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WAVE RUN-UP ON STEEP SLOPES - A literature review

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Abstract

A literature review has been conducted into wave run-up on steep slopes. Both smooth and armoured rubble slopes have been considered, as have both regular and random wave attack. The effects of oblique wave attack have been explored.

The report considers a number of run-up prediction methods, and contrasts values of relative run-up calculated by different methods. The report makes a number of recommendations for further work, and suggests model studies to test various empirical prediction formulae.

Notation

R	run-up, expressed as a height above static water level
R_s	significant wave run-up, mean of highest 1/3 run-up crests
R_2	run-up level exceeded by only 2% of run-up crests
$\bar{R}$	mean run-up level
H	wave height, crest to trough
H_o	offshore wave height, in deep water
H_s	significant wave height
H_2	wave height exceeded by only 2% of the waves
T_z	mean wave period, zero crossing
T_p	wave period of maximum spectral energy
L	wave length
L_o	wave length in deep water
α	structure slope angle to the horizontal
β	incident wave angle, wave crests to seawall
ε	spectral width parameter, defined in section 3.3
K_s	shoaling coefficient, defined in section 2.1
K_r	structure reflection coefficient
k	wave number, $= \frac{2\pi}{L}$

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1 INTRODUCTION

1.1 Wave run-up

Coastal structures such as sea walls and breakwaters, when subject to wave attack, will experience wave run-up. If the structure crest is lower than the maximum run-up, the structure will suffer overtopping. This may in turn lead to flooding and/or damage to the rearward face of the structure. In the planning and design of coastal structures, especially sea walls, wave run-up and overtopping are often the two primary factors dictating the crest level of the wall. As the cross sectional area and the cost increases approximately with the square of the structure height, a clear understanding of the processes of wave run-up and overtopping is essential to the economic design of such structures.

Historically the designers of sea walls and breakwaters have often attempted to design the crest level of their structure high enough to prevent overtopping. This was done by calculating the maximum run-up level and setting the crest level above it. This does, however, presuppose that a maximum run-up level may be identified. With a fuller understanding of the random nature of wind waves, it has become clear that overtopping cannot be wholly prevented, although the mean expected overtopping discharge for a design event may be reduced to negligible proportions. The design approach for simple sea walls in the UK has therefore recently been altered to one of designing for various levels of tolerable discharge under the extreme events considered, using the method proposed by Owen(1,2). This method was derived from analysis of the results of model tests on simple, and bermed, smooth seawall slopes, without wave return walls. However, for structures with such parapet walls, the prediction of overtopping discharge becomes extremely complex. Owen(3) argues that model tests are needed to determine the hydraulic performance of such crest details. Calculations of wave run-up levels are therefore still required for most preliminary designs, and for further analysis of walls of complex form.

Recent work by Losada & Gimenez-Curto(4) and by Allsop(5) has, however, shown that wave run-up behaviour on rough and/or permeable slopes is not well described by the application of a single roughness correction factor to the run-up on an equivalent smooth slope.

1.2 Purpose and scope of the review

As part of continuing work on the design principles for sea walls and breakwaters, it was decided therefore that Hydraulics Research would conduct a series of random wave model tests on rubble slopes, armoured with a number of different armour units. These tests would determine the levels and probability distributions of run-up crests on such slopes, and on the equivalent smooth slopes. However, before model tests to measure and describe the run-up performance of random waves on armoured rubble slopes could be designed it was clear that a review of the available design guidelines and of recent research findings should be conducted.

The purpose of this literature review was to look at wave run-up on the steep slopes commonly used for sea wall and breakwater construction around the UK and elsewhere. Work was therefore concentrated on reports and papers dealing principally with slopes of 1:1 to 1:3, although shallower slopes were considered where appropriate. Similarly, it was hoped to concentrate on those publications covering the influence of random waves. However, as relatively few articles have been published on the effects of random waves, the earlier work with regular waves was also considered.

Two major documents summarise much of the work in wave run-up. The Shore Protection Manual(6) is produced in the USA by the Corps of Engineers Coastal Engineering Research Centre (CERC) and is revised periodically. The latest edition at the time of writing was that published in 1984. The Shore Protection Manual (SPM), as amended and expanded by the Coastal Engineering Technical Aids (CETA), may be taken as summarising current American design practice.

In Holland the Technical Advisory Committee on Protection against Inundation (TACPI) published a major review on wave run-up and overtopping in 1974(7). This was based on a Dutch language report published in 1972. The TACPI report is a primary source of information on wave run-up, and overtopping, covering most sources of field and test results published before 1972. A full review of regular wave measurements is given, as are the few irregular wave model results and field measurements reported before 1972. In general, however, the structure slopes considered tend to be relatively shallow. The test results and prediction formulae therefore may have limited use in the design of steeper slope structures. The prediction formulae suggested are generally based upon the Hunt formula, with a number of modifying coefficients.

Whilst some work dated earlier than 1972 will be referred to in this report, in the main the review will concentrate on work published later. Similarly, most relevant American work prior to about 1982 will have been incorporated into the 1984 edition of the SPM. Run-up is also dealt with by some of the standard text books. Of these Horikawa[8] summarises much of the Japanese work up to about 1970.

1.3 Outline of the report

This review considers wave run-up from both theoretical and experimental standpoints, and summarises the methods available to predict wave run-up under various conditions.

Theoretical considerations of run-up have been divided principally into the behaviour of non-breaking waves on steep slopes, and breaking waves on shallow slopes. Efforts have also been made to describe the probability distributions of random wave run-up levels. These three aspects are covered in chapter 2.

It has not been possible to describe wave run-up from a theoretical basis over the complete range of wave conditions. Many authors have therefore turned to empirical expressions fitted to the results of experimental work. Most of this work has used regular wave model tests, but recently tests with random waves, and some field measurements, have yielded more realistic results. Chapter 3 covers experimental work, including run-up performance on smooth and rubble slopes, under both regular and irregular waves. Also included is the effect of angled wave attack.

As a result of the theoretical and experimental work, a number of prediction methods have been suggested and these are summarised in chapter 4. These allow the prediction of wave run-up on various slopes, under a wide variety of wave conditions. The effects of some of the different prediction formulae are compared, and a number of simple tables show relative run-up values for a number of wave steepnesses and structure slopes. Two different expressions describing the effect of angled wave attack are contrasted.

The last chapter of this report seeks to draw together the conclusions of the review, and to identify those remaining areas of uncertainty. Recommendations for further research are made, particularly into the probability distributions of run-up levels under random wave attack on smooth or rubble slopes.

1.4 Literature considered

In order to simplify the use of this review, the formulae and expressions from the literature have been expressed using a single set of symbols, and in SI units where appropriate. In particular this may mean that some American and Japanese expressions with coefficients having dimensions will have been re-worked. A list of notation used in this review is given at the start of this report.

In common with the practice in some other reviews of this type, reports and papers considered have been referred to in one of two ways. Those articles that have actually been considered in detail, and to which the reader may be referred, have been classified as references. References have been listed at the end of this report, in the order in which they have been considered. They have been identified in the text by the reference number in elevated brackets, e.g. (8). The branch literature, and many classic papers and reports to which time did not allow further study, have been listed in the bibliography. These items are identified in the text by the use of the author's name and the date of publication, e.g. Pocklington (1921). The bibliography is arranged in alphabetical order of author's name.

2 THEORETICAL APPROACH

2.1 General

Many attempts have been made to describe wave run-up from a theoretical standpoint. The major impediment to the evolution of a full description of the behaviour of a wave, or series of waves, running up a slope, is the difficulty of describing the extremely complex behaviour of wave breaking. On many of the slopes considered, the behaviour of the previous wave may be critically important in determining the peak run-up level reached by the following wave. The flow processes involved are highly complex and variable. Theoretical descriptions of the turbulent and stochastic behaviour of random waves are not yet available in a form that may be used by the designer. Two distinct approaches to this problem can, however, be distinguished. The first deals with non-breaking waves on steep slopes; the second considers breaking waves on shallow slopes. However, in this analysis, all such slopes are considered to be essentially smooth and impermeable.

