain discrete values of  $f$  and  $\theta$  and (4) will be approximated by

$$(K_d)_{\text{ran}} \approx \frac{\left[ \sum_{i=1}^n \sum_{j=1}^m K_d^2(f_i, \theta_j) S(f_i, \theta_j) \Delta\theta_j \Delta f_i \right]^{\frac{1}{2}}}{\left[ \sum_{i=1}^n \sum_{j=1}^m S(f_i, \theta_j) \Delta\theta_j \Delta f_i \right]^{\frac{1}{2}}},$$

where  $n$  frequency and  $m$  direction components are being considered, and  $\Delta f_i$  and  $\Delta\theta_j$  are the width of the  $i$ th frequency and  $j$ th direction interval. The frequency and direction components used in (5) must be selected so as to fully cover the range of the incident wave spectrum.

Therefore to calculate the diffraction coefficient in random waves we need to specify the incident wave spectrum, and determine the diffraction coefficients for each of the discrete frequency and direction components in (5).

A typical spectrum is of the form,

$$S(f, \theta) = S(f) G(\theta),$$

where  $S(f)$  is the frequency distribution of wave energy and  $G(\theta)$  is the directional distribution of wave energy which is assumed to be independent of frequency. The frequency spectrum will normally be calculated using established formula for deep water waves such as Pierson-Moskowitz or JONSWAP. A summary of the formulae which are in common use is given in Chakrabarti (Ref 5). In the present work a Pierson-Moskowitz spectrum is used to provide examples of the application of this theory.

We also require a function describing the directional distribution of wave energy. Much research has been done on the choice of a function describing directional spreading of energy. In particular the work of Hasselman et al (Ref 6) which resulted in the formulation of the JONSWAP wave spectrum which uses a distribution based on  $\cos^2(\theta - \theta_m)$ , where  $\theta_m$  is the mean wave direction, to describe the directional spread of waves. There is some evidence to suggest that the use of a narrower directional spread, see Mitsuyasu (Ref 7), may be appropriate in shallow water, but in the present work the  $\cos^2$  spreading function is retained.

To find the random wave diffraction coefficients for a particular breakwater configuration it remains to calculate the diffraction coefficients for each of the frequency and direction components. The method which is employed here to calculate diffraction coefficients for a breakwater gap and an island breakwater uses an integral equation technique which is described in Gilbert and Brampton (Ref 3).

An example of the application of the technique described above to calculate diffractions coefficients in the lee of a breakwater gap in random waves is given in the following section.

## 2.2 Application to a breakwater gap

In conventional diffraction diagrams the gap width or breakwater length is specified in terms of the wavelengths of the incident wave. This allows them to be used in a more general way than if the gap width were given in dimensional variables, as both the water depth and incident wave period would then need to be specified. A similar approach is used here for random waves with the gap width being given in terms of the

wavelength corresponding to the peak period of the incident spectrum.

Once the gap width of interest has been selected it remains to calculate the diffraction coefficients for this layout for the various frequency and direction components. The frequency components in equation (5) should be selected to cover the range  $f_p/2$  to  $2f_p$ , where  $f_p$  is the frequency corresponding to the peak period. For a straight breakwater the direction range will be  $\pm 90^\circ$  from the centre line of the breakwater. Typically, 10 frequency and 11 direction components will be sufficient to ensure accurate results, whilst maintaining moderate computational effort. Although more components may be required where the frequency range of the incident spectrum is large, or there is likely to be a significant variation in the diffraction coefficients caused by a small change in incident direction.

To provide an example, the method described above was used to calculate diffraction coefficients in the lee of a breakwater gap. To examine the influence of introducing a frequency and directional spread on diffraction coefficient the following cases were considered.

- a) Gap width one wavelength, normally incident waves (unidirectional, monofrequency incident waves).
- b) Gap width one wavelength, mean incident direction normal,  $\cos^2$  directional spread (monofrequency incident waves).
- c) Gap width one wavelength at the peak period of the incident (Pierson-Moskowitz) spectrum, normally incident waves (unidirectional).