2.2 Steep slopes

On the steep slopes of interest to the designer of seawalls, or breakwaters, theoretical work has been concentrated on combinations of structure slope and wave steepness that lead to surging waves, that do not break on the slope. Researchers considering wave run-up on smooth, steep walls have included Pocklington (1921), Sainflou (1928), Miche (1944), Isaacson (1950) and Rundgren (1958). A simplified expression for the run-up level, R, of a regular wave of deep water wave height H_o, on a steep slope of angle, α, has been suggested:

$$\frac{R}{H_o} = \left(\frac{\pi}{2\alpha} \right)^{\frac{1}{2}} \qquad (2.1)$$

Sainflou (1928) argued that the maximum water level at a vertical (or near) wall was higher by an amount, Δ, such that:-

$$\frac{R}{H_o} = \left(\frac{\pi}{2\alpha} \right)^{\frac{1}{2}} + \Delta \qquad (2.2a)$$

$$\text{where} \quad \Delta = \frac{\pi H_\bullet^2}{L_o} \coth \frac{2\pi d_s}{L_\bullet} \qquad (2.2b)$$

where d_s is the depth of water at the structure toe, and L_o the incident deep water wave length.

More recently, Le Méhauté, Koh and Hwang (1968) proposed a different expansion, giving:-

$$\frac{R}{H_o} = \left(\frac{\pi}{2\alpha} \right)^{\frac{1}{2}} + D \qquad (2.3a)$$

$$\text{where } D = \frac{kH}{2} \left[1 + \frac{3}{4} \sinh^{-2} (kd) - \frac{1}{4} \cosh^{-2} (kd) \right] \coth kd \qquad (2.3b)$$

where the wave number, $k = 2\pi/L$.

A similar modification has been proposed by Nagai and Takada(10) who considered second, third and fourth order modifications to equation 2.1. For situations where the sea steepness s_o was less than $(2\alpha/\pi)^{\frac{1}{2}}(\sin\alpha)/\pi$, they suggested:-

$$\frac{R}{H_o K_s} = \left(\frac{\pi}{2\alpha} \right)^{\frac{1}{2}} + D \qquad (2.4a)$$

$$\text{where } D = \frac{k\,h}{8} (3 \coth^3 k\,d + \tanh k\,d) \qquad (2.4b)$$

$$\text{and} \quad K_s = (2\,n \tanh kd)^{-\frac{1}{2}}, \qquad (2.4c)$$

$$n = \frac{1}{2} + k\,d/(\sinh 2kd) \qquad (2.4d)$$

Keller and Keller (1965) used linear wave theory for waves on a plane slope, with a horizontal sea bed, and produced a complex expression including Bessel functions of zeroth and first order. Using asymptotic approximations to these functions, their expression tends to:-

$$\frac{R}{H} = \pi \left(\frac{2}{\alpha T} \left(\frac{d}{g}\right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \qquad (2.5)$$

2.3 Shallow slopes

Some researchers have analysed wave behaviour on slopes which, taken with certain sea steepnesses, lead to wave breaking. A particularly simple approach is to assign a water particle velocity, v, to the mass of water as it arrives at the structure. If all the kinetic energy was to be converted to potential energy, the water would be expected to ascend to a height equal to $v^2/2g$. However, some fraction of the kinetic energy might be expected to be dissipated in wave breaking. The true run-up level might then be calculated by applying a reduction factor, f. No convenient way of determining values of this factor appear in the literature, the equations suggested by Freeman and Le Méhauté (1964) and by Bullock (1968) being too complex to evaluate and use for design purposes.

More sophisticated methods have been used recently by researchers including Peregrine (1978,1980), Svendsen (1978,1983), Hawkes[9] and Stive[11]. In general these approaches have considered each wave as the propagation of a bore in shallowing water. Each bore is followed from breaking, through the run-up and back-wash cycle. Its behaviour then determines the flow conditions near the shoreline for the approach of the following wave.

2.4 Probability distributions of run-up

Wind waves are of an essentially stochastic nature, as are wave heights and periods. Wave run-up is therefore also of a stochastic nature, and a distinct maximum cannot be defined. Instead a level exceeded by only a few run-up crests is defined for the purposes of design. One such level that is frequently used is the 2% exceedance level, R_2. For a fuller description of the wave run-up behaviour, or in order to determine R_2 from some other measure of run-up level, the correct probability distribution must be found.

This problem is usually approached from an empirical basis, some theoretical work has, however, been done. Saville[12] introduced a hypothesis of equivalence, which postulates that, on average, a wave in a random wave train will have a run-up level equal to that of uniform waves of the same height and period. A distribution of run-up levels may then be derived given:-

(a) a relationship between incident waves and run-up levels;
(b) a relationship between wave height and period;
(c) independence between successive waves.

Saville[12] used run-up prediction curves, together with descriptions of wave heights and periods by Bretschneider (1958,1959), to produce a frequency distribution of wave run-up. A number of other authors have used the basic assumptions of the hypothesis of equivalence to evolve other distributions of run-up crests.

Battjes (1971) used Saville's hypothesis to derive the distribution of run-up crests of ocean waves. Battjes assumed high correlation between wave height and deep water wave length, and a Rayleigh distribution for each. From these it was shown that run-up crests also follow a Rayleigh probability curve. This was confirmed by measurements on the Ijesselake Dike on a slope of 1:3.6.

Thompson[23] has used the analytical framework for the calculation of run-up distribution, produced by Battjes (1971), for both simple and composite slopes. The Hunt formula is used with bivariate Rayleigh distributions of wave height and length. [The Rayleigh distribution of wave heights is well verified in nature, but there is little evidence for a Rayleigh distribution of wave lengths. Thompson argues however, that Battjes shows that the use of these distributions leads to satisfactory results.] Thompson considers the two cases when wave heights and lengths are wholly dependent, given by the correlation parameter $\rho = 1$, and wholly independent, given by $\rho = 0$. A value of $\rho = 1$ is thought to apply to a sea in the early stages of growth and implies higher run-up levels. For simple slopes Thompson concluded that:-

$$R_2 = 0.6\ Tz\ (g\ H_s)^{\frac{1}{2}} \tan\alpha \qquad \text{for } \rho = 0 \text{ and}$$

$$R_2 = 0.75\ Tz\ (g\ H_s)^{\frac{1}{2}} \tan\alpha \qquad \text{for } \rho = 1 \qquad (2.6)$$

For fully developed seas in deep water, where $\rho = 0$, and a mean sea steepness, $s = H/L$, of around 0.05:-

$$R_2 \sim 6.7\ H_s \tan\alpha\ , \qquad (2.7)$$

which is reasonably close to the formula given by Wassing (1957) for that sea steepness:

$$R_2 = 8\ H_s \tan\alpha.$$

Other authors have considered the distributions of wave heights and periods (or wave lengths). Longuet-Higgins (1975) gave a theoretical joint distribution of heights and periods, based on assumptions of zero correlation between deep water wave heights and periods, and a normally distributed sea surface. The probability contours were symmetrical about the mean period, and the larger waves tended to occur near the mean period, which agreed with the data presented. Cavanie, Arhan and Ezraty (1976) considered more data than Longuet-Higgins, mainly from storm records in the North Sea, and found a definite correlation between the wave height in deep water, H_s, and the wave period, T. Their equation for the joint distribution of wave height and period required the second and fourth moments of the energy spectrum to be known, as well as the zeroth moment, and the mean wave period. The resulting probability contours were not symmetrical, and longer waves tended to have larger heights, in agreement with their data. Arhan and Ezraty (1978) subsequently analysed this data set, and detected a correlation between successive wave heights, particularly between the larger waves. The joint distributions of wave heights and periods have also been considered by Goda (1970), and Overick and Houmb (1977). The lack of a complete correlation between wave height and period, together with the apparent correlation between successive waves, both contradict the basic assumptions of the hypothesis of equivalence. Taken with the less-than-complete applicability of the run-up curves (or expressions) used, these have led to the replacement of this approach by empirical methods based upon the results of experimental work.

3 EXPERIMENTAL WORK

3.1 Regular waves, smooth slopes

Rather than attempt to derive theoretical expressions for wave run-up, based on an incomplete understanding of the complex hydrodynamics, many researchers have fitted empirical expressions for run-up levels to the results of model tests. In the USA, Saville (1955, 1956), Savage (1958) and Hunt (1959) conducted and/or analysed run-up measurements from regular wave model tests. Many of these results, and those from subsequent studies, have been incorporated into the prediction curves in the Shore Protection Manual[6]. Re-analysis of many of the early test results by Stoa[13] has led to the production of revised wave run-up prediction curves[14,15].