- d) Gap width one wavelength at the peak period of the incident (Pierson-Moskowitz) spectrum, mean incident direction normal,  $\cos^2$  directional spread.

The results from these tests are presented in Figure 1 as contours of equal diffraction coefficients.

Figure 1(a) shows the wave height coefficients obtained for unidirectional monofrequency incident waves. It can be seen that the values of wave height coefficient are symmetric about the centre line through the gap, as should be expected for normally incident waves. The largest values along any line parallel to the gap also occur along this centre line. At distances in the lateral direction greater than one wavelength from the gap centre the wave height coefficients are less than 0.3.

The effect of introducing a directional spread of the incident waves can be seen by comparing Figures 1(a) and 1(b). The most significant feature is that along the gap centre line the effect of directional spreading is to decrease the wave height coefficient. This decrease will be due to there being less energy in the direction of normal incident, when compared with 1(a). There is however more energy penetrating into the areas either side of the centre line outside the immediate vicinity of the gap. This will be due to energy being directed along other directions than normal, and influencing the diffraction coefficient through the directional spreading.

The diffraction coefficients for normally incident waves without any directional spreading for a Pierson-Moskowitz spectrum are shown in Figure 1(c). It can be seen that the line of equal diffraction coefficient are much closer and more sharply defined than either 1(a) or 1(b). This will be a feature of

most of the energy being directed along the centre line through the gap. At most locations the diffraction coefficient is higher than for cases (a) or (b).

For the case with a Pierson-Moskowitz incident spectrum with a  $\cos^2$  directional spread about the mean normal direction the results are shown in Figure 1(d). As should be expected the effect of introducing directional spreading, when compared with 1(c), is to decrease wave heights at most locations, and to broaden the contours of diffraction coefficients. The general shape of the contours is similar to those in 1(b), but the values of wave height coefficients at most points are slightly higher. On comparing the results from 1(d) with 1(a) it can be seen that for this case using the monochromatic diffraction coefficients instead of the random diffraction coefficients would result in wave heights in the lee of the breakwater being slightly overestimated. This should not be generalised to other breakwater configurations and incident spectra, as each situation will have different characteristics.

### 3 DIFFRACTION DIAGRAMS FOR RANDOM WAVES

In this chapter we present diffraction diagrams for random incident waves for both a breakwater gap, an an island breakwater. These are primarily intended to provide examples of the type of diagrams that can be produced using the method outline in the previous chapter. All of the diagrams presented are for a Pierson-Moskowitz incident wave spectrum with  $\cos^2$  directional spreading about a specified mean direction.

Diffraction diagrams for breakwater gaps of one and two wavelengths (corresponding to the peak period of the incident wave spectrum) are given in Figures 2 and 3. In both figures diffraction coefficients are given for mean incident directions of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ . In all cases the characteristic pattern is displayed with the contours of equal coefficients forming in the shape of a parabola around the gap. The axis of this parabola being along the extension of the incident wave direction. As a result of this the maximum wave height on a line parallel to the breakwater will always occur along the line of the incident direction continued into the lee of the breakwater.

Similar diagrams for island breakwaters of length one and two wavelengths are given in Figures 4 and 5. These again display some typical features. For example, there is again a parabolic shape to the contours of equal wave height coefficient. In both cases at relatively short lateral distances from the breakwater the wave height coefficient become equal to one. That is, diffraction effects have become small and the incident wave height is being maintained. It should also be noted that such breakwaters only offer good shelter in their immediate lee, and that this varies considerably as the wave direction changes.

#### 4 CONCLUSIONS AND RECOMMENDATIONS

1. A method has been developed for the calculation of diffraction effects in random waves. Examples have been given of the use of this technique for predicting wave conditions in the lee of the breakwater gap, or an offshore breakwater.
2. Diagrams, such as those shown in Figures 2 and 3, will be useful tools for the engineer involved in the design of island breakwaters. Similar

diagrams could easily be produced for different incident frequency spectra, directional spreading functions and breakwater configurations.

3. Consideration should be given to the production of a series of diffraction diagrams which could be routinely used by engineers (similar to those in Refs 1 and 2) in site-specific studies. One possibility is that such diagrams could be generated by the engineers themselves using the present model implemented on a micro-computer.