The most widely quoted of the early studies are those of Hunt[16] in which different slopes, compound slopes, and beaches, were subjected to attack by regular waves of various heights and periods. Measurements showed that permeability and roughness reduced run-up, and that a berm at the base of a sea wall also considerably lowered the run-up. An increase of either structure slope or wave period increased the run-up caused by waves of a particular height. Hunt described the relative run-up by:-

$$\frac{R}{H} = 2.3 \tan \alpha/(H/T^2)^{\frac{1}{2}}, \text{ for } \tan \alpha < (H/T^2) \qquad (3.1)$$

This is, however, dimensionally inconsistent, and has since been re-written:-

$$R/H = \tan \alpha/(H/L_o)^{\frac{1}{2}}, \text{ for } \tan \alpha/(H/L_o)^{\frac{1}{2}} < 2.26 \qquad (3.2)$$

where $L_o = g\,T^2/2\pi$, R is the limit of run-up, H and T the height and period of the (regular) wave train considered, and α is the angle of the structure slope to the horizontal. The term $\tan \alpha/(H/L_o)^{\frac{1}{2}}$ is variously known as the surf similarity parameter or the Iribarren number, Ir. Battjes(17) suggests that it should be known as the Iribarren number "in honour of the man who introduced it" Iribarren (1949) "and who has made many other valuable contributions to our knowledge of water waves".

It has been shown that the Hunt formula is remarkably accurate for many natural beaches, which are relatively smooth and for which Ir < 2.5. On steeper slopes, however, it has been noted by Günbak(19) Losada and Gimenez-Curto(4) and by Sawaragi et al(20), that the Hunt formula is only valid for values of the Iribarren number less than about 2.5. Three expressions have been proposed by Losada(4) to cover run-up on smooth slopes over the full range of Ir:-

$$\begin{aligned} 0 < Ir < 2.5, &\quad R/H = Ir \\ 2.5 < Ir < 4.0, &\quad R/H = 2.5-(Ir-2.5)/3.0 \\ 4.0 < Ir, &\quad R/H = 2.0 \end{aligned} \qquad (3.3)$$

Similar trends have been shown by Günbak and Sawaragi, who also identify the range over which resonance on the slope will occur as $2.0 < Ir < 3.0$.

Various other relationships have been proposed to describe run-up behaviour on smooth slopes. Tautenhaim(34) proposes an equation including the coefficient of reflection, Kr:-

$$R = 1.29\,(H\,L_o)^{\frac{1}{2}}\left(\frac{1-Kr}{\cos\alpha}\right) \qquad (3.4)$$

Chue(21) combines and adapts the expressions for run-up on steep and shallow slopes to produce a single formula of wider applicability:-

$$R/H = 1.8\ (1-3.111\ \frac{H_o}{L_o})\ \xi_o\ (1-\exp[-(\frac{\pi}{2\alpha})^{\frac{1}{2}}\ \frac{1}{\xi_o}])$$

$$\text{where } \xi_o = \tan\ \alpha/(H/L_o)^{0.4} \qquad (3.5)$$

The use of the exponent 0.4 is justified as "found to fit remarkably well" but no supporting reference or data are given. The main formula is however tested against regular wave results of Savage, Saville, Hunt, Whalin (1971) and Hudson (1958). Chue's expression is discussed by Ahrens and Titus(22). The trend of relative run-up decreasing with increasing wave steepness, proposed by Chue, is contrasted with data from Savage (1958) and with the work of Le Méhauté, Koh and Hwang (1968). Ahrens and Titus then use data from regular wave tests by Saville (1956) and Savage to develop a general equation for run-up on smooth slopes, under non-breaking waves. This equation may be written:-

$$R/H_o = C_o\ (\pi/2\alpha)^{C_1}\ \exp\ [C_2\ ((\eta/H)\ -0.5)^2]$$
$$\text{for Ir} < 3.5, \qquad (3.6)$$

where H_o is the unrefracted deep water wave height, η is the elevation of the wave crest above still water level, and C_0, C_1 and C_2 are dimensionless coefficients. Values for these coefficients are calculated by fitting the data considered, by regression analysis. Ahrens and Titus suggest $C_0 = 1.093$, $C_1 = 0.449$, and $C = 6.354$. They suggest that the crest elevation, η, may be determined using stream function wave theory.

3.2 Regular waves, rubble slopes

The behaviour of wave run-up on armoured rubble slopes has also been studied with regular waves. Many researchers including Hudson (1958) and Savage (1958) have reported such tests, and a number of different empirical formulae have been derived to fit the results. This work has led to the definition of a rough slope reduction factor, r, to be applied to values of run-up levels calculated for a smooth slope to obtain predicted levels on the corresponding rubble slope.

Stevenson(18) has reported tests with regular waves on shingle beaches of slopes of 1:6, 1:8 and 1:10.45. The results are compared with the Hunt formula for smooth slopes. Over the range of Iribarren numbers considered, the run-up on the shingle beaches was reduced to about 35% of that predicted on the equivalent smooth slope, thus giving R/H = 0.35 Ir for shingle beaches. Günbak[19] presents a comprehensive review of wave behaviour on smooth and armoured slopes. Flow characteristics are related to the surf similiarity parameter. Results of run-up measurements on both smooth and armoured slopes are plotted as relative run-up, R/H, against the Iribarren number. On the armoured slopes considered, run-up is compared with an expression ascribed to CERC:-

$$R/H = 0.8\ Ir/(1 + 0.5\ Ir) \tag{3.7}$$

Measured relative run-up on rock armour was generally slightly greater than predicted by equation 3.7. Günbak also proposes a two-part expression for relative run-up on rock armoured slopes:-

$$\begin{aligned} R/H &= 0.4\ Ir \quad \text{for} \quad 0 < Ir < 3.0, \text{ and} \\ R/H &= 1.2 \quad \text{for} \quad 3.0 < Ir. \end{aligned} \tag{3.8}$$

Losada and Gimenez-Curto[4] also consider flow conditions on armoured slopes under regular wave attack. They examine many author's measurements of run-up on slopes armoured with various types of armour units. They present graphs of relative run-up against Iribarren number for each armour unit. To these results have been fitted a generalised expression for run-up on armoured slopes:-

$$R/H = A(1-\exp\ (B\ Ir)) \tag{3.9}$$

Values of the coefficients A and B are presented for various armour units. Losada and Gimenez-Curto note that run-up on smooth slopes does not follow this general trend, and it is concluded that it is not therefore correct simply to apply a reduction factor depending only on the type of armour unit. [It should be noted that the run-up measurements used by these authors in their analysis were made by a variety of different researchers and are quoted at second or third hand. Caution is advised in using the absolute values derived from such an analysis, as the results of different researcher's work may not be directly comparable.]

Burcharth[24] measured run-up on a 1:5.4 smooth, impermeable slope, and on a 1:1.5 dolos armoured slope. Three different wave patterns were used. Pattern 1 consisted of regular sinewaves. Pattern 2 consisted of alternate large and small waves. Pattern 3 was produced by two closely grouped regular waves, giving a beating pattern. The tests on the smooth slope exhibited marked differences in the run-up behaviour of the different wave patterns. For patterns 1 and 3, the relative run-up was given by

$R/H = 1.1$ Ir, but for pattern 2 higher run-up levels resulted giving $R/H = 1.65$ Ir, all for Ir < 2.5. On the 1:1.5 dolos armoured slope, pattern 2 again gave greater relative run-up levels. The general trend of relative run-up on the dolos slope is similar to that measured earlier by Whillock and Thompson (1970) whose results are used by Günbak(19) and Losada(4).

3.3 Irregular Waves

Run-up on smooth slopes has also been measured under irregular or random wave attack. For shallow structure slopes, expressions similar to Hunt's formula have been developed by various authors. A modified Iribarren number, Ir' may also be defined:-

$$Ir' = \tan \alpha / (2\pi H_s / g T_p^2)^{\frac{1}{2}} \qquad (3.10)$$

where H_s is the significant wave height, and T_p is the period of peak spectral energy. Van Oorschot and d'Angremond(25) present the results of tests with irregular waves on smooth slopes of 1:4 and 1:6. Using fixed values of significant wave height, H_s, and peak period, T_p, they tested the effect of varying the width of the spectrum, and hence its shape, by using spectra of different spectral widths, ε, where:-

$$\varepsilon^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4} \qquad (3.11)$$

(m_0, m_2 and m_4 being the zeroth, second and fourth moments of the spectrum, cut off at an energy density of 5% of that at the peak.) It should be noted that, whilst T_p was held constant, the changes in spectral shape inherent in altering the spectral width will also have altered the mean wave period, T_z. From the results of these tests the authors concluded that wider spectra gave rise to higher extreme run-up levels, and conversely narrow spectra gave lower run-up levels. They suggested a modified version of the Hunt formula for the 2% run-up level R_2:-

$$R_2 = C_2 \tan \alpha \, (H_s \, g \, T_p^2)^{\frac{1}{2}} \qquad (3.12)$$

This may be written as $R_2/H_s = (2\pi)^{\frac{1}{2}} C_2 \, Ir'$. The coefficient C_2 is determined by the spectral width, ε. A single graph of C_2 against ε is presented in the TACPI report (7). Günbak has suggested that the following values may be used(19):-

Spectral Width, ε	Coefficient, C_2
0.3	0.55
0.4	0.61
0.5	0.67
0.6	0.73

From the results of the regular wave tests considered earlier, equation 3.12 would only be expected to be valid for values of Ir' less than about 2.5. It should be noted, however, that the slopes tested by van Oorschot and d'Angremond were 1:4 and 1:6, both significantly shallower than might be expected in sea wall construction around the UK. For the steeper slopes, and typical storm conditions with mean sea steepness, $s = H/L_o$, of 0.03 - 0.05, the range for Ir' might be between 1.8 and 4.6. Much of that range is outside of the range of validity of the Hunt formula. Furthermore, whilst each wave spectrum used in these tests had the same peak period, (T_p), as the spectral width, and hence spectral shape, was altered, so it is likely that the mean period, (T_z), also varied.

Grüne[26] presents results of field measurements on the German north sea coast on simple slopes of 1:4 and composite slopes of 1:4 and 1:6. Waves and run-up levels were measured. An equation of the form of 3.12 was fitted to the results. For the 1:4 slope, mean values of C_2 of 0.71 to 0.92 were calculated, somewhat higher than the values usually quoted in the range 0.6-0.8.

Webber and Bullock[27,28] describe the mechanisms of run-up on, mainly, shallow slopes. Different prediction methods are reviewed and the limitations of each technique are discussed. They report the results of small scale tests with 1:2, 1:4 and 1:10 smooth slopes, with wind generated random waves. Dimensionless run-up levels, R', are calculated using the mean run-up level, $\bar{R}$, and the standard deviation of run-up levels, σ_R:-

$$R' = (R - \bar{R})/\sigma_R \qquad 3.13$$

R' was best described by a Gaussian distribution (with $\mu = 0$, $\sigma = 1.0$) with good agreement for all three slopes.

Hashimoto[29] used a steeper slope (1:1) with paddle generated random waves. Graphs of wave heights and run-up levels, and of wave and run-up spectra are presented. Measured values of $R_s/\bar{R}$ varied from 1.3 to 1.8 for $\bar{R}/\bar{H}_o$ varying from 0.95 to 2.74. [For a Rayleigh distribution $R_s/\bar{R} = 1.60$.] Relatively steep structure slopes were also considered by Kamphuis and Mohamed[30]. They review the earlier work of Pocklington (1921) and Hunt (1959) and report results of tests on slopes of 1:1, 1:1.5, 1:2 and 1:3, using limited component irregular waves. From these tests they conclude that both wave heights and run-up levels are approximately Rayleigh distributed, but that their run-up results give $R_2/\bar{R} = 2.4$ rather than 2.23 which would be predicted for a Rayleigh distribution. For non-breaking irregular waves they conclude that the expression ascribed to Pocklington and Miche (equation 2.1) may also be valid for irregular waves when written:-

$$\bar{R}/\bar{H} = \left(\frac{\pi}{2\alpha}\right)^{\frac{1}{2}} \quad (3.14)$$

Kamphuis and Mohamed did not detect any significant effect of spectral width on run-up levels.

Ahrens [31] also presents results of wave run-up on a 1:1.5 smooth slope under irregular waves. He concludes, however, that his results agree generally with those of van Oorschot and d'Angremond, noting that a wider range of incident spectra were used than by Kamphuis and Mohamed. A good fit to the results is given by the Gamma cumulative distribution function. This distribution describes the higher run-up levels well. The Rayleigh distribution, however, tends to under-predict the more extreme run-up levels. In a further paper, Ahrens [32] considers run-up caused by both regular and irregular waves. The results of tests with non-breaking regular waves on slopes of 1:1.5 and 1:4 are fitted to a version of equation 3.6, with different values of C_0, C_1 and C_2:-

$$R/H = 1.18 \left(\frac{\pi}{2\alpha}\right)^{0.38} \exp\left[3.19\,(\eta_c/H - 0.5)^2\right] \quad (3.15)$$

This equation is claimed to fit the measurements on the shallower slope (1:4.0) well, but not those on the steeper slope (1:1.5). Use of equation 3.15 requires the computation of the wave crest elevation, η_c. This calculation may be performed using stream function wave theory, but may be complex and slow. For non-breaking irregular waves, the relative run-up generally follows the trends given by equation 3.14 and thus supports a conclusion drawn by Kamphuis and Mohamed[30]. Ahrens also examines the statistical distribution of run-up levels. A two-parameter Weibull distribution is found to fit the data very well. The general distribution is given by:-

$$P(R > Ri) = \exp\left[-\frac{1}{B}\left(\frac{Ri}{H_s}\right)^A\right] \quad (3.16)$$

Allsop[5] measured random wave run-up on both smooth and armoured slopes at 1:1.33, 1.5 and 2.0. The armoured slopes used SHED single layer armour units laid on 4 D_{50} of rock underlayer. Run-up distributions were measured from video recordings. Probability distributions of Rayleigh form were fitted to the results. The run-up on the SHED armoured slopes fitted this distribution form very closely with minimal divergence even at high run-up levels. The run-up on smooth slopes, however, exhibited a similar trend to that identified by Ahrens in his earlier paper[31], in that the Rayleigh distribution tended to under-predict the extreme run-up levels.

3.4 Effect of angled wave attack

The Shore Protection Manual[6] does not consider the effect of wave attack at any angle of incidence β other than 0°, that is with the wave crests parallel to the structure. The implicit assumption is that normal wave attack represents the most serious case. A similar general conclusion is drawn in the TACPI report[7], but is extended to give a reduction factor of cos β for plain slopes. Two references were considered by the TACPI report, one postulates the reduction factor of cos β, the other, ascribed to Hosoi and Shuto[33], presents experimental results for β = 0°, 30°, 45° and 60° on a 1:2 slope and indicates a lower reduction for most wave steepnesses for β < 45°, than would be given by multiplying normal run-up by cos β. Hosoi and Shuto's[33] results lie in the main between two limits:-

$$\frac{1 + \cos \beta}{2} > k_\beta > \frac{1}{1 + \cos^2 \alpha \tan^2 \beta} \tag{3.17}$$

The lower limit is ascribed to a Russian researcher, Shidorova. It should be noted that no incident angles within the range 0-30° were considered in any of the above studies.

More recent work has however shown that normal wave attack may not give the greatest run-up (or overtopping). Test results, apparently originating from CSIR tests on dolosse, and quoted by Günbak[19] but not referenced, illustrate that for waves of steepness s = H/L_o of 0.03 - 0.04, wave run-up is greater for $\beta \sim 30°$ than for β = 0° or 45°. This was not commented upon, in fact Günbak concludes that run-up may be reduced by the cos β factor. Work by Owen[1] and Tautenhaim et al[34] has however shown that run-up, and overtopping may increase over an incident angle range of around β = 10°-30°. Tautenhaim et al report the results of model tests on a 1:6 slope under regular wave attack at incident angles β between 0° and 60°. It is argued, on the basis of the test results, that the effect of oblique incidence is simply to modify the run-up at normal incidence R_n, by a factor k_β where:-

$$k_\beta = \cos \beta \, (2 - \cos^3 2\beta)^{1/3} \tag{3.18}$$

A similar effect was noticed by Owen in tests measuring the overtopping of sea walls(1). This comprehensive set of tests considered sea walls of slopes of 1:1, 1:2 and 1:4. Tests with waves at incident angles of 0°, 15°, 30°, 45° and 60° were run against walls of 1:1 and 1:4 slope. It was shown that generally the mean overtopping discharge at 15°, and sometimes 30°, exceeded that at $\beta = 0°$.