## 5 ACKNOWLEDGEMENTS

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FIGURES.



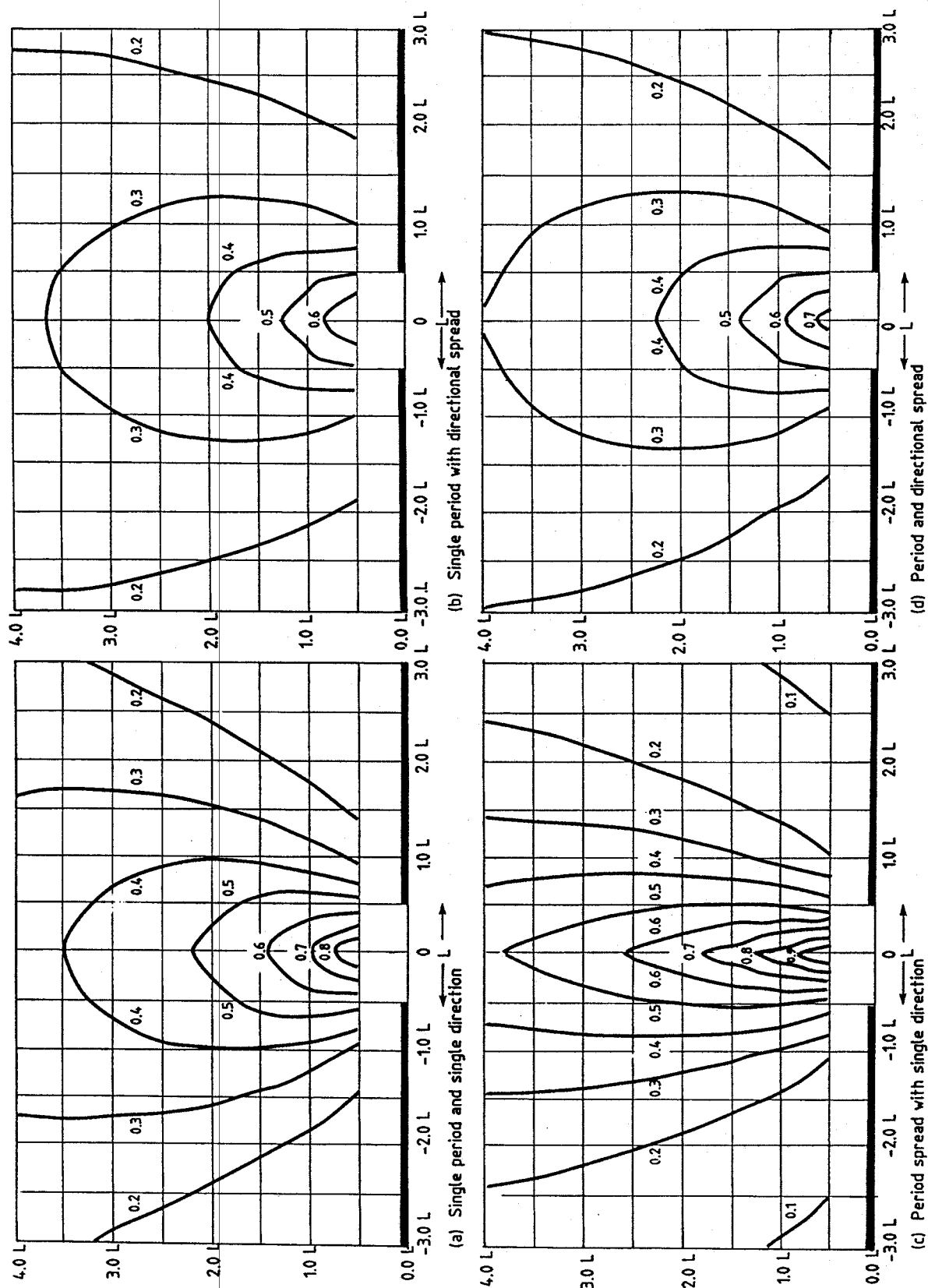


Fig 1 Effect of frequency and directional spread on diffraction coefficients for breakwater gap of one wavelength, normal incidence

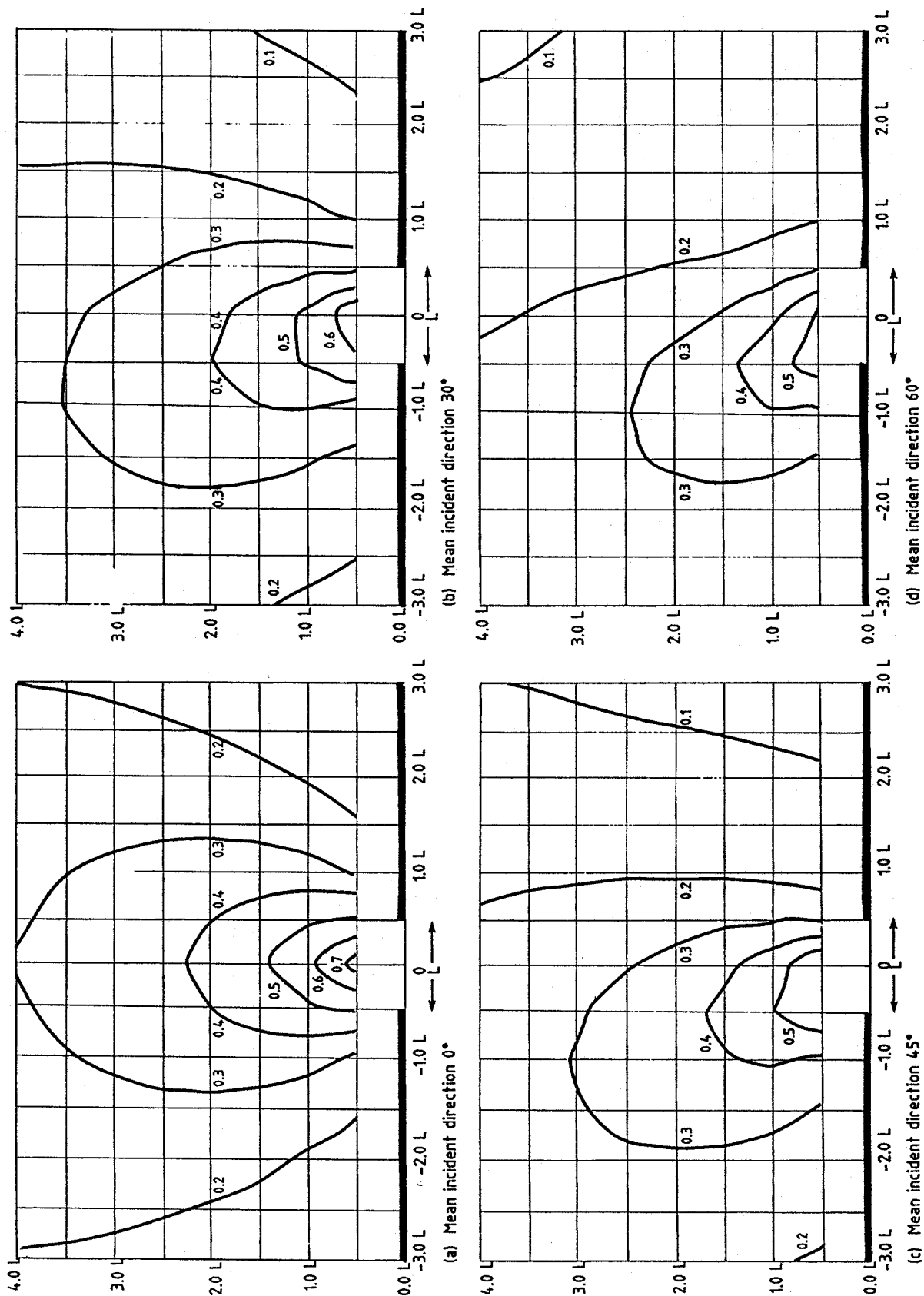


Fig 2 Diffraction coefficients for breakwater gap of one wavelength, Pierson-Moskowitz spectrum,  $\text{Cos}^2$  directional spread