It seems likely that, whilst the expression given by Tautenhaim was only derived for walls of 1:6 slope, a similar effect may be seen for sea walls of steeper slope. It would appear from these two studies that the angle giving greatest run-up (or overtopping) will be around $\beta = 15\text{-}20°$. This angle, and the level of increase itself, may depend upon the mean sea steepness and the structure slope.

4 PREDICTION OF WAVE RUN-UP

4.1 Methods available

Wave run-up levels on a sloping structure under given conditions may be estimated by various procedures. The designer may use any of three different methods (theoretical, empirical or experimental) to predict the run-up behaviour of waves on a seawall or breakwater. The research project, of which this review is a part, is concerned principally with the theoretical or empirical methods available to a designer. The principles and procedure of modelling seawalls, or other coastal structures, are not discussed here. However, it should be noted that, in some circumstances, site specific model tests of the hydraulic performance of a structure may be, not only more certain, but also quicker and more economic, than a complicated analysis of wave behaviour.

In all instances, however, the prediction of wave run-up levels requires that the characteristics of both the incident waves and the structure must be known. For estimates of the run-up level under regular waves, the data needed for the simplest prediction methods may reduce to values of the wave height, H; the offshore wave length, L_o ; and the structure slope, α. However, for a full description of the distribution of run-up crests on a seawall under random wave attack, the data needed will be considerably more complex. The rest of this chapter will summarise both the analytical methods available, and the input data required by each method. Where appropriate, the effect of the different prediction formulae has been illustrated by calculating the relative run-up, R/H, for a range of wave steepnesses ($0.01 < H/L_o < 0.07$) and structure slopes ($1.33 < \cot \alpha < 3.0$). The results are presented in a number of simple tables.

4.2 Regular waves, smooth slopes

The simplest prediction expressions available for waves on smooth slopes are those ascribed to Hunt (equation 3.2) and to Miche (equation 2.1). These two equations may be regarded as covering the range of wave conditions on a slope, from surging to breaking waves (but without a transition). It has been noted that the Hunt formula, $R/H = Ir$, is valid for $Ir < 2.26$. It may be argued that equation 2.1, being valid for steep slopes, will cover the rest of the range of Ir. For the range of wave steepnesses, s, and structure slopes, cot α, considered, values of Ir have been calculated, and are shown in Table 1. It may be seen that, for the steep structure slopes considered in this study, much of the range of conditions used falls outside the range of the Hunt formula. For these combinations of wave steepness and structure slope for which $Ir > 2.3$, the relative run-up may then be calculated by equation 2.1. Within the range of validity of the Hunt formula, the relative run-up might then be given simply by equation 3.2. The results of this approach are summarised in Table 2. Over the range covered by equation 2.1, the values of R/H are significantly less than would be predicted by equation 3.2 (used outside its range of validity).

As discussed in section 3.1, Losada and Gimenez-Curto(4) have argued that regular wave run-up on smooth slopes may be more accurately estimated by the three part expression given by equation 3.3. Calculations of relative run-up over the same range of values as used above, with equation 3.3, give values of relative run-up summarised in Table 3. It may be seen that equation 3.3 gives generally larger values of R/H than does equation 2.1 used for part of Table 2. Taken with the comparisons given in their paper(4), Losada and Gimenez-Curto's expression appears to represent relative run-up better than a simple combination of Hunt's and Miche's formulae, especially over the transition zone between surging and breaking waves.

Chue(21) has proposed a single expression covering the range of wave behaviour on a smooth slope. Values of relative run-up have been calculated using equation 3.5, and are summarised in Table 4. It should be noted that Ahrens and Titus(22) have expressed severe reservations about the use of this equation, and it may be seen that the effect of wave steepness, s, on R/H is opposite to that shown in Table 3 for the steeper structure slopes.

The Shore Protection Manual (SPM) presents run-up prediction curves, derived from the results of regular wave tests. For waves in deep water, relative run-up is given by Figure 7-12 in the SPM (4th edition, 1984). Values of relative run-up have been read off these curves for the conditions used earlier, and the results are summarised in Table 5. Over much of the range considered, these values are reasonably similar to those calculated from equation 3.3 (Table 3). For steep waves on steep structure slopes, the SPM curves give relative run-up values less than those predicted by Losada, and greater than predicted by Chue.

The SPM also presents similar prediction curves for shallow water, curves for relative depths at the structure, d_s/H_o, of 0.0, 0.45, 0.8 and 2.0 are given. Table 6 presents values of relative run-up, R/H_o, for waves in water of $d_s/H_o \sim 0.45$ (SPM Figure 7.9). The significant influence of water depth may be seen by comparing Tables 5 and 6. Some of the difference may be explained, however, by noting that the wave height used in these definitions of relative run-up is the deep water wave height, H_o. Both wave height and wavelength will have been transformed during the approach into shallow water.

Of these expressions for run-up on smooth slopes, only Losada's expression, equation 3.3, gives results (see Table 3) that are in agreement with the trend identified by Ahrens(31) that, for cot α = 1.50, relative run-up increases with increasing wave steepness.

It should be also noted that the Shore Protection Manual suggests that all values of run-up on smooth slopes, predicted from its curves, should be increased by a "scale correction factor", ranging from about 1.14 to 1.21 for the range of structure slopes considered here.

4.3 Regular waves, rubble slopes

The literature describes two different approaches to the calculation of regular wave run-up on rough permeable slopes. A prediction method favoured by Stoa(15), the TACPI report(7) and the Shore Protection Manual(6), involves the use of a "roughness factor", r, to reduce the run-up predicted for smooth slopes. Suggested values of r are given by each of those references.

The alternative method uses prediction curves or formulae for the particular type of armour unit, or slope construction considered. The SPM gives prediction curves for rough, impermeable, slopes (SPM Figures 7-15, 7-16) and for rough, permeable slopes (7-19,20) based upon tests by Saville, Hudson and Jackson. In the latest edition of the SPM (4th edition, 1984), a further set of curves is given of relative run-up against Iribarren number for riprap, rock and dolos armoured breakwaters (Figure 7-44).

British Standard BS 6349(35) also presents two sets of run-up prediction curves for riprap embankments and for rubble mounds. Neither figure is attributed, nor is any reference given.

Prediction formulae have been suggested by Losada and Gimenez-Curto(4) (equation 3.9) and by Günbak (equations 3.7 and 3.8). These expressions, together with the curves in BS 6349 Figure 15 and SPM Figure 7-44, suggest a different relationship between relative run-up and wave conditions on rough, permeable, slopes, than would be given by applying a reduction factor to any of the run-up prediction formulae considered in section 4.2 of this report.

To illustrate the effect, described clearly in Figure 8 of the paper by Losada & Gimenez-Curto(4), values of relative run-up have been calculated using equation 3.9, with values of the coefficients A and B derived from tests on riprap armoured slopes by Ahrens & McCartney (1975). These values of R/H are summarised in Table 7. In contrast, the values of relative run-up in Table 8 have been calculated using a roughness factor for quarrystone, r = 0.65, with the values of smooth slope run-up given in Table 5. Whilst agreement is reasonable for the shallower structure slopes, at cot α = 1.5 or 1.33 there is considerable discrepancy between relative run-up calculated by these different methods. That used for Table 8 gives markedly lower values than are predicted by Losada's regression formula, equation 3.9. The prediction curve in BS6349, Figure 15, gives relative run-up values that appear to be consistent with those predicted by Losada's formula, but this could be due to the use of a similar data set.

4.4 Irregular waves, smooth slopes

Two different approaches may be seen in the methods used to predict run-up under irregular, or random, wave attack. One method assumes the theory of equivalence, viewing irregular wave run-up simply as the result of many, independent, regular waves. A simplified version of this approach is that advanced in the Shore Protection Manual, based upon work by Ahrens. In this method, a typical run-up level say the significant run-up, R_s, or the mean run-up level, $\bar{R}$ is determined by one of the methods for regular waves discussed in section 4.2. Other run-up levels such as the 2% exceedance level, R_2 are then determined from R_s or $\bar{R}$ by assuming a Rayleigh distribution of run-up levels. Similar approaches involving the assumption of probability distributions for both wave heights and periods, have been suggested by Saville[12], Battjes (1971) and Thompson[23].

An alternative approach is based upon the measurement and empirical description of the probability distribution of wave run-up under random wave conditions. The parametisation of these measurements then allows the full description of irregular wave run-up under random wave attack. As discussed earlier in this review, a number of authors have attempted to fit standard probability distributions to measured random wave run-up results. Rayleigh, Gamma, Weibull and log Normal distributions have all been tried, with varying degrees of success. Kamphuis and Mohamed[30], Ahrens[31] and Allsop[5] have all fitted Rayleigh distributions to random wave run-up measurements on smooth slopes, and have all concluded that the Rayleigh distribution tends to under-predict the extreme run-up levels. Ahrens has also tried the Gamma distribution[31], but has since recommended the use of the two-parameter Weibull distribution[32]. Ahrens[32] has also noted that the non-linear effects that have a strong influence on regular wave run-up identified by Le Méhauté et al (1968), Nagai & Takada[10], and Ahrens & Titus[22] do not have a strong influence on the run-up of irregular waves.

Another method considers only the run-up parameters, R_s, R_2 and $\bar{R}$, ignoring the form of the run-up distribution. Ahrens has presented a run-up prediction method (37) that supersedes some of the earlier work by Stoa[14,15], Ahrens[36] and the SPM[6]. This new method is based upon the results of random wave tests on smooth slopes. From these, Ahrens suggests the use of a prediction formula:-

$$R_x/H_s = C_1 + C_2\ (H_s/g\ T_p^{\,2}) + C_3\ (H_s/g\ T_p^{\,2})^2 \qquad (4.1)$$

where R_x represents R_s, R_2 or $\bar{R}$, and C_1, C_2 and C_3 are dimensionless coefficients determined by a regression analysis of the data considered. Tables of values of C_1, C_2 and C_3 are given for each of R_s, R_2 and $\bar{R}$, for a range of slopes, cot α.

4.5 Irregular waves, rubble slopes

Very little data on random wave run-up on rubble slopes is available. The prediction methods are generally, therefore, based upon the results of regular wave tests. The Shore Protection Manual does not specifically cover irregular wave run-up on rubble slopes. Instead it presents methods to predict run-up on rough slopes from that predicted for smooth slopes,and to predict irregular wave run-up from regular wave run-up, but only on smooth slopes. Presumably the methods could be combined! Ahrens[37], however suggests that irregular wave run-up may be predicted on rough slopes, by applying the roughness coefficients presented in the SPM (after the TACPI report[7], and Stoa[15]) to smooth slope run-up calculated by equation 4.1.

Allsop[5,38] has measured random wave run-up on rubble slopes armoured with hollow cube concrete armour units. The results gave very good agreement with the Rayleigh distribution. Values of R_s and R_2 on the armoured slopes were compared with those predicted by Ahrens' method[37], and gave a relative roughness factor of around 0.54. Comparing values of R_s and R_2 measured on smooth slopes in the same study, with those measured on the equivalent armoured slopes, however, suggested a roughness factor of 0.62. This implies some discrepancy between Allsop's measured run-up levels on smooth slopes, and those predicted by using Ahrens' method.

4.6 Angled wave attack

Both Owen[1] and Tautenheim et al[34] have shown that wave run-up, and overtopping discharge, may be greatest at incident angles around 10-20°. Similar conclusions may be drawn from test results quoted by Günbak[11]. It is therefore surprising that, in a recent review of wave run-up and overtopping, Gadd, Potter, Safie and Resio[39] should state "Wave run-up will be diminished as the wave incidence angle to the slope increases." Gadd et al seem to be unaware of Owen's work on overtopping[1,2,3] and that of Tautenhaim et al[34] on run-up! They therefore suggest the use of a factor k_β, suggested by Hosoi & Shuto[33], applied to the run-up for normal wave attack, where β is the incident wave angle:-

$$k_\beta = (1 + \cos \beta)/2 \qquad (4.2)$$

Tautenhaim et al, however, suggest a factor k_β calculated by:-

$$k_\beta = \cos \beta \, (2 - \cos^3 2\beta)^{1/3} \qquad (4.3)$$

The effects of these different expressions may be illustrated by determining values of k_β for angles between 0 and 30° using equations 4.2 and 4.3:

Angle, β	0°	5°	10°	15°	20°	30°
k_β (Tautenhaim)	1.0	1.01	1.04	1.07	1.09	1.07
k_β (Hosoi & Shuto)	1.0	1.00	0.99	0.98	0.97	0.93

Until further research information is available on the influence of angled wave attack on run-up, it would appear to be prudent to allow for enhancement over β = 10 - 30° of the order of that suggested by Tautenhaim et al.

5 CONCLUSIONS AND RECOMMENDATIONS

This literature review has identified a number of methods for predicting wave run-up on simple, smooth, slopes. Of these, the methods given by the Shore Protection Manual(6) (typified by SPM Figure 7.12), and by Losada & Gimenez-Curto (equation 3.3), appear to give reliable results. In the main, however, the prediction of random wave run-up levels is based upon methods with relatively little validation. It is recommended, therefore, that a series of tests be conducted to measure smooth slope run-up levels under random wave attack, and to relate those results to the prediction methods available, such as that proposed by Ahrens(36).

It is clear from the literature that wave run-up on rubble slopes exhibits a different form of response to wave characteristics, than does run-up on smooth slopes. The use of a simple roughness factor applied to smooth slope run-up will, therefore, over-predict the run-up on rubble slopes under some circumstances, and under-predict under others. There is a need for significantly more data to allow the derivation of more reliable empirical expressions to predict wave run-up levels on rubble slopes. It is recommended that model tests on a number of different armoured rubble slopes should be performed to measure random wave run-up levels under a number of incident wave conditions.

A number of attempts have been made to describe the probability distribution of wave run-up crests in terms of the wave and structure parameters. The literature does not, however, clearly identify any particular probability distribution as describing the data well. A knowledge of such probability distributions is needed to relate run-up levels of different probabilities to typical run-up levels, such as $\bar{R}$ or R_s, calculated from the wave and structure parameters. It is recommended that any further model tests should try to fit a number of different probability distributions to the run-up levels measured.

The effect on run-up levels of oblique wave attack is particularly little understood. Recent work by two authors has indicated that wave run-up, or overtopping, may be greater for angles of incidence, β, around 10-30°, than under normal wave attack, $\beta = 0°$. This implies that the common assumption of normal wave attack may give rise to underestimates of the likely run-up levels, or overtopping discharges. Only one set of test results, on a structure slope of 1:4, is available to help the designer to estimate wave run-up under oblique wave attack. It is therefore, recommended that a series of model tests be run to measure the effects of oblique wave attack at angles β around 10-30° on run-up levels, or overtopping discharges.

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7 REFERENCES

1. Owen M W "Design of seawalls allowing for wave overtopping" Hydraulics Research, Report EX 924, Wallingford, June 1980.

2. Owen M W "Overtopping of sea defences" Proc. conf Hydraulic modelling of civil engineering structures. BHRA, Coventry, 1982.

3. Owen M W "The hydraulic design of sea-wall profiles" Proc. conf. Shoreline protection, ICE, Southampton, 1982.

4. Losada M A & Gimenez-Curto LA "Flow characteristics on rough permeable slopes under wave action" Coastal Engineering 4, Elsevier, 1981.

5. Allsop N W H "The SHED breakwater armour unit, model tests in random waves" Hydraulics Research, Report EX 1124, Wallingford, April 1983.

6. Coastal Engineering Research Centre "Shore Protection Manual" US Government Printing Office, 2 vols, 4th edition, Washington 1984.

7. Technical Advisory Committee on Protection Against Inundation "Wave run-up and overtopping" Government Publishing Office, The Hague, 1974.

8. Horikawa K "Coastal engineering, an introduction to ocean engineering "University of Tokyo Press, 1978.

9. Hawkes P J "Experimental study and computer simulation of waves on beaches" PhD Thesis, University of Liverpool, 1982.