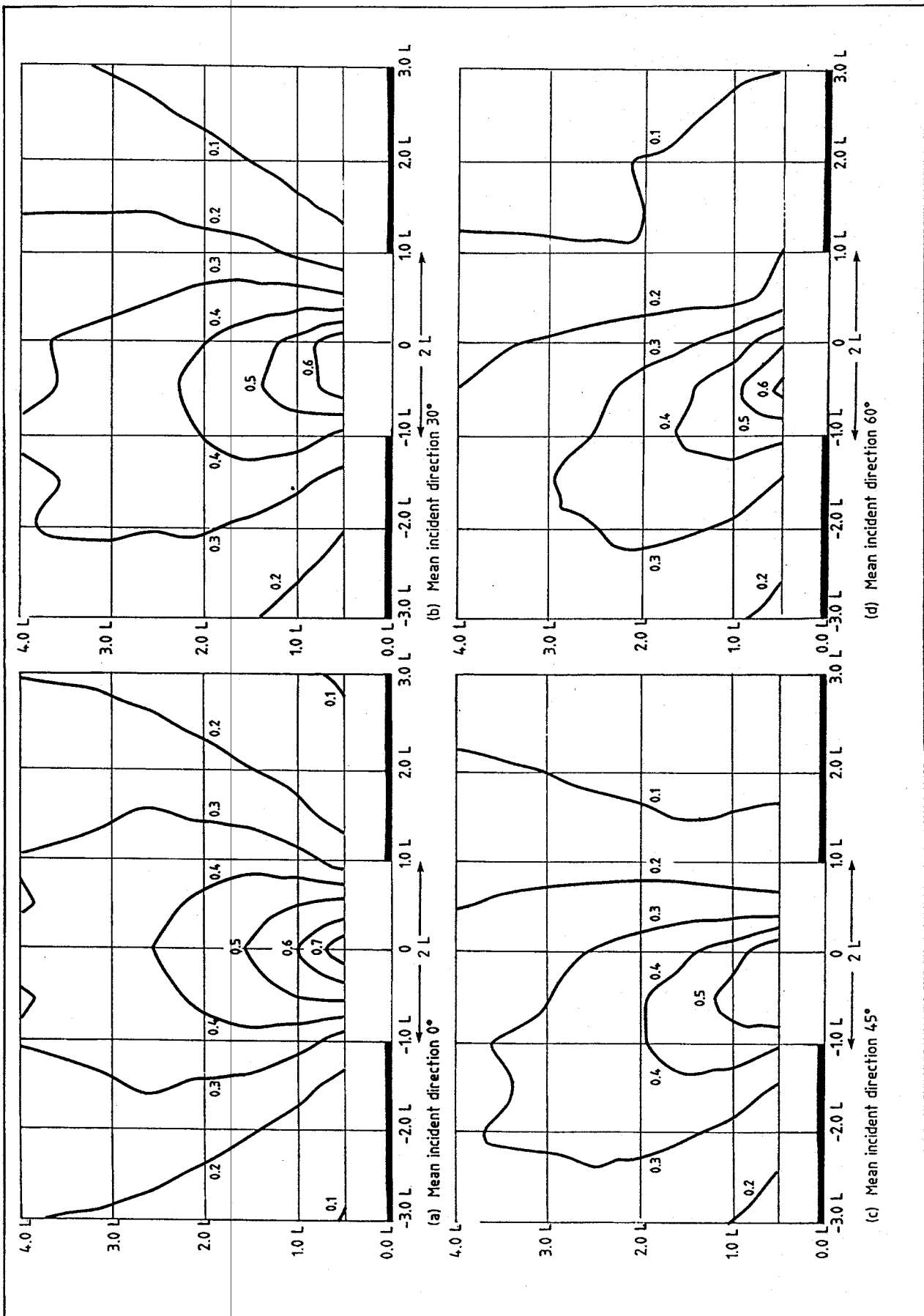


Fig 3 Diffraction coefficients for breakwater gap of wavelengths, Pierson-Moskowitz spectrum,  $\text{Cos}^2$  directional spread