10. Nagai S and Takada A "Relations between the run-up and overtopping of waves" Proc. 13th Coastal Eng. Conf., Vancouver, 1972.

11. Stive M J F "Energy dissipation in waves breaking on gentle slopes" Coastal Engineering 8, Elsevier, 1984

12. Saville T "An approximation of the wave run-up frequency distribution Proc, 8th Coastal Eng. Conf. Mexico, 1962.

13. Stoa P N "Reanalysis of wave run-up on structures and beaches" CERC Technical Paper No 78-2, Fort Belvoir, March 1978.

14. Stoa P N "Revised wave run-up curves for smooth slopes" CERC, CETA 78-2, Fort Belvoir, July 1978.

15. Stoa P N "Wave run-up on rough slopes" CERC CETA 79-1, Fort Belvoir, July 1979.

16. Hunt I A "Design of seawalls and breakwaters" US Lake Survey, US Army, January 1958.

17. Battjes J A "Surf similarity" Proc. 14th Coastal Eng. Conf., Copenhagen, 1974.

18. Stevenson T "Wave run-up on shingle beaches" Hydraulics Research, report IT 245, Wallingford, March 1983.

19. Günbak A R "Rubble mound breakwaters" Norwegian Institute of Technology, report no 1, 1979.

20. Sawaragi T, Iwata K, & Kobayashi M "Condition and probability of occurrence of resonance on steep slopes of coastal structures" Coastal Eng in Japan, Vol 25, 1982.

21. Chue S H "Wave run-up formula of universal applicability" Proc. ICE Pt 2, Vol 69, TN 274, December 1980.

22. Ahrens J P & Titus M F "Wave run-up formulae for smooth slopes" Proc ASCE, Jo W'way Port Coastal Div. Vol III No 1, January 1985.

23. Thompson D M "Hydraulic design of sea dikes" Hydraulics Research, HRS Report DE6, January 1973.

24. Burcharth H F "The effect of wave grouping on on-shore structures" Coastal Engineering 2, Elsevier, 1979.

25. van Oorschot J H & d'Angremond K "The effect of wave energy spectra on wave run-up" Proc. 11th Coastal Eng. Conf. London, 1968.

26. Grüne J "Wave run-up caused by natural storm surge waves" Proc. 18th Coastal Eng. Conf. Cape Town, 1982

27. Webber N B & Bullock G N "A model study of the distribution of run-up of wind-generated waves on sloping sea walls" Proc. 11th Coastal Eng. Conf. London 1968.

28. Webber N B & Bullock G N "Run-up of waves on sloping faces, a review of the present state of knowledge". Proc. Conf. Soc. Earthquake & Civil Eng. Dynamics, Swansea, 1970.

29. Hashimoto H "An experimental study of irregular wave run-up on a coastal dike" Coastal Engineering in Japan, Vol 17, 1974.

30. Kamphuis J W & Mohamed N "Run-up of irregular waves on plane smooth slope" Proc. ASCE Vol 104, WW2, May 1978.

31. Ahrens J P "Irregular wave run-up" Proc. Coastal Structures 79, ASCE, Alexandria, 1979.

32. Ahrens J P "Wave run-up on idealized structures" Proc. Coastal Structures 83, ASCE, Arlington, 1983.

33. Hosoi M and Shuto N "Run-up height on a single slope dike due to waves coming obliquely" Coastal Engineering in Japan, Vol 7, 1964.

34. Tautenhaim E, Kohlase S, and Partenscky H W "Wave run-up at sea dikes under oblique wave approach" Proc. 18th Coastal Eng. Conf. Cape Town, 1982.

35. British Standards Institution "British standard code of practice for maritime structures; part 1, general criteria" BS 6349: Part 1 : 1984.

36. Ahrens J P "Prediction of irregular wave run-up" CERC, CETA 77-4, Fort Belvoir, July 1977.

37. Ahrens J P "Irregular wave run-up on smooth slopes" CERC, CETA 81-17, Fort Belvoir, December 1981.

38 Wilkinson A R and Allsop N W H "Hollow block breakwater armour units" Proc. Coastal Structures 83, ASCE, Arlington 1983.

8 BIBLIOGRAPHY

Ahrens J P & McCartney B L "Wave period effect on the stability of riprap" Proc. Civil Engineering in the Oceans, III ASCE. 1975.

Arhan M and Ezraty R "Statistical relations between successive wave heights". Oceanologica Acta, Vol 1, No 2, 1978.

Battjes J A "Run-up distributions of waves breaking on slopes" Jo. W'way, Harbour & Coastal Eng. Div. Proc. ASCE No WW1, February 1971.

Bretschneider C L "Revisions in wave forecasting, deep and shallow water" Proc. 6th Coastal Eng. Conf., Council on Wave Research, 1958.

Bretschneider C L "Wave variability and wave spectra for wind-generated gravity waves". Technical Memo. 118, Beach Erosion Board, 1959.

Bullock G N "A study of the run-up of irregular waves on uniform slopes". Ph.D. Thesis, Southampton University, 1968.

Cavanié A, Arhan M & Ezraty R "A statistical relationship between individual heights and periods of storm waves" Proc. BOSS 76, Vol II, Trondheim, 1976.

Erchinger H F "Wave run-up in field measurements with new developed instrument". Coastal Engineering Conference, 1976.

Freeman J C and Le Méhauté B "Wave breakers and surges on a dry bed" Jo. Hyd. Div. Proc ASCE No HY2, March 1968.

Goda Y "Numerical experiments on wave statistics with spectral simulation "Port and Harbour Research Institute, Vol 9 1970.

Hudson R Y "Design of quarrystone cover layers of rubble-mound breakwater - hydraulic laboratory investigation" WES Vicksburg, Research report 2-2, 1958.

Hunt I A "Design of seawalls and breakwaters" Proc. ASCE Vol 85, WW3, September 1959.

Iribarren R and Nogales C "Protection des ports" 17th International Navigation Congress, Lisbon, 1949.

Isaacson E "Water waves on a sloping bottom" Comm. Pure and Applied Math. Vol 3, 195●.

Keller J and Keller H B "Water wave run-up on a beach" Office of Naval Research, Washington, 1965.

Kirkgoz M S "Breaking waves: their action on slopes and impact on vertical seawalls". Ph.D. Thesis, University of Liverpool, 1978.

Le Méhauté B, Koh RCY and Hwang L "A synthesis of wave run-up" Jo. W'Ways & Harbour Div. Proc ASCE, Vol 94, No WW1, February 1968.

Longuet-Higgins M S "On the joint distribution of the periods and amplitudes of sea waves" J. Geophys. Res. Vol 80, 1975.

Miche R "Mouvements ondulatoires de la mer en profondeur constant our décroissante" Annales des Ponts et Chaussées, 114e Annee, 1944.

Miller R L "Experimental determination of run-up of undular and fully developed bores". Journal of Geophysical Research, Vol. 73, No 14, July 1968, pp 4497-4510.

Overick T & Houmb O G "A note on the distribution of wave steepness" Norwegian Institute of Technology Report 11, 1977.

Packwood A R & Peregrine D H "The propagation of solitary waves and bores over a porous bed" Coastal Engineering Vol 3, Elsevier 1980.

Peregrine D H & Svendsen I A "Spilling breakers, bores and hydraulic jumps" Proc 16th Coastal Eng. Conf. Hamburg, 1978.

Pocklington H C "Standing waves parallel to a plane beach" Proc. Cambridge Phil. Soc. Vol. 20, 1921.

Rundgren L "Water wave forces" Royal Institute of Technology, Bulletin no 54, Stockholm, 1958.

Sainflou M "Essai sur les digues maritime verticales" Annales des Ponts et Chaussées, Vol 2 No 4, 1928.

Savage R P "Wave run-up on roughened and permeable slopes" Jo. W'way & Harbours Div., Proc ASCE WW3 1958 (and Beach Erosion Board TM 109, 1959).

Saville T "Laboratory data on wave run-up and overtopping on shore structures" Beach Erosion Board, TM-64, October 1955.

Saville T "Wave run-up on shore structures, Jo. W'ways & Harbour Div. Proc. ASCE, Vol 82, WW2, 1956.

Sonu C J et al "Measurement of swash profile and orbital motion on the beach". Ocean Wave Measurement and Analysis Conference, 1974.