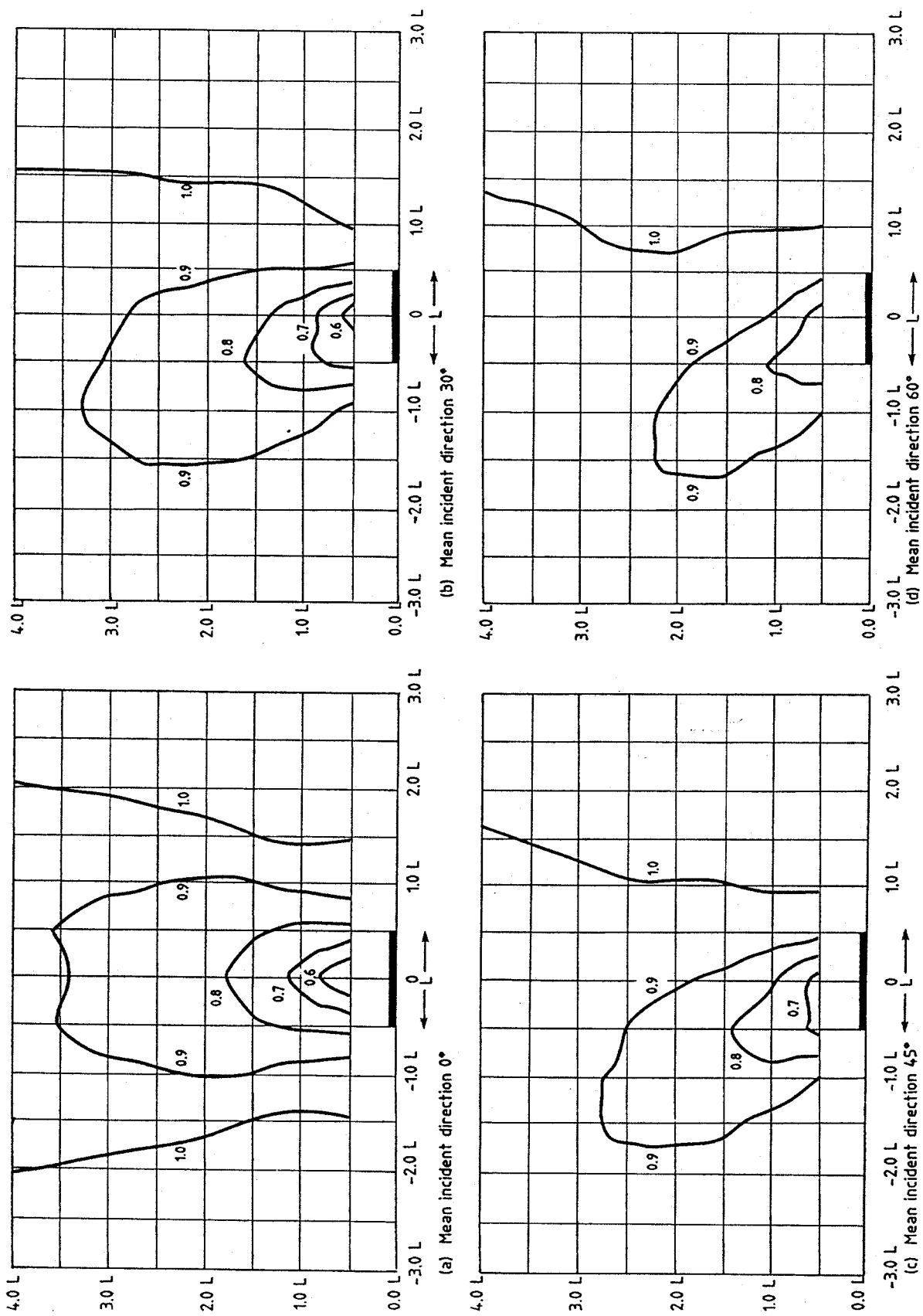


Fig 4 Diffraction coefficients for island breakwater of one wavelength, Pierson-Moskowitz spectrum,  $\text{Cos}^2$  directional spread