Svendsen I A, Madsen P A, Hansen J B "Wave characteristics in the surf zone" Proc. 16th Coastal Eng. Conf. Hamburg, 1978.

Svendsen I A, Madsen P A "Turbulent bores and hydraulic jumps" Jo. Fluid Mechanics, Vol 129, April 1983.

Wassing F "Model investigations of wave run-up on dikes carried out in the Netherlands in the last 20 years." Proc. 6th Coastal Eng. Conf. Florida, 1957.

Whalin R W "Run-up, set-up, and stability of intermediate period water waves at Monterey, California" In Dynamic waves in civil engineering, Wiley-Interscience, 1971.

Whillock A F and Thompson D M "High Island water scheme, Hong Kong - a study on the use of dolos armour units for wave protection" Hydraulics Research Station, Report Ex 532, October 1970.

Tables

TABLE 1

Values of Ir for a range of structure slopes and sea steepnesses

	Ir	cot α				
		1.333	1.5	2.0	2.5	3.0
	0.01	7.5	6.7	5.0	4.0	3.3
	0.02	5.3	4.7	3.5	2.8	2.4
	0.03	4.3	3.8	2.9	2.3	1.9
$s = H/L_o$	0.04	3.8	3.3	2.5	2.0	1.7
	0.05	3.4	3.0	2.2	1.8	1.5
	0.06	3.1	2.7	2.0	1.6	1.4
	0.07	2.8	2.5	1.9	1.5	1.3

TABLE 2

Relative run-up using Miche's & Hunt's expressions

	R/H	cot α				
		1.333	1.5	2.0	2.5	3.0
	0.01	1.6	1.6	1.8	2.0	2.2
	0.02	1.6	1.6	1.8	2.0	2.2
	0.03	1.6	1.6	1.8	2.0	1.9
$s = H/L_o$	0.04	1.6	1.6	1.8	2.0	1.7
	0.05	1.6	1.6	2.2	2.0	1.5
	0.06	1.6	1.6	2.0	1.8	1.4
	0.07	1.6	1.6	1.9	1.6	1.3

TABLE 3

Relative run-up using Losada's expressions

	R/H	cot α				
		1.333	1.50	2.0	2.5	3.0
	0.01	2.0	2.0	2.0	2.0	**2.2**
	0.02	2.0	2.0	**2.2**	**2.4**	2.4
	0.03	2.0	**2.1**	**2.4**	2.3	1.9
$s = H/L_o$	0.04	**2.1**	**2.2**	2.5	2.0	1.7
	0.05	**2.2**	**2.3**	2.2	1.8	1.5
	0.06	**2.3**	**2.4**	2.0	1.6	1.4
	0.07	**2.4**	2.5	1.9	1.5	1.3

TABLE 4

Relative run-up using Chue's expression

	R/H	cot α				
		1.333	1.50	2.0	2.5	3.0
	0.01	2.3	2.4	2.4	2.4	2.4
	0.02	2.1	2.2	2.2	2.1	2.0
	0.03	2.0	2.0	2.0	1.9	1.8
$s = H/L_o$	0.04	1.9	1.9	1.8	1.7	1.6
	0.05	1.8	1.8	1.7	1.6	1.5
	0.06	1.7	1.7	1.6	1.5	1.3
	0.07	1.6	1.6	1.5	1.3	1.2

TABLE 5

Relative run-up, smooth slope, from SPM Fig 7.12.

	R/H_o	cot α				
		1.333	1.50	2.0	2.5	3.0
$s = H/L_o$	0.01	2.0	2.1	2.3	2.4	2.5
	0.02	2.0	2.0	2.1	2.2	2.1
	0.03	2.0	2.0	2.0	2.0	1.9
	0.04	2.0	2.0	2.0	1.8	1.7
	0.05	2.0	2.0	1.9	1.7	1.5
	0.06	2.0	2.0	1.8	1.6	1.4
	0.07	2.0	2.0	1.7	1.4	1.2

TABLE 6

Relative run-up $d_s/H_o \sim 0.45$ (SPM Fig 7.9)

	R/H_o	cot α				
		1.333	1.50	2.0	2.5	3.0
$s = H/L_o$	0.01	3.6	3.5	3.2	2.8	2.6
	0.02	2.6	2.5	2.2	1.9	1.8
	0.03	2.0	1.9	1.7	1.5	1.4
	0.04	1.6	1.6	1.4	1.3	1.2
	0.05	1.4	1.4	1.2	1.1	1.1
	0.06	1.2	1.2	1.1	1.0	0.95
	0.07	1.0	1.0	0.95	0.9	0.8

TABLE 7

Relative run-up on riprap armour slopes, Losada, equation, data Ahrens & McCartney (1975)

	R/H	cot α				
		1.333	1.5	2.0	2.5	3.0
	0.01	1.73	1.70	1.60	1.50	1.40
	0.02	1.63	1.58	1.43	1.29	1.19
	0.03	1.54	1.47	1.31	1.16	1.04
$s = H/L_o$	0.04	1.47	1.39	1.22	1.07	0.96
	0.05	1.41	1.33	1.13	1.00	0.88
	0.06	1.35	1.27	1.07	0.93	0.84
	0.07	1.29	1.22	1.04	0.88	0.80

TABLE 8

Run-up on quarrystone, r = 0.65

	R/H	cot α				
		1.333	1.5	2.0	2.5	3.0
	0.01	1.3	1.4	1.5	1.6	1.6
	0.02	1.3	1.3	1.4	1.4	1.4
	0.03	1.3	1.3	1.3	1.3	1.2
$s = H/L_o$	0.04	1.3	1.3	1.3	1.2	1.1
	0.05	1.3	1.3	1.2	1.1	0.98
	0.06	1.3	1.3	1.2	1.0	0.91
	0.07	1.3	1.3	1.1	0.91	0.78

Appendix

APPENDIX
Measurement Techniques

1 Laboratory measurement

Several methods of measuring run-up have been described in the literature. The simplest of these is direct observation of the waves on the structure. Other methods may be divided into those that use a visual assessment, and those that use instrumentation. The first of these may be affected by recording the image from a television camera placed perpendicular to the slope. The recording may then be played back counting the number of run-up crests above certain, arbitrary levels. This method was used by Allsop(5), and, whilst slow, directly yields a description of the exceedance levels of the run-up. Direct visual observation was also used by Kirkgoz (1979), but only with regular waves. Miller (1968) and Svendsen (1978) measured the change in bore shape and celerity on shallow slopes using film cameras.

Instrumented methods may use either continuous or stepped gauges. The type of gauge used is dependent upon whether a continuous description of wave run-up and draw-down is required, as well as of the run-up crests. The gauge type will also depend on the nature of the structure surface. Location of a continuous gauge on an armoured slope, or mobile beach, is particularly difficult. Bullock (1968) and Hawkes(9) experimented with different types of gauge. They both rejected continuous resistance or capacitance gauges, possibly due to the shallow slopes that they were studying. Bullock, reported by Webber and Bullock(27,28), used a stepped run-up gauge consisting of 40 pairs of probes suspended just above the slope. The presence of water at each pair of probes closed an electrical circuit, giving a stepped output. The large number of probes used provided a near-continuous record. This method was found to be reliable, regardless of beach slope.

Ahrens used a similar approach, but his gauge consisted of pairs of studs mounted on the (plastic) structure slope. Stepped gauges were also used by Kamphuis and Mohamed(30), van Oorschot and d'Angremond(25) and Hawkes(9).

2 Field measurement

Whilst many authors have studied wave behaviour on beaches and structures in the laboratory, few have made any such measurements in the field.

Battjes (1971) presents run-up measurements collected on a dike of the Ijesselake in the Netherlands. Waves were not measured, but the distribution of run-up crests on a slope of 1:3.6 is presented. The Rayleigh probability distribution describes the data well. Sonu et al (1974) measured waves and swash using four vertical capacitance gauges on a beach slope of 1:45. Erchinger (1976) measured run-up on a dike of 1:6 slope during storm conditions. Grüne[26] also, measured run-up on German dikes, and used a stepped run-up gauge. Hawkes[9] used 50 separate studs on the surface of the Wallasey embankment to measure run-up. Waves were also recorded. Analysis showed a marked lack of correlation between waves and run-up on the (shallow) slope studied.