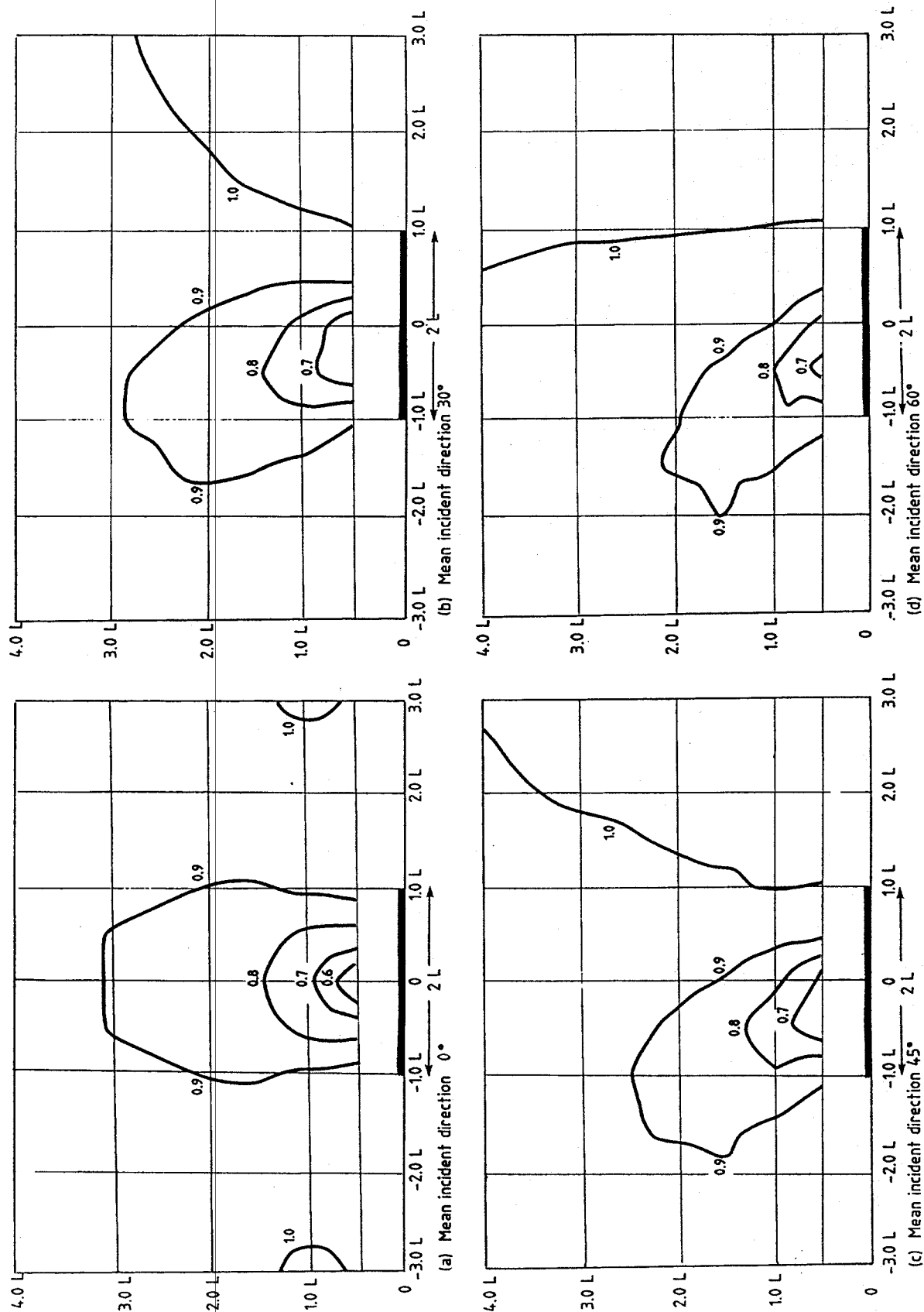


Fig 5 Diffraction coefficients for island breakwater of two wavelengths, Pierson-Moskowitz spectrum,  $\text{Cos}^2$  directional spread